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Abstract

The paper addresses the role of education policies for institutional change. We focus on two contrasting effects of education and human capital accumulation. On the one side, education prompts economic growth and increases the wealth managed by an autocratic elite. On the other side, education increases the "awareness" of citizens (capturing their reluctance to accept a dictatorship and their labor-market aspirations) and requires the elite to devote more resources to income redistribution. Along the lines of this trade-off, our theory provides a Lipsetian explanation of the positive relationship between education and institutional change, the positive relationship between development and institutional change, and the negative relationship between inequality and institutional change. Furthermore, we obtain new insights on the natural resources curse hypothesis and on the design of development aid programs.

Keywords: Democratization; Human capital; Lipset's theory; Natural resource curse.

JEL Classification: D72; I25; O11; O43.

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1 Introduction

Whether education is a prerequisite of democratization or one of its side products is an unsettled question in both political sciences and economics. Perhaps the most influential contemporaneous work on this theme is due to Lipset (1960). According to Lipset, democracy requires a significant civic engagement, a political culture of negotiation, and the recognition of the need for compromises. As these values are typical of educated people, he concludes that education is more compatible with democracies.

A broader connected debate is the origin and evolution of institutions.The viewpoint that socio-economic development is conducive to democratization, commonly known as "modernization theory," is another major contribution of Lipset (1960). An alternative viewpoint has been recently advanced by Acemoglu et al. (2001), who put forward the colonial origins of institutions. Their message is that institutions are the ultimate cause of development, and not the reverse (see also Rodrik et al., 2004). Glaeser et al. (2004) dispute their result and conclude that "human capital is a more basic source of growth than are institutions," in line with the Lipsetian view and the earlier results by Barro (1999). They argue that the typical institutional indicators considered by Acemoglu et al. have a serious problem: they rather capture volatile outcomes than durable norms (as should proxys for institutions do).[1] While the econometric debate is still undecided (see, among others, Acemoglu et al., 2005; Castello-Climent, 2008; and Przworski et al., 2000), it's fair to say that the Lipsetian theory of democratization is up to now a valid and powerful framework for the analysis of institutional transformations. It's for example essential to understand the divergent economic patterns of North and South Korea, which have been part of the same country until the early 1950s (see details of the argument in Glaeser et al., 2004).

In contrast to the abundant empirical literature, there are only few attempts to capture the Lipsetian view in a theoretical framework. Bourguignon and Verdier (2000) introduce an endogenous political economy decision mechanism that depends on the education of citizens: the ruling oligarchs set the education policy anticipating their effects on the economic growth, on inequality, on the political participation of citizens, and on the structure of political power. They show that a high initial per capita income is associated with a larger likelihood of a country to be in a democracy and to a quicker transition; initial inequality has, instead, the opposite effects. On a similar

[1]For example, certain popular measures of constraints on governments are shown to be twice as volatile (in terms of average within country deviations) as Barro's measure of years of schooling.

line, Glaeser et al. (2007) convey the idea that education raises the benefits of civic engagement pretty much as social capital, therefore leading to a larger social and political involvement. They further argue that education does not only favor the emergence of democracy, but also helps stabilizing it. More recently, other aspects of Lipset theory are being the object of theoretical developments. For example, Jung and Sunde (2014) have investigated the Lipset claim that democracy is more likely in countries with more equal distributions of resources.

In this paper, we develop a model of institutional change that allows us to explore simultaneously the three essential features of Lipset's theory: the link between education and democracy; the link between economic development and democracy; and the link between inequality and democracy. In this sense, our theory is a more comprehensive exploration of the implications of the Lipsetian thought. Concretely, we study the paradigmatic case of an autocratic elite with full political and economic power. In line with the recent literature on democratization games (see Acemoglu and Robinson, 2006), the elite anticipate the existence and extent of revolutionary threats. They can act to avoid revolutions in two ways. They can either introduce an appropriate redistribution policy and, thus, placate the incentives to revolt of the citizens. Or, they can start the democratization process and dismiss their power.

Citizens are hand-to-mouth workers: they are employed in the national industry and consume in each period their labor income and transfers. Should their consumption not reach a specific threshold, workers would revolt. This threshold is endogenous in the model and depends on their level of human capital. The idea is that as citizens become more educated, they also become more aware of the political situation (see Zaller, 1992) and they tend to be more politically sophisticated (see Luskin, 1990, and Neuman, 1986). Moreover, as suggested by Campante and Chor (2012) to explain the Arab Spring events, they also have higher income expectations and require better working opportunities.[2] Consequently, the education policy set by the elite has two opposite effects. On the one side, human capital is a production factor in the economy, so education enhances labor productivity and triggers economic growth. On the other side, human capital accumulation has the above-described "awareness" cost, making workers more demanding and possibly leading to the disruption of the incumbent elite.

[2]Education is certainly more multifaceted than what economists generally assume. As argued by Chomsky (1996), education can be paradoxically used to obtain "ignorance," with the aim of standardization and domination of populations. This view is supported empirically by Castello-Climent and Mukhopadhyay (2013). For the sake of tractability, we abstract from non-monotonic effects of education on peoples desire for freedom.

The latter effect has been already put forward by several authors to explain the lack of support for mass education and/or its late set-up in England (see Galor and Moav on the 1870 Forster act, 2006), in India (see Pal and Ghosh, 2012, on the role of the landed elite against investment in basic education), or in Southern US (see Ager, 2013, on the planters elite's lack of support for mass schooling in the nineteenth century).

A major distinctive feature of our model is the resource-dependence of the economy under scrutiny, which fits a largely significant number of autocratic regimes in the world. We assume that the economic power of the elite originates from a windfall of resources (natural resources or other). The elite can either export such resources on the international market or supply them to the national industry. In the first case, the revenues from export can be directly allocated for their own benefit or for redistribution and education policies. In the second case, the supply of resources to the national industry provides the elite with a further instrument to control citizens: by manipulating the supply of resources, they can influence the relative scarcity of citizens' labor and, consequently, also their wage.

Our theory predicts two possible scenarios. A particularly painful post-dictatorship period would not encourage the Lipsetian elite to democratize the country: they would decide to keep investment in education low, rely massively on resource export, and redistribute to citizens just enough to prevent any revolution. More surprisingly, a Lipsetian elite—despite their full political and economic power—might undertake a path leading to an institutional change in a finite time horizon. This path is characterized by high investment in education, a progressive reduction of the dependence on resource export, increasing claims of citizens, reducing inequality, and, eventually, a voluntary power dismissal by the elite.

These predictions match well the Lipsetian traits of institutional change. The democratization path is triggered by the correct balance between economic returns to citizens' education and their increasing claims to consumption. Thus, a high level of human capital is both a prerequisite and a consequence of institutional change: it is a prerequisite as the elite would dismiss their power only if the economy is rich enough to guarantee sufficient economic returns; it is a consequence as the elite anticipates the costs and benefits of the democratization path and optimally decides to reach such human capital level at the cost of their political power. This provides a new explanation of the Lipsetian link between education and democratization. More mechanically, the Lipsetian link between economic development and democratization builds on education being the engine of growth. The Lipsetian link between inequality and democratization follows from the citizens becoming more aware and demanding as their human capital increases.

Relation to the literature

With respect to the existing scarce economic theory literature on Lipset's theory cited above (in particular, Bourguignon and Verdier, 2002, and Glaeser et al., 2007), three main differences should be highlighted. First, by attributing **full** political and economic power to the Lipsetian elite, we further emphasize how powerful education policies can be for institutional change. Second, by eliminating the possibility of unanticipated and violent revolutions—as well as the corresponding institutional changes—our setting is more likely to capture those institutional changes that lead to stable democracies; confirming, from a different perspective, the results by Glaeser et al. (2007). Third and quite importantly, the presence of resources enriches the model with a further dimension. Our comparative statics with respect to windfall resources are consistent with Lipset's theory, stating that higher incomes are favorable for democracy: in our model, a larger amount of resources endows the elite with the necessary wealth to take the path of workers' education and guarantees a sufficient long-run payoff, despite this is associated with an institutional change. Beyond making democratization more likely, a larger amount of resources also leads to a shorter duration of the authoritarian regime.

Moreover, the windfall of resources allows us to compare our predictions with game-theoretic models of institutional change or of natural resources-induced conflicts. Our conclusions are in line with those obtained within the former stream of literature where dictatorships are ended through revolutions and not voluntarily as in our model. In particular, the idea that the initial wealth of a country leads to earlier conflicts has also been put forward also by Boucekkine et al. (2014). This result seems at odds with the resource curse hypothesis, according to which resource abundance may undermine the democratization process. However, the resource curse hypothesis receives only a moderate empirical support: some economist argue that resource wealth strengthens autocratic regimes (see among others, Ross 2001, and Tsui, 2011); others find no evidence of a positive relationship between resource wealth and the stability of autocratic regimes (Alexeev and Conrad, 2009, and Haber and Menaldo, 2011). The new mechanism presented in this paper—through the investment in education and the development of citizens' aspiration for democracy—can rationalize these contrasting opinions: resource windfalls cannot alone trigger the democratization process. This also explains why in certain countries with large endowment in natural resources, investment in education is particularly weak (see Gylfason, 2001). In our setting, even though a country has a given large level of windfall rents, human capital accumulation (and therefore democratization) is suboptimal at least in two cases: when the effectiveness of education investment is not large enough and/or when the share of wealth accruing to the elite after democratization is not large enough. Finally, our work can be related to the broader debate on the effectiveness of large aid programs to poor countries (see Kraay and Raddatz, 2007). Our theory predicts that even massive education policies devoted to improve access to schools may not trigger sustainable development, nor can they promote democratization. Indeed, their effectiveness is merely a matter of whether they pursue the correct target, which according to our analysis is the quality of the schooling system.

