

On Stabilization Policy in Sunspot-Driven Oligopolistic Economies

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Abstract

Economies with oligopolistic markets are prone to inefficient sunspot fluctuations triggered by autonomous changes in firms equilibrium conjectures. We show that a well designed taxation-subsidization scheme can eliminate these fluctuations by coordinating firms in each sector on a single efficient equilibrium. At the macroeconomic level, implementing this stabilization policy leads to significant welfare gains, attributable to a quantitatively dominant "efficient stabilization effect". This effect, while important, is typically ignored in the traditional computations of the welfare costs of aggregate fluctuations (e.g., Lucas, 2003).

Keywords: Business cycles; Stabilization policy; Indeterminacy; Sunspot equilibria; Oligopolistic competition.

JEL classification number: E32, E62, D43, D61.

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"The market economy is a social system. In attempting to optimize her own actions, each agent must attempt to predict the actions of the other agents. A, in forecasting the market strategy of B, must forecast B's forecasts of the forecasts of others including those of A herself. An entrepreneur is uncertain about the moves of his rivals, and they of his moves. It is not surprising that this process may generate uncertainty in outcomes even in the extreme case in which the fundamentals are non-stochastic." (Shell, "Sunspot Equilibrium", *The New Palgrave Dictionary of Economics*, 2008).

1 Introduction

In this paper, we analyze efficiency and welfare issues in dynamic economies with imperfectly competitive product markets. We emphasize the importance of designing appropriate stabilization policies for improving consumer welfare. As is well known, one typical feature of such economies is that they are likely to exhibit sunspot-driven fluctuations. This is true in economies where the degree of market power is sufficiently large that the model dynamics becomes *locally indeterminate*, as in the canonical RBC model with imperfect competition and increasing returns analyzed in Benhabib and Farmer (1994) and Farmer and Guo (1994). This is also true when increasing returns to scale are small, if the strategic interactions between firms lead to a multiplicity of equilibrium configurations in each oligopolistic sector (a situation which we may call *strategic indeterminacy*). In this case, as emphasized in Karl Shell's quotation above, endogenous fluctuations may emerge as a result of exogenous *self-fulfilling* changes in firms' conjectures on competitors' actions. Dos Santos Ferreira and Dufourt (2006) introduced such conjecture changes in an otherwise standard RBC model with oligopolistic markets and showed that they could generate significant and qualitatively relevant business cycles.¹

Because sunspot-driven fluctuations are usually considered detrimental to risk averse consumers, several papers, starting with Grandmont (1986) and Reichlin (1986), have explored the possibility of immunizing the economy from these fluctuations using standard policy tools. In particular, Guo and Lansing (1998) showed

¹For an early application of this idea within a simple overlapping generations model, see Chatterjee *et al.* (1993).

that the introduction of a sufficiently progressive income taxation rate can eliminate sunspot equilibria in the Benhabib and Farmer (1994) model.² The general idea of the taxation scheme is to distort equilibrium conditions in order to make the steady state locally determinate. However, taxation schemes which eliminate dynamic indeterminacy do not necessarily rule out sunspot equilibria implied by *strategic indeterminacy*. Indeed, as we will show below, endogenous fluctuations can occur in this case even if the steady-state is locally determinate.

In this paper, we thus explore the possibility of eliminating sunspot fluctuations linked to the strategic interactions between firms, and we analyze the welfare effects of implementing such stabilization policies. Building on the general framework developed in Dos Santos Ferreira and Dufourt (2007) to analyze equilibrium configurations in oligopolistic markets with free entry and exit, we show that there exist well-designed taxation-subsidization schemes which can achieve this objective by coordinating firms' conjectures in each sector on an efficient equilibrium. The particular taxation scheme we propose has the feature of distorting payoff functions, making unsustainable all equilibrium configurations other than the desired (efficient) equilibrium, which is left unaffected. Hence, while distortive *ex-ante*, it acts *ex-post* as a pure selection mechanism.

A good policy would then select a desired equilibrium according to some welfare or efficiency criterion. To gain some insights on the potential welfare gains that such regulation policies would provide, we develop a simple macroeconomic model with overlapping generations of consumers and a large number of differentiated sectors composed of several Cournot competitors. We prove that, in the laissez-faire economy, the multiplicity of free entry equilibria in any sector is consistent with a large set of aggregate dynamics – influenced by firms' expectations –, infinitely many of them involving fluctuating macroeconomic variables. This set of equilibria is very large and includes, among others, equilibria with smooth fluctuations around a long-run production level, equilibria with endogenous deterministic cycles, and equilibria with stochastic regime switches. All these trajectories are associated with variations in the aggregate markup factor – which roughly depends on the aggregate (average) number of firms per sector – and, as such, imply the existence of a time-varying gap between the efficient and the realized activity levels. Our theory is thus consistent with the findings of Chari *et al.* (2007), who report that a significant proportion of actual fluctuations are associated with

²See also Christiano and Harrison (1999), and Guo and Lansing (2002).

time-varying wedges between the marginal rate of substitution of consumption for leisure and the marginal productivity of labor.

Turning to normative issues, we then compare these potential trajectories in the laissez-faire economy to those that would be chosen by a benevolent social planner in first-best and second-best environments. We show that, in each case, the efficient trajectory is a stationary deterministic path associated with a well-determined (fixed) number of active firms per sector. Thus, our framework gives clear rationale for attempting to stabilize the economy on more stable and efficient activity levels. The application to all sectors of the particular taxation-subsidization scheme analyzed at the microeconomic level provides the appropriate incentives to achieve this objective. We show that implementing this policy would generate two kinds of welfare gains: first, by reducing the variance of aggregate fluctuations, it would increase the welfare of risk averse consumers. Second, by stabilizing the economy on an efficient (second-best) production level, it would provide direct efficiency gains to these consumers. A calibrated version of the model, in which all aggregate uncertainty results from exogenous stochastic changes in firms equilibrium conjectures, enables us to quantify these potential gains. In our benchmark economy, consistent with a variance of aggregate production similar to those of the US economy, we obtain welfare gains equivalent to a permanent increase in aggregate consumption ranging between 0.91 and 2.31 percent – an order of magnitude which is approximately 15 times larger than the corresponding estimates in Lucas (2003). We decompose these gains into a "pure stabilization effect" and an "efficient stabilization effect" and show that, from a quantitative point of view, most of the welfare gains result from the efficient stabilization effect. This effect, while potentially important, is typically ignored in the traditional computations of the welfare costs of aggregate fluctuations.

The remaining of the paper is organized as follows. In section 2, we present the general concept of free entry equilibrium at the industry level, and examine how a well-defined taxation scheme can help selecting the desired equilibrium in the set of potential free entry equilibria. In section 3, we insert this conceptual framework into a standard overlapping generations general equilibrium model, and prove that strategic indeterminacy is consistent with a large class of deterministic and stochastic (endogenous and/or sunspot-driven fluctuations) equilibria. We characterize the first and second-best activity levels and analyze the trade-offs involved in regulation. In section 4, we use our framework to reconsider the

traditional issues of stabilization policy and quantification of the welfare costs of fluctuations. Section 5 concludes.

2 Microeconomics: taxation policy in contestable markets

In this section, we start by briefly recalling some of the results obtained in Dos Santos Ferreira and Dufourt (2007), where a general framework was developed in order to analyze equilibrium configurations within perfectly contestable markets. Specifically, we show that under Cournot competition and internal economies of scale (here originating in a non-sunk fixed cost), free entry and exit conditions are typically consistent with multiplicity of oligopolistic equilibria characterized by different numbers of active firms and different individual production levels. In other words, free entry and exit are not enough to ensure uniqueness of the equilibrium with zero profits – an equilibrium which in any case is not necessarily optimal.

We then proceed to show that there exists a taxation-subsidization scheme which can eliminate all undesired equilibria. The specific scheme that we propose is required to have two desirable properties: first, to be balanced at the industry level; second, to select the desired equilibrium without introducing any additional distortions (which would partially or totally offset the efficiency gains obtained from selecting the right equilibrium). This taxation scheme would thus act *ex-post* as a pure *selection mechanism*.

2.1 Cournot free entry equilibria under laissez-faire

Consider, as an example of the general framework developed in Dos Santos Ferreira and Dufourt (2007), an industry with a large number N of identical firms producing a homogeneous good and competing à la Cournot. The demand to the industry at price p is b/p , with $b > 0$. Each firm can produce the homogenous good with a constant marginal cost c and a positive non-sunk fixed cost $c\phi$. The technology can thus be described by a cost function C such that $C(0) = 0$ and $C(y) = c(\phi + y)$ for positive output y . Contestability is expressed by the existence, at any equilibrium, of potential entrants, able to produce the same good under the same technological conditions as the incumbents, yet obtaining a max-

imum profit by remaining inactive. The *Cournotian price*

$$p^*(n) = \frac{n}{n-1}c, \quad (1)$$

equal to the markup factor applied to the marginal cost by any of n active firms, results from the necessary (and sufficient) first order condition for profit maximization by producing firms,³ given their correct conjecture that $n-1$ competitors are producing $y_n = b/np^*(n)$. However, for this price to be an equilibrium price, two additional conditions must be fulfilled. First, it must be *profitable* to produce y_n rather than stay inactive, the fixed cost being at least covered by the gross profit. In other words, the Cournotian price must be at least equal to the *break-even price* (the price ensuring zero profit), which is easily obtained as:

$$\underline{p}(n) = \frac{c}{1 - nc\phi/b}, \quad (2)$$

an increasing function of the number n of active firms, of the marginal cost c , and of the share $c\phi/b$ of individual fixed cost in aggregate expenditure (which is also a measure of the degree of increasing returns to scale).

The second condition for the Cournotian price to characterize an equilibrium is that inactivity be optimal for any potential entrant, given the correct conjecture that active firms are producing an aggregate quantity ny_n , so that a strategy profile with n firms choosing y_n , all others choosing zero, be *sustainable*. Starting from the first order condition for profit maximization by a potential entrant ($y = \sqrt{(b/c)ny_n} - ny_n$), it is easy to check that the corresponding profit $by/(y + ny_n) - c(y + \phi)$ will indeed be non-positive provided ny_n is large enough or, equivalently, provided the Cournotian price $p^*(n)$ is small enough, namely not larger than the *limit price*

$$\bar{p}(n) = \frac{c}{\left(1 - \sqrt{c\phi/b}\right)^2}. \quad (3)$$

This price appears to be constant in the number n of active firms, but is again an increasing function of the marginal cost c and of the share $c\phi/b$ of individual fixed cost in aggregate expenditure.

