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Public Housing Quotas and Segregation

Benoît Schmutz, Aix-Marseille University (Aix-Marseille School of Economics), CNRS & EHESS
and Georgetown University

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Benoît Schmutz†

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Abstract

This paper adapts a framework à-la Hotelling to an urban context in order to study the impact of public housing on the level of segregation in a fixed-size city where consumers differ both in income and taste. In this city, the market allocation of the population is characterized by partial segregation: both rich and poor consumers can be found in both neighborhoods. Public authorities replace a fraction of the housing stock with public housing. This policy will not decrease segregation if applicants are not screened according to their income level. Any departure from the optimal level of screening has to be compensated for by a larger program. The final policy mix will then be determined by the extent to which public authorities have the ability, either to screen applicants, or to fund more public units. However, this trade-off will be softened when taking neighborhood externalities into account, thanks to a snowball effect of public housing on neighborhood quality.

JEL codes: R2, R3.
Keywords: public housing, segregation, sorting, Hotelling, rationing.

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†Aix-Marseille University (Aix-Marseille School of Economics), CNRS & EHESS, Georgetown University, http://sites.google.com/site/benoitschmutz

Introduction

In this paper, I use a framework à la Hotelling to show that public housing policies aiming at reducing socioeconomic stratification through rent control may only succeed if the allocation of public housing units is not social-blind, but rather favors applicants whose kind is under-represented in the neighborhood. Without screening, and if total housing supply is fixed, public housing has no impact on overall housing affordability because it also increases prices in the private units nearby. If, for external reasons, the city-planner does not set the screening rule herself, her only instrument left is the size of the program: in that case, the lower the screening, the higher the minimum program size required to reduce segregation. A simple condition on the respective sizes of the two neighborhoods and the two groups of consumers will then determine in which neighborhood the program will be implemented. Finally, the introduction of neighborhood externalities in the form of peer effects does not alter the message of the model. On the contrary, the public housing policy is better at addressing segregation in this context, because its indirect impact on neighborhood valuation mitigates the polarizing trends at work in the city.

In 2000 the Law "Solidarité et Renouvellement Urbains" (SRU) is established in France. It states that most municipalities must progressively reach 20% of public housing (Habitations à Loyers Modérés, HLM) in their total stock, under the threat of financial sanctions. The main goal of the SRU law is to address urban economic segregation. The rationale behind such regulation by quota is that public housing is an efficient tool against economic segregation, provided the following two conditions are met: first, the municipal scale is relevant to ensure that public housing units are scattered enough between and within metropolitan areas; second, the rent gradient according to neighborhood quality must be less steep on the public housing market. While this double assumption is not unrealistic, it is not sufficient to ensure that public housing will reduce segregation. In a context of very high eligibility thresholds (in France, according to Wasmer in Mistral and Plagnol (2009), four fifths of the population are eligible), the result will crucially depend on the characteristics of the matching process between applicants and vacancies. Indeed, an increase in the quality of the public housing supply is quite likely to increase entry, hence competition, at the expense of the lowest types of applicants. More generally, this kind of reasoning neglects general equilibrium effects of a change in the public housing supply on the level of rents in the private housing market as well as on neighborhood characteristics, through externalities. The purpose of this paper is to study the following dilemma: anti-segregation public housing policies may not be able to contribute to vertical equity at the same time; they suffer from being a single instrument with two conflicting objectives, namely an increase in overall housing affordability and a decrease in segregation. This is all the more embarrassing that place-

based policies are currently under siege (Sarpca, Leung, and Yilmaz (2010)), whereas the most compelling argument in their favor is maybe that, thanks to their somewhat more rigid design, they are more efficient against economic segregation, which often emerges as the market outcome when people are free to move.

The model presented here is not an urban economic model per se: for the introduction of rent control into an urban economic model, see Heffley (1998), which however does not focus on segregation issues. Here, the core of the model is a market à-la Hotelling with fixed housing supply and where agents consume one housing unit. The market is both vertically and horizontally differentiated. There are two types of agents, who differ in terms of price sensitivity. Any asymmetric equilibrium yields an allocation of the population which is characterized by incomplete sorting: members of both groups will be found in both neighborhoods. Until recently, partial income stratification was seldom a result obtained by economic modelling (Moretti (2011)). Two noteworthy exceptions were Epple and Platt (1998) and Schmidheiny (2006), who present models where households, who both differ in income and preferences, vote on the tax-expenditure package the community will provide. The former focuses on property taxes and aims at describing the US, whereas the latter, which aims at describing Switzerland, focuses on local income taxes. These two papers are very efficient at explaining incomplete income stratification and the variability in local public good provision. However, I choose here an alternative modelling strategy, for two reasons: first, the local public good approach is not fully relevant in the case of public housing, which is a very specific public good in that it directly impacts people's location and it is generally not funded at the local level; second, whereas the authors' richer framework compels them to resort to numerical simulations, the intent of this paper is to study the equilibrium properties of the model from a fully analytical viewpoint.

Public authorities introduce public housing in the city, the price of which is fixed exogenously below market prices. This leads to rationing because everybody is eligible and anyone who lives in one neighborhood unambiguously prefers to live in public housing in the same neighborhood. I consider two allocation processes, which differ according to whether people locate before or after the lottery. Both processes are relevant, depending on the geographical scale of the location choice: the first one deals with a larger scale, such as the metropolitan area, whereas the second one applies to the choice between two neighborhoods which would be close to each other. I mostly focus on the case when neighborhood quality is exogenous, city size is fixed and public housing is made of preexisting stock which is preempted by public authorities. I show that if both types of applicants are as likely to be selected (random allocation), the policy has strictly no impact on the allocation of the population because its direct impact on affordability is exactly counterbalanced by its indirect effect on private market prices. Namely, the introduction of public housing in the more (resp., less) expensive neigh-

borhood will unambiguously increase (resp., decrease) the price differential between the two neighborhoods. In order to succeed in changing the allocation of the population, one has to implement unequal treatment of the two types of applicants. To achieve a predetermined reduction in the level of segregation, there is a trade-off between the level of screening and the size of the public housing program. In particular, there is a minimum level of screening that public housing authorities have to maintain in order to minimize the total cost of the public housing program.

I discuss a first extension where I describe what happens when, due to externalities, the quality of a neighborhood also depends on its social make-up. Whereas a random allocation of public housing still cannot reduce segregation in this case, the screening of applicants now has snowball effects on neighborhood quality, which loosen the trade-off between the level of screening in the allocation process and the size of the public housing program. Finally, I relax the assumption that public housing units are as desirable as private ones and I introduce the notion of indirect screening whereby households decide whether to enroll or not in the program depending on the quality gap between the two segments. In this case, the quality of public housing becomes a policy parameter, which can be used to improve the targeting of the program. I provide an equivalence relationship between indirect and direct screening in terms of segregation and I discuss the respective relevance of these two mechanisms.

The rest of the paper is organized as follows: in section 1, I introduce the baseline model of the private market and I describe the allocation of public housing. Section 2 discusses the impact of public housing programs when neighborhood quality is taken as given and is the same across the public/private border; Section 3 describes what happens when either of these two assumptions is relaxed and Section 4 concludes.

1 The framework

1.1 Set-up

I consider a city of population normalized to 1 with two neighborhoods $j = 0, 1$ of fixed size $n_0 \equiv n$ and $n_1 \equiv 1 - n$ respectively.[1] Neighborhoods differ in quality q_j and price p_j. In the baseline case, I assume that q_j is exogenous, whereas p_j is always determined at equilibrium. Housing is supplied by an absentee construction sector facing a unit cost c. A necessary condition for the city to be built is $\sum_{j=0,1} n_j p_j \geq c$.[2]

[1] As opposed to what is generally assumed in the industrial organization literature, there are capacity constraints: supply is here fixed to a limit given by exogenous factors, such as land availability.

[2] This modelisation of the supply side of the market is made as simple as possible and will not play a large role. Without it, the model would be written in differential terms (only the price differential would be determined at equilibrium) but would lead to the same conclusions.

The city is filled by a population of size 1. Consumers differ along two dimensions: (i) their type $i = H, L$ (for high/low income) with α_i the proportion of type i in the economy ($\alpha_H \equiv \alpha$); (ii) an idiosyncratic term of heterogeneity x uniformly distributed on a segment $[0;1]$ of unit density. Therefore, the market exhibits both vertical differentiation (group-average valuation q_j) and horizontal differentiation (dispersion of individual preferences x). An interpretation of these two dimensions could be that vertical differentiation relates to an universally orderable neighborhood characteristic, such as sun exposure or air pollution, whereas horizontal differentiation relates to a binary set of non-orderable neighborhood characteristics, such as the presence of gardens in one neighborhood and movie theaters in the other. The "taste parameter" t gives the relative weight of heterogeneity in individual preferences. Everyone consumes one unit of housing and pays p_j. The two types of consumers have a different coefficient of disutility β^i with respect to p_j. Indirect utility functions are defined by $\forall \{i, j, x\} \in \{H, L\} \times \{0, 1\} \times [0;1]$,

$$U_j^i(x) = q_j - \beta^i p_j - t|x - j| \tag{1}$$

Following Tirole (1988), β^i may be interpreted as an approximation of the marginal rate of substitution between income and quality. If I assume that $0 < \beta^H < \beta^L$, this means that type-H consumers are wealthier than type-L. I note $\hat{\beta} = \sum_{i=H,L} \alpha_i \beta^i$ the average value of β^i in the economy. As for the term $-t|x - j|$, it gives the cost of having to choose between the two options given by the horizontal dimension of neighborhood differentiation, when a consumer x would in fact prefer benefiting from a combination of these two options.[3]

For simplicity, I assume that the market is always covered: no consumer is better-off refusing to participate in the housing market. In Appendix A.1, I provide a sufficient condition for the market to be covered, which states that the expected utility of the median type-L consumer with an equal probability to live in either neighborhood, must be greater than its reservation utility. This assumption is not costly when q_j is exogenous and may be chosen as high as necessary.

Let $x_i(p_0, p_1)$ denote the type-i indifferent consumer between two neighborhoods, i.e. such that $U_0^i(x_i(p_0, p_1)) = U_1^i(x_i(p_0, p_1))$. All the results will be written as a function of 0, in order to indicate that no public housing unit is funded for now. Later, results will be written as a function of s_j, the size of the public housing program in neighborhood j. The equilibrium is given by $(p_0^*(0), p_1^*(0))$, solution to the system formed by the market-clearing equation (2) and the investors' free-entry condition (3):

[3] A more classical interpretation would be to consider that x gives the location of the consumer's job. If so, $-t|x - j|$ becomes commuting costs. However, this is less relevant in this model without real labor market.

$$n = \sum_{i=H,L} \alpha_i x_i^* (0) \tag{2}$$

$$c = \sum_{j=0,1} n_j p_j^* (0) \tag{3}$$

with $x_i^* (0) = x_i (p_0^* (0), p_1^* (0))$. This yields the following equilibrium prices:

$$p_j^* (0) = c + (1 - n_j) [q_j - q_{-j} + t (1 - 2n_j)] / \hat{\beta} \tag{4}$$

Straightforward comparative statics derived from equation 4 allow to write the following proposition:

Proposition 1 *The price differential between the two neighborhoods: (i) always increases with the quality differential between the two neighborhoods, the proportion of type-H consumers in the economy, and the level of income differentiation between the two types of consumers; (ii) increases with the size of the larger neighborhood and with the weight of individual heterogeneity if and only if the better-quality neighborhood is also the smaller.*

The first three effects are straightforward, whereas the last two derive from the fact that there is substitutability between neighborhood quality and size: both features make the neighborhood more attractive, hence more expensive in relative terms. In this respect, different equilibria should be distinguished: in the "symmetric" kind, prices are the same in both neighborhoods; however, this may either be due to the fact that both neighborhoods are perfectly identical, or that their features compensate each other; conversely, the equilibrium is "asymmetric" if prices are different in the two neighborhoods, and it is "fully asymmetric" if both features go in the same direction, i.e. the better quality neighborhood is also the smaller. In the rest of the paper, I choose to focus on this latter class of cities, for two reasons: first, it is more realistic to assume that the better-quality good is also the relatively scarce resource; second, the possibility of a trade-off between neighborhood quality and size would blur the mechanisms behind the impact of public housing in a not very relevant manner.

