On the Timing and Optimality of Capital Controls

Public Expenditures, Debt Dynamics and Welfare

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On the timing and optimality of capital controls: Public expenditures, debt dynamics and welfare[*]

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Abstract

This paper solves a second-best problem where a government has in particular to choose whether to tax financial inflows (capital controls) or not, and **when**. A multi-stage optimal control technique is used to this end. First, it is shown that it is optimal to switch in finite time from capital controls to full financial liberalization (zero tax on capital inflows) whenever a measure of total wealth is above a certain threshold. In particular, a too large initial debt makes financial liberalization sub-optimal. Second, our analysis suggests that capital controls should be used countercyclically: booms should be responded by more financial liberalization while recessions should rather lead to more stringent capital controls. Third, when public expenditure is chosen in order to maximize social welfare, financial liberalization is not unaffordable only for poor countries, even wealthy countries might find it optimal to implement capital controls if they aim to keep a large amount of public expenditure. In short, the preservation of the welfare states might require a more frequent use of capital controls.

Key words: Capital controls, second-best, debt, public expenditures, multi-stage optimal control

JEL classification: F 34, F 43, C 61

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1   Introduction

The recent turmoil in the eurozone, originating in the massive public debts of the country members, and the associated inexorable contagion phenomenon have led many researchers and practitioners to come back to the basics. In particular, the "trilemma" principle inherent to the traditional Mundell-Fleming model (Mundell, 1963), *i.e* fixed exchange rate, perfect capital mobility and independent monetary policy cannot all coexist, has recovered all its relevance in the more recent related literature (see the excellent survey of Gallagher, 2012). As a corollary, capital controls are re-emerging as a potential valuable tool in the face of financial and economic instability triggered by financial globalization. Several countries in the world have already taken this step: for example, Brazil has introduced a tax on international capital inflows from october 2009; Taiwan, South Corea and Thailand, among others, have also managed to limit these inflows.

Beside the recent events, there is now a widely shared view that full financial integration is not necessarily beneficial for all countries in all circumstances. In particular, it has been shown that the impact of financial globalization on economic growth depends on the countries and time spans under scrutiny (see among others, Kose et al., 2009). More recently, Aizenman et al. (2011) have performed a thorough empirical analysis of the impact of the recent financial crisis on the economic growth on 100 countries from 1990 to 2010, using a disaggregated data on capital flows (inflows vs outflows, FDIs, equity investment, short-term debts...etc). Among several highly interesting findings, they have identified a negative growth effect of short-term debt and a definitely worse performance of countries with weak institutions and larger short-term debt in the crisis period. The latter finding is consistent with the conclusion of Kose et al. (2009) according to whom financial liberalization promotes growth only in countries which are already above a certain institutional threshold. On the theoretical ground, while there are obvious arguments in favor of financial integration as an engine of growth (like the well-known risk-sharing argument, see Acemoglu and Zilibotti, 1997), recent papers more concerned with welfare considerations have identified cases against capital mobility by stressing a possible negative effect on the level of consumption. In particular, Boucekkine et al. (2012) (see also Boucekkine and Pintus, 2012) have shown that collateral-*constrained* borrowing is welfare-increasing provided the growth rate of the economy under autarky (that's prior to liberalization) is large enough. Again the latter condition features a kind of threshold below which financial liberation worsens welfare, as in the empirical literature on growth and globalization quoted above.

While the latter threshold arguments do not constitute *per se* a case for capital controls, they do open the door for alternative managements of capital flows. More explicit arguments in favor of capital control can be found in the related and so called *new welfare economics* literature surveyed by Gallagher (2012). Along this line of research, key contributors are Korinek and Jeanne (see Korinek, 2011, and Jeanne and Korinek, 2010). The main idea is that free capital flows generate negative (pecuniary) externalities that should be corrected by a Pigouvian tax mechanism; capital controls play exactly this role (as they could be modeled, for instance, as taxes levied on capital inflows). In Korinek (2011), the externality acts through exchange rate fluctuations, which are only internalized at the central planner level. On the contrary, in Schmitt-Grohé and Uribe (2012), the externality comes from pegging the nominal exchange rate in a small open economy subject to downward nominal wage rigidity, which is supposed to describe the actual picture in countries like Greece. This peg-induced externality is shown to cause unemployment, over-borrowing, and depressed levels of consumption. In such a situation, optimal capital control policy should be *procyclical* according to these authors: it should restrict capital inflows in good times and subsidize external borrowing in bad times.

In this paper, we take an intermediate step and consider a second-best optimum problem in which a central planner *takes the decisions of the representative agent as given and* has to decide whether and when to use capital controls modeled as a tax on capital inflows. Borrowing in international markets at the international interest rate (small open economy assumption) helps financing public expenditures which are assumed to increase the welfare of the economy. But depending on initial conditions (like the initial debt stock) and borrowing conditions, resorting to financial markets without further regulation may lead to non-desirable paths for public debt and ultimately harm consumption and welfare. This is likely to make a case for optimal capital controls in a second-best context. We study the problem using a two-stage optimal control technique first proposed by Tomiyama (1985) and already applied to a variety of optimal switching problems (for optimal technology adoption problems, see Boucekkine et al., 2004 and 2011, and Saglam, 2011). An application to the optimal implementation of capital controls is suggested in Makris (2001). We elaborate on the latter contribution to build a broader second-best optimization problem allowing to discuss the optimality and timing of capital controls in the light of the most recent related developments in the theory of capital controls mentioned above.