The remainder of the paper is organized as follows. Section 2 presents the model of the economy. Section 3 studies in details the elite maximization problem. Section 4 and 5 study all the possible solutions and then compare them to determine the optimal choices. Implications of the model are discussed in Section 6 whereas Section 7 concludes. All the proofs are gathered in the appendix.

2 The model

Time is continuous, let $t \in [0, \infty)$. For notational simplicity, the time index is omitted when there is no risk of confusion. In each period, the elite manage a constant windfall of natural resources $R > 0$. Resources have two possible uses: a part of it can be exported on the international primary good market –let the exported quantity be denoted by X– and for the remaining part, it can be supplied internally to the manufacturing sector –let this quantity be denoted by $Q \leq R - X$. The international price of the resources is constant over time and is denoted $p^x > 0$.

National firms operate in a competitive environment and produce a homogeneous commodity Y using two inputs: resources Q and human capital H. With a Cobb-Douglas specification $Y = F(Q, H) = AQ^{\alpha}H^{1-\alpha}$, resource price and wage (per unit of human capital) are:

$$p = \alpha \frac{Y}{Q} \tag{1}$$

and

$$w = (1 - \alpha) \frac{Y}{H}. \tag{2}$$

The resource rent of the elite, obtained from export X and national supply Q, is allocated between the elite's consumption C, transfers to workers Θ, and education investment E:

$$p^x X + pQ \geq C + \Theta + E. \tag{3}$$

Investment in the education sector increases human capital according to the following accumulation function:

$$\dot{H} = h(E, H) = hE - \delta H \tag{4}$$

where $h > 0$ measures the effectiveness of the education investment and $\delta \geq 0$ is the depreciation rate of human capital.

In each period, a unitary mass of workers –or citizens– inelastically supply their human capital H to national firms and earn an equilibrium wage w, determined by (2). Their income is completed by the transfers Θ and is entirely consumed in each period.

If workers find their consumption not large enough, they can decide to contest the power of the elite. The threshold consumption that triggers a revolt is given by the sum of a subsistence consumption—let $s > 0$ denote such level—and an awareness component, which depends on their human capital level. The important point is that this threshold is endogenous. As outlined in the introduction, this conveys the idea that as workers get more and more educated, they become more aware of the political situation in the country, they become more politically sophisticated, require better working opportunities, and have higher claims for democracy. As a result, they would not revolt only if their consumption were sufficiently large. Let the "awareness" component of consumption be linear in human capital, with political awareness parameter (the multiplier of human capital) $\phi > 0$. Then, workers decision to revolt at each t is:

$$\begin{cases} \text{revolt} & \text{if } wH + \Theta < s + \phi H \\ \text{no revolt} & \text{otherwise} \end{cases} \tag{5}$$

We assume that a revolt, if any, leads to a more democratic regime. In this situation, workers represent a permanent threat to the elite. However, a key aspect of our model is the capacity of the elite to fully internalize their incentives to revolt. This capacity manifests itself in the additional – no revolt – constraint (5) faced by the elite who will ultimately be the ones to decide whether or not to instigate the democratization process.

Let $T \in \mathbb{R}_+ \bigcup \{\infty\}$ be the time at which the autocratic regime comes to an end (permanent dictatorship holds when $T = \infty$). This is a control variable in the hands of the elite. The decision to democratize is based on their situation in the regime following the institutional change. A sharing rule is then used to describe how the wealth of the economy is distributed to agents at T; in particular, since wealth is an increasing function of human capital, we shall assume that the elite expect a factor $\pi > 0$ of the amount of human capital at T. As explained in the introduction, such an assumption may also be interpreted as a Lipsetian trait: human capital is tightly connected with negotiation and absence of violence in Lipset's theory; thus, a scrap value function increasing with human capital is a natural way to summarize what is going on in the second regime. Indeed, Elite highly regard the possibility of leaving the power without being exposed to full expropriation and political violence. The larger parameter π, the larger this impact of human capital on the post-autocratic regime is valued by the elite. An alternative explanation is that the elite expects to control (some part of the) resources also after democratization and, given some complementarity, their value will depend on human capital.

The intertemporal well-being of the elite is given by:

$$U^e = \int_0^T e^{-\rho t} u\left(C\right) dt + e^{-\rho T} \pi H_T \tag{6}$$

where the instantaneous utility function is $u\left(C\right) \equiv \frac{(C)^{1-\gamma}}{1-\gamma}$ with $\gamma \in (0,1)$ and $\rho > 0$ is the discount rate.[3]

Before moving to the analysis, it is worth mentioning three final points.

First, the elite are particularly powerful. They are able to control the consumption/income of workers, and thus their willingness to revolt, in three different ways: *(i)* directly, by setting the transfer Θ; *(ii)* indirectly, by deciding how many resources to supply to the national industry Q; and *(iii)* dynamically, by investing more or less in education E and thus setting their level of human capital. Furthermore, they control the political transition process and choose the timing T (possibly infinite) for the

[3] The assumption that the elasticity of consumption be positive ensures that the utility is positive for any value of consumption. This is needed for the continuation payoff at the time of institutional change, i.e. $\pi H_T \geq 0$, to be intrapersonal comparable. When instead $\gamma > 1$, utility levels are strictly negative and an immediate institutional change (independently of the human capital level) is always optimal.

institutional change.

Second, we shall assume that the resource windfall is sufficiently large so that it is possible to sustain a dictatorial regime when human capital is zero.

Assumption 1. $p^x R > s$: *The value of resources is larger than the subsistence consumption of the workers and gives the elite some freedom in how to allocate such wealth.*

Third, an alternative approach would have been to *(i)* explicitly model the features of the economy following the institutional change, *(ii)* solve the corresponding optimization problem, and *(iii)* retrieve the resulting value function and use it as a scrap value. Proceeding like this is both interesting and feasible (simply see the second regime problem – with no more distinction between the elite and the workers – as an optimal growth problem) but rises a number of additional issues (related to the two groups' respective size in the first regime etc.). Besides, it would make the optimality analysis really tricky without changing much the results.

3 The elite maximization problem

The elite seek to maximize utility (6), subject to the budget constraint (3), equilibrium prices (1) and (2), the revolution decisions of workers (5), and the dynamics of human capital (4). The decision variables are the use of resources Q, own consumption C, transfers Θ, and education E. Substituting Θ from the non-revolt condition of workers (5), the optimization problem of the elite can be written as an optimal stopping problem, where T is the time until which the constraint is met. Formally:

$$\max_{\{Q,E,T\}} \int_0^T e^{-\rho t} u\left(p^x\left(R-Q\right) + AQ^\alpha H^{1-\alpha} - E - s - \phi H\right) dt + e^{-\rho T}\pi H(T)$$

s.t.

$$\begin{cases} \dot{H} = hE - \delta H \\ H(T) = H_T \text{ is free} \\ E \geq 0 \\ \text{with } H(0) = H_0 \text{ given} \end{cases}$$

Maximizing the criterion with respect to national resource supply Q, requires that national prices equalize international ones, i.e. $p = p^x$, and sets the optimal ratio between resources and human capital as follows:[4]

$$\frac{Q}{H} = \left(\frac{\alpha A}{p^x}\right)^{\frac{1}{1-\alpha}} \tag{7}$$

Let λ be the costate variable associated with human capital. Its equilibrium dynamics is:

$$\dot{\lambda} = (\delta + \rho)\,\lambda - C^{-\gamma}\left((1-\alpha)\,A\left(\frac{Q}{H}\right)^{\alpha} - \phi\right) \tag{8}$$

At the equilibrium, optimal investment in education is such that the marginal benefit from education equals the marginal cost of investing in education (in terms of foregone consumption). Let μ be the Lagrange multiplier associated with the positivity constraint on education, this yields the optimal consumption of the elite:

$$C = (\lambda h + \mu)^{-\frac{1}{\gamma}} \tag{9}$$

The slackness conditions on education investment E require:

$$\mu \geq 0 \qquad \text{and } \mu E = 0 \tag{10}$$

The optimal time $T < \infty$ for violating the no-revolt condition and inducing a regime change is such that the current value of the Lagrangian be equal to the value of the salvage function; i.e.:

$$u\,(C(T)) + \lambda\,[hE(T) - \delta H(T)] = \rho\pi H(T) \tag{11}$$

Finally, the transversality condition requires that:

$$\lambda(T) = \frac{\partial S\,(H)}{\partial H}|_{H=H(T)} = \pi \tag{12}$$

Convexity of the problem with respect to optimal education investment E and internally supplied resources Q guarantees that the corresponding second order conditions are

[4]The optimal resource supply of the elite to the national industry would determine a price wedge between international and internal resource prices in case of costly redistributive transfers. In this case, the elite would find it more profitable to redistribute income to workers by oversupplying resources and, indirectly, determining a wage increase. While this extension is potentially relevant for an empirical assessment, the results discussed are not affected.

always satisfied. The second order conditions for the optimal stopping problem are not necessarily met and will be discussed in more details in the following.

