aix marseille school of economics

Working Papers / Documents de travail

About Polluting Eco-Industries: Optimal Provision of Abatement Goods and Pigouvian Fees

Damien Sans
Sonia Schwartz
Hubert Stahn

WP 2014 - Nr 53

About Polluting Eco-Industries: Optimal Provision of Abatement Goods and Pigouvian Fees

Damien SANS[1]

Aix-Marseille University (Aix-Marseille School of Economics), CNRS, & EHESS.

Sonia SCHWARTZ[2]

CERDI, Université d'Auvergne.

Hubert STAHN[3]

Aix-Marseille University (Aix-Marseille School of Economics), CNRS, & EHESS.

Abstract

In this article we introduce a polluting eco-industry. Depending on the level of the damage, we find one of two optimal equilibria. If the damage is low, we generalize the usual results of the economic literature to the polluting eco-industry: the dirty firm partially abates their emissions, only efficient eco-industry firms produce and the abatement level increases with the damage. However, we obtain very specific results if the damage is high. In this case, not all efficient eco-industry firms produce. The abatement level and the number of active eco-industry firms both decrease as the damage increases. We finally show that a well-designed Pigouvian tax implements these equilibria in a competitive economy.

Key words: Polluting Eco-Industry, Heterogeneous firms, Welfare Analysis, Pigouvian Tax
JEL Classification: D62, Q58, L11

Email addresses: damien.sans@univ-amu.fr (Damien SANS), sonia.schwartz@udamail.fr (Sonia SCHWARTZ), hubert.stahn@univ-amu.fr (Hubert STAHN)

[1]Postal address: GREQAM, Centre de la Vieille Charité, 2, rue de la Charité, 13236 Marseille, France

[2]Postal address: CERDI, 65 boulevard François Mitterrand 63000 Clermont-Ferrand

[3]Postal address: GREQAM, Chateau Lafarge, Route des Milles, 13290 Les Milles, France

Preprint submitted to DT-AMSE *November 17, 2014*

1. Introduction

Pharmaceuticals and other organic wastewater contaminants are found in treated wastewater in Europe (Ternes, 1998; Comoretto and Chiron, 2005), the US, (Boyd et al., 2003) and Asia (Nozaki et al., 2000). These observations suggest that wastewater treatment plants do not totally abate pollution; they are only partially efficient. Moreover, their production process may also be polluting. According to Kyung et al. (2013), wastewater treatment plants (and incineration facilities) have been reported to emit significant amounts of GHGs, and water treatment plants have also been categorized as one of the significant public facilities emitting important amounts of CO2 by consuming immense amounts of electricity and chemicals (Raucher et al., 2008; Rothausen and Conway, 2011). Along the same lines, a debate has emerged about the energy balance of the photovoltaic industry. This raises the question of grey energy. Grey energy is the hidden energy associated with a product, meaning the total energy consumed throughout the product life cycle from its production to its disposal. The issue is whether the reduction in pollution is greater than the grey energy consumed. If this is the case, we can infer that the eco-industry is efficient.

Wastewater treatment, air treatment, waste treatment plants and the photovoltaic industry are all part of the eco-industry sector. This is a new industrial sector covering pollution and resource management activities, ranging from the development of clean technologies to the optimization of methods for monitoring and managing environmental impacts.[4] It appears that this sector can be partially efficient and polluting. This point is crucial for policy-makers because the emergence of eco-industry firms is often conditional on environmental policy.

The eco-industry is well-documented in the economic literature, but nevertheless mainly focuses on the fact that it is highly concentrated. The research can be divided in two main branches. The first branch considers innovative firms investing in R&D to obtain a patent for a pollution-reducing new technology. The performance of taxes and tradable permits are compared in various contexts. Denicolo (1999) and Requate (2005) make these comparisons under different timing and commitment regimes. A threat of imitation is introduced by Fisher et al. (2003), while Perino (2008) studies green horizontal innovation, where new technologies reduce pollution of one type while causing a new type of damage. More recently, Perino (2010a) focuses on the second-best policies for all combinations of emission intensity and marginal abatement costs.

The second branch analyses how eco-industry modifies the usual results of the economic literature. It takes market power as a given and suggests the optimal design of environmental policy within this context. Most of these papers consider the Pigouvian tax as environmental policy tool: see for instance Canton (2008), Canton et al. (2007), Canton et al. 2012, David and Sinclair-Desgagné (2005, 2010), David et al. (2011), Nimubona

[4]See Ernst and Young (2006), Sinclair-Desgagné (2008), European Commission Report (1999, 2007, 2009) and OECD (1999) for an analysis and description of eco-industry.

(2012) and Nimubona and Sinclair-Desgagné (2005, 2010). Greaker (2006) and Greaker and Rosendahl (2008) introduce non-tradable quotas, while Schwartz and Stahn (2014) study a pollution permit market

None of the studies mentioned above explores *polluting* eco-industry. The aim of this paper is to investigate whether the standard results in economic literature are challenged if the eco-industry is polluting. Following almost all the papers cited, we consider a vertical structure composed of a downstream polluting sector and an upstream eco-industry. Contrary to the existing literature, in this article eco-industry firms are polluting and heterogeneous, i.e., they are more or less polluting. This situation may correspond to different generations of plants having different technologies. We also assume that they cannot reduce their emissions. To focus on this original assumption, we do not consider market power. Under these new assumptions, we first seek to define the centralized solution. Next, we examine whether this optimal policy can be decentralized using a traditional economic instrument: the Pigouvian tax. On these points, our article is in keeping with the second branch of the economic literature described above.

We find that two kinds of equilibrium can emerge. The first equilibrium occurs if the marginal damage is not too high. In this case, we extend the usual results of the economic literature to polluting eco-industry. We find that the optimal level of abatement is such that the marginal social benefit and marginal social cost of abatement are equal to the marginal damage. The dirty firm partially abates its emissions and only efficient eco-industry firms produce. The greater the marginal damage, the less the dirty firm produces and the higher the abatement level.

The second kind of equilibrium occurs if the marginal damage is high: the dirty firm abates all its emissions and not all active firms in the first equilibrium produce. As pollution is very harmful for the environment, the only way to prevent even more damage is to reduce the pollution produced by eco-industry. To do this in an efficient way, the regulator should not only reduce the number of active firms but also modify the distribution of abatement in the eco-industry. We also find the counter-intuitive result that the number of active firms and the level of abatement decrease with the marginal damage. The optimal abatement level is such that the marginal social benefit is equal to the marginal social cost, but they are both lower than the marginal damage. This second equilibrium is very specific to polluting eco-industry.

Finally, we show that a competitive economy reaches these optimal equilibria if the regulator implements a Pigouvian tax. The rule is very simple, because it is the same whatever the level of the damage: the Pigouvian tax must be equal to the marginal damage. However, depending on the damage level, the functioning of the economy will be different, as we will see in the paper.

In Section 2, we present the model. Section 3 defines the social benefits and social costs of pollution abatement. In Section 4 we determine the efficient level of abatement. Policy issues are presented in Section 5 and some concluding remarks are given in Section 6. Proofs are relegated to the Appendix

2. The basic assumptions

To keep the assumptions as simple as possible, we assume that the standard *polluting industry* is characterized by a representative firm that produces a quantity Q at a given cost $C(Q)$. This cost is increasing and convex (*i.e.,* $C'(Q) > 0$ and $C''(Q) > 0$) and inaction is allowed (*i.e.,* $C(0) = 0$). This activity is polluting. Emissions are given by $\varepsilon(Q)$, an increasing and convex function (*i.e.,* $\varepsilon'(Q) > 0$ and $\varepsilon''(q) > 0$) with $\varepsilon(0) = 0$. This "end-of-pipe" pollution can be reduced by an abatement activity provided by the specialized external firms which comprise the eco-industry. So if we denote by A the total abatement realized by the polluting firm, the remaining pollution will be $\max\{\varepsilon(Q) - A, 0\}$.

The *eco-industry* is composed of a continuum[5] $[0, 1]$ of firms indexed by i. Each of them supplies $a(i)$ pollution reduction services produced at some cost $\kappa(a(i))$. They share the same increasing and convex cost function and inaction is allowed (*i.e.,* $\kappa'(a) > 0$, $\kappa''(a) > 0$ and $\kappa(0) = 0$) We also assume that $\kappa'(0) = 0$ in order to ensure that there is, in a competitive setting, an offer for each positive price[6].

owever, we assume that this activity pollutes and that these firms are heterogeneous with respect to their emissions. Emissions of firm i are a proportion $\beta(i) \in [\beta_{\min}, \beta_{\max}]$ of its production and are considered as unavoidable (they cannot be abated). Since one unit of abatement good reduces the pollution of the downstream firm in the same proportion, the coefficient $[1 - \beta(i)]$ measures the net contribution of firm i to pollution reduction. Firms in the abatement good sector are also heterogenous: they are ranked from the least to the most polluting. We also assume that $\beta : [0, 1] \rightarrow [\beta_{\min}, \beta_{\max}]$ is a continuous and differentiable function, and because they are ranked, $\beta'(i) > 0$.