In particular, three important features will be carefully highlighted. First of all, we shall show that our two-stage optimal control technique yields quite naturally explicit thresholds values (related to the initial values of

debt stocks, international interest rates, local technologies...etc) determining whether it is optimal or not to run capital controls, and when a country *should* start liberalizing capital flows. In this sense, our theoretical findings are perfectly consistent with the above mentioned "thresholds" literature (Kose et al., 2009, Aizenman et al., 2011, and Boucekkine et al., 2012). Second, our set-up allows to analyze to which extent the cycle (as measured by an exogenous flow of income) affects the decision to keep on using capital controls or to allow for full capital liberalization. Third, by incorporating endogenous welfare-enhancing public expenditure, we are able to evaluate to which extent the presence of this ingredient shapes the optimal capital control policy.

The paper is organized as follows. The next section describes the second-best problem, section 3 solves the problem with exogenous public expenditures, section 4 examines an extension where the latter *increases* welfare. Finally section 5 concludes.

2　The problem

The second-best problem extends the one suggested by Makris (2001). It models a small open economy trading foreign assets in an international financial markets at a constant interest rate $\theta > 0$ (only foreign assets are internationally traded). Initially, the government levies a tax $\tau(t) > 0$ on the returns to foreign assets acquired by domestic agents (capital inflows). The resulting income is divided between public expenditures $g(t)$ and the payment of the interests on public debt, $h(t)$. The government has to deal with the budget constraint:

$$\dot{h} = \theta h + g - \tau a \tag{1}$$

with $h_0 \geq 0$, the initial debt stock, given. $a(t)$ is private wealth.[1] Last but not least, we abstract away from any potential uncertainty and information asymmetry in what follows. This is clearly a benchmark but as we will show along the way, this is enough to make the three points announced in the introduction.

The representative agent derives utility both from public and private good consumption. She takes the policy (that is, the variable g and τ) as given

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[1]Precisely, private wealth $a(t)$ consists of public debt and private holdings of foreign assets. But since only the latter are internationally tradable, equation (1) holds at equilibrium.

and solves the problem of maximizing

$$\int_0^\infty e^{-\rho t} \left(\frac{c^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} + \alpha \frac{g^\beta}{\beta} \right) dt \tag{2}$$

with $\rho > 0$, the discount rate, $\beta < 1$, $\sigma < 1$, subject to the following budget constraint:

$$\dot{a} = (\theta - \tau)a + y - c \tag{3}$$

with $a_0 \geq 0$ given and $y > 0$ an exogenous flow of income. As outlined in the introduction, the exogenous variable y can be taken as a close indicator of the economic cycle and as the state of production technology as well. An additional technical comment is in order here. One has to observe that the problem of the representative agent is not well-posed whatever the position of the international interest rate with respect to the discount rate. Indeed, from the representative household's program, one gets the standard Euler equation:

$$\dot{c} = \sigma(\theta - \rho - \tau)c,$$

meaning that consumption growth at rate $\sigma(\theta - \rho - \tau)$ for any t. For the objective function of the agent's problem to be asymptotically bounded, further parametric conditions have to imposed. If we assume as usual that the net of tax rate of return must be non-negative, that is $\tau(t) \leq \theta$, then full liberalization ($\tau(t) = 0$) may not be asymptotically sustainable when $\rho > \theta$ under $\sigma < 1$. We shall therefore impose $\theta \geq \rho$. [2]

Let us describe now our second-best problem. In our second best economy, the government takes the decisions of the representative agent (captured by the Euler equation above) as given and chooses the sequence of tax rates and public expenditures so as to maximize the welfare of the agent. Regarding the choice of the tax rates sequence, we specifically formulate an optimal timing problem consistently with multi-stage optimal control (Tomiyama, 1985). The government has the choice between full liberalization ($\tau = 0$) and capital controls ($0 < \tau \leq \theta$). We virtually assume that the government starts with the latter and has to decide whether she switches to the second regime with $\tau(t) = 0$ for all $t \geq t_1$. t_1 is therefore a decision variable. Needless to say, the corner solution $t_1 = 0$ may be optimal, in such case the government has to choose to liberalize capital flows from $t = 0$. So, the government's

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[2]Makris (2001) takes $\theta = \rho$ and $\alpha = 0$. The former is not needed in this case.

program consists in maximizing (2) with respect to $\{g(t), \tau(t), t_1\}$ subject to

$$
\begin{cases}
\dot{a} = (\theta - \tau)a + y - c \\
\dot{h} = \theta h + g - \tau a \\
\dot{c} = \sigma(\theta - \rho - \tau)c \\
\tau \leq \theta, a_0, h_0 \text{ given}
\end{cases}
\tag{4}
$$

In our problem, $0 < \tau \leq \theta$ in the time interval $[0, t_1)$ and $\tau = 0$ after t_1. Hereafter, we shall refer to the economic regime chosen before t_1 (Resp. after t_1) as regime 1 (Resp. regime 2). Accordingly regime 1 is the capital controls regime while regime 2 refers to full liberalization. The induced optimization problem involves a standard optimal switching time choice, and it can be therefore treated with the multi-stage optimal control proposed by Tomiyama (1985) as explained just below. We start with the case where public expenditures are exogenous.

