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Public Education Spending, Sectoral Taxation and Growth*

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Abstract

This paper examines the interplay between public education expenditure and economic growth in a two-sector model. We reveal that agents' preferences for services, education and savings play a major role in the relationship between growth and public education expenditures, as long as production is taxed at a different rate in each sector.

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1 Introduction

When should a government direct more resources to education? There is no consensus in empirical studies regarding the relationship between government spending in education and economic growth. The contribution of this paper is to explore the link between the level of public expenditure on education and economic growth, in a two-sector overlapping generation growth model. The desegregation of production into a manufacturing and a services sector, allows us to assess the growth implications of new tax schemes to finance an increase in public education expenditure, namely sectoral taxes.

Since the emergence of the new growth theory initiated by Lucas (1988), human capital accumulation has been identified as a major determinant of long-term economic growth and a growing literature has focused on the link between the level of public expenditure on education and economic growth (Glomm and Ravikumar 1992, 1997, 1998). In more recent studies, authors point out the factors influencing the effect of public education and it appears that the role of agents' preference does not matter. Blankenau and Simpson (2004) and Blankenau et al. (2007) emphasize that the effect of government spending on education depends on the level of government spending, the tax structure and the parameters of production technologies. Basu and Bhattarai (2012) emphasize that the elasticity of human capital to public education is a key parameter. When this

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elasticity is high, countries with a greater government involvement in education experience lower growth.

In our study, there are two consumption goods: a manufactured good and services. The government allocates a fixed share of GDP to education policy leaving a tax on manufacturing output, on services, or on the aggregate production. Public education expenditures has a non-monotonic effect on the long-term growth. It enhances directly human capital accumulation but it may crowd out the private education spending and the investment in physical capital. The magnitude of the two opposite effects is highly conditional on the fiscal policy that the government chooses and on agents' preferences. When sectoral taxes are used, additional channels through which the policy influences growth emerge. A tax on output of the manufactured good favors education making it cost cheaper relative to services, whereas a tax on services makes manufactured good more attractive. Consequently, as long as production is taxed at a different rate in each sector, agents' preferences for time, for education and taste for services shape the effect of public policy. According to the literature on growth and development, developing countries are characterized by a consumption oriented toward manufactured goods rather than services (see e.g. Heish and Klenow (2007)). Thus, we reveal that when policy is financed by a tax on manufacturing output, governments in developed economy should devote a higher share of GDP to education. Conversely, when policy is financed by a tax on services, the government should allocate a lower share of output to education to observe a positive relationship between growth and public education spending. We also prove that the relative price adjustment reduces or reinforces the costs and benefits of public education policies such that sectoral taxes may perform better than a standard production tax regarding long-term growth. In a country where the taste for services is low, a tax on the manufacturing output does better than a tax on the aggregate production to finance an increase in public education. When taste for services is high enough, a tax on the services sector performs better than a tax on the aggregate output.

The rest of this paper is organized as follows. In Section 2, we set up the theoretical model. Section 3 is devoted to the impact of sectoral taxation regarding the relationship between growth rate and public education. In section 4, we compare the long-term growth rate with the different funding systems. Finally, Section 5 concludes.

2 The Model

We develop a two-sector overlapping generations (OLG) model in which individuals live for three periods. All individuals are identical within each generation and we assume there is no population growth. Population size is normalized to unity. The initial adult is endowed with K_0 units of physical capital and H_0 units of human capital.

2.1 Production technologies

We use the two-sector production structure proposed by Erosa et al. (2010). The representative firm produces in the manufacturing (Y_M) and the services (Y_S) sector. Production in both sectors results from Cobb-Douglas production technologies, using two inputs, human capital or effective labor supply H, and physical capital K. Let K_i and H_i, $i = M, S$, be respectively the quantities of capital and effective labor used by sector i, production is given by:

$$Y_{Mt} = A_M K_{Mt}^{\alpha} H_{Mt}^{1-\alpha} \qquad A_M > 0 \tag{1}$$

$$Y_{St} = A_S K_{St}^{\alpha} H_{St}^{1-\alpha} \qquad A_S > 0 \tag{2}$$

with $\alpha \in (0,1)$, the elasticity of output with respect to capital, which is assumed equal across sectors. Manufacturing output can be either consumed or invested in physical capital while services are consumed or invested in human capital. Physical capital investment is only private whereas human capital investment results from both public and private investment. Since in the OECD countries, on average, 90% of the current expenditure on public education is devoted to teacher salaries[1], we consider that educational expenditures in terms of services are empirically relevant. Both inputs are perfectly mobile between the two sectors provided that:

$$H_T + H_N \leq H, \qquad K_T + K_N \leq K \tag{3}$$

K being the total stock of physical capital and H the total amount of human capital.
Let $k_i = Ki/Hi$ be the capital of sector i, $h_i = H_i/H$ be the share of human capital allocated to sector $i, i = M, S$, and $k = K/H$ the physical to human capital ratio. Equations (1), (2) and (3) can be rewritten:

$$Y_{Mt} = A_M k_{Mt}^{\alpha} h_{Mt} H_t \tag{4}$$

$$Y_{St} = A_S k_{St}^{\alpha} h_{St} H_t \tag{5}$$

$$h_M + h_S \leq 1, \qquad k_M h_M + k_S h_S \leq k \tag{6}$$

The government collects revenue through a sector specific tax on output $\tau_i \in [0,1)$, $i = M, S$. We normalize the price of manufactured good to one. Denoting by w the wage rate, R the gross rental rate of capital and P_S the price of services, profit maximization over the two sectors implies that production factors are paid at their net-of-tax marginal product:

$$R_t = (1 - \tau_{Mt}) A_M k_{M_t}^{\alpha-1} = (1 - \tau_{St}) P_{St} A_S k_{N_t}^{\alpha-1} \tag{7}$$

$$w_t = (1 - \tau_{Mt}) A_M k_{M_t}^{\alpha} = (1 - \tau_{St}) P_{St} A_S k_{S_t}^{\alpha} \tag{8}$$

[1]See OECD Indicator B6: "On what resources and services is education funding spent?", available at http://www.oecd.org/education/eag.htm

From which we have:

$$k_{Mt} = k_{St} = k_t \quad ; \quad P_{St} = \frac{A_M(1-\tau_{Mt})}{A_S(1-\tau_{St})} \tag{9}$$

Equation (9) shows that the price of services increases with the tax on services whereas it decreases with the tax on manufacturing output.

We assume that physical capital fully depreciates after one period. In line with Blankenau and Simpson (2004), the human capital accumulation of is given by:

$$h_{t+1} = A_H e_t^a v_t^b h_t^{1-a-b} \qquad\qquad a, b \in [0,1], A_H > 0 \tag{10}$$

Parameters a and b are respectively the elasticity of human capital to private (e_t) and public education (v_t) expenditure. Public and private education expenditures are imperfect substitutes in producing human capital. In line with Keane and Wolpin (2001), e_t represents resources that households invest in their children outside school (individual teachers, tuitions payments...). To keep the impact of the stock of parental knowledge (h_t) on children's human capital positive, we restrict $a + b < 1$.

2.2 Government

We assume that a fixed share (θ) of GDP (Y_t) is devoted to public education, i.e $P_{St}v_t = \theta Y_t$ where $Y_t = Y_{Mt} + P_{St}Y_{St}$. From equations (4) and (5), the public expenditure on education is:

$$P_{St}v_t = \theta k_t^\alpha H_t \left(A_M h_{Mt} + A_S h_{St} P_{St}\right) \tag{11}$$

Production taxes supported by the firms are the only source of government income. Government policy is the set $\{\tau_S, \tau_M, \theta\}$ and government budget constraint is given by:

$$P_{St}v_t = \tau_{St} P_{St} Y_{St} + \tau_{Mt} Y_{Mt} \tag{12}$$

Using (4), (5), (9) and (11), the government budget constraint can be written:

$$\theta \left(h_{Mt}(1-\tau_{St}) + h_{St}(1-\tau_{Mt})\right) = \tau_{St} h_{St}(1-\tau_{Mt}) + \tau_{Mt} h_{Mt}(1-\tau_{St}) \tag{13}$$

2.3 Preferences

The economy is populated by finite-lived agents living for three periods. We consider a paternalistic altruism, according to which parents value the level of human capital of their children. In their first period of life, agents are young and benefit from education. In their second period of life, agents are adult and they are endowed with h_t efficiency units of labor that they supply inelastically to firms. Their income is allocated between current consumption, c_t, savings, s_t and investment in children's education, e_t.

$$w_t h_t = \pi_t c_t + P_{St} e_t + s_t \tag{14}$$

In their third period of life, agents are old and retire. They consume the proceeds of their savings:

$$R_{t+1}s_t = \pi_{t+1}d_{t+1} \tag{15}$$

We denote by π the price of the composite good c, which is an aggregate of the manufactured and the service goods. Let $x = c, d$ denote the individual consumption at each period of life, x_M and x_S be respectively the quantities allocated to manufactured goods and services. Instantaneous preferences over the two goods are defined according to:

$$x = x_M^{\mu} x_S^{1-\mu} \tag{16}$$

with $\mu \in (0,1)$ the share of manufactured goods in consumption. The optimal allocation of total expenditure between consumption of manufactured goods and services is obtained by solving the following static problem:

$$\max_{x_M, x_S} x_M^{\mu} x_S^{1-\mu}$$

$$s.t \;\; \pi x = P_S x_S + x_M$$

and leads to:

$$x_M = \mu \pi x$$

$$P_S x_S = (1-\mu)\pi x \tag{17}$$

$$\pi = \phi(\mu) \equiv \mu^{-\mu}(1-\mu)^{-(1-\mu)}$$

An individual born in period $t-1$ chooses e_t and s_t so as to maximize his life-cycle utility:

$$U(c_t, d_{t+1}, h_{t+1}) = \ln c_t + \beta \ln d_{t+1} + \gamma \ln h_{t+1} \tag{18}$$

$$0 < \beta < 1 \; ; \; 0 < \gamma < 1$$

subject to (10), (14) and (15). Parameters β and γ are respectively the discount factor and the degree of paternalistic altruism.

