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Matching with Phantoms

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Abstract: Searching for partners involves informational persistence that reduces future traders' matching probability. In this paper, traders who are no longer available but who left tracks on the market are called phantoms. We examine a discrete-time matching market in which phantoms are a by-product of search activity, no coordination frictions are assumed, and non-phantom traders may lose time trying to match with phantoms. The resulting aggregate matching technology features increasing returns to scale in the short run, but has constant returns to scale in the long run. We embed this matching function in the canonical equilibrium search unemployment model. Although the model may feature sunspot fluctuations, its typical calibration on monthly US data displays the saddle-path property. The model predicts the same monthly job-finding probability and quarterly aggregate volatility as the standard model with a Cobb-Douglas matching function.

Keywords: Information persistence; Endogenous matching function; Business cycle

JEL classification: J60

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1 Introduction

The matching technology is a popular tool among labor market specialists and macroeconomists. The technology gives the number of jobs formed as an increasing function of the numbers of job-seekers and vacancies. This function is generally well-behaved in that it is strictly concave and has constant returns to scale. Such properties have strong empirical relevance (see Petrongolo and Pissarides, 2001) and are associated with strong model outcomes, as with the independence of the unemployment rate vis-à-vis workforce size, and the saddle-path and uniqueness properties of equilibrium under rational expectations. Most of the time, the functional form of the matching technology is exogenous and can hardly be derived from elementary principles. The problem goes beyond the labor market case and arises whenever people must meet before trade activities take place.

Several papers provide an explicit scenario behind the aggregate matching technology. As noted by Stevens (2007), these papers rely on an implicit limited mobility assumption with an associated coordination problem.[1] Given that workers cannot readily transfer their attention from one job (or sub-market) to another, lack of coordination generates frictions. However, another property is also involved: matching frictions result from intratemporal congestion externalities. Traders on one side of the market deteriorate search prospects for those who are *currently* on the same side, and improve prospects for those who are *currently* on the other side. In this paper, we follow the general trend in the rest of the literature as we assume that individuals have limited mobility between potential partners. However, the source of market frictions is no longer contemporaneous. We examine the complementary idea whereby matching frictions can result from informational persistence on the market about traders who have already found a match. We refer to these traders as phantom traders, or phantoms for short. Phantoms are a by-product of the search activity: when exiting the market, each trader may leave a trace that disappears over time. Phantoms result in a loss of time and resources for future traders who want to find an adequate partner. We argue that a matching technology endowed with reasonable properties can be derived from this single source of information imperfection.

Phantom traders are ubiquitous in search markets. In everyone's search experience for a job, a house, or even a partner, there is anecdotal evidence where information on the object of the search was clearly outdated. A friend who talks about a job that is already filled, an ad for a dream bike that is already sold, a showcased house that has just been rented. Headhunters also have specific stories involving obsolete profiles of job-seekers. Internet is a very useful resource to find direct evidence on phantom traders. Agents who post ads on matchmaker websites are usually invited to withdraw them once obsolete, which they probably do not do. A simple proof of this behavior can be found on Craigslist, a major job board in the

[1] In mismatch models, workers are imperfectly mobile between sub-markets, and the distribution of traders across sub-markets governs the shape of the aggregate matching technology (see Drèze and Bean, 1990, Lagos, 2000, 2003, and Shimer, 2007). In stock-flow matching models, traders can only match with newcomers (see e.g. Taylor, 1995, Coles and Muthoo, 1998, Coles and Smith, 1998, Coles, 1999, Gregg and Petrongolo, 2005, Coles and Petrongolo, 2008, and Ebrahimy and Shimer, 2010). In urn-ball matching models, buyers independently send one buy order to each seller. Buyers do not coordinate and some sellers receive several buy orders, while others do not receive any order (see, e.g., Butters 1977, Hall 1977, Burdett et al 2001, Albrecht et al 2004, 2006, and Galenianos and Kircher 2009). Stevens (ibid) makes explicit the time-consuming nature of search and endogenizes search investments. The resulting technology is CES.

US. On Wednesday March 26 2014, we computed the distribution of job listings by age for the 23 US cities listed under the heading "Cities" on the Craigslist website. The support of this distribution cannot exceed 30 days, because ads are destroyed after this age. Table 1 reports the different quartiles of this distribution in each city. Two consecutive quartiles are systematically separated by 7 or 8 days. The median of the distribution corresponds to day 15 or day 16. Thus the distribution is uniform by quartile. On the basis that job creation did not dramatically decline over the month, this shows that very few employers delete obsolete ads. If outdated ads were destroyed, the density of job listings by age would be decreasing, and the quartiles of the distribution would be separated by increasing number of days.

	Atlanta	Austin	Boston	Chicago	Dallas	Denver	Detroit	Houston
Q1	7	7	8	7	8	8	8	7
Q2	15	15	15	15	15	15	15	15
Q3	22	23	23	23	23	23	23	23
Q4	30	30	30	30	30	30	30	30
# ads	16000	15000	27000	29900	27900	21700	9400	19000
	Las Vegas	Los Angeles	Miami	Minneapolis	New York	Orange cty	Philadelphia	Phoenix
Q1	8	7	7	8	8	8	8	7
Q2	15	15	15	15	15	15	16	15
Q3	23	22	22	23	23	23	23	23
Q4	30	30	30	30	30	30	30	30
# ads	8500	41600	24400	15100	45000	16600	12300	21300
	Portland	Raleigh	Sacramento	San Diego	San Francisco	Seattle	Washington	
Q1	8	8	8	8	7	8	8	
Q2	16	16	15	15	15	16	16	
Q3	23	23	23	23	23	23	23	
Q4	30	30	30	30	30	30	30	
# ads	17500	6400	8700	18200	41600	30200	20900	

Figure 1: Table 1: Quartiles of the distribution of Craigslist's job listings by age. The lines headed Q1 to Q4 report the different daily quartiles of the empirical distribution of vacancies by age. The line # ads gives the total number of job listings. Numbers of jobs are rounded to the hundred. Source: Craigslist website, March 26 2014.

We consider a generic situation (Section 2). Time is discrete and buyers and sellers try to contact each other on a unique search place. To disentangle the impacts of phantoms on the search market from more standard congestion externalities, we assume that each buyer meets one seller at most and vice-versa, whereas every trader on the short side of the market is sure to meet someone. Unfortunately, that someone may be a phantom trader. No trade takes place in such cases. We assume that the populations of phantoms obey simple flow-stock equations, the inflow of new phantoms being proportional to the past outflow of successful traders. We examine the resulting matching pattern between the two populations of traders.

We refer to the aggregate matching technology as the phantom matching technology, or PMT for short. The PMT features intratemporal and intertemporal externalities. Intratemporal externalities result from the fact that an increase in the number of agents on the long side of the market reduces the proportion of phantom traders. A larger proportion of contacts leads to matches as a result. Intratemporal externalities imply that the PMT displays increasing returns to scale in the short run. Intertemporal externalities result from the fact that current matches fuel future phantom traders. Although period-t number of traders may have an ambiguous impact on period-$(t+k)$ number of matches, intertemporal externalities combine so as to negatively affect the current number of matches. Intratemporal and intertemporal externalities balance each other, and the PMT features constant returns to scale vis-à-vis the whole history of traders.

We study the stationary phantom matching technology (SPMT) that emerges as the steady-state PMT of an environment where the populations of traders are themselves stationary. The SPMT obeys a simple parametric form that only depends on the ratio of the entry rate of new phantoms to the phantom death probability. The SPMT exhibits constant returns to scale. The elasticity of the matching technology negatively depends on the seller-to-buyer ratio when phantom and non-phantom sellers are on the short side, and positively otherwise.

We further discuss the PMT through three extensions to the basic model. The first extension focuses on the case of atomistic traders. The second extension considers another popular source of market frictions, namely coordination frictions. This allows a distinction to be made between the respective contributions of phantom traders and coordination frictions to overall matching frictions. The final extension discusses the case of on-the-match search. Like phantom traders, the pool of on-the-match seekers is fueled by match formation. By neglecting phantom vacancies, papers studying the labor market and highlighting the role played by on-the-job search may overestimate the extent of job competition between employed and unemployed seekers.

We then apply the PMT to the labor market (Section 3). We embed the PMT in the canonical model of equilibrium search unemployment where there is free entry on the vacancy side. Here buyers are unemployed and sellers are vacancies. Due to the property of constant returns to scale in the long run, this modified model admits a unique steady state, which standardly consists of finding an unemployment rate and a vacancy-to-unemployed ratio that jointly solve the usual Beveridge curve and the free-entry condition. Thus, the long-run impacts of the different parameters is like the standard model with a Cobb–Douglas matching function.

However, unlike the canonical model, the short-run dynamics is non-trivial. On the one hand, phantom proportions vary along the business cycle. Such changes affect vacancy profitability, which in turn can magnify or reduce the direct effects of the cycle. On the other hand, information persistence may create fluctuations driven by self-fulfilling prophecies.[2] This configuration arises in one of the two possible regimes of the model. Then, entry decisions do not impact the contemporaneous recruitment probability, but affect the future one through induced changes in unemployment and the future number of phantoms.

We finally calibrate the model on monthly US data for the 2000–2008 period. The calibration strategy implies that the steady state is saddle-path stable. We first compare the predicted monthly job-finding probability with the actual one. We then simulate productivity

[2] Mortensen (1989, 1999) shows that indeterminacy may occur in continuous-time models when the production function or the matching function display increasing returns to scale. In discrete time, indeterminacy may occur under constant returns to scale, but the economic mechanism leading to indeterminacy differs from ours (see Krause and Lubik, 2010). In a different context, Farmer (2012) advocates self-fulfilling prophecies as a reliable source of labor market fluctuations.

shocks in the spirit of Shimer (2005) and focus on unemployment, vacancies and the vacancy-to-unemployed ratio. We adopt the small surplus calibration of Hagedorn and Manovskii (2008). We report the relative volatility of each variable with respect to labor productivity and the auto-correlation parameter. All these predictions are very close to the ones obtained with a Cobb–Douglas technology.

2 The Phantom Matching Technology

2.1 The model

Time is discrete and denoted by $t \geq 0$. A population of nonatomistic buyers and sellers want to trade with each other. But they have to meet before trade takes place. Matching takes place every period. Every time a buyer and a seller meet and agree on match formation, they exit the market.

Let B denote the (mass) number of buyers, S the number of sellers, P^B the number of phantom buyers, and P^S the number of phantom sellers.

The matching mechanism involves two steps. In a first step, each trader on the short side of the market is assigned to a trader on the long side. This results in the following number of contacts:

$$\min\left\{B_t + P_t^B, S_t + P_t^S\right\}. \tag{1}$$

In a second step, matches are derived from contacts. The rule is that only contacts between non-phantom traders lead to effective trade. The number of matches is

$$M_t = \frac{B_t}{B_t + P_t^B}\frac{S_t}{S_t + P_t^S}\min\left\{B_t + P_t^B, S_t + P_t^S\right\}. \tag{2}$$

The number of contacts is multiplied by the product of the two proportions of non-phantom traders. We assume that phantoms cannot be distinguished from non-phantoms by the matching mechanism.

The matching probabilities are μ_t for buyers and η_t for sellers. We have

$$\mu_t = \frac{M_t}{B_t} = \frac{S_t}{S_t + P_t^S}\min\left\{1, \frac{S_t + P_t^S}{B_t + P_t^B}\right\}, \tag{3}$$

$$\eta_t = \frac{M_t}{S_t} = \frac{B_t}{B_t + P_t^B}\min\left\{\frac{B_t + P_t^B}{S_t + P_t^S}, 1\right\}. \tag{4}$$

We assume that the numbers of phantoms obey the following laws of motion:

$$P_t^B = \beta^B M_{t-1} + \left(1 - \delta^B\right) P_{t-1}^B, \tag{5}$$

$$P_t^S = \beta^S M_{t-1} + \left(1 - \delta^S\right) P_{t-1}^S, \tag{6}$$

with $0 < \beta^j \leq 1$, and $0 < \delta^j \leq 1$, $P_0^i \geq 0$, $j = B, S$. The inflow of new phantoms is proportional to former matches. The parameter β^j can be interpreted as the probability that a match gives birth to a phantom trader, or as the relative search efficiency of phantoms vis-à-vis non-phantoms. The outflow results from constant depreciation at rate δ^j. Phantoms face a constant probability of dying δ^j each period. Their life expectancy is $1/\delta^j$, which measures the degree of persistence of obsolete information.