3 Optimal policy under exogenous public expenditures

Consider the government's problem *with $\alpha = 0$ and $g \geq 0$*, an exogenous (constant) flow of public expenditures. Define $\boldsymbol{x}(t)$ as the vector of state variables, that are assumed to be continuous functions with first derivatives piecewise continuous on $\mathcal{R}$: $\boldsymbol{x}(t) = (a(t), h(t), c(t))$ with corresponding law of motions $f_x^i(\boldsymbol{x}(t), \tau(t))$ in regime $i = 1, 2$. Let $\mathcal{D}$ be the control region in regime 1: $\mathcal{D} = (0, \theta]$. $\tau(t)$ is assumed to be piecewise continuous on $\mathcal{D}$. The necessary conditions for $(\boldsymbol{x}^*(t), \tau^*(t), t_1^*)$ to be an optimal solution are summarized in the following proposition.

Proposition 3.1 *There exist real numbers λ_0^i and a vector of continuous functions $\boldsymbol{\lambda_x^i}(t)$, $i = 1, 2$, $x = a, h, c$, where $\boldsymbol{\lambda_x^1}(t)$ (resp. $\boldsymbol{\lambda_x^2}(t)$) is piecewise differentiable for all $t \in [0, t_1]$ (resp. for all $t \in [t_1, \infty)$), such that:*
Define the current-valued Hamiltonian of regime i, $i = 1, 2$, as

$$
H^i(\boldsymbol{x}(t), \tau(t), \boldsymbol{\lambda_x^i}(t)) = \lambda_0^i U(c(t)) + [\boldsymbol{\lambda_x^i}(t)]^T f_x^i(\boldsymbol{x}(t), \tau(t)),
$$

and the current-valued Lagrangian as

$$
L^i(\boldsymbol{x}(t), \tau(t), \boldsymbol{\lambda_x^i}(t), \pi(t)) = H^i(\boldsymbol{x}(t), \tau(t)\boldsymbol{\lambda_x^i}(t)) + \pi(t)(\theta - \tau(t))
$$

with $\pi(t) \geq 0$ for all t, the Kuhn-Tucker multiplier. Then,
1/ $\lambda_0^i = 0$ or 1 and $(\lambda_0^i, \boldsymbol{\lambda_x^i}(t))$ is never $(0, \boldsymbol{0})$,

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2/ $\lambda_c^i(0) = 0$,
3/ For all τ in $\mathcal{D}$:

$$\frac{\partial H^1(\boldsymbol{x}(t), \tau(t), \boldsymbol{\lambda_x^i}(t))}{\partial \tau} \lessgtr 0, \ \left(\frac{\partial L^1(\boldsymbol{x}(t), \tau(t), \boldsymbol{\lambda_x^i}(t))}{\partial \tau} = 0\right), \ \pi(t) \geqq 0 \ and \ \pi(t)(\theta - \tau) = 0,$$

4/ Let $H^{i*}(t)$ be the maximized Hamiltonian: $H^{i*}(t) = H^i(\boldsymbol{x}^*(t), \tau^*(t), \boldsymbol{\lambda_x^i}(t))$, the functions $\lambda_x^i(t)$ satisfy $\dot{\lambda}_x^i(t) = \rho\lambda_x^i(t) - \frac{\partial H^{i*}(t)}{\partial x(t)}$ for $x = a, h, c$.
5/ We have $\dot{x}^*(t) = \frac{\partial H^{i*}(t)}{\partial \lambda_x^i(t)}$ for $x = a, h, c$.
6/ Suppose $t_1^* < \infty$. If $t_1 > 0$ then it must hold that

$$\lambda_x^1(t_1^*) = \lambda_x^2(t_1^*) \ for \ x = a, h, c \ and \ H^{1*}(t_1^*) = H^{2*}(t_1^*). \tag{5}$$

Otherwise, $t_1 = 0$ and $\lim_{t_1^* \to 0} H^{1*}(t_1^*) \leq \lim_{t_1^* \to 0} H^{2*}(t_1^*)$. The transversality conditions are given by $\lim_{t \to \infty} e^{-\rho t}\lambda_x^2(t)x(t) = 0$ for $x = h, c, a$.
7/ Assume now that $t_1^* = \infty$. Then, $\lim_{t_1^* \to \infty} H^{1*}(t_1^*) \geq \lim_{t_1^* \to \infty} H^{2*}(t_1^*)$. The transversality conditions are $\lim_{\to \infty} e^{-\rho t}\lambda_x^1(t)x(t) = 0$ for $x = h, c, a$.