Finally we assume $\beta_{\min} < 1$ to ensure that at least some firms have a net contribution to global pollution reduction. $\beta_{\max} > 1$ means that at least some of these firms contribute to pollution abatement in an inefficient way since their global contribution to the emissions reduction per unit of output, $(1 - \beta(i))$, is negative.

The *global emissions,* $E = \max\{\varepsilon(Q) - A, 0\} + \int_0^1 \beta(i)a(i)di$, are comprised of the remaining pollution from the dirty industry and the emissions generated by the abatement activity. This means that we can have situations in which the "dirty" industry is clean and some pollution remains. So, contrary to most of the literature which does not consider polluting eco-industry, it is now crucial to take into account the fact that the abatement activity becomes inefficient when the pollution of the dirty industry is completely removed. We assume that these emissions create social damage, measured by $D(E) = v.E$ with $v > 0$. Hence v is the marginal damage.

Finally, to close the model, we introduce an *inverse demand function* for the polluting

[5]This continuum assumption is essentially introduced to simplify the treatment of an industry composed of heterogeneous agents. The same arguments hold with a finite number of firms.

[6]A discussion about the emergence of an eco-industry related to the fact that $\kappa'(0) > 0$ can be found in Canton *et al.* (2007).

goods $P(Q)$. This function is decreasing (*i.e.* $P'(Q) < 0$) and verifies that $\lim_{Q\to 0} P(Q) = +\infty$ and $\lim_{Q\to+\infty} P(Q) = 0$.

3. Social benefits and costs from pollution abatement

This section is rather traditional. We fix a production level A of the abatement good and define, within our setting, the social benefits and costs of this abatement choice. The main difference with the usual approach is that the eco-industry is polluting. This has two consequences: (i) these goods only reduce the emissions of the polluting industry and (ii) the residual pollution must be included in the social cost of the abatement production. We then obtain the marginal social benefit and marginal social cost of abatement.

The social benefit of abatement The *social benefit* from a level A of pollution abatement is obtained by choosing the production of the dirty industry. This production level maximizes the welfare of consumers net of the production costs and of the pollution induced by this activity. This function is given by:

$$SB(A, v) = \max_{Q\geq 0} \left(\int_0^Q P(q)dq \right) - C(Q) - v \cdot \max\{\varepsilon(Q) - A, 0\} \tag{1}$$

This definition of the social benefit is very conventional, especially for "end-of-pipe" pollution. But in most treatments of this problem, the condition stating that the emission of the dirty industry must be non-negative (*i.e.*, $\max\{\varepsilon(Q) - A, 0\}$ in our article) is quickly forgotten simply because this corner solution in which no pollution occurs is not really interesting. However, this is far from being the case when the eco-industry also pollutes, because there is now a possible arbitrage between upstream and downstream pollution, *i.e.*, between the emissions of the abaters and those of the final goods producers. This is why we have to solve this non-smooth optimization problem globally. The method (see the proof of Lemma 1 in the appendix) essentially makes use of the sub-differential introduced by Rockafellar (1979). In any event, non-smooth optimization involves case studies and thresholds. In this article, if we solve this program for all levels of pollution abatement $A \in \mathbb{R}_+$, there are three possible outcomes.

The first situation, the usual one, is characterized by partial abatement: $\varepsilon(Q)-A > 0$. In this case, the first order condition is given by $P(Q) - C'(Q) - v.\varepsilon'(Q) = 0$. If we set $\xi(Q) = \frac{P(Q)-C'(Q)}{\varepsilon'(Q)}$, the optimal level of production $Q = \xi^{-1}(v)$, that solves this condition is simply a decreasing function of the marginal damage v. But this solution only occurs if $\varepsilon(Q) - A > 0$, which requires that $\xi^{-1}(v) > \varepsilon^{-1}(A)$ or that the fixed level of abatement good verifies $A < \varepsilon\left(\xi^{-1}(v)\right)$.

The second situation corresponds to full abatement of the emissions of the downstream industry:. $Q = \varepsilon^{-1}(A)$. This requires that the previous condition is not met *i.e.*, $\xi^{-1}(v) \leq \varepsilon^{-1}(A)$ or $A \geq \varepsilon\left(\xi^{-1}(v)\right)$. But if we now bear in mind that this full abatement condition means that $\max\{\varepsilon(Q) - A, 0\} = 0$, this production level is optimal as long as we do not reach the production level $Q_{\max}$which is efficient when there is any damage. In other words, this also requires that $\left(P(Q)-C'(Q)\right)|_{Q=\varepsilon^{-1}(A)} \leq 0$ or that $A < \varepsilon\left(Q_{\max}\right)$.

Finally if $A \geq \varepsilon\left(Q_{\max}\right)$, the optimal production level that solves program (1) will be equal to $Q_{\max}$. As we will see later, this last case never occurs, simply because pollution is not taken into account. It is given here for the sake of completeness.

From all these observations, we can construct the social benefit $SB(A, v)$ of pollution abatement. It is a piecewise continuous function depending on the fixed level of abatement A. However what really matters is the marginal social benefit:. $\frac{\partial SB(A,v)}{\partial A}$:

- if there is only partial reduction, this marginal benefit will be, as usual, equal to the marginal damage v;

- if there is full abatement with $A > \varepsilon(Q_{\max})$, additional abatement is fully inefficient since the optimal production level does not depend on A. The marginal social benefit is clearly 0;

- if there is full abatement with $A \in \left[\varepsilon\left(Q(v)\right), \varepsilon(Q_{\max})\right)$, the optimal production level is positively correlated with A. The social marginal benefit is then given by

$$\frac{P(\varepsilon^{-1}(A)) - C'(\varepsilon^{-1}(A))}{\varepsilon'(\varepsilon^{-1}(A))} = \xi(\varepsilon^{-1}(A)) \tag{2}$$

  We know that this case occurs if $\xi^{-1}(v) \leq \varepsilon^{-1}(A)$, which is equivalent to $v \geqslant \xi(\varepsilon^{-1}(A))$. In this last case the marginal social benefit is therefore smaller than the marginal damage.

More formally, we can state that:

Lemma 1. *If* $\xi(Q) = \frac{P(Q)-C'(Q)}{\varepsilon'(Q)}$, *inspection of program 1 shows that:*

(i) The optimal production level is given by:

$$Q(v, A) = \begin{cases} \max\left\{\xi^{-1}(v), \varepsilon^{-1}(A)\right\} \ \textit{if}\ A < \varepsilon\left(Q_{\max}\right) \\ Q_{\max}\ \textit{if}\ A \geq \varepsilon\left(Q_{\max}\right) \end{cases} \tag{3}$$

(ii) The marginal social benefit is given by:

$$\frac{\partial SB(v, A)}{\partial A} = \max\left\{\min\left\{v, \xi(\varepsilon^{-1}(A))\right\}, 0\right\} \tag{4}$$

The social cost of abatement The *social cost* induced by the production of abatement goods in quantity A is obtained, as usual, by choosing an optimal distribution of the production between the different plants which comprise the eco-industry. But in our case, this process involves not only the cost structure of these firms but also their pollution structure. This cost is defined by:

$$SC(A) = \min_{a(\cdot)\geq 0} \int_0^1 \kappa\left(a(i)\right) di + v \cdot \int_0^1 \beta(i) a(i) di \tag{5}$$

$$\text{subject to } \int_0^1 a(i) di = A \tag{6}$$

If we denote by λ the Lagrangian multiplier associated with the constraint, the first order conditions of this convex minimization problem are given by:

$$\begin{cases} \forall i \in [0,1]\,, \ \kappa'(a(i)) + v\beta(i) - \lambda \geq 0 \text{ (with equality if } a_i > 0) \\ \text{and } \ \int_0^1 a(i)di = A \end{cases} \tag{7}$$

From these FOC we see that a given firm i is active if $\lambda - v\beta(i) > 0$ and in this case its production level is of $(\kappa')^{-1}(\lambda - v\beta(i))$. Since the emissions of these firms are increasing with their index i, his also means that there exists a pivotal firm[7] $i_0 = \beta^{-1}\left(\min\left\{\frac{\lambda}{v}, \beta_{\max}\right\}\right)$ which is the first for which it is optimal to stop the production. If we now keep bear in mind that the total production level A of production of abatement goods is given, the index of this firm can be obtained by making sure that the total level of production of firms $i \leq i_0$ is equal to A.

In other words, even if these firms share the same cost function, the optimal distribution of the global production is not symmetrical because they are heterogeneous in their contribution to pollution. We can therefore expect that not all firms will be selected at the optimal allocation. In order to define this allocation, we also need some information about the marginal social cost of the production of an additional unit of abatement goods. This quantity is given by $\dfrac{\partial SC(v,A)}{\partial A}$. Since the constraint to the problem is $\int_0^1 a(i)di = A$, the envelop theorem immediately tells us that the social marginal cost is equal to the Lagrangian multiplier associated with this program.