³The first order condition for (interior) maximization of profit $by/(y + \bar{Y}) - c(y + \phi)$, under the conjecture that the competitors' aggregate supply is $\bar{Y}$, can be stated as $b\bar{Y}/(y + \bar{Y})^2 = c$. Clearly, this necessary condition is also sufficient, and leads to a symmetric solution, so that the Cournotian price indeed verifies: $((n-1)/n)p^*(n) = c$.

Observe that profitability ($p^*(n) \geq \underline{p}(n)$) requires, by (1) and (2),

$$n \leq \sqrt{b/c\phi} \equiv \bar{n}. \quad (4)$$

The upper bound $\bar{n}$ must be at least equal to 2, in order for an equilibrium to exist, so that $c\phi/b \leq 1/4$ is a necessary condition for existence. Sustainability ($p^*(n) \leq \bar{p}(n)$) in turn requires, by (1) and (3),

$$n \geq \max \left\{ \frac{\bar{n}}{2 - 1/\bar{n}}, 2 \right\} \equiv \underline{n}. \quad (5)$$

A specific equilibrium outcome will thus be associated with any number n of active firms in the admissible interval $[\underline{n}, \bar{n}]$. This interval contains more than one integer as soon as $\bar{n} \geq 4.562$, or $c\phi/b \leq 0.048$. In this case, the zero-profit equilibrium, corresponding to the highest integer in the admissible interval, is only one particular equilibrium among many others. Figure 1 gives an illustration (for $c\phi/b = 0.04$) of this equilibrium indeterminacy, with three equilibrium configurations characterized by 3, 4 and 5 active firms.

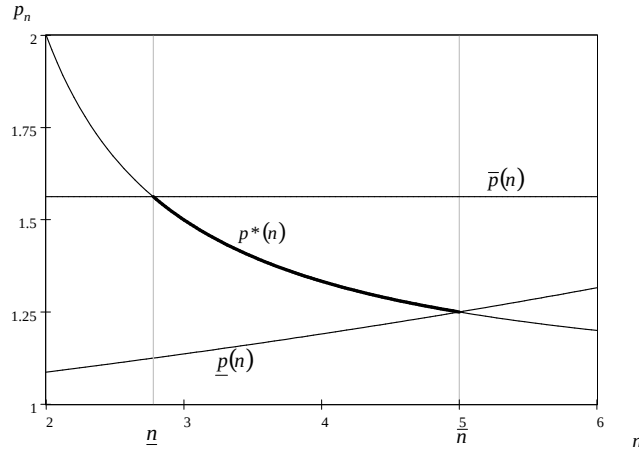


Figure 1: Equilibrium conditions: the *Cournotian*, *break-even* and *limit* price schedules

2.2 Taxation as a selection device

As long as several strategy profiles are sustainable as equilibrium configurations, there is no reason to suppose that firms always coordinate on the zero profit equilibrium, which is the *least* profitable equilibrium for active firms. There is no more reason to believe that the zero profit equilibrium is efficient. Indeed, if an

increase in the equilibrium number of active firms reduces the distortion introduced by the markup factor $\mu_n \equiv n/(n-1)$, it also generates scale inefficiencies through the duplication of the non-sunk fixed cost $c\phi$ (which is incurred by all active firms). Clearly, a proper characterization of an efficient equilibrium would require the specification of a well-defined selection criterion which should be based on economic fundamentals, preferences and deep parameters of the model (a point which will be examined in the next section). In any case, government intervention may be welcome if it can ensure stabilization on the efficient equilibrium in the sector.

There are many ways which can be thought about in order to achieve this objective. However, distortive taxation appears as the most natural tool. The aim of the taxation policy would be to distort one or more of the three schedules of Cournotian, break-even and limit prices so as to make the admissible interval $[\underline{n}, \bar{n}]$ contain at most one integer. While this policy seems attractive, it has a potential pitfall: if all equilibria are distorted, the efficiency gains obtained through the coordination on the best equilibrium might be canceled out by the efficiency losses generated by the price distortion. Thus, we would ideally want the taxation policy to be a strict *selection* instrument, ensuring coordination on the best equilibrium but avoiding any distortion or redistribution effects among sectors or among types of agents (firms and consumers). These conditions can in turn be fulfilled by imposing several restrictions on the taxation scheme. First, the taxation policy should be *balanced* at the sectoral level, the proportional taxes collected from firms in a sector being entirely redistributed to these firms by lump-sum transfers. Second, the taxation-subsidization scheme should be "state-dependent" (defined with respect to the ratio n/n^* of the number n of active firms to its target value n^*),⁴ in order to distort all equilibria except the efficient one, which should be left unaffected. In other words, the taxation scheme should be *inoperative* at the optimal equilibrium, meaning that neither taxes nor subsidies are applied at that equilibrium. We thus define:

Definition 1 *The set of desirable fiscal policies is the set of proportional taxes on sales at rate $\tau(n/n^*)$ and of lump sum subsidies $T(n/n^*)$ to active firms satisfying:*

(i) *the sectoral balanced budget (SBB) condition: $T(n/n^*)n = \tau(n/n^*)b$;*

⁴Notice that taxation can equivalently be defined as dependent on the ratio $(b/n^*)/(b/n) = n/n^*$ of the target revenue of the firm to its actual value.

(ii) the inoperativeness at the efficient equilibrium (IEE) condition: $T(1) = \tau(1) = 0$.

We can now prove existence of policies satisfying these properties. In order to take into account taxes and subsidies, we express as follows the profit at state n/n^* of a firm producing y and conjecturing $\bar{Y}$ as the competitors' aggregate output:

$$\Pi(y, \bar{Y}, n/n^*) = \left(\frac{(1 - \tau(n/n^*))b}{y + \bar{Y}} - c \right) y - (c\phi - T(n/n^*)). \quad (6)$$

The first order condition for maximizing this profit function (together with the condition $\bar{Y} = (n - 1)y$) gives the modified *Cournotian price*

$$p^*(n, n^*) = \frac{n}{n - 1} \frac{c}{1 - \tau(n/n^*)}, \quad (7)$$

a price clearly affected by the proportional sales tax, which augments the marginal cost by a factor $1/(1 - \tau(n/n^*))$.

A simple way to achieve the coordination objective is to make *unsustainable* the strategy profiles with less than the desired number of active firms, by transforming fixed costs into variable costs. This can be done by subsidizing any producing firm through a lump sum transfer $T(n/n^*)$ depending on the ratio n/n^* of the actual to the target number of active firms, and by financing this subsidy through a proportional sales tax at the rate $\tau(n/n^*)$. Similarly, strategy profiles with more than the desired number of active firms should be made *unprofitable* by accomplishing the reverse transformation.

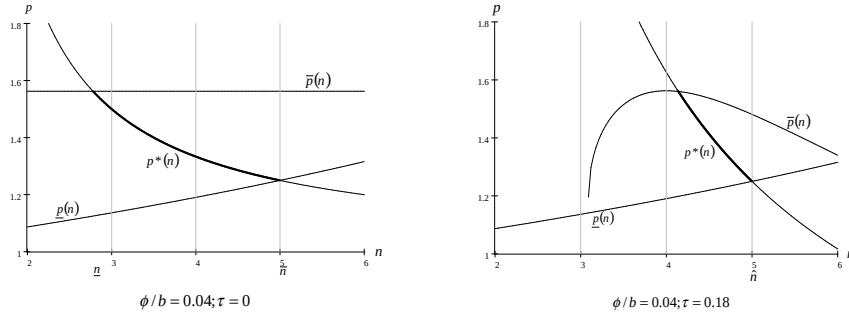
We can accordingly establish two propositions:

Proposition 1 *Any free entry equilibrium with $n < n^*$ in the laissez-faire economy can be ruled out by the choice of a high enough taxation rate $\tau(n/n^*)$ on sales (redistributed through a lump-sum transfer equal to $T(n/n^*)$).*

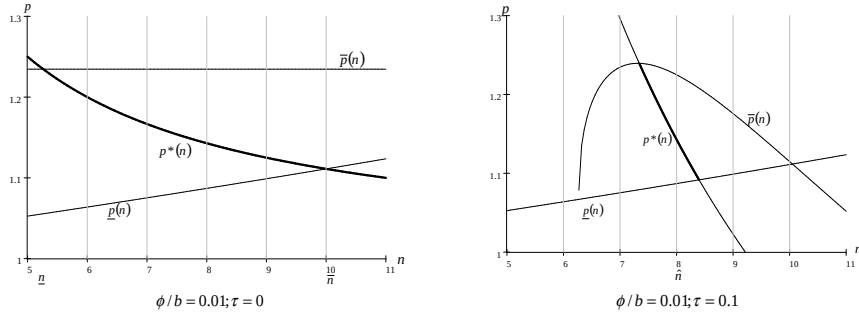
Proof. See appendix. ■

Proposition 2 *Any free entry equilibrium with $n > n^*$ in the laissez-faire economy can be ruled out by the choice of a high enough subsidization rate $-\tau(n/n^*)$ applied to sales (financed by a lump-sum tax equal to $-T(n/n^*)$).*

Proof. See appendix. ■



Equilibria without and with taxation for $n^* = 5 \in \{3, 4, 5\}$



Equilibria without and with taxation for $n^* = 8 \in \{6, \dots, 10\}$

Figure 2: Equilibrium selection through taxation

The balanced tax-subsidy schemes defined in Propositions 1 and 2 eliminate all possible equilibrium outcomes under laissez-faire except the efficient (or the desired) one, associated with zero taxes and subsidies. Many such schemes can be designed. Here, we just give the example of the linear affine scheme $\tau(n/n^*) = \tau n^*(1 - n/n^*)$ in order to show, in Figure 2, how the price schedules are distorted by the taxation policy (relative to the laissez-faire regime) so as to leave one single possible equilibrium outcome ($n^* = 5 \in \{3, 4, 5\}$ on the top panel, and $n^* = 8 \in \{6, 7, 8, 9, 10\}$ on the bottom one).

3 Macroeconomics: strategic indeterminacy and implications

Up to now we have proved that the government could select any particular equilibrium outcome as its preferred one without referring to an explicit social welfare criterion. Taking social welfare into account requires to move from partial to general equilibrium analysis, and to provide a complete description of the economy (preferences, technology, and so on). Accordingly, we now develop a simple macro-

economic model enabling us to derive some important results concerning welfare analysis and the desirability of adopting policy rules of the type described above. The model presented in section 3.1 describes a stationary economy with a large number of sectors producing different goods under Cournot competition.