Without loss of generality, I will assume that $\Delta q \equiv q_0 - q_1 > 0$ and $n < 1/2$. In the private market case, these assumptions are compatible with a fully asymmetric city in favor of neighborhood 0.

1.2 Segregation and welfare in the private city

Segregation – I consider a simple segregation indicator: the dissimilarity index D, which gives the mass of people who would need to swap neighborhoods for the city to be perfectly integrated. It is the sum of two equal quantities: the excess of type-H in 0

and the excess of type-L in 1: $D(0) = \alpha(x_H^*(0) - n) + (1-\alpha)(n - x_L^*(0))$. In this framework, segregation is partial: both types of consumers are observed in both neighborhoods, as opposed to total segregation, where one group of consumers is confined to one single neighborhood. In analytical terms, this means that for both $i = H, L$, $x_i^*(0) \in (0,1)$. The upper bound of D will then be given by the size of the smaller of the two groups of consumers: $D \leq \min_i \{\alpha_i\}$. After simplification, one can show that $D(0)$ is a simple linear increasing function of $\Delta p_{01}^*(0)$, given by:

$$D(0) = \alpha(1-\alpha)\left(\beta^L - \beta^H\right)\Delta p_{01}^*(0)/t \tag{5}$$

and straightforward comparative statics yield the following proposition:

Proposition 2 *The level of segregation (i) increases with the quality differential between the two neighborhoods and the level of income differentiation between the two types of consumers; (ii) decreases with the size of the better-quality neighborhood and the weight of individual heterogeneity; (iii) increases then decreases with the proportion of type-H consumers.*

The gap between consumer's type as well as the relative attractiveness and scarcity of neighborhood 0 tend to increase segregation, as opposed to horizontal differentiation, which tends to reduce it. As far as α, its effect stems from the way segregation is measured by the dissimilarity index, which requires both groups to be large: this is easily understood when one considers total segregation, with $D(0) = \min_i \{\alpha_i\}$. Segregation is maximal when $\alpha = 1 - \sqrt{\beta^H}/(\sqrt{\beta^L} + \sqrt{\beta^H})$.

Welfare – In this city without production, welfare W is confounded with consumer surplus and is equal to the average utility level in the city. We have:

$$W = \sum_{i=H,L} \alpha_i \left(\int_0^{x_i^*} U_0^i(x)\,dx + \int_{x_i^*}^1 U_1^i(x)\,dx \right) \tag{6}$$

This expression can also be written as $W = \bar{q} - \bar{p} - \bar{t}$, where $\bar{q}$ is the average neighborhood quality in the city, $\bar{p}$ is the average price component of utility and $\bar{t}$ is the average taste component of utility. Let $W(0)$ the level of welfare in the private city. For the corresponding expressions of $\bar{q}$, $\bar{p}$ and $\bar{t}$, see Appendix A.2. As one could expect, the function $W(0)$ exhibits the following properties:

Proposition 3 *Welfare increases with neighborhood quality and decreases with construction cost and the weight of individual heterogeneity.*

The impact of the other parameters on total welfare is ambiguous. However, this is no longer the case if the analysis is broken between the two groups of consumers. Let $W_i(0)$ the average utility of a type-i consumer, such that $W(0) = \sum_{i=H,L} \alpha_i W_i(0)$.

Proposition 4 *(i) The group of type-H consumers benefits from type-L consumers being poorer, whereas the group of type-L consumers benefits from type-H consumers being richer; (ii) Each group of consumers benefits from being a smaller share of the city population; (iii) whereas the group of type-H consumers benefits from an increase in the size of the better-quality neighborhood, the group of type-L consumers benefits from such an increase if and only if the group of type-H consumers is not too large.*

Proof: see Appendix A.2.

These last results call for some remarks. The mechanism behind the respective impact of β^H and β^L is twofold. First, the impact of β^i on $W_i(0)$ is ambiguous, whereas the impact of β^i on $W_{-i}(0)$ is not. This comes from the fact that β^i impacts type-i consumers' utility both directly and indirectly (through the price and the taste components) while it impacts type-$(-i)$ consumers' utility only indirectly. Second, there also are differences between the two types of consumers: type-H benefit from type-L facing a larger β^L whereas type-L suffer from type-H facing a larger β^H. In other words, the externality of one group's wealth on the other group's welfare is asymmetric: beneficial for the rich and detrimental for the poor. The reason is the following: Equation (4) shows that any increase in β^i yields a decrease in $p_0^*(0)$ and an increase in $p_1^*(0)$. Both phenomena affect both groups of consumers, but since type-H consumers are over-represented in neighborhood 0, the decrease in $p_0^*(0)$ overcomes the increase in $p_1^*(0)$ whereas the contrary happens for type-L consumers. The mechanisms behind the respective impact of α and n are similar. The impact of α may be interpreted as a within-group competition effect. As far as the impact of n, it may be interpreted as a between-group competition effect: for type-H consumers, who are over-represented in neighborhood 0, an increase in n always benefits enough people to overcome the negative impact of this increase on those who still live in neighborhood 1. For type-L consumers, however, this will be the case if and only if a sufficient fraction of them live in neighborhood 0, a condition which will be met if an only if $\alpha < (\beta^L - \beta^H)/\hat{\beta}$.

1.3 Public housing policy

Reducing segregation at the expense of welfare – The planner's only goal is to reduce segregation. This goal is pursued at the expense of the welfare of the population in this model. In Appendix A.3, I show that the first-best allocation of the population is achieved for total segregation. Moreover, one can show that the differential between the level of welfare of a population living in a city with no segregation and the level of welfare of a population living in the private city is an increasing function of the level of market-driven segregation $D(0)$: Let $W^{No} = \bar{q} - \hat{\beta}c - t[1/2 - n(1-n)]$ denote the

welfare of the population in the absence of segregation. This level of welfare verifies $W(0) - W^{No} = t \times [D(0)]^2/4\alpha(1-\alpha)$.

The assumption that the planner only cares about reducing segregation is normative in the sense that it is drawn from outside of the model. There is a comprehensive body of literature on the impact of socioeconomic stratification on growth, stemming from Benabou (1996) and Epple and Romano (1998), which involves strategic complementarities or external effects. However, these mechanisms are not included here. Providing a micro-funded rationale of the planner's goal is beyond the scope of this paper. The fact that public authorities seek to reduce socioeconomic segregation through public housing programs is a political reality in many countries. While taking this political reality as granted, the purpose of this paper is to question the ability of these programs to achieve such a goal.

The planner's program – City size is fixed.[4] The planner may only decide whether to buy a fraction s_j of pre-existing housing in j at price p_j, $s_j < n_j$, which will be allocated to applicants after a lottery. The price of public housing is equal to $k \leq \min\{p_0^*(s_j); p_1^*(s_j)\}$ and is the same in both neighborhoods.[5] There is no use in funding public housing in both neighborhoods at the same time in this static framework. For this reason, the city planner simultaneously selects one neighborhood in which to fund public housing and sets the size of the program. The program can be written as follows:

$$\min_{j=0,1}\left\{\min_{s_j\in[0;n_j]} D(s_j) \text{ s.t. } s_j\left(p_j^*(s_j)-k\right)\leq G\right\} \tag{7}$$

where $D(s_j)$ is the level of segregation in the city once the program s_j has been implemented. Optimal neighborhood choice derives from the comparison between the two potential outcomes of s_0 and s_1. In Section 2, I will first describe the impact of s_j on $D(s_j)$ for a given G before discussing optimal neighborhood choice with a cost-minimization criterion.

Note that I assume that k is exogenously determined, as well as public resources G. One may want G to be endogenized, for example through a tax on consumers' income. However, such feature is not easy to include into a framework à la Hotelling, where incomes are not modeled explicitly: indeed, taxing income would here increase β^H and β^L in a non-trivial way. Moreover, assuming this technical difficulty is solved, such tax would by itself have an impact on segregation, because it would affect the level of

[4]It may seem surprising to consider that a public housing policy has no impact on city size, but this is relevant in a context of very constrained supply, such as central cities of the largest metropolitan areas in France. In addition, this assumption has been empirically verified in some US settings. For instance, Eriksen and Rosenthal (2010)) document a 100% crowd-out effect of public housing construction on private construction.

[5]Note that this condition on the price of public housing yields more restrictions on the maximum size of the program, which is in fact given by $n_j\{1-n_j[\Delta q + t(1-2n)]/[\hat{\beta}(c-k)]\}$.

vertical differentiation between consumers, unless strong assumptions are made on the relationship between β^i and the taxation rate. The impact of the program on segregation would then be twofold, which would make the interpretation of the specific impact of public housing on segregation more difficult. This partial equilibrium framework may be easier to justify in countries where the financing of public housing program is highly centralized: this is clearly the case of France, in sharp contrast with the US (Laferrère and LeBlanc (2006)).

Allocation rules – Public housing is rationed, because public housing units only differ from private ones with respect to their lower price. I now describe the allocation rules which can be used to address this rationing problem. Consider that the planner decides to fund public housing in neighborhood j: $s_j > 0$ and $s_{-j} = 0$. Let $\Pr_j^i$ the probability that a type-i applicant to public housing in j receives an offer. If there are A_j applicants to public housing in j, this probability is defined by a function f_i such that $\Pr_j^i = f_i(s_j/A_j)$, with $f_i' > 0$. In addition to s_j, the government can then choose the level of screening in the allocation of the program by fixing a set of functions $(f_i(.), f_{-i}(.))$ which will ensure that all public housing units are allocated eventually.

I consider two allocation processes $\lambda = a, p$. The ex ante allocation process a means that everybody chooses location before the lottery takes place. It is relevant when one considers that the level of public housing supply has a magnetic effect on location decisions between metropolitan areas (Verdugo (2011)) or if people need to be already living in the local area in order to be allowed to apply. Under the ex post allocation process p, people are allowed to relocate in another neighborhood after the result of the lottery. It is relevant when one considers location decisions at a smaller geographical scale, for example between the different jurisdictions of the same metropolitan area.

Ex ante allocation is easily tractable because it takes $A_j = n_j$ as exogenous. Let $\tilde{p}_j$ the stochastic price paid for a housing unit in j. Agents choose where to locate according to their expected utility in each neighborhood, which depends on the probabilities of the lottery $\Pr_j^i$, with $E^i\tilde{p}_j(s_j) = \Pr_j^i k + (1 - \Pr_j^i)p_j < p_j$ and $E^i\tilde{p}_j(s_{-j}) = p_{-j}$. Let $x_i(E^i\tilde{p}_0(s_j); E^i\tilde{p}_1(s_j))$ denote a type-i indifferent consumer between a lottery on housing in j and private housing in $-j$. The equilibrium is given by $(p_0^a(s_j), p_1^a(s_j))$, solution to a system formed by a new market-clearing equation and a new free-entry condition. Expressions of the corresponding equations (32) and (33) are given in Appendix A.4). Investors fully anticipate the impact of the public housing program on prices and the free-entry condition changes accordingly.

