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Kazuo Nishimura
Carine Nourry
Thomas Seegmuller
Alain Venditti

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Kazuo NISHIMURA

Institute of Economic Research, Kyoto University

Carine NOURRY

Aix-Marseille University (Aix-Marseille School of Economics), CNRS-GREQAM, EHESS & Institut Universitaire de France

Thomas SEEGMULLER

Aix-Marseille University (Aix-Marseille School of Economics), CNRS-GREQAM & EHESS

and

Alain VENDITTI

Aix-Marseille University (Aix-Marseille School of Economics), CNRS-GREQAM, EHESS & EDHEC

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Abstract: *We examine the impact of balanced-budget consumption taxes on the existence of expectations-driven business cycles in two-sector economies with infinitely-lived households. We prove that, whatever the relative capital intensity difference across sectors, aggregate instability can occur if the consumption tax rate is not too low. Moreover, we show through a numerical exercise based on empirically plausible tax rates that endogenous business-cycle fluctuations may be a source of instability for all OECD countries, including the US.*

Keywords: *Aggregate instability, indeterminacy, expectations-driven fluctuations, consumption taxes, balanced-budget rule, infinite-horizon two-sector model*

Journal of Economic Literature Classification Numbers: C62, E32, H20, O41.

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1 Introduction

The last financial crisis and the large increase of public debt in the US and within EMU countries, that have been observed during the last four years, have generated a revival of the debate on balanced-budget fiscal policies. The adoption of a "golden rule" is now seen as a solution to ensure government debt sustainability and to prevent macroeconomic instability of the kind observed recently in most OECD countries.

But, as shown by Schmitt-Grohé and Uribe [21], balanced budgets may on the contrary lead to indeterminacy, and thus inducing beliefs-driven aggregate fluctuations. Using labor income taxes under a balanced-budget rule, Schmitt-Grohé and Uribe have proved that locally indeterminate equilibria arise for empirically plausible tax rates. Such strong conclusions have been weakened by Giannitsarou [11] who considers instead consumption taxes still under a balanced-budget rule. She shows that contrary to labor income taxes, local indeterminacy cannot occur. She then suggests that consumption taxation must be preferred to labor income taxation if the government has a stabilization target for public policies.[1] However, the main limitation of this result is that Giannitsarou [11] assumes log-linear preferences in consumption. Using instead a Greenwood-Hercovitz-Huffman [10] (GHH) utility function characterized by no-income effect, Nourry *et al.* [19] prove that expectations-driven fluctuations may still occur under large enough consumption tax rates which are empirically plausible for European countries provided government spendings are sufficiently counter-cyclical.[2]

The basic intuition for this result is the following: If households expect a higher future consumption tax rate, they will reduce their future labor supply through a substitution effect. But, households also expect to decrease their future consumption which increases labor at equilibrium through an income effect. If this last effect is dominated by the first one, which is obviously satisfied with GHH preferences, a higher expected consumption tax rate implies a lower labor supply which reduces the real interest rate and has a negative effect on current investment. Needing less income today, households work less and decrease their consumption. This generates a beliefs-driven increase of the current consumption tax rate in order to match the balanced-budget rule.

The existence of local indeterminacy in an aggregate model is then based on the effect of consumption taxes on the trade-off between consumption and leisure. In two-sector models, there is, beside this one, an additional mechanism that relies on the impact of the tax rate on the relative price between consumption and investment goods.[3] The basic intuition is the following: The expectation of an increase in the consumption tax rate yields a lower level of production of goods as a response to the decrease of private consumption spendings. Then the capital invested in the current period mechanically decreases, and so does the labor supply. As the agents income is lower, they consume less, and the current consumption tax increases to match the balanced budget rule.

The aim of this paper is to show that this new mechanism alone is sufficient to generate local indeterminacy even in cases in which it is ruled out within aggregate models. We will then prove

[1] See also Saïdi [20] who shows that even with labor income taxes, determinacy of the equilibrium trajectory is generically preserved for a non-empty range of initial values of the capital stock.

[2] Lane [14] shows that most OECD countries are characterized by counter-cyclical government spendings.

[3] See also Yano [25] and Bond *et al.* [7] for recent analysis of two-sector optimal growth models in a closed economy or with trade.

that considering a two-sector economy allows to extend the plausibility of expectations-driven fluctuations based on consumption taxation.

This claim is based on the conclusions derived in the literature dealing with local indeterminacy generated by external effects in production. As initially established by Benhabib and Farmer [2], in one-sector models local indeterminacy requires strongly unrealistic increasing social returns to scale based on large aggregate capital and labor externalities. On the contrary, as shown in Benhabib and Farmer [3] or Benhabib and Nishimura [5], in two-sector models with mild sector-specific externalities, local indeterminacy becomes compatible with weakly increasing or even constant social returns. Capital and, more importantly, labor allocations across sectors, that are generated through Rybczynski and Stolper-Samuelson effects, allow indeed to get locally indeterminate equilibria under plausible parameterizations.

In this paper, we follow Benhabib and Nishimura [5] and consider a two-sector economy with Cobb-Douglas sectoral technolgies. We assume that the households' preferences are characterized by some utility function homogeneous of degree one. This specification is again justified by recent conclusions on the existence of indeterminacy in two-sector models with externalities. Nishimura and Venditti [18] prove indeed that homogenous preferences provide large degrees of freedom to get sunspot fluctuations under realistic parameters' values. As in Nourry *et al.* [19] we finally assume balanced-budget consumption taxes with government spendings that can be counter-cyclical, constant or weakly pro-cyclical.

We show that expectations-driven fluctuations can arise for any capital intensity difference across sectors provided the consumption tax rate is not too low and the wage elasticity of the labor supply is low enough. Moreover, the lower bound of the tax rate is decreasing with the degree of counter-cyclicality of government spendings. We also provide a numerical illustration based on a CES specification of preferences. Under a standard parameterization of the model compatible with quarterly data, we show that the lower bound of the consumption tax rate above which local indeterminacy arises is less than 5%. These results show that consumption taxation in two-sector models may explain business-cycle fluctuations based on beliefs in all OECD countries, including the US.

The rest of this paper is organized as follows. We present the model in the next section. In Section 3, we discuss the existence of a normalized steady state. In Section 4, we provide our main conditions for the existence of aggregate instability and endogenous sunspot fluctuations under linear homogenous preferences, with a special emphasis on the CES specification. Section 5 gives economic interpretations of our results. Some concluding remarks are provided in Section 6, and all the proofs are gathered in a final Appendix.

2 The model

2.1 The production structure

We consider an economy producing a consumption good y_0 and a capital good y_1. Each good is produced by capital k_j and labor l_j, $j = 0, 1$, through a Cobb-Douglas production function. The representative firm in each industry indeed faces the following technology:

$$y_0 = k_0^{\alpha} l_0^{1-\alpha}, \quad y_1 = k_1^{\beta} l_1^{1-\beta} \tag{1}$$

with $\alpha, \beta \in (0,1)$. Each firm maximizes its profit given the price of the consumption good p_0, the price of the investment (capital) good p_1, the rental rate of capital w_1 and the wage rate w_0. The first order conditions subject to the private technologies (1) give

$$\begin{aligned} k_0/y_0 = p_0\alpha/w_1 \equiv a_{10}(w_1, p_0), \quad l_0/y_0 = p_0(1-\alpha)/w_0 \equiv a_{00}(w_0, p_0) \\ k_1/y_1 = p_1\beta/w_1 \equiv a_{11}(w_1, p_1), \quad l_1/y_1 = p_1(1-\beta)/w_0 \equiv a_{01}(w_0, p_1) \end{aligned} \tag{2}$$

We call a_{ij} the input coefficients.