More precisely, we can say:

Lemma 2. *If $A = 0$ then $a(i) = 0$ for almost all i and $\lambda(A) \in (-\infty, v.\beta_{\min}]$ and for $A > 0$:*

(i) The productions of abatement goods are given by:

$$a^*(i,A,v) = \begin{cases} (\kappa')^{-1}\left(\lambda(A,v) - v.\beta(i)\right) \ \textit{if } i \leq \beta^{-1}\left(\frac{\lambda}{v}\right) \\ 0 \ \textit{else} \end{cases} \tag{8}$$

[7] As long as i_0 is smaller than one. This is why we introduce the $\min\left\{\frac{\lambda}{v}, \beta_{\max}\right\}$ in the definition of this pivotal firm.

(ii) $\lambda(A,v)$ *is implicitly defined by:*

$$\int_0^{\beta^{-1}\left(\min\left\{\frac{\lambda}{v},\beta_{\max}\right\}\right)} (\kappa')^{-1}(\lambda - v.\beta(i))\,di = A \tag{9}$$

(iii) The marginal social cost is given by:

$$\frac{\partial SC(v,A)}{\partial A} = \lambda(A,v) \tag{10}$$

As we have defined the marginal social cost and benefit, it remains for us to find the optimal level of abatement.

4. Optimal outcome

With regard to our previous results, let us first identify the optimal provision of abatement goods. This level results from a trade-off between the marginal social benefit and the marginal social cost. It is given by:

$$A^{opt}(v) = \arg\max_{A\geq 0} SB(A,v) - SC(A,v) \tag{11}$$

As expected, this program shows that the optimal production of abatement goods never exceed $\varepsilon^{-1}(Q_{\max})$, the level of abatement which maximizes the social benefit without damage. If the last case occurs, the social benefit is constant while the costs are increasing with the abatement effort (see (ii) of Lemma 1), which contradicts optimality. We can also observe that the optimal provision of abatement goods is always positive, *i.e.,* $A^{opt}(v) > 0$. Otherwise, the marginal cost of abatement is, by Lemma 2, smaller than $v.\beta_{\min} < v$ while the marginal benefit of an additional unit of abatement when there is no abatement at all is of v since $\lim_{A\to 0}\xi(\varepsilon^{-1}(A)) = +\infty$ (see Lemma 1).

Following these observations, we can affirm that the FOC associated with program (11) is:

$$\frac{\partial SB(A,v)}{\partial A} - \frac{\partial SC(A,v)}{\partial A} = \min\left\{v, \xi\left(\varepsilon^{-1}(A)\right)\right\} - \lambda(A,v) = 0 \tag{12}$$

This condition clearly suggests that two kinds of efficient outcomes occur, depending on the level of the marginal damage. The first situation is rather classical: the dirty firm partially abates its emissions and the marginal damage of pollution is equal to both the marginal benefit and the marginal cost of abating pollution. The second case occurs if the pollution of the dirty industry is totally removed but some pollution persists due to the activity of the eco-industry. In this case, the marginal benefit remains equal to the marginal cost, but lower than the marginal damage induced by pollution.

Intuition suggests that the marginal damage v admits a threshold for which we switch from one situation to the other. To get this intuition, let us start with a case in which there is partial abatement in the dirty industry or, more formally from Equation (12), in which $\xi\left(\varepsilon^{-1}(A)\right) > v = \lambda(A,v)$. By equation (9), we can compute the optimal provision $A^{opt}(v)$ of the abatement good simply by replacing $\lambda(A,v)$ by v. This quantity, given by:

$$A^{opt}(v) = \int_0^{\beta^{-1}(1)} (\kappa')^{-1}(v \times (1-\beta(i)))\,di \tag{13}$$

is obviously increasing with the marginal damage. But this case only holds seeing that $\xi\left(\varepsilon^{-1}(A^{opt}(v))\right) > v$. So if on has in mind that ξ is a decreasing function, the left hand side of the previous condition is decreasing in v while the right hand side in increasing. This rather intuitive argument therefore suggests that the case of partial abatement disappears for sufficiently high marginal damage levels.

Lemma 3. *There exists a unique threshold $\bar{v}$ given by:*

$$\varepsilon\left(\xi^{-1}(\bar{v})\right) - \int_0^{\beta^{-1}(1)} (\kappa')^{-1} (\bar{v} \times (1 - \beta(i)))\, di = 0 \tag{14}$$

with the property that if $v < \bar{v}$ there is only partial pollution reduction in the downstream industry, while in the other case there is full abatement of the pollution emitted by the dirty industry. The pollution of the eco-industry nevertheless remains in the last case.

If the marginal damage of pollution is lower than $\bar{v}$, we are in the standard case described by the literature. There is, at the optimal allocation, partial abatement of the pollution emitted by the dirty industry. The only difference is that there are now some additional emissions due to the eco-industry. The aggregated level of abatement is a usually chosen such that the marginal cost and benefit are both equal to the marginal damage of pollution and the aggregated level of production of the final industry is commonly decreasing with the marginal damage. In other words, there is a traditional arbitrage between the reduction of the final production and the increase in production abatement: both quantities are negatively correlated when the marginal damage v changes.

However, the remaining emissions of the eco-industry contribute to a selection of which firms in this sector should produce. This selection is not based solely on private cost-minimizing considerations, but also takes into account the emissions of the eco-industry. Since the aggregated marginal social cost is equal to the marginal damage, only the firms which have a positive net contribution to pollution abatement (*i.e.,* $[1 - \beta(i) > 0]$ produce, and because these firms are heterogeneous in their emissions, the less polluting firms contribute more.

Moreover, since one unit of the abatement good removes one unit of pollution, we observe that at the efficient allocation, the marginal cost and benefit from the aggregate level of abatement are both equal to the marginal damage of pollution. Finally, we have the usual arbitrage between the reduction of the final production and the increase in pollution abatement, which depends on the level of the marginal damage, since these quantities are negatively correlated when v changes. More precisely, we can say that:

Proposition 1. *If the marginal damage is not too high, i.e, $v \leq \bar{v}$,only partial abatement of the pollution of the dirty firm occurs, and efficient allocation has the usual properties: (i) the marginal benefit and marginal cost of pollution abatement are both equal to the marginal damage of pollution, i.e. $\frac{\partial SB(A^{opt},v)}{\partial A} = \frac{\partial SC(A^{opt},v)}{\partial A} = v$,*
(ii) the optimal level of production $Q^{opt} = \xi^{-1}(v)$ is decreasing with the marginal damage of pollution while the total production of abatement good

$$A^{opt}(v) = \int_0^{\beta^{-1}(1)} (\kappa')^{-1} (v \times (1 - \beta(i)))\, di \tag{15}$$

and the individual production are increasing with the level of the marginal damage.
(iii) all firms in the eco-industry that efficiently reduce pollution, i.e, $\beta(i) \leq 1$,*contribute to the abatement but to different extents depending on their own emissions, i.e.,* $\forall i \leq \beta^{-1}(1)$, $a^{opt}(i) = (\kappa')^{-1}(v \times (1 - \beta(i)))$ *and this quantity is decreasing with* $\beta(i)$,

The second case in which the marginal damage is higher than the threshold, $\bar{v}$, is less usual. Its interpretation largely depends on the assumption that the eco-industry pollutes. In this situation, it becomes optimal to remove all the emissions of the upstream industry, even if some pollution persists due to the activity of the eco-industry. As this irreducible pollution is harmful, this may not be sufficient to improve the environment. This is why it is also optimal (i) to reallocate the production of abatement goods toward the less polluting firms in the eco-industry and (ii) to slow down the production of abatement goods and therefore also the production of the final good, since the emissions from this activity are totally abated thanks to the eco-industry production. So it is not really surprising that (i) the number of active firms decreases with the marginal damage, contrary to the previous case in which all efficient firms produce, and (ii) the level of final production is now positively correlated with the level of abatement, simply because the maximal abatement level is reached and both are decreasing with the level of marginal damage.