Proposition 1 (The PMT) In each period t, the number of matches is given by

$$\ln M_t = \ln S_t + \ln B_t - \ln \left[X_t + \beta_t \sum_{k=0}^{t-1} (1-\delta_t)^k M_{t-k-1} + (1-\delta_t)^t P_0^t \right], \qquad \text{(PMT)}$$

$$\textit{with} \begin{cases} \beta_t = \beta^B,\ \delta_t = \delta^B,\ X_t = B_t \text{ and } P_0^t = P_t^B \text{ if } B_t + P_t^B > S_t + P_t^S \\ \beta_t = \beta^S,\ \delta_t = \delta^S,\ X_t = S_t \text{ and } P_0^t = P_t^S \text{ if } B_t + P_t^B < S_t + P_t^S \end{cases}.$$

Proof. Suppose that $\min\left\{B_t + P_t^B, S_t + P_t^S\right\} = S_t + P_t^S$. Then $M_t = S_t B_t/(B_t + P_t^B)$. We have

$$P_t^B = \beta^B \sum_{k=0}^{t-1} \left(1-\delta^B\right)^k M_{t-k-1} + (1-\delta^B)^t P_0^S. \qquad (7)$$

This gives

$$\ln M_t = \ln S_t + \ln B_t - \ln \left[B_t + \beta^B \sum_{k=0}^{t-1} \left(1-\delta^B\right)^k M_{t-k-1} + (1-\delta^B)^t P_0^S \right]. \qquad (8)$$

Now suppose that $\min\left\{B_t + P_t^B, S_t + P_t^S\right\} = B_t + P_t^B$. Then $M_t = \frac{B_t S_t}{S_t + P_t^S}$. We have

$$P_t^S = \beta^S \sum_{k=0}^{t-1} \left(1-\delta^S\right)^k M_{t-k-1} + (1-\delta^S)^t P_0^B. \qquad (9)$$

This gives

$$\ln M_t = \ln S_t + \ln B_t - \ln \left[S_t + \beta^S \sum_{k=0}^{t-1} \left(1-\delta^S\right)^k M_{t-k-1} + (1-\delta^S)^t P_0^B \right]. \qquad (10)$$

The phantom matching technology (PMT) collapses into the usual non-frictional technology whenever $\beta_t = 0$. The novelty comes from the inclusion of the weighted sum of former matches in the last term. The weights depend on survival probabilities $(1-\delta_t)^k$ and entry rate of new phantoms β_t.

The role played by market history is parameterized by δ_t. As δ_t tends to 0, phantoms are almost infinite-lived and old phantoms have a large impact on current matches. Conversely, with full depreciation $\delta^B = \delta^S = 1$, phantoms live for one period and the PMT reduces to

$$\ln M_t = \ln S_t + \ln B_t - \ln \left[X_t + \beta_t M_{t-1}\right]. \qquad (11)$$

2.2 Intratemporal vs intertemporal externalities

We examine the matching externalities featured by the PMT. The combination of intratemporal and intertemporal externalities implies that the technology has increasing returns to scale in the short run and constant returns in the long run.

Proposition 2 (Intratemporal and intertemporal externalities) *The following properties hold.*

(i) Assume that $S_t + P_t^S > B_t + P_t^B$*. Then,*

$$d\ln M_t/d\ln B_t = 1, \quad (12)$$
$$d\ln M_t/d\ln S_t = 1-\mu_t, \quad (13)$$
$$d\ln M_t/d\ln P_t^S = -(1-\mu_t). \quad (14)$$

(ii) Assume that $S_t + P_t^S \neq B_t + P_t^B$ *for all* $t \geq 0$ *and there is* $\tau \geq 0$ *such that* $S_t + P_t^S > B_t + P_t^B$ *or* $S_t + P_t^S > B_t + P_t^B$ *for all* $t \geq \tau$*. Then,*

$$\lim_{t\to\infty} \sum_{k=0}^{t} \{d\ln M_t/d\ln B_k + d\ln M_t/d\ln S_k\} = 1. \quad (15)$$

Proof. See Appendix A.1.

Part (i) examines intratemporal externalities. It focuses on the case where phantom and nonphantom sellers are on the long side of the market, but similar though inverted results hold when buyers are on the long side. Part (i) shows that the PMT has constant returns with respect to the number of traders on the short side of the market. This property is typical of non-frictional matching models. Meanwhile, the PMT has positive returns with respect to the number of traders on the long side. The reason is that additional traders reduce the proportion of phantom traders. The more phantoms there are, the greater the effect.

Intratemporal externalities imply that the matching technology exhibits increasing returns to scale in the short run. Indeed, $d\ln M_t/d\ln S_t + d\ln M_t/d\ln B_t = 2-\mu_t > 1$. The magnitude of increasing returns to scale is parameterized by β^S (driving phantom births), δ^S (governing phantom deaths), and by the sequence of matching flows $\{M_{t-k-1}\}_{k=0}^{\infty}$ (fueling potential phantoms).

Part (ii) describes intertemporal externalities. Former matches generate phantom traders. In turn, phantoms deteriorate the current matching process. These intertemporal externalities imply that the whole market history of buyers and sellers affects the current number of matches. Part (ii) shows that intratemporal and intertemporal externalities combine so that the matching technology has constant returns to scale in the long run with respect to the history of traders. The restriction $S_t + P_t^S \neq B_t + P_t^B$ is made because the PMT is not differentiable when the two numbers coincide. The other restriction is made for tractability. Though the long side of the market can alternate for some time, it does not fluctuate in the long run.

Current matches may positively or negatively alter future matches. To understand this property, we consider the case where phantoms only last one period, i.e. $\delta^B = \delta^S = 1$. Assuming that phantom and nonphantom sellers are always on the long side, we have

$$d\ln M_{t+k}/d\ln S_t = (-1)^k \prod_{j=0}^{k-1} \frac{\beta^S M_{t+j}}{S_{t+j} + \beta^S M_{t+j}}. \quad (16)$$

The magnitude of this elasticity decreases with horizon period k. Its sign depends on $(-1)^k$, which is negative for even k and positive for odd k. An increase in the number of period-t traders increases the number of period-$t+1$ phantoms, thereby reducing the flow of matches in period $t+1$. For a similar reason, this increases the flow of matches in period $t+2$.

2.3 Stationary phantom matching technology

We assume that whenever a buyer and a seller match they are replaced by a similar pair of agents. This situation corresponds to a steady state where there is a constant inflow of new agents into the pools of buyers and sellers. We show that the phantom matching technology (PMT) converges towards a stationary technology, the stationary phantom matching technology (SPMT).

The numbers of traders are $B_t = B$ and $S_t = S$ for all $t \geq 0$. The number of matches follows the PMT.

Proposition 3 (The SPMT) *Let* $\sigma^B = \beta^B/\delta^B$, $\sigma^S = \beta^S/\delta^S$, *and* $\theta = S/B$.

(i) If $B + P_t^B > S + P_t^S$ *for all* $t \geq 0$, *then the sequence* M_t *converges towards the stationary number of matches*

$$M = m^B(B, S) = B\frac{-1 + (1 + 4\sigma^B S/B)^{1/2}}{2\sigma^B}, \tag{17}$$

and the elasticity of this function vis-à-vis S *is*

$$\varepsilon^B(\theta) = \frac{2\sigma^B\theta}{-(1 + 4\sigma^B\theta)^{1/2} + 1 + 4\sigma^B\theta} \in (1/2, 1); \tag{18}$$

(ii) If $B + P_t^B < S + P_t^S$ *for all* $t \geq 0$, *then the sequence* M_t *converges towards the stationary number of matches*

$$M = m^S(B, S) = S\frac{-1 + (1 + 4\sigma^S B/S)^{1/2}}{2\sigma^S}, \tag{19}$$

and the elasticity of this function vis-à-vis S *is*

$$\varepsilon^S(\theta) = 1 - \frac{2\sigma^S\theta^{-1}}{-\left(1 + 4\sigma^S\theta^{-1}\right)^{1/2} + 1 + 4\sigma^S\theta^{-1}} \in (0, 1/2); \tag{20}$$

Proof. We prove (i) and omit the proof of (ii), which is similar. In steady state, $M_t = M$ and solves $\sigma^B M^2 + BM - BS = 0$. Resolution gives (17). To establish convergence, note that $M_t = BS/(B + P_t)$. This implies that the sequence $\{M_t\}$ converges towards M if and only if the sequence $\{P_t\}$ converges towards $P = \sigma^B M$. But, $P_{t+1} = \phi(P_t)$, with $\phi(x) = \beta^B BS/(B + x) + (1 - \delta^B)x$. As $\phi(0) > 0$ and $0 < \phi'(x) < 1$ for all $x \geq 0$, $\{P_t\}$ converges towards P for all $\beta^B \in (0, 1]$ and all $\delta^B \in (0, 1]$. The elasticity $\varepsilon^B = \theta m_2^B(B, S)/m^B(B, S)$. The computation gives (18).

The SPMT features standard properties. First, it is strictly increasing in the numbers of traders on each market side. Second, it has constant returns to scale. This property results from the constant intertemporal returns to scale discussed by Proposition 2. Third, the elasticity of the matching technology with respect to the ratio of sellers to buyers depends on whether buyers or sellers are on the short side of the market. It is larger than 1/2 when phantom and nonphantom sellers are on the short side, whereas it is lower than 1/2 when buyers are. This property is useful in applied work when we can guess or estimate the elasticity ε, whereas we cannot observe phantom traders. We use it in Section 3.4.

At given seller-to-buyer ratio θ, the stationary number of matches decreases with $\sigma^i = \beta^i/\delta^i$. This parameter governs the magnitude of search frictions induced by information obsolescence. It consists of the product of phantoms' life expectancy $1/\delta^i$ by their relative search efficiency β^i. Thus σ^i captures the amount of outdated information and its degree of visibility for match seekers. When σ^i tends to 0, the non-frictional case obtains and the number of matches tends to $m^B(B, S) = S$ or $m^S(B, S) = B$. When σ^i tends to infinity, the number of matches tends to 0 and the elasticity of the matching function ε tends to 1/2.

2.4 Discussions

We discuss several aspects of the PMT: the case of atomistic traders, the interplay between information obsolescence and coordination frictions, alternative phantom motions, and the case of on-the-match search. The non-interested reader can directly skip this part and go to Section 3 where we apply the PMT to the labor market.

Atomistic traders.—Propositions 1 to 3 apply for nonatomistic agents. We here relax this assumption and examine the case of atomistic agents. The model is unchanged, but B, S, P^B, and P^S are now integers. Let M be the stochastic number of matches. When phantom and nonphantom sellers are on the long side, this number follows a hypergeometric law such that

$$\Pr[M = k] = \frac{\binom{S}{k}\binom{P^S}{B-k}}{\binom{P^S+S}{B}}, \tag{21}$$

with $M \leq S$. The denominator is the total number of combinations involving the B nonphantom buyers and the $P^S + S$ sellers, and the denominator is the total number of combinations where exactly k nonphantom buyers are matched with the same number of nonphantom sellers.

The mean and the variance of M are

$$\mathbb{E}(M) = \mu B, \tag{22}$$

$$V(M) = B\mu(1-\mu)\frac{S + P^S - B}{S + P^S - 1} \tag{23}$$

The mean coincides with the PMT given by Proposition 1 when phantom and nonphantom sellers are on the long side.

Let $B = bN$, $S = sN$ and $P^S = p^S N$, where N is the population size. As N becomes arbitrarily large, the variance of M/N tends to 0. This implies that Propositions 1 to 3 still hold for a large population of non-atomistic agents provided we define per capita variables (i.e. B/N, S/N, or P^S/N). For instance, the matching probability M/B tends to the deterministic number μ and the matching probability M/S tends to $\mu B/S$.

The reasoning is the same when buyers are on the long side.