Except Conditions 2/, 6/ and 7/, the first-order conditions displayed above are completely standard. Condition 2/ is quite unusual: it is indeed the transversality condition induced by the fact that the initial value of a state variable, namely consumption, is free. Conditions 6/ and 7/ are the necessary conditions regarding the optimal switching time problem. They have been essentially developed by Tomiyama (1985). In particular, (5) are the necessary conditions for an interior optimal time switching to occur.[3] At the optimal $t_1 > 0$, the co-state variables are all continuous. Moreover, the Hamiltonian should be also continuous at t_1. The latter is indeed the optimality condition for an optimal interior t_1 to occur. The rest of conditions stated in 6/ and 7/ are immediate necessary (non sufficient) conditions for the corner solutions $t_1 = 0$ (immediate full liberation) and $t_1 = \infty$ (no full liberalization at all) to hold. The corresponding economic interpretations are also immediate. For example, Condition 7/ delivers that if the optimal regime is capital controls for ever, then total welfare under this regime is always bigger than under full liberalization for large *enough* time horizons.

We shall now use the explicit specifications of our problem to develop the conditions above. For our particular problem, the necessary optimality conditions include (when there is no risk of confusion, regime and time indexes

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[3]See detailed proofs in Tomiyama (1985).

are dropped for convenience):

$$
\begin{aligned}
\pi &= -((\lambda_h + \lambda_a)a + \lambda_c \sigma c) \\
\dot{\lambda}_a &= (\rho - \theta + \tau)\lambda_a + \tau \lambda_h \\
\dot{\lambda}_c &= (\rho - \sigma(\theta - \rho - \tau))\lambda_c + \lambda_a - c^{-\frac{1}{\sigma}} \\
\dot{\lambda}_h &= (\rho - \theta)\lambda_h
\end{aligned}
\tag{6}
$$

Let us write the necessary condition with respect to the switching time t_1 corresponding to (5), given that $\tau(t) = 0$ for $t \in (t_1, \infty)$ and that both the co-state and state variables are continuous at t_1 (their levels being denoted by hats):

$$
H^1(t_1) = \frac{\hat{c}^{1-\frac{1}{\sigma}}}{1 - \frac{1}{\sigma}} + \hat{\lambda}_a \left[(\theta - \tau(t_1))\hat{a} + y - \hat{c} \right] + \hat{\lambda}_c \sigma [\theta - \rho - \tau(t_1)]\hat{c} + \hat{\lambda}_h \left[\theta \hat{h} + g - \tau(t_1)\hat{a} \right]
$$

$$
H^2(t_1) = \frac{\hat{c}^{1-\frac{1}{\sigma}}}{1 - \frac{1}{\sigma}} + \hat{\lambda}_a \left[\theta \hat{a} + y - \hat{c} \right] + \hat{\lambda}_c \sigma [-\rho]\hat{c} + \hat{\lambda}_h \left[\theta \hat{h} + g \right]
$$

Thus, $H^1(t_1) = H^2(t_1)$ is equivalent to:

$$
\tau(t_1)(\sigma \hat{\lambda}_c \hat{c} + \hat{\lambda}_a \hat{a} + \hat{\lambda}_h \hat{a}) = 0
\tag{7}
$$

If $\tau(t_1) > 0$, then it imposes $\pi(t_1) = -(\sigma \hat{\lambda}_c \hat{c} + \hat{\lambda}_a \hat{a} + \hat{\lambda}_h \hat{a}) = 0$.

From now on, consider the case $\tau(t_1) > 0$ implying $\pi(t_1) = 0$: the government stops taxing capital when the constraint on the tax rate becomes unbinding. We argue that $\pi(t_1) > 0$ for all $t < t_1$ that is, the constraint is binding till t_1: $\tau(t) = \theta$ for all $t \in (0, t_1)$.

If $0 < t_1 < \infty$ exists, then the economy will end up in regime 2 with $\tau = 0$. The system (6) can easily be solved for each regime taken separately that is, for the first regime valid for all $t \in [0, t_1]$, where $\tau = \theta$, and for the second one, valid for $t \in (t_1, \infty)$ with $\tau = 0$. Then, we can use the set of boundary conditions $(a_0, h_0, \lambda_c(0) = 0$ because $c(0)$ is free and the continuity of c, h, a and their co-states at $t_1)$ to obtain the equation that defines the optimal switching time:

$$
\frac{\sigma \rho}{\sigma \rho + \theta}[a_0 + y(\theta^{-1} + t_1)] = [a_0 + \theta^{-1}(y - g) - h_0] \left[\frac{1 - \frac{(1-\sigma)\theta e^{-\sigma \rho t_1}}{\sigma \rho + \theta(1-\sigma)}}{1 + \frac{\sigma \theta e^{-(\theta + \sigma \rho)t_1}}{\sigma \rho + \theta(1-\sigma)}} \right]
\tag{8}
$$

This leads to the following result:

Proposition 3.2   *i/ A necessary condition for the existence of $0 < t_1 < \infty$ is*

$$\frac{\theta}{\theta + \sigma\rho}\left(a_0 + \frac{y}{\theta}\right) > h_0 + \frac{g}{\theta}. \tag{9}$$

ii/ In the special case where $y = 0$, this condition, that reduces to $\frac{\theta}{\theta + \sigma\rho}a_0 > h_0 + \frac{g}{\theta}$, is also sufficient.