Following our partial equilibrium analysis, the free-entry equilibrium number of active firms within each sector is indeterminate under laissez-faire and can take any integer value in some admissible interval $[\underline{n}, \bar{n}]$. Therefore, the realized equilibrium in each sector will depend on firms' mutually consistent conjectures about their rivals' actions. But there is no reason to believe that the implicit scheme ensuring this coordination process on a particular equilibrium should remain the same across sectors and through time. As Shell (2008) emphasizes in the quotation at the beginning of this paper, it is perfectly conceivable that outcomes of such strategic games with multiple equilibria may fluctuate randomly, or according to some extraneous or subjective events unrelated to the economy, even in the extreme case in which the environment is stable and non-stochastic. Accordingly we prove in section 3.2 that, in accordance with Shell's predictions, at the intertemporal general equilibrium of the economy, solutions to the coordination problem encompass a large class of admissible dynamic trajectories, infinitely many of them involving fluctuating aggregate variables.

Facing this fundamental indeterminacy inherent to free entry equilibria, the government may find profitable to use the taxation policy described above to ensure coordination on a particular equilibrium within each sector. We prove in section 3.3 that this is indeed the case. Compared to the laissez-faire situation, a well-designed taxation policy can ensure coordination on a second-best intertemporal equilibrium which is welfare improving. The welfare gains are obtained for two main reasons. First, by ensuring coordination on a particular equilibrium – which may now remain invariant – the taxation policy can totally eliminate sunspot driven fluctuations in aggregate variables, improving the welfare of a risk-averse social planner: this is the *pure stabilization* effect of the taxation scheme. Second, by coordinating the economy on a second-best equilibrium, closer in its properties to the first-best allocation than the laissez-faire economy, the policy can provide direct efficiency gains to the social planner: this is the *efficient stabilization* effect of taxation.

3.1 A simple macroeconomic model

In order to maintain analytical tractability, the model we develop is an overly simplified dynamic macroeconomic model composed of overlapping generations of ‘young’ and ‘old’ consumers and of a large number of differentiated industries exhibiting identical characteristics.

At each date t , a generation of identical consumers of unit mass is born and lives for two periods. Consumers work only when young, receiving in this period wage earnings and dividends from firms (which are equally held by young consumers) and consume only when old. Young consumers can only save in the form of money, which bears no interest, and use their money savings when old to pay for their consumption purchases. We assume that old consumers’ preferences are defined over goods $i = 1, \dots, m$ produced by m industries, with a constant elasticity of substitution between goods that we take equal to unity. This implies that consumption may be represented by the aggregate index $Y = m \left(\prod_{i=1}^m y_i^{1/m} \right)$, which can be purchased at the corresponding price index $P = \prod_{i=1}^m p_i^{1/m}$.

Since equally aged consumers are identical and are identically treated at equilibrium, we can simply refer to the choices of an aggregate representative young consumer, born at t and choosing present labor supply L_t and future aggregate consumption Y_{t+1} in order to maximize expected utility $E_t U(Y_{t+1}) - V(L_t)$ subject to the budget constraints $P_{t+1} Y_{t+1} \leq M_t$ and $M_t \leq w_t L_t + \mathcal{D}_t$, where M_t is money demand, w_t the nominal wage and $\mathcal{D}_t$ the total amount of dividends received from firms. The first-order condition for this program may be written as

$$E_t \left(U'(Y_{t+1}) \frac{w_t}{P_{t+1}} \right) = V'(L_t), \quad (8)$$

with the two budget constraints binding at the optimum.

For simplicity, we shall restrict our attention to the case of isoelastic sub-utility functions $U(Y) = Y^{1-\sigma}/(1-\sigma)$ and $V(L) = vL^{1+\chi}/(1+\chi)$, with positive parameter values and $\sigma \neq 1$. With these assumptions, it is easy to see from the optimality condition (8) that the young consumer will save in the form of money all its income available at t , supplying an amount of labor

$$L_t = \left(E_t (Y_{t+1}^{1-\sigma}) \frac{w_t}{vM_t} \right)^{1/\chi}. \quad (9)$$

As regards the productive sector, a number N of potential producers compete

in each one of the m contestable oligopolistic industries, deciding in particular to be active or not according to their conjectures about other producers' decisions (required to be consistent at equilibrium). The individual production function is $y_t = l_t - \phi$, where y_t is output, l_t is labor used in production and $\phi > 0$ is a fixed (non-sunk) cost. Thus, we consider similar assumptions as in the previous section, except that in switching from partial to general equilibrium, the parameter c becomes an endogenous variable, w_t . By contrast, if we assume a constant stock of money M for all periods and because of the unit elasticity of intersectoral substitution, we may continue to take expenditure per sector as a parameter $b = M/m$, corresponding to the money holdings of old consumers divided by the number of industries.

The condition for output market equilibrium is the equality of aggregate supply and demand:

$$Y_t = \frac{M}{P_t}. \quad (10)$$

Individual sectoral prices are given by Cournotian prices, according to equation (7), so that the general price index P_t satisfies:

$$P_t = w_t \prod_{i=1}^m \left(\frac{n_{it}}{n_{it} - 1} \frac{1}{1 - \tau(n_{it}/n^*)} \right)^{1/m} \equiv w_t \prod_{i=1}^m \mu_{it}^{1/m}, \quad (11)$$

where, by symmetry, we have assumed in every industry the same target n^* and the same taxation scheme τ (with $\tau \equiv 0$ under laissez-faire and $\tau(1) = 0$ generally). Of course, the optimal target n^* will be derived endogenously when considering the second-best intertemporal allocation.

Equilibrium in the labor market also requires the equality of the corresponding aggregate demand (given by equation (9)) and supply ($\sum_{i=1}^m l_{it} = \sum_{i=1}^m y_{it} + \phi \sum_{i=1}^m n_{it}$, with $y_{it} = M/mw_t\mu_{it}$):

$$\left(E_t(Y_{t+1}^{1-\sigma}) \frac{w_t}{vM} \right)^{1/\chi} = \frac{M}{mw_t} \sum_{i=1}^m \frac{1}{\mu_{it}} + \phi \sum_{i=1}^m n_{it}. \quad (12)$$

The money market can be ignored since it clears by Walras law when both the output and the labor markets are in equilibrium.

Combining the market equilibrium conditions (10)–(12), it is easy to show that the general equilibrium of the economy may be represented by the following

non-autonomous one-dimensional dynamic system:

$$v\mu_t^G \left(\frac{\mu_t^G}{\mu_t^H} Y_t + \phi m n_t^A \right)^x Y_t = E_t (Y_{t+1}^{1-\sigma}), \quad (13)$$

where $\mu_t^G = \prod_{i=1}^m \mu_{it}^{1/m}$ and $\mu_t^H = m / \sum_{i=1}^m (1/\mu_{it})$ are the geometric and arithmetic markup means, respectively, and $n_t^A = (1/m) \sum_{i=1}^m n_{it}$ is the arithmetic mean of the numbers of active firms in each sector. Equation (13) is the fundamental equation defining the set of admissible dynamic trajectories consistent with the intertemporal general equilibrium of our economy. Clearly, as noticed earlier, equilibrium trajectories are strongly influenced by how coordination on a particular free entry equilibrium is obtained within each sector, through its effects on the geometric and harmonic markup means and on the total number of active firms. In the following subsection, we shall consider stationary deterministic and stochastic trajectories satisfying equation (13) under *laissez-faire*. The use of regulation through the taxation policy previously described would then rule out all but the deterministic trajectories by ensuring coordination on a specific equilibrium configuration within each sector.

3.2 Expectation-driven fluctuations in the *laissez-faire* economy

Assume as a starting step that the coordination process selects a time-invariant number of active firms n_i in each sector. Then it is easy to show that equation (13) has a unique deterministic stationary equilibrium Y obtained by solving the equation

$$v\mu^G \left(\frac{\mu^G}{\mu^H} Y + \phi m n^A \right)^x Y^\sigma = 1. \quad (14)$$

It can easily be verified that if $\sigma \leq 2$, the eigenvalue of the dynamic equation (13) has a modulus larger than 1 and the deterministic equilibrium Y exhibits the saddle path stability. In this case, the unique non-explosive equilibrium trajectory requires that output jump instantaneously and permanently to its unique (but not necessarily optimal) long-run stationary value Y . The dynamic system is then said to be *determinate in the dynamic sense*, since any coordination device that keeps constant the number of active firms in each industry excludes endogenous fluctuations. Since we are not interested in this paper in endogenous fluctuations resulting from dynamic indeterminacy, we shall from now on impose the parameter

restriction $\sigma \leq 2$.

However, as noted above, there is no reason to believe that a decentralized coordination scheme would spontaneously select a time-invariant number of active firms in each sector. When this is not the case, we shall prove that the set of admissible dynamics implied by equation (13) includes infinitely many trajectories with fluctuating aggregate variables.

To be explicit, consider K integers $\{n_1, \dots, n_K\}$ such that $2 \leq n_1 < \dots < n_K$ and $K > 1$, and define a *state* of the economy at time t as a vector $F_t = (f_{t1}, \dots, f_{tK})$ of proportions of industries that have coordinated in this period on the number n_k of active firms ($k = 1, \dots, K$). Obviously, there are infinitely many such vectors. A coordination scheme would then describe the set of vectors that are allowed in each period and also specify the transition process between states across time.

An interesting benchmark consists in assuming that the coordination process is a simple Markov chain. Specifically, we assume that there is an arbitrary number $R \in \mathbb{N} \setminus \{0, 1\}$ of possible states, indexed by $r = 1, \dots, R$ and characterized for each r by the vector of proportions F_r . The transition between states is governed by a $(R \times R)$ row-stochastic matrix $\mathbf{T}$ with elements T_{ij} satisfying $T_{ij} \equiv \Pr(r' = j \mid r = i)$, where a prime stands for next period. It is straightforward to verify that equation (13) may in this case be reformulated as a system of R equations (associated with states $r = 1, \dots, R$) characterizing a *stationary stochastic equilibrium*:

$$v\mu_r^G \left(\frac{\mu_r^G}{\mu_r^H} Y_r + \phi m n_r^A \right)^x Y_r = \sum_{r'=1}^R T_{rr'} Y_{r'}^{1-\sigma}, \quad (15)$$

where the markup means in state r are rewritten as $\mu_r^G = \prod_{k=1}^K \mu_k^{f_{rk}}$ and $\mu_r^H = 1 / \sum_{k=1}^K (f_{rk} / \mu_k)$, with $\mu_k = n_k / (n_k - 1)$, and the corresponding average number of active firms is also rewritten as $n_r^A = \sum_{k=1}^K f_{rk} n_k$.