What is perhaps more surprising is that the marginal cost and benefit from abatement are now lower than the marginal damage of pollution. In order to understand this property, let us start with a level of production in the eco-industry corresponding to the total abatement, for which the marginal social benefit of pollution reduction is equal to the marginal damage. If the damage is high, this often requires a large reduction in the final output. So it is possible that the marginal social cost of producing enough abatement goods to totally remove downstream emissions remains lower than the marginal damage of the pollutants. This provides an incentive to produce more abatement goods and to expand the production of the final good in a way that ensures full upstream emission abatement. More precisely, we observe that:

Proposition 2. *If the marginal damage is high, i.e.* $v > \bar{v}$, *the pollution of the dirty firm is totally abated. The efficient allocation has the following less usual properties:*
(i) the marginal benefit remains equal to the marginal cost of abatement, but this value is now smaller than the marginal damage:

$$\frac{\partial SB(A^{opt}, v)}{\partial A} = \frac{\partial SC(A^{opt}, v)}{\partial A} = \lambda^{opt}(v) < v \tag{16}$$

(ii) the optimal level of production $Q^{opt} = \varepsilon^{-1}(A^{opt})$ *bis now positively correlated with the optimal level of abatement. The total production of abatement goods is now decreasing with the marginal damage, since the pollution of the eco-industry can only be reduced by reducing the production of these goods, i.e.,* $\frac{dQ^{opt}(v)}{dv} \leq 0$ *and* $\frac{dA^{opt}(v)}{dv} \leq 0$.
(iii) some firms that can reduce pollution efficiently are not active at the optimal allocation. The firm i is active if $\beta(i) < \frac{\lambda^{opt\prime}(v)}{v} < 1$.*Moreover, the number of active firms*

decreases with the marginal damage: $i^{opt}(v) = \beta^{-1}\left(\frac{\lambda^{opt\prime}(v)}{v}\right)$ *verifies* $\frac{di^{opt}(v)}{dv} < 0$. *However, their contribution to the production of abatement good* $a^{opt}(i) = (\kappa')^{-1}(\lambda^{opt}(v) - v\beta(i))$, *remains decreasing with their emission rate.*
(iv) the Lagrangian multiplier $\lambda^{opt}(v) \in (v\beta_{\min}, v)$ *is the unique solution to:*

$$\varepsilon\left(\xi^{-1}(\lambda)\right) = \int_0^{\beta^{-1}\left(\frac{\lambda}{v}\right)} (\kappa')^{-1}(\lambda - v.\beta(i))\, di \tag{17}$$

It remains for us to analyze how to decentralize these two optimal equilibria.

5. The policy issues

We have shown that the model exhibits two kinds of efficient allocation, depending on the level of the marginal damage (v) and that these allocations have rather different properties. In this section, we investigate whether a standard instrument like the Pigouvian tax can implement each of these equilibria in a competitive setting. This last point will be verified in two steps. We first assume that there is a price signal τ representing the emission tax and we compute the competitive allocation for each value of τ. In the second step, we determine the level of the Pigouvian tax that implements the efficient allocation in each case.

5.1. The competitive behaviors

We first analyze the competitive behavior of the dirty firm, and then that of the eco-industry firms. Lastly, we expose the abatement market equilibrium.

The dirty firm So let us start with the dirty firm. If there is a price signal $\blacksquare$ associated with the emission of pollution, this firm will choose its production supply and its demand for the abatement good by solving the profit equation:

$$\pi(Q, A) = \max_{Q \geq 0}\left\{ p.Q - C(Q) - \underbrace{\min_{A \geq 0}\{p_a.A + \tau.\max\{\varepsilon(Q) - A, 0\}\}}_{=C_A(p_a,\tau,Q)} \right\} \tag{18}$$

We see that the objective function is linear in A on $[0, \varepsilon(Q)]$. This implies that the optimal conditional demand for abatement goods will be 0 if $p_a > \tau$, $\varepsilon(Q)$ if $p_a < \tau$, and any quantity within $[0, \varepsilon(Q)]$ if $p_a = \tau$. It follows that the abatement cost is given by $C_A(p_a, \tau, Q) = \min\{p_a, \tau\}.\varepsilon(Q)$ and that the FOC characterizing the product supply is:

$$p - C'(Q) - \min\{p_a, \tau\}\varepsilon'(Q) \leq 0 \text{ (with equality if } Q > 0) \tag{19}$$

Since we know that the inverse demand is given by $P(Q)$, the quantity which clears the commodity market is obtained by:

$$Q(p_a, \tau) = \xi^{-1}(\min\{p_a, \tau\}) \tag{20}$$

while the demand for abatement goods is:

$$A^d(p_a,\tau) = \begin{cases} 0 \text{ if } p_a > \tau \\ \left[0, \varepsilon\left(\xi^{-1}(\tau)\right)\right] \text{ if } p_a = \tau \\ \varepsilon\left(\xi^{-1}(\tau)\right) \text{ if } p_a < \tau \end{cases} \tag{21}$$

The eco-industry firms Let us now study the supply of the abatement good. Each firm $i \in [0,1]$ in the eco-industry maximizes its profit:

$$\pi_i(a(i)) = \max_{a(i)\geq 0} \{p_a.a(i) - \kappa(a(i)) - \tau.\beta(i).a(i)\} \tag{22}$$

the first-order condition of which is given by:

$$p_a - \kappa'(a(i)) - \tau.\beta(i) \leq 0 \text{ (with equality if } a(i) > 0) \tag{23}$$

So, if $p_a < \beta_{\min}.\tau$, no abatement good is supplied while if $p_a \geq \beta_{\max},\tau$ each firm produces and its production level is given by $a(i) = (\kappa')^{-1}(p_a - \tau.\beta(i))$. Finally if $p_a \in [\beta_{\min}.\tau, \beta_{\max}.\tau)$, only the firms with an index $i \leq \beta^{-1}\left(\frac{p_a}{\tau}\right)$ produce. Hence, the aggregated supply of abatement goods is:

$$A^s(p_a,\tau) = \begin{cases} 0 \text{ if } \frac{p_a}{\tau} < \beta_{\min} \\ \int_0^{\beta^{-1}\left(\min\left\{\frac{p_a}{\tau},\beta_{\max}\right\}\right)} (\kappa')^{-1}(p_a - \tau.\beta(i))\, di \text{ else} \end{cases} \tag{24}$$

The abatement good market It now remains for us to study the equilibrium of the abatement good market for any given price τ of pollution. So let us denote by $z(p_a,\tau) = A^d(p_a,\tau) - A^s(p_a,\tau)$ the excess demand correspondence. A first look at this correspondence tells us that for any $p_a > \tau$ there is always an excess supply: when the price of the abatement good is higher than τ, nobody is willing to buy abatement goods and therefore no equilibrium can be reached. We can now investigate whether $p_a = \tau$ and $\beta_{\min}.\tau < p_a \leq \tau$ can each be an equilibrium.

We begin by analyzing if $p_a = \tau$ clears the market. This requires that the upper bound of the demand $\varepsilon\left(\xi^{-1}(\tau)\right)$ at price τ is higher than the supply at this price, *i.e.*:

$$\varepsilon\left(\xi^{-1}(\tau)\right) \geq \int_0^{\beta^{-1}(1)} (\kappa')^{-1}(\tau(1-\beta(i)))\, di \tag{25}$$

This conditions is similar to condition (14). This means that there exists a threshold $\bar{\tau} = \bar{v}$, with the property that for all implicit pollution prices $\tau \leq \bar{v}$, $p_a = \tau$ is the market clearing price of the abatement good market. With this observation we can affirm that:

Lemma 4. *If $t \leq \bar{v}$, the equilibrium production levels of the aggregated market are given by $Q^c(\tau) = \xi^{-1}(\tau)$ and $A^c(\tau) = \int_0^{\beta^{-1}(1)} (\kappa')^{-1}(\tau(1-\beta(i)))\, di$. The equilibrium prices are $p^c(\tau) = P\left(\xi^{-1}(\tau)\right)$ and $p_a^c(\tau) = \tau$. Moreover each efficient firm in the eco-industry is active and its production is given by $a^c(i,\tau) = (\kappa')^{-1}(\tau(1-\beta(i)))$.*

In the opposite case, *i.e.* $\tau > \bar{v}$, the equilibrium price of the abatement good market is lower than τ but nevertheless higher than $\beta_{\min}.\tau$ because there is no supply of abatement at any price lower then $\beta_{\min}.\tau$ (see equation (24)). In fact this price solves:

$$\varepsilon\left(\xi^{-1}\left(p_a\right)\right) = \int_0^{\beta^{-1}\left(\frac{p_a}{\tau}\right)} \left(\kappa'\right)^{-1}\left(p_a - \tau.\beta(i)\right) di \tag{26}$$

This also implies (i) that not all efficient firms in terms of pollution reduction are active at equilibrium since $\frac{p_a}{\tau} < 1$ and (ii) the pollution of the dirty downstream firm is completely abated. More precisely, we can say:

Lemma 5. *If $\tau > \bar{v}$, the price of the abatement goods $p_a^c(\tau)$ is the unique solution to equation (26) and is lower than the implicit price for emissions. The quantities traded on the markets are given by:*

$$\begin{cases} Q^c(\tau) = \xi^{-1}\left(p_a^c(\tau)\right) = \varepsilon^{-1}\left(A^c(\tau)\right) \\ A^c(\tau) = \int_0^{\beta^{-1}\left(\frac{p_a(t)}{\tau}\right)} \left(\kappa'\right)^{-1}\left(p_a(\tau) - \tau.\beta(i)\right) di \end{cases} \tag{27}$$

The number of active firms in the eco-industry is given by $i^c(\tau) = \beta^{-1}\left(\frac{p_a^c(\tau)}{\tau}\right) < 1$ each of them producing $a^c(i,\tau) = \left(\kappa'\right)^{-1}\left(p_a^c(\tau) - \tau.\beta(i)\right)$. Finally the equilibrium of the commodity market is $p^c(\tau) = P\left(Q^c(\tau)\right)$

5.2. The level of the Pigouvian tax

In the previous section we analyzed competitive behaviors following the implementation of a Pigouvian tax. In this section we ask if there is a tax rate which implements the efficient allocation obtained either in Proposition 1 or Proposition 2.