The proof is in the Appendix.

Some conclusions can immediately be drawn from proposition 3.2. The likelihood of a switch to the regime of full liberalization of capital movements stems from the comparison between on the one hand, the sum of the initial debt and the discounted flows of future expenditure and on the other hand, the sum of initial wealth and the discounted flows of future income where discounting is made using the interest rate. Resources should be sufficiently larger than expenses for the government to start liberalizing capital flows at some point in time. Indeed, the second regime of full liberalization becomes worthwhile only when the public debt has turned into an asset. This finding is consistent with Kose et al. (2009)'s conclusion that stopping capital control is desirable only if some measure of total wealth, defined as the sum between initial wealth and discounted flows of income, is above some threshold. Incidentally, our compact switching condition entails some other plausible implications. For example, a large initial debt h_0 makes full liberalization sub-optimal, which is somehow consistent with Aizenman et al. (2011) findings on the bad performance of countries with high short term debt ratio during the crisis period. On the other hand, when a country can count on a large enough initial stock of domestic private assets (that's a_0), switching to full liberalization can be (second-best) optimal. Also, our switching condition can be used to discuss the optimal policy depending on the cycle. Indeed, one can rewrite condition (8) for any given exogenous path of income, $y(t)$, $t \geq 0$. Although our analysis is limited in such a case by the absence of uncertainty, some preliminary lessons can be extracted. In particular, our second-best analysis suggests that booms should stimulate financial liberalization while recessions should rather lead to push for capital controls. This contrasts with the prudential use of capital controls recommended by Schmitt-Grohé and Uribe (2012) in their model with exchange rate pegging and downward wage nominal rigidity. Our setting is of course minimal but we believe it is interesting to notice than in such a benchmark, second-best capital control policies should rather be countercyclical.

Last but not least, our optimal switching condition depends on both the interest rate and the preference parameters, and therefore one can examine to which extent the second-best policy is sensitive to these parameters. As to

the interest rate, note that it is equal to the tax rate before the switch. An increase in θ implies both a more costly debt service and higher tax revenues. Hence, its effect on the opportunity to switch is ambiguous. Regarding the impact of the other parameters, it turns out that the higher the intertemporal elasticity of substitution and the discount rate, the less likely is the liberalization of capital flows. Under the regime of capital control, optimal consumption is decreasing because there is no point in accumulating assets whose returns are fully taxed. The higher the intertemporal elasticity and the larger the discount factor, the steeper this decrease and the higher initial consumption. Tax revenues are then lower and it is less likely that a regime switch occurs.

Finally, we examine the possibility of corner solutions to be optimal. Typically this occurs when Condition (9) no longer holds. This question is treated in the next proposition.

Proposition 3.3 *i/ The corner solution $t_1 = 0$ (full liberalization from $t = 0$) is always sub-optimal.*

ii/ If Condition (9) does not hold, the corner solution $t_1 = \infty$ is optimal. Along this solution, consumption converges to zero.

The proof is in the Appendix. The first property of the model is rather good news. Because full liberalization only increases debt given the way this regime is modeled in this model ($\tau = 0$), applying it on a permanent basis leads to sub-optimal explosive dynamics *ceteris paribus*. This is largely consistent with the empirical literature (see Aizenman et al., 2011, for example) which points out the negative impact of liberalization precisely through this bias. In contrast, the empirical literature emphasizes the positive impact of liberalization, notably in developing countries, when it operates through a substantial increase in FDIs. We don't have such a channel in our simplistic model. Concerning the second corner solution, that's permanent capital controls, it turns out to be optimal when the switching condition does not hold, which happens when intertemporal expenditures exceed intertemporal income of the economy (symmetrically to our interpretation of (9)). Because optimal capital controls lead to choose $\tau = \theta$, the optimal consumption rule, considered as a constraint in our second-best problem, implies that consumption goes to zero asymptotically (while the objective function of the optimization problem is still bounded despite $\sigma < 1$, see the Appendix). Again this extreme property derives from the simplicity of our benchmark and the second-best setting adopted. But the whole exercise examplifies the effects of permanent full liberalization Vs capital controls in the case where liberalization only operates through financial inflows, resulting in an increasing debt, and capital controls show up through linear taxation schemes.

4 Optimal policy under endogenous public expenditures

Let us now consider g as a control variable of the government. In order to simplify the computations, we set $y = 0$. As one can see from Proposition (??) ii/ under exogenous public expenditures, the switching conditions are qualitatively the same in this case as when $y > 0$. So we can analyze the impact of endogenizing public expenditures under $y = 0$ without loss of generality. The optimization program reads

$$\max_{g,\tau,t_1} \int_0^\infty e^{-\rho t} \left(\frac{c^{1-\frac{1}{\sigma}}}{1 - \frac{1}{\sigma}} + \alpha \frac{g^\beta}{\beta} \right) dt$$

subject to (4). The necessary conditions are similar to (6). In addition, the optimization with respect to g yields:

$$g(t) = g(\lambda_h(t)) \text{ with } g(\lambda_h(t)) = \left(-\frac{\lambda_h(t)}{\alpha} \right)^{\frac{1}{\beta-1}}$$