We solve the model in three steps: we first study the dynamics of the system when education investments are strictly positive; we then study the case of either zero investments in education or alternating periods of positive and zero investments; and finally we characterize optimality by combining the results.

4 Optimality candidates

4.1 Education-driven institutional change

Define the instantaneous returns on human capital Ω and the instantaneous return on education investment χ as follows:

$$\begin{aligned} &\Omega \equiv \tfrac{1-\alpha}{\alpha} p^x \left(\tfrac{\alpha A}{p^x}\right)^{\frac{1}{1-\alpha}} - \phi, \\ &\chi \equiv h\Omega - \delta, \end{aligned}$$

Instantaneous returns on human capital are defined as the difference between a gross return and the feedback effect exerted by human capital on workers' claims for democracy, which is represented by the political awareness parameter, ϕ.

Straightforward manipulation of the necessary optimality conditions yields the general solution, valid for any solution with strictly positive education $E > 0$ (the superscript 1 indicates this regime):

$$\begin{cases} C^1(t) = C_0^1 e^{\frac{(\chi-\rho)t}{\gamma}} \\ H^1(t) = \left(H_0 + \frac{h(p^x R - s)}{\chi} - \frac{\gamma h C_0^1}{\rho - \chi(1-\gamma)}\right) e^{\chi t} + \frac{\gamma h C_0^1}{\rho - \chi(1-\gamma)} e^{\frac{(\chi-\rho)t}{\gamma}} - \frac{h(p^x R - s)}{\chi} \\ \lambda^1(t) = \frac{\left(C_0^1\right)^{-\gamma}}{h} e^{(\rho-\chi)t} \end{cases} \tag{13}$$

Then, we can establish that:

Proposition 1. *(i) There is no solution combining permanent dictatorship and positive education.*

(ii) There may be a solution combining institutional change and positive education.

(a) This solution is characterized by an increasing stock of human capital, an institutional change in finite time $T = T(H_0, R, \pi)$, and a corresponding

end-point stock of human capital:

$$H_T = \frac{h}{\rho - \chi}\left(\frac{\gamma(h\pi)^{-\frac{1}{\gamma}}}{1-\gamma} + p^x R - s\right) > 0. \tag{14}$$

(b) Necessary conditions for the existence of such solution for all $H_0 \in [0, H_T]$ *are:*

$$\begin{cases} \rho > \chi \\ p^x R - s > (h\pi)^{-\frac{1}{\gamma}}. \end{cases} \tag{15}$$

(c) Sufficient conditions for the existence of a unique solution are (15) and $\chi > \underline{\chi}$*, with* $\underline{\chi} \in (0, \rho)$ *the unique solution of:*

$$e^{\frac{\gamma\chi}{\rho-\chi}} = \frac{\rho[(1-\gamma)(\rho - \chi(1-\gamma)) + \gamma^2\chi]}{(1-\gamma)(\rho-\chi)^2}. \tag{16}$$

The condition $\rho > \chi$ sets an upper bound on the instantaneous returns to human capital and is a necessary condition for the existence of a solution with education-driven institutional change. Under the opposite condition $\chi \geq \rho$, autocracy is too growth-friendly. The elite can invest in education and, due to the high returns, this stimulates growth of output (and citizens' consumption) while being compatible with the respect of the no-revolution constraint, which is in fact never binding. However, the solution with permanent dictatorship and positive education is not relevant because it would either violate the resource constraint, or imply resource imports to become infinite (with X tending to $-\infty$). The second necessary existence condition in (15) states that resource windfalls net of the intrinsic subsistence consumption level should be larger than the level of consumption the elite just enjoy at the date dictatorship ceases, $C^1(T)$. Finally, a sufficient condition for existence requires that the returns to human capital be higher than a threshold $\underline{\chi}$, defined by (16). High enough returns to human capital logically guarantees that it is worthwhile for the elite to engage in the path of education and sustained capital accumulation.

Under $\rho > \chi$, the time path of consumption is decreasing whereas the stock of human capital is increasing. The intuition runs as follows. For the elite to find it optimal to democratize they should be able to accumulate a sufficient amount of human capital, which will directly affect the wealth they will hold in the post-dictatorship regime, and will also guarantee that they can enjoy their wealth in a peaceful environment. Thus investment in human capital should be favored over consumption. Moreover, by investing a lot in human capital, the elite foster the development of citizens' claims for a freer system through the increasing awareness mechanism. In order to delay the political regime change the elite have no other option but to transfer a lot of resources to the citizens. This also comes at the expense of their own consumption.

Note that under the conditions of Proposition 1, solutions that combine positive education and a revolution in finite time exist for any $H_0 \leq H_T$. In other words, the stated conditions guarantee the existence of a solution with education-driven institutional change independently of the initial endowment in human capital. This is a reasonable feature of our model: it would otherwise be difficult to explain why some countries are doomed to dictatorial regimes exclusively based on their initial stock of human capital and would also raise the issue of identifying this initial period (of the development process). Importantly, this doesn't mean that the initial stock of human capital is irrelevant to our analysis. As far as the optimality analysis is concerned, one expects that this variable will be crucial to determine which one of the optimality candidates yields the optimum.

The results stated in Proposition 1 also give a first insight into the predictions of our model concerning the Lipsetian links between human capital and democracy, and between resources and democracy. An obvious implication of the proposition is the incompatibility between permanent dictatorship and education *(i)*, which is in accordance with Lipset's theory. The reason of this incompatibility follows from the no-revolt constraint: it's impossible to satisfy this constraint with a positive level of education investment as the consumption aspirations associated with positive education will end up exceeding the redistribution capacities of the elite. In addition, it is worth emphasizing some other interesting features of the first optimality candidates. They are summarized in the next two corollaries.

Corollary 1. *The solution with education-driven institutional change is possible only if, ceteris paribus,*

(i) Resource wealth, $p^x R$, is large enough.

(ii) Elite's incentives to democratize, that are provided by the share of wealth accruing to the elite after they give up power, π, are important enough.

(iii) The effectiveness of the education, h, needs to be important enough too. But, in contrast to resource wealth and the sharing rule, it should not take an excessive value since the instantaneous returns to education, χ, cannot be too high.

These properties are in line with Lipset's theory in two essential aspects, the link between democratization and education and the link between resources (or income) and democratization.[5] First of all, the model predicts that a large amount of resources (or of their export price) is a precondition for the emergence of a non-dictatorial regime through human capital accumulation. However, the resource wealth of a country (measured by $p^x R$) is not the unique relevant determinant of democratization. Two further factors enter the necessary conditions for the emergence of a democratization path: a sufficient return to investment in education, h, and a sufficient reward for the elite at the time of institutional change, π. Democratization may not occur under large resource revenues because one of the two latter parameters is too small (leading to violating conditions (15)). Importantly, the interaction between the resource wealth and these factors is likely to be responsible for the mixed support for the natural resource curse hypothesis (see the debate opposing proponents of this hypothesis, Ross, 2001, and Tsui, 2011, and detractors, Alexeev and Conrad, 2009, and Haber and Menaldo, 2011) and is in line with the empirical studies pointing at the mis-management of education in several oil-exporting countries (see Gylfason, 2001). Finally, notice that the role of the return to education is tricky: it should be high enough *ceteris paribus* for democratization via education to arise but it should not be too high as the induced wealth in the hands of the elite in such a case could be sufficient to compensate for the larger awareness of the workers. In this case, a developing dictatorship could be sustained, although no equilibrium paths exist (see the interpretation of (15)).

Beyond the existence of a solution with institutional change, it is also useful to stress how the time-to-democratization is affected by the parameters of the model (see the comparative statics exercise at the end of Appendix A.1).

Corollary 2. *The optimal time for institutional change, $T = T(H_0, R, \pi)$, is decreasing in both the initial endowment in human capital, H_0, the constant flow of resource windfall, R, and the sharing rule, π.*

The first two features strengthen the correlation between wealth and democratization discussed before. The larger is the initial stock of human capital (another possible measure of human wealth) or the windfall of resources, the quicker are the Lipsetian elite in driving the country into an institutional change. While the larger windfall is

[5] Note that they are a direct consequence of the second necessary condition in (15). Also note that the parameters R and π do not show up in the sufficient condition $\chi > \underline{\chi}$ since they don't enter into the expressions of Ω. In contrast, the parameters h and p^x enter this condition through Ω.

also associated to a larger level of human capital at the time of institutional change, such an effect is absent for the initial human capital level. Finally, the optimal time-to-democratization is decreasing in π. The elite compensates a less favorable sharing rule by increasing the human capital of the country at the institutional change, H_T. This requires a longer period of investment in education.

To end up this discussion, it is important to measure the elite's payoff associated with the solution with education-driven institutional change. Let such optimality candidate be referred to as regime 1; then the present value (for the elite) of following this regime is given by (hereafter, the optimal time for institutional change is expressed in terms of H_0 only):

$$V^1(H_0) = e^{-\rho T(H_0)} \left(\frac{\gamma (h\pi)^{-\frac{(1-\gamma)}{\gamma}}}{(1-\gamma)(\rho - \chi(1-\gamma))} (e^{\frac{(\rho-\chi(1-\gamma)T(H_0)}{\gamma}} - 1) + \pi H_T \right).$$

The typical dynamics corresponding to this first possible solution is depicted in Figure 1.