The problem of ex ante location choice is that it forces a fraction of consumers who would be better off moving after the lottery to stay in place. The second allocation process takes this issue into account. Agents move according to the result of the lottery. Let $x_i(k; p_1(s_0))$ (resp., $x_i(p_0(s_1); k)$) denote the type-i indifferent consumer between public

housing in 0 (resp., 1) and private housing in 1 (resp., 0) and $x_i(p_0(s_j); p_0(s_j))$ the type-i indifferent consumer between private housing in 0 and private housing in 1. The size of the two potential groups of applicants is now given by $A_0 = \sum_{i=H,L} \alpha_i x_i(k; p_1(s_0))$ and $A_1 = \sum_{i=H,L} \alpha_i [1 - x_i(p_0(s_1); k)]$. The equilibrium is given by $(p_0^p(s_j), p_1^p(s_j))$, solution to a new system formed by the same free-entry condition (33) as before and new market-clearing equations (34) if $j = 0$ or (35) if $j = 1$ (for expressions, see Appendix A.4).

2 Public housing under exogenous neighborhood quality

2.1 Random allocation of public housing

I first consider that public housing is randomly allocated across types: $\forall i \in (H, L)$, $f_i = Id$. If public housing is funded in neighborhood j, the price differential, defined by $\Delta p_\gamma^\lambda(s_j) = p_\gamma^\lambda(s_j) - p_\gamma^*(0)$, gives the impact of public housing on private prices in neighborhood $\gamma \in \{j; -j\}$. It can be shown that this differential is positive in neighborhood j and negative in neighborhood $(-j)$. This is the case for both allocation processes. (for proof, see Appendix A.5.1 and A.5.2. The impact of public housing on private market prices may then be summarized as follows:

Proposition 5 *The introduction of public housing in one neighborhood increases the private market price in this neighborhood and decreases the private market price in the other neighborhood.*

This effect of public housing on prices comes directly from the fact that supply is fixed and there are no externalities of public housing on the private market. The introduction of public housing in j reduces private housing supply in j, which raises prices, whereas it makes neighborhood $(-j)$ less attractive. Whereas it may seem counterintuitive, this positive effect of public housing on prices nearby is relatively consistent with US evidence. For instance, Baum-Snow and Marion (2009) document a positive impact of low income tax housing credit developments on housing prices in declining areas (and no impact elsewhere). On the Parisian housing market, Goujard (2011) also finds evidence of a positive impact of public housing on the price of private units located very close-by.

The previous price mechanism leads to the following fundamental result:

Proposition 6 *If public housing is randomly allocated between household types, the impact of public housing on private market prices will perfectly counterbalance the increase in affordability for elected households. As a result, segregation will stay unchanged at the neighborhood level.*

Proof: see Appendix A.5.3.

Proposition 6 is the central result of this model. If everyone is eligible, the random allocation of public housing will not impact segregation at the neighborhood level, despite the direct effect of public housing, i.e., the provision of a stock of housing units below market rents. This calls for some remarks. First, the fact that the indirect price effect goes in the opposite direction to the direct effect of public housing is driven by a pure supply effect on the private market. Second, the fact that both compensate each other perfectly is due to the equilibrium conditions of the model (fixed supply in each neighborhood, fixed number of consumers and a covered-market assumption): because of these conditions, expected prices are kept constant in the ex ante allocation case and private prices adjust completely in the ex post allocation case. Finally, whereas both allocation processes yield the same between-neighborhood allocation of the population, they differ when considering their impact on the allocation between private and public units within the neighborhood. Under ex post allocation, while the proportion of type-L consumers increases in public housing because of their greater price sensitivity, it decreases in the private units of the neighborhood because of the increase in private prices and both mechanisms perfectly compensate each other. On the contrary, under ex ante allocation, the shares of type-H and type-L consumers also remain exactly the same across the private/public boundary.

Since the impact of randomly-allocated public housing is the same on segregation at the neighborhood level through both allocation processes, I will focus on ex ante allocation (and drop the superscript a) in the rest of the paper. This choice is motivated by the need to keep the analytical results easily interpretable. However, as stated above, both allocations processes are not equivalent and such restriction does come at a cost.

2.2 Screening

I now introduce the possibility for the public housing agency to screen applicants with respect to their type. The allocation of public housing can be made type-dependent, in order to reduce segregation. For a public housing program s_j, screening is defined by the vector $\boldsymbol{\phi}_j = (\phi_j^H, \phi_j^L) \neq (1,1)$ such that $f_i\left(s_j/A_j\right) = \phi_j^i s_j/A_j$. This setting enables me to describe the link between the minimum size of the public housing program and the minimum level of screening that must be implemented to reduce market-driven segregation by a given factor δ. If the planner could more easily play on the level of screening because program size s_j is fixed exogenously (lack of public resources, limited capacity of eviction, etc.), this setting would give the minimum level of screening that will have to be implemented to reduce the level of segregation by a factor δ. Reciprocally, if screening $\boldsymbol{\phi}_j$ is fixed exogenously, it will give the minimum program size s_j that needs

to be funded in order to reduce the level of segregation by a given factor δ. The two problems are analytically equivalent. Since many political and technical constraints are likely to impede the screening of applicants, I choose to present the latter.

Consider that one of the two weights ϕ_j^{-i} is fixed: it gives the level of acceptance of type-$-i$ applicants in public housing funded in neighborhood j. The two unknowns of the problem are s_j and ϕ_j^i. The two solutions, denoted $s_j(\phi_j^{-i})$ and $\phi_j^i(\phi_j^{-i})$, give the minimum size of the public housing program that is needed to reach the goal δ and the extent to which type-$(-i)$ applicants will be favored along the way, under the political constraint ϕ_j^{-i}. Again, type-i agents choose where to locate according to expected prices $E^i\tilde{p}_j(s_j) = (\phi_j^i s_j/n_j)k + (1 - \phi_j^i s_j/n_j)p_j$ and $E^i\tilde{p}_j(s_{-j}) = p_{-j}$. The type-$i$ indifferent consumer between a lottery on housing in j and private housing in $-j$ is now denoted $x_i(E^i\tilde{p}_0(s_j, \phi_j^i); E^i\tilde{p}_1(s_j, \phi_j^i])$. The objective function is given by Equation (8):

$$D\left(s_j, \phi_j^i\right) = (1 - \delta)\, D\,(0) \tag{8}$$

which is solved under a similar system as before and a new constraint, which stipulates that no public housing unit should remain vacant at the end of the allocation process. The solutions to this system enable me to write the following proposition:

Proposition 7 *(i) Under screening, one type of applicant is always favored at the expense of the other. (ii) There is a possibility of substitution between the level of screening and the size of the public housing program, (iii) but there is a minimum level of screening below which the planner will not be able to reduce segregation.*

Proof: see Appendix A.6.2

If public housing is funded in the better (resp., worse) neighborhood, then type-L (resp., type-H) applicants must be favored at the expense of type-H (resp., type-L) for public housing to reduce segregation. Conversely, a planner wishing to reduce the level of screening in the allocation of public housing while keeping the same goal in terms of segregation will have to compensate by increasing the size of the public housing program. However, since neighborhood size is fixed, this trade-off does not allow for perfectly random allocation. A minimum level of screening needs to be enforced, which corresponds to the situation where the whole neighborhood needs to be transformed into public housing. Let $\phi_0^{H\max}$ and $\phi_1^{L\max}$ the weights such that $s_0(\phi_0^{H\max}) = n$ and $s_1(\phi_1^{L\max}) = 1 - n$. From now on, I will consider that this condition is met, ie, $\phi_0^H \leq \phi_0^{H\max}$ and $\phi_1^L \leq \phi_1^{L\max}$. Finally, the new level of segregation $D^*(\phi_j^i, G)$ is equal to $(1 - \delta^*(\phi_j^i, G))D(0)$, where $\delta^*(\phi_j^i, G) = \min\{\delta^{\sup}(G), 1\}$ is obtained by solving equation $s_j(\phi_j^i)p_j(s_j(\phi_j^i)) = G$ in δ. This equation admits a unique positive solution $\delta^{\sup}(G)$. If the planner's resources are above a threshold $G^{\max}$, then $\delta^{\sup}(G) > 1$. (for proof, see Appendix A.7).

I develop a numerical example of the impact of screening when public housing is funded in the better neighborhood. It shows how much of the neighborhood must be turned into public housing to achieve a reduction of the level of market-driven segregation by a given factor δ. The values of the model parameters are given in Table 1. These values were simply selected in order to meet the parametric conditions that have been assumed so far and they do not relate to real-life values in a simple way. They yield the following equilibrium: $p_0^*(0) = 3.7$, $p_1^*(0) = 2.5$, $x_H^*(0) = 0.69$, $x_L^*(0) = 0.16$. The value of the dissimilarity index is $D(0) = 0.26$, meaning that under the free market allocation, a quarter of the population would have to move to the other neighborhood to achieve perfect integration. The value of total welfare is $W(0) = 3.7$, a weighted average between $W_H(0) = 5.2$ and $W_L(0) = 2.5$.

Table 1: Parameter values

β^L	β^H	q_0	q_1	α	n	t	c	k
1	0.1	6	5.5	0.45	0.4	1	3	0

Figure 1 illustrates the property of substitutability between the level of screening in the allocation of public housing and the size of the public housing program. For any goal δ, moving to the right of the graph, which means increasing the access to public housing for type-H applicants, has to be compensated by funding more public housing units, in the limit fixed by the size of the neighborhood. As a consequence, very low levels of screening (ie, high values of φ_0^H) may be out of reach. Finally, one can see that if the planner only seeks a modest decrease in segregation, the substitution effect will only be large for very low levels of screening: the higher the goal in terms of reducing segregation, the more needed the screening. The conclusions are the same if public housing is funded in neighborhood 1, except that the goal $\delta = 1$ is out of reach for any value of φ_1^L (see Figure 5 in Appendix C).

Figure 2 gives the corresponding minimum reachable level of segregation $D^*(\phi_j^i, G)$ as a function of ϕ_j^i, for different values of G . The new level of segregation is equal to $D\,(0)$ for $G = 0$ or $\phi_0^H = 1$ (represented by the dotted line in Figure 2): while completely random allocation remains incompatible with any decrease in segregation, a planner wishing to implement as little screening as possible may always marginally substitute screening with more public housing, but at an increasing marginal cost. Finally, when public housing is funded in neighborhood 0, as soon as $G > \hat{G}_0 \approx 1.35$, there is a minimum value of ϕ_0^H under which the program is saturated, meaning that public resources are too abundant. The conclusions are the same if public housing is funded in neighborhood 1, except that $\hat{G}_1 \approx 1.7$ (see Figure 6 in Appendix C).