Considering that total labor is given by $\ell = l_0 + l_1$, and the total stock of capital is given by $k = k_0 + k_1$, the factor market clearing equation is directly obtained from the input coefficients as defined by (2). Denoting $x = (\ell, k)'$, $y = (y_0, y_1)'$ and $A(w,p) = [a_{ij}(w_i, p_j)]$, we get

$$A(w,p)y = x \tag{3}$$

From (2), substituting the expressions of (k_j, l_j), $j = 0, 1$, into the production functions (1) and solving with respect to p_j gives the factor-price frontier, which provides a relationship between input prices and output prices. Denoting $p = (p_0, p_1)'$ and $w = (w_0, w_1)'$, we get

$$p = A'(w,p)w \tag{4}$$

From now on, we choose the consumption good as the numeraire, i.e. $p_0 = 1$. It follows from (3) and (4) that at the equilibrium, the wage rate and the rental rate are functions of the price of the capital good only, i.e. $w_i = w_i(p_1)$, $i = 0, 1$, while outputs are functions of the capital stock, total labor and the price of the capital good, $y_j = \tilde{y}_j(k, \ell, p_1)$, $j = 0, 1$.

Profit maximization in both sectors gives demands for capital and labor as linear homogeneous functions of the capital stock, the output of the investment good and total labor, namely $\tilde{k}_j = k_j(k, y_1, \ell)$, $\tilde{l}_j = l_j(k, y_1, \ell)$ $j = 0, 1$. The production frontier is then

$$y_0 = T(k, y_1, \ell) = \tilde{k}_0^\alpha \tilde{l}_0^{1-\alpha} \tag{5}$$

Note that $T(k, y_1, \ell)$ is a linear homogeneous function. From the envelope theorem we get $w_1 = T_1(x_1, y_1, \ell)$, $p_1 = -T_2(x_1, y_1, \ell)$ and $w_0 = T_3(x_1, y_1, \ell)$.

2.2 Government

In our simple neoclassical economy, the only source of government revenue is a consumption tax supported by the households, government purchases are variable, the initial shock of public debt is zero, and the government is subject to a balanced-budget requirement. The endogenous level of public spending $\mathcal{G}(t)$ satisfies the following resource constraint

$$c(t) + \mathcal{G}(t) = y_0(t) = T(k(t), y_1(t), \ell(t)) \tag{6}$$

Given the level of government income $\Omega\left(c(t)\right)$ derived from the tax function $\tau(t)$, the government expenditures $\mathcal{G}(t)$ also satisfy the following balanced-budget rule:

$$\mathcal{G}(t) = \Omega\left(c(t)\right) = \tau(t)c(t) \tag{7}$$

In the following we will consider a sufficiently general formulation of public expenditures $\mathcal{G}(t)$ that encompasses the case of Giannitsarou [11], in which the government faces an exogenous stream of constant expenditure $\mathcal{G}(t) = \mathcal{G}$ that is financed by levying proportional taxes on consumption. Our formulation allows indeed for counter or pro-cyclical government spendings. We choose consumption c as a proxy of the measure of the business cycle and we assume that public spending is a function of c, namely $\mathcal{G}(t) \equiv \mathcal{G}(c(t))$.A word of caution is necessary here. Usually government spendings are expressed as a function of output. But since consumption is almost perfectly correlated with output, our formulation is not restrictive. Using our notations we get

$$\mathcal{G}(c(t)) = \Omega(c(t)) = \tau(t)c(t) \;\Leftrightarrow\; \tau(t) = \tau(c(t)) = \mathcal{G}(c(t))/c(t) \tag{8}$$

This expression endogenously determines the level of the consumption tax rate. The tax function is assumed to have the following properties:

Assumption 1. $\tau(c)$: $[0, +\infty) \to [0, +\infty)$ *is continuous, and* C^1 *on* $(0, +\infty)$.

Let us then denote:

$$\eta \equiv \mathcal{G}'(c)c/\mathcal{G}(c)$$

We will say that public expenditures are counter(pro)-cyclical when $\eta < 0$ ($\eta > 0$). The Giannitsarou's case of constant government spending is obtained when $\eta = 0$. Using the balanced-budget rule, let us introduce the following elasticity:

$$\zeta \equiv \frac{\tau'(c)c}{1+\tau(c)} = \frac{\tau(\eta-1)}{1+\tau} \tag{9}$$

The tax rate $\tau(c) = \mathcal{G}(c)/c$ is therefore counter-cyclical for $\zeta < 0$, or $\eta < 1$, pro-cyclical for $\zeta > 0$, or $\eta > 1$, and constant for $\zeta = 0$, or $\eta = 1$. In the following, the most important parameter will be the elasticity of the tax rate ζ. The elasticity of government spendings η will be used only as a way to increase or decrease the value of ζ. Note that even when government spendings are pro-cyclical with $\eta \in (0, 1)$, the consumption tax rate remains counter-cyclical with $\zeta < 0$. We further assume that the public expenditures $\mathcal{G}(t)$ neither affect the consumers' preferences nor the production function. In the following, we will mainly focus on a counter-cyclical consumption tax rate since, as shown by Lane [14], government spendings in most OECD countries are counter-cyclical ($\eta < 0$).

2.3 Households' behavior

The economy is populated by a large number of identical infinitely-lived agents. We assume without loss of generality that the total population is constant and normalized to one. At each period a representative agent supplies elastically an amount of labor $\ell \in (0, \bar{\ell})$, with $\bar{\ell} > 1$ his time endowment. He derives utility from consumption c and leisure $\mathcal{L} = \bar{\ell} - \ell$ according to a function $U(c, \mathcal{L}/B)$, where $B > 0$ is a scaling parameter,[4] which satisfies:

Assumption 2. $U(c, \mathcal{L}/B)$ *is* $\mathbf{C}^r$ *over* $\mathbb{R}_{++} \times (0, \bar{\ell})$ *for* $r \geq 2$, *increasing in each argument, concave and homogeneous of degree* 1. *Moreover, for all* $(c, X) \in \mathbb{R}^2_{++}$, $\frac{U_{12}(c,X)X}{U_1(c,X)} - \frac{U_{22}(c,X)X}{U_2(c,X)} \neq 1$, $\lim_{X\to 0} U_2(c, X)/U_1(c, X) = +\infty$ *and* $\lim_{X\to+\infty} U_2(c, X)/U_1(c, X) = 0$.

Consumption and leisure are then normal goods. Building on the homogeneity of degree 1, we introduce the share of consumption within total utility $\theta(c, \mathcal{L}/B) \in (0, 1)$ defined as follows:[5]

$$\theta(c, \mathcal{L}/B) = \frac{U_1(c,\mathcal{L}/B)c}{U(c,\mathcal{L}/B)} \tag{10}$$

[4]The constant B is used to prove the existence of a normalized steady state which remains invariant with respect to preference parameters such that the elasticity of intertemporal substitution in consumption or the wage elasticity of the labor supply.

[5]The share of leisure within total utility is similarly defined as $1 - \theta(c, \mathcal{L}/B) \in (0, 1)$.

the elasticity of substitution between consumption and leisure

$$\phi(c, \mathcal{L}/B) = \frac{\frac{U_2(cB/\mathcal{L},1)/U_1(cB/\mathcal{L},1)}{cB/\mathcal{L}}}{\frac{\partial(U_2(cB/\mathcal{L},1)/U_1(cB/\mathcal{L},1))}{\partial(cB/\mathcal{L})}} \tag{11}$$

and the elasticity of intertemporal substitution in consumption

$$\epsilon_{cc} = -\frac{U_1(c,\mathcal{L}/B)}{U_{11}(c,\mathcal{L}/B)c} \tag{12}$$

that will be useful in the rest of the analysis. It is worth noting that ϕ is linked with the elasticity ϵ_{cc} through the share θ: $\phi = (1-\theta)\epsilon_{cc}$.