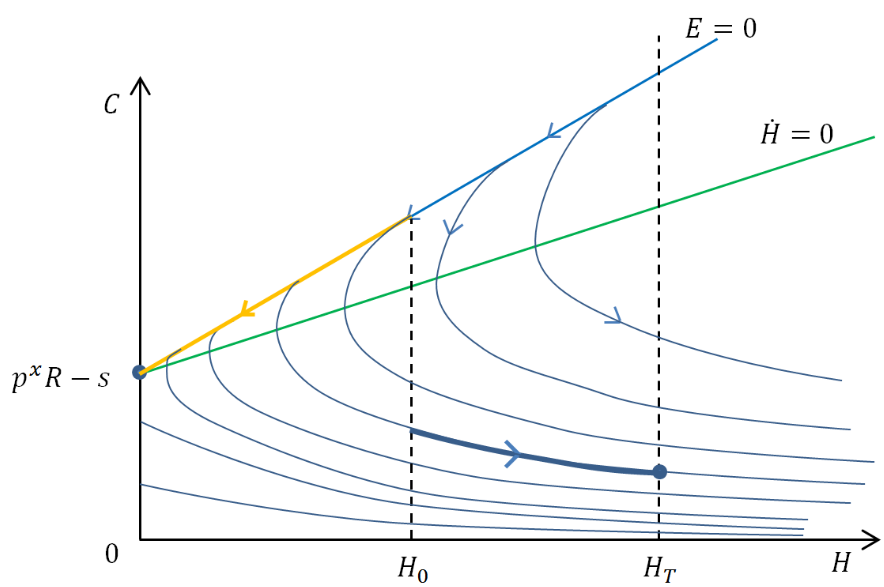

Figure 1: Phase diagram.

In the next section the other optimality candidates are briefly reviewed.

4.2 No-education and permanent dictatorship

The general solution corresponding to no investments in education, i.e. $E = 0$, is given by (the superscript 2 refers to the regime with no education):

$$\begin{aligned}
&H^2(t) = He^{-\delta t}\\
&C^2(t) = p^x R - s + \Omega H^2(t)\\
&\lambda^2(t) = e^{(\rho+\delta)t}\left(L - \int \Omega C^2(u)^{-\gamma} e^{-(\rho+\delta)u} du\right)
\end{aligned}$$

with H, L two constants to be determined. It can easily be shown that:

Proposition 2. *(i) There always exists a solution combining permanent dictatorship with no investment in education.*

(ii) There may be a solution alternating periods of investment in education with periods of no investment, but these never provide a candidate for optimality.

The solution with permanent dictatorship and no education is characterized by a decreasing flow of consumption and a decreasing stock of human capital. Consumption asymptotically converges toward $C^2(\infty) = p^x R - s$, while the stock of human capital vanishes. This is the path taken by the elite that attach greatest importance to being in power and prefer not to invest in human capital in order to avoid the risk of being overthrown. Solutions with no education exist for any level of the stock of human capital; whereas, solutions featuring a regime change from positive to zero education can be disregarded because their are always dominated by the former type, i.e., they are associated with lower present values (see Appendix A.2).

At this stage of the analysis, we are left with two optimality candidates, which makes the options available to the elite very clear. Either they choose to rely on resource wealth and not to invest in education in order to keep the labor force uneducated and docile. But this requires to sacrifice education-driven economic growth. Or, the elite engage in a policy of sustained investment in education, which promotes the accumulation of human capital at the cost of giving up political power in finite time because of the development of citizens' claims for democracy. As expected, variables like the returns to education (and human capital), the initial stock of human capital, the discount rate but also the share of wealth accruing to the elite after a revolution will play a central role in explaining what is the elite's best option.

For the remainder of the analysis, it is useful to retrieve the value function corresponding to the optimality candidate with no education. The present value is given by:

$$V^2(H_0) = \int_0^\infty \frac{1}{1-\gamma} \left(p^x R - s + \Omega H_0 e^{-\delta t}\right)^{1-\gamma} e^{-\rho t} dt.$$

Before determining the optimal choice of the elite, we compare regime 1 and 2 in terms of their implications for the link between inequalities and institutional change.

4.3 Inequality implications of a Lipsetian elite

So far, we have addressed the links income-institutional change and human capital-institutional change. In this respect, we have shown that the predictions of the model are consistent with Lipset's theory. It remains to study the link inequalities-institutional change.

As workers are a homogeneous mass of individuals, the only way to appraise inequalities in a direct and elementary way is by tracking the consumption of the elite vs. the consumption of workers. Although this is not completely in the spirit of Lipset's theory concerning this aspect (see Jung and Sunde, 2014, for a tighter connection), this exercise turns out to be worthwhile. Recall that the workers' income is entirely devoted to consumption. At any solution, we have $C_W^i(t) = s + \phi H^i(t)$ for $i = 1, 2$. Let $I^i(t) \equiv \frac{C_W^i(t)}{C^i(t)}$ be the index of inequalities at solution $i = 1, 2$. Then, we can establish the following result.

Proposition 3. *At the solution with education-driven institutional change, inequalities continuously shrink. At the solution with permanent dictatorship and no investment in education, the opposite result holds if:*

$$R > \frac{(1-\alpha)s}{\phi\alpha} \left(\frac{\alpha A}{p^x}\right)^{\frac{1}{1-\alpha}}. \tag{17}$$

A couple of comments are in order here. First, we observe that along the transition process to non-dictatorship, inequalities do decrease. It is as if in order to prepare the ground for a democratic regime, the elite have to progressively reduce the income (consumption) gap between the two groups until the institutional change. Intuitively, since the elite invest in human capital along this path, growth is stimulated. But the positive growth effect is dominated by the negative effect due to increasing awareness and the elite have no option but to sacrifice part of their consumption to satisfy the no-revolt constraint and delay the date of leaving office. Second and not surprisingly, permanent dictatorship imply a widening of inequalities if resource windfalls are high, the awareness cost is large, the international resource price is high, and the level of subsistence consumption is low. Under these conditions, the dictator is able to fill the revolt constraint at least cost. By not investing in human capital, the people are maintained under control while the elite become richer and richer relative to the workers.

The next section investigates the optimality of the above-identified solutions.

5 Optimal solution

The optimality analysis boils down to a study of the relative performance of the solution with education-driven institutional change vs. the solution with permanent dictatorship and no-education. To conduct this analysis, we proceed to a comparison of the present values associated with our two optimality candidates. The results can be summarized as follows:

Proposition 4. *Let $H_0 \in [0, H_T]$. The following cases can arise:*

(i) The solution with permanent dictatorship and no-education is optimal for all H_0 iff $V^2(H_T) > V^1(H_T)$;

(ii) The solution with education-driven institutional change is optimal for all H_0 iff $V^1(0) > V^2(0)$;

(iii) Otherwise, a human capital poverty trap arises. There exists $\bar{H} \in [0, H_T]$ such that the solution with education-driven institutional change is optimal iff $H_0 \geq \bar{H}$.

Both the no-education regime with persistent dictatorship and the education regime with democratization can arise. Depending on the parameters, it might be possible that: *(i)* the first alternative is chosen independently of the initial stock of human capital; *(ii)* the second alternative is chosen independently of the initial stock of human capital; and *(iii)* the regime choice depends on the initial human capital stock, a low stock is associated to no-education investment and infinite horizon dictatorship while a large stock is associated to education investment and democratization in finite time.

This result sheds light on the relationship between education, development, and democratization. First, education is necessary for both development and democratization: it is the engine of economic growth and, by increasing the workers awareness, it is also responsible for the institutional change. Second, education investments might be optimal for the ruling elite, despite it might lead to more democratic institutions, as their political power gets substituted by economic returns. Third, the existence of a poverty trap is particularly interesting for it teaches that development aid leading to "small" increases in human capital might not be sufficient for a regime switch and thus fails to have permanent effects on development and institutions of the recipient country.

The next result further emphasizes the conditions under which the elite find the democratization path optimal.

Proposition 5. *The solution with education-driven institutional change is optimal for all $H_0 \in [0, H_T]$ if:*

$$p^x R - s > e^{\frac{\gamma}{1+\gamma}} (h\pi)^{-\frac{1}{\gamma}}. \tag{18}$$

This sufficient condition can easily be interpreted once one observes that it is a stronger version of the second necessary existence condition (15). It confirms the previous intuition about which factors are crucial for the decision of a Lipsetian elite to educate the population and drive the country out of autocracy. Indeed, Proposition 5 illustrates that institutional change initiated by the Lipsetian elite is a matter of having the right conditions. A large stock of resources might not trigger education policies and democratization if the education sector doesn't ensure good enough economic returns to the elite. A permanent positive shock to international resource prices might give the elite the wealth needed to invest in education and human capital accumulation, but this opportunity will not be taken if the wealth prospects at the time of institutional change are not sufficiently compelling.

Now assume that condition (18) doesn't hold but a path with democratization remains an optimality candidate (conditions (15) are met). Would the elite still find it optimal to democratize? The answer is yes, provided the initial human capital H_0 is high enough, consistently with Proposition 4, item (iii). In such a situation, democratization cannot be optimal for any initial level of human capital, and poverty traps may be the optimal outcomes for the least endowed countries. It's possible to establish the following sufficient condition result.