The necessary optimality condition (7) related to the optimal switching time t_1 is unchanged and, under the same argument as before, we can claim that during the regime of *capital controls* the government sets the tax rate at the maximum possible level that is, $\tau = \theta$ for all $t \leq t_1$. It is pretty easy to solve the dynamic system for the two possible regimes. Given that $\lambda_h(t) = \bar{\lambda}_h e^{(\rho-\theta)t}$, the general solution under the first regime of capital control is:

$$\lambda_a(t) = \lambda_{a1} e^{\rho t} - \lambda_h e^{(\rho-\theta)t}$$
$$\lambda_c(t) = \lambda_{c1} e^{\rho(1+\sigma)t} + \frac{\bar{\lambda}_h}{\theta + \rho\sigma} e^{(\rho-\theta)t} + \frac{(c_1^{-\frac{1}{\sigma}} - \lambda_{a1})}{\sigma\rho} e^{\rho t}$$
$$c(t) = c_1 e^{-\sigma\rho t}$$
$$a(t) = a_1 + yt + \frac{c_1}{\sigma\rho} e^{-\sigma\rho t}$$
$$h(t) = h_1 e^{\theta t} + a_1 + y(\theta^{-1} + t) + \frac{(\beta-1)g(\bar{\lambda}_h)e^{\frac{\rho-\theta}{\beta-1}t}}{\rho - \beta\theta} + \frac{\theta c_1}{\sigma\rho(\sigma\rho+\theta)} e^{-\sigma\rho t}$$

Under full liberalization ($\tau = 0$), solutions are given by

$$\lambda_a(t) = \lambda_{a2} e^{(\rho-\theta)t}$$
$$\lambda_c(t) = \lambda_{c2} e^{(\rho(1+\sigma)-\theta\sigma)t} - \frac{\lambda_{a2} - c_2^{-\frac{1}{\sigma}}}{\sigma\rho + \theta(1-\sigma)} e^{(\rho-\theta)t}$$
$$c(t) = c_2 e^{\sigma(\theta-\rho)t}$$
$$a(t) = a_2 e^{\theta t} - \frac{y}{\theta} + \frac{c_2}{\sigma\rho + \theta(1-\sigma)} e^{\sigma(\theta-\rho)t}$$
$$h(t) = h_2 e^{\theta t} + \frac{(\beta-1)g(\bar{\lambda}_h)e^{\frac{\rho-\theta}{\beta-1}t}}{\rho - \beta\theta}$$

When a change in policy occurs in finite time ($0 < t_1 < \infty$), the second regime is the terminal one. Suppose $\theta > \rho$. Then the transversality conditions impose $\lambda_{c2} = a_2 = h_2 = 0$. In addition, it must be true that $\rho > \beta$. In this situation, both consumption and private assets tend to $+\infty$ as time goes to infinity. Since $\beta < 1$, we also have the public debt going to minus infinity. This is an irrelevant solution that could have been prevented from the initial statement of the problem by requiring the non-negativity of the public debt asymptotically (like a No-Ponzi game condition). To do things in a much simpler way, we set $\rho = \theta$ herafter: the interest rate is equal to the discount rate. As a consequence, it appears that both the co-state of the public debt λ_h and public expenditures are constant over time. Direct calculations reveal that if a $0 < t_1 < \infty$ exists then it solves

$$\frac{\sigma a_0}{1+\sigma} \frac{1 + \sigma e^{-\theta(\sigma+1))t_1}}{1 - (1-\sigma)e^{-\sigma\theta t_1}} = a_0 - \frac{g(\lambda_h(t_1))}{\theta} - h_0. \tag{10}$$

This equation is similar to (8) with $\rho = \theta$, $y = 0$ and $g = g(\lambda_h(t_1))$ defined above. With exogenous public spending, one was able to solve the system of state variables separately from the system of co-states and to obtain the simple equation (8) defining the existence of the optimal t_1. This is no longer the case because the amount of public good provided by the government now depends on the co-state of the public debt. So, it is necessary to solve the system of co-states in order to derive the expression of $\lambda_h(t_1)$. After straightforward computations, one gets:

$$\lambda_h(t_1) = \frac{1+\sigma}{\sigma} \frac{(\sigma\theta a_0)^{-\frac{1}{\sigma}}(1 - (1-\sigma)e^{-\sigma\theta t_1})^{\frac{1+\sigma}{\sigma}}}{(1-\sigma)e^{-\theta(1+\sigma)t_1}(1 - e^{\theta(1+\sigma)t_1}) - (1+\sigma)e^{-\theta t_1}}, \tag{11}$$

with, $c_1(t_1) = \sigma\theta a_0(1 + (\sigma-1)e^{-\sigma\theta t_1})^{-1}$. Hence, we can establish the following.

Proposition 4.1 *A necessary and sufficient condition for the existence of $0 < t_1 < \infty$ is*

$$\frac{a_0}{1+\sigma} > h_0 + \frac{1}{\theta}\left[\frac{(1+\sigma)(\sigma\theta a_0)^{-\frac{1}{\sigma}}}{\alpha\sigma(1-\sigma)}\right]^{\frac{1}{\beta-1}}. \tag{12}$$

The proof is in the Appendix.