A possible illustration of such an utility function is provided by the following CES formulation

$$U(c, \mathcal{L}/B) = \left[\nu c^{-\psi} + (1-\nu)(\mathcal{L}/B)^{-\psi}\right]^{-1/\psi} \tag{13}$$

with $\nu \in (0,1)$ and $\psi > -1$. In this case we have

$$\theta(c, \mathcal{L}/B) = \frac{\nu}{\nu+(1-\nu)(Bc/\mathcal{L})^{\psi}} \tag{14}$$

and the elasticity of substitution between consumption and leisure is constant and given by $\phi = 1/(1+\psi)$.

We assume that the representative household considers as given the tax rate on consumption. The intertemporal optimization problem of the representative household is given by:

$$\begin{aligned} \max_{\{c(t), y_1(t), \ell(t)\}} \quad & \int_0^{+\infty} U(c(t), (\bar{\ell}-\ell(t))/B) e^{-\rho t} dt \\ s.t. \quad & (1+\tau(\bar{c}(t)))c(t) = T(k(t), y_1(t), \ell(t)) \\ & \dot{k}(t) = y_1(t) - \delta k(t) \\ & k(0) \textit{ and } \tau(\bar{c}(t)) \textit{ given} \end{aligned} \tag{15}$$

where $\rho \geq 0$ is the discount rate and $\delta > 0$ the depreciation rate of the capital stock.

2.4 Intertemporal equilibrium

The modified Hamiltonian in current value is given by:

$$\begin{aligned} \mathcal{H} \; = \; & U(c(t), (\bar{\ell}-\ell(t))/B) + \lambda(t)\left[T(k(t), y_1(t), \ell(t)) - (1+\tau(\bar{c}(t)))c(t)\right] \\ + \; & q_1(t)\left[y_1(t) - \delta k(t)\right] \end{aligned}$$

with $q_1(t)$ the co-state variable which corresponds to the utility price of the capital good in current value and $\lambda(t)$ the Lagrange multiplier associated with the Government budget constraint. The first order conditions of problem (15) are given by the following equations:

$$U_1(c(t), (\bar{\ell}-\ell(t))/B) = \lambda(t)(1+\tau(c(t))) \tag{16}$$

$$q_1(t) = p_1(t)\lambda(t) \tag{17}$$

$$U_2(c(t), (\bar{\ell}-\ell(t))/B)/B = w_0\lambda(t) \tag{18}$$

$$\dot{k}(t) = y_1(t) - \delta k(t) \tag{19}$$

$$\dot{q}_1(t) = (\delta+\rho)q_1(t) - w_1(t)\lambda(t) \tag{20}$$

As shown in Section 2.1, we have $w_i = w_i(p_1)$, $i = 0,1$, $y_1 = \tilde{y}_1(k, \ell, p_1)$ and $y_0 = \tilde{y}_0(k, \ell, p_1) = T(k, \tilde{y}_1(k, \ell, p_1), \ell)$. Therefore, solving equation (18) describing the labor-leisure trade-off at the equilibrium, we express the labor supply as a function of the capital stock and the output price, $\ell = \ell(k, p_1)$. Then, we get $y_0 = y_0(k, p_1) \equiv \tilde{y}_0(k, \ell(k, p_1), p_1)$, $y_1 = y_1(k, p_1) \equiv \tilde{y}_1(k, \ell(k, p_1), p_1)$

and thus $c(1+\tau(c)) = y_0(k,p_1)$. This last equation allows therefore to express consumption as $c = c(k,p_1)$.

Let us introduce the following additional elasticities:[6]

$$\epsilon_{\ell c} \equiv -\frac{U_2(c,\mathcal{L}/B)}{U_{21}(c,\mathcal{L}/B)c} = -\frac{(1-\theta)\epsilon_{cc}}{\theta}, \ \epsilon_{c\ell} \equiv -\frac{U_1(c,\mathcal{L}/B)B}{U_{12}(c,\mathcal{L}/B)\ell} = \Phi\epsilon_{\ell c} = -\frac{(1-\theta)\Phi\epsilon_{cc}}{\theta},$$
$$\epsilon_{\ell\ell} \equiv -\frac{U_2(c,\mathcal{L}/B)B}{U_{22}(c,\mathcal{L}/B)\ell} = \frac{(1-\theta)^2\Phi\epsilon_{cc}}{\theta^2} \tag{21}$$

with $\Phi = [\delta(1-\beta)+\rho]/[\delta(1-\beta)+\rho(1-\alpha)]$. Note that $\epsilon_{\ell\ell}$ provides the wage elasticity of the labor supply.

Considering (16)-(20), the equations of motion are finally derived as

$$\dot{k} = y_1(k,p_1) - \delta k$$
$$\dot{p}_1 = \frac{(\delta+\rho)p_1 - w_1(p_1) + \left[\left(\zeta+\frac{1}{\epsilon_{cc}}\right)\frac{p_1}{c}\frac{\partial c}{\partial k} - \frac{p_1}{\epsilon_{c\ell}\ell}\frac{\partial \ell}{\partial k}\right](y_1(k,p_1)-\delta k)}{E(k,p_1)} \tag{22}$$

with

$$E(k,p_1) = 1 - \left[\left(\zeta + \frac{1}{\epsilon_{cc}}\right)\frac{p_1}{c}\frac{\partial c}{\partial p_1} - \frac{p_1}{\epsilon_{c\ell}\ell}\frac{\partial \ell}{\partial p_1}\right] \tag{23}$$

Any solution $\{k(t),p_1(t)\}_{t\geq 0}$ that satisfies the transversality condition

$$\lim_{t\to+\infty} \frac{e^{-\rho t}U_1(c(t),(\bar{\ell}-\ell(t))/B)p_1(t)k(t)}{(1+\tau(c(t)))} = 0 \tag{24}$$

is called an equilibrium path.

3 Steady state and characteristic polynomial

3.1 A normalized steady state

A steady state of the dynamical system (22) is defined by a pair (k^*, p_1^*) solution of

$$y_1(k,p_1) = \delta k, \quad w_1(p_1) = (\delta+\rho)p_1 \tag{25}$$

Let us denote $\kappa = k/\ell$ the capital/labor ratio.

Proposition 1. *There exist unique values of κ^* and p_1^* such that $y_1(\ell\kappa^*, p_1^*) = \delta\ell\kappa^*$ and $w_1(p_1^*) = (\delta+\rho)p_1^*$.*

Proof: See Appendix 7.1. □

However, given κ^* and p_1^*, we need to prove the existence of c, ℓ and k. Using the balanced-budget rule (8), a steady state can be defined by a solution (c,ℓ) satisfying:

$$\mathcal{G}(c) = \Omega(c) = \tau c \tag{26}$$
$$(1+\tau)c = \ell T(\kappa^*, \delta\kappa^*, 1) \equiv \ell T^* \tag{27}$$
$$\frac{(1+\tau)U_2(c,(\bar{\ell}-\ell)/B)}{U_1(c,(\bar{\ell}-\ell)/B)B} = w_0(p_1^*) \equiv w_0^* \tag{28}$$

We have shown previously that consumption can be expressed as $c = c(k,p_1) = c(\ell\kappa, p_1)$. Substituting this expression in (28), we use the scaling parameter $B > 0$ in order to ensure the existence of a normalized steady state (NSS in the sequel) such that $\ell^* = 1$. We have indeed:

$$\frac{(1+\tau(c(\kappa^*,p_1^*)))U_2(c(\kappa^*,p_1^*)),(\bar{\ell}-1)/B)}{U_1(c(\kappa^*,p_1^*)),(\bar{\ell}-1)/B)B} = w_0^* \tag{29}$$

Using this expression together with Proposition 1 we can show:

[6] These expressions are derived from Lemma 1 in Appendix 7.3, $U_{12} = -(c/\mathcal{L})U_{11}$ and $U_{22} = (c/\mathcal{L})^2 U_{11}$.