Proposition 6. *Assume $\delta = 0$. Then, the solution with education-driven institutional change is optimal for a high enough initial resource stock if:*

$$p^x R - s > \frac{(h\pi)^{-\frac{1}{\gamma}}}{\rho(1-\gamma)} ((\rho - \chi)(1-\gamma) e^{\frac{\gamma}{1+\gamma}} - \chi\gamma). \tag{19}$$

The proof is algebraically cumbersome as it consists in comparing the stream of value functions attainable in each institutional regime for any initial level of human capital.

The condition $\delta = 0$ is a simplification that allows us to get an explicit sufficient condition, comparable to those generated along the text. The sufficient condition (19) is weaker than condition (18). At first glance, however, the right hand side of (19) looks more complex than the corresponding side of (18) as new terms show up: the discount rate ρ and the return on education investment χ. However, one can straightforwardly check that the right hand side of (19) is increasing in ρ and converges exactly to the right hand side of (18) when ρ goes to infinity. Not surprisingly, the larger the discount rate, the lower the incentives to save and invest, and the less likely is democratization: larger discount rates make it more difficult to satisfy the sufficient condition for democratization (19). The latter also tells us that if the fundamentals of the economy are not too bad –the resource windfall, the return on education, and the elite's share parameters– and if the initial human capital is not too low, there may exist an optimal path to democratization via education.

6 Implications for development policies

In terms of aid policy, our model delivers some interesting mixed recommendations. We believe they are specially interesting because our institutional set-up, while simplistic, covers the case of many autocracies in the South. In first place, our setting implies that aid policy in support for education, even if massive, may not be always effective in fostering economic growth and/or democratization. Even tough a massive aid policy of education systems can temporarily increase human capital (when improving access to education and therefore raising the enrollment rates), it may not have a permanent positive effect either because the efficiency of local education systems is hard to improve (that's parameter h cannot be raised above a minimal level), or more seriously, because the institutional conditions are not good enough (here for example, π should be big enough). This is consistent with the view questioning the efficiency of large aid to the poorest countries (see Kraay and Raddatz, 2007, for example).

Of course, aid programs may (and should) target education efficiency, especially in the situation where the economy is poor (low windfall revenue in our theory). Our model produces a clear hierarchy in this respect: if education efficiency (our parameter h) is above a certain threshold value for education, development and democratization will turn optimal *ceteris paribus* irrespective of the initial value of capital (H_0) and even though the country is run by an autocratic regime (as one can infer from Proposition

5). Therefore our model suggests an immediate way to settle the traditional tradeoff between expanding school enrollments versus improving school quality faced by development agencies. Again this is consistent with the view expressed by many development economists from the 80s. For example, Fuller (1986) already noted that "...school quality (indicated by per pupil expenditures) has suffered most in those developing countries which have expanded enrollments rapidly in the past decade. No progress in improving school quality is evident in-the poorest developing countries since 1970. The gap in school quality between low-income and middle-income Third World countries also has widened during this period." The situation is not that improved in the current century. While primary school enrollment has increased from 59 percent to 77 percent in sub-Saharan Africa over the past decade, education experts still point at very low learning figures as reflected for example in the Africa Learning Barometer.[6] Of course, a development agency cannot have full control of school quality in developing countries as the notion of education quality itself is connected to a bulk of determinants and circumstances in these countries. It's far from a pure statistical debate (see Hanushek et al. 2007, for a quick view of this type of debate): the efficiency of the education sector in these countries does depend on various factors which are not all directly related to the se! ctor, say to the quality of the teachers or of the education infrastructures. These external factors range from those affecting the learning capacity of children (like health and nutrition, see Strauss and Thomas, 1998) to institutional and political factors, some related to the story told in our paper as a *better* education is likely to stimulate decisively political awareness, not speaking about the so-called dictatorship in education where the education system is used by the autocracies to annihilate the critical skills of the individuals (see again Chomsky, 1996) and not to prepare them for high-skilled and productive jobs. While the first set of external factors may be accounted for (within accompanying health and nutrition aid programs), the second set is much trickier as it may lead to face the opposition of the ruling autocracies.

The link between education and democratization, as exemplified in our Lipsetian theory, may lead to ask more philosophical and ethical questions about the ultimate targets of education policies: if education aid policies (ideally targeting quality and not only access to school) can fuel democratization, then aid polices can be used as an ideological tool to westernize the world. This raises a series of ethical questions that go

[6] In certain cases, policies favoring access to education have led to worsen the school quality as in the case of the implementation of free primary education in Kenya in 2003 as reported in a 2005 UNESCO report.

much beyond the scope of this paper (see Tabulawa, 2003, for a discussion). We shall only observe that if the ultimate objective is democratization (that's equal access to political and economic rights for all the individuals in the same country), then acting on autocracies via education is probably more ethical than encouraging coups and killings.

7 Conclusion

In this paper we develop a Lipsetian theory of institutional change. The key mechanism at play is the feedback effect of education policies on the awareness of workers, measuring their understanding of the political system, their political sophistication, and, more in general, their reluctance to accept a dictatorship. Human capital makes national industry more productive and is the engine of economic growth, but has a political cost in terms of the larger services/transfers that workers require to be refrained from revolting. The main result of the paper is to show that two possible regimes can arise, depending on the relative magnitude of the education incentives. The first case is that of countries where the ruling elite support a permanent dictatorship characterized by low (no) education, low growth, low level of worker's life conditions. The second case is that of countries where the elite favor long-term economic interests by investing in education and human capital accumulation, achieve a high growth path and improving life conditions; the cost of this development is an unavoidable removal of dictatorship at finite time.

Our theory is compatible with Lipset in the three essential dimensions: the positive link between human capital and institutional change, the positive link between income and institutional change and, in a more stylized fashion, the negative link between inequality and institutional change. It also gives new insights on the "resource curse" hypothesis. The –largely empirical– debate whether or not resource wealth impedes democracy is still undecided. Our analysis explains the mixed support for the resource curse. Resource wealth may promote the transition from autocratic regimes to democracies, but only if combined with other crucial ingredients, such as the quality of the education sector and of the institutional system. When these conditions are met, we also highlight that the higher the windfalls the sooner the transition. Our theory finally has some implications in terms of aid programs intended to promote development. Massive education support is a good candidate to trigger the process of sustained growth of human capital and, as a side-product, democratization. However, its effectiveness requires development programs be directed to the improvement of the education system, that has many determinants beside the quality of teaching and school infrastructure.

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A Proofs

A.1 Proof of Proposition 1

A.1.1 Item (i)

When regime 1 is permanent, from the transversality condition we must have:

$$C_0^1 = \frac{\rho - \chi(1-\gamma)}{\gamma h}\left(H_0 + \frac{h(p^x R - s)}{\chi}\right)$$

So existence requires $\rho > \chi(1-\gamma)$. More generally, the ordering between χ and ρ is crucial to discuss the nature of any potential solution. First assume that $\rho > \chi$. Then, from the expression of the stock of human capital

$$H^1(t) = \frac{\gamma h C_0^1}{\rho - \chi(1-\gamma)} e^{\frac{(\chi-\rho)t}{\gamma}} - \frac{h(p^x R - s)}{\chi},$$

we obtain that $H_\infty^1 = -\frac{h(p^x R - s)}{\chi} < 0$, which is impossible. Thus, in this case, necessarily the system reaches the frontier $E^1 = 0$ in finite time. Next, consider the alternative, $\chi(1-\gamma) < \rho < \chi$ ($\rho = \chi$ is a knife-edge situation). In this case, both consumption and human capital are varying at the constant rate $(\chi - \rho)/\gamma$. Hereafter we disregard this case because either it is not compatible with the resource constraint or, if resource imports are allowed, it implies that $X^1 \to -\infty$, which is not a reasonable feature of the model.

A.1.2 Item (ii)

From the second optimality condition 12, for the stopping time T, we have:

$$C_0^1 = (h\pi)^{-\frac{1}{\gamma}} e^{-\frac{(\chi-\rho)T}{\gamma}}.$$

Substituting the expression in (13), we obtain:

$$C^1(t) = (h\pi)^{-\frac{1}{\gamma}} e^{\frac{1}{\gamma}(\chi-\rho)(t-T)}$$
$$H^1(t) = \varphi(T) e^{\chi t} + \frac{\gamma h (h\pi)^{-\frac{1}{\gamma}}}{\rho - \chi(1-\gamma)} e^{\frac{(\chi-\rho)(t-T)}{\gamma}} - \frac{h(p^x R - s)}{\chi}$$

where $\varphi(T) \equiv \left(H_0 + \frac{h(p^x R-s)}{\chi} - \frac{\gamma h(h\pi)^{-\frac{1}{\gamma}} e^{-\frac{(\chi-\rho)T}{\gamma}}}{\rho-\chi(1-\gamma)}\right)$. The second optimal stopping condition (11) can be rewritten as:

$$\frac{\gamma(h\pi)^{-\frac{1-\gamma}{\gamma}}}{1-\gamma} + h\pi(p^x R - s) + \chi\pi H^1(T) = \rho\pi H^1(T) \tag{20}$$

where the LHS is the marginal benefit of waiting (achieving utility from consumption $C(T)$ and the advantage from increasing human capital $\pi\dot{H}^1(T)$), while the LHS is the marginal cost of waiting (delaying the scrap value). Since $\frac{\gamma(h\pi)^{-\frac{1-\gamma}{\gamma}}}{1-\gamma} + h\pi(p^x R - s) > 0$, $\chi \geq \rho$ implies that LHS>RHS for each level of human capital.