Neglecting the profitability and sustainability conditions, existence of a solution $(Y_1, \dots, Y_R)$ to this system of equations can be established by a straightforward application of Brouwer's fixpoint theorem.

Lemma 1 *Assume $\sigma \leq 2$. Let $\{n_1, \dots, n_K\}$ be any set of integers larger than 1. Also let the family $(F_1, \dots, F_R)$ of possible states in the economy (a set of R points in the standard $(K - 1)$ -simplex Δ_{K-1}) and the transition matrix $\mathbf{T}$ be given. Then there exists a solution $(Y_1, \dots, Y_R)$ to the equation system (15).*

Proof. See appendix. ■

However, for such a solution to be an equilibrium, the profitability and sustainability conditions must also be satisfied, imposing restrictions on the set $\{n_1, \dots, n_K\}$ of admissible numbers of active firms (or else on the subset of Δ_{K-1} which contains the possible states in the economy). Remind that in the partial equilibrium context of section 2, these numbers were required to belong to some properly defined interval $[\underline{n}, \bar{n}]$. However, in the present general equilibrium context, this interval ceases to be determined by the sole parameters of the model. Indeed, according to conditions (4) and (5) in the previous section, $\bar{n} = \sqrt{b/c\phi}$ and $\underline{n} = \max \left\{ \sqrt{b/c\phi} / \left(2 - \sqrt{c\phi/b} \right), 2 \right\}$, with c now equal to w , an endogenous variable. More precisely, recalling that

$$b = \frac{M}{m} = \frac{P_r Y_r}{m} = \prod_{k=1}^K \left(\frac{n_k}{n_k - 1} \right)^{f_{rk}} \frac{w_r Y_r}{m}, \quad (16)$$

we must in fact refer to the conditions (imposed on all states $r = 1, \dots, R$):

$$n_k \leq \sqrt{\prod_{k'=1}^K \left(\frac{n_{k'}}{n_{k'} - 1} \right)^{f_{rk'}} \frac{Y_r}{\phi m}} \equiv \bar{n}_r \quad (17)$$

for profitability, and

$$n_k \geq \frac{\bar{n}_r}{2 - 1/\bar{n}_r} \equiv \underline{n}_r \quad (18)$$

for sustainability. As $\bar{n}_r$ and $\underline{n}_r$ depend on the aggregate output Y_r , which is also an equilibrium quantity, these conditions define, *for each state* r , an admissible interval $[\underline{n}_r, \bar{n}_r]$, containing the integer numbers n_k of active firms for which $f_{rk} > 0$ is allowed in state r , with $f_{rk} = 0$ for all numbers outside the interval. It is this interdependence which makes far from trivial the task of determining general conditions for existence of a solution $(Y_1, \dots, Y_R)$ to the system (15) that also satisfies the profitability and sustainability conditions at any state r .

Notice however that when there is *full symmetry* across industries as regards the number of active firms in each state (F_r belongs to the canonical basis of $\mathbb{R}^K$ for any state r) and when the present state is expected to last with probability one (the transition matrix is the identity matrix), it results from equation (15) that the conditions (17) and (18) can be stated as $\underline{n}(n) \leq n_k \leq \bar{n}(n)$, where $\underline{n}$ and $\bar{n}$ are state-independent. One can then uniquely define, as in the partial equilibrium

context, an admissible interval $[\underline{n}, \bar{n}]$ depending on the sole parameter values, at least under a condition stated in the following lemma.

Lemma 2 *Consider a set of numbers $n_k \in \mathbb{N} \setminus \{0, 1\}$ ($k = 1, \dots, K$), and let $F_r = e_r$ ($r = 1, \dots, R$, with $R = K$) and $\mathbf{T} = I$. If $\sigma \geq (1 - \chi)/3$, there exists a state-independent admissible interval $[\underline{n}, \bar{n}]$ such that profitability and sustainability require n_k to belong to that interval.*

Proof. See appendix. ■

Existence of a state-independent admissible interval $[\underline{n}, \bar{n}]$, in which the set $\{n_1, \dots, n_K\}$ should be included in order to ensure profitability and sustainability of a solution $(Y_1, \dots, Y_R)$ to the equation system (15), cannot be established in general. Thus, it is only after that solution is determined (we know by Lemma 1 that it exists) that we must verify, for each state r , that the interval $[\underline{n}_r, \bar{n}_r]$ as defined by (17) and (18) contains the set $\{n_1, \dots, n_K\}$ (or the subset which corresponds to the non-zero elements of F_r). However, we can find a range of parameter values for which a state-independent admissible interval can still be defined, whatever the family of states $(F_1, \dots, F_R)$ and the transition matrix $\mathbf{T}$. This is stated in the following proposition.

Proposition 3 *Assume $(1 - \chi)/3 \leq \sigma \leq 2$, and take any family of possible states in the economy $(F_1, \dots, F_R) \in (\Delta_{K-1})^R$ and any transition probabilities $(T_1, \dots, T_R) \in (\Delta_{R-1})^R$. Consider the set $\{n_1, \dots, n_K\}$ of integers introduced in Lemma 1 and the interval $[\underline{n}, \bar{n}]$ referred to in Lemma 2. Then, for*

$$\chi \geq \frac{1 + \sqrt{1 - 1/\underline{n}}}{\underline{n}} + \frac{1}{\underline{n} - 1} \equiv X(\underline{n}) \text{ or } \chi \leq X(\bar{n}),$$

a stationary stochastic equilibrium $(Y_1, \dots, Y_R)$ exists if the set $\{n_1, \dots, n_K\}$ is included in some admissible state-independent interval $[n_{\inf}, n_{\sup}] \subset [\underline{n}, \bar{n}]$. Moreover, $[n_{\inf}, n_{\sup}] = [\underline{n}, \bar{n}]$ if $\sigma < 1$ and $\chi \geq X(\underline{n})$.

Proof. See appendix. ■

With non-degenerate transition matrices, if the admissible interval contains more than one integer, aggregate real output will fluctuate stochastically among its R potential (generically different) values. With degenerate transition matrices, in particular when all the rows of $\mathbf{T}$ belong to the canonical basis of $\mathbb{R}^R$, real output

will tend to a deterministic cycle of order $q \leq R$ (possibly after a transition period of finite time). Clearly, even when there is no extrinsic uncertainty affecting economic fundamentals, the implicit coordination procedures called for by the multiplicity of free entry equilibria may be the source of substantial economic instability.

3.3 Regulation through efficient coordination

To avoid inefficient sunspot fluctuations, the government may find desirable to intervene. An extreme form of regulation would impose optimal production plans and resource utilization to all agents – a situation which will be referred to as the *first best allocation*. Analyzing the first best allocation is important in order to understand the nature of the distortions involved in a competitive (decentralized) economy and the trade-offs involved in public regulation. However, the first best allocation requires an extreme form of government intervention which is in many ways unrealistic or undesirable. A less extreme form of regulation would preserve market structure and the freedom of firms to set their price, while using taxation/subsidization schemes in order to affect individual incentives. By selecting a well-defined taxation scheme (as described in section 2), the government may then ensure coordination on a (more plausible) second best allocation.

First best allocation. Consider a social planner with utilitarian preferences over current and future generations:

$$W(Y_1, \dots, L_0, \dots) = \sum_{t=0}^{\infty} \frac{[U(Y_{t+1}) - V(L_t)]^{1-\rho}}{1-\rho}, \quad (19)$$

where $\rho > 0$ may be seen as the inverse of the elasticity of substitution between generations in the social planner's preferences or, alternatively, as the degree of relative risk aversion of the social planner. As ρ increases, the social planner cares proportionately more about the welfare of the poorer generations over time, so that ρ can also be interpreted as a measure of inequality aversion. In the limit, when ρ tends to infinity, the welfare function ends up in the Rawls criterion, with a social planner only concerned about the utility of the poorest generation.

We define the *first best* allocation as the solution in $(Y_{t+1}, L_t, (n_{it})_i)_{t=0}^{\infty}$ to the maximization⁵ of welfare $W(Y_1, \dots, L_0, \dots)$ under the technological constraint (for

⁵Strictly speaking, the maximization problem over an infinite horizon is not well defined since

$t = 0, \dots, \infty$):

$$Y_t \leq L_t - \phi \sum_{i=1}^m n_{it}, \quad \text{with } n_{it} \in \mathbb{N}^* \text{ for } i = 1, \dots, m. \quad (20)$$

A particular feature of the first best allocation is that the social planner can "choose" the number of active firms in each sector and in each period, avoiding the difficulties raised by the implicit coordination problem in a competitive equilibrium. Characterizing the first best allocation is then easy. Since there is no predetermined variable and the economy is symmetric across periods and sectors, successive generations confront a time invariant environment. The risk-averse social planner would thus eliminate sunspot fluctuations by confining the economy to stationary symmetric trajectories with the same constant number of active firms in any industry i and any date t . Also, economies of scale resulting from fixed costs ϕ imply that each industry is a natural monopoly, so that a social planner would obviously require that output be produced by a single firm in each industry along an optimal trajectory ($n_{it} = n = 1$ for $i = 1, \dots, m$ and $t = 0, \dots, \infty$). Finally, an optimal stationary trajectory with one active firm per industry must also satisfy the first-order condition for the representative consumer's maximization problem, namely the equality of the marginal utility of consumption Y^* and the marginal disutility of labor $L^* = Y^* + \phi m$

$$U'(Y^*) = V'(Y^* + \phi m) \quad (21)$$

Using the specification for consumers' preferences, this condition may be written as

$$(Y^*)^{-\sigma} = v(Y^* + \phi m)^x, \quad (22)$$

implicitly defining the optimal (stationary) output Y^* .

To sum up, in a first best solution the social planner would fully exploit increasing returns to scale by allocating all the production activity to a *single* productive firm in each industry. Naturally, the social planner would also want to eliminate

it concerns an undiscounted series. Following von Weizsäcker (1965) and Gale (1967), we should more properly refer to the choice of a sequence $(Y_{t+1}, L_t, (n_{it})_i)_{t=0}^\infty$ that *overtakes* all the other feasible sequences, meaning a sequence that at some point of time "has provided more utility than the other[s] and [...] continues to do so from that point on" (Gale, 1967, p.3). Applying this criterion is however trivial in the present context of an invariant environment.

any market power distortion resulting from this monopoly position. Hence, the first best solution can be implemented in a market economy only if the regulatory agency is capable of imposing marginal cost pricing to monopolistic firms, while subsidizing them to cover their fixed costs.