Proposition 2. *There exists a unique value B^* solution of equation (29) such that when $B = B^*$, $(\kappa^*, p_1^*, \ell^*)$, with κ^*, p_1^* as given by Proposition 1 and $\ell^* = 1$, is a normalized steady state.*

Proof: See Appendix 7.2. □

Remark 1: Using a continuity argument we derive from Proposition 2 that there exists an intertemporal equilibrium for any initial capital stock $k(0)$ in the neighborhood of κ^*.

3.2 Characteristic polynomial

Linearizing the dynamical system (22) around the NSS leads to the characteristic polynomial

$$\mathcal{P}(\lambda) = \lambda^2 - \mathcal{T}\lambda + \mathcal{D} \tag{30}$$

with

$$\begin{aligned} \mathcal{D} &= \frac{\left(\frac{\partial y_1}{\partial k} - \delta\right)\left(\delta + \rho - \frac{\partial w_1}{\partial p_1}\right)}{E(k^*, p_1^*)} \\ \mathcal{T} &= \frac{\frac{\partial y_1}{\partial k} + \rho - \frac{\partial w_1}{\partial p_1} + \left[\left(\zeta + \frac{1}{\epsilon_{cc}}\right)\frac{p_1}{c}\frac{\partial c}{\partial k} - \frac{p_1}{\epsilon_{c\ell}\ell}\frac{\partial \ell}{\partial k}\right]\frac{\partial y_1}{\partial p_1} - \left[\left(\zeta + \frac{1}{\epsilon_{cc}}\right)\frac{p_1}{c}\frac{\partial c}{\partial p_1} - \frac{p_1}{\epsilon_{c\ell}\ell}\frac{\partial \ell}{\partial p_1}\right]\left(\frac{\partial y_1}{\partial k} - \delta\right)}{E(k^*, p_1^*)} \end{aligned} \tag{31}$$

Most of these partial derivatives are functions of ζ, ϵ_{cc}, $\epsilon_{c\ell}$, $\epsilon_{\ell c}$ and $\epsilon_{\ell\ell}$. The role of $\epsilon_{\ell c}$ and $\epsilon_{\ell\ell}$ occurs through the presence of endogenous labor but remains implicit at that stage because of our methodology to derive the dynamical system (22) from the first order conditions (16)-(20).

Any solution of (22) that converges to the NSS satisfies the transversality condition (24) and is an equilibrium. Therefore, given $k(0)$, if there is more than one initial price $p_1(0)$ in the stable manifold of the NSS, the equilibrium path from $k(0)$ will not be unique. In particular, if (30) has two roots with negative real parts, there will be a continuum of converging paths and thus a continuum of equilibria. The NSS is locally indeterminate and there exist expectation-driven endogenous fluctuations. Local indeterminacy requires therefore that $\mathcal{D} > 0$ and $\mathcal{T} < 0$. Obviously saddle-point stability is obtained when $\mathcal{D} < 0$ while total instability holds (with both eigenvalues having positive real parts) if $\mathcal{D} > 0$ and $\mathcal{T} > 0$. In this last case, there is no equilibrium. In the following, we will focus on a locally indeterminate equilibrium and we will refer to the destabilizing effect of the tax rate which generates expectation-driven fluctuations.

4 Local indeterminacy with consumption taxes

Our aim is to study the existence of local indeterminacy, i.e. business-cycle fluctuations based on self-fulfilling prophecies. As initiated by Benhabib and Nishimura [4, 5], the analysis in two-sector models is based on capital intensity differences across sectors. From the input coefficients defined in Section 2.1, we derive that the consumption (investment) good is capital intensive when $\beta - \alpha < (>) 0$.

4.1 Main result

It is well-known that if consumption taxes are set equal to zero, the NSS is saddle-point stable for any capital intensity difference. We show in the following that with balanced-budget consumption taxation, aggregate instability and sunspot fluctuations easily arise in a standard two-sector optimal growth model. In the rest of the analysis, in order to avoid unrealistic configurations, we assume that $\zeta > -1$, or equivalently $\eta > -1/\tau$.

Proposition 3. *Under Assumptions 1-2, for any* $\alpha, \beta \in (0,1)$*, there exist* $\underline{\theta} \in (0,1)$ *and* $\underline{\tau} > 0$ *such that the NSS is locally indeterminate if* $\theta \in (\underline{\theta}, 1)$*,* $\tau > \underline{\tau}$ *and* $\eta \in (-1/\tau, 1)$*.*

Proof: See Appendix 7.3. □

Considering the expression of the lower bound $\underline{\tau}$ as given in Appendix 7.3, we easily derive that $\partial\underline{\tau}/\partial\eta > 0$. It follows that the more counter-cyclical government spendings are, the lower the consumption tax rate can be for the existence of local indeterminacy.

A striking property of our result is that sunspot fluctuations can arise for any capital intensity difference, even in the limit case with identical technologies in the two sectors, i.e. $\alpha = \beta$. This conclusion can be easily explained. Actually, a two-sector model with identical technologies across sectors producing therefore identical goods is not equivalent to a standard one-sector model. As suggested by the contributions of Benhabib and Farmer [2, 3] in models with productive externalities, the main difference between these two formulations is that in two-sector models, even with symmetric technologies, there exist capital and labor allocations across sectors, that are generated through Rybczynski and Stolper-Samuelson effects and that may produce expectation-driven fluctuations even when these are ruled-out in aggregate models. The same type of mechanism also arises in our model through counter-cyclical consumption taxes.

It is also worth pointing out that a not too low consumption tax rate and a low enough wage elasticity of the labor supply $\epsilon_{\ell\ell}$ appear as necessary conditions for local indeterminacy. Moreover, sunspot fluctuations compatible with constant government spendings, i.e. $\eta = 0$. Finally, by adjusting the elasticity of substitution between consumption and leisure, sunspot fluctuations can arise with any value for the elasticity of intertemporal substitution in consumption. We show in the following section that our conclusions are compatible with empirically plausible values of these structural parameters.