When $\chi < \rho$, the RHS increases faster than the LHS: marginal benefit is first larger and then smaller than marginal cost. Then, we can define the optimal end-point $H^1(T) = H_T$ as:

$$H_T = \frac{h}{\rho-\chi}\left(\frac{\gamma(h\pi)^{-\frac{1}{\gamma}}}{1-\gamma} + p^x R - s\right) > 0. \tag{21}$$

The second order condition (SOC) for the optimal stopping problem is satisfied iff $(\chi-\rho)\,\dot{H}^1(T) < 0$, which requires $\dot{H}^1(T) > 0$.

From the continuity of the state variable, we have:

$$H_T = \varphi(T)\,e^{\chi T} + \frac{\gamma h\,(h\pi)^{-\frac{1}{\gamma}}}{\rho-\chi(1-\gamma)} - \frac{h(p^x R - s)}{\chi} \tag{22}$$

Rearranging we obtain:

$$\varphi(T)\,e^{\chi T} = \frac{\rho h}{\rho-\chi}\left(\frac{p^x R - s}{\chi} + \frac{\gamma^2\,(h\pi)^{-\frac{1}{\gamma}}}{(1-\gamma)(\rho-\chi(1-\gamma))}\right) \tag{23}$$

Let $F(T)$ be the LHS and $G > 0$ the RHS. By the monotonicity of the path $\{H^1(t)\}_{t=0}^T$ (see the next item) and the SOC, $H_0 \leq H_T$ needs to hold. Hereafter, let's pay attention to an interior solution, i.e. $H_0 < H_T$. This is equivalent to $F(0) < G$. Moreover, $F(\infty) = -\infty$. The sign of the derivative of $F(T)$:

$$F'(T) = e^{\chi T}\left(\chi H_0 + h(p^x R - s) - h(h\pi)^{-\frac{1}{\gamma}} e^{-\frac{(\chi-\rho)T}{\gamma}}\right).$$

For the existence of an optimal interior T for democratization, it's first necessary that $F'(0) > 0$, which equivalent to:

$$H_0 > \underline{H}_0 = \frac{h(h\pi)^{-\frac{1}{\gamma}} - h(p^x R - s)}{\chi}, \tag{24}$$

and for the interval $(\underline{H}_0, H_T)$ to be non-empty, we must impose:

$$p^x R - s + \frac{(h\pi)^{-\frac{1}{\gamma}}(\chi - \rho(1-\gamma))}{\rho(1-\gamma)} > 0. \tag{25}$$

This is equivalent to the SOC.

One way to tackle (and simplify) the existence issue is to make sure that the optimality candidate exists for any initial level of the stock of human capital $H_0 \in [0, H_T]$. Otherwise, we would have to discuss and justify the relevance of some particular intervals for existence. Thus, hereafter we scrutinize the conditions under which such a candidate exists for $H_0 = 0$. Put $H_0 = 0$ in all the expressions above. Then, $F'(0) > 0$ simplifies to

$$p^x R - s > (h\pi)^{-\frac{1}{\gamma}}, \tag{26}$$

and under this condition the SOC is satisfied.

Next we define the value that maximizes $F(T)$, say $\tilde{T}$, as:

$$\tilde{T} = \frac{\gamma}{\rho - \chi} \ln\left(\frac{p^x R - s}{(h\pi)^{-\frac{1}{\gamma}}}\right), \tag{27}$$

and for the existence of (at most two) $T^* > 0$ that solve(s) (23), it must hold that $F(\tilde{T}) > G$. This condition can be restated as a condition on the parameters of the technology and preferences. Indeed, $F(\tilde{T}) > G \Leftrightarrow$

$$\frac{(\rho-\chi)^2}{\rho(\rho-\chi(1-\gamma))} e^{\frac{\gamma\chi}{\rho-\chi}} \left[\frac{p^x R - s}{(h\pi)^{-\frac{1}{\gamma}}}\right]^2 > \frac{p^x R - s}{(h\pi)^{-\frac{1}{\gamma}}} + \frac{\gamma^2\chi}{(1-\gamma)(\rho-\chi(1-\gamma))},$$

which is a simple polynomial of degree 2 in $\frac{p^x R - s}{(h\pi)^{-\frac{1}{\gamma}}}$. Now, under (26), this ratio is larger than 1. So, a sufficient condition for $F(\tilde{T}) > G$ is:

$$e^{\frac{\gamma\chi}{\rho-\chi}} > \frac{\rho[(1-\gamma)(\rho-\chi(1-\gamma)) + \gamma^2\chi]}{(1-\gamma)(\rho-\chi)^2}, \tag{28}$$

where both terms of the inequality above are greater than 1. A quick inspection of the properties of the LHS and RHS of (28), seen as functions of parameter χ, reveals that there exists a unique threshold $\underline{\chi} \in (0, \rho)$ such that (28) holds iff $\underline{\chi} < \chi$.

The reasoning above is also valid for any $H_0 > 0$. Actually, the existence of a solution for the particular value $H_0 = 0$ implies that such a solution exists for any $H_0 > 0$. In general, the optimal stopping time can be expressed as a function H_0: $T^* = T(H_0)$ that satisfies, by differentiating (23):

$$\frac{\partial T}{\partial H_0} = -\frac{1}{\chi\varphi\left(T(H_0)\right) + \varphi'\left(T(H_0)\right)}.$$

Given that we want $\frac{\partial T}{\partial H_0} < 0$ (uniqueness of the optimal trajectory), only the solution corresponding the increasing part of $F(T)$ is relevant, i.e. one has $F'(T) > 0$, which is equivalent to:

$$p^x R - s > e^{\frac{(\rho-\chi)T}{\gamma}}(h\pi)^{-\frac{1}{\gamma}}. \tag{29}$$

Comparative statics on T^* that solves (23), given that this equation can be rewritten as $F(T, R, \pi) = G(R, \pi)$. By the implicit function theorem, we have:

$$\frac{\partial T}{\partial R} = \frac{\frac{\partial G}{\partial R} - \frac{\partial F}{\partial R}}{\frac{\partial F}{\partial T}} \text{ and } \frac{\partial T}{\partial \pi} = \frac{\frac{\partial G}{\partial \pi} - \frac{\partial F}{\partial \pi}}{\frac{\partial F}{\partial T}},$$

given that $\frac{\partial G}{\partial R} - \frac{\partial F}{\partial R} = \frac{hp^x(\rho-1)}{\chi} < 0$ and $\frac{\partial G}{\partial \pi} - \frac{\partial F}{\partial \pi} = -\frac{h^2(h\pi)^{-\frac{1}{\gamma}}}{\rho-\chi(1-\gamma)}(\frac{\rho\gamma}{(\rho-\chi)(1-\gamma)} + e^{\frac{(\rho-\chi)T}{\gamma}} < 0$. Finally, since the solution satisfies $\frac{\partial F}{\partial T} > 0$ (because we want $T'(H_0) < 0$ for all $H_0 \geq 0$), we can conclude that T^* is decreasing w.r.t. both R and π.

A.1.3 Monotonicity of trajectories

The value function at any $H^1(t_i) = H_i$ taken on the optimal path is given by:

$$V^1(H_i) = e^{-\rho\theta(H_i)}\left(\frac{\gamma(h\pi)^{-\frac{(1-\gamma)}{\gamma}}}{(1-\gamma)(\rho-\chi(1-\gamma))}\left[e^{\frac{(\rho-\chi(1-\gamma))}{\gamma}\theta(H_i)} - 1\right] + \pi H_T\right)$$

with $\theta(H_i) = T(H_i) - t_i$, the optimal time-to-go before stopping, which doesn't depend on t_i.

If there exists an optimal trajectory of type 1 from some H_0 with $H(t)$ non monotone, it must be true that H is decreasing first, then increasing. This implies that there exists

(t_1, t_2), with $t_1 < t_2$, such that: $H^1(t_1) = H^1(t_2)$. Thus, we have $\theta(H_1) = \theta(H_2)$: The time that elapses between t_1 and $T(H_1)$ must be the same as the one between t_2 and $T(H_2)$. This yields a contradiction because the optimal trajectory is uniquely defined (H_T is invariant) and (initial) consumptions at t_1 and t_2 necessarily differ.

Finally note that the solution with positive education and a revolution in finite time yields the following present value to the elite:

$$V^1(H_0) = e^{-\rho T(H_0)} \left(\frac{\gamma (h\pi)^{-\frac{(1-\gamma)}{\gamma}}}{(1-\gamma)(\rho - \chi(1-\gamma))} (e^{\frac{(\rho-\chi(1-\gamma))}{\gamma} T(H_0)} - 1) + \pi H_T \right) \tag{30}$$

This completes the proof of Proposition 1.