This result is generally in the line with the outcomes of our analysis of the exogenous public expenditures case. Assuming $\rho = \theta$ and $y = 0$, condition (9) reduces to $\frac{a_0}{1+\sigma} > h_0 + \frac{g}{\theta}$. Similarly, condition (12) can be rewritten as

$\frac{a_0}{1+\sigma} > h_0 + \frac{g_{\min}}{\theta}$, with $g_{\min} = \left[\frac{(1+\sigma)(\sigma\theta a_0)^{-\frac{1}{\sigma}}}{\alpha\sigma(1-\sigma)}\right]^{\frac{1}{\beta-1}}$ the lower value $g(\lambda_h)$ can

reach since it corresponds to the limit of $g(\lambda_h(t_1))$ when t_1 tends toward infinity and $\frac{\partial g}{\partial \lambda_h} \frac{\partial \lambda_h}{\partial t_1} < 0$. Therefore, the decision to liberalize capital revenues at some point in time is dictated by the gap between initial wealth and the sum of the initial debt and the discounted flows of minimal future public expenditures, where discounting is made using the interest rate. However, $g_{\min}$ is endogenous, driving other effects of the fundamental parameters of the model. In particular, it increases with the initial tax rate (*that is* the international interest rate in this case) and the sensitivity of utility with respect to public expenditures. In other words, the higher these parameters, the less likely the switch in finite time to full financial liberalization. Moreover, the minimum level of public expenditures also positively depends on initial wealth. As a result the total impact of a_0 is now unclear: a wealthier country is more likely to be able to afford capital control elimination for a given level of public expenditures. But, due to the endogenous g, higher a_0 translates into higher public expenditures, which reduces the desirability of financial liberalization. It can easily be shown that it is only for intermediate wealth levels that a switch to a regime with no taxation may occur. A very important conclusion of this exercise is then to qualify substantially one of our findings under exogenous public expenditures. In this case, the richer the country in terms of private assets (*that is* a_0), the more profitable the switch to full liberalization at a given finite optimal date. When public expenditures are chosen in order to maximize social welfare, things are more complicated: financial liberalization is not unaffordable only for poor countries, even wealthy countries might find it optimal to implement capital controls if they aim to keep a large (optimally chosen) amount of public expenditure. In other words, the preservation of the welfare states in rich countries (say the current industrialized countries) might require a more frequent use of capital controls.[4]

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[4]Note that when condition (12) is not verified, we necessarily have a corner solution. Once again, the corner solution where the government liberalize capital movements from the period $t = 0$ is not relevant. From the boundary conditions, it must hold that $h_0 = -g(\lambda_h)/\theta$, which cannot be true as long as the initial stock of debt in the economy is non negative. So, as in the case of exogenous public expenditures, the only alternative to the regime where the government switches from full capital control to full liberalization is the regime of permanent capital control that is, the regime where $\tau = \theta$ lasts forever. The dynamic and asymptotic behavior of the economy can thus be derived from the analysis of the corner of the preceding section by simply substituting ρ and y respectively with θ and 0 in the solution and noticing that the value of λ_h, and consequently $g(\lambda_h)$, exists iff $a_0 > h_0$ and is uniquely and implicitly given by the system $g(\lambda_h) = \theta(a_0 - h_0 - \frac{\hat{c}}{\theta(1+\sigma)})$ and $\hat{c}^{-\frac{1}{\sigma}} = -\frac{\sigma \lambda_h}{1+\sigma}$ with $\hat{c}$ the constant of integration of consumption.

5 Concluding remarks

We have solved a benchmark second-best problem where a government has in particular to choose whether to tax financial inflows (capital controls) or not, and **when**. We have done the job having in mind the recent developments in the empirical and theoretical literature on financial liberalization. In particular, we have taken care of three important aspects. First of all, we have analytically highlighted the existence of thresholds (on initial debt, on initial wealth...etc) above or below which capital controls are optimal. Second, we have shown that second-best capital controls should be used counter-cyclically: booms should be responded by more financial liberalization while recessions should rather lead to push for more stringent capital controls. Last but not least, we have found that even wealthy countries might find it optimal to implement capital controls if they aim to keep a large amount of public expenditure.

Clearly enough, our model is too simple in several respects, and some essential ingredients have to be added to make it more relevant given the recent trends. In particular, adding FDIs as an essential form of liberalization is a necessary step to take consistently with the data. This addition might significantly alter some of the conclusions reached in this paper.