If the level of the damage is such that it is optimal to partially abate the pollution of the dirty downstream firm ($v \leq \bar{v}$), we are back to the traditional case largely covered by the literature: the Pigouvian tax has to be equal to the marginal damage. If $\tau = v \leq \bar{v}$ the quantities traded at the competitive equilibrium ($Q^c(v)$, $A^c(v)$ and $a^c(i,v)$ in Lemma 4) are exactly the same as the optimal quantities obtained in proposition 1. In this case the price of the abatement good also reflects the marginal damage since $p_a^c(v) = v$.

If the marginal damage is higher than the threshold identified in Lemma 3 ($v > \bar{v}$) the value of τ is less obvious. Assuming that the policy maker keeps the same rule (*i.e.* $\tau = v$). Eq (26) tells us that the equilibrium price of the abatement good has to bebe equated with the optimal Lagrangian multiplier representing the marginal social cost (see Eq (17) in proposition 2). Hence, the quantities traded at the competitive equilibrium are exactly the same as the efficient quantities (see lemma 5). In other words, the policy rule remains the same but the mechanism leading to the efficient allocation is totally different. By setting the Pigouvian tax at the level of the marginal damage, the adjustment of the abatement good market results in a price corresponding to the social cost of abatement. Hence, the ratio between this equilibrium price and the Pigouvian tax selects the number of active firms in an optimal way.

Proposition 3. *As usual, the efficient allocation is reached if the Pigouvian tax is set at the level of the marginal damage. However, if the marginal damage is high ($v > \bar{v}$) this tax is higher than the marginal benefit from abatement, which is given by the equilibrium price of the abatement good.*

6. Concluding remarks

In this article, we have investigated whether the hypothesis of a polluting eco-industry challenges the usual results in economic literature. To this purpose, we considered a vertical structure composed of a polluting downstream firm and an upstream eco-industry. We assumed that eco-industry firms are heterogeneous and that they cannot reduce their pollution level. Under these assumptions, we obtained two kinds of equilibrium. The first equilibrium, with a lower level of damage, extended the standard results of economic literature to the case of a polluting eco-industry, but our results are very different when the damage is high. In this case, the dirty firm must totally abate its emissions. To reduce the remaining pollution, produced by the eco-industry, not all efficient eco-industry firms produce and the level of production among these firms is different to that of the first equilibrium. We found that the greater the damage, the lower the abatement level and the smaller the number of producing eco-industry firms.

We finally show that both equilibria can be decentralized in a competitive economy by means of a Pigouvian tax. Whatever the equilibrium, the regulator can follow a very simple rule, because the Pigouvian tax should always be equal to the marginal damage. However, this rule plays a different role in reaching each equilibrium.

Our results suggest that a polluting eco-industry is not a problem for the regulator, because the competitive equilibrium selects the right firms to be in production, provided that the regulator sets the correct level of the Pigouvian tax. However, this optimistic conclusion depends on the crucial assumption of perfect information that we implicitly make in our model. In the real world, the regulator cannot define this tax so well, and our results may not hold. Moreover, eco-industry is characterized by the fact that it is highly concentrated. In this respect, one may wonder what level of the Pigouvian tax would decentralize the optimum. Finally, this article takes as given the pollution features of each firm. Taking into account the innovative process would make it possible to endogenize the pollution distribution among firms. Further research is needed to investigate these different questions.

References

[1] Boyd, G.R., H. Reemtsma, D.A. Grimm, S. Mitra (2003) "Pharmaceuticals and personal care products (PPCPs) in surface and treated waters of Louisania, USA and Ontario, Canada" *Science of the total Environment*, 311, 135-149.

[2] Canton J. (2008) "Redealing the cards: how an eco-industry modifies the political economy of environmental taxes" *Resource and Energy Economics* 30 (3), 295-315.

[28] Canton J., M. David and B. Sinclair-Desgagné (2012) "Environmental regulation and horizontal mergers in the eco-industry" *Strategic Behavior and the environment*, 2, 107-132.

[4] Canton J., A. Soubeyran and H. Stahn (2008) "Environmental taxation and vertical Cournot oligopolies: how eco-industries matter" *Environmental and Resource Economics* 40 (3), 369-382.

[5] Comoretto, L., S. Chiron (2005) "Comparing pharmaceutical and pesticide loads into a small Mediterranean river" *Science of the total Environment*, 349, 201-210.

[6] David M. and B. Sinclair-Desgagné (2005) "Environmental regulation and the eco-industry" *Journal of Regulatory Economics* 28 (2), 141-155.

[7] David M. and B. Sinclair-Desgagné (2010) "Pollution abatement subsidies and the eco-industry" *Environmental and Resource Economics* 45 (2), 141-155.

[8] David M., A. D. Nimubona and B. Sinclair-Desgagné (2011) "Emission taxes and the market for abatement goods and services" *Resource and Energy Economics* 33, 179-191.

[9] Denicolo V. (1999), "Pollution-reducing innovations under taxes or permits", *Oxford Economic Papers* 51, 184-199.

[10] Ernst and Young (2006) "Eco-industry, its size, employment, perspectives and barriers to growth in an enlarged EU" Final Report to the DG environment of the European Commission available at:
http://ec.europa.eu/environment/enveco/jobs/index.htm#_Toc240786990

[11] European Commission Report (1999) "The EU eco-industry's export potential" available at: http://ec.europa.eu/environment/enveco/jobs/index.htm#_Toc240786990

[12] European Commission Report (2007) "Facts and figures: the link between EU's economy and environment" available at:
http://ec.europa.eu/environment/enveco/pdf/facts.pdf

[13] European Commission Report (2009) "Study on the Competitiveness of the EU eco-industry" available at:
http://ec.europa.eu/environment/enveco/jobs/index.htm#_Toc240786990

[14] Greaker M. (2006) "Spillovers in the development of new pollution abatement technology: a new look at the Porter-hypothesis" *Journal of Environmental Economics and Management* 52, 411-420.

[15] Greaker M. and K. E. Rosendahl (2008) "Environmental policy with upstream pollution abatement technology firms" *Journal of Environmental Economics and Management* 56, 246-259.

[16] Kyung D., D. Kim, N. Park, W. Leeand (2013) "Estimation of CO2 emission from water treatment plant - Model development and application, *Journal of Environmental Management*, 131, 74-81.

[17] Nawaz I. and Tiwari G.N. (2006) "Embodied energy analysis of photovoltaic (PV) system based on macro- and micro-level" *Energy Policy,* 34, 3144–3152.

[28] Nimubona, A.D. (2012) "Pollution policy and trade liberalization of environmental goods" *Environmental and Resource Economics*, 52, 323-346.

[19] Nimubona, A. D. and B. Sinclair-Desgagné (2005) "The Pigouvian tax rule in the presence of an eco-industry" *FEEM, Nota di Lavoro* 57.

[20] Nimubona, A. D. and B. Sinclair-Desgagné (2011) "Polluters and abaters" *Annales d'Economie et de Statistiques,* 103 / 104, 9-24..

[21] Nosaky, Y., D. Lerche, D. S. Alibo, M. Tsutsumi (2000) "Dissolved indium and rare earth elements in three japonese rivers and Tokyo bay: evidence for anthropogenic Gd and In." Géochimie Cosmochimie Acta, 64, 975-3982.

[22] OECD (1999) *The environmental goods and services industry: Manual for data collection and analysis*, OCDE Editions, Paris.

[23] Raucher R.S., J. E Cromwell, K. Cooney, P. Thompson, L. Sullivan, B. Carrico, M. Macphee, R. Wilkinson (2008) "Risk and benefits of energy management for drinking water utilities", . AwwaRF, Denver.

[24] R. Tyrrell Rockafellar (1997), *Convex Analysis* Princeton landmarks in mathematics (Reprint of the 1979 Princeton mathematical series 28 ed.). Princeton, NJ: Princeton University Press

[25] Rothausen S. and D. Conway (2011) "Greenhouse-gas emissions from energy use in the water sector" *Nature Climate Change*, 1, 210–219.

[26] Schwartz S., H. Stahn (2014) "Competitive permit markets and vertical structures: the relevance of imperfectly competitive eco-industries, *Journal of Public Economic Theory*, 16 (1), 69–95.