From the first order conditions, we obtain individual's optimal choices:

$$s_t = \frac{\beta}{1 + \gamma a + \beta} w_t h_t \tag{19}$$

$$e_t = \frac{\gamma a}{P_S(1 + \gamma a + \beta)} w_t h_t \tag{20}$$

2.4 Equilibrium

Definition 1 *Given a set of initial conditions $\{K_0, H_0\}$, an equilibrium is a sequence of prices $\{w_t, R_t, P_{St}\}_{t=0}^{t=\infty}$, decision rules $\{c_{Mt}, c_{St}, d_{Mt+1}, d_{St+1}, s_t, e_t, h_{t+1}\}_{t=0}^{t=\infty}$ and quantities $\{K_t, h_t, Y_t\}_{t=0}^{t=\infty}$ such that, for all $t \geqslant 0$:*

i) A period t adult chooses $c_{Mt}, c_{St}, d_{Mt+1}, d_{St+1}, s_t, e_t, h_{t+1}$ to solve the agent's problem taking prices and government policy as given;

ii) (w_t, R_t, P_{St}) is given by (7) and (8);

iii) the effective labor supply in t is $H_t = h_t$;

iv) the service goods market clears: $Y_{St} = c_{St} + d_{St} + e_t + v_t$;

v) the physical capital market clears: $K_{t+1} = s_t$;

vi) the budget constraint clears: $\theta(Y_{Mt} + P_{St}Y_{St}) = \tau_{St}P_{St}Y_{St} + \tau_{Mt}Y_{Mt}$;

The clearance of the goods markets in period t requires the demand for services (i.e., the sum of consumption of service goods and public and private spending on education) to be equal to the supply of the service goods:

Lemma 1 *The clearance of the service goods market in period t*

$$Y_{St} = c_{St} + d_{St} + e_t + v_t \tag{21}$$

gives the share of human capital allocated to each sector:

$$h_{St} = \mathcal{X} + \tau_{Mt}(1 - \mathcal{X}) \;\; ; \;\; h_{Tt} = (1 - \tau_{Mt})(1 - \mathcal{X}) \tag{22}$$

$$\text{with} \quad \mathcal{X} = \frac{(1-\alpha)\gamma a + (1-\mu)(1 + \alpha(\gamma a + \beta))}{1 + \gamma a + \beta} < 1 \tag{23}$$

Proof. See Appendix B. ■

By substituting equation (22) in (13) we deduce the relationship between θ, τ_M and τ_S :

$$\theta = \frac{\tau_{St}\mathcal{X} + \tau_{Mt}(1 - \mathcal{X})}{1 + (\tau_{Mt} - \tau_{St})(1 - \mathcal{X})} \tag{24}$$

We study alternatively a tax on manufacturing ($\tau_S = 0$) and services production ($\tau_M = 0$). As a result, a constant share θ means that tax rates are time invariant. The capital market clearing condition with equation (10) gives:

$$k_{t+1} = \frac{s_t}{A_H e_t^a v_t^b h_t^{1-a-b}}$$

Using equations (11), (19), (20) and (22) we finally obtain the dynamic equation characterizing equilibrium paths:

$$k_{t+1} = \frac{A_M \beta (1 - \tau_M)(1 - \alpha)^{1-a} k_t^{\alpha(1-a-b)}}{A_H (\gamma a)^a A_S^{a+b} (1 - \tau_S)^a (1 + \gamma a + \beta)^{1-a} \theta^b (1 + (1 - \mathcal{X})(\tau_M - \tau_S))^b} \tag{25}$$

The dynamic path of k_t is monotonic and converges toward the following steady state value:

$$\bar{k} = \left(\frac{A_M \beta (1-\tau_M)(1-\alpha)^{1-a}}{A_H (\gamma a)^a A_S^{a+b} (1-\tau_S)^a (1+\gamma a+\beta)^{1-a} \theta^b (1+(1-\mathcal{X})(\tau_M-\tau_S))^b} \right)^{\frac{1}{1-\alpha(1-a-b)}}$$

Then, we obtain a balanced growth path equilibrium along which the variables chosen by individuals (s_t, e_t, c_t and d_{t+1}) and public education expenditure (v_t) grow at the same constant rate as human capital:

$$1+g = \frac{h_{t+1}}{h_t} = A_H \left(A_S \bar{k}^\alpha \right)^{a+b} \left(\frac{\gamma a (1-\alpha)(1-\tau_S)}{1+\gamma a+\beta} \right)^a \theta^b (1+(1-\mathcal{X})(\tau_M-\tau_S))^b$$

In the following we focus on the balanced growth path.

3 Public education funding and long-term growth rate

We examine the relationship between public education expenditure and long-run growth considering different types of funding. By decomposing the aggregate economy into two-sector splits, we can consider sectoral taxation. We define ε_{ij} as the elasticity of i with respect to j and z as the private education spending per unit of human capital e/h. We examine three alternative policies to finance public education.