Information obsolescence and coordination frictions.—Information persistence can be combined with alternative sources of matching frictions. We here consider the popular case of coordination frictions. We assume that each buyer sends a buy order to one of the sellers, including phantoms. The probability that a particular seller receives a buy order from a particular buyer is $1/\left(S+P^S\right)$. The number of matches is

$$M = S\left[1-\left(1-\frac{1}{S+P^S}\right)^B\right]. \tag{24}$$

As $B, S, P^S \to \infty$, this gives

$$M = S\left[1-\exp\left(-\frac{B}{S+P^S}\right)\right]. \tag{25}$$

This technology still features increasing returns to scale vis-à-vis B and S because increasing S allows the phantom proportion on the sellers' side to be reduced. In the long run, the SPMT is

$$M = S\left[1-\exp\left(-\frac{B}{S+\beta^S M/\delta^S}\right)\right]. \tag{26}$$

This equation implicitly defines $M = m(B,S)$. The SPMT has constant returns to scale.

This scenario could be enriched. Buyers could observe each seller's time spent in the market and use this information to direct their search. Sellers could signal they are still active. We leave these extensions to future research.

Alternative technologies of phantom formation / dissolution.—The phantom stock obeys a linear dynamic equation, i.e. current phantom stock equals former stock minus depreciation plus new phantoms. A more general formulation would be

$$P_t = f\left(M_{t-1}, P_{t-1}\right). \tag{27}$$

Additional restrictions need to be imposed on function f. First, f must be increasing in the number of matches M_{t-1} and in the former phantom stock P_{t-1}. Second, to avoid the phantom stock growing to infinity, the partial derivative with respect to P_{t-1} must be less than one. Moreover, f must have constant returns to scale. Indeed, the stationary number of phantoms solves $P = f(M,P)$. Owing to the fact that $0 < f_P < 1$, P can be expressed as an increasing function of the stationary number of matches M. That is $P = g(M)$. The SPMT can be written

$$M = \frac{BS}{B+g(M)}. \tag{28}$$

The PMT has constant returns to scale if and only if $g(M) = Mg(1)$. Thus g must be linear in M. In turn, this restriction implies that f must have constant returns to scale.

On-the-match search.—On-the-match search occurs when matched traders go on searching for alternative partners. When on-the-match seekers compete with the unmatched traders, they act as phantoms from the perspective of those unmatched. This implies that usual strategies to quantify the impact of on-the-match search on the unmatched may fail to distinguish matched seekers from phantoms.

In the remaining, we assume that phantom and nonphantom buyers are on the short side. We denote by E_t the number of matched buyers. Such agents separate with exogenous probability δ^E, and a fraction $\beta^E \leq 1$ search for alternative partners and compete with unmatched buyers. If the matching odds are the same for matched and unmatched traders, then the number of matches between unmatched buyers and sellers is given by the following modified PMT:

$$\ln M_t = \ln S_t + \ln B_t - \ln(S_t + \beta^E E_t + \beta^S P_t^S), \tag{29}$$

where phantoms obey equation (6) and the motion for matched buyers is

$$E_t = M_{t-1} + (1 - \delta^E)E_{t-1}. \tag{30}$$

On-the-match search and information obsolescence have very different implications for match formation. However, it is hard to disentangle their respective impacts on the unmatched. Suppose indeed that the true specification of the matching function is (29). Manipulating equation (29) and adding an error component e_t orthogonal to the other covariates gives

$$\frac{S_t B_t}{M_t} - S_t = \beta^E E_t + \beta^S P_t^S + e_t. \tag{31}$$

Omitting P_t^S should bias upward the estimate of parameter β^E. The problem is that E_t and P_t^S are positively correlated: they both depend on the sequence of previous matches. They are actually identical when $\beta^E = \beta^S$ and $\delta^E = \delta^S$. In other words, we may wrongly attribute the whole effect of β^E to on-the-match search, whereas part of this effect is also due to phantoms. This leads to overestimating the extent of competition between matched and unmatched seekers. For instance, the OLS estimate of parameter β^E is

$$\hat{\beta}^E = \sum_{t=1}^{T} E_t S_t \frac{B_t - M_t}{M_t} \left(\sum_{t=1}^{T} E_t^2 \right)^{-1}, \tag{32}$$

where T is the sample size. The expectation of $\hat{\beta}^E$ is

$$\mathbb{E}\left(\hat{\beta}^E\right) = \beta^E + \beta^S \frac{\mathbb{E}\left(E_t P_t^S\right)}{\mathbb{E}\left(E_t^2\right)} > \beta^E. \tag{33}$$

This problem goes beyond the particular specification (29). The statement casts doubt on the interpretation of estimated matching functions for the unemployed that explicitly account for on-the-job search. Such estimates typically find evidence of job competition between employed and unemployed job-seekers (see, e.g., Broersma and van Ours, 1999, and Anderson and Burgess, 2000). The fact that traders of the past affect current recruitments does not prove that employees create congestion effects for the unemployed. Moreover, there are reasons to believe that unemployed and employees compete for different jobs (see Delacroix and Shi, 2006, for a directed search model with homogenous agents in which unemployed and employees search in segmented markets). Crowding-out effects reported in the literature may also result from information obsolescence.[3]

[3]The literature mentioned above already points out the problem of other missing variables like unregistered vacancies or the search intensity of the various job-seekers (see also Sunde, 2007).

3 Labor market phantoms

We embed the phantom matching technology (PMT) in the canonical equilibrium search unemployment model. We study the local properties of the steady state, and then calibrate the model on US labor market data.

3.1 Equilibrium search unemployment with a PMT

We describe a standard matching model of unemployment with aggregate productivity shocks. The only non-standard element of the model is the PMT. There is a continuum of homogenous workers whose total size is normalized to unity. These workers are either employed—in mass $1-u$—, or unemployed—in mass u. Unemployed workers seek for jobs, whereas employed workers do not. On the other side of the market, there is a continuum of jobs. These jobs are either vacant—in mass v—, or filled—in mass $1-u$. Unemployed and vacancies are brought together by pair by the PMT. In the terminology of the previous section, unemployed are buyers and vacancies are sellers, i.e. $B = u$ and $S = v$. Worker-firm pairs produce output y_t in each period. The log output follows an AR(1) process, i.e. $\ln y_t = (1-\lambda)\ln y + \lambda \ln y_{t-1} + e_t$, where $\lambda \in (0,1)$ is a persistence parameter and e_t is an i.i.d. shock that follows the normal distribution of mean 0 and variance ${\sigma_e}^2$.

The timing of events in each period is as follows. First, workers who were hired in the previous period enter employment relationships. Second, an aggregate productivity shock occurs. Third, firms post jobs and matching takes place between unemployed and vacancies. These matches lead to new employment relationships in the next period. Fourth, wages are bargained. Fifth, production takes place. Sixth, separation may occur.

The aggregate number of matches is given by the PMT:

$$M_t = \min\{u_t + P_t^u, v_t + P_t^v\} \frac{u_t}{u_t + P_t^u} \frac{v_t}{v_t + P_t^v}, \tag{34}$$

with

$$P_t^i = (1-\delta_i) P_{t-1}^i + \beta_i M_{t-1}, \; i = u, v. \tag{35}$$

Following Proposition 3, the PMT is associated to the following SPMTs, depending on whether phantom and nonphantom vacancies are on the short side. Thus

$$m^u(u,v) = u \frac{-1 + (1 + 4\sigma^u v/u)^{1/2}}{2\sigma^u}, \tag{36}$$

$$m^v(u,v) = v \frac{-1 + (1 + 4\sigma^v u/v)^{1/2}}{2\sigma^v}. \tag{37}$$

with $\sigma^i = \beta^i/\delta^i$.

The law of motion for unemployment is

$$u_t = u_{t-1} + s(1-u_{t-1}) - M_{t-1}, \tag{38}$$

where s is the job loss probability. Workers hired at date t become productive in $t+1$; similarly, workers who were dismissed at the end of t stay unemployed at least one period and can only start working again in $t+2$.

We denote by J and V the values of, respectively, having a filled job and having a vacancy. We have

$$J_t = y_t - w_t + \rho(1-s)\mathbb{E}_t J_{t+1}, \quad (39)$$
$$V_t = -c + \rho\eta_t \mathbb{E}_t J_{t+1}, \quad (40)$$

with $\rho \in (0,1)$ the discount factor, c the posting cost, and $\eta_t = M_t/v_t$. The cost c may be controversial in our framework where there is information persistence and firms may receive applications well after the cost has been paid. Thus there is another standard interpretation: job creation is costly and firms repay the creation cost until the job is destroyed.

There is free entry of new jobs. It follows that $V_t = 0$ for all t and

$$\frac{c}{\eta_t} = \rho\mathbb{E}_t J_{t+1} = \rho\mathbb{E}_t \left[y_{t+1} - w_{t+1} + \rho(1-s)\mathbb{E}_{t+1} J_{t+2}\right].$$

Since the free-entry condition holds in each period, we obtain

$$\frac{c}{\eta_t} = \rho\mathbb{E}_t \left[y_{t+1} - w_{t+1} + (1-s)\frac{c}{\eta_{t+1}}\right]. \quad (41)$$

Equation (41) is an Euler equation relating the expected search cost c/η_t of period t to the expected cost c/η_{t+1} of period $t+1$.

The values of being employed and unemployed are, respectively:

$$W_t = w_t + \rho\left[(1-s)\mathbb{E}_t W_{t+1} + s\mathbb{E}_t U_{t+1}\right], \quad (42)$$
$$U_t = z + \rho\left[\mu_t \mathbb{E}_t W_{t+1} + (1-\mu_t)\mathbb{E}_t U_{t+1}\right], \quad (43)$$

where $\mu_t = M_t/u_t$ is the job-finding probability and z is unemployment income.

Nash bargaining implies that workers obtain a constant share of match surplus:

$$W_t - U_t = \gamma\left[W_t - U_t + J_t - V_t\right], \quad (44)$$

where $\gamma \in (0,1)$ is worker's bargaining power.

Since $\mu_t/\eta_t = \theta_t$ and $V_t = 0$ for all t, we obtain

$$\rho\mathbb{E}_t J_{t+1} = \frac{\gamma}{1-\gamma}\rho\mathbb{E}_t\left[W_{t+1} - U_{t+1}\right] = \frac{c}{\eta_t}. \quad (45)$$

Straightforward manipulation yields the usual wage equation, that is

$$w_t = \gamma\left(y_t + c\theta_t\right) + (1-\gamma)z. \quad (46)$$

3.2 Steady state

Definition 1 A stochastic *intertemporal equilibrium* is a sequence $\{y_t, u_t, M_t, v_t, P_t^u, P_t^v\}_{t=0}^{\infty}$ such that:

(i) (y_0, u_0, P_0^u, P_0^v) is given by history, v_0 is a forward variable and $M_0 = \min\{\frac{u_0 v_0}{v_0 + P_0^v}, \frac{u_0 v_0}{u_0 + P_0^u}\}$;

(ii) the following equations hold for all $t \geq 1$, and $i = u, v$:

$$
\begin{aligned}
\ln y_t &= (1-\lambda)\ln \overline{y} + \lambda \ln y_{t-1} + e_t, && (47)\\
M_t &= \min\{u_t + P_t^u, v_t + P_t^v\} \frac{u_t}{u_t + P_t^u} \frac{v_t}{v_t + P_t^v}, && (48)\\
u_t &= u_{t-1} + s(1-u_{t-1}) - M_{t-1}, && (49)\\
P_t^i &= (1-\delta^i) P_{t-1}^i + \beta^i M_{t-1}, && (50)\\
c\frac{v_{t-1}}{M_{t-1}} &= \rho \mathbb{E}_{t-1}\left[(1-\gamma)(y_t - z) - \gamma c \frac{v_t}{u_t} + (1-s)c\frac{v_t}{M_t}\right]. && (51)
\end{aligned}
$$

We proceed in two steps. We first examine the steady state and then turn to the study of its local properties.

Definition 2 A *steady state* is a vector $(y^\star, u^\star, M^\star, v^\star, P^{v\star}, P^{u\star})$ such that $(y_t, u_t, M_t, v_t, P_t^v, P_t^u) = (y^\star, u^\star, M^\star, v^\star, P^{v\star}, P^{u\star})$ for all t.

There are two cases: either $v^\star + P^{v\star} > u^\star + P^{u\star}$ or $v^\star + P^{v\star} < u^\star + P^{u\star}$, the unlikely case of equality being left aside. We call regime v the former case, whereas regime u is the latter one. To find the steady state, we first assume that we are in one of the two possible regimes, say i. Then the PMT is replaced by the corresponding SPMT m^i for $i = \{u, v\}$. The solving strategy is usual: it consists in finding a number of unemployed $u^\star$ and a vacancy-to-unemployed ratio $v^\star/u^\star$ that jointly solve the following Beveridge curve and the free-entry condition:

$$
\begin{aligned}
u &= s/(s + m^i(1, v/u)), && (52)\\
c\frac{v/u}{m^i(1,v/u)} &= \rho\left[(1-\gamma)(y_t - z) - \gamma c\frac{v}{u} + (1-s)c\frac{v/u}{m^i(1,v/u)}\right]. && (53)
\end{aligned}
$$

Finally, we compute $v^\star + P^{v\star}$ and $u^\star + P^{u\star}$ in both cases and we find which regime holds.