Figure 1: Optimal public housing share in neighborhood 0 as a function of the level of acceptance of type-H applicants

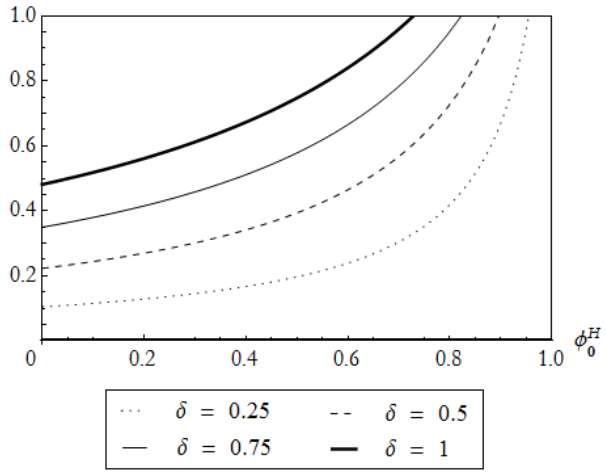
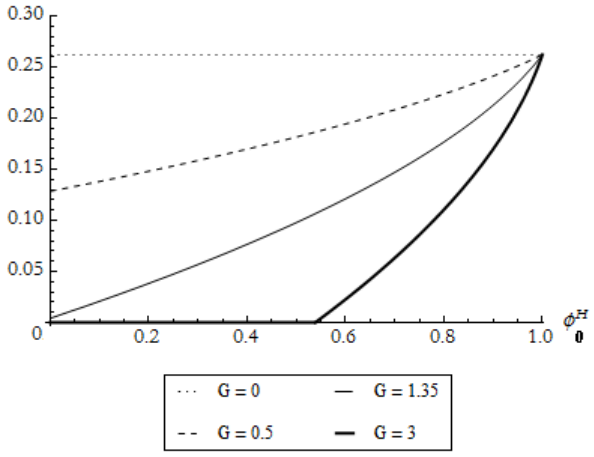


Figure 2: Optimal level of segregation when public housing funded in neighborhood 0 as a function of the level of acceptance of type-H applicants



2.3 The planner's choice

I now come back to the planner's program. If the planner, for external reasons, is facing a fixed level of public rents k and a fixed screening process, its only instrument left is the choice of the neighborhood in which public housing should be funded. Under no other constraint, the factor behind the choice of one neighborhood over the other will be the cost of the program, given by the number of public housing times the rent differential between private rent in the neighborhood and k. Whereas this differential is larger in neighborhood 0, it may well be the case that this effect is overcome by the need to fund more units in neighborhood 1 in order to achieve the same goal in terms of segregation.

Under random allocation, the program has no impact on segregation and the planner does not fund any public housing in either neighborhood and there is indetermi-

nacy in the choice of the neighborhood. Conversely, if the planner decides to implement a screening rule, a very simple condition on optimal neighborhood choice can be given, under one additional assumption on the management of screening. The level of screening is given by ϕ_0^H if public housing is funded in neighborhood 0 and by ϕ_1^L if public housing is funded in neighborhood 1. Let us assume that $\phi_0^H = \phi_1^L = \phi \in \left[0; \min\left\{\phi_0^{H\max}; \phi_1^{L\max}\right\}\right]$. Such constraint means that the public housing agency in charge of the allocation process is only able to enforce one simple screening rule at its disposal, whether it is at the expense of type-H applicants in neighborhood 0 or at the expense of type-L applicants in neighborhood 1. A general expression for the cost of the program in neighborhood j is then $c_j(\phi) = s_j(\phi)\left[p_j\left(s_j(\phi)\right) - k\right]$ and the expression for the cost differential verifies $c_0(\phi) > c_1(\phi) \Leftrightarrow n > \alpha$. This result can be summarized by the following proposition:

Proposition 8 *When facing the same screening rule in both neighborhoods, a planner will choose to fund public housing in the better-quality neighborhood if and only if this neighborhood is too small compared to the population of rich consumers.*

proof: see Appendix A.7.

This is the other important result of the paper: in a city where high-quality neighborhoods are in relative shortage compared to the number of wealthy residents, it is more efficient to fund public housing in these high-quality neighborhoods. Think of the two possible situations of total segregation in the city: all type-H consumers living in 0 or all type-L consumers living in 1. Proposition 8 states that if the former is in place, then it is cheaper to move some type-H consumers into public housing in neighborhood 1, where none of them lives, than to try and relocate type-L consumers into public housing in neighborhood 0, where some of them already reside. The same reasoning applies to intermediate situations. The comparison of n and α gives a measure of the market relative capacity of answering the needs of the two social groups. If $n > \alpha$, the market answers the needs of the group of type-H more adequately and, as a result, the public housing program must counterbalance this feature by locating where type-L are overrepresented. In my numerical example, the chosen values for n and α are such that public housing is funded in neighborhood 0.

3 Extensions

3.1 Endogenous neighborhood quality

Set-up– Up to now, I have considered that the degree of attractiveness of a neighborhood was driven by the combination of two features, its size, or relative scarcity, and its qual-

ity, taken as exogenous: this was the Fully Exogenous Quality case, henceforth FEXQ. While it is possible to think of orderable exogenous neighborhood characteristics, such as natural attributes, a neighborhood is also characterized by the social make-up of its inhabitants. When talking about a "good" or a "bad" neighborhood, one generally includes a statement regarding the neighborhood's level of wealth. To put it bluntly, it may not be entirely realistic to assume that funding a large public housing complex in a wealthy neighborhood and filling it with poor tenants may only have an upward impact on prices of the private housing nearby. In this section, I investigate what can be said about the impact of public housing policies when consumers account for the externalities created by their neighbors in the valuation of their neighborhood.

One might want to consider two potentially conflicting external effects: peer effects and network effects. Peer effects mean here that any consumer, regardless of her type, prefers a neighborhood with a higher proportion of type-H residents. They are relevant when one thinks of the quality of local public goods, especially schools. Network effects mean that the valuation of the neighborhood by a consumer depends on his or her type and increases with the proportion of residents of the same type who live in the neighborhood. They are more relevant when economic status overlaps with other features, such as immigration status.

With neighborhood externalities, the valuation of neighborhood quality becomes a combination of an exogenous component (amenities) and an endogenous component (social make-up). In order to keep the differences between the two types of households to a minimum, I assume that the respective importance of these two components is given by a type-independent scalar η. The expressions for the neighborhood valuation are then given by v_j^{PE} if one considers peer effects and v_{ij}^{NE} if one considers network effects, with

$$v_j^{PE} = q_j + \eta \frac{\alpha \left| x_H - j \right|}{n_j} \quad \text{and} \quad v_{ij}^{NE} = q_j + \eta \frac{\alpha_i \left| x_i - j \right|}{n_j}, \tag{9}$$

where q_j is the same as before and x_i is the position of the type-i indifferent consumer between the two neighborhoods, which is determined at the equilibrium and is also equal to the mass of type-i consumers who end up living in neighborhood 0.

I choose to focus on peer effects. This does not mean that network effects do not exist, but I assume that their relative magnitude is probably small enough for them to be neglected as a first-order approximation. Moreover, peer effects are far more relevant to account for NIMBY-like behavior, such as coalitions of existing residents who oppose public housing projects in their neighborhood for fear of a drop in neighborhood quality and property values. Note that since neighborhood sizes are fixed, I could equivalently consider that consumers put a negative weight on the proportion of type-L residents in the neighborhood.

I still focus on the situation which led to a fully asymmetric market equilibrium in favor of neighborhood 0 in the absence of externalities, i.e. such that $\Delta q > 0$ and $n < 0.5$. Let $p_j^{PE}(0)$ the market price in neighborhood j assuming consumers take the proportion of type-H residents among their neighbors into account. I solve, in $p_0^{PE}(0)$, $p_1^{PE}(0)$, $x_H^{PE}(0)$ and $x_L^{PE}(0)$, a four-equation system formed by a market-clearing equation, the investors' participation constraint and the two equations defining $x_H^{PE}(0)$ and $x_L^{PE}(0)$. The price differential $\Delta p_{01}^{PE}(0) \equiv p_0^{PE}(0) - p_1^{PE}(0)$ is now given by:

$$\Delta p_{01}^{PE}(0) = \frac{2n(1-n)t[\Delta q + t(1-2n)]}{2\hat{\beta}n(1-n)t - \eta\alpha(1-\alpha)(\beta^L - \beta^H)} \tag{10}$$

As for the new dissimilarity index $D^{PE}(0)$, it is given by the same function of $\Delta p_{01}^{PE}(0)$ as in equation 5. The complete expressions of $p_0^{PE}(0)$ and $p_1^{PE}(0)$ are provided in Appendix B.1. There is an existence condition to this equilibrium: $\eta \neq \eta_0$, with $\eta_0 = 2\hat{\beta}n(1-n)t/[\alpha(1-\alpha)(\beta^L - \beta^H)]$. However, another condition on η is in fact more restrictive than this existence condition. Recall that the dissimilarity index cannot be larger than the size of the smaller of the two groups of consumers. This yields two threshold values η_α (if $\alpha < 1/2$) and $\eta_{1-\alpha}$ if ($\alpha > 1/2$), which are both lower than η_0 (for complete expressions, see Appendix B.1). Under the condition $\eta \leq \max_{i=H,L}\{\eta_{\alpha_i}\}$, it is always verified that $\Delta p_{01}(0) < \Delta p_{01}^{PE}(0)$: the introduction of the externality exacerbates the polarization of the city.

The impact of public housing under peer effects– One may wonder if the introduction of externalities is enough to alter the previous result that random allocation of public housing under FEXQ cannot reduce the level of segregation in the city. I focus on ex ante allocation and show that the expressions for the price differentials $\Delta p_\gamma^{PE}(s_j) = p_\gamma^{PE}(s_j) - p_\gamma^{PE}(0)$ are similar to what is obtained under FEXQ (for complete expressions, see Appendix B.2). The consumers who are indifferent between living in either of the two neighborhoods stay unchanged, regardless of the size of the public housing program, and segregation remains at its market level. Since there is no screening of applicants, the population of public tenants does not alter the social make-up of the neighborhood, hence the endogenous component of neighborhood valuation does not affect the equilibrium. The only change with respect to Section 2 is about the cost of the program: the private market price is higher (resp., lower) in neighborhood 0 (resp., in neighborhood 1) than under FEXQ.

Introducing the screening of applicants under endogenous neighborhood quality has both a direct effect on the social make-up of the public housing complex, and an indirect effect on the quality of the neighborhood. As a result, public housing policy will be more efficient. By "efficient", I mean that the same reduction in the level of

segregation will require a smaller public housing program, for any level of screening.

I solve a similar problem as in Subsection 2.2: after getting the equilibrium as a function of $\boldsymbol{\phi}_j$, I solve the system formed by the public housing market clearing equation and the equation giving the planner's goal in terms of segregation. This yields the solutions $\phi_j^{iPE}(\phi_j^{-i}, \eta)$ and $s_j^{PE}(\phi_j^{-i}, \eta)$. The functions $\phi_j^{iPE}(\phi_j^{-i}, \eta)$ are exactly the same as under FEXQ. As for the functions $s_j^{PE}(\phi_j^{-i}, \eta)$, they give the optimal size of the public housing program needed to reach the goal δ. Then, the final step is to compare $s_j^{PE}(\phi_j^{-i}, \eta)$ and $s_j(\phi_j^{-i})$ and show that $s_j^{PE}(\phi_j^{-i}, \eta) < s_j(\phi_j^{-i})$. This yields the following proposition:

Proposition 9 *The minimum size of the public housing program required to reduce segregation by a fixed fraction is lower when taking the impact of neighborhood externalities into account.*

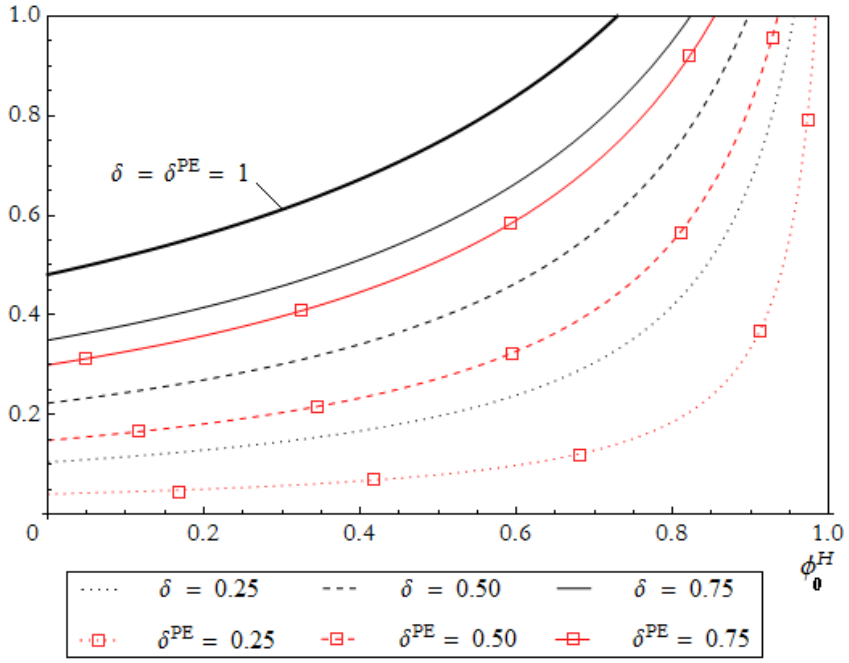
Proof: see Appendix B.3.