Appendix

Proof of Proposition 3.2

Part i/ Denote the LHS of (8) by $F(t_1)$. This function has the following features: $F(0) = \frac{\sigma\rho}{\sigma\rho+\theta}(a_0 + y/\theta) > 0$, $F(\infty) = +\infty$ and $F(t_1), F'(t_1) > 0$ for all $t_1 > 0$. Let $G(t_1)$ be the RHS. This function verifies: $G(0) = \frac{\sigma\rho}{\sigma\rho+\theta}(a_0 + \frac{y}{\theta} - \frac{\sigma\rho+\theta}{\sigma\rho+(1-\sigma)\theta}(h_0 + \frac{g}{\theta}))$ and $G(\infty) = a_0 + \theta^{-1}(y - g) - h_0 < \infty$. A necessary condition for existence is $a_0 + \theta^{-1}(y - g) - h_0 > 0$. Otherwise, $G(t_1) < 0$ for all t_1. Under this condition, one has $G(t_1), G'(t_1) > 0$ for all $t_1 > 0$. A comparison between these boundary values yields: $F(0) > G(0)$ and $F(\infty) > G(\infty)$. Hence, it is clear that if $F(0) > G(\infty)$, which is equivalent to $\frac{\theta}{\theta+\sigma\rho}\left(a_0 + \frac{y}{\theta}\right) < h_0 + \frac{g}{\theta}$, there is no solution. Thus, (9) is a necessary condition for existence (condition (9) implies that $a_0 + \theta^{-1}(y - g) - h_0 > 0$).

Part ii/ Now, take $y = 0$. The LHS of (8) is now constant and equal to $\frac{\theta}{\theta+\sigma\rho}a_0$. Let $H(t_1)$ be the simplification of $G(t_1)$ with y put equal to 0. One has $H(0) = \frac{\sigma\rho}{\sigma\rho+\theta}(a_0 - \frac{\sigma\rho+\theta}{\sigma\rho+(1-\sigma)\theta}(h_0 + \frac{g}{\theta}))$ and $H(\infty) = a_0 - \theta^{-1}g - h_0$. A necessary condition for existence is $a_0 - \theta^{-1}g - h_0 > 0$. If this condition holds, then $H(t_1), H'(t_1) > 0$ for all $t_1 > 0$ and a necessary and sufficient condition for existence is $H(\infty) > \frac{\theta}{\theta+\sigma\rho}a_0$, which is equivalent to $\frac{\theta}{\theta+\sigma\rho}a_0 > h_0 + \frac{g}{\theta}$. $\square$

Proof of Proposition ?? It should be clear that a corner solution with $\tau = 0$ for all t is not possible. To see this, note that in this case $h(t) = -\frac{g}{\theta}$ for all t and the boundary condition $h_0 = -\frac{g}{\theta}$ does not hold in general. Let us now consider the case $\tau = \theta$ for all t, the general solution reads: $c(t) = \hat{c}e^{-\sigma\theta t}$, $a(t) = \hat{a} + yt + \frac{\hat{c}e^{-\sigma\theta t}}{\sigma\rho}$ and $h(t) = \hat{a} + y(t + \theta^{-1}) - g\theta^{-1} + \frac{\theta\hat{c}e^{-\sigma\theta t}}{\sigma\rho(\sigma\rho+\theta)}$ for the state variables and, for the co-states: $\lambda_a(t) = -\lambda_h(t)$, $\lambda_h(t) = \hat{\lambda}_h e^{(\rho-\theta)t}$ and $\lambda_c(t) = \frac{\hat{\lambda}_h e^{(\rho-\theta)t}}{\sigma\rho+\theta} + \frac{\hat{c}^{-\frac{1}{\sigma}}e^{\rho t}}{\sigma\rho}$ where $\hat{c} = (\theta+\sigma\rho)(a_0+\theta^{-1}(y-g)-h_0)$ which is assumed to be positive even when (9) does not hold, $\hat{a} = -\frac{\theta a_0-(\theta+\sigma)(h_0+\theta^{-1}(g-y))}{\sigma\rho}$ and $\hat{\lambda}_h = -\frac{(\theta+\sigma\rho)^{1-\frac{1}{\sigma}}(a_0+\theta^{-1}(y-g)-h_0)^{-\frac{1}{\sigma}}}{\sigma\rho}$. To sump up, in a regime of permanent capital control, consumption asymptotically goes to zero whereas both the private and public assets tend toward infinity because of the permanent flow of exogenous income $y > 0$. In the particular case where $y = 0$, consumption still converges to zero but now a and h achieve constant values respectively given by: $a(\infty) = -\frac{\theta a_0-(\theta+\sigma)(h_0+\theta^{-1}g)}{\sigma\rho} > 0$ and $h(\infty) = -\theta^{-1}g + a(\infty) \lesseqgtr 0$. □

Proof of Proposition 4.1

Denote the LHS of (10) by $J(t_1)$. $J(.) > 0$ is monotonically decreasing with $J(0) = a_0$ and $J(\infty) = \frac{\sigma a_0}{1+\sigma}$. Let $f(t_1)$ be the RHS. This function is defined in terms of $\lambda_h(t_1)$, whose expression is given by (11), with $\lambda'_h(t_1) < 0$ for all t_1. In addition $\lambda_h(0) = -(\theta a_0)^{-\frac{1}{\sigma}}$ and $\lambda_h(\infty) = -\frac{1+\sigma}{\sigma}\frac{(\sigma\theta a_0)^{-\frac{1}{\sigma}}}{1-\sigma}$. From the definition of $f(t_1)$, this in turn implies that $f'(t_1) > 0$. Since $f(0) < J(0)$, a necessary and sufficient condition for the existence of a $0 < t_1 < \infty$ is $f(\infty) > J(\infty)$, which is equivalent to (12). □

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