Remark 2: It can be shown that even in the case of additively separable preferences such that

$$U(c, \mathcal{L}/B) = \tfrac{c^{1-\sigma}}{1-\sigma} - \tfrac{[(\bar{\ell}-\mathcal{L})/B]^{1+\chi}}{(1+\chi)} \tag{32}$$

with $\sigma, \chi \geq 0$, local indeterminacy arises for some range of tax rates if the wage elasticity of labor is low and the elasticity of intertemporal substitution in consumption is larger than 1, although saddle-point stability always prevails in an aggregate models (see Giannitsarou [11] and Nourry *et al.* [19]). As explained above, this result follows from the existence of capital and labor allocations across sectors in two-sector models.[7]

4.2 A CES illustration

Let us consider the CES specification as given by (13). Within all the parameters involved in our analysis, two are of particular importance. On the one hand, the elasticity of the aggregate labor supply is shown to be significantly lower than one. Most econometric analysis available in the literature conclude indeed that the wage elasticity of labor belongs to $(0, 0.5)$ for men and to $(0.5, 1)$ for women (see Blundell and MaCurdy [6]). On the other hand, while the elasticity of intertemporal substitution in consumption is usually shown to be low, recent estimates provide divergent views. In a first set of contributions, Campbell [8] and Kocherlakota [13] suggest the

[7] It can be shown that with GHH preferences local indeterminacy also occurs for any capital intensity difference in our two-sector framework as in aggregate models (see Nourry *et al.* [19]).

interval $(0.2, 0.8)$. On the contrary, Mulligan [17], Vissing-Jorgensen [23] and Vissing-Jorgensen and Attanasio [24] repeatedly obtain estimates of this elasticity which are significantly larger than one. More recently, Gruber [11] and Kapoor and Ravi [12] provide robust estimates in the range $(2, 3)$. According to these various empirical estimates, we focus on the following ranges $\epsilon_{cc} \in (0.5, 2.5)$ and $\epsilon_{\ell\ell} \in (0, 1)$.

According to the consumption tax rates provided by Mendoza *et al.* [15, 16],[8] we have the following ranges for each OECD countries:

$\tau \in (0.05, 0.1)$	Australia, Japan, US, Switzerland
$\tau \in (0.1, 0.15)$	Canada, Italy, Spain
$\tau \in (0.15, 0.2)$	Belgium, Germany, Greece, Netherlands, New Zealand, Portugal, UK
$\tau \in (0.2, 0.25)$	Austria, France, Iceland, Luxembourg, Sweden
$\tau > 0.25$	Denmark, Finland, Ireland, Norway

Table 1: Consumption tax rates of OECD countries

Let us then consider parameters' values that match quarterly data: $\delta = 2.5\%$, $\rho = 0.04$, $\alpha = 0.33$, $\beta = 0.32$.[9] In the case of constant government spendings ($\eta = 0$) we find that the NSS is locally indeterminate for all $\tau > \underline{\tau}$, with $\underline{\tau}$ depending on the value of ϵ_{cc}, if $\theta \in (0.72736, 1)$. Let $\theta = 0.73$ and assume first an elasticity of intertemporal substitution in consumption $\epsilon_{cc} = 0.5$ in the range provided by Campbell [8] and Kocherlakota [13]. We conclude that $\underline{\tau} \approx 4.76\%$ and the corresponding wage elasticity of labor is $\epsilon_{\ell\ell} = 0.089$. Assuming now an elasticity of intertemporal substitution in consumption $\epsilon_{cc} = 2.2$ as recently suggested by Gruber [11] and Kapoor and Ravi [12], we find that $\underline{\tau} \approx 4.34\%$ and $\epsilon_{\ell\ell} = 0.39$.[10] In both cases, the lower bound $\underline{\tau}$ is compatible with the consumption tax rates of all OECD countries including the US and Japan, as mentioned in Table 1.

This numerical exercise is based on a capital intensive consumption good sector, i.e. $\alpha > \beta$. It is worth pointing out that our results still hold in the configuration with a reverse capital intensity difference, i.e. $\alpha < \beta$. However, we can show that the lower bound of the tax rate, above which local indeterminacy arises, satisfies $\partial\underline{\tau}/\partial(\beta - \alpha) > 0$. This means that if the investment good is capital intensive, expectations-driven fluctuations occur for higher tax rates. Considering the same basic parameters' values but setting $\alpha = 0.32$ and $\beta = 0.33$, we find indeed slightly larger lower bounds such that $\underline{\tau} = 7.26\%$ when $\epsilon_{cc} = 0.5$ and $\underline{\tau} = 6.57\%$ when $\epsilon_{cc} = 2.2$.

Let us finally complete this exercise by discussing the value of η. Using annual data over the interval $1960 - 1998$ for 22 OECD countries, Lane [14] provides some empirical estimates of the elasticity of government expenditure with respect to output growth. Since consumption is almost perfectly correlated with output, we then have an approximated measure of η. Lane shows that in most OECD countries government spendings are counter-cyclical with $\eta \in (-0.66, 0)$. Considering that the lower bound $\underline{\tau}$ decreases with η, we conclude that the above results are reinforced

[8] Updated estimates up to 1996 are available online from the authors.

[9] In this parameterization the consumption good is slightly more capital intensive than the investment good. This configuration appears to be the most plausible in the OECD countries over the last 40 years as shown by Baxter [1] and Takahashi *et al.* [22].

[10] The values of $\epsilon_{\ell\ell}$ in both illustrations belong to the lower ranges of estimated wage elasticities for men.

when η becomes negative. We have then proved that with linear homogeneous preferences, aggregate instability based on expectations-driven fluctuations can be generated by balanced-budget consumption taxes under empirically plausible parameters' configurations.

5 Economic interpretations

To simplify the exposition, we restrict our attention to a balanced-budget rule with constant spendings ($\eta = 0$). Such a simplification is not restrictive. In Proposition 3 we focus indeed on a counter-cyclical tax rate ($\zeta < 0$) which is compatible, as shown by equation (9), with counter-cyclical ($\eta < 0$), constant ($\eta = 0$) or pro-cyclical ($\eta \in (0, 1)$) government spendings. Actually, the value of η only affects the size of the elasticity ζ of the tax rate.

As shown by Nourry *et al.* [19], in aggregate models if the utility function is characterized by sufficiently larger substitution effects with respect to income effects, a higher expected consumption tax rate implies a lower labor supply. This decrease of next period labor reduces the real interest rate, which has a negative effect on current investment. Needing less income today, households work less. This induces a lower consumption, and thus an increase of the current consumption tax rate in order to match the balanced-budget rule. Therefore, expectations are self-fulfilling.

When additively separable preferences, or linear homogeneous preferences with reasonable parameters' values are considered, income effects remain too large to allow for such a mechanism and expectations-driven fluctuations are ruled out (see Giannitsarou [11] and Nourry *et al.* [19]). On the contrary, with GHH preferences characterized by no income effect, local indeterminacy easily arises under plausible parameterizations of the model (see Nourry *et al.* [19]).

In two-sector models, there exists an additional mechanism that can also explain the existence of expectations-driven fluctuations even in the case of symmetric technologies across sectors. Let us indeed assume that $\alpha = \beta$. It can be easily shown that in this configuration, $k_0 = k_1 = k/2$, $l_0 = l_1 = \ell/2$, $p_1 = 1$ and thus $y_0 = y_1$. The capital accumulation equation then becomes

$$\dot{k} = y_1 - \delta k = y_0 - \delta k = (1 + \tau)c - \delta k \tag{33}$$

Consider linear homogeneous preferences. Using the first order conditions (16) and (18) together with (6)-(8) and (21), we get

$$\frac{dc}{d\tau}\frac{\tau}{c} = -\frac{\tau}{1+\tau}\frac{(1-\theta)^2\Phi\epsilon_{cc}-\theta}{(1-\theta)\Phi-\theta}$$

It follows that if, as in Proposition 3, θ is close enough to 1, we derive

$$\frac{\partial}{\partial\tau}(1 + \tau)c < 0$$

Starting from some equilibrium at date t, assume that the agents expect a rise in the consumption tax rate. We conclude from (33) that the productions of both goods have to decrease. The corresponding lower investment level generates a lower labor supply and thus a decrease of the real interest rate w_1 which has a negative impact on current investment. Needing less income today, households work less. The implied fall of consumption then induces an increase of the current consumption tax rate in order to match the balanced-budget rule. The initial expectation becomes self-fulfilling leading to another equilibrium. This additional mechanism, clearly based on the property $y_0 = y_1$, explains why local indeterminacy may arise in a two-sector model with symmetric technologies.