Proposition 4 (Existence of steady state) *Let* $a = [1-\rho(1-s)](\rho\gamma)^{-1}$ *and* $b = (1-\gamma)(y - z)(\gamma c)^{-1}$. *The following statements hold.*

(i) In a steady state of regime $i = u, v$, *the job-finding probability is* μ^i *such that*

$$
\begin{aligned}
\mu^u &= \frac{-(\sigma^u a + 1) + [(\sigma^u a + 1)^2 + 4\sigma^u(b-a)]^{1/2}}{2\sigma^u}, && (54)\\
\mu^v &= \frac{-(\sigma^v a + b) + [(\sigma^v a + b)^2 + 4\sigma^v b]^{1/2}}{2\sigma^v}. && (55)
\end{aligned}
$$

(ii) There exists a steady state of regime u *if and only if* $b > a$ *and*

$$
\mu^u < \bar{\mu} \equiv \frac{-(1+\sigma^v - \sigma^u) + [(1+\sigma^v-\sigma^u)^2 + 4\sigma^u]^{1/2}}{2\sigma^u}. \qquad (56)
$$

(iii) There exists a steady state of regime v *if and only if* $\mu^v > \bar{\mu}$.

Proof. Part (i). Suppose we are in a steady state of regime i and let θ^i denote the corresponding tightness. Equation (51) implies that θ^i solves

$$a\frac{\theta^i}{m^i(1,\theta^i)} = b - \theta^i. \tag{57}$$

We actually solve this equation in $\mu^i = m^i(1,\theta^i)$. When $i = u$, inverting the SPMT gives $\theta = \mu(1+\sigma^u\mu)$. We replace it in equation (57), and we solve the resulting equation. This gives (54). The proof is similar in the case $i = v$. The only difference is that $\theta = \sigma^v\mu^2/(1-\mu)$.

Part (ii). Regime u holds if and only if μ^u exists and $P^u + u > P^v + v$. The former condition is equivalent to $b > a$. We then divide both sides of the latter inequality by u and we replace P^i by M_i/σ^i. This gives the following condition

$$P^u + u > P^v + v \Leftrightarrow (\sigma^v - \sigma^u)\mu^u + \theta^u - 1 < 0. \tag{58}$$

Replacing θ^u by $\mu(1+\sigma^u\mu)$ and solving in μ gives (56).

Part (iii). The proof is similar, except that we replace θ^v by $\sigma^v\mu^2/(1-\mu)$.

The PMT has constant returns to scale in the long run. This explains why we solve the steady state in unemployment rate and labor market tightness. The steady state, whether in regime u or v, has standard properties. Equilibrium vacancy-to-unemployed ratio $v^\star/u^\star$ decreases with workers' bargaining power γ, unemployment benefit z, job destruction probability s, and vacancy cost c. It also decreases with the ratio of phantom birth rate to phantom death rate $\sigma^i = \beta^i/\delta^i$, which governs the magnitude of matching frictions created by information persistence.

Corollary 1 (Normalized steady state) *Let* $\mu \in (0,1)$. *Then,*

(i) $\mu^u = \mu$ *if and only if* $b = \sigma^u\mu^2 + (\sigma^u a + 1)\mu + a$;

(ii) $\mu^v = \mu$ *if and only if* $b = \sigma^v\mu(\mu + a)(1-\mu)^{-1}$.

Suppose μ is the observed stationary job-finding probability. Corollary 1 states that given a subset of parameters summarized by a, we can always find the complementary subset of parameters summarized by b so that μ is the predicted steady-state job-finding probability. Corollary 1 also has implications for calibration strategies. We can compensate any change in a by a change in b according to (i) and (ii). For instance, the cost of holding a vacancy c makes b vary from 0 to infinity as it goes from infinity to 0. Thus we can set any combination of parameters and compute c so that μ stays fixed. We use this property in the next subsection.

3.3 Local stability

We consider each regime separately. We thus forbid cases where the dynamics starts in one of the regimes and enters the other one at some time. We fix the normalized steady state μ and examine how the local stability properties depend on the different parameters. Beyond

μ, the steady-state Jacobian matrix depends on ρ, s, γ, δ^i, and σ^i. There is not much uncertainty about ρ and s in a given country and for a given period. Thus we focus on the bargaining power γ and the two parameters of the PMT δ^i and σ^i (hence $\beta^i = \sigma^i \delta^i$).

Given there are three dimensions and three parameters of interest, the local properties of the steady state are difficult to discuss in the general case. A close look at the Jacobian matrix suggests the following strategy: we restrict our theoretical analysis to the case $\delta^i = s$. Thus the phantom stock and employment depreciate at the same rate. As we shall see in the next subsection, this case is not the most realistic. However, this is interesting for two reasons. First, it reduces the state space to two dimensions and leaves us with only two parameters of interest: γ and σ^i. Second, numerical simulations show that the analytical results discussed hereafter can be extended to all plausible parameter combinations.

Proposition 5 (Stability properties of regimes u and v) *Let μ be the normalized steady-state job-finding probability and let also $\delta^i = s$, $i = u, v$. The following properties hold.*

(i) Regime u: the steady state is locally a sink when $\gamma > \frac{\sigma^u(\mu+s)}{\rho(1+\sigma^u\mu)}$ and a source when $\gamma < \frac{\sigma^u(\mu+s)}{\rho(1+\sigma^u\mu)}$;

(ii) Regime v: if $s/\mu > 1$, then the steady state is locally a source; if $s/\mu < 1 < s/\mu + \frac{\rho(\mu+s)(1-\mu)^2}{(1-\rho s)\mu}$, then there is a unique $\bar{\gamma} \in (0,1)$ such that the steady state is locally a saddle when $\gamma < \bar{\gamma}$ and a source when $\gamma > \bar{\gamma}$; if $1 > s/\mu + \frac{\rho(\mu+s)(1-\mu)^2}{(1-\rho s)\mu}$, then the steady state is locally a saddle.

Proof. See Appendix A.2.

Regime u can feature indeterminacy or instability, whereas regime v is a source or is saddle-path stable. The surprising result here is the possibility of having indeterminacy and associated self-fulfilling prophecies. In the continuous-time model where the matching function has constant returns to scale, indeterminacy never arises. It only occurs when the matching function has increasing returns to scale (Mortensen, 1989, 1999). In our framework the PMT has long-run constant returns to scale, but it features short-run increasing returns to scale. In the discrete-time model, Krause and Lubik (2010) show that indeterminacy may arise despite the fact that the matching function has constant returns to scale. Thus indeterminacy is not new in itself, but the mechanism that drives self-fulfilling prophecies is original.

To understand indeterminacy, consider the free-entry condition in the standard case, i.e. when the matching function is the SPMT:

$$c/\eta(\theta) = \rho J_{+1} = \rho \left[(1-\gamma)(y-z) - \gamma c \theta_{+1} + (1-s) c/\eta(\theta_{+1}) \right], \tag{59}$$

where the index + refers to lead variables and $\eta(\theta)$ is the recruitment probability. Firms create new jobs until the expected search cost $c/\eta(\theta)$ equals the discounted future value of a filled job. This gives a relationship between current tightness, which affects the current recruitment probability, and future tightness θ_{+1} through the free-entry condition. In the standard case, $0 < d\theta/d\theta_{+1} < 1$ in the neighborhood of $\theta^\star$. Expecting $\theta_{+1} > \theta^\star$ leads to explosive dynamics, and so there is saddle-path stability. Krause and Lubik (2010) show that we can actually have $d\theta/d\theta_{-1} < 0$. It can even be so negative that $d\theta/d\theta_{-1} < -1$ in particular configurations. Then, any initial value of θ leads to the steady state with oscillatory pattern of decreasing amplitude. Thus there can be indeterminacy. Appendix B explains this argument more formally.

Indeterminacy in regime u obeys a different economic mechanism. It relies on the intertemporal congestion externalities displayed by the PMT. The free-entry condition is now:

$$c\left(1+P^u/u\right)=\rho J_{+1}=\rho\left[(1-\gamma)\left(y-z\right)-\gamma c\theta_{+1}+(1-s)c\left(1+P^u_{+1}/u_{+1}\right)\right]. \qquad (60)$$

Unlike the standard model, entry decisions affect the future expected search cost and not the current one. More vacancies in the current period raise the number of matches, which fuel the future phantom stock and decrease future unemployment. Thus the ratio P^u_{+1}/u_{+1} goes up. Starting from steady state, expecting $\theta_{+1} > \theta^\star$ must be balanced by an increase in the future continuation value of a match J_{+2}. This in turn requires $\theta > \theta^\star$. When γ is sufficiently large, a small increase in θ_{+1} is compensated by a large increase in θ. Thus, $d\theta/d\theta_{+1} > 1$ and there is indeterminacy. Otherwise, the steady state is locally unstable.

Indeterminacy never occurs in regime v. The free-entry condition is

$$c\left(\theta+P^v/u\right)=\rho J_{+1}=\rho\left[(1-\gamma)\left(y-z\right)-\gamma c\theta_{+1}+(1-s)c\left(\theta_{+1}+P^v_{+1}/u_{+1}\right)\right]. \qquad (61)$$

As in regime u, entry decisions affect the future recruitment probability through changes in unemployment and phantom vacancies. However, entry decisions also affect the current recruitment probability through usual congestion effects. Thus the dynamics is closer to the standard model, but without the exotic configuration leading to indeterminacy.

3.4 PMT for the US labor market

We use monthly US data. We assume that the length of a period in our model is also one month. Thus the data-generating process requires the amount of time needed to observe changes in the different aggregate variables.

The Bureau of Labor Statistics (BLS) provides data on unemployed people. The number of vacancies is taken from the Job Openings and Labor Turnover Survey. This dataset only includes existing and distinct vacancies from data collected by the BLS from 16,000 establishments. The variable is available since December 2000. We cut the data in December 2008, just before the anomaly observed in the Beveridge curve—unemployment stays constant for a while, whereas vacancies strongly increase. We do not consider the Help-Wanted index, which is available over a much longer period. This index is based on counting the number of ads in 36 newspapers. Unfortunately, the increasing role played by internet in job search strategies has negatively affected the index over the recent years. This downward trend does not reflect a potential fall in actual vacancies.

We assume that each observation corresponds to the true numbers of available agents on each side of the market. Unlike statisticians, real people seek for each other on decentralized local labor markets. Information persistence becomes a problem at this level of decision, and this is why it may cause matching frictions.

The BLS also provides data on short-term unemployed (less than one month). We follow Shimer (2007) and use these data to compute the monthly job-finding probability. In our sample, the mean job-finding probability is $\bar{\mu} = 0.403$, whereas the mean ratio of vacancies to unemployed workers is $\bar{\theta} = 0.524$.

The PMT may have two forms, depending on whether phantom and nonphantom vacancies are on the short side of the market or not (see Proposition 1). The empirical value of θ is lower than one, and thus unemployed workers outnumber vacancies. However, what matters is the sum of phantoms and nonphantoms on each side of the market. Nonphantom vacancies may well be much more numerous than phantom unemployed.

We use another piece of information to choose the PMT. Proposition 3 states that the elasticity of the matching technology is larger than .5 when sellers (vacancies) are on the short side and lower than .5 when they are not. Empirically, the elasticity of the matching technology differs across datasets and estimation techniques (see Petrongolo and Pissarides, 2001). However, neglecting endogeneity issues, the best fit of the US job-finding probability on the vacancy-to-unemployed ratio implies that the elasticity with respect to the number of vacancies is lower than .5 (see also Shimer, 2005). When we regress the log of the matching probability on the log tightness, we obtain

$$\ln \frac{M_t}{u_t} = \underset{(.018)}{-.684} + \underset{(.025)}{.339} \ln \frac{v_t}{u_t} + \hat{\nu}_t, \tag{62}$$

where the standard errors are between brackets. It follows that an estimate of the elasticity $\varepsilon^v(\theta)$ is $0.339 < 0.5$. Therefore, we assume that phantom and nonphantom vacancies are on the long side of the market. Following Proposition 5, this calibration choice forbids self-fulfilling prophecies.