Second best allocation. By contrast, we define a second best allocation as an allocation preserving market structure and the freedom of firms to set their price, but resulting from the capacity of the government to commit to a predetermined (optimal) taxation/subsidization scheme that ensures coordination on a particular equilibrium at each date and in each sector.

Compared to the first best allocation, the intervention possibilities of the government are limited in two dimensions. First, in a competitive economy characterized by free entry in each sector, the government can only select (through appropriate taxation) a number of active firms in each sector which belongs to the admissible interval $[\underline{n}, \bar{n}]$, defined according to Lemma 2. Second, the social planner must take into account, in addition to the technology constraints $y_{it} \leq l_{it} - \phi n_{it}$, the three additional incentive compatibility and market equilibrium constraints (10)–(12) leading to the dynamic system (13). Formally, the second best allocation is obtained as the solution in $((y_{it}, l_{it}, n_{it})_i)_{t=0}^\infty$ to the maximization of welfare $W(Y_1, \dots, L_0, \dots)$, with $Y_t = m \Pi_{i=1}^m y_{it}^{1/m}$ and $L_t = \sum_{i=1}^m l_{it}$, under the constraints (for $t = 0, \dots, \infty$):

$$\begin{aligned} y_{it} &\leq l_{it} - \phi n_{it}, \quad \text{with } n_{it} \in (\mathbb{N} \setminus \{0, 1\}) \cap [\underline{n}, \bar{n}] \text{ for } i = 1, \dots, m, \\ v \mu_t^G \left(\frac{\mu_t^G}{\mu_t^H} Y_t + \phi m n_t^A \right)^x Y_t &= Y_{t+1}^{1-\sigma}, \end{aligned} \quad (23)$$

where $n_t^A = (1/m) \sum_{i=1}^m n_{it}$, $\mu_t^G = \Pi_{i=1}^m \mu_{it}^{1/m}$ and $\mu_t^H = m / \sum_{i=1}^m (1/\mu_{it})$ (with $\mu_{it} = n_{it} / (n_{it} - 1)$).

Notice that in this problem the coordination process in each sector is no longer ensured by the exogenous transition matrix $\mathbf{T}$. By committing to a predetermined taxation/subsidization scheme for any sector and any date, the government can make the dynamic equilibrium path deterministic. Characterizing the second best allocation is then simple. As in the former case, the strict concavity of the social planner's utility function implies that stationary paths are preferred to fluctuating paths. Along a symmetric stationary trajectory with $n_{it} = n$ for all industries and all dates, the second best aggregate production level $Y(n)$ is then determined by

the welfare maximizing value of $n \in [\underline{n}, \bar{n}]$ along a stationary path, i.e. the one satisfying the optimality condition

$$U'(Y(n)) = \frac{n}{n-1} V'(Y(n) + mn\phi) \quad (24)$$

Comparing (24) to (21), we see that the social planner faces two sources of inefficiencies in the second-best environment: (i) a *market power distortion*, reflected by the term $n/(n-1)$ in (24), which reduces labor supply compared to the first-best allocation ; (ii) a *scale inefficiency effect*, which results from the duplication of the fixed cost of production as the number of competitors in each sector increases. Obviously, the first distortion is decreasing in n while the second one is increasing in n , so that there is a trade-off involved in regulation. We cannot tell in general which of the two effects is going to dominate, but it is clear that the relative shapes of the functions $U(\cdot)$ and $V(\cdot)$ of the representative consumer's preferences are crucial in the arbitrage. Using the specific functional form retained, the optimality condition (24) becomes:

$$Y(n)^{-\sigma} = \frac{n}{n-1} v(Y(n) + \phi mn)^{\chi} \quad (25)$$

and the problem of the social planner, equivalent to maximizing *in the number n of active firms* the utility of the representative agent of a representative generation along a stationary path:

$$\mathcal{U}(Y(n), n) = \frac{Y(n)^{1-\sigma}}{1-\sigma} - v \frac{(Y(n) + \phi mn)^{1+\chi}}{1+\chi} \quad (26)$$

can be written as, using (25),

$$\mathcal{U}(Y(n), n) = \frac{Y(n)^{1-\sigma}}{1-\sigma} - \frac{((1 - 1/n) Y(n)^{-\sigma})^{1+1/\chi}}{v^{1/\chi} (1+\chi)}. \quad (27)$$

From inspection of (26) and (27), it is clear that labor supply elasticity (as measure by $1/\chi$) plays a dominant role. Equation (26) shows that, for any level of production $Y(n)$, an increase in χ increases the disutility of labor, making the utility cost of the scale inefficiency effect higher. Meanwhile, equation (27) shows that, as χ increases, labor supply becomes less and less sensitive to the real wage and, thus, to the market power distortion effect. It is then clear that when labor supply is relatively inelastic (i.e., for large χ 's) welfare

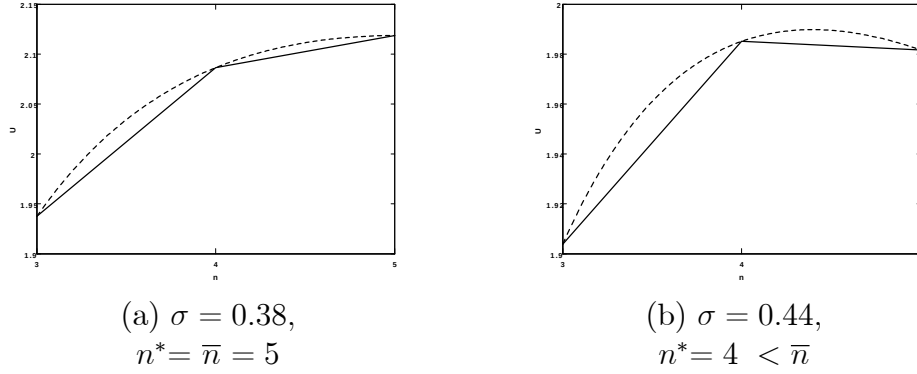


Figure 3: Utility of the representative agent in a symmetric economy with n active firms per sector.

is unambiguously maximized at the *smallest* integer in the admissible interval $[\underline{n}, \bar{n}]$. By contrast, when χ is small, the market power distortion effect becomes significant. So, when the labor supply elasticity is high, there is a real trade-off involved. For instance, in the limit case of a perfectly elastic labor supply ($\chi = 0$) we obtain, by (25) and (26), $Y(n) = ((1 - 1/n)/v)^{1/\sigma}$ and $\mathcal{U}(Y(n), n) = Y(n)^{1-\sigma} / (1 - \sigma) - v(Y(n) + \phi mn)$. Aggregate production is increasing in n , but utility $\mathcal{U}(Y(\cdot), \cdot)$ is strictly quasi-concave.⁶ Depending on whether the value $\tilde{n}$ maximizing $\mathcal{U}(Y(\cdot), \cdot)$ on the unconstrained interval $[2, \infty)$ is outside or in the interior of the admissible interval $[\underline{n}, \bar{n}]$, the representative young consumer's utility $\mathcal{U}$ may have a maximum either at the lowest, the largest or an interior integer of this interval. Figure 3 illustrates the latter two possibilities when labor supply is elastic, using the following calibration: $\chi = 0$, $v = 0.4$, $\phi = 0.2$ and $\sigma = 0.38$ (case (a)) or $\sigma = 0.44$ (case (b)).

4 Welfare gains of stabilization

The generic existence of sunspot-driven fluctuations in economies with imperfectly competitive markets also sheds new light on the traditional issues of stabilization policy and of the welfare costs of fluctuations. Since the well-known paper by Lucas (2003), it is often argued that the benefits from reducing further the variability

⁶We easily obtain:

$$\frac{d\mathcal{U}(Y(n), n)}{dn} = v \left(\frac{(n-1)^{1/\sigma-2}}{\sigma v^{1/\sigma} n^{1+1/\sigma}} - \phi m \right),$$

a decreasing function of n , tending to $-v\phi m$ as n tends to ∞ . Hence, $\mathcal{U}(Y(\cdot), \cdot)$ is strictly quasi-concave and eventually decreasing.

of aggregate consumption are very small, typically of an order of magnitude which is less than a tenth of a percent of total consumption. In addition, stabilization (or equivalently, in the present model, the reduction of inequality among generations) is often seen as conflicting with efficiency since the main instrument of redistribution — distortive taxation — is believed to generate an inefficient reallocation of resources. In this section, we show that, in the present context of strategic indeterminacy, these two objectives, far from being conflicting, are instead complementary. Because stabilization can be obtained *together with* efficiency gains, it could lead to substantial welfare gains. We use our framework to quantify these potential gains. We decompose these gains into two easily interpretable components: a "pure stabilization effect", which merely captures the effect of a reduction in the variance of consumption for stochastic consumption streams of given mean, and an "efficient stabilization effect" which measures the effects associated with an efficient reorganization of production activities. In line with Lucas (2003), we show that the welfare gains resulting from the pure stabilization effect are very small. However, the efficient stabilization effect leads to sizeable welfare gains that are quantified below.

Before going further, we shall mention at this point that, while the present framework of overlapping generations has the obvious advantage of enabling us to straightforwardly derive clearcut and rigorous existence results on stationary equilibria influenced by sunspots, it is not casted into the standard DSGE-type model which is typically used to evaluate the welfare costs of fluctuations. We nonetheless believe that this framework is a good first approximation to illustrate the potentially significant welfare gains that can be obtained from applying the proposed economic policy. While, strictly speaking, the parameter ρ in the social welfare function is a natural measure of the social planner's aversion toward inequality between generations, it could also be considered as a standard measure of relative risk aversion of an infinitely-lived representative agent maximizing intertemporal utility, as in standard business cycle models. Under both interpretations, as ρ increases, the social planner dislikes more and more fluctuations in aggregate consumption paths with identical means.