A.2 Proof of Proposition 2

A.2.1 Item (i)

If regime 2 is permanent, then from the transversality condition $L = 0$, and the solution reduces to (using the superscript 2):

$$\begin{aligned}
&H^2(t) = H_0 e^{-\delta t} \\
&C^2(t) = p^x R - s + \Omega H^2(t) \\
&\lambda^2(t) = -e^{(\rho+\delta)t} \textstyle\int_0^t \Omega C(u)^{-\gamma} e^{-(\rho+\delta)u} du (< 0).
\end{aligned}$$

The value function is given by:

$$V^2(H_0) = \int_0^\infty \frac{1}{1-\gamma} \left(p^x R - s + \Omega H_0 e^{-\delta t} \right)^{1-\gamma} e^{-\rho t} dt. \tag{31}$$

A.2.2 Item (ii)

Consider a trajectory $(\{H^1\}, \{C^1\})$ that reaches the locus $E = 0$ at date t_1 for some stock $H^1(t_1) = \tilde{H}$ and consumption $C^1(t_1) = \tilde{C}$. From the dynamical system, both H^1 and C^1 are all decreasing w.r.t. time. The approach is to consider a solution with permanent $E = 0$ as a limit case of the solution with a regime change from $E > 0$ to $E = 0$. Let's work with the general solution obtained by combining regimes 1 and 2.

For the time being, let t_1 be given. Recall that the general solution in each regime is:

$$\begin{aligned} C^1(t) &= C_0 e^{\frac{(\chi-\rho)t}{\gamma}} \\ H^1(t) &= \left(H_0 + \frac{h(p^xR-s)}{\chi} - \frac{\gamma h C_0}{\rho-\chi(1-\gamma)}\right) e^{\chi t} + \frac{\gamma h C_0}{\rho-\chi(1-\gamma)} e^{\frac{(\chi-\rho)t}{\gamma}} - \frac{h(p^xR-s)}{\chi} \end{aligned}$$

and,

$$\begin{aligned} H^2(t) &= \tilde{H} e^{-\delta(t-t_1)} \\ C^2(t) &= p^xR - s + \Omega H^2(t) \end{aligned}$$

From the continuity of consumption at t_1, we obtain: $C_0 = \left(p^xR - s + \Omega\tilde{H}\right) e^{-\frac{(\chi-\rho)}{\gamma}t_1}$ and $C^1(t) = \left(p^xR - s + \Omega\tilde{H}\right) e^{\frac{(\chi-\rho)(t-t_1)}{\gamma}}$.

From the continuity of the state variable at t_1, $\tilde{H}$ can be expresses as a function of t_1: $\tilde{H} = \zeta(t_1)$ with:

$$\zeta(t_1) = \frac{(\rho-\chi(1-\gamma))\left[\left(H_0 + \frac{h(p^xR-s)}{\chi} - \frac{\gamma h(p^xR-s)}{\rho-\chi(1-\gamma)} e^{-\frac{\chi-\rho}{\gamma}t_1}\right) e^{\chi t_1} - \frac{h(\rho-\chi)(p^xR-s)}{\chi(\rho-\chi(1-\gamma))}\right]}{\rho-\chi(1-\gamma) + \gamma\Omega h\left(e^{\frac{(\rho-\chi(1-\gamma))}{\gamma}t_1} - 1\right)}$$

So the value corresponding to this trajectory can be written as:

$$\begin{aligned} V(t_1) \;&=\; \tfrac{1}{1-\gamma}\Big[\int_0^{t_1} \left(p^xR - s + \Omega\zeta(t_1)\right)^{1-\gamma} e^{\frac{(1-\gamma)(\chi-\rho)}{\gamma}(t-t_1)} e^{-\rho t} dt + \\ &+ \int_{t_1}^{\infty} \left(p^xR - s + \Omega\zeta(t_1)e^{-\delta(t-t_1)}\right)^{1-\gamma} e^{-\rho t} dt\Big] \end{aligned}$$

Taking the derivative w.r.t t_1 yields:

$$\begin{aligned} \tfrac{\partial V}{\partial t_1} \;&=\; \tfrac{1}{1-\gamma}\left(p^xR - s + \Omega\zeta(t_1)\right)^{1-\gamma} e^{-\rho t_1} + \\ &+ \tfrac{1}{1-\gamma}\int_0^{t_1} e^{-\frac{(\rho-\chi(1-\gamma)}{\gamma}t} e^{\frac{(1-\gamma)(\rho-\chi)}{\gamma}t_1} \left(p^xR - s + \Omega\zeta(t_1)\right)^{-\gamma}\Big[(1-\gamma)\Omega\zeta'(t_1) + \\ &+ \tfrac{(1-\gamma)(\rho-\chi)}{\gamma}\left(p^xR - s + \Omega\zeta(t_1)\right)\Big] - \\ &+ \tfrac{1}{1-\gamma}\left(p^xR - s + \Omega\zeta(t_1)\right)^{1-\gamma} e^{-\rho t_1} + \\ &+ \int_{t_1}^{\infty} \Omega(\zeta'(t_1) + \delta\zeta(t_1)) e^{-\delta(t-t_1)} \left(p^xR - s + \Omega\zeta(t_1)e^{-\delta(t-t_1)}\right)^{-\gamma} e^{-\rho t} dt \end{aligned}$$

Taking the limit when $t_1 \to 0$, we obtain:

$$\lim_{t_1\to 0} \frac{\partial V}{\partial t_1} = \int_0^{\infty} \Omega(\zeta'(0) + \delta\zeta(0)) \left(p^xR - s + \Omega\zeta(t_1)e^{-\delta t}\right)^{-\gamma} e^{-(\delta+\rho)t} dt,$$

and it's clear that the sign of the limit is determined by the sign of the expression

$\zeta'(0)+\delta\zeta(0)$. Direct computations yield: $\zeta(0) = H_0$ and the derivative of $\zeta(t_1)$ evaluated at $t_1 = 0$ is given by: $\zeta'(0) = -\delta H_0$. Thus, we obtain $\zeta'(0) + \delta\zeta(0) = 0$. From multi-stage optimal control theory (interpreting the change from $E > 0$ to $E = 0$ as a regime switching problem), we know that a necessary condition for an immediate switch $t_1 = 0$ is $\lim_{t_1 \to 0} \frac{\partial V}{\partial t_1} \leq 0$ (see Amit [6], Theorem 1). Thus trajectories of the 1-2 type are always dominated by the ones associated with permanent $E = 0$.

The last eventuality is a candidate with a regime change from 2 to 1. Suppose the economy starts in regime $E = 0$ and enters the region with positive education at $t_1 < \infty$. Then, there are two options. Either the economy stays in region 1 till the institutional change. This would implies the crossing of the locus $\dot{H} = 0$ in finite time. But this is excluded by the argument provided in Appendix A.1 because the trajectory $\{H^1(t)\}$ must be monotonous. Or, the economy stays for a while in regime 1 before going back in regime 2. But this is not optimal if one refers to the reasoning developed just above. The economy prefers to directly settle on the locus $E = 0$ rather than to start in regime 1 before a switch, in finite time, to regime 2.

This completes the proof of Proposition 2.

A.3 Proof of Proposition 3

At solution 1 (democratization), the index of inequalities is:

$$I^1(t) = \frac{(s\chi - \phi h(p^x R - s))}{\chi}(h\pi)^{\frac{1}{\gamma}} e^{\frac{(\rho-\chi)(t-T)}{\gamma}} + \phi\varphi(T)(h\pi)^{\frac{1}{\gamma}} e^{\frac{(\rho-\chi(1-\gamma))t}{\gamma}} e^{\frac{(\chi-\rho)T}{\gamma}} + \frac{\phi\gamma h}{\rho - \chi(1-\gamma)}$$

Take the derivative w.r.t time:

$$\dot{I}^1(t) = \frac{(h\pi)^{\frac{1}{\gamma}}}{\gamma} e^{\frac{(\rho-\chi)(t-T)}{\gamma}} \left[(\rho - \chi)(s\chi - \phi h(p^x R - s)) + \phi(\rho - \chi(1-\gamma)\varphi(T)e^{\chi T}\right].$$

The sign of the derivative is given by the sign of the term, denoted Ψ, between squared brackets. Evaluating this coefficient at $t = 0$, we obtain:

$$\Psi = (\rho - \chi)s + \phi(\rho - \chi(1-\gamma))H_0 + \gamma\phi h \left[p^x R - s - (h\pi)^{-\frac{1}{\gamma}} e^{\frac{(\rho-\chi)T}{\gamma}}\right],$$

which is positive according to (29). Thus $\dot{I}^1(t) > 0$ for all $t \in [0, T]$.

At solution 2 (permanent dictatorship), the index is simply given by:

$$I^2(t) = \frac{s + \phi H^2(t)}{p^x R - s + \Omega H^2(t)},$$

with derivative:

$$\dot{I}^2(t) = \frac{p^x \dot{H}}{(p^x R - s + \Omega H^2(t)^2} \left[\phi R - \frac{1-\alpha}{\alpha} \left(\frac{\alpha A}{p^x} \right)^{\frac{1}{1-\alpha}} s \right].$$

Thus,

$$R > \frac{(1-\alpha)s}{\phi \alpha} \left(\frac{\alpha A}{p^x} \right)^{\frac{1}{1-\alpha}}$$

is sufficient to conclude that $\dot{I}^2(t) < 0$ for all $t \in [0, \infty)$.

A.4 Proof of Proposition 4

The proof of the first item of Proposition 4 works by contradiction and relies on a time consistency requirement for optimal trajectories.