If we had credible estimates of the stock of phantom vacancies, we could directly estimate the phantom birth and death rates. Instead, we start from the PMT and we invert it

$$\frac{M_t}{u_t} = \frac{v_t}{v_t + P_t^v} \Leftrightarrow P_t^v = v_t \frac{u_t - M_t}{M_t}. \tag{63}$$

Assuming that the linear dynamics of the phantom stock is correct up to an i.i.d. error term of mean 0, we have

$$v_t \frac{u_t - M_t}{M_t} = (1 - \delta^v) v_{t-1} \frac{u_{t-1} - M_{t-1}}{M_{t-1}} + \beta^v M_{t-1} + e_t. \tag{64}$$

Parameters δ and β are estimated by OLS. This gives

$$v_t \frac{u_t - M_t}{M_t} = \underset{(.065)}{.553} v_{t-1} \frac{u_{t-1} - M_{t-1}}{M_{t-1}} + \underset{(.122)}{.846} M_{t-1} + \hat{e}_t. \tag{65}$$

Thus $\beta^v = 0.846$ and $\delta^v = 0.447$. Each new match gives birth to 0.846 phantom vacancy, and such phantoms live for two months on average. The ratio $\hat{\sigma}^v = 0.846/0.447 = 1.892$ and Proposition 3 implies that the corresponding SPMT is

$$m^v(u, v) = v \frac{-1 + (1 + 7.58u/v)^{1/2}}{3.79}. \tag{66}$$

At the mean empirical value $\bar{\theta} = 0.524$, the elasticity of the SPMT with respect to vacancies is

$$\hat{\varepsilon}^v = 1 - \frac{2\hat{\sigma}^v \bar{\theta}^{-1}}{-\left(1 + 4\hat{\sigma}^v \bar{\theta}^{-1}\right)^{1/2} + 1 + 4\hat{\sigma}^v \bar{\theta}^{-1}} = 0.373. \tag{67}$$

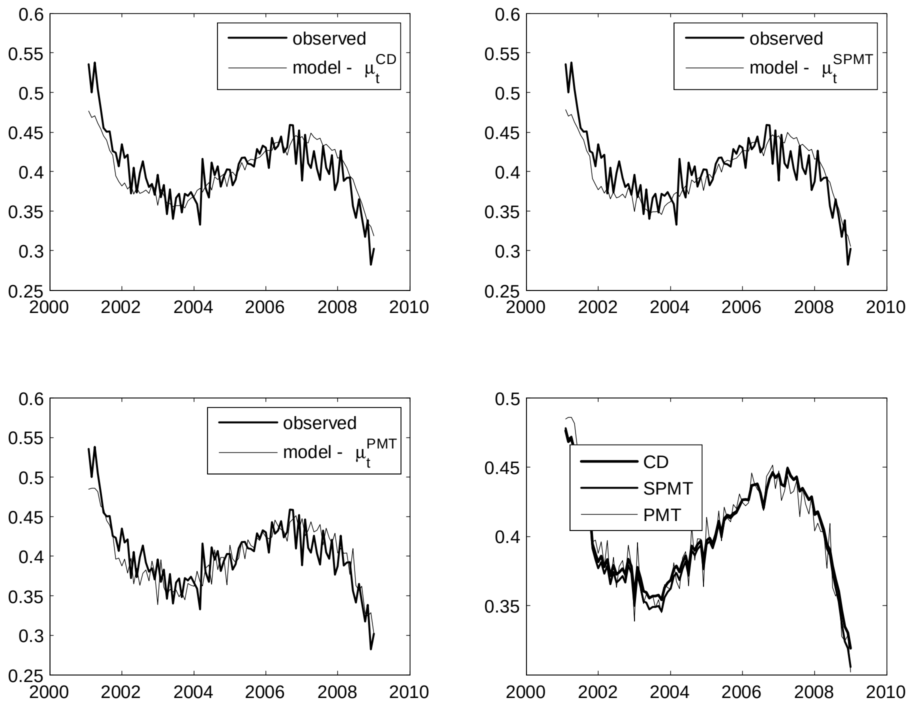

Figure 2: Actual and predicted monthly job-finding probability

This elasticity is close to the empirical value reported above.

For each month between 2000-12 and 2008-12, we predict the job-finding probability using the different matching technologies. The Cobb-Douglas function and the SPMT predictions are computed from contemporaneous market tightness $\theta_t = v_t/u_t$. As for the PMT prediction, we replace the period-t phantom number by the deterministic part of equation (65). This computation only involves period-$t-1$ variables. However, the computation of the period-t number of matches involves the use of the period-$t+1$ short-term unemployed. Thus the different predictions require the same information set.

To summarize,

$$\hat{\mu}_t^{CD} = \exp(-0.684)\theta_t^{0.339}, \quad (68)$$

$$\hat{\mu}_t^{SPMT} = \theta_t \frac{-1 + (1 + 7.58\theta_t^{-1})^{1/2}}{3.79}, \quad (69)$$

$$\hat{\mu}_t^{PMT} = \frac{v_t}{v_t + 0.553 v_{t-1} \frac{u_{t-1} - M_{t-1}}{M_{t-1}} + 0.846 M_{t-1}}. \quad (70)$$

Figure 2 confronts these different predictions with actual values of the job-finding probability. These predictions are very similar. The PMT seems slightly more volatile, but the actual job-finding probability is also more volatile than the SPMT and Cobb-Douglas predictions. When we compute the standard deviation of the prediction error, we obtain 2.5% in the three cases.[4]

[4] When we perform a similar computation for regime u, we obtain $\beta^u = 0.109$ and $\delta^u = 0.188$. The prediction error of the matching probability is 50% higher.

3.5 Aggregate fluctuations

Calibration.—The identification strategy of the parameters of the PMT is based on a monthly version of the scenario that leads to the PMT. We respect this strategy and calibrate the model on a monthly basis. The monthly calibration has undesired (but quantitatively small) implications for the short-run dynamics of the different variables. Thus we also report results for a weekly calibration in the end of this section. We use the estimated PMT and simulate the propagation of productivity shocks. We normalize labor productivity to $\overline{y} = 1$ and set the autocorrelation parameter to the standard value $\lambda = 0.97$. To overcome the unemployment volatility puzzle (Shimer, 2005), we adopt the small surplus calibration of Hagedorn and Manovskii (2008). Unemployment income is fixed to $z = 0.9$ or even $z = 0.95$, whereas the bargaining power is set to $\gamma = 0.05$. We also report results for $z = 0.4$ and for higher values of γ. As emphasized by Proposition 4, γ cannot be set too high because the corresponding steady state would feature explosive dynamics. This occurs for $\gamma > 0.27$. The remaining parameters are as follows. The discount factor is $\rho = 0.9967$. Concerning the job loss probability s, we compute the short-term unemployed to employed ratio. This naive estimate has been criticized by Shimer (2005) because it suffers from the time-aggregation bias. In our model, transitions can only take place after one month. So we deliberately neglect the problem and we fix s to its naive sample mean. We obtain $s = 0.023$ over the period 2000-12 to 2008-12. The reference parameters for the PMT are taken from our previous estimates, $\beta^v = 0.846$, $\delta^v = 0.447$ hence $\sigma^v = 0.846/0.447 = 1.892$. To study the role of phantom persistence, we also discuss a case where phantoms last much longer with $\delta^v = 0.10$.

There is only one parameter remaining, c the search cost. We use Corollary 1, part (ii), to compute it. This gives

$$c = \frac{\rho(1-\gamma)(1-z)(1-\mu)}{\sigma^v \mu(\gamma\mu\rho + 1 - \rho(1-s))}, \tag{71}$$

with $\mu = \bar{\mu} = 0.403$. The associated steady-state tightness $\theta^\star = 0.515$. This value is by construction very close to the empirical mean $\bar{\theta} = 0.524$ reported above.

Impulse-Response Functions.—The PMT has phantom and nonphantom vacancies on the long side. With our calibration, the steady state has the local saddle-path property and there cannot be sunspot fluctuations. We start with Impulse-Response simulations to examine the propagation of a one-period shock on aggregate productivity. The panel of Figures 3 depicts the impacts of a one-period 1% productivity shock on different variables. We compute quarterly averages of monthly responses. The SPMT case is identical to the Cobb-Douglas case: tightness monotonically decreases over time, which mirrors the monotonic decline in output per worker. The job-finding probability and the number of vacancies follow a similar monotonic pattern. The unemployment rate starts decreasing, reaches a minimum, and then slowly returns to its long-run value.

The PMT case produces similar dynamics. However, it displays oscillations around the SPMT pattern. These oscillations are due to the birth and death of phantom vacancies along the adjustment. When tightness is above the SPMT pattern, job creation is stronger and this fuels the phantom stock. Thus tightness is lower the next period. This reduces job and phantom creation, etc. The magnitude of such oscillations decreases over time, as the PMT converges to the SPMT. As we show below, simulating the model at higher frequency

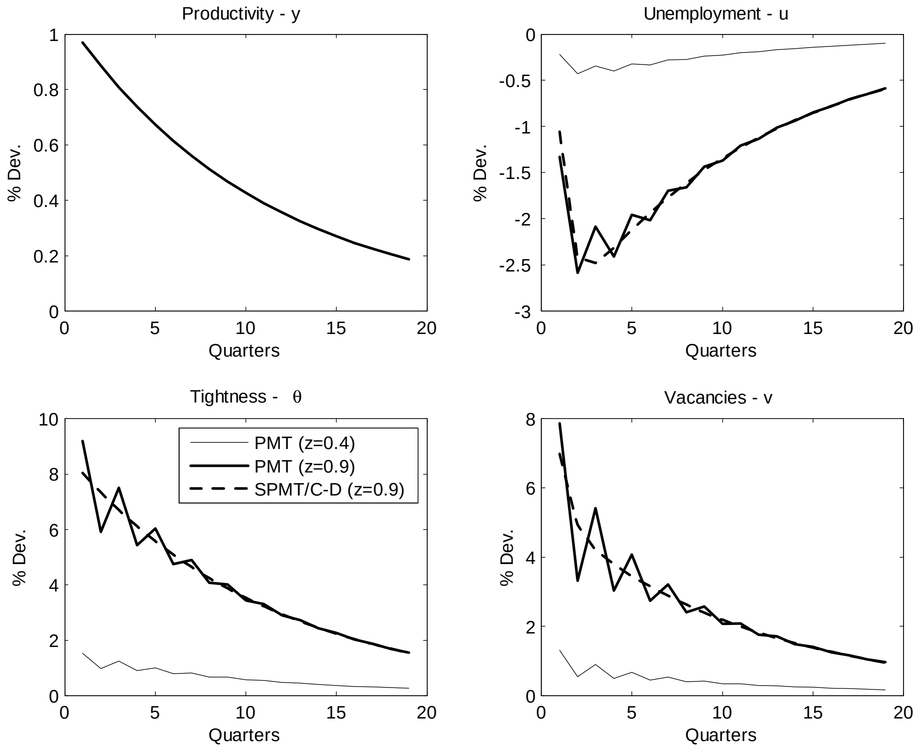

Figure 3: Impulse Response Function to a 1% productivity shock, monthly calibration

eliminates these oscillations. Finally, as demonstrated by Hagedorn and Manovskii (2008), the short-run elasticities of tightness and vacancies strongly depend on the value of leisure z. When $z = 0.4$, the instantaneous elasticity is about 2, whereas it is close to 10 when $z = 0.9$.

Simulations, not reported here, show that the magnitude of oscillations around the SPMT increases with phantom persistence δ^v. The reason is that σ^v being fixed, the fall in δ^v must be compensated by an equivalent decrease in β^v. Thus the share of new phantoms in the overall phantom stock is much smaller than in the reference calibration. It follows that the phantom stock is less sensitive to job creation, and the extra volatility of the different aggregate variables is reduced. Conversely, workers' bargaining power tends to increase the magnitude of oscillations.