3.1 Public education financed by a tax on the aggregate production

Assume that $\tau_M = \tau_S = \tau_Y$, the fiscal policy is equivalent to a tax on the aggregate production. Factor returns given by equations (7) and (8) are negatively affected by the tax, whereas the relative price of goods remains unchanged. From equation (24), the tax rate is equal to the share of output devoted to education spendings:

$$\tau_Y = \theta$$

The physical to human capital ratio, private education per unit of human capital, and long-term growth rate are respectively given by:

$$\bar{k} = \left(\frac{A_M \beta (1-\theta)^{1-a} (1-\alpha)^{1-a}}{A_H (\gamma a)^a A_S^{a+b} \theta^b (1+\gamma a+\beta)^{1-a}} \right)^{\frac{1}{1-\alpha(1-a-b)}}$$

$$z = \frac{\gamma a}{1+\gamma a} A_S (1-\alpha) \bar{k}^\alpha (1-\theta)$$

$$g = A_H \left(A_S \bar{k}^\alpha \right)^{a+b} \left(\frac{\gamma a (1-\alpha)}{1+\gamma a+\beta} \right)^a \theta^b (1-\theta)^a \tag{26}$$

The impact of an increase in the share of GDP devoted to public education on private choices and growth is deduced from the elasticities:

Lemma 2 *When the government taxes the aggregate production, the elasticities are given by:*

$$\varepsilon_{\bar{k},\theta} = -\frac{1}{1-\alpha+\alpha(a+b)}\left(\frac{(1-a)\theta}{1-\theta}+b\right) < 0$$

$$\varepsilon_{z,\theta} = \alpha\varepsilon_{\bar{k},\theta} - 1 < 0$$

$$\varepsilon_{g,\theta} = \alpha(a+b)\varepsilon_{\bar{k},\theta} + b - \frac{a\theta}{1-\theta} \lesseqgtr 0$$

Proof. See Appendix B. ■

We obtain results similar to Blankenau and Simpson (2004), the differences being due to our formalization of agents' preferences for education and the scheme of public policy.[2] Elasticities do not depend on agents' preferences and an increase in public education spendings crowds out both physical capital accumulation and private human capital investment. Thus, there is a growth-maximizing level of public expenditures:

Proposition 1 *When the government taxes the whole production at the same rate, the level of public expenditure maximizing the growth rate is given by:*

$$\theta_Y^{max} = \frac{a+b}{(1-\alpha)a}$$

Policy is growth enhancing (resp. reducing) when $\theta < \theta_Y^{max}$ (resp. $\theta > \theta_Y^{max}$).

The relationship between growth and public education spending is not affected by agents' preferences as long as the level of tax is the same in both sectors.

3.2 Public education financed by a tax on manufacturing output

Assume that $\tau_S = 0$. We focus on the growth effect of public education spending on the long-term growth rate when public intervention is financed by a tax on the production of manufactured goods only. This positive tax causes a fall in factor returns. Moreover, it creates a distortion making education more attractive. Indeed, education and service goods become cheaper relative to manufactured goods. From equation (24), a balanced budget constraint requires:

$$\tau_M = \frac{\theta}{(1-\theta)(1-\mathcal{X})}$$

Policy is sustainable ($\tau_M < 1$) if the following condition is satisfied $\theta < \frac{1-\mathcal{X}}{2-\mathcal{X}} \equiv \bar{\theta}$. It is essential that θ be not too high. A higher θ is associated with a lower share of human capital allocated to the production of manufacturing output and a higher tax on this output. With $\mathcal{X}$ given by (23), examining the expression of τ_M we emphasize the following properties:

[2] In Blankenau and Simpson (2004), agents borrow for education when young and the government allocates a share of its budget to unproductive spendings.

Proposition 2 *The tax rate on manufacturing production required to balance the public budget is increasing with altruism factor (γ) and decreasing with time preference (β) and taste for manufactured goods (μ).*

An economy characterized by a low taste for education (γ), a high degree of time preferences (β) and a high taste for manufactured goods (μ), will be more oriented toward the consumption of manufactured goods. Since the government taxes this sector to finance public education expenditure, the required tax rate will be lower.

The physical to human capital ratio, private education per unit of human capital, and long-term growth rate are respectively given by the expressions:

$$\bar{k} = \left(\frac{A_M \beta \left(\frac{(1-\theta)(1-\mathcal{X})-\theta}{(1-\theta)(1-\mathcal{X})} \right) (1-\alpha)^{1-a}}{A_H (\gamma a)^a A_S^{a+b} (1+\gamma a+\beta)^{1-a} \left(\frac{\theta}{1-\theta}\right)^b} \right)^{\frac{1}{1-\alpha(1-a-b)}}$$

$$z = \frac{\gamma a}{1+\gamma a} A_S (1-\alpha) \bar{k}^\alpha$$

$$g = A_H \left(A_S \bar{k}^\alpha \right)^{a+b} \left(\frac{\gamma a (1-\alpha)}{1+\gamma a+\beta} \right)^a \left(\frac{\theta}{1-\theta} \right)^b \tag{27}$$

