Environmental Policy and Inequality: A Matter of Life and Death

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July 7, 2015

Abstract

This paper analyzes the economic implications of an environmental policy when we take into account the life expectancy of heterogeneous agents. In a framework where everyone suffers from pollution, but health status depends also on individual human capital, we find that the economy may be stuck in a trap where inequalities persistently grow, when the initial level of pollution is too high. Therefore, we study whether a tax on pollution associated with an investment in pollution abatement can be used to reduce inequalities and to improve endogenous growth. We obtain that a tighter environmental policy may allow the economy to escape the inequality trap and hence to converge to a long-term equilibrium without inequality, while it enhances the long-term growth rate. However, if inequalities or pollution are initially too high, such a result does not hold for reasonable tax rates.

JEL Classification: I15; O44; Q58

Keywords: Environmental Policy, Endogenous Growth, Human Capital, Inequality, Longevity.

1 Introduction

While the average life expectancy has globally increased during the last decades, health inequalities have not only persisted but widened sharply. For example, Singh and Siahpush (2006) highlight that the absolute difference in life expectancy between less-deprived groups and more deprived groups has risen by over 60% between 1980 and 2000 in the United States. Such disparities in terms of life expectancy represent a worldwide phenomenon.

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OECD (2013) reports that on average the gap in expected years of life between men with the highest level and the lowest level of education was of 7.8 years in 2010.[1] In Europe, the excess risk of dying among middle-aged adults in the lowest socioeconomic groups ranges from 25% to 150% (Mackenbach , 2006). The extent of this problem has crucial impacts on our societies, not only because health inequalities are unfair and costly in terms of wellbeing, but also because it has important economic consequences, through increased health and social costs, lowered productivity, discouraged investments in education and savings, etc.. Therefore, addressing health disparities has become a major political issue and many governments explicitly aimed to eliminate such inequalities, so far without success (see e.g. reports of the U.S. Department of Health and Human Services , 2000 or of the U.K. Department of Health , 2003).

In this paper, we aim to study if an environmental policy can represent a useful tool for removing existing health inequalities. The reason why we are interested in the role of the environment in this issue is twofold. First, regarding determinants of life expectancy, there is considerable evidence that pollution has a positive and significant effect on mortality (see e.g. Bell and Davis , 2001 ; Pope et al. , 2002 ; Bell et al. , 2004 or Evans and Smith , 2005). In particular, air pollution was found to be responsible for around 7 million of premature deaths in 2012, representing 1 in 8 of total global deaths (WHO , 2014).[2] At an aggregate level, Pimentel et al. (1998) even show that 40% of the world deaths each year can be attributed to direct and indirect effects of environmental degradation.

Second, a key characteristic of the relationship between the environment and health seems to be the unequal repartition of the health effects of pollution across population. In this sense, Stiglitz argues that "environmental degradation is everyone's problem but it is especially a problem for the poor, who are less able to respond effectively".[3] This observation is broadly supported by empirical studies, which provide evidence of an increased susceptibility to mortality from pollution of disadvantaged populations in terms of education and income (see e.g. Cifuentes et al. , 1999 ; Health Effects Institute , 2000 ; Pope et al. , 2002 ; O'Neill et al , 2003 ; Laurent et al. , 2007 or Cakmak et al. , 2011). For the United States, Zeka *et al.* (2006) reveal that the mortality risk associated with particulate matter PM_{10} for low educated individuals is more than twice the size of the risk for individuals with high education.[4]

Among the socioeconomic indicators, education is found to have the strongest effect

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[1]More precisely, this value corresponds to the average expected years of life remaining at age 30 among