Simulations with random productivity shocks.—We then simulate the monthly model with random productivity shocks. For each parameter configuration we proceed to 10,000 Monte-Carlo simulations. We consider means of log variables over quarters. We finally HP-filter these variables with parameter 10^5. To begin with, Figure 4 shows the business cycle components of the aggregate variables of interest for a given sequence of productivity shocks. The figure shows a very strong positive correlation between tightness, vacancies and productivity, whereas unemployment is negatively related to these variables. Moreover, changes in labor market variables are broadly of the same order of magnitude as changes in productivity. The figure also shows that the deterministic oscillations displayed by Figure 3 cannot easily be seen once mixed with a realistic sequence of productivity shocks.

We then proceed to a more systematic study of the business-cycle properties. Table 2 reports the relative volatility of different aggregate variables with respect to output volatility.

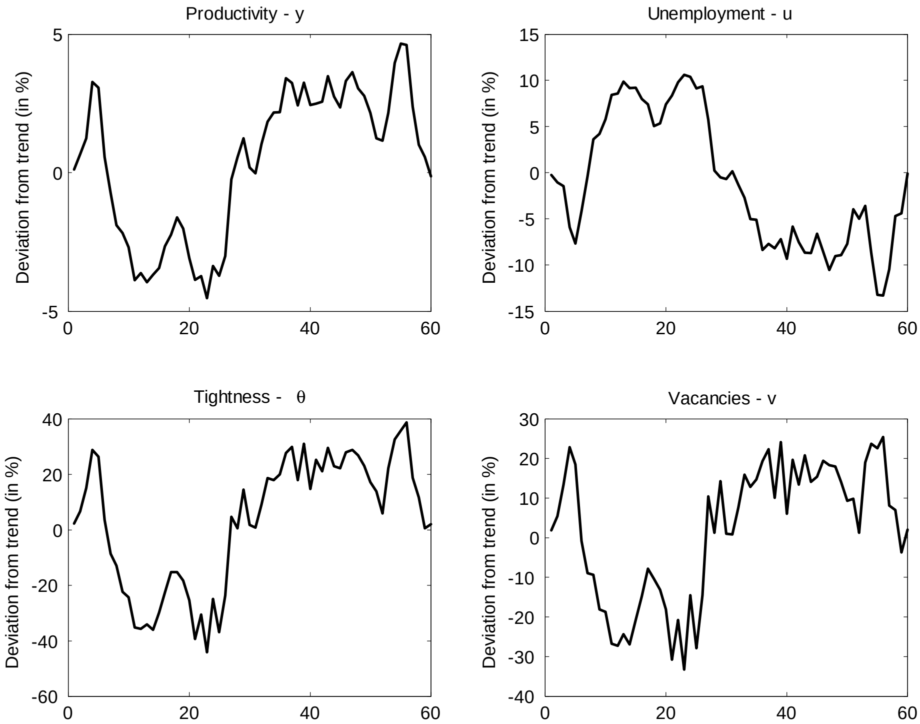

Figure 4: Business cycle quarterly dynamics of aggregate variables

It also displays the quarterly parameter of auto-correlation of each variable. Table 2 shows that the PMT and the SPMT behave like the Cobb-Douglas function. The SPMT produces the same statistics as the Cobb-Douglas technology. The simulations with the PMT display slightly more volatility and slightly less auto-correlation. The PMT has propagation mechanisms of its own and this slightly raises the predicted volatility. Such propagation mechanisms also involve some negative auto-correlation (more matches today, more phantoms tomorrow), which explains why quarterly auto-correlation is lower.

Table 2: Business cycle volatility and autocorrelation

	u	v	θ	y	u	v	θ	y
				Volatility with respect to y				
Facts	9.5	10.1	19.1	1	9.5	10.1	19.1	1
	SPMT-CD				PMT			
$z = 0.4$	0.47	0.94	1.38	1	0.47	0.98	1.39	1
$z = 0.9$	2.85	5.67	8.29	1	2.81	5.85	8.34	1
$z = 0.95$	5.69	11.34	16.58	1	6.35	14.26	19.14	1
				Auto-correlation				
Facts	0.94	0.94	0.94	0.88	0.94	0.94	0.94	0.88
	SPMT-CD				PMT			
$z = 0.4$	0.92	0.82	0.88	0.88	0.89	0.69	0.83	0.88
$z = 0.9$	0.92	0.82	0.88	0.88	0.89	0.69	0.83	0.88
$z = 0.95$	0.92	0.82	0.88	0.88	0.89	0.69	0.83	0.88

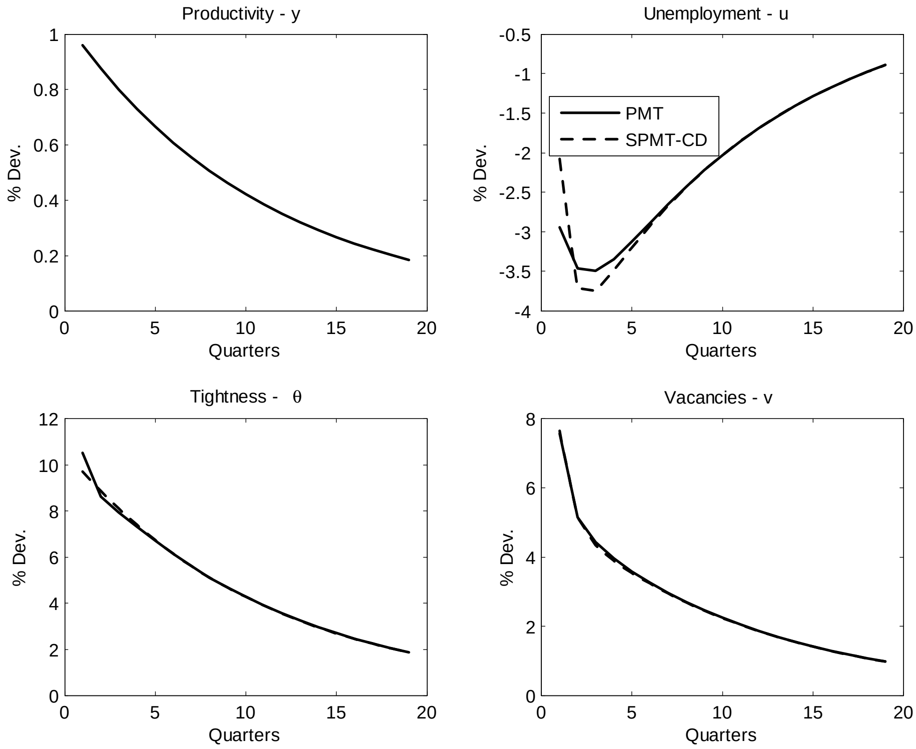

Figure 5: Impulse Response Function to a 1% productivity shock (weekly calibration)

Weekly calibration.—We finally report the IRFs of a weekly calibration. This calibration is less cautious because we cannot identify parameters β^v and δ^v at this frequency. The parameters that do not depend on time stay the same. Thus $y = 1$, $z = 0.9$ and $\gamma = 0.05$. We fix μ, ρ and λ so that the monthly job-finding probability, monthly discount factor, and monthly autocorrelation parameter are unchanged. Thus $\mu = 1 - (1 - 0.403)^{1/4} = 0.130$, $\rho = 0.97^{1/4} = 0.992$, and $\lambda = 0.97^{1/4} = 0.99$. The job loss probability is such that the steady-state unemployment rate remains the same. This leads to $s = 0.006$. As for the parameters of the PMT, we fix $\beta^v = 0.846$ as in the monthly calibration. The average duration of phantoms could stay the same. This would give $\delta^v = 0.138$. However, the steady state becomes a source in such a case. Thus we slightly reduce the phantom death probability to $\delta = 0.100$. Note that these choices are arbitrary: each time we set a different ratio σ^v, we implicitly modify the steady-state tightness. Finally, c adjusts following Corollary 1.

Figure 5 depicts the IRFs following a 1% increase in labor productivity. The PMT and the SPMT can hardly be distinguished. The oscillations displayed by Figure 3 still exist. However, they take place at higher frequency and so most of the convergence to the SPMT is achieved in a month.

4 Conclusion

This paper focuses on information persistence as a source of market frictions. It shows that this single source of frictions gives birth to a matching technology that has convenient properties from an applied perspective. The key idea is that each new match gives birth to a pair of phantom traders. In turn, phantom traders haunt the search place for some random period, inducing wasted resources spent by unmatched traders that desperately try to contact them. The resulting aggregate matching technology features increasing returns to scale in the short run, and constant returns in the long run. We apply the technology to US unemployment and vacancy data. We predict the monthly job-finding probability and aggregate volatility à la Shimer (2005). The results are very similar to those obtained with a Cobb–Douglas matching function.

The research can be extended in two main directions. On the macro side, information persistence sometimes leads to sunspot fluctuations. It would be interesting to analyze the contribution of such types of fluctuations in a more general setting where there are alternative sources of search frictions. The interplay between intratemporal and intertemporal externalities should also have implications for turnover externalities. On the micro side, the parameters that govern phantom birth and death could be endogenized. Match makers spend time and money to clean their websites or to advertise for available trade partners. Time spent in the market may also signal whether the trader is available or not.

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A Proofs

A.1 Proof of Proposition 2

Points (i) to (iii) result from direct computation. To prove (iv), we first show that M_t has constant returns to scale with respect to $S^t = (S_0, S_1, ..., S_t)$, $B_t = (B_0, B_1, ..., B_t)$, P_0^B and P_0^S. Let $H^t = (S^t, B^t, P_0^B, P_0^S)$ be the *market history*. Let also $\mathbf{M}_t : H^t \to \mathbb{R}_+$ and $\mathbf{P}_t^i : H^{t-1} \to \mathbb{R}_+$, $i = B, S$, be such that $M_t = \mathbf{M}_t(H^t)$ and $P_t^i = \mathbf{P}_t^i(H^{t-1})$. We first remark that $\mathbf{M}_t$ has constant returns to scale with respect to H^t if and only if $\mathbf{P}_t^B$ and $\mathbf{P}_t^S$ have constant returns to scale with respect to H^{t-1}. Then, suppose $t = 0$. We have $M_0 = B_0 S_0/(S_0 + P_0^S)$ if $S_0 + P_0^S > B_0 + P_0^B$ and $M_0 = B_0 S_0/(B_0 + P_0^B)$ if $S_0 + P_0^S < B_0 + P_0^B$. Multiplying each term by a similar factor $\lambda > 0$, we obtain that $\lambda S_0 + \lambda P_0^S > \lambda B_0 + \lambda P_0^B$ if and only if $S_0 + P_0^S > B_0 + P_0^B$. Thus the long side of the market stays the same. Moreover, in both cases we obtain $\mathbf{M}_0(\lambda H^t) = \lambda \mathbf{M}_0(H^t)$. So the property is true for $t = 0$. Now suppose the property holds for $t > 0$ and consider $t + 1$. We multiply all components of the market history by the same factor $\lambda > 0$. We have

$$\begin{aligned}
\lambda S_{t+1} + \mathbf{P}_{t+1}^S(\lambda H^t) &= \lambda S_{t+1} + \beta^S \mathbf{M}_t(\lambda H^t) + (1-\delta)\mathbf{P}_t^S(\lambda H^t) \\
&= \lambda(S_{t+1} + \mathbf{P}_{t+1}^S(H^t)), \qquad (72)
\end{aligned}$$

because $\mathbf{M}_t$ and $\mathbf{P}_t$ are homogenous of degree one by assumption. Similarly, we can show that

$$\lambda B_{t+1} + \mathbf{P}_{t+1}^B(\lambda H^t) = \lambda(B_{t+1} + \mathbf{P}_{t+1}^B(H^t)). \qquad (73)$$

Thus the difference $\lambda S_{t+1} + \mathbf{P}_{t+1}^S(\lambda H^t) - (\lambda B_{t+1} + \mathbf{P}_{t+1}^B(\lambda H^t))$ has the sign of the difference $S_{t+1} + \mathbf{P}_{t+1}^S(H^t) - (B_{t+1} + \mathbf{P}_{t+1}^B(H^t))$ and the long side of the market is the same again.

If $S_{t+1} + P_{t+1}^S > B_{t+1} + P_{t+1}^B$, then

$$\begin{aligned}
\mathbf{M}_{t+1}(\lambda H^{t+1}) &= \frac{\lambda^2 B_{t+1} S_{t+1}}{\lambda S_{t+1} + \beta \mathbf{M}_t(\lambda H^t) + (1-\delta^S)\mathbf{P}_t^S(\lambda H^{t-1})} \\
&= \lambda \mathbf{M}_{t+1}(H^{t+1}), \qquad (74)
\end{aligned}$$

because $\mathbf{M}_t$ has constant returns to scale with respect to H^t by assumption and this implies that $\mathbf{P}_t^S$ has constant returns to scale with respect to H^{t-1}. The reasoning is very similar when $S_{t+1} + P_{t+1}^S < B_{t+1} + P_{t+1}^B$. Thus the property holds in $t + 1$ and the function $\mathbf{M}_t$ is homogenous of degree one with respect to the market history H^t.