Thus, we compute the following elasticities:

Lemma 3 *When public policy is financed by a tax on manufacturing production, the elasticities are given by:*

$$\varepsilon_{\bar{k},\theta} = -\frac{1}{(1-\theta)(1-\alpha+\alpha(a+b))} \left(\frac{\theta}{(1-\theta)(1-\mathcal{X})-\theta} + b \right) < 0$$

$$\varepsilon_{z,\theta} = \alpha \varepsilon_{\bar{k},\theta} < 0$$

$$\varepsilon_{g,\theta} = \alpha(a+b)\varepsilon_{\bar{k},\theta} + \frac{b}{1-\theta} \lesseqgtr 0$$

Proof. See Appendix B. ■

Similar to the case where sectors are taxed at the same rate, an increase in public education spending crowds out investment in physical capital because taxation reduces wage, and therefore, the amount of saving. Regarding private education choices, the ratio w/P_S is determining. Since the tax decreases the price of education, this ratio is influenced by policy only through the modification of physical to human capital ratio. Consequently, a tax on the production of manufactured goods allows to reduce the crowding out effect on private education choices. Using $\varepsilon_{g,\theta}$ and $\varepsilon_{\bar{k},\theta}$ we easily see that policy has a non-monotonic impact on the growth rate which crucially depends on the agents' preferences.

Proposition 3 *Under manufacturing-tax funding system, the policy maximizing the long-term growth rate is:*

$$\theta_M^{max} = \frac{b(1-\alpha)(1-\mathcal{X})}{b(1-\alpha)(1-\mathcal{X})+a\alpha+b} < \bar{\theta} \;\; ; \;\; \tau_M^{max} = \frac{b(1-\alpha)}{a\alpha+b}$$

Policy is growth enhancing (resp. reducing) when $\theta < \theta_M^{max}$ (resp. $\theta > \theta_M^{max}$).

The elasticity of the growth rate to public education ($\varepsilon_{g,\theta}$) is decreasing with preferences for education (γ) and the share of services in total consumption ($1-\mu$). This is directly linked to the higher level of output taxation required when economy is oriented toward services. Countries with a high preferences for manufactured goods experience higher growth rate when the tax is imposed on manufacturing production. Therefore, policy recommendations are taste-dependent.

Corollary 1 *In the manufacturing-tax funding system, the higher the consumption taste for manufactured goods relative to services, the more the government has to devote resources to education to maximize the growth rate.*

3.3 Public education financed by a tax on services

We assume now the case where $\tau_M = 0$, that is public intervention is exclusively financed through a tax on services. From the firm's optimization program, taxation of services differs from taxation of the manufacturing sector in two ways. A tax on services does not affect the factor return, however, it raises the price of education. From (24), the tax rate balancing the budget constraint is the following:

$$\tau_S = \frac{\theta}{\mathcal{X} + (1-\mathcal{X})\theta}$$

Proposition 4 *The tax rate on services required to balance public budget is decreasing with altruism factor (γ) and increasing with time preference (β) and taste for manufactured goods (μ).*

An economy characterized by a high taste for education (γ), a low degree of time preferences (β) and a low share of manufactured goods in consumption expenditure (μ), will be more oriented toward the consumption of services. A high demand for services entails a large scale of the production of this good. In a services-tax funding system this guarantees that the tax rate is not too high. As previously, we compute the physical to human capital ratio, private education per unit of human capital, and long-term growth rate:

$$\bar{k} = \left(\frac{A_M \beta (1-\alpha)^{1-a}}{A_H (\gamma a)^a A_S^{a+b} \left(\frac{\mathcal{X}(1-\theta)}{\mathcal{X}+(1-\mathcal{X})\theta}\right)^a (1+\gamma a+\beta)^{1-a} \left(\frac{\theta\mathcal{X}}{\mathcal{X}+(1-\mathcal{X})\theta}\right)^b} \right)^{\frac{1}{1-\alpha(1-a-b)}}$$

$$z = \frac{\gamma a}{1+\gamma a} A_S (1-\alpha) \bar{k}^{\alpha} \left(\frac{\mathcal{X}(1-\theta)}{\mathcal{X}+(1-\mathcal{X})\theta} \right)$$

$$g = A_H \left(A_S \bar{k}^{\alpha}\right)^{a+b} \left(\frac{\gamma a (1-\alpha)}{1+\gamma a+\beta}\right)^a \left(\frac{\mathcal{X}(1-\theta)}{\mathcal{X}+(1-\mathcal{X})\theta}\right)^a \left(\frac{\theta\mathcal{X}}{\mathcal{X}+(1-\mathcal{X})\theta}\right)^b \tag{28}$$

The elasticities are derived in the following Lemma:

Lemma 4 *When public education expenditure is financed by a tax on services, elasticities are given by:*

$$\varepsilon_{\bar{k},\theta} = \frac{a\theta - b(1-\theta)\mathcal{X}}{(1-\theta)(1-\alpha+\alpha(a+b))(\mathcal{X}+(1-\mathcal{X})\theta)} \lesseqgtr 0$$

$$\varepsilon_{z,\theta} = \alpha\varepsilon_{\bar{k},\theta} - \frac{1}{(1-\theta)(\mathcal{X}+(1-\mathcal{X})\theta)} < 0$$

$$\varepsilon_{g,\theta} = \alpha(a+b)\varepsilon_{\bar{k},\theta} + \frac{\mathcal{X}b(1-\theta) - a\theta}{(1-\theta)(\mathcal{X}+(1-\mathcal{X})\theta)} \lesseqgtr 0$$

Proof. See Appendix B. ∎

In contrast to the case where policy is financed by a manufacturing output tax, an increase in public education expenditures does not always reduce the physical to human capital ratio. The introduction of a tax on the production of services increases the price of services (and the price of education) by the full amount of the tax. Thus, it generates opposite effects on the physical to human capital ratio: a negative effect coming from the increase in public spending on education and a positive effect arising because the education expenditure becomes more costly relative to savings. The positive effect dominates when the economy is more oriented toward manufactured goods ($\mathcal{X}$ low). The private education choice per unit of human capital goes down when the government increases public education ($\varepsilon_{z,\theta} < 0$). Even when policy favors the return of human capital through the raise in the physical to human capital ratio ($\varepsilon_{\bar{\kappa},\theta} > 0$), the negative effect generated by the increase in the relative price is higher. The global impact of a raise in θ on the long-term growth rate is given by $\varepsilon_{g,\theta}$. It is ambiguous and depends on agents' preferences:

Proposition 5 *Under service-tax funding system, the policy maximizing the long-term growth rate is:*

$$\theta_S^{max} = \frac{b\mathcal{X}}{a+b\mathcal{X}} \;\; ; \;\; \tau_S^{max} = \frac{b}{a+b}$$

Policy is growth enhancing (resp. reducing) when $\theta < \theta_S^{max}$ (resp. $\theta > \theta_S^{max}$).

The elasticity of the growth rate to public education ($\varepsilon_{g,\theta}$) is increasing with agents' taste for education and the share of services in total consumption.[3] As previously, this is because of a lower level of output taxation required when economy is services sector oriented. Thus, we emphasize the following result:

Corollary 2 *In the service-tax funding system, the government has to spend a higher share of aggregate output on education in countries with higher preferences for services in order to maximize the growth rate.*

[3]Note that θ_S^{max} does not depend on the elasticity of output to physical capital (α). The direct impact of policy on the long-term growth rate, captured by the second term of the right hand side of $\varepsilon_{g,\theta}$, always neutralizes the indirect impact generated by the adjustment of k.

In this model we highlight that government has to adjust its policy according to the pattern of consumption. Based on the literature on growth and development, it appears that rich countries are more oriented toward services than developed countries (see e.g Heish and Klenow (2007)). As a result, taking into account sectoral taxation, we can conclude that the relationship between growth and public education expenditure is not the same along the process of development. Conversely, a tax on aggregate production predicts that the relationship between growth and public education does not depend on agents' tastes.

4 Sectoral tax versus aggregate output tax

We have shown in the previous section that the relationship between growth and public education spending is not monotonous. It crucially depends on the sectors the government decides to tax and on the consumption behavior. When sectoral taxes are implemented, an increase in government education spending shapes the long-term growth through an additional channel. Distortionary taxation affects the relative price of goods and may amplify or weaken the positive effect of a policy. We compare the long-term growth rate in different fiscal regimes presented previously. More precisely, we examine if a distortionary sectoral production tax has to be preferred to a tax on aggregate production to finance public education expenditure. We define by g_M, g_S and g_Y the growth rates with manufacturing, services and aggregate production taxes respectively. The tradeoff depends on the share of manufactured goods in consumption expenditure (μ):

Proposition 6 *For a given θ,*

j) There exists a critical level $\bar{\mu}_M$ such that: when $\mu > \bar{\mu}_M$ (resp. $\mu < \bar{\mu}_M$), $g_M > g_Y$ (resp. $g_M < g_Y$).

jj) There exists a critical level $\bar{\mu}_S$ such that: when $\mu < \bar{\mu}_S$ (resp. $\mu > \bar{\mu}_S$), $g_S > g_Y$ (resp. $g_S < g_Y$).

with critical values $\bar{\mu_M}$ and $\bar{\mu_S}$ decreasing in β and increasing in γ.[4]

Proof. See Appendix C. ■

The long-term growth rate is higher when public education policy is financed by a tax on the services sector rather than a tax on the aggregate production, as long as the consumption of services is important (μ low), the taste for children education is high (γ high) and the time preference is low enough (β low). When government taxes the production of services only rather than the aggregate production, two additional opposite effects arise. On the one hand, the factor returns are not directly affected by taxation, making the return of human capital higher. On the other hand, education becomes more expensive. When demand for services is important taxation is