Using the same numerical example as in Section 2, one finds $\eta_0 \approx 1.28$. The maximum value of η for which the problem is still meaningful solves $D^{PE}(0) = \alpha$. This is obtained for $\eta = \eta_\alpha \approx 0.54$. Prices $p_j^{PE}(0)$ are now between $p_0^*(0) \approx 3.7$ and $p_1^*(0) \approx 2.5$ (for $\eta = 0$) and extreme values $p_0^{PE}(0) \approx 4.2$ and $p_1^{PE}(0) \approx 2.2$ when $\eta = \eta_\alpha$. By definition of η_α, market segregation goes from $D(0) = 0.26$ for $\eta = 0$ to $D^{PE}(0) = 0.45$ for $\eta = \eta_\alpha$. The value of total welfare goes from $W(0) = 3.7$ to $W^{PE}(0) = 4.1$, the weighted average between $W_H^{PE}(0) = 5.6$ and $W_L^{PE}(0) = 2.8$. When $\eta > 0$, the condition under which neighborhood 0 is chosen over neighborhood 1 is more complicated than in the exogenous case and may depend on δ (complete expression is provided in Appendix B.4). However, in the numerical example, neighborhood 0 is chosen for any value of δ and ϕ, since the maximum value of the difference between the cost of the program in neighborhood 0 and the cost of the program in neighborhood 1 is equal to -0.315, value reached for $\phi = 0$ and $\delta = 1$.

Figure 3 illustrates the impact of externalities on the minimum program size $s_0^{PE}\left(\phi_0^H, \eta\right)$ required to reduce segregation by 25%, 50%, 75% and 100%. The black lines, which correspond to the FEXQ case ($\eta = 0$), are the same as in Figure (1). The red lines with squares, which are referenced by the superscript PE in order to be clearly distinguished from the regular lines, describe the other polar case when $\eta = \eta_\alpha$, i.e. when neighborhood valuation is largely driven by its endogenous component. One can check that the red lines with squares are always below the black lines, except for $\delta = \delta^{PE} = 1$, when they are confounded. Indeed, when there is no segregation at all, the endogenous component of neighborhood valuation does not impact location decisions.

While peer effects increase the polarization of the city, they also increase the efficiency of the public housing policy: reducing segregation by the same factor δ will require fewer public resources, or a lower level of screening, than in the FEXQ case.

Figure 3: Optimal public housing share in neighborhood 0 as a function of the level of acceptance of type-H applicants, without and with externalities



The black lines correspond to the FEXQ case. The red lines with squares correspond to the case where the relative importance of peer effects in consumers' valuation of a neighborhood is maximum.

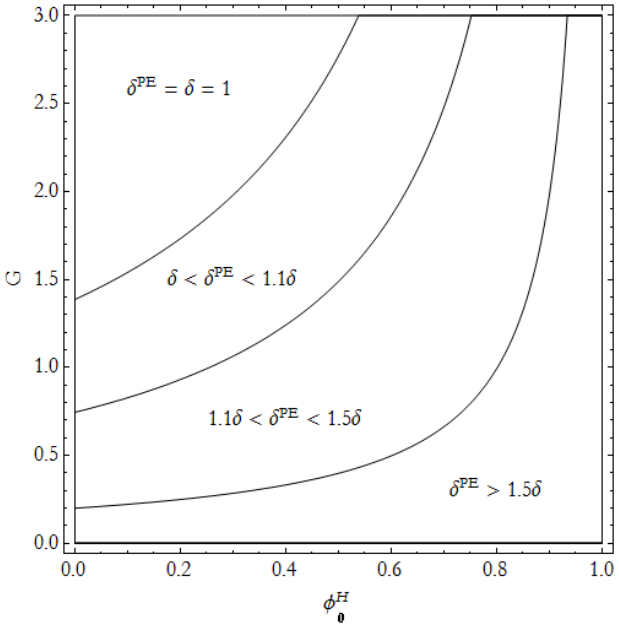
Figure 4 illustrates this by comparing the relative optimal decreases in segregation $\delta^*(\phi, G, \eta)$ when $\eta = 0$ and $\eta = \eta_\alpha$ as a function of combinations of (ϕ, G), using the same parametrization as before when public housing is funded in neighborhood 0.

Except when the program is overfunded and segregation goes down to zero (the top-left corner of the figure), it is always the case that $\delta^*(\phi, G, \eta_\alpha) > \delta^*(\phi, G, 0)$, for any combination (ϕ, G) where $\phi < 1$ and $G > 0$. The relative distance between $\delta^*(\phi, G, \eta_\alpha)$ and $\delta^*(\phi, G, 0)$ increases when $\phi \to 1$ and $G \to 0$: in case the program is both small in magnitude and tends to random allocation, the presence of neighborhood externalities makes it all the more efficient.

3.2 Indirect screening

Whereas public housing agencies may not be allowed to directly screen applicants because the political cost would be too high, there are no reasons to think that public housing units may not be designed such that they are only attractive for a certain type of consumers. This will be the case if a housing unit is not only defined by its location but also by other characteristics -such as its size and its comfort, and if these additional characteristics are not valued equally by both types of consumers. For example, it is

Figure 4: Comparison of $\delta = \delta^* \left(\phi_0^H, G, 0\right)$ and $\delta^{PE} = \delta^* \left(\phi_0^H, G, \eta_\alpha\right)$ as a function of public resources and the level of acceptance of type-H applicants



possible to imagine that only richer households will value certain dwelling patterns, such as space for representation. Similarly, the assumption that everyone, regardless of type, consumes one unit of housing, may also hide an indirect screening channel based on the size distribution of public housing units. If public housing units, even in a good location, mostly attract poor households, the policy will reduce segregation, and all the more so if externalities then exacerbate this initial differential.

One can easily amend the model to incorporate these features. Consider that the characteristics of the dwelling may be summarized by an orderable index, independent of location, equal to F if the unit is private and to f if it is public, with $F > f$. The assumption that $F > f$ may only reflect the fact that the level of mismatch is higher in public housing, whereas there is a continuum of possible combinations in the private market which makes it easier for households to find a dwelling more adapted to their needs. The expressions for the neighborhood valuation are now given by v_{ij}^{ϵ}, with

$$v_{ij}^{\epsilon} = \begin{cases} q_j + \epsilon^i F & \text{if the unit is private} \\ q_j + \epsilon^i f & \text{if the unit is public} \end{cases} . \tag{11}$$

where q_j is the same as before and $\epsilon^i > 0$ measures the relative importance of comfort characteristics with respect to location.

Assume that $\epsilon^H > \epsilon^L$, ie richer households are more sensitive to the intrinsic characteristics of their dwelling. A public housing program funded in neighborhood 0, where s_0 units are randomly allocated to both types of applicants, will reduce segregation

down to a level of dissimilarity $D^{\epsilon}(s_0)$ defined by:

$$D^{\epsilon}(s_0) = D(0) - \frac{\alpha(1-\alpha)(\beta^L \epsilon^H - \beta^H \epsilon^L)}{\hat{\beta} nt}(F - f)s_0 \tag{12}$$

Since $\beta^L > \beta^H$ and $\epsilon^H > \epsilon^L$, one can check that, indeed, $D^{\epsilon}(s_0) < D(0)$. Assume further that f is a policy parameter, whereas F is exogenously determined by the construction sector. Provided public housing agencies are able to set f low enough, they will only attract type-L applicants, and the program will be as able to reduce segregation as a program with direct screening. In that sense, an equivalence relationship between f and ϕ can be found.

However, despite how attractively simple this indirect screening channel may seem, it is far from obvious that it is relevant, for at least two reasons. First, from a theoretical viewpoint, the assumption that $\epsilon^H > \epsilon^L$ may strike as completely ad-hoc: whereas it derives from the marginal utility of income that $\beta^L > \beta^H$, this is not the case of this latter assumption. Moreover, even if it is likely that richer consumers are pickier regarding the comfort of their dwelling, one must ensure that this difference stands true relative to their valuation of neighborhood quality q_j, which is unclear. The second problem of the indirect screening channel is a practical one: leaving aside the question of the observability of ϵ^i, how can agencies set a value of f such that they reach their goal in terms of reduction of segregation? if, for example, it is by letting public housing buildings deteriorate, or by allowing the production of low-quality public housing buildings, why would this strategy be more politically feasible, especially in wealthy neighborhoods, than directly screening applicants? On the contrary, for external political reasons, what should be most likely observed is a positive correlation between q_j and f; and in that case, the indirect screening channel will increase segregation.

4 Conclusion

In this paper, I have shown that implementing public housing quotas do not always reduce segregation. In a context of very constrained city size, when public housing units come from preexisting housing stock, public housing quotas will be effective if and only if a minimum level of screening is enforced at the expense of the applicants whose kind is already overrepresented in the neighborhoods where public housing is funded. The final message to be drawn from this model then depends on whether public authorities are able, either politically or technically, to impose this kind of screening.

Since the main alternative to public housing programs is housing vouchers, it would seem natural to try and compare the impact of the two policies. However, because income is not modeled explicitly, vouchers will here be assimilated to gross utility trans-

fers, conditional on people living in the targeted neighborhood. Such conditional utility transfer policy may prove more efficient if consumers are less sensitive to housing prices than to the price of other goods; moreover, and somewhat more importantly, if those transfers are fully divisible, this policy will not suffer from the potential inefficiencies induced by rationing.

There are several possible extensions to this simple framework. First of all, total housing supply is not always fixed and a city-planner may often choose whether public housing should come from existing stock or be created ex nihilo; in this case, public housing programs will increase the city population and possibly change its social make-up. If included, this feature would allow to investigate the political economy behind public housing programs. In particular, if the planner has to seek reelection, there are incentives for her to buy votes with housing. Such a model might help understand the patterns of apparently exogenous increases in the population of some cities, for instance in the Paris region in the 1960's and 1970's. Another interesting extension in terms of political economy would be to no longer assume that investors perfectly anticipate future public housing programs when they make their investment decision and are subject to a hold-up problem. I leave this for future work.

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A Analytical expressions from Section 2

A.1 Covered-market condition

I provide here a sufficient condition for the market to be covered under exogenous neighborhood quality. I focus on $i = L$, because, provided both types have the same level of reservation utility, if the market is covered for type-L consumers, then it is also covered for type-H consumers. Let $U^{\min}$ the reservation utility that consumers would get outside the market. If the market is not covered, this means that $\forall j = 0, 1$, $\exists x \in [0, 1]$, $U_j^L(x) < U^{\min}$. I derive the implications of these inequalities and provide a condition under which they cannot be verified. We have:

$$\left\{ \begin{array}{l} q_0 - \beta^L p_0 - tx < U^{\min} \\ q_1 - \beta^L p_1 - t(1-x) < U^{\min} \end{array} \right. \Rightarrow \left\{ \begin{array}{l} x > \left(q_0 - \beta^L p_0 - U^{\min}\right)/t \\ x < 1 - \left(q_1 - \beta^L p_1 - U^{\min}\right)/t \end{array} \right. \tag{13}$$

As a consequence, a sufficient condition for the market to be covered is the following:

$$1 - (q_1 - \beta^L p_1 - U^{\min})/t < (q_0 - \beta^L p_0 - U^{\min})/t, \tag{14}$$

which can also be written as:

$$\frac{1}{2}\left[U_0^L\left(\frac{1}{2}\right) + U_1^L\left(\frac{1}{2}\right)\right] > U^{\min} \tag{15}$$

The expected utility of the median type-L consumer entering the market with equiprobability to live in either neighborhood must be positive. This can always be achieved by setting q_j high enough.