Consider finally the case of asymmetric technologies across sectors with $\alpha \neq \beta$. The additional mechanism is now based on Stolper-Samuelson effects that can amplify or lower the above mentioned effects depending on the sign of the capital intensity difference. Assume first that the consumption good is capital intensive, i.e. $\alpha > \beta$. In this case we know that the relative price of the investment good p is a decreasing function of the rental rate of capital w_1. As previously, the expected rise in the consumption tax rate generates a decrease of the labor supply and thus a fall of w_1. But now, the Stolper-Samuelson effect implies an increase of p_1 which stimulates the production of the investment good but lowers the production of the consumption good $y_0 = (1+\tau)c$. This amplifies the decrease of the labor supply and thus the fall of the real interest rate. Current investment is then negatively affected, leading to a fall of current labor supply and consumption. All this results in a self-fulfilling increase of the consumption tax rate in order to match the balanced-budget rule. As a consequence local indeterminacy becomes compatible with lower tax rates as shown by the numerical exercise. Obviously, this overall effect is opposite when the investment good sector is capital intensive, i.e. $\alpha < \beta$ and this explains why local indeterminacy requires larger tax rates in this case.

6 Concluding comments

In this paper we precisely examine the impact of balanced-budget consumption taxes on the existence of expectations-driven business cycles in two-sector economies. Contrary to the initial conclusion of Giannitsarou [11], Nourry *et al.* [19] have recently proved that in aggregate growth models, balanced-budget fiscal policy rules based on consumption taxation may generate aggregate instability through self-fulfilling expectations when the utility function is characterized by low enough income effects.

The main contribution of this paper is to prove that in a two-sector economy, there exists an additional mechanism based on sectoral allocations of capital and labor that provides new room for the occurrence of locally indeterminate equilibria. We prove indeed that for any sign of the capital intensity difference expectations-driven fluctuations occur when the wage elasticity of labor is low and the consumption tax rate is not too low. Moreover, under a standard parameterization of the model compatible with quarterly data, we show that the lower bound of the consumption tax rate above which local indeterminacy arises is less than 5%. We then conclude that consumption taxation in two-sector economies may explain business-cycle fluctuations based on beliefs in all OECD countries, including the US.

7 Appendix

7.1 Proof of Proposition 1

Maximizing profit subject to the private technologies (1) gives the first order conditions

$$\begin{aligned} p_0\alpha y_0/k_0 = w_1, \quad p_0(1-\alpha)y_0/l_0 = w_0 \\ p_1\beta y_1/k_1 = w_1, \quad p_1(1-\beta)y_1/l_1 = w_0 \end{aligned} \tag{34}$$

with $p_0 = 1$. Considering the steady state with $y_1 = \delta k$ and $w_1 = (\delta+\rho)p_1$, we get

$$k_1 = \tfrac{\beta}{\delta+\rho}\delta k \tag{35}$$

Using the production function (1) for the investment good we derive

$$l_1 = \left(\tfrac{\beta}{\delta+\rho}\right)^{-\frac{\beta}{1-\beta}}\delta k \text{ and thus } \tfrac{k_1}{l_1} = \left(\tfrac{\beta}{\delta+\rho}\right)^{\frac{1}{1-\beta}} \tag{36}$$

Finally we obtain from (34):

$$\frac{\alpha(1-\beta)}{\beta(1-\alpha)} = \frac{l_1 k_0}{k_1 l_0} \quad \Leftrightarrow \quad \frac{k_0}{l_0} = \frac{\alpha(1-\beta)}{\beta(1-\alpha)} \left(\frac{\beta}{\delta+\rho}\right)^{\frac{1}{1-\beta}} \tag{37}$$

Considering (36), (37) and $l_0 + l_1 = \ell$, $k = k_0 + k_1$, we get

$$k^* \quad = \quad \ell \frac{\frac{\alpha(1-\beta)}{\beta(1-\alpha)}\left(\frac{\beta}{\delta+\rho}\right)^{\frac{1}{1-\beta}}}{1-\frac{\beta\delta}{\delta+\rho}\left[1-\frac{\alpha(1-\beta)}{\beta(1-\alpha)}\right]} \equiv \ell\kappa^* \tag{38}$$

Equation (34) with (37) gives

$$w_1^* = \alpha \left(\frac{\alpha(1-\beta)}{\beta(1-\alpha)}\right)^{-(1-\alpha)} \left(\frac{\beta}{\delta+\rho}\right)^{-\frac{1-\alpha}{1-\beta}} \text{ and } w_0^* = w_1^* \frac{1-\beta}{\beta} \left(\frac{\beta}{\delta+\rho}\right)^{\frac{1}{1-\beta}} \tag{39}$$

Considering then the fact that $w_1 = (\delta+\rho)p_1$ implies

$$p_1^* \quad = \quad \frac{\alpha}{\delta+\rho} \left(\frac{\alpha(1-\beta)}{\beta(1-\alpha)}\right)^{-(1-\alpha)} \left(\frac{\beta}{\delta+\rho}\right)^{-\frac{1-\alpha}{1-\beta}}$$

□

7.2 Proof of Proposition 2

To establish the existence of the normalized steady state with $\ell^* = 1$, we have to prove the existence and uniqueness of a solution B^* of the following equation

$$\Gamma(B) \equiv \frac{(1+\tau(c(\kappa^*,p_1^*))))U_2(c(\kappa^*,p_1^*)),(\bar{\ell}-1)/B)}{U_1(c(\kappa^*,p_1^*)),(\bar{\ell}-1)/B)B} = w_0^*$$

Under Assumption 2 we have $\lim_{B\to 0}\Gamma(B) = 0$, $\lim_{B\to+\infty}\Gamma(B) = +\infty$ and $\Gamma'(B)B/\Gamma(B) \neq 0$. The existence and uniqueness of B^* follow. □

7.3 Proof of Proposition 3

Let us first introduce a useful relationship between $\epsilon_{\ell c}$ and $\epsilon_{c\ell}$.

Lemma 1. *Let Assumption 2 hold. Then at the steady state*

$$\epsilon_{c\ell} = \frac{\delta(1-\beta)+\rho}{\delta(1-\beta)+\rho(1-\alpha)}\epsilon_{\ell c} \equiv \Phi\epsilon_{\ell c} \tag{40}$$

Proof: Using the first order conditions (16) and (18), we get $\epsilon_{c\ell} = \epsilon_{\ell c}(c(1+\tau)/w_0\ell)$. At the steady state we have $(1+\tau)c = y_0 = \ell T(\kappa, \delta\kappa, 1) \equiv \ell T$ and (34) implies $w_0 = (1-\alpha)y_0/l_0$ with $l_0 = \ell - l_1$. Considering then (38), the result follows. □