Let's assume that $V^1(H_T) < V^2(H_T)$. Regarding the ordering between the value functions at the other boundary, two case are possible.

Case 1. Let $V^1(0) < V^2(0)$. If the curves $V^1(H_0)$ and $V^2(H_0)$ intersect then the number of intersections must be even. For instance, consider that there are two intersections $\hat{H}$ and $\tilde{H}$, with $0 < \tilde{H} < \hat{H} < H_T$. By construction, we have $V^1(H_0) > V^2(H_0)$ for all $H_0 \in (\tilde{H}, \hat{H})$, $V^1(H_0) < V^2(H_0)$ for all $H_0 \in [0, \tilde{H}) \cup (\hat{H}, H_T]$, $V^1(\tilde{H}) = V^2(\tilde{H})$ and $V^1(\hat{H}) = V^2(\hat{H})$.

At $H_0 = \hat{H}$, there exist two optima, i.e. the elite are indifferent between following path 1 (with positive education) or path 2 (no education). If the economy settles on path 1, then from what has been establish in Appendix A.1, human capital increases. But by construction again, for any H varying in $(\hat{H}, H_T]$, $V^1(H) < V^2(H)$: The elite would prefer to follow path 2 rather than path 1, which implies that the solution considered is not time consistent. This yields a contradiction. If the elite chooses path 2, then from Appendix A.2, human capital decreases monotonically. But $V^1(H) > V^2(H)$ for all $H \in (\tilde{H}, \hat{H})$. There is a non degenerate period of time during which the elite would prefer in fact being in regime 1. Based on the time consistency requirement, we obtain a contradiction.

Case 2. Let $V^1(0) > V^2(0)$. This implies that the number of intersections between

$V^1(H_0)$ and $V^2(H_0)$ (if any) is odd. Let's work with a unique intersection at $\check{H}$ that is such that $V^1(H_0) > V^2(H_0)$ for all $H_0 \in [0, \check{H})$, $V^1(H_0) < V^2(H_0)$ for all $H_0 \in (\check{H}, H_T]$ and $V^1(\check{H}) = V^2(\check{H})$.

At $H_0 = \check{H}$, there is multiplicity of optima. Either, the elite may adopt the regime with positive education and human capital increases. Again by construction, we have $V^1(H) < V^2(H)$ for all $H \in (\check{H}, H_T]$, which yields a contradiction. Or, they may choose not to invest in education, in which case H decreases and a contradiction arises too.

Proofs of the remaining items are left to the reader since they exactly follow the same line. In particular the reasoning of the second item is symmetric when one works with $V^1(0) > V^2(0)$. As for the third item, assuming that $V^1(0) \leq V^2(0)$ and $V^1(H_T) \geq V^2(H_T)$ (with one strict inequality), it's easy to show that there exists a unique intersection between the two value functions, for a critical initial stock of human capital $\bar{H}$ such that $V^1(H_0) \gtreqless V^2(H_0) \Leftrightarrow H_0 \gtreqless \bar{H}$.

A.5 Proof of Proposition 5

For the sake of exposition, let $T(0)$ be denoted by T. Then, $V^1(0) > V^2(0)$ if and only if:

$$e^{-\rho T}\left(\frac{\gamma(h\pi)^{-\frac{(1-\gamma)}{\gamma}}}{(1-\gamma)(\rho-\chi(1-\gamma))}\left[e^{\frac{(\rho-\chi(1-\gamma))}{\gamma}T}-1\right]+\pi H_T\right) > \frac{(p^xR-s)^{1-\gamma}}{\rho(1-\gamma)};$$

which, by the definition of H_T in (21) and by (23), yields:

$$\frac{(p^xR-s)^{1-\gamma}}{\rho(1-\gamma)} < \frac{\gamma(h\pi)^{-\frac{(1-\gamma)}{\gamma}}}{\rho(1-\gamma)}e^{\frac{(\rho-\chi)(1-\gamma)}{\gamma}T} + \frac{h\pi(p^xR-s)}{\rho}e^{(\chi-\rho)T}. \tag{32}$$

Denote the RHS of (32) by $J(T)$. It follows that $J(0) = \frac{h\pi}{\rho}\left(\frac{\gamma}{1-\gamma}(h\pi)^{-\frac{1}{\gamma}} + p^xR - s\right) > 0$, $\lim_{T\to\infty} J(T) = \infty$ and:

$$J'(T) = \frac{h\pi(\rho-\chi)}{\rho}e^{(\chi-\rho)T}\left[(h\pi)^{-\frac{1}{\gamma}}e^{\frac{(\rho-\chi)T}{\gamma}} - (p^xR-s)\right].$$

We observe that $J'(T) \leqq 0 \Leftrightarrow T \leqq \tilde{T}$, where $\tilde{T}$ has been defined in (27). Thus, imposing $J(\tilde{T}) > \frac{(p^xR-s)^{1-\gamma}}{\rho(1-\gamma)}$, which is equivalent to:

$$p^xR - s > e^{\frac{\gamma}{1+\gamma}}(h\pi)^{-\frac{1}{\gamma}}, \tag{33}$$

is sufficient to conclude that $V^1(0) > V^2(0)$.

Moreover, whatever the regime, the value functions are strictly increasing in H_0, it's clear that a sufficient condition for having $V^2(H_0) > V^1(H_0)$ for all $H_0 \in [0, H_T]$ is $V^2(0) \geq V^1(H_T)$, which is equivalent to:

$$\frac{(p^x R - s)^{1-\gamma}}{\rho(1-\gamma)} \geq \pi H_T \Leftrightarrow \frac{(p^x R - s)^{1-\gamma}}{\rho(1-\gamma)} \geq \frac{\gamma (h\pi)^{-\frac{(1-\gamma)}{\gamma}}}{(\rho-\chi)(1-\gamma)} + \frac{\pi h (p^x R - s)}{\rho - \chi}. \tag{34}$$

A.6 Proof of Proposition 6

For any $H_0 \in [0, H_T]$, regime 1's value function (30) can be rewritten as:

$$V^1(H_0) = \frac{\pi e^{-(\rho-\chi)T(H_0)}}{\rho} \left(\frac{h\gamma (h\pi)^{-\frac{1}{\gamma}}}{1-\gamma} e^{\frac{(\rho-\chi)}{\gamma} T(H_0)} + \chi H_0 + h(p^x R - s) \right),$$

denote this function by $W^1(T(H_0))$, with $W^{1\prime} < 0$ and define:

$$\tilde{T}(H_0) = \frac{\gamma}{\rho - \chi} \ln \left(\frac{\chi H_0 + h(p^x R - s)}{h(h\pi)^{-\frac{1}{\gamma}}} \right),$$

this generalizes (27) for any $H_0 \in [0, H_T]$. From Appendix A.1, we know that $T(H_0) \in (0, \tilde{T}(H_0)) \Leftrightarrow W^1(T(H_0)) > W^1(\tilde{T}(H_0))$ for all H_0, with:

$$W^1(\tilde{T}(H_0)) = \frac{\pi e^{-\gamma}}{\rho(1-\gamma) h (h\pi)^{-\frac{1}{\gamma}}} (\chi H_0 + h(p^x R - s))^2,$$

denote this lower bound as $\underline{W}^1(H_0)$. We have $\underline{W}^1(0) = \frac{(h\pi)^{\frac{1+\gamma}{\gamma}} e^{-\gamma}}{\rho(1-\gamma)} (p^x R - s)^2$ and $\underline{W}^{1\prime}(H_0), \underline{W}^{1\prime\prime}(H_0) > 0$.

Now, note that for any $H_0 \in [0, H_T]$, regime 2's value function (31) is bounded from above by a function $\overline{W}^2(H_0)$ defined as follows:

$$\overline{W}^2(H_0) = \frac{(p^x R - s + \Omega H_0)^{1-\gamma}}{\rho(1-\gamma)},$$

with $\overline{W}^2(0) = \frac{(p^x R - s)^{1-\gamma}}{\rho(1-\gamma)}$, $\overline{W}^{2\prime}(H_0) > 0$ and $\overline{W}^{2\prime\prime}(H_0) < 0$.

Suppose that condition (18) of Proposition 5, item *(i)*, doesn't hold. This is equivalent to $\overline{W}^2(0) \geq \underline{W}^1(0)$. Given that $\lim_{H_0 \to \infty} \frac{\overline{W}^2(H_0)}{\underline{W}^1(H_0)} = 0$, we can conclude that there exists a critical level of human capital $\underline{H}_0 \in (0, H_T)$ such that for all $H_0 \geq \underline{H}_0$,

$\underline{W}^1(H_0) \geq \overline{W}^2(H_0)$ if and only if:

$$\overline{W}^2(H_T) < \underline{W}^1(H_T),$$

with H_T defined by (21).

This condition implies that $V^1(H_0) > V^2(H_0)$ for all $H_0 \geq \underline{H}_0$, i.e., the path with democratization is optimal for a high enough resource stock. This condition is not easily interpretable since it depends on an endogenous variable H_T. This condition can however be rewritten in terms of the parameters when assuming that $\delta = 0$. In this case, it reduces to:

$$p^x R - s > \frac{(h\pi)^{-\frac{1}{\gamma}}}{\rho(1-\gamma)}((\rho - \chi)(1-\gamma)e^{\frac{\gamma}{1+\gamma}} - \chi\gamma).$$