A.2 Welfare in the private city

The expressions $\bar{q}$, $\bar{p}$ and $\bar{t}$ are the following:

$$\bar{q} = \sum_{j=0,1} n_j q_j \tag{16}$$

$$\bar{p} = \sum_{i=H,L} \alpha_i \beta^i \left[x_i^*(0)\, p_0^*(0) + (1 - x_i^*(0))\, p_1^*(0)\right] \tag{17}$$

$$\bar{t} = \frac{t}{2} \sum_{i=H,L} \alpha_i \left[1 - 2\left(x_i^*(0)\right)\left(1 - \left(x_i^*(0)\right)\right)\right] \tag{18}$$

The complete expression for welfare is:

$$W(0) = \frac{1}{4\hat{\beta}^2 t} \left\{ \begin{array}{c} 4ct\left[\alpha^3\left(\beta^L - \beta^H\right)^3 - \left(\beta^L\right)^3\right] \\ +\alpha(1-\alpha)\left(\beta^L - \beta^H\right)^2\left\{\left[\Delta q + t(1-2n)\right]^2 + 12\beta^L ct\right\} \\ +12\alpha ct\left(\beta^H\left(\beta^L\right)^2 - \beta^L\left(\beta^H\right)^2\right) \\ +2\hat{\beta}^2 t\left\{nq_0 + (1-n)q_1 - t\left[1 - 2n(1-n)\right]\right\} \end{array} \right\} \tag{19}$$

From Equation (19), I can compute the following expressions for the comparative statics for $W(0)$:

$$\frac{\partial W(0)}{\partial c} = -\hat{\beta} \tag{20}$$

$$\frac{\partial W(0)}{\partial q_0} = n + \alpha(1-\alpha)\left(\beta^L - \beta^H\right)^2 \left[\Delta q + t(1-2n)\right] / \left(2t\hat{\beta}^2\right) \tag{21}$$

$$\frac{\partial W(0)}{\partial q_1} = 1 - n - \alpha(1-\alpha)\left(\beta^L - \beta^H\right)^2 \left[\Delta q + t(1-2n)\right] / \left(2t\hat{\beta}^2\right) \tag{22}$$

$$\frac{\partial W(0)}{\partial t} = -\frac{1}{\left(2t\hat{\beta}\right)^2} \left\{ \begin{array}{c} \alpha(1-\alpha)\left(\beta^L - \beta^H\right)^2 \left[(\Delta q)^2 - t^2(1-2n)^2\right] \\ +\hat{\beta}^2 t^2 \left[1 - 2n(1-n)\right] \end{array} \right\} \tag{23}$$

When I consider separately the welfare of each group of consumer, I get the following:

$$\frac{\partial W_L(0)}{\partial \beta^H} = -\alpha^2 \beta^L \left(\beta^L - \beta^H\right) \left[\Delta q + t(1-2n)\right]^2 / \left(2t\hat{\beta}^3\right) \tag{24}$$

$$\frac{\partial W_H(0)}{\partial \beta^L} = (1-\alpha)^2 \beta^H \left(\beta^L - \beta^H\right) \left[\Delta q + t(1-2n)\right]^2 / \left(2t\hat{\beta}^3\right) \tag{25}$$

$$\frac{\partial W_L(0)}{\partial \alpha} = \alpha\beta^L \left(\beta^L - \beta^H\right)^2 \left[\Delta q + t(1-2n)\right]^2 / \left(2t\hat{\beta}^3\right) \tag{26}$$

$$\frac{\partial W_H(0)}{\partial \alpha} = -(1-\alpha)\beta^H \left(\beta^L - \beta^H\right)^2 \left[\Delta q + t(1-2n)\right]^2 / \left(2t\hat{\beta}^3\right) \tag{27}$$

$$\frac{\partial W_L(0)}{\partial n} = \beta^L \left[\beta^L - 2\alpha\left(\beta^L - \beta^H\right)\right] \left[\Delta q + t(1-2n)\right] / \hat{\beta}^2 \tag{28}$$

$$\frac{\partial W_H(0)}{\partial n} = \beta^H \left[\beta^H + 2(1-\alpha)\left(\beta^L - \beta^H\right)\right] \left[\Delta q + t(1-2n)\right] / \hat{\beta}^2 \tag{29}$$

A.3 Optimality

The first-best allocation of the population is distinct from the market allocation. The allocation of the population that would maximize the welfare of consumers is given by the consumers $(\tilde{x}_H, \tilde{x}_L)$, who represent the consumers who live in neighborhood 0 and are the furthest on the right of the $[0,1]$ segment, and the prices $(\tilde{p}_0, \tilde{p}_1)$. Given the two constraints $\alpha\tilde{x}_H + (1-\alpha)\tilde{x}_L = n$ and $n\tilde{p}_0 + (1-n)\tilde{p}_1 = c$, finding the first-best allocation amounts to finding a pair $(\tilde{x}_H^*, \tilde{p}_0^*)$ which maximizes

$$\begin{aligned} \tilde{W}(\tilde{x}_H, \tilde{p}_0) \;=\; & \int_0^{\tilde{x}_H} \left[q_0 - \beta^H p_0 - tx\right] dx + \int_{\tilde{x}_H}^1 \left[q_1 - \beta^H \left(\frac{c - np_0}{1-n}\right) - t(1-x)\right] dx \\ & + \int_0^{\frac{n-\alpha\tilde{x}_H}{1-\alpha}} \left[q_0 - \beta^L p_0 - tx\right] dx + \int_{\frac{n-\alpha\tilde{x}_H}{1-\alpha}}^1 \left[q_1 - \beta^L \left(\frac{c - np_0}{1-n}\right) - t(1-x)\right] dx \end{aligned} \tag{30}$$

under the constraints $\tilde{p}_0 \in [0, c/n]$ and $\tilde{x}_H \in \left[\max\left\{\frac{n-(1-\alpha)}{\alpha}, 0\right\}, \min\left\{\frac{n}{\alpha}, 1\right\}\right]$. A simple way to rewrite $\tilde{W}(\tilde{x}_H, \tilde{p}_0)$ is:

$$\tilde{W}(\tilde{x}_H, \tilde{p}_0) = W^{No} + \frac{\alpha}{(1-\alpha)(1-n)}(\tilde{x}_H - n)\left[(1-\alpha)\left(\beta^L - \beta^H\right)(\tilde{p}_0 - c) - t(1-n)(\tilde{x}_H - n)\right] \tag{31}$$

Note that the function $\tilde{W}(\tilde{x}_H, \tilde{p}_0)$ is not concave and the maximization problem will yield corner solutions, ie, total segregation.

Using the same numerical example as in sections 2 and 3, one finds that $\tilde{W}^* = \tilde{W}(\tilde{x}_H^*, \tilde{p}_0^*)$ is obtained for the maximum value of $\tilde{x}_H^* = n/\alpha = 0.89$ and for $\tilde{p}_0^* = 7.5$. In that case, $\tilde{W}^* = 4.94$, whereas $W(0) = 3.72$ and $W^{No} = 3.66$.

A.4 Allocation process

The market clearing equation and the free entry condition for ex ante allocation are given by:

$$n = \sum_{i=H,L} \alpha_i x_i \left(E^i \tilde{p}_0(s_j); E^i \tilde{p}_1(s_j)\right) \tag{32}$$

$$c = \sum_{k=0,1} n_k p_k(s_j) \tag{33}$$

The market clearing equations for ex post allocation are given by:

$$n = \sum_{i=H,L} \alpha_i \left[x_i(p_0(s_0); p_1(s_0)) + \Pr_0^i \langle x_i(k; p_1(s_0)) - x_i(p_0(s_0); p_1(s_0))\rangle\right] \tag{34}$$

$$n = \sum_{i=H,L} \alpha_i \left[x_i(k; p_0(s_1)) + \left(1 - \Pr_1^i\right)\langle x_i(p_0(s_1); p_1(s_1)) - x_i(p_0(s_1); k)\rangle\right] \tag{35}$$

A.5 Prices and segregation under random allocation

A.5.1 Pricing under ex ante location choice

The expressions for the $\Delta p_\gamma^a(s_j)$ are the following:

$$\Delta p_j^a(s_j) = \frac{s_j n_{-j}}{n_j - s_j n_{-j}}\left[p_j^*(0) - k\right] \quad \text{and} \quad \Delta p_{-j}^a(s_j) = -\frac{s_j n_j}{n_j - s_j n_{-j}}\left[p_j^*(0) - k\right] \tag{36}$$

Since $n_j - s_j n_{-j} > 0$, one can then immediately check that $\Delta p_j^a(s_j) > 0$ and $\Delta p_{-j}^a(s_j) < 0$.

A.5.2 Pricing under ex post location choice

Under ex post allocation, the probability $\Pr_j^i$ is proportional to the stock of public housing in the neighborhood and inversely proportional to the number of applicants $A_j > n_j$. Since A_j is endogenous, it also depends on prices and the market clearing equation is of degree 2. The

relevance of the two pairs of solutions depends on the parameter space. I hereby provide the analytical expressions for one of them.

Let $Z(s_0) = \hat{\beta}(p_0^*(0) - k) + 2t[n + s_0(1-n)]$ and $Y(s_0) = 8ns_0t\hat{\beta}(p_0^*(0) - k)$. Assuming $(Z(s_0))^2 > Y(s_0)$, the relevant pair of solutions for public housing in 0 yields the following:

$$\Delta p_0^p(s_0) = \frac{1-n}{2n\hat{\beta}}\left(Z(s_0) - \sqrt{(Z(s_0))^2 - Y(s_0)}\right) \tag{37}$$

$$\Delta p_1^p(s_0) = -\frac{1}{2\hat{\beta}}\left(Z(s_0) - \sqrt{(Z(s_0))^2 - Y(s_0)}\right) \tag{38}$$

Since $Z(s_0) > 0$ and $Y(s_0) > 0$, one can then immediately check that $\Delta p_0^p(s_0) > 0$ and $\Delta p_1^p(s_0) < 0$.

Let $Z(s_1) = \hat{\beta}(p_1^*(0) - k) + 2t[1 - n + s_1 n]$ and $Y(s_1) = 8(1-n)s_1t\hat{\beta}(p_1^*(0) - k)$. Assuming $(Z(s_1))^2 > Y(s_1)$, the relevant pair of solutions for public housing in 1 yields the following:

$$\Delta p_0^p(s_1) = -\frac{1}{2\hat{\beta}}\left(Z(s_1) - \sqrt{(Z(s_1))^2 - Y(s_1)}\right) \tag{39}$$

$$\Delta p_1^p(s_1) = \frac{n}{2(1-n)\hat{\beta}}\left(Z(s_1) - \sqrt{(Z(s_1))^2 - Y(s_1)}\right) \tag{40}$$

Since $Z(s_1) > 0$ and $Y(s_1) > 0$, one can then immediately check that $\Delta p_0^p(s_1) < 0$ and $\Delta p_1^p(s_1) > 0$.