[27] Sinclair-Desgagné B. (2008) "The environmental goods and services industry" *International Review of Environmental and Resource Economics*, 2 (1), 69-99.

[28] Ternes, T.A. (1998) "Occurrence of drugs in German sewage treatment plants and rivers" *Water Resources*, 32, 3245-3260.

APPENDIX

A. Proof of Lemma 1

Step 0: Some notations.
(i) $Q_{\max} = \arg\max_{Q\geq 0}\left(\left(\int_0^Q P(q)dq\right) - C(Q)\right)$ is given by $P(Q_{\max}) - C'(Q_{\max}) = 0$. This quantity exists and is unique since $(P(Q) - C'(Q))$ is, under our assumptions, a continuous and decreasing function with the property that $\lim_{Q\to 0}(P(Q) - C'(Q)) = +\infty$ and $\lim_{Q\to+\infty}(P(Q) - C'(Q)) = \lim_{Q\to+\infty}(-C'(Q)) < 0$
(ii) $\forall Q \in [0, Q_{\max}]$, $\xi(Q) := \frac{P(Q)-C'(Q)}{\varepsilon'(Q)}$ is invertible and $\xi^{-1} : \mathbb{R}_+ \to [0, Q_{\max}]$. This follows, from the fact that $\forall Q \in [0, Q_{\max}]$

$$\xi'(Q) = \frac{(P'(Q) - C''(Q))\,\varepsilon'(Q) - \varepsilon''(Q)\,(P(Q) - C'(Q))}{(\varepsilon'(Q))^2} < 0 \tag{28}$$

and $\lim_{Q\to 0}\xi(Q) = +\infty$ and $\xi(Q_{\max}) = 0$.

Step 1: The existence of a solution $Q(v, A)$ to program (1).

To prove this point, let us verify that we maximize (i) a strictly concave function on (ii) a domain which can be reduced to the compact convex set $[0, Q_{\max}]$.
(i) Let us first observe that $\left(\left(\int_0^Q P(q)dq\right) - C(Q)\right)$ is a strictly concave function since its second derivative is given by $(P'(Q) - C"(Q)) < 0$. Now, note that $(\varepsilon(Q) - A)$ is convex in Q while $v \times \max\{x, 0\}$ is convex and increasing in x (for $v > 0$), hence their combination $v \times \max\{(\varepsilon(Q) - A), 0\}$ is convex. We therefore conclude that:

$$\phi_1(Q; A, v) = \left(\left(\int_0^Q P(q)dq\right) - C(Q)\right) - v \times \max\{(\varepsilon(Q) - A), 0\} \tag{29}$$

is strictly concave.
(ii) By (i) of step 0, and since $v \times \max\{(\varepsilon(Q) - A), 0\}$ is non decreasing in Q, $\phi_1(Q; A, v)$ decreases after $Q_{\max}$. We can therefore reduce the maximization domain to $[0, Q_{\max}]$.

Step 2: The characterization of the solution $Q(v, A)$.

Even if this problem is non-smooth but nevertheless concave, we can always define the subdifferential (see Rockafellar 1979 part V) of $\phi_1(Q; A, v)$ (see Eq (29)) with respect to Q. This quantity is given by:

$$\partial_Q\phi_1 = \begin{cases} P(Q) - C'(Q) \text{ if } Q < \varepsilon^{-1}(A) \\ \left[P(\varepsilon^{-1}(A)) - C'(\varepsilon^{-1}(A)) - v \times \varepsilon'\left(\varepsilon^{-1}(A)\right), P(\varepsilon^{-1}(A)) - C'(\varepsilon^{-1}(A))\right] \text{ if } Q = \varepsilon^{-1}(A) \\ P(Q) - C'(Q) - v \times \varepsilon'(Q) \text{ if } Q > \varepsilon^{-1}(A) \end{cases} \tag{30}$$

Since a maximum is reached if and only if $0 \in \partial_Q\phi_1$, this one is given by:

$$Q(v, A) = \begin{cases} Q_{\max} \text{ if } P(\varepsilon^{-1}(A)) - C'(\varepsilon^{-1}(A)) < 0 \\ \varepsilon^{-1}(A) \text{ if } P(\varepsilon^{-1}(A)) - C'(\varepsilon^{-1}(A)) \geq 0 \text{ and } P(\varepsilon^{-1}(A)) - C'(\varepsilon^{-1}(A)) - v \times \varepsilon'\left(\varepsilon^{-1}(A)\right) \leq 0 \\ \xi^{-1}(v) \text{ if } P(\varepsilon^{-1}(A)) - C'(\varepsilon^{-1}(A)) - v \times \varepsilon'\left(\varepsilon^{-1}(A)\right) > 0 \end{cases} \tag{31}$$

Now note that:

- $\left[P(\varepsilon^{-1}(A)) - C'(\varepsilon^{-1}(A)) \lesseqgtr 0\right] \Leftrightarrow \left[\varepsilon^{-1}(A) \gtreqless Q_{\max}\right]$ (by (i) of step 0)
- $\left[P(\varepsilon^{-1}(A)) - C'(\varepsilon^{-1}(A)) - v \times \varepsilon'\left(\varepsilon^{-1}(A)\right) \lesseqgtr 0\right] \Leftrightarrow \left[\varepsilon^{-1}(A) \gtreqless \xi^{-1}(v)\right]$ (by (ii) of step 0)

Thus, we deduce that:

$$Q(v,A) = \begin{cases} Q_{\max} \text{ if } A > \varepsilon(Q_{\max}) \\ \max\left\{\xi^{-1}(A), \xi^{-1}(v)\right\} \text{ if } A \leq \varepsilon(Q_{\max}) \end{cases} \tag{32}$$

<u>*Step 3:*</u> The computation of $\frac{\partial SB(A,v)}{\partial A}$.

If we replace Q by $Q(v,A)$ in $\phi_1(Q;A,v)$ (see Eq (29)) and remember, by step 0, that $\forall v > 0$, $\xi^{-1}(v) < Q_{\max}$, we obtain:

$$SB(A,v) = \begin{cases} \left(\int_0^{\xi^{-1}(v)} P(q)dq\right) - C(\xi^{-1}(v)) - v \times \left(\varepsilon\left(\xi^{-1}(v)\right) - A\right) \text{ if } A < \varepsilon\left(\xi^{-1}(v)\right) \\ \left(\int_0^{\varepsilon^{-1}(A)} P(q)dq\right) - C(\varepsilon^{-1}(A)) \text{ if } A \in \left[\varepsilon\left(\xi^{-1}(v)\right), \varepsilon(Q_{\max})\right] \\ \left(\int_0^{Q_{\max}} P(q)dq\right) - C(Q_{\max}) \text{ if } A > \varepsilon(Q_{\max}) \end{cases} \tag{33}$$

Moreover if we differentiate this function piecewise with respect to A, we can see that:

$$\frac{\partial SB(A,v)}{\partial A} = \begin{cases} v \text{ if } A < \varepsilon\left(\xi^{-1}(v)\right) \\ \frac{P\left(\varepsilon^{-1}(A)\right) - C(\varepsilon^{-1}(A))}{\varepsilon'(\varepsilon^{-1}(A))} \text{ if } A \in \left[\varepsilon\left(\xi^{-1}(v)\right), \varepsilon(Q_{\max})\right] \\ 0 \text{ if } A > \varepsilon(Q_{\max}) \end{cases} \tag{34}$$

is a continuous function (remember step 0) which can be summarized by $\frac{\partial SB(A,v)}{\partial A} = \max\left\{\min\left\{v, \xi(\varepsilon^{-1}(A))\right\}, 0\right\}$.

B. Proof of Lemma 2

<u>*Step 1:*</u> The solution to program (5).