The function $\mathbf{M}_t$ is differentiable by assumption. The Euler theorem implies that

$$\sum_{k=1}^{t} \left\{ \frac{\partial \mathbf{M}_t(H^t)}{\partial B_k} B_k + \frac{\partial \mathbf{M}_t(H^t)}{\partial S_k} S_k \right\} + \frac{\partial \mathbf{M}_t(H^t)}{\partial P_0^B} P_0^B + \frac{\partial \mathbf{M}_t(H^t)}{\partial P_0^S} P_0^S = \mathbf{M}_t(H^t), \qquad (75)$$

Now we show that $P_0^B \partial \mathbf{M}_t(H^t)/\partial P_0^B$ and $P_0^S \partial \mathbf{M}_t(H^t)/\partial P_0^S$ tend to 0 as t tends to infinity. Let $I_t = 1$ if $S_t + P_t^S > B_t + P_t^B$ and $I_t = 0$ if $S_t + P_t^S < B_t + P_t^B$. For $j = B, S$, we have

$$\frac{\partial \mathbf{M}_t(H^t)}{\partial P_0^j} = -(1-\mu_t)\left(I_t \frac{\partial P_t^S}{\partial P_0^j} + (1 - I_t)\frac{\partial P_t^B}{\partial P_0^j} \right). \qquad (76)$$

Let $P_t = (P_t^B, P_t^S)'$ and A_t be the following 2×2 matrix:

$$A_t = \begin{bmatrix} 1 - \delta^B - (1 - I_t)\frac{\beta^B M_t}{B_t + P_t^B} & -I_t \frac{\beta^B M_t}{S_t + P_t^S} \\ -(1 - I_t)\frac{\beta^S M_t}{B_t + P_t^B} & 1 - \delta^S - I_t \frac{\beta^S M_t}{S_t + P_t^S} \end{bmatrix}. \tag{77}$$

We have $\partial P_t / \partial P_0^j = \Pi_{k=0}^{t-1} A_k \partial P_0 / \partial P_0^j$, with $\partial P_0 / \partial P_0^B = (1, 0)'$ and $\partial P_0 / \partial P_0^S = (0, 1)'$. Suppose $I_t = 1$ for all $t \geq \tau$. Then, for all $t > \tau$:

$$\Pi_{k=\tau}^t A_k = \begin{bmatrix} 1 - \delta^B & -\frac{\beta^B M_t}{S_t + P_t^S} \\ 0 & 1 - \delta^S - \frac{\beta^S M_t}{S_t + P_t^S} \end{bmatrix} \Pi_{k=\tau}^{t-1} A_k. \tag{78}$$

This gives

$$\Pi_{k=\tau}^t A_k = \begin{bmatrix} (1 - \delta^B)^{t-\tau} & Z_t \\ 0 & \Pi_{k=\tau}^t \left(1 - \delta^S - \frac{\beta^S M_t}{S_t + P_t^S}\right) \end{bmatrix}, \tag{79}$$

where $Z_t = (1 - \delta^B) Z_{t-1} - \frac{\beta^B M_t}{S_t + P_t^S} \Pi_{k=\tau}^{t-1} \left(1 - \delta^S - \frac{\beta^S M_k}{S_t + P_k^S}\right)$. As $M_k / (S_t + P_k^S) < 1$ for all $k \geq \tau$, we have $\lim_{t \to \infty} Z_t = 0$, and all the coefficients of the matrix above tend to 0 when t tends to infinity. It follows that $\partial P_t / \partial P_0^j = \Pi_{k=0}^{t-1} A_k \partial P_0 / \partial P_0^j$ tends to $(0, 0)$. We can reach the same conclusion when $I_t = 0$ for all $t \geq \tau$. Thus $\lim_{t \to \infty} \partial \mathbf{M}_t(H^t) / \partial P_0^i = 0$ for $i = B, S$ and

$$\lim_{t \to \infty} \sum_{k=1}^t \left\{ \frac{\partial \ln \mathbf{M}_t(H^t)}{\partial \ln B_k} + \frac{\partial \ln \mathbf{M}_t(H^t)}{\partial \ln S_k} \right\} = \mathbf{1}. \tag{80}$$

A.2 Proof of Proposition 5

Hereafter, we omit the index i as much as possible. We also omit the time index t and simply refer to a lagged variable by the subscript -1., i.e. $u_{-1} \equiv u_{t-1}$. Let $x = \beta u + P$. We have

$$x = \beta s + (1 - \delta) x_{-1} + \beta(\delta - s) u_{-1}, \tag{81}$$

which does not involve M_{-1}.

In regime u, we have $M = uv/(u + P) = uv/((1 - \beta)u + x) = \theta u^2 / ((1 - \beta)u + x)$. Hence $M/v = u/((1 - \beta)u + x)$. The dynamic system is

$$\begin{aligned} x &= \beta s + (1 - \delta) x_{-1} + \beta(\delta - s) u_{-1}, & (82) \\ u &= s + (1 - s) u_{-1} - \theta_{-1} u_{-1}^2 / ((1 - \beta) u_{-1} + x_{-1}), & (83) \\ \theta &= A^u - B^u x_{-1} / u_{-1} + C^u x / u, & (84) \end{aligned}$$

with $A^u = a - (1 - \beta) b > 0$, $B^u = (\rho\gamma)^{-1}$, $C^u = (1 - s)\gamma^{-1} = \rho(1 - s) B^u$.

In regime v, we have $M = uv/(v + P) = uv/(v - \beta u + x) = \theta u^2 / (\theta u - \beta u + x)$. Hence $v/M = \theta - \beta + x/u$. The dynamic system is

$$\begin{aligned} x &= \beta s + (1 - \delta) x_{-1} + \beta(\delta - s) u_{-1}, & (85) \\ u &= s + (1 - s) u_{-1} - \theta_{-1} u_{-1}^2 / (\theta_{-1} u_{-1} - \beta u_{-1} + x_{-1}), & (86) \\ \theta &= A^v - B^v x_{-1} / u_{-1} + C^v x / u - B^v \theta_{-1}, & (87) \end{aligned}$$

with $A^v = \gamma(a+\beta b)(\gamma+s-1)^{-1}$, $B^v = (\rho(\gamma+s-1))^{-1}$, $C^v = (1-s)(\gamma+s-1)^{-1} = \rho(1-s)B^v$.

In steady state, we have $M = s(1-u)$, $x = \sigma(s+(\delta-s)u)$, $u = s/(s+\mu)$, $\theta^u = \mu(1+\sigma\mu)$, $\theta^v = \sigma\mu^2/(1-\mu)$. Thus all these variables can be expressed as functions of the steady-state job-finding probability μ.

In both regimes, the Jacobian matrix evaluated in steady state is

$$J^i = \begin{bmatrix} 1-\delta & \sigma\delta(\delta-s) & 0 \\ u_1^i & u_2^i & u_3^i \\ \theta_1^i & \theta_2^i & \theta_3^i \end{bmatrix}, \tag{88}$$

where

$$\begin{aligned}
u_1^u &= \frac{\mu}{1+\sigma\mu}, \\
u_2^u &= 1-s-\frac{\mu}{1+\sigma\mu}(1-\sigma\delta+2\sigma(\mu+\delta)), \\
u_3^u &= -\frac{s}{(\mu+s)(1+\sigma\mu)}, \\
\theta_1^u &= \frac{s+\mu}{s}\left\{-B^u + C^u[1-\delta-\frac{\sigma\mu}{1+\sigma\mu}(\mu+\delta)]\right\}, \\
\theta_2^u &= \frac{s+\mu}{s}\left\{\sigma(\mu+\delta)[B^u - C^u(1-\mu-s-\frac{\sigma\mu}{1+\sigma\mu}(\mu+\delta))] + C^u\sigma\delta(\delta-s)\right\}, \\
\theta_3^u &= \frac{\sigma(\mu+\delta)}{1+\sigma\mu}C^u,
\end{aligned}$$

and

$$\begin{aligned}
u_1^v &= \frac{1-\mu}{\sigma}, \\
u_2^v &= (1-\mu)^2 - s - \delta(1-\mu), \\
u_3^v &= \frac{-s(1-\mu)^2}{\sigma\mu(\mu+s)}, \\
\theta_1^v &= \frac{s+\mu}{s}\left\{-B^v + C^v[1-\delta-(\mu+\delta)(1-\mu)]\right\}, \\
\theta_2^v &= \frac{s+\mu}{s}\left\{\sigma(\mu+\delta)[B^v - C^v((1-\mu)^2 - s - \delta(1-\mu))] + C^v\sigma\delta(\delta-s)\right\}, \\
\theta_3^v &= C^v\frac{(\mu+\delta)(1-\mu)^2}{\mu} - B^v.
\end{aligned}$$

Remark: As for the terms in x/u in equations (84) and (87). We simply replace x and u by their corresponding motions given by equations (82)-(83) and (85)-(86).

The steady state is locally saddle-path stable when two eigenvalues of J^i belong to the unit circle and the other is outside the circle. There is indeterminacy when all the eigenvalues belong to the unit circle, and the steady state is locally unstable when at least two eigenvalues lie outside the unit circle.

When $\delta = s$, $1-s$ is an eigenvalue of the Jacobian matrix. The variable x monotonically converges towards its steady-state value, here $x = \sigma s$. It follows that we can concentrate on the sub-system involving the joint dynamics of unemployment u and market tightness θ. The corresponding sub-matrix of the Jacobian matrix is

$$I^i = \begin{bmatrix} u_2^i & u_3^i \\ \theta_2^i & \theta_3^i \end{bmatrix}. \tag{89}$$

In the standard matching model, the rate of unemployment does not enter the free-entry condition and so $\theta_2^i = 0$. This case does not arise with the PMT because changes in unemployment rate affect the phantom proportion and, therefore, the efficiency of the matching technology. Thus the matrix I^i is not triangular. Its eigenvalues are the roots of the characteristic polynomial $P(\lambda) = \lambda^2 - T_i\lambda + D_i$, where the trace $T_i = u_2^i + \theta_3^i$ and the determinant $D_i = u_2^i\theta_3^i - u_3^i\theta_2^i$.

In regime u, we have

$$T(\gamma) = 1 - \mu - s - \frac{\mu\sigma}{1+\mu\sigma}(\mu+s) + \frac{\sigma(\mu+s)}{1+\sigma\mu}\frac{1-s}{\gamma}, \tag{90}$$

$$D(\gamma) = \frac{\sigma(\mu+s)}{1+\sigma\mu}\frac{1}{\rho\gamma}. \tag{91}$$

Thus the relationship between T and D is the following straight line:

$$D = -\frac{1-\mu-s-\frac{\mu\sigma}{1+\mu\sigma}(\mu+s)}{\rho(1-s)} + \frac{1}{\rho(1-s)}T \tag{92}$$

In regime v, we have

$$T(\gamma) = (1-\mu)^2 - 2s + s\mu + [(\mu+s)((1-\mu)^2/\mu)\rho(1-s) - 1]\frac{1}{\rho(\gamma+s-1)} \tag{93}$$

$$= \alpha_1 + \frac{\alpha_2\rho(1-s)-1}{\rho(\gamma+s-1)}, \tag{94}$$

$$D(\gamma) = [(\mu+s)(1-\mu)^2/\mu - (1-\mu)^2 + 2s - s\mu)]B$$

$$= \frac{\alpha_2-\alpha_1}{\rho(\gamma+s-1)}. \tag{95}$$

The relationship between T and D is now

$$T = \alpha_1 + \mu\frac{\alpha_2\rho(1-s)-1}{s}D. \tag{96}$$

We use the geometrical method advocated by Grandmont et al (1998). To this goal, we define the following functions: $f_1(\gamma) = D(\gamma) - T(\gamma) + 1$, and $f_2(\gamma) = D(\gamma) + T(\gamma) + 1$. Figure A1 singles out a triangle whose bounds are associated to cases where at least one of the eigenvalues is of modulus one. Along the lines $f_1(\gamma) = 0$, which is equivalent to $D = -T-1$, and $f_2(\gamma) = 0$, which is equivalent to $D = T-1$, there are two eigenvalues of which one is equal to one in absolute value. Along the line $D = 1$, there are two conjugated complex eigenvalues of modulus one. Inside the triangle, named *sink*, both eigenvalues belong to the unit circle and the steady state is locally indeterminate. The other areas are indicated as follows: *source*, associated to local instability, and *saddle*, where the steady state has the local saddle-path property. Changes in parameter γ and σ, holding the steady-state job-finding rate constant thanks to Corollary 1, describe a relationship between D and T. By locating this relationship in the plane, we can deduce the stability properties of the dynamic system.