4.1 Calibration

In order to get some magnitude about the welfare gains obtained by applying the proposed stabilization policy, we need to find a calibration for the structural

parameters and for the states and transition matrices that generate a realistic stochastic process for aggregate output. The number m of sectors is only a scale parameter, and we arbitrarily set it to $m = 1$. Following Hansen (1985), we assume an infinitely elastic aggregate labor supply by setting $\chi = 0$. We know from the last section that with this assumption, sectoral output is increasing in the number of producing firms in each sector but welfare may be maximized for any integer in the admissible interval $[\underline{n}, \bar{n}]$, depending on the values of the other structural parameters. We chose a calibration for σ , v and ϕ that ensures that the admissible interval for the number of active firms in each sector always includes $[3, 5]$ (in each state), while the optimal number of active firms per sector is *interior* and equal to $n^* = 4$. There are some degrees of freedom in this choice, but the quantitative results are only marginally affected. A reasonable calibration accomplishing this task is $v = 0.4$, $\phi = 0.2$ and $\sigma = 0.44$. This is the configuration illustrated in Figure 3(b). With this calibration, output increases from 3.19 to 4.83 as n increases from 3 to 5 firms per sector, but the optimal level of aggregate output, obtained when the number of active firms per sector is equal to $n^* = 4$ in each sector, is $Y^* = 4.17$.

A more difficult task in the calibration step consists in specifying the states and transition probabilities adequately. In order to achieve this step, we adapted the numerical procedure described in Tauchen (1986) so as to obtain a set of states $(F_1, \dots, F_R) \in (\Delta_{K-1})^R$ and transition probabilities $(T_1, \dots, T_R) \in (\Delta_{R-1})^R$ that generate a simulated path for the log of aggregate output that mimics as closely as possible a standard AR(1) process, while the number of active firms in each sector and each state $r = 1, \dots, R$ remains in the corresponding admissible interval $[\underline{n}_r, \bar{n}_r] \supset [3, 5]$. The number of states R can be chosen arbitrarily, but precision is of course increased for larger R 's. In our simulations, we found that imposing $R = 7$ different states was sufficient to obtain a reasonable approximation.

Using annual US data for the period 1947-2010, the log of real aggregate output about a linear trend is reasonably well approximated by an AR(1) process with autoregressive parameter $\varphi = 0.90$ and standard deviation $\sigma_\varepsilon = 0.023$. In our benchmark scenario, we thus computed the states and transition matrix so that simulated aggregate production mimics an identical AR(1) process around a mean value which we set at $Y = 4.17$, the same aggregate production level as in the second-best economy. The following matrix F (containing the vectors of proportions of sectors producing with $n = 3, 4$ and 5 firms, respectively, in each

Table 1 - Aggregate data for the simulated economy

States: $r =$	1	2	3	4	5	6	7
Y_r	3.86	3.96	4.07	4.17	4.28	4.39	4.51
n_r^A	3.75	3.91	4.05	4.19	4.33	4.48	4.67
μ_r^G	1.39	1.37	1.35	1.33	1.32	1.30	1.28
Φ_r	.20	.20	.20	.20	.20	.20	.21

state) and transition matrix T accomplish this task, for $r = 1, \dots, 7$:

$$F = \begin{bmatrix} .47 & .30 & .23 \\ .38 & .32 & .30 \\ .30 & .34 & .36 \\ .23 & .35 & .42 \\ .16 & .34 & .50 \\ .10 & .32 & .58 \\ .04 & .24 & .72 \end{bmatrix} \quad T = \begin{bmatrix} .59 & .32 & .08 & .01 & 0 & 0 & 0 \\ .22 & .41 & .29 & .07 & .01 & 0 & 0 \\ .04 & .21 & .42 & .27 & .06 & 0 & 0 \\ 0 & 0.05 & .24 & .42 & .24 & .05 & 0 \\ 0 & 0 & .06 & .27 & .42 & .21 & .04 \\ 0 & 0 & .01 & .07 & .29 & .41 & .22 \\ 0 & 0 & 0 & .01 & .08 & .32 & .59 \end{bmatrix}.$$

Thus, for example, when the current state is $r = 4$, there are 23% of sectors which produce with 3 active firms, 35% of them producing with 4 active firms, and 42% of them with 5 active firms. The probability of remaining in the same state for the next period is 42%, while the probability of moving to, *e.g.*, state $r' = 5$ is 24%. With this specification, as noticed, the long run production level is $\bar{Y} = Y_4 = 4.17$. The long run average number of firms per sector is $n_4^A = (.23, .35, .42) \cdot (3, 4, 5) = 4.19$, the long run markup factor is $\bar{\mu}^G = 1.33$ and the long run degree of increasing returns to scale, as measured by $\bar{\Phi} = (\bar{Y} + \bar{n}^A \phi m) / \bar{Y} - 1$, is 0.20.⁷ Table 1 provides these summary statistics for each state $r = 1, \dots, 7$:

4.2 Welfare gains

We simulated $T = 10000$ times the Markov process described above, and computed the social planner's welfare $\widetilde{W}_T \equiv \sum_{t=1}^T ([U(Y_{t+1}) - V(L_t)]^{1-\rho} / (1-\rho))$ over this

⁷Note that, in this model, increasing returns to scale appear under the weak form of fixed costs in production. Aggregate increasing returns to scale of about 20% are consistent with the findings of Burnside *et al.* (1997) when they take the form of fixed costs. Markup rates of 33% are typically in the upper range of available empirical estimates. Note however that, in this literature, markup rates are typically obtained as a by-product of the estimated degree of aggregate increasing returns to scale, *imposing the zero profit assumption*. Relaxing this assumption would have led these studies to conclude to significantly larger estimated markups.

horizon. Then, following Lucas' approach, we calculated the proportion λ_c of permanent consumption that the social planner would be willing to sacrifice in each period in order to get rid off aggregate uncertainty and have the economy stabilized around an aggregate production level $\bar{Y}$, i.e. the value of λ_c for which $\widetilde{W}_T = \sum_{t=0}^T \left([U((1 - \lambda_c)\bar{Y}) - V(\bar{L})]^{1-\rho} / (1 - \rho) \right)$. We provide this coefficient for two hypothetical configurations. In the first situation, we assume that output is stabilized on the *median* state $r = 4$, corresponding to a situation in which output is stabilized by "freezing" permanently the equilibrium configuration prevailing in each sector. In the second situation, we assume that output is stabilized *efficiently* by imposing that each sector produces with $n^* = 4$ active firms (*e.g.*, by imposing the proposed taxation scheme). The first experiment enables us to compute the welfare gains resulting from the "pure stabilization" effect, *i.e.* the effect of removing consumption uncertainty without improving average efficiency in the economy. The second experiment enables us to estimate the total stabilization effect, including the effect of stabilizing the economy efficiently together with the pure stabilization effect. Results from these experiments are given in Table 2, for different values of the risk aversion parameter ρ .

As Table 2 shows, the benefits implied by the "pure stabilization" effect are very small and in line with those obtained in Lucas (2003). For example, when $\rho = 1$, aggregate fluctuations generate a cost for the social planner which is equivalent to a permanent reduction of its consumption level of about one seventeenth of a percent ($\lambda_c = 0.06$). However, the welfare gains resulting from the total stabilization effect (including that of the efficient reorganization of production) are much more significant, with λ_c now equal to 0.93. This means that the same social planner would now be willing to sacrifice almost 1 percent of its consumption level in each period in order to have the economy stabilized on the efficient production level — 15.5 times as much as in the first configuration. As the relative risk aversion of the social planner increases, the ratio of the total to the pure stabilization effects decreases, but it remains equal to more than 3.5 at $\rho = 10$.

4.3 Understanding the welfare gains

How can we explain this sharp discrepancy between our results and those obtained by Lucas (2003), while preferences are similar and the variance of aggregate consumption is approximately the same? Clearly, the difference comes from the *efficient stabilization effect*. To get a better sense of this effect, notice that

Table 2 — Welfare gains (λ_c , in percent of consumption)

	$\rho = 1$	$\rho = 2$	$\rho = 4$	$\rho = 10$
Stabilization on median state (pure stabilization effect)	0.06	0.09	0.15	0.34
Stabilization on $n^* = 4$ (total stabilization effect)	0.93	0.96	1.02	1.21

Benchmark economy $Y(4) = Y^*$

when the economy is stabilized on the median state $r = 4$ (pure stabilization,) the aggregate production level $Y_4 = Y^* = 4.17$ is reached while several sectors produce at a suboptimal individual scale of production (23% of sectors produce with 3 active firms, and 42% of them produce with 5 active firms). At the aggregate level, this inefficiency can be seen by observing that this production level Y^* is obtained with an average number of active firms, $n_4^A = (.23, .35, .42) \cdot (3, 4, 5) = 4.19$, which is larger than the optimal number of firms per sector, $n^* = 4$. Thus, the aggregate quantity of labor required to produce Y^* in the median state is also larger than the corresponding quantity of labor in the second-best economy: $L_4 = (\mu_4^G/\mu_4^H)Y^* + n_4^A m\phi > L^* = Y^* + n^* m\phi$. Following Lucas (2003), we can then quantify this inefficiency cost in utility equivalents for the social planner. For example, the utility cost of having the economy stabilized on the median state $r = 4$ (rather than on the second-best equilibrium) can be measured by calculating the additional quantity of labor, expressed in percentage points of labor, required to produce the same aggregate production level $Y_4 = Y^*$. This amounts to compute the parameter λ_L such that $\tilde{\mathcal{U}}(Y^*, L_4) = \tilde{\mathcal{U}}(Y^*, (1 + \lambda_L)L^*)$, where L^* is the efficient quantity of labor. By additivity ($\tilde{\mathcal{U}}(Y, L) = Y^{1-\sigma}/(1 - \sigma) - \nu L$), we simply obtain:

$$1 + \lambda_L = \frac{L_4}{L^*} = \frac{(\mu_4^G/\mu_4^H)Y^* + n_4^A m\phi}{Y^* + n^* m\phi}.$$

Using $m = 1$, $\phi = 0.2$, $Y^* = 4.17$ and $F_4 = (.23, .35, .42)$ in state 4, we easily compute $\lambda_L = 0.0091$, a utility cost equivalent to a 0.91 percent permanent increase in aggregate labor supply. To make this measure more directly comparable to the one obtained in Lucas (2003), we can easily translate this cost in terms of consumption equivalents using the formula $\tilde{\mathcal{U}}(Y^*, (1 + \lambda_L)L^*) = \tilde{\mathcal{U}}((1 - \lambda_C)Y^*, L^*)$. With our specification for preferences and $\chi = 0$, we then obtain

$$\lambda_C = 1 - \frac{(Y^{*1-\sigma} - v(1-\sigma)\lambda_L(Y^* + n^*\phi))^{\frac{1}{1-\sigma}}}{Y^*},$$

a number which is equal to $\lambda_C = 0.0087$ for the calibration considered. Thus, looking at Table 2 and considering for example the case $\rho = 1$, our results indicate that the welfare gains resulting from the "pure stabilization effect" are equivalent to a 0.06 percent permanent increase in consumption while the welfare gains resulting from the "efficient stabilization effect" amount to a 0.87 percent such increase. We obtain in total $\lambda_c = 0.06 + 0.87 = 0.93$, which is consistent with the numerical findings reported in Table 2.