Let us remember that the FOCs of program (5) are given by:

$$\forall i \in [0,1], \quad \kappa'(a(i)) + v \times \beta(i) - \lambda \geq 0 \ (= \text{ if } a_i > 0) \tag{35a}$$

$$\int_0^1 a(i)di = A \tag{35b}$$

It is a matter of fact to observe that if $A = 0$, almost all $a(i) = 0$, and Eq (35a) requires that $\lambda \leq v \times \beta_{\min}$ since $\kappa'(0) = 0$ and $\beta(i)$ increasing. So let us us concentrate on the situations in which $A > 0$ and $\lambda > v \times \beta_{\min}$. From Eq (35a), we observe (i) that for $\lambda < v \times \beta_{\max}$, only the firms $i \in \left[0, \beta^{-1}\left(\frac{\lambda}{v}\right)\right)$ produce while, for $\lambda \geq v \times \beta_{\max}$, each firms is active, and (ii) that their individual production is given by $(\kappa')^{-1}(\lambda - v \times \beta(i))$. It remains to use Eq (35b) to get λ. This quantity is implicitly defined by:

$$\phi_2(\lambda, A, v) = A - \int_0^{\beta^{-1}\left(\min\left\{\frac{\lambda}{v}, \beta_{\max}\right\}\right)} (\kappa')^{-1}(\lambda - v \times \beta(i))\, di = 0 \tag{36}$$

Let us now note that $\forall (A,v) \gg 0$, (i) $\lim_{\lambda \to v\beta_{\min}} \phi_2 = A > 0$, (ii) $\lim_{\lambda \to +\infty} \phi_2 = -\infty$ since $\lim_{a \to +\infty} \kappa'(a) = +\infty$ and (iii) this function is continuous and decreasing in λ since:

$$\partial_\lambda \phi_2 = -\int_0^{\beta^{-1}\left(\min\left\{\frac{\lambda}{v}, \beta_{\max}\right\}\right)} \left(\kappa"\left((\kappa')^{-1}(\lambda - v \times \beta(i))\right)\right)^{-1} di < 0 \tag{37}$$

for $\lambda \neq v.\beta_{\max}$ (remember that $\kappa'(0) = 0$). It therefore exists a unique $\lambda(A,v)$ which solves Eq (36) for each (A,v) and the optimal solution to program (5) is given by:

$$\forall i \in [0,1], \quad a^*(i,A,v) = \begin{cases} (\kappa')^{-1}(\lambda(A,v) - v \times \beta(i)) \text{ if } i \leq \beta^{-1}\left(\min\left\{\frac{\lambda}{v}, \beta_{\max}\right\}\right) \\ 0 \text{ else} \end{cases} \tag{38}$$

Step 2: The computation of $\frac{\partial SC(A,v)}{\partial A}$.

$$\frac{\partial SC(A,v)}{\partial A} = \int_0^{\beta^{-1}\left(\min\left\{\frac{\lambda(A,v)}{v},\beta_{\max}\right\}\right)} \left(\kappa'\left(a^*(i,A,v)\right) + v\times\beta(i)\right)\frac{\partial a^*(i,A,v)}{\partial A}di$$
$$+\left(\kappa\left(a^*(i,A,v)\right)+v\times\beta(i)\times a^*(i,A,v)\right)\Big|_{i=\beta^{-1}\left(\frac{\lambda(A,v)}{v}\right)}\times\frac{1}{v}\times\frac{\partial\lambda(A,v)}{\partial A}\ \left(\text{if } \frac{\lambda(A,v)}{v}<\beta_{\max}\ \right)$$

But for $\frac{\lambda(A,v)}{v}<\beta_{\max}$, $i=\beta^{-1}\left(\frac{\lambda(A,v)}{v}\right)$ is the pivotal agent so that $a^*(i,A,v)=0$ and since $\kappa(0)=0$, the second term vanishes. Moreover by Eq (35a):

$$\begin{aligned}\frac{\partial SC(A,v)}{\partial A} &= \lambda(v,A)\int_0^{\beta^{-1}\left(\min\left\{\frac{\lambda(A,v)}{v},\beta_{\max}\right\}\right)}\frac{\partial a^*(i,A,v)}{\partial A}di\\ &= \lambda(v,A) \qquad \text{since } \int_0^{\beta^{-1}\left(\min\left\{\frac{\lambda(A,v)}{v},\beta_{\max}\right\}\right)} a^*(i,A,v)di = A\end{aligned}$$

Step 3: Additional results for latter use.
Let us observe that for $\lambda < v\times\beta_{\max}$ we have:

- $\frac{\partial\phi_2(\lambda,A,v)}{\partial\lambda} = -\int_0^{\beta^{-1}\left(\frac{\lambda}{v}\right)}\left(\kappa"\left(\left(\kappa'\right)^{-1}\left(\lambda - v\times\beta(i)\right)\right)\right)^{-1}di<0$ (remember that $\kappa'(0)=0$).
- $\frac{\partial\phi_2(\lambda,A,v)}{\partial A} = 1>0$
- $\frac{\partial\phi_2(\lambda,A,v)}{\partial v} = \int_0^{\beta^{-1}\left(\frac{\lambda}{v}\right)}\beta(i)\left(\kappa"\left(\left(\kappa'\right)^{-1}\left(\lambda - v\times\beta(i)\right)\right)\right)^{-1}di>0$ (remember that $\kappa'(0)=0$).

It follows that:

$$\frac{\partial\lambda(v,A)}{\partial v} = -\frac{\partial_v\phi_2(\lambda,A,v)}{\partial_\lambda\phi_2(\lambda,A,v)}>0 \text{ and } \frac{\partial\lambda(v,A)}{\partial A} = -\frac{1}{\partial_\lambda\phi_2(\lambda,A,v)}>0 \tag{39}$$

C. Proof of Lemma 3

Step 1: There exists a unique threshold $\bar{v}$

Let us verify that:

$$\phi_3(v)=\varepsilon\left(\xi^{-1}(v)\right)-\int_0^{\beta^{-1}(1)}\left(\kappa'\right)^{-1}\left(v\times(1-\beta(i))\right)di=0 \tag{40}$$

admits a unique solution $\bar{v}$. This result is rather immediate since:

- $\phi_3(v)$ is continuous and decreasing since:

$$\phi_3'(v)=\frac{\varepsilon'\left(\xi^{-1}(v)\right)}{\xi'\left(\xi^{-1}(v)\right)}-\int_0^{\beta^{-1}(1)}\left(\frac{(1-\beta(i))}{\kappa"\left(\left(\kappa'\right)^{-1}\left(v\times(1-\beta(i))\right)\right)}\right)di \tag{41}$$

  Moreover we know that (i), by assumption, $\varepsilon'(Q)>0$, (ii), by step 0 of the proof of lemma 1, $\xi'(Q)<0$, and (iii), by the range of the integral, $1-\beta(i)\geq 0$. Hence $\phi_3'(v)<0$.

- $\lim_{v\to 0}\phi_3(v)=\varepsilon\left(Q_{\max}\right)>0$. More precisely $\lim_{v\to 0}\phi_3(v)=\lim_{v\to 0}\varepsilon\left(\xi^{-1}(v)\right)$ since $\kappa'(0)=0$. Using again step 0, we know that $\xi^{-1}(0)=Q_{\max}>0$. The result follows.

- $\lim_{v\to+\infty}\phi_3(v)<0$. By step 0 of the proof of lemma 1, we know that $\lim_{Q\to 0}\xi(Q)=+\infty$. It remains to remember that $\varepsilon(0)=0$ in order to conclude that :

$$\lim_{v\to+\infty}\phi_3(v)=-\lim_{v\to+\infty}\int_0^{\beta^{-1}(1)}\left(\kappa'\right)^{-1}\left(v\times(1-\beta(i))\right)di<0 \tag{42}$$

Step 2: If $v < \bar{v}$ then the optimal abatement provision only partially reduces the emissions of the dirty firm

By contraposition, assume that the efficient solution requires full pollution abatement of the dirty industry. At this optimal allocation λ^{opt} and A^{opt} verifies:

$$\lambda^{opt} = \xi\left(\varepsilon^{-1}(A^{opt}\right) \leq v \text{ and } \underbrace{A^{opt} - \int_0^{\beta^{-1}\left(\frac{\lambda^{opt}}{v}\right)} (\kappa')^{-1}\left(\lambda^{opt} - v \times \beta(i)\right) di}_{=\phi_2(\lambda^{opt}, A^{opt}, v) \text{ (see Eq (36))}} = 0 \tag{43}$$

From the first equation, we get that $\lambda^{opt} \leq v$ and $A^{opt} \geq \varepsilon\left(\xi^{-1}(v)\right)$. It follows, from step 3 of the proof of lemma 2, that:

$$0 = \phi_2(\lambda^{opt}, A^{opt}, v) \geq \phi_2(v, \varepsilon\left(\xi^{-1}(v)\right), v) = \phi_3(v) \tag{44}$$

Now remember by Step 2 that $\phi_3(\bar{v}) = 0$. Since $\phi_3'(v) < 0$, this implies that $v \geq \bar{v}$.

Step 3: If $v \geq \bar{v}$ then the optimal abatement provision requires full pollution abatement of the dirty industry

By contraposition, assume now that the efficient solution requires partial abatement. At this optimal allocation λ^{opt} and A^{opt} verifies:

$$\lambda^{opt} = v < \xi\left(\varepsilon^{-1}(A^{opt}\right) \text{ and } A^{opt} = \int_0^{\beta^{-1}(1)} (\kappa')^{-1}\left((1 - \beta(i)) \times v\right) di \tag{45}$$

Since now $A^{opt} < \varepsilon\left(\xi^{-1}(v)\right)$, we can say by using the second condition and step 3 of the proof of lemma 2 that:

$$0 = \phi_2(v, A^{opt}, v) < \phi_2(v, \varepsilon\left(\xi^{-1}(v)\right), v) = \phi_3(v) \tag{46}$$

Now remember by Step 2 that $\phi_3(\bar{v}) = 0$. Since $\phi_3'(v) < 0$, this implies that $v < \bar{v}$.