[Insert Figure A1]

Part (i) Regime u. The slope of the line (92) is larger than 1. Moreover $D(\gamma) > \frac{\sigma(\mu+s)}{1+\sigma\mu} > 0$ and

$$f_1(\gamma) = [1-\rho(1-s)]D(\gamma) + \mu + s + \frac{\mu\sigma(\mu+s)}{1+\mu\sigma} > 0, \tag{97}$$

$$f_2(\gamma) = [1+\rho(1-s)]D(\gamma) + 2 - \mu - s - \frac{\mu\sigma(\mu+s)}{1+\mu\sigma} \tag{98}$$

$$> \frac{\sigma(\mu+s)(1-\mu)}{1+\mu\sigma} + 2 - \mu - s > 0. \tag{99}$$

Thus the steady state is a sink when $D(\gamma) > 1$, whereas it is a source when $D(\gamma) < 1$. The result follows. Figure A2 shows the case where $D(1) < 1$. The steady state is a sink when γ is close to 1. As γ decreases, the line crosses the triangle and the steady state becomes a source.

[Insert Figure A2]

Part (ii) Regime v. The determinant is linear in $B(\gamma)$. However, B is first negative and strictly decreasing in γ, tends to minus infinity as γ approaches $1-s$, then starts from plus infinity and decreases to $1/(\rho s) > 1$ when $\gamma = 1$. Thus we distinguish two cases: $\gamma < 1-s$ and $\gamma > 1-s$. Then we collect all possible configurations.

We have

$$f_1(\gamma) = \{\alpha_2[1-\rho(1-s)] + 1 - \alpha_1\}B(\gamma) + 1 - \alpha_1, \tag{100}$$

$$f_2(\gamma) = \{\alpha_2[1+\rho(1-s)] - (1+\alpha_1)\}B(\gamma) + 1 + \alpha_1. \tag{101}$$

We start with $\gamma > 1-s$. Then $D(\gamma) > 1$ $f_1(\gamma) > 0$. In Figure A3, the hatched area corresponds to values of T and D that cannot be reached when $\gamma > 1-s$. Thus the steady state is saddle-path stable when $f_2(\gamma) < 0$ and it is a source when $f_2(\gamma) > 0$.

[Insert Figure A3]

When $1+\alpha_1 \leq \alpha_2[1+\rho(1-s)]$, $f_2(\gamma) > 0$ and the steady state is a source. When $1+\alpha_1 > \alpha_2[1+\rho(1-s)]$, the function f_2 strictly increases with γ. Two cases arise. If $f_2(1) > 0$, which is equivalent to $1+\alpha_1 < \frac{\alpha_2[1+\rho(1-s)]}{1-\rho s}$, then there is a unique $\bar{\gamma} \in (1-s, 1)$ such that $f_2(\bar{\gamma}) = 0$. The steady state is a source for values of γ above $\bar{\gamma}$, and a saddle for values below $\bar{\gamma}$. If $f_2(1) < 0$, then $f_2(\gamma) < 0$ for all γ and the steady state is always a saddle.

Now suppose $\gamma < 1-s$. Then $D(\gamma) < D(0) < 0$ and $f_1(\gamma) < 0$. Figure A4 shows the corresponding area of the (T, D) plane. Thus the steady state is a saddle when $f_2(\gamma) > 0$ and a source when $f_2(\gamma) < 0$.

[Insert Figure A4]

When $1+\alpha_1 \leq 0$, $f_2(\gamma) < 0$ and the steady state is a source. When $1+\alpha_1 \geq \alpha_2[1+\rho(1-s)]$, $f_2(\gamma) > 0$ and the steady state is a saddle. When $0 < 1+\alpha_1 < \alpha_2[1+\rho(1-s)]$, the function f_2 strictly decreases with γ. Here again two cases arise. If $f_2(0) < 0$, which is equivalent to $1+\alpha_1 < \alpha_2$, then $f_2(\gamma) < 0$ for all γ and the steady state is a source. If $f_2(0) > 0$, which is equivalent to $1+\alpha_1 > \alpha_2$, then there exists a unique $\bar{\gamma} \in (0, 1-s)$ such that $f_2(\bar{\gamma}) = 0$. The steady state is a saddle for values of γ below $\bar{\gamma}$ and a source for values above $\bar{\gamma}$.

Combining the different cases and noting that $\alpha_2 - \alpha_1 = s/\mu$ establishes claim (ii).

B Local stability in the standard case

Differentiating (59) with respect to θ and θ_{+1} in the neighborhood of the steady state gives:

$$\frac{d\theta}{d\theta_{+1}} = \rho\left(1 - s - \frac{\gamma\mu}{1-\varepsilon}\right) < 1, \tag{102}$$

where ε is the elasticity of the job finding-probability with respect to tightness. A closer look at the derivative shows that indeterminacy is very unlikely. It requires large values of γ, s, μ and ε. The main problem is that values of s and μ depend on the period length, i.e. the shorter the period, the smaller μ and s. If we consider the values of μ and s used in our monthly calibration, we obtain

$$\frac{d\theta}{d\theta_{+1}} = 0.9967\left(0.977 - \frac{\gamma 0.403}{1-\varepsilon}\right), \tag{103}$$

The derivative is lower than -1 when $\gamma > 4.898(1-\varepsilon)$. This requires ε to be above 0.8. With a weekly calibration, ε must even be larger than 0.95.

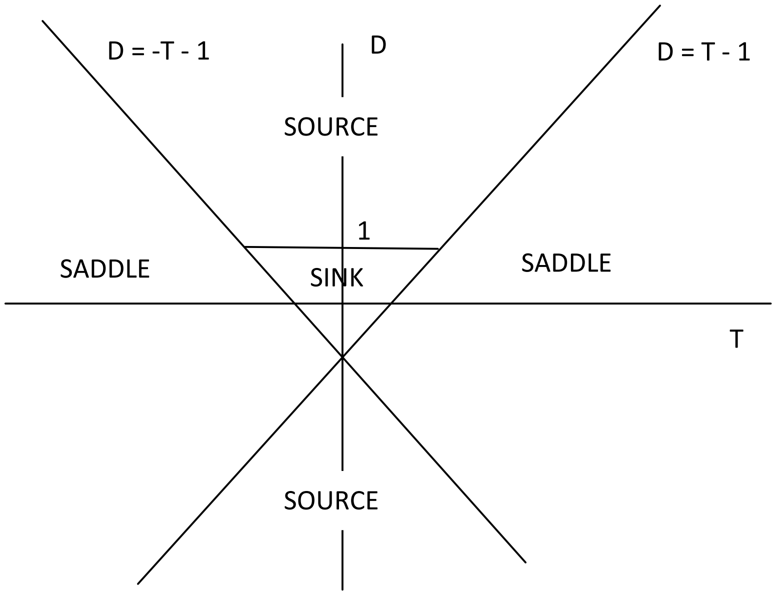

Figure A1 : Local stability properties in the Trace –Determinant (T,D) plane.

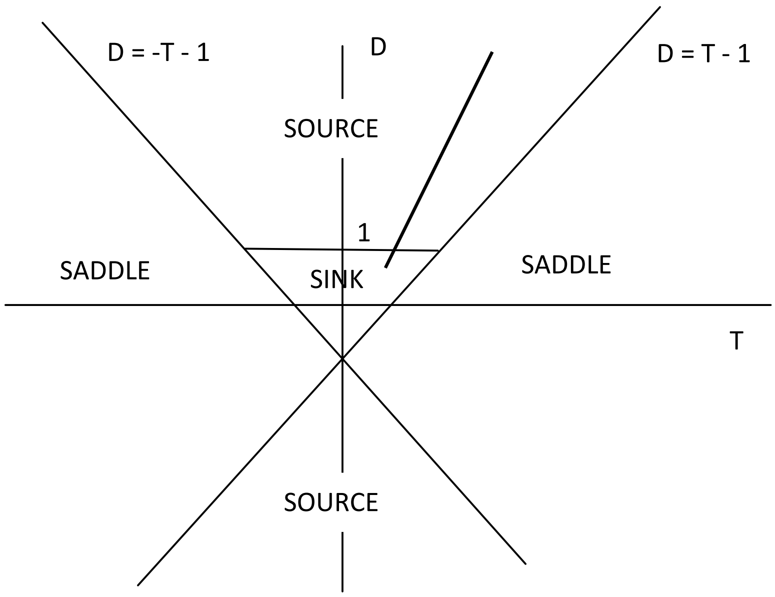

Figure A2: Local stability properties of the regime u. Case D(1) < 1

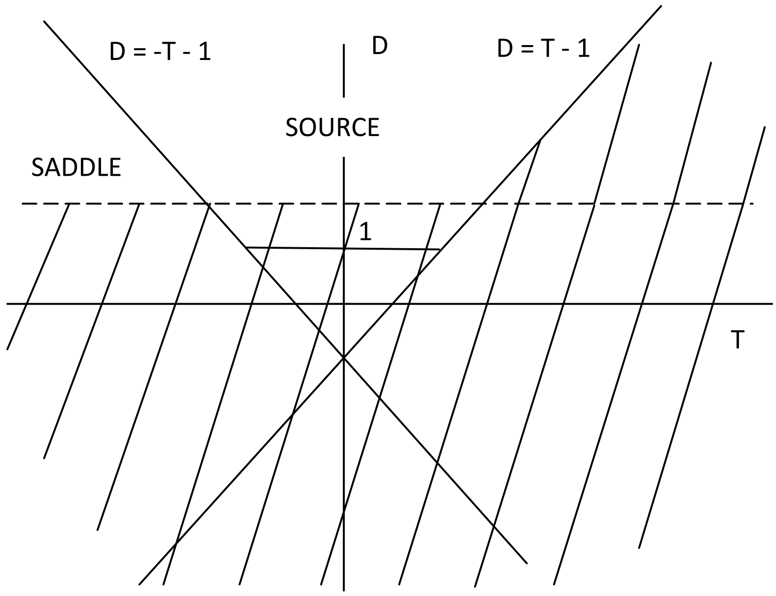

Figure A3: Local stability properties of the regime v when γ > 1-s. The hatched area cannot be reached

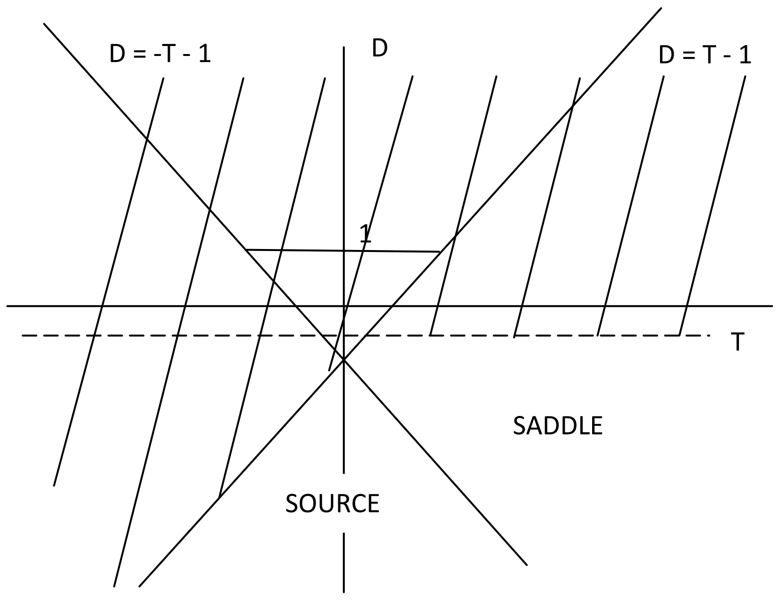

Figure A4: Local stability properties of the regime v when γ < 1-s. The hatched area cannot be reached