As a final remark, observe that these results were obtained while assuming that the economy was fluctuating around a mean value for output which was identical to the second-best optimal value $Y^* = 4.17$. This assumption was made for simplicity, but it should be clear that it only provides a *lower bound* on the size of welfare gains that are obtained from the efficient stabilization effect. To see this, consider for example an alternative economy in which output fluctuates according to the same AR(1) process (with identical autoregressive coefficient and variance) but around a mean value which is now 12% lower than the efficient output level ($Y_4 = 3.67 < Y^* = 4.17$). This situation can be obtained, keeping the same transition matrix T , by using the following matrix F' of vectors of proportions of sectors producing with 3, 4 or 5 active firms in each state $r = 1, \dots, 7$:

$$F' = \begin{bmatrix} .85 & .10 & .05 \\ .74 & .17 & .09 \\ .65 & .23 & .12 \\ .57 & .26 & .17 \\ .49 & .29 & .22 \\ .41 & .32 & .27 \\ .32 & .32 & .35 \end{bmatrix}$$

The welfare gains obtained in this situation are reported in Table 3. As the table shows, the welfare gains corresponding to the pure stabilization effect are still very small and are actually smaller than in the reference economy (for example, $\lambda_c = 0.02$ when $\rho = 1$, as opposed to 0.06 in the reference economy). However, the gains resulting from the efficient stabilization effect are now much larger, exceeding 2 percent of permanent increase in average consumption.

Table 3 — Welfare gains (λ_c , in percent of consumption)

	$\rho = 1$	$\rho = 2$	$\rho = 4$	$\rho = 10$
Stabilization on median state (pure stabilization effect)	0.02	0.04	0.10	0.27
Stabilization on $n^* = 4$ (total stabilization effect)	2.08	2.10	2.15	2.31

Alternative economy: $Y(4) < Y^*$

5 Concluding remarks

We have explored *strategic indeterminacy*, arising in the context of oligopolistic competition, as a convenient alternative to dynamic indeterminacy in generating macroeconomic sunspot fluctuations. The case for this alternative is here presented in a very basic framework, that of standard Cournot competition with free entry in the presence of fixed costs (as concerns microeconomics), and of a simple overlapping generations macro model. Since strategic indeterminacy is shown to be the source of costly sunspot fluctuations, a straightforward way of stabilizing the economy is to ensure determinacy by destroying undesirable, inefficient, equilibria. This is achieved, in the present context, by converting fixed into variable costs (or vice versa) through a taxation-subsidization policy which happens to be inactive, hence non distortive, in the only remaining equilibrium. Corresponding policies, acting as pure selection mechanisms designed to stabilize the economy at the efficient equilibrium, might also apply in other contexts of strategic indeterminacy. Finally, we have quantified the welfare gains of this stabilization policy in a calibrated version of our model, fitting US data. These gains appear to be much larger than those estimated by Lucas (2003) on the basis of a *pure* stabilization effect benefiting risk averse consumers. The difference between Lucas's estimation and our own results can be entirely ascribed to *efficient* stabilization, that is, to elimination of inefficient states – an effect that would be praised even by risk neutral consumers.

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Appendix

Proof of Proposition 1

Taking the expression for profit (6), making it equal to zero and referring to symmetric profiles with $y = y_n = \bar{Y}/(n-1)$, we see that the corresponding *break-even price* is

$$\underline{p}(n, n^*) = \frac{c}{1 - \tau(n/n^*) - (c\phi - T(n/n^*))n/b}, \quad (28)$$

which, taking condition (SBB) into account, simplifies to

$$\underline{p}(n, n^*) = \frac{c}{1 - nc\phi/b}. \quad (29)$$

When the government balances its budget at the sectoral level, this price is thus left unaffected by the taxation policy, in contrast to the *Cournotian price* given by (7). By using the first order condition for maximization in y of $\Pi(y, ny_n, (n+1)/n^*)$ by a potential entrant, and making the corresponding (interior) maximum equal to zero, we may easily compute the *limit price*

$$\bar{p}(n, n^*) = \frac{c}{\left(\sqrt{1 - \tau((n+1)/n^*)} - \sqrt{(c\phi - T((n+1)/n^*))}/b\right)^2} \quad (30)$$

or, taking (SBB) into account,

$$\bar{p}(n, n^*) = \frac{c}{\left(\sqrt{1 - \tau((n+1)/n^*)} - \sqrt{c\phi/b - \tau((n+1)/n^*)/(n+1)}\right)^2}, \quad (31)$$

a price which is also affected by the taxation policy.

It is now easy to verify that an equilibrium with $n < n^*$ active firms can always be made unsustainable by the choice of a high enough taxation rate. Indeed, take for any $n/n^* < 1$ the boundary value $\tau(n/n^*) = nc\phi/b = n/\bar{n}^2$ which, by condition (SBB), leads to a complete subsidization of the fixed cost ($T(n/n^*) = c\phi$). The unsustainability condition $\bar{p}(n, n^*) < p^*(n, n^*)$ for $n < n^* - 1$ then translates into $2n < \bar{n}^2$, an inequality which is always satisfied, since $2 \leq n < n^* \leq \bar{n}$. For $n = n^* - 1$, as $\tau((n+1)/n^*) = 0$, the unsustainability condition is equivalent to $(1 - n/\bar{n})^2 > 0$, which again is always satisfied.

Proof of Proposition 2

The expressions for the Cournotian, break-even and limit prices are the same as before, even if the taxation rate τ is now negative. From (29) and (7), together with condition (SBB), it is easy to see that the condition $p^*(n, \hat{n}) < \underline{p}(n, \hat{n})$ is fulfilled for any high enough rate of subsidy on sales:

$$-\tau(\hat{n}) > \frac{1 - n^2 c \phi / b}{n - 1} = \frac{1 - (n/\bar{n})^2}{n - 1}. \quad (32)$$

However, we must also check that the equilibrium associated with n^* subsists. The profitability condition under *laissez-faire* is not modified, since $\tau(0) = 0$. By contrast, sustainability requires the Cournotian price $p^*(n^*, 0)$ to be at most equal to the modified limit price $\bar{p}(n^*, 0)$ given by (31). If we replace the subsidy $-\tau(-1)$ by its upper lower bound in (32) with $n = n^* + 1$, the sustainability condition can be easily shown to be always (strictly) satisfied.

Proof of Lemma 1

Take the r -th equation of the system (15). Given the set of possible numbers of active firms $\{n_1, \dots, n_K\}$ (with $2 \leq n_1 < \dots < n_K$) and the r -th state $F_r = (f_{r1}, \dots, f_{rK})$ of the economy together with the corresponding transition probability vector $(T_{r1}, \dots, T_{rR})$, the output value Y_r at this state can be determined as a function η_r of the output values $(Y_1, \dots, Y_R)$ at all the states of the economy (through the weighted arithmetic mean on the RHS of the equation). For consistency, we must of course look for a fixed point of the continuous mapping $\eta = (\eta_1, \dots, \eta_R)$. Existence of such a fixed point results from Brouwer's theorem, provided we can establish that η maps some non-empty set $[\underline{Y}, \bar{Y}]^R$ into itself.

First consider the case $\sigma < 1$. Since $Y^{1-\sigma}$ is then increasing in Y , $\underline{Y}^{1-\sigma} \leq \sum_{r'=1}^R T_{rr'} Y_{r'}^{1-\sigma} \leq \bar{Y}^{1-\sigma}$ for any $(Y_1, \dots, Y_R) \in [\underline{Y}, \bar{Y}]^R$, any $\mathbf{T}$ and any r . Given $\{n_1, \dots, n_K\}$, the means μ_r^G and μ_r^H are functions of F_r , and their ratio is a continuous function defined on the compact set Δ_{K-1} , which has a maximum $\max_{F \in \Delta_{K-1}} (\mu^G(F) / \mu^H(F)) = \hat{\mu} > 1$ and a minimum $\min_{F \in \Delta_{K-1}} (\mu^G(F) / \mu^H(F)) = 1$. As $n_1 \leq n_k \leq n_K$ and $n_K / (n_K - 1) \leq n_k / (n_k - 1) \leq n_1 / (n_1 - 1)$ for any k , and as the LHS of any equation in (15) is increasing in Y , we may choose the values $\underline{Y}$ and $\bar{Y}$ that solve, respectively, the equations

$$v \frac{n_1}{n_1 - 1} (\hat{\mu} \underline{Y} + \phi m n_K)^x \underline{Y} = \underline{Y}^{1-\sigma} \text{ and } v \frac{n_K}{n_K - 1} (\bar{Y} + \phi m n_1)^x \bar{Y} = \bar{Y}^{1-\sigma}. \quad (33)$$

Indeed, for any $(Y_1, \dots, Y_R) \in [\underline{Y}, \overline{Y}]^R$ and any r , $\underline{Y} < \eta_r(Y_1, \dots, Y_R) < \overline{Y}$, as required. Notice that $\underline{Y}$ is always smaller than $\overline{Y}$, so that $[\underline{Y}, \overline{Y}]^R$ is non-empty (and non-degenerate).

We use the same kind of argument for $\sigma > 1$ but, as $Y^{1-\sigma}$ is now decreasing in Y , we take the values $\underline{Y}$ and $\overline{Y}$ that solve simultaneously the equations:

$$v \frac{n_1}{n_1 - 1} (\hat{\mu} \underline{Y} + \phi m n_K)^\chi \underline{Y} = \overline{Y}^{1-\sigma} \text{ and } v \frac{n_K}{n_K - 1} (\overline{Y} + \phi m n_1)^\chi \overline{Y} = \underline{Y}^{1-\sigma}. \quad (34)$$

However, by dividing both sides of the first equation by the corresponding sides of the second, we see that they together imply:

$$1 < \frac{1 - 1/n_K}{1 - 1/n_1} = \left(\frac{\overline{Y} + \phi m n_1}{\hat{\mu} \underline{Y} + \phi m n_K} \right)^\chi \left(\frac{\overline{Y}}{\underline{Y}} \right)^{2-\sigma} < \left(\frac{\overline{Y}}{\underline{Y}} \right)^{2-\sigma+\chi}.$$

Since we want $\underline{Y}$ to be smaller than $\overline{Y}$, the two inequalities imply $2 - \sigma + \chi > 0$, which is only possible, for χ arbitrarily close to 0, if $\sigma \leq 2$ (the condition which entails dynamic determinacy).