We start by the computation of $\mathcal{D}$ and $\mathcal{T}$ using a general formulation for $U(c, \mathcal{L}/B)$. Consider the expressions (31). We need to compute the following five derivatives: $\partial c/\partial k$, $\partial c/\partial p_1$, $\partial y_1/\partial k$, $\partial y_1/\partial p_1$ and $\partial w_1/\partial p_1$. Let $b = (a_{11}a_{00} - a_{10}a_{01}) = a_{00}(\beta-\alpha)/(\delta+\rho)(1-\alpha)$. Total differentiation of the factor-price frontier (4) and the factor market clearing equation (3) gives:

$$\frac{dw_1}{dp_1} \quad = \quad \frac{a_{00}}{b} \tag{41}$$

$$\frac{dy_0}{dk} \quad = \quad -\frac{a_{01}}{b} + \frac{a_{11}}{b}\frac{d\ell}{dk} \tag{42}$$

$$\frac{dy_1}{dk} \quad = \quad \frac{a_{00}}{b} - \frac{a_{10}}{b}\frac{d\ell}{dk} \tag{43}$$

$$\frac{dy_1}{dp_1} \quad = \quad \frac{a_{00}\ell^*(\mathcal{Z}_1+\mathcal{Z}_2)}{bp_1^*} - \frac{\delta\ell^*\kappa^*}{p_1^*} - \frac{a_{10}}{b}\frac{d\ell}{dp_1} \tag{44}$$

$$\frac{dy_0}{dp_1} \quad = \quad \frac{-a_{01}\ell^*(\mathcal{Z}_1+\mathcal{Z}_2)}{bp_1^*} + \frac{a_{11}}{b}\frac{d\ell}{dp_1} \tag{45}$$

with

$$\mathcal{Z}_1 = \frac{\kappa^*(1-\alpha)}{\beta-\alpha}, \quad \mathcal{Z}_2 = \frac{a_{11}\alpha}{a_{01}(\beta-\alpha)} \tag{46}$$

Consider the constraint $c(1+\tau(c)) = T(k, y_1, \ell(k, p_1)) = y_0(k, p_1)$. Total differentiation gives:

$$\begin{aligned} \frac{dc}{dk} &= \frac{dy_0}{dk}\frac{1}{(1+\tau(c))(1+\zeta)} = \frac{1}{h(c)(1+\zeta)}\left[-\frac{a_{01}}{b} + \frac{a_{11}}{b}\frac{d\ell}{dk}\right] && (47)\\ \frac{dc}{dp_1} &= \frac{dy_0}{dp_1}\frac{1}{(1+\tau(c))(1+\zeta)} = \frac{1}{h(c)(1+\zeta)}\left[\frac{-a_{01}\ell^*(\mathcal{Z}_1+\mathcal{Z}_2)}{bp_1^*} + \frac{a_{11}}{b}\frac{d\ell}{dp_1}\right] && (48) \end{aligned}$$

We finally need to compute $d\ell/dk$ and $d\ell/dp_1$. Total differentiation of equation (28) gives:

$$\frac{d\ell}{\ell}\left(\frac{1}{\epsilon_{\ell\ell}} - \frac{1}{\epsilon_{c\ell}}\right) = -dp_1\left[\frac{dc}{dp_1}\frac{1}{c}\left(\zeta + \frac{1}{\epsilon_{cc}} - \frac{1}{\epsilon_{\ell c}}\right) - \frac{1}{w_0}\frac{dw_0}{dp_1}\right] - dk\frac{dc}{dk}\frac{1}{c}\left(\zeta + \frac{1}{\epsilon_{cc}} - \frac{1}{\epsilon_{\ell c}}\right)$$

When $dp_1 = 0$, using (47) we derive:

$$\frac{d\ell}{dk} = \frac{\frac{a_{01}}{(1+\zeta)bT^*}\left(\zeta+\frac{1}{\epsilon_{cc}}-\frac{1}{\epsilon_{\ell c}}\right)}{\frac{1}{\epsilon_{\ell\ell}}-\frac{1}{\epsilon_{c\ell}}+\frac{a_{11}}{(1+\zeta)bT^*}\left(\zeta+\frac{1}{\epsilon_{cc}}-\frac{1}{\epsilon_{\ell c}}\right)} \tag{49}$$

When $dk = 0$, consider the factor-price frontier (4). Solving for w_0 gives:

$$w_0 = \frac{a_{10}p_1(\beta-\alpha)}{b\alpha} \tag{50}$$

Equations (41) and (50) then imply

$$\frac{1}{w_0}\frac{dw_0}{dp_1} = \frac{-\alpha}{p_1(\beta-\alpha)}$$

Therefore, considering (44) we derive:

$$\frac{d\ell}{dp_1} = \frac{\frac{\ell^*}{p_1^*}\left[\frac{a_{01}(\mathcal{Z}_1+\mathcal{Z}_2)\left(\zeta+\frac{1}{\epsilon_{cc}}-\frac{1}{\epsilon_{\ell c}}\right)}{(1+\zeta)bT^*} - \frac{\alpha}{\beta-\alpha}\right]}{\frac{1}{\epsilon_{\ell\ell}}-\frac{1}{\epsilon_{c\ell}}+\frac{a_{11}}{(1+\zeta)bT^*}\left(\zeta+\frac{1}{\epsilon_{cc}}-\frac{1}{\epsilon_{\ell c}}\right)} \tag{51}$$

Consider then a linearly homogeneous utility function $U(c, \mathcal{L}/B)$ and assume first that $\alpha \neq \beta$. Substituting (21), (42), (44), (47), (48), (49) and (51) into (31) finally gives:

$$\begin{aligned} \mathcal{D} &= -\frac{\left[\frac{\theta[\delta(1-\beta)+\rho(1-\alpha)]}{(1-\theta)^2\epsilon_{cc}\Phi} + \frac{[\delta(1-\beta)+\rho](1-\alpha)\left(\zeta+\frac{1}{(1-\theta)\epsilon_{cc}}\right)}{(1+\zeta)a_{00}T^*}\right]\frac{(\delta+\rho)(1-\beta)}{(\beta-\alpha)^2}}{\frac{\theta}{(1-\theta)^2\epsilon_{cc}\Phi} + \frac{a_{11}\left[\zeta\beta+\frac{\beta-\theta\alpha}{(1-\theta)\epsilon_{cc}}\right]}{(1+\zeta)b(\beta-\alpha)T^*} + \frac{\zeta\theta^2[a_{01}\kappa^*(1-\alpha)+a_{11}\alpha]}{(1+\zeta)(1-\theta)^2\epsilon_{cc}b(\beta-\alpha)T^*\Phi} + \frac{\theta\alpha}{(1-\theta)\epsilon_{cc}(\beta-\alpha)\Phi}} \\ \mathcal{T} &= \frac{\frac{\rho\theta}{(1-\theta)^2\epsilon_{cc}\Phi} + \frac{\zeta\beta^2\rho+\frac{\beta^2\rho-\theta\alpha[\delta(1-\beta)+\rho]}{(1-\theta)\epsilon_{cc}}}{(1+\zeta)(\delta+\rho)b(\beta-\alpha)T^*} - \frac{\zeta\theta^2\delta[a_{01}\kappa^*(1-\beta)+a_{11}\alpha]}{(1+\zeta)(1-\theta)^2\epsilon_{cc}b(\beta-\alpha)T^*\Phi} + \frac{\theta\alpha[\delta(1-\beta)+\rho(1-\alpha)]}{(1-\theta)\epsilon_{cc}(\beta-\alpha)^2\Phi}}{\frac{\theta}{(1-\theta)^2\epsilon_{cc}\Phi} + \frac{a_{11}\left[\zeta\beta+\frac{\beta-\theta\alpha}{(1-\theta)\epsilon_{cc}}\right]}{(1+\zeta)b(\beta-\alpha)T^*} + \frac{\zeta\theta^2[a_{01}\kappa^*(1-\alpha)+a_{11}\alpha]}{(1+\zeta)(1-\theta)^2\epsilon_{cc}b(\beta-\alpha)T^*\Phi} + \frac{\theta\alpha}{(1-\theta)\epsilon_{cc}(\beta-\alpha)\Phi}} \end{aligned} \tag{52}$$

It is worth noting at this point that $(\beta - \alpha)b > 0$.