[4] Expression for $\bar{\mu}_M$ and $\bar{\mu}_S$ are given in Appendix.

not too high. In this case the positive effect dominates and allows to achieve a higher growth rate. Concerning the comparison between aggregate taxation and a tax on manufacturing output a symmetric result emerges. The wage and the price of education are lower with a tax on manufacturing sector than with an aggregate one. Consequently, the reduction of physical to human capital ratio is more severe, whereas the opposite result holds for private education spending. Financing an increase of public education policy by imposing a tax on the manufacturing output performs better, provided that the tax is not too high. It is the case when consumption of manufactured goods is sufficiently important (μ high, γ low and β high).[5]

5 Concluding remarks

The effects of a public education policy financed by distortionary sectoral taxes differ from those of standard production tax examined in a one-sector model. A sectoral tax shapes the relative price of goods, making the agents' tastes become a major determinant of the relationship between growth and public education expenditure. Cross-country heterogeneity in preferences for education, services and savings has to be considered to design a growth enhancing education policy.

6 Appendix

A. Proof of Lemma 1

From Eq. (14), (15), (17), (19) and (20) we have:

$$P_{St}c_{St} = \frac{(1-\mu)w_t h_t}{1+\gamma a+\beta} \quad ; \quad P_{St}d_{St} = (1-\mu)s_{t-1}R_t$$

Using these expressions and Eq. (12) and (20), the clearance of the service good's market is:

$$P_{St}Y_{St} = \frac{(1-\mu)w_t h_t}{1+\gamma a+\beta} + (1-\mu)s_{t-1}R_t + \frac{\gamma a w_t h_t}{1+\gamma a+\beta} + \tau_{St}Y_{St}P_{St} + \tau_{Mt}Y_{Mt}$$

Including (4), (5) and factor returns:

$$A_M(1-\tau_{Mt})k_t^{\alpha}h_{Nt}h_t = \frac{A_M(1-\alpha)(1-\tau_{Mt})k_t^{\alpha}h_t((1-\mu)+\gamma a)}{1+\gamma a+\beta} + s_{t-1}A_M\alpha(1-\tau_{Mt})k_t^{\alpha-1} + \tau_{Mt}A_M k_t^{\alpha}h_{Tt}h_t$$

Simplifying and using the equilibrium on the physical capital market $s_{t-1}h_t = k_t$:

$$h_{Nt} = \frac{(1-\alpha)((1-\mu)+\gamma a)}{1+\gamma a+\beta} + \alpha(1-\mu) + \frac{\tau_{Mt}}{1-\tau_{Mt}}h_{Tt}$$

As $h_{Tt} = 1-h_{Nt}$, we easily obtain Eq. (22).

[5] Regarding threshold levels $\bar{\mu}_M$ and $\bar{\mu}_S$, a situation where both kinds of distortionary sectoral taxes perform better than the tax on aggregate production ($\bar{\mu}_M < \mu < \bar{\mu}_M$) is not excluded. Nevertheless, analytically, we can not conclude in favor of one type of sectoral taxation or the other.

B. Proof of Lemma 2, 3 and 4

Elasticities presented in Lemma 2, 3 and 4 are computed using derivatives $\frac{\partial k}{\partial \theta}$, $\frac{\partial z}{\partial \theta}$ and $\frac{\partial g}{\partial \theta}$. We determine these derivatives for each regime.
Tax on aggregate production:

$$\frac{\partial k}{\partial \theta} = \left(\frac{1-a}{1-\theta} - \frac{b}{\theta}\right) \frac{k}{1-\alpha+\alpha(a+b)}$$

$$\frac{\partial z}{\partial \theta} = \alpha \frac{\partial k}{\partial \theta} \frac{z}{k} - \frac{z}{1-\theta}$$

$$\frac{\partial g}{\partial \theta} = \alpha(a+b) \frac{\partial k}{\partial \theta} \frac{g}{k} + g\left(\frac{b}{\theta} - \frac{a}{1-\theta}\right) g$$

Manufacturing-tax funding system:

$$\frac{\partial k}{\partial \theta} = -\left(\frac{1}{(1-\theta)(1-\mathcal{X})-\theta} + \frac{b}{\theta}\right) \frac{k}{(1-\theta)(1-\alpha+\alpha(a+b))}$$

$$\frac{\partial z}{\partial \theta} = \alpha \frac{\partial k}{\partial \theta} \frac{z}{k}$$

$$\frac{\partial g}{\partial \theta} = \alpha(a+b) \frac{\partial k}{\partial \theta} \frac{g}{k} + g \frac{b}{(1-\theta)\theta}$$