Proof of Lemma 2

In state $r = k$, there are n_k firms actually producing in any industry, and the aggregate output $Y(n_k)$ (which, as $T_k = e_k$, is expected to be realized in next period with probability 1) is determined by equation (14), with $\mu^G = \mu^H = n_k / (n_k - 1)$ and $n^A = n_k$. For simplicity of notation, we will omit the subscript k in the following. By condition (17), profitability requires:

$$n \leq \sqrt{\frac{nY(n)}{(n-1)\phi m}} \equiv \bar{n}(n). \quad (35)$$

If the elasticity of $\bar{n}$ is smaller than 1 for $n \geq 2$, the equation $n = \bar{n}(n)$ uniquely determines the least upper bound $\bar{n}$ for an admissible n . Similarly, sustainability requires by equation (18):

$$n \geq \frac{\bar{n}(n)}{2 - 1/\bar{n}(n)} \equiv \underline{n}(n). \quad (36)$$

Again, if the elasticity of $\underline{n}$ is smaller than 1 for $n \geq 2$, the equation $n = \underline{n}(n)$ uniquely determines the greatest lower bound $\underline{n}$ for an admissible n . The elastic-

ities of $\bar{n}$ and $\underline{n}$ are:

$$\epsilon \bar{n}(n) = \frac{1}{2} \left(\epsilon Y(n) - \frac{1}{n-1} \right) \quad (37)$$

$$\epsilon \underline{n}(n) = 2\epsilon \bar{n}(n) \frac{\bar{n}(n) - 1}{2\bar{n}(n) - 1}. \quad (38)$$

The inequality $\epsilon \underline{n}(n) < 1$ is implied by the inequality $\epsilon \bar{n}(n) < 1$: hence, both inelasticities are smaller than 1 if $\epsilon Y(n) < 1/(n-1) + 2$. The elasticity of $Y(n)$, by equation (14), is:

$$\epsilon Y(n) = \frac{1}{n-1} \frac{Y(n)/\phi m - ((n-1)\chi - 1)n}{(\sigma + \chi)Y(n)/\phi m + \sigma n}. \quad (39)$$

It can be easily checked that $3\sigma + \chi \geq 1$ is a sufficient condition for this elasticity to be smaller than $1/(n-1) + 2$ when $n \geq 2$.

Proof of Proposition 3

By Lemma 1, we know that there exists a solution $(Y_1, \dots, Y_R)$ to the system (15). For this solution to be an equilibrium, we must show that profitability ($n_k \leq \bar{n}_r$, by (17)) and sustainability ($n_k \geq \underline{n}_r$, by (18)) are ensured for any $k = 1, \dots, K$ and any $r = 1, \dots, R$. In order to do that, let us find $Y_{\inf}$ and $Y_{\sup}$ such that

$$Y_{\inf} \leq \min \{Y_1, \dots, Y_R\} \leq \max \{Y_1, \dots, Y_R\} \leq Y_{\sup}, \quad (40)$$

and such that, for any k ,

$$\frac{\sqrt{\frac{n_{\inf}}{n_{\inf}-1} \frac{Y_{\sup}}{\phi m}}}{2 - 1/\sqrt{\frac{n_{\inf}}{n_{\inf}-1} \frac{Y_{\sup}}{\phi m}}} = n_{\inf} \leq n_k \leq n_{\sup} = \sqrt{\frac{n_{\sup}}{n_{\sup}-1} \frac{Y_{\inf}}{\phi m}}. \quad (41)$$

Clearly, if such pair $(Y_{\inf}, Y_{\sup})$ exists, $n_{\sup} \leq \bar{n}_r$ and $n_{\inf} \geq \underline{n}_r$ for any r , so that $(Y_1, \dots, Y_R)$ is indeed an equilibrium. Moreover, we want the choice of $(Y_{\inf}, Y_{\sup})$, and hence that of the interval $[n_{\inf}, n_{\sup}]$, to be made independently of the specific family of states $(F_1, \dots, F_R)$ and of the specific transition probabilities $(T_1, \dots, T_R)$. So, take the r -th equation of the system (15), with the means μ_r^G , μ_r^H and n_r^A expressed as functions of $(n_{1r}, \dots, n_{mr})$. The elasticity of its LHS, denoted

$H(Y_r, n_{1r}, \dots, n_{mr})$, with respect to n_{ir} is

$$\begin{aligned} \epsilon_i H(Y_r, n_{1r}, \dots, n_{mr}) &= \chi \frac{(\mu_r^G / \mu_r^H) Y_r (\epsilon_i \mu_r^G - \epsilon_i \mu_r^H) + \phi n_{ir}}{(\mu_r^G / \mu_r^H) Y_r + \phi m n_r^A} + \epsilon_i \mu_r^G \\ &= \frac{1}{m n_{ir}} \left(\chi \frac{(\mu_r^G / \mu_r^H) Y_r (\mu_r^H - \mu_{ir}) + \phi m n_r^A (n_{ir} / n_r^A) n_{ir}}{(\mu_r^G / \mu_r^H) Y_r + \phi m n_r^A} - \mu_{ir} \right). \end{aligned} \quad (42)$$

As $\mu_{ir} = n_{ir} / (n_{ir} - 1)$ is decreasing in n_{ir} and as the effects of a change of n_{ir} on the markup means are dominated (at least for large m) by its direct effects on μ_{ir} , the expression inside the parentheses is increasing in n_{ir} , so that H is a strictly quasi-convex function of n_{ir} .

If H is an increasing function of n_{ir} , the maximum (resp. the minimum) of H over $[n_{\inf}, n_{\sup}]$, given Y_r , is obtained when $n_{ir} = n_{\sup}$ (resp. $n_{ir} = n_{\inf}$) for any i . Then, if $\sigma < 1$, we can determine $Y_{\inf}$ and $Y_{\sup}$ from (15):

$$v \frac{n_{\sup}}{n_{\sup} - 1} (Y_{\inf} + \phi m n_{\sup})^\chi Y_{\inf} = Y_{\inf}^{1-\sigma} \leq \sum_{r'=1}^R T_{rr'} Y_{r'}^{1-\sigma} \leq Y_{\sup}^{1-\sigma} \quad (43)$$

and

$$v \frac{n_{\inf}}{n_{\inf} - 1} (Y_{\sup} + \phi m n_{\inf})^\chi Y_{\sup} = Y_{\sup}^{1-\sigma} \geq \sum_{r'=1}^R T_{rr'} Y_{r'}^{1-\sigma} \geq Y_{\inf}^{1-\sigma}. \quad (44)$$

These two equations, which would apply to a deterministic symmetric equilibrium so that $Y_{\inf} = Y(n_{\sup})$ and $Y_{\sup} = Y(n_{\inf})$. By (41) and according to the proof of Lemma 2, $n_{\inf} = \underline{n}$ and $n_{\sup} = \bar{n}$, so that $[n_{\inf}, n_{\sup}] = [\underline{n}, \bar{n}]$. Also, again by (41), $Y_{\sup} / \phi m = Y(\underline{n}) / \phi m = (\underline{n} - 1) \left(1 + \sqrt{1 - 1/\underline{n}} \right)$. By (42), $\epsilon_i H(Y(\underline{n}), \underline{n}, \dots, \underline{n}) \geq 0$ if $\chi \geq X(\underline{n})$, so that H is indeed increasing in n_{ir} as assumed and also, by (39), Y is a decreasing function, as it should for having $Y(\bar{n}) \leq Y(\underline{n})$.

If $\sigma > 1$, the two equations (43) and (44) must be replaced by the equations:

$$v \frac{n_{\sup}}{n_{\sup} - 1} (Y_{\inf} + \phi m n_{\sup})^\chi Y_{\inf} = Y_{\sup}^{1-\sigma} \leq \sum_{r'=1}^R T_{rr'} Y_{r'}^{1-\sigma} \leq Y_{\inf}^{1-\sigma} \quad (45)$$

and

$$v \frac{n_{\inf}}{n_{\inf} - 1} (Y_{\sup} + \phi m n_{\inf})^\chi Y_{\sup} = Y_{\inf}^{1-\sigma} \geq \sum_{r'=1}^R T_{rr'} Y_{r'}^{1-\sigma} \geq Y_{\sup}^{1-\sigma}. \quad (46)$$

Hence, $Y_{\inf} \leq Y(n_{\sup})$ and $Y_{\sup} \geq Y(n_{\inf})$, and also $n_{\inf} \geq \underline{n}$ and $n_{\sup} \leq \bar{n}$, so that we obtain $[n_{\inf}, n_{\sup}] \subset [\underline{n}, \bar{n}]$. The condition for H to be an increasing function

of n_{ir} is now $\chi \geq X(n_{\inf})$ but, since X is a decreasing function, it is satisfied if $\chi \geq X(\underline{n})$.

Let us now consider the case where H is a decreasing function of n_{ir} . The argument is similar. If $\sigma < 1$, $Y_{\inf} = Y(n_{\inf})$ and $Y_{\sup} = Y(n_{\sup})$ by (43) and (44) with $n_{\sup}$ and $n_{\inf}$ interchanged. Also, $n_{\inf} \geq \underline{n}$ and $n_{\sup} \leq \bar{n}$ by (41), hence $[n_{\inf}, n_{\sup}] \subset [\underline{n}, \bar{n}]$. By (41) and (42), $\epsilon_i H(Y(n_{\sup}), n_{\sup}, \dots, n_{\sup}) \leq 0$, so that H is indeed a decreasing function of n_{ir} as assumed, if $\chi \leq X(n_{\sup})$, a weaker condition than $\chi \leq X(\bar{n})$. The same condition on χ entails by (39) that Y is increasing, so that $Y(n_{\inf}) < Y(n_{\sup})$. Finally, if $\sigma > 1$, we obtain by the same kind of argument: $Y_{\inf} \leq Y(n_{\inf}) \leq Y(n_{\sup}) \leq Y_{\sup}$, $n_{\inf} \geq \underline{n}$ and $n_{\sup} \leq \bar{n}$, hence $[n_{\inf}, n_{\sup}] \subset [\underline{n}, \bar{n}]$, with the same condition on χ .