A.5.3 Segregation index

I compute the dissimilarity indexes after the program is implemented and show that $\forall(j,\pi) \in \{0,1\} \times \{a,p\}$, $D^\pi(s_j) = D(0)$. For ex ante location, we have : $\forall j = 0, 1$

$$\begin{aligned} D^a(s_j) &= \alpha\left[x_H\left(E^i\tilde{p}_0^a(s_j); E^i\tilde{p}_1^a(s_j)\right) - n\right] \\ &\quad + (1-\alpha)\left[n - x_L\left(E^i\tilde{p}_0^a(s_j); E^i\tilde{p}_1^a(s_j)\right]\right) \end{aligned} \tag{41}$$

Under equiprobability, $\forall(i,j) \in \{H,L\} \times \{0,1\}$, $E^i\tilde{p}_j^a(s_j) = \frac{s_j}{n_j}k + \frac{n_j - s_j}{n_j}p_j^a(s_j)$. Straightforward computations show that $x_i\left(E^i\tilde{p}_0^a(s_j); E^i\tilde{p}_1^a(s_j)\right) = x_i(p_0^*(0), p_1^*(0))$. For ex post location, we have:

$$\begin{aligned} D^p(s_0) &= \alpha\left\{\begin{array}{c} \Pr_0^H x_H(k; p_1^p(s_0)) - n \\ + \left(1 - \Pr_0^H\right) x_H(p_0^p(s_0); p_1^p(s_0)) \end{array}\right\} \\ &\quad + (1-\alpha)\left\{\begin{array}{c} 1 - x_L(k; p_1^p(s_0)) - (1-n) \\ + \left(1 - \Pr_0^L\right)(x_L k; p_1^p(s_0)) - x_L(p_0^p(s_0); p_1^p(s_0))) \end{array}\right\} \end{aligned} \tag{42}$$

$$D^p(s_1) = \alpha \left\{ \begin{array}{c} x_H(p_0(s_1);k) - n \\ + \left(1 - \Pr_1^H\right)\left(x_H\left(p_0^p(s_1);p_1^p(s_1)\right) - x_H(p_0(s_1);k)\right) \end{array} \right\} + (1-\alpha) \left\{ \begin{array}{c} 1 - x_L\left(p_0^p(s_1);p_1^p(s_1)\right) - (1-n) \\ + \Pr_1^L\left(x_L\left(p_0^p(s_1);p_1^p(s_1)\right) - x_L\left(p_0^p(s_1);k\right)\right) \end{array} \right\} \quad (43)$$

Under equiprobability, $\forall i \in \{H, L\}$,

$$\underset{0}{\overset{i}{\Pr}} = s_0 / \left[\alpha x_H\left(k; p_1^p(s_0)\right) + (1-\alpha) x_L\left(k; p_1^p(s_0)\right)\right] \quad (44)$$

and

$$\underset{1}{\overset{i}{\Pr}} = s_1 / \left[\alpha\left(1 - x_H(p_0(s_1);k)\right) + (1-\alpha)\left(1 - x_L(p_0(s_1);k)\right)\right]. \quad (45)$$

Simplifying $D^p(s_j)$ is computationally more burdensome but it leads to the same conclusion that $\forall j = 0, 1$, $D^p(s_j) = D(0)$.

A.6 The effects of screening public housing applicants

A.6.1 Constraints

The constraints under which Equation (8) is solved are the following:

$$n = \sum_{i=H,L} \alpha_i x_i\left(E^i\tilde{p}_0\left(s_j, \phi_j^i\right); E^i\tilde{p}_1\left(s_j, \phi_j^i\right)\right) \quad (46)$$

$$n_j = \sum_{i=H,L} \phi_j^i \alpha_i \left| x_i\left(E^i\tilde{p}_0\left(s_j, \phi_j^i\right); E^i\tilde{p}_1\left(s_j, \phi_j^i\right)\right) - j \right| \quad (47)$$

A.6.2 The trade-off between screening and program size

In this section, I prove proposition 7 on the trade-off between the level of screening in the allocation of public housing and the size of the program. In analytical terms, this proposition derives from the following inequalities and equivalence relationships:

$$\partial\phi_j^i\left(\phi_j^{-i}\right) / \partial\phi_j^{-i} \leq 0, \quad (48)$$

$$\phi_j^i \leq 1 \Leftrightarrow \phi_j^{-i}\left(\phi_j^i\right) \geq 1, \quad (49)$$

$$\left[\partial s_j\left(\phi_j^i\right) / \partial\phi_j^i \geq 0 \text{ and } \phi_j^i \leq 1\right] \Leftrightarrow \left[\partial s_j\left(\phi_j^i\right) / \partial\phi_j^{-i}\left(\phi_j^i\right) \leq 0 \text{ and } \phi_j^{-i}\left(\phi_j^i\right) \geq 1\right], \quad (50)$$

I focus on the case where public housing is introduced in the better neighborhood since the computations would be almost identical in the other case. I show that $\partial s_0\left(\phi_0^H\right) / \partial\phi_0^H \geq 0$ and conversely, that $\partial s_0\left(\phi_0^H\right) / \partial\phi_0^L \leq 0$. For this purpose, I first solve for equilibrium private market prices, as a function of $\boldsymbol{\phi}_0$. I find:

$$p_1(s_0, \boldsymbol{\phi}_0) = \frac{\hat{\beta}cn - (c - kn) s_0\left(\alpha\beta^H\phi_0^H + (1-\alpha)\beta^L\phi_0^L\right) - n^2\left[\Delta q + t(1-2n)\right]}{\hat{\beta}n - (1-n) s_0\left(\alpha\beta^H\phi_0^H + (1-\alpha)\beta^L\phi_0^L\right)\alpha\beta^H} \quad (51)$$

$$p_0\left(s_0,\boldsymbol{\phi}_0\right)=p_1^a\left(s_0,\boldsymbol{\phi}_0\right)+\frac{n\left[\Delta q+t\left(1-2n\right)\right]+\left(c-k\right)s_0\left[\alpha\beta^H\phi_0^H+\left(1-\alpha\right)\beta^L\phi_0^L\right]}{\hat{\beta}n-\left(1-n\right)s_0\left(\alpha\beta^H\phi_0^H+\left(1-\alpha\right)\beta^L\phi_0^L\right)\alpha\beta^H} \tag{52}$$

Then, I consider the system formed by equations (47) and (8). For clarity purposes, I introduce a variable ξ_0 which represents by how much type-H tenants will be over-represented in neighborhood 0 and I replace equation (8) by the following:

$$x_H\left(E^H\tilde{p}_0^a\left(s_0,\boldsymbol{\phi}_0\right);E^H\tilde{p}_1\left(s_0,\boldsymbol{\phi}_0\right)\right)=\left(1+\xi_0\right)n \tag{53}$$

The problem is equivalent to the previous one as soon as $\xi_0=\left(1-\delta\right)D\left(0\right)/\left(2n\alpha\right)$. Restrictions on ξ_0 are the following: first, I assume that the policy will never lead to type-H being under-represented in neighborhood 0; in other words, $\xi_0\geq 0$. Second, the number of type-H ready to enter neighborhood 0 must be lower than the total number of spots in neighborhood 0:

$$\alpha x_H\left(E^H\tilde{p}_0\left(s_0,\boldsymbol{\phi}_0\right);E^H\tilde{p}_1\left(s_0,\boldsymbol{\phi}_0\right)\right)\leq n\Longleftrightarrow\xi_0\leq\left(1-\alpha\right)/\alpha \tag{54}$$

Finally, the policy is supposed to reduce segregation. This will be the case as soon as

$$\xi_0\leq\frac{D\left(0\right)}{2n\alpha}=\left(1-\alpha\right)\left(\beta^L-\beta^H\right)\left[\Delta q+t\left(1-2n\right)\right]/\left(2\hat{\beta}nt\right) \tag{55}$$

I solve the system (47),(53)) in $\phi_0^L\left(\phi_0^H\right)$ and $s_0\left(\phi_0^H\right)$. This yields the following solutions:

$$\phi_0^L\left(\phi_0^H\right) = 1+\frac{\alpha\left(1+\xi_0\right)\left(1-\phi_0^H\right)}{1-\alpha\left(1+\xi_0\right)} \tag{56}$$

$$s_0\left(\phi_0^H\right) = \frac{A_0}{B_0-C_0\phi_0^H} \tag{57}$$

with:

$$A_0=n\left[1-\alpha\left(1+\xi_0\right)\right]\left\{\left(1-\alpha\right)\left(\beta^L-\beta^H\right)\left[\Delta q+t\left(1-2n\right)\right]-2\hat{\beta}nt\xi_0\right\} \tag{58}$$

$$B_0=\left(1-\alpha\right)\beta^L\left\{\beta^H\left(c-k\right)+\left(1-n\right)\left[\Delta q+t\left(1-2n\left(1+\xi_0\right)\right)\right]\right\} \tag{59}$$

$$\begin{aligned}C_0 &= \left(1-n\right)\left(1+\xi_0\right)\alpha\left\{\left(1-\alpha\right)\left(\beta^L-\beta^H\right)\left[\Delta q+t\left(1-2n\right)\right]-2\hat{\beta}nt\xi_0\right\}\\ &\quad+\beta^H\left(1-n\right)\left\{\left(1-\alpha\right)\left[\Delta q+t\left(1-2n\right)\right]+\alpha\left(2\hat{\beta}nt\xi_0\right)\right\}+\beta^H\beta^L\left(1-\alpha\right)\left(c-k\right)\end{aligned} \tag{60}$$

Expression 48 is established by Equation (56) under condition (54). As far as Expression 49, given conditions (54) and (55), it is straightforward to verify that $A_0\geq 0$, $B_0>0$ and $C_0>0$. Consequently, we have $\partial s_0\left(\phi_0^H\right)/\partial\phi_0^H=A_0C_0/\left(B_0-C_0\phi_0^H\right)^2\geq 0$. Conversely, $\partial s_0\left(\phi_0^H\right)/\partial\phi_0^L\left(\phi_0^H\right)\leq 0$. One can check that $s_0\left(\phi_0^H\right)$ is well-defined for all possible values of ϕ_0^H, since $B_0-C_0=\{(1-\alpha)(\beta^L-\beta^H)[\Delta q+t(1-2n)]-2\hat{\beta}nt\xi_0\}(1-n)[1-\alpha(1+\xi_0)]>0$. However, $A_0/\left(B_0-C_0\right)=\frac{n}{(1-n)}>1$, which is impossible. The minimum level of screening is defined by $\phi_0^{H\max}$ such that $s_0\left(\phi_0^{H\max}\right)=n$. This will be the case when $\phi_0^{H\max}=1-A_0/C_0$. This proves Expression 50.

A.7 The planner's decision

The cost $c_0\left(\phi_0^H\right) = s_0\left(\phi_0^H\right)\left[p_0\left(s_0\left(\phi_0^H\right)\right) - k\right]$ of the public housing program in neighborhood 0 simplifies to:

$$c_0\left(\phi_0^H\right) = \frac{A_0}{(1-\alpha)\left(1-\phi_0^H\right)\beta^L\beta^H} \tag{61}$$

and it does not depend on k. The cost of the public housing program in neighborhood 1 is obtained similarly, this time using ϕ_1^L as a parameter, with $\xi_1 = \frac{(1-\delta)D(0)}{2(1-n)(1-\alpha)} \leq \frac{\alpha}{1-\alpha}$ and

$$A_1 = (1-n)\left[\alpha - (1-\alpha)\xi_0\right]\left\{\alpha\left(\beta^L - \beta^H\right)\left[\Delta q + t\left(1-2n\right)\right] - 2\hat{\beta}\left(1-n\right)t\xi_1\right\} \tag{62}$$

This yields

$$c_1\left(\phi_1^L\right) = \frac{A_1}{\alpha\left(1-\phi_1^L\right)\beta^L\beta^H} \tag{63}$$

If I consider that the same level of screening must be enforced against type-H applicants in neighborhood 0 and against type-L applicants in neighborhood 1, I can therefore directly compare these two costs, as a function of $\phi = \phi_0^H = \phi_1^L$. Straightforward computations lead to the following expression of the cost differential:

$$c_0\left(\phi\right) - c_1\left(\phi\right) = \frac{\delta\left(\beta^L - \beta^H\right)\left[\Delta q + t\left(1-2n\right)\right]}{\left(1-\phi\right)\beta^L\beta^H}\left(n-\alpha\right) \tag{64}$$

which proves Proposition 7.