Service-tax funding system:

$$\frac{\partial k}{\partial \theta} = \left(\frac{(a+b)(1-\mathcal{X})}{\mathcal{X}+\theta(1-\mathcal{X})} - \frac{b}{\theta} + \frac{a}{1-\theta}\right) \frac{k}{1-\alpha+\alpha(a+b)}$$

$$\frac{\partial z}{\partial \theta} = z\left(\frac{\alpha}{k}\frac{\partial k}{\partial \theta} - \frac{1}{(1-\theta)(\mathcal{X}+(1-\mathcal{X})\theta)}\right)$$

$$\frac{\partial g}{\partial \theta} = \alpha(a+b) \frac{\partial k}{\partial \theta} \frac{g}{k} - g\left(\frac{(a+b)(1-\mathcal{X})}{\mathcal{X}+\theta(1-\mathcal{X})} - \frac{b}{\theta} + \frac{a}{1-\theta}\right)$$

C. Proof of Proposition 6

We give the condition which guarantees $g_M > g_Y$, using Eq. (27) and (26):

$$\left(\frac{\theta}{1-\theta}\right)^{b\left(1-\frac{\alpha(a+b)}{1-\alpha(1-a-b)}\right)} \left(\frac{(1-\theta)(1-\mathcal{X})-\theta}{(1-\theta)(1-\mathcal{X})}\right)^{\frac{\alpha(a+b)}{1-\alpha(1-a-b)}} > \theta^b(1-\theta)^a \left(\frac{(1-\theta)^{1-a}}{\theta^b}\right)^{\frac{\alpha(a+b)}{1-\alpha(1-a-b)}}$$

By simplifying this expression we obtain:

$$\frac{(1-\theta)(1-\mathcal{X})-\theta}{(1-\theta)(1-\mathcal{X})} > (1-\theta)^{\frac{1+\alpha}{\alpha}}$$

By replacing expression $\mathcal{X}$ by (23), we finally get:

$$\mu > 1 - \left(\frac{(1-\theta)\left(1-(1-\theta)^{\frac{1}{\alpha}}\right)-\theta}{(1-\theta)\left(1-(1-\theta)^{\frac{1}{\alpha}}\right)}\right) \frac{1+\gamma a+\beta}{1+\alpha(\gamma a+\beta)} + \frac{(1-\alpha)\gamma a}{1+\alpha(\gamma a+\beta)} \equiv \bar{\mu}_M$$

Then, we determine the condition which guarantees $g_S > g_Y$, using Eq. (28) and (26):

$$\left[\left(\frac{\mathcal{X}(1-\theta)}{(1-\mathcal{X})\theta+\mathcal{X}}\right)^a\left(\frac{\mathcal{X}\theta}{(1-\mathcal{X})\theta+\mathcal{X}}\right)^b\right]^{1-\frac{\alpha(a+b)}{1-\alpha(1-a-b)}} > \theta^b(1-\theta)^a\left(\frac{(1-\theta)^{1-a}}{\theta^b}\right)^{\frac{\alpha(a+b)}{1-\alpha(1-a-b)}}$$

After simplifications, we obtain:

$$\frac{\mathcal{X}}{\mathcal{X}+(1-\mathcal{X})\theta} > (1-\theta)^{\frac{\alpha}{1-\alpha}}$$

and with (23), we finally get:

$$\mu < 1 - \frac{(1-\theta)^{\frac{\alpha}{1-\alpha}}\theta}{1-(1-\theta)^{\frac{\alpha}{1-\alpha}}}\frac{1+\gamma a+\beta}{1+\alpha(\gamma a+\beta)} + \frac{(1-\alpha)\gamma a}{1+\alpha(\gamma a+\beta)} \equiv \bar{\mu}_S$$

References

Basu, P., and Bhattari, K. (2012), "Government Bias in Education, Schooling Attainment, and Long-Run Growth", *Southern Economic Journal* 79(1), 127-143.

Blankenau, F.B., and Simpson, N.B. (2004), "Public Education Expenditure and Growth", *Journal of Development of Economics* 73, 583-605.

Blankenau, F.B., Simpson, N.B., and Tomljanovich, M. (2007), "Public Education Expenditure, Taxation, and Growth: Linking Data to Theory", *American Economic Review, Paper and Proceeding* 97(2), 393-397.

Erosa, A., Koreshkova, T. and Restuccia, D. (2012), "How Important Is Human Capital? A Quantitative Theory Assessment of World Income Inequality", *The Review of Economic Studies* 77, 1421-1449.

Glomm, G., and Ravikumar, B. (1992), "Public versus Private Investment in Human Capital: Endogenous Growth and Income Inequality", *Journal of Political Economy* 100(4), 818-833.

Glomm, G., and Ravikumar, B. (1997), "Productive Government Expenditures and Long Run Growth", *Journal of Economic Dynamics and Control* 21, 183-204.

Glomm, G., and Ravikumar, B. (1998), "Flate-Rate Taxes, Government Spending on Education, and Growth," *Review of Economic Dynamics* 1, 306-325.

Hsieh, C.T., and Klenow, P. (2007), "Relative Prices and Relative Prosperity", *American Economic Review* 97, 562–585.

Keane, M.P., and Wolpin, K.I. (2001), "The Effect of Parental Transfers and Borrowing Constraints on Educational Attainment", *International Economic Review* 42(4), 1051-1103.

Lucas, R. (1988): "On The Mechanics of Economic Development", *Journal of Monetary Economics* 22, 3-42.