D. Proof of proposition 1

Point (i): This result follows from the definition of the case.

Point (ii): By step 0 of the proof of lemma 1, we can say that $\frac{dQ^{opt}}{dv} = \left(\xi'\left(\xi^{-1}(v)\right)\right)^{-1} < 0$ and by computation:

$$\frac{dA^{opt}}{dv} = \int_0^{\beta^{-1}(1)} \frac{(1 - \beta(i))}{\kappa"\left((\kappa')^{-1}\left(v \times (1 - \beta(i))\right)\right)} di > 0 \tag{47}$$

since $\forall i \in \left[0, \beta^{-1}(1)\right)$ we have $(1 - \beta(i)) > 0$ and $\kappa" > 0$.

Point (iii): This follows from the proof of step 1 of lemma 2 for $\lambda = v$

E. Proof of proposition 2

Point (i): This result follows from the definition of the case.

Point (ii): Since $Q^{opt} = \varepsilon^{-1}(A^{opt})$ it is obvious that if $\frac{dA^{opt}(v)}{dv} \leq 0$ then $\frac{dQ^{opt}(v)}{dv} \leq 0$. So let us check that $\frac{dA^{opt}(v)}{dv} \leq 0$. To verify this point, let us remember that an optimal allocation is in this case defined by:

$$\phi_2(\lambda^{opt}, A^{opt}, v) = 0 \text{ and } \lambda^{opt} = \xi\left(\varepsilon^{-1}\left(A^{opt}\right)\right) \tag{48}$$

By differentiation, this implies that:

$$\left(\underbrace{\frac{\partial \phi_2(\lambda, A, v)}{\partial \lambda}}_{<0} \times \underbrace{\frac{\xi'\left(\varepsilon^{-1}(A)\right)}{\varepsilon'\left(\varepsilon^{-1}(A)\right)}}_{<0} + \underbrace{\frac{\partial \phi_2(\lambda, A, v)}{\partial A}}_{>0}\right) dA + \underbrace{\frac{\partial \phi_2(\lambda, A, v)}{\partial v}}_{>0} dv = 0 \tag{49}$$

and the result follows from the proof of Lemma 1 (step 0) and Lemma 2 (step 3).

Point (iii): Since $\lambda^{opt}(v) < v$, the proof of the first part of the result directly follows from equation (38). It remains to verify that $\frac{di^{opt}(v)}{dv} = \frac{d\beta^{-1}\left(\frac{\lambda^{opt\prime}(v)}{v}\right)}{dv} < 0$. Since $\beta(i)$ is increasing let us compute:

$$\frac{d\left(\lambda\left(v, A^{opt}(v)\right)/v\right)}{dv} = \frac{1}{v}\left(\underbrace{\left(\left.\frac{\partial\lambda\left(v,A\right)}{\partial A}\right|_{A=A^{opt}(v)} \cdot \frac{dA^{opt}(v)}{dv}\right)}_{<0} + \underbrace{\left(\left.\frac{\partial\lambda\left(v,A\right)}{\partial v}\right|_{A=A^{opt}(v)} - \frac{\lambda\left(v, A^{opt}(v)\right)}{v}\right)}_{=W}\right) \tag{50}$$

By point (ii) of this proof and Eq (39), we know that the first term of the previous equation is negative. Now let us note, by Eq (39), that the second term can be written as:

$$W = \left(-\frac{\partial\phi_2(\lambda, A, v)}{\partial\lambda}\right)^{-1}\left(\frac{\partial\phi_2(\lambda, A, v)}{\partial v} + \frac{\lambda}{v}\frac{\partial\phi_2(\lambda, A, v)}{\partial\lambda}\right) \tag{51}$$

If we now replace the derivatives of ϕ_2 by its value (see step 3 of the proof of Lemma 2), we obtain:

$$W = \underbrace{\left(-\frac{\partial\phi_2(\lambda, A, v)}{\partial\lambda}\right)^{-1}}_{>0}\left(\int_0^{\beta^{-1}\left(\frac{\lambda}{v}\right)}\underbrace{\left(\beta(i) - \frac{\lambda}{v}\right)}_{<0}\underbrace{\left(\kappa"\left(\left(\kappa'\right)^{-1}\left(\lambda - v\times\beta(i)\right)\right)\right)^{-1}}_{>0} di\right) < 0 \tag{52}$$

Point (iv): In this case, the marginal social benefit is $\dfrac{\partial SB(v, A)}{\partial A} = \xi(\varepsilon^{-1}(A^{opt}(v))) = \lambda^{opt}(v)$. It follows that $A^{opt}(v) = \varepsilon\left(\xi^{-1}\left(\lambda^{opt}(v)\right)\right)$. But $A^{opt}(v) = \int_0^{\beta^{-1}\left(\frac{\lambda}{v}\right)}\left(\kappa'\right)^{-1}\left(\lambda - v\times\beta(i)\right) di$. We can therefore say that $\lambda^{opt}(v)$ is implicitly defined by:

$$\phi_4(\lambda) = \varepsilon\left(\xi^{-1}(\lambda)\right) - \int_0^{\beta^{-1}\left(\frac{\lambda}{v}\right)}\left(\kappa'\right)^{-1}\left(\lambda - v\times\beta(i)\right) di = 0 \tag{53}$$

and this equation admits a unique solution $\lambda \in (v\beta_{\min}, v)$ since:

- $\phi_4'(\lambda) = \frac{\varepsilon'\left(\xi^{-1}(\lambda)\right)}{\xi'\left(\xi^{-1}(\lambda)\right)} - \int_0^{\beta^{-1}\left(\frac{\lambda}{v}\right)}\left(\kappa"\left(\left(\kappa'\right)^{-1}\left(\lambda - v\times\beta(i)\right)\right)\right)^{-1} di < 0$ (remember that $\xi' < 0$, $\varepsilon' > 0$ and $\kappa" > 0$)
- $\lim_{\lambda\to v\beta_{\min}}\phi_4(\lambda) = \varepsilon\left(\xi^{-1}(v.\beta_{\min})\right) > 0$ because there is no production in the eco-industry (see the proof of Lemma 2)
- $\lim_{\lambda\to v}\phi_4(\lambda) = \varepsilon\left(\xi^{-1}(v)\right) - \int_0^{\beta^{-1}(1)}\left(\kappa'\right)\left(v\times\left(1-\beta(i)\right)\right) di < 0$. In fact by Eq (46) we know that $\left[0 \le \phi_2(v, \varepsilon\left(\xi^{-1}(v)\right), v)\right] \Longrightarrow [v \le \bar{v}]$ hence $[v > \bar{v}] \Longrightarrow \left[\phi_2(v, \varepsilon\left(\xi^{-1}(v)\right), v) < 0\right]$. It remains for us to observe that $\lim_{\lambda\to v}\phi_4(\lambda) = \phi_2(v, \varepsilon\left(\xi^{-1}(v)\right), v)$.

F. Proof of Lemma 4

This result directly follows from our discussion.

G. Proof of Lemma 5

We simply have to make sure that for all $\tau > \bar{v}$, there exists a unique price $p_a^c(\tau) \in [\beta_{\min}.\tau, \tau]$ which solves $\phi_5(p_a, \tau) = \varepsilon\left(\xi^{-1}(p_a)\right) - A^s(p_a, \tau) = 0$. To verify this point let us observe that:
(i) $\forall \tau > \bar{v}$, $\lim_{p_a \to \beta_{\min}.t} \phi_5(p_a, \tau) = \varepsilon\left(\xi^{-1}(\beta_{\min} \times \tau)\right) > 0$ since $A^s(\beta_{\min} \times \tau, \tau) = 0$ (see Eq (24).
(ii) $\forall \tau > \bar{v}$, $\lim_{p_a \to \tau} \phi_5(p_a, \tau) = \varepsilon\left(\xi^{-1}(\tau)\right) - \int_0^{\beta^{-1}(1)} (\kappa')^{-1} (\tau \times (1 - \beta(i)))\, di < 0$ by the definition of this case.
(iii)$\forall p_a \in (\tau \times \beta_{\min}, \tau)$, $\partial_{p_a} \phi_5(p_a, \tau) < 0$ since:

$$\frac{\partial \phi_5(p_a, \tau)}{\partial \tau} = \frac{\varepsilon'\left(\xi^{-1}(p_a)\right)}{\xi'\left(\xi^{-1}(p_a)\right)} - \int_0^{\beta^{-1}\left(\frac{p_a}{\tau}\right)} \left(\kappa\text{''}\left((\kappa')^{-1}(p_a - \tau \times \beta(i))\right)\right)^{-1} di \tag{54}$$

(remember $\kappa(0) = 0$), $\kappa\text{''} > 0$, $\varepsilon' > 0$ by assumptions and $\xi' < 0$ by step 0 of the proof of Lemma 1.

H. Proof of Proposition 3

This result directly follows from our discussion.