Note first that when $\theta = 0$ we get

$$\mathcal{D} = -\frac{[\delta(1-\beta)+\rho](1-\alpha)(\delta+\rho)(1-\beta)b}{a_{00}a_{11}\beta(\beta-\alpha)} < 0 \tag{53}$$

and saddle-point stability always holds. Consider now the denominator of $\mathcal{D}$ and $\mathcal{T}$. Since $\zeta = \tau(\eta-1)/(1+\tau)$, this expression is negative if and only if

$$\begin{aligned} \frac{\theta b(\beta-\theta\alpha)}{\epsilon_{cc}\Phi} + \frac{a_{11}(1-\theta)(\beta-\theta\alpha)}{\epsilon_{cc}T^*} \quad &< \quad \tau\left\{\frac{(1-\theta)a_{11}[\beta(1-\theta)\epsilon_{cc}-(\beta-\theta\alpha)]}{\epsilon_{cc}T^*} + \frac{\theta^2[a_{01}\kappa^*(1-\alpha)+a_{11}\alpha]}{\epsilon_{cc}\Phi T^*}\right. \\ &\quad \left. - \quad \eta\left[\frac{\theta b(\beta-\theta\alpha)}{\epsilon_{cc}\Phi} + \frac{\beta(1-\theta)^2a_{11}}{T^*} + \frac{\theta^2[a_{01}\kappa^*(1-\alpha)+a_{11}\alpha]}{\epsilon_{cc}\Phi T^*}\right]\right\} \end{aligned}$$

Obviously, there exists $\underline{\theta}^1 \in (0,1)$ such that the right-hand side of this inequality is positive when $\theta \in (\underline{\theta}^1, 1)$, or equivalently, the denominator of $\mathcal{D}$ and $\mathcal{T}$ is negative if $\theta \in (\underline{\theta}^1, 1)$ and

$$\tau > \frac{\frac{\theta b(\beta-\theta\alpha)}{\epsilon_{cc}\Phi}+\frac{a_{11}(1-\theta)(\beta-\theta\alpha)}{\epsilon_{cc}T^*}}{\frac{(1-\theta)a_{11}[\beta(1-\theta)\epsilon_{cc}-(\beta-\theta\alpha)]}{\epsilon_{cc}T^*}+\frac{\theta^2[a_{01}\kappa^*(1-\alpha)+a_{11}\alpha]}{\epsilon_{cc}\Phi T^*}-\eta\left[\frac{\theta b(\beta-\theta\alpha)}{\epsilon_{cc}\Phi}+\frac{\beta(1-\theta)^2a_{11}}{T^*}+\frac{\theta^2[a_{01}\kappa^*(1-\alpha)+a_{11}\alpha]}{\epsilon_{cc}\Phi T^*}\right]} \equiv \tilde{\tau}$$

Moreover, in the limit case $\theta = 1$ we find

$$\mathcal{D} = -\frac{\frac{[\delta(1-\beta)+\rho(1-\alpha)](\delta+\rho)(1-\beta)}{(\beta-\alpha)^2}}{1+\frac{\zeta[a_{01}\kappa^*(1-\alpha)+a_{11}\alpha]}{(1+\zeta)b(\beta-\alpha)T^*}} \quad \text{and} \quad \mathcal{T} = \frac{\rho-\frac{\zeta\delta[a_{01}\kappa^*(1-\beta)+a_{11}\alpha]}{(1+\zeta)b(\beta-\alpha)T^*}}{1+\frac{\zeta[a_{01}\kappa^*(1-\alpha)+a_{11}\alpha]}{(1+\zeta)b(\beta-\alpha)T^*}} \tag{54}$$

Therefore, there exists $\underline{\theta}^2 \in [\underline{\theta}^1, 1)$ such that $\mathcal{D} > 0$, $\mathcal{T} < 0$ and thus the NSS is locally indeterminate if $\theta \in (\underline{\theta}^2, 1)$ and $\tau > \tilde{\tau}$.

Consider now the limit case $\alpha = \beta$. We derive from (52):

$$\begin{aligned}
\lim_{\beta \to \alpha} \mathcal{D} &= -\frac{\left[\frac{\theta(\delta+\rho)(1-\beta)}{(1-\theta)^2\epsilon_{cc}\Phi} + \frac{[\delta(1-\beta)+\rho](1-\beta)\left(\zeta+\frac{1}{(1-\theta)\epsilon_{cc}}\right)}{(1+\zeta)a_{00}T^*}\right]a_{00}}{\frac{a_{11}\beta\left(\zeta+\frac{1}{\epsilon_{cc}}\right)}{(1+\zeta)T^*} + \frac{\zeta\theta^2[a_{01}\kappa^*(1-\beta)+a_{11}\beta]}{(1+\zeta)(1-\theta)^2\epsilon_{cc}T^*\Phi}} \\
\lim_{\beta \to \alpha} \mathcal{T} &= \frac{\frac{\zeta\beta^2\rho+\frac{\beta^2\rho-\theta\beta[\delta(1-\beta)+\rho]}{(1-\theta)\epsilon_{cc}}}{(1+\zeta)(\delta+\rho)T^*} - \frac{\zeta\theta^2\delta[a_{01}\kappa^*(1-\beta)+a_{11}\beta]}{(1+\zeta)(1-\theta)^2\epsilon_{cc}T^*\Phi} + \frac{\theta\beta a_{00}}{(1-\theta)\epsilon_{cc}\Phi}}{\frac{a_{11}\beta\left(\zeta+\frac{1}{\epsilon_{cc}}\right)}{(1+\zeta)T^*} + \frac{\zeta\theta^2[a_{01}\kappa^*(1-\beta)+a_{11}\beta]}{(1+\zeta)(1-\theta)^2\epsilon_{cc}T^*\Phi}}
\end{aligned} \quad (55)$$

When $\theta = 0$ we get $\mathcal{D} < 0$. We also easily derive

$$\lim_{\beta \to \alpha} \tilde{\tau} = \frac{\frac{a_{11}\beta(1-\theta)^2}{\epsilon_{cc}}}{\beta(1-\theta)^2 a_{11}\left[1-\eta-\frac{1}{\epsilon_{cc}}\right]+(1-\eta)\frac{\theta^2[a_{01}\kappa^*(1-\alpha)+a_{11}\alpha]}{\epsilon_{cc}\Phi}}$$

Obviously, there exists $\underline{\theta}^3 \in (0,1)$ such that $\lim_{\beta\to\alpha} \tilde{\tau} > 0$ when $\theta \in (\underline{\theta}^3, 1)$. Moreover, in the limit case $\theta = 1$ we find

$$\lim_{\beta \to \alpha} \mathcal{D} = -\frac{(\delta+\rho)(1-\beta))a_{00}}{\frac{\zeta[a_{01}\kappa^*(1-\beta)+a_{11}\beta]}{(1+\zeta)T^*}} \quad \text{and} \quad \lim_{\beta \to \alpha} \mathcal{T} = -\delta \quad (56)$$

with $\lim_{\beta\to\alpha} \tilde{\tau} = 0$. Therefore, there exist $\underline{\theta}^4 \in (0,1)$ such that the NSS is locally indeterminate if $\theta \in (\underline{\theta}^4, 1)$ and $\tau > \lim_{\beta\to\alpha} \tilde{\tau}$. The result follows considering $\underline{\theta} = \max\{\underline{\theta}^1, \underline{\theta}^2, \underline{\theta}^3, \underline{\theta}^4\}$ and $\underline{\tau} = \tilde{\tau}$.

□

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