To search for $\delta^*\left(G\right)$, I then solve equation $c_{j^*}\left(\phi\right) = G$ in δ. This yields two solutions

$$\delta^{\inf}\left(G\right) = \frac{\Phi_{j^*} - \Omega - \sqrt{\left(\Omega - \Phi_{j^*}\right)^2 + 4\Omega G}}{2\Omega} \tag{65}$$

$$\delta^{\sup}\left(G\right) = \frac{\Phi_{j^*} - \Omega + \sqrt{\left(\Omega - \Phi_{j^*}\right)^2 + 4\Omega G}}{2\Omega} \tag{66}$$

with

$$\Phi_0 = \frac{n\hat{\beta}D\left(0\right)t}{\alpha\left(1-\phi\right)\beta^L\beta^H} \tag{67}$$

$$\Phi_1 = \frac{\left(1-n\right)\hat{\beta}D\left(0\right)t}{\left(1-\alpha\right)\left(1-\phi\right)\beta^L\beta^H} \tag{68}$$

$$\Omega = \frac{\hat{\beta}D\left(0\right)t}{2\alpha\left(1-\alpha\right)\left(1-\phi\right)\beta^L\beta^H} \tag{69}$$

Since $\Phi_0 > 0$, $\Phi_1 > 0$ and $\Omega > 0$, the solution $\delta^{\inf}\left(G\right)$ is negative and can be ruled out. The other solution $\delta^{\sup}$ is positive. A necessary and sufficient condition on G for $\delta^{\sup}\left(G\right) < 1$ is $G \leq G^{\max} = 2\Omega - \Phi_{j^*}$. If $G > G^{\max}$, $\delta^*\left(G\right) = 1$.

B Analytical expressions from Section 3

B.1 Private market Equilibrium

The expressions for the market prices under endogenous neighborhood quality are the following:

$$p_1^{PE}(0) = c - \frac{2(1-n)n^2t[\Delta q + t(1-2n)]}{2\hat{\beta}(1-n)nt - \eta\alpha(1-\alpha)(\beta^L - \beta^H)} \tag{70}$$

$$p_0^{PE}(0) = c + \frac{2(1-n)^2nt[\Delta q + t(1-2n)]}{2\hat{\beta}(1-n)nt - \eta\alpha(1-\alpha)(\beta^L - \beta^H)} \tag{71}$$

The expressions for η_α and $\eta_{1-\alpha}$ verify:

$$\eta_\alpha = \eta_0 - 2n(1-n)[\Delta q + t(1-2n)]/\alpha \tag{72}$$

$$\eta_{1-\alpha} = \eta_0 - 2n(1-n)[\Delta q + t(1-2n)]/(1-\alpha) \tag{73}$$

B.2 Equilibrium with no screening of applicants

The expressions for the market prices under endogenous neighborhood quality when public housing is allocated at random verify:

$$\Delta p_j^{PE}(s_j) = \frac{s_j n_{-j}}{n_j - s_j n_{-j}}\left[p_j^{PE}(0) - k\right] \tag{74}$$

$$\Delta p_{-j}^{PE}(s_j) = -\frac{s_j n_j}{n_j - s_j n_{-j}}\left[p_j^{PE}(0) - k\right] \tag{75}$$

B.3 The snowball effects of screening

I provide here the expressions for public housing in neighborhood 0, but similar expressions could be given for public housing in neighborhood 1 as well. I solve a system similar to ((47),(53)) in $\phi_0^{L^{PE}}\left(\phi_0^H\right)$ and $s_0^{PE}\left(\phi_0^H\right)$. This yields the following solutions:

$$\phi_0^{L^{PE}}\left(\phi_0^H\right) = 1 + \frac{\alpha\left(1+\xi_0^{PE}\right)\left(1-\phi_0^H\right)}{1-\alpha\left(1+\xi_0^{PE}\right)} \tag{76}$$

$$s_0^{PE}\left(\phi_0^H\right) = \frac{A_0^{PE}}{B_0^{PE} - C_0^{PE}\phi_0^H} \tag{77}$$

with

$$A_0^{PE} = n\left[1-\alpha\left(1+\xi_0^{PE}\right)\right]\left\{\begin{array}{c}(1-\alpha)\left(\beta^L-\beta^H\right)\{(1-n)[\Delta q + t(1-2n)]\} \\ +\left\{\alpha(1-\alpha)\left(\beta^L-\beta^H\right)\eta - 2\hat{\beta}n(1-n)t\right\}\xi_0^{PE}\end{array}\right\} \tag{78}$$

$$B_0^{PE} = (1-\alpha)\beta^L(1-n)\left\{\begin{array}{c}\beta^H(c-k) + (1-n)[\Delta q + t(1-2n)] \\ +\{\alpha\eta - 2n(1-n)t\}\xi_0^{PE}\end{array}\right\} \tag{79}$$

$$C_0^{PE} = (1-n)\left\{ \begin{array}{c} \beta^H \left[\begin{array}{c} \beta^L (1-\alpha)(c-k) \\ +\left[1-\alpha\left(1+\xi_0^{PE}\right)\right] \left[\begin{array}{c} (1-\alpha)(1-n)\left[\Delta q + t(1-2n)\right] \\ +\alpha\left[(1-\alpha)\eta + 2n(1-n)t\xi_0^{PE}\right] \end{array} \right] \end{array} \right] \\ +\alpha(1-\alpha)\beta^L\left(1+\xi_0^{PE}\right) \left\{ \begin{array}{c} (1-n)\left[\Delta q + t(1-2n)\right] \\ +\left\{\alpha\eta - 2n(1-n)t\right\}\xi_0^{PE} \end{array} \right\} \end{array} \right\} \tag{80}$$

Once again, it is straightforward to verify that $A_0^{PE} \geq 0$, $B_0^{PE} > 0$, $C_0^{PE} > 0$, hence, $\partial s_0^{PE}(\phi_0^H) / \partial \phi_0^H \geq 0$. In addition, since $\xi_0^{PE} < D^{PE}(0) / (2n\alpha)$, we have $\forall \phi_0^H \in [0,1]$, $B_0^{PE} - C_0^{PE}\phi_0^H > 0$ and $s_0^{PE}\left(\phi_0^H\right)$ is well-defined for all values of ϕ_0^H. The snowball effects will be established if $\forall \phi_0^H \in [0,1]$, $s_0^{PE}\left(\phi_0^H\right) \leq s_0\left(\phi_0^H\right)$. I proceed as follows: I first use that $s_0^{PE}(1) = s_0(1)$. Indeed,

$$s_0^{PE}(1) = s_0(1) = \frac{n}{1-n} \tag{81}$$

This equality leads to

$$A_0^{PE}B_0 - A_0B_0^{PE} = A_0^{PE}C_0 - A_0C_0^{PE} \tag{82}$$

I then show that $\forall \eta \in [0, \eta_1(0)]$,

$$A_0^{PE}B_0 - A_0B_0^{PE} \leq 0 \tag{83}$$

Indeed, we have

$$A_0^{PE}B_0 - A_0B_0^{PE} = -\frac{\Theta_0}{\Xi_0} \tag{84}$$

with

$$\begin{aligned} \Theta_0 \quad = \quad & \eta(1-\alpha)^4\alpha\left(\beta^L - \beta^H\right)^2\beta^L(1-\delta)\delta(1-n)\left[\Delta q + t(1-2n)\right]^2 \\ & \times\left\{2\beta^H(1-n)nt + \alpha\left(\beta^L - \beta^H\right)\left[\beta^H(c-k) + \delta(1-n)\left[\Delta q + t(1-2n)\right]\right]\right\} \end{aligned} \tag{85}$$

which is always positive, and

$$\Xi_0 = 2\hat{\beta}t\left[2\hat{\beta}n(1-n)t - \eta\alpha(1-\alpha)\left(\beta^L - \beta^H\right)\right] \tag{86}$$

which is positive as soon as $\eta \leq \eta(0)$. This inequality leads to

$$A_0^{PE}B_0 - A_0B_0^{PE} < \phi_0^H\left(A_0^{PE}C_0 - A_0C_0^{PE}\right) \tag{87}$$

hence, with $B_0 - C_0 > 0$ and $B_0^{PE} - C_0^{PE} > 0$, we get that:

$$s_0^{PE}\left(\phi_0^H\right) \leq s_0\left(\phi_0^H\right) \tag{88}$$

B.4 Neighborhood choice

The cost differential under screening and endogenous neighborhood valuation is such that:

$$\begin{aligned}
\Delta c_{01}(\phi,\eta) \;>\; & 0 \Leftrightarrow \\
& \alpha\{2\hat{\beta}(1-n)nt - \eta\alpha(1-\alpha)\left(\beta^L-\beta^H\right)\} \\
& \times\{2\hat{\beta}(1-n)nt\delta + \eta\alpha(1-\alpha)\left(\beta^L-\beta^H\right)(1-\delta)\} \\
& \times\left\{\begin{array}{c}\alpha(1-\alpha)\left(\beta^L-\beta^H\right)(1-\delta)\Delta q \\ -t\{\hat{\beta}[1+\delta(1-2n)]+\beta^H(1-\delta)(1-2n)\}\end{array}\right\} \\
\;>\; & 4(1-\alpha)\hat{\beta}^2\delta(1-n)n^2t^2 \\
& \times\left\{\begin{array}{c}\alpha\left(\beta^L-\beta^H\right)[\eta(1-a)\;+(1-\delta)\Delta q(1-n)] \\ +(1-n)t\left[2\beta^L n\;-\alpha\left(\beta^L-\beta^H\right)(1-\delta(1-2n))\right]\end{array}\right\}.
\end{aligned} \tag{89}$$

C Additional figures

Figure 5: Optimal share of public housing in neighborhood 1 as a function of the level of acceptance of type-L applicants

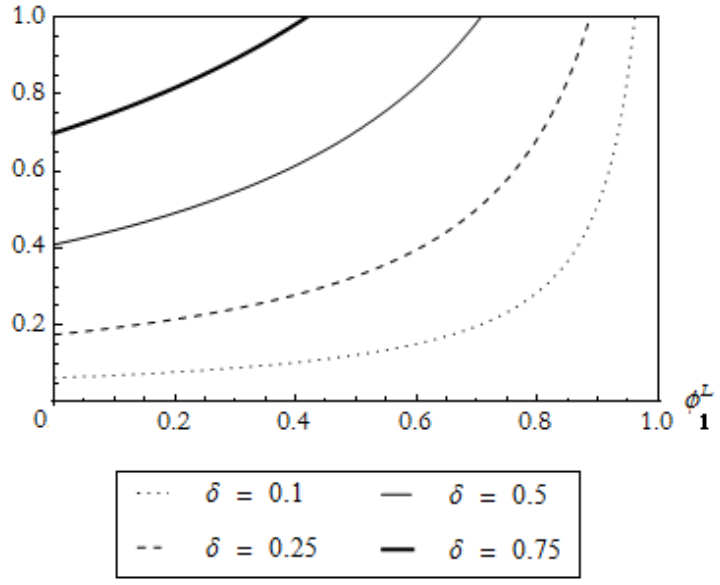


Figure 6: Optimal level of segregation when public housing funded in neighborhood 1 as a function of the level of acceptance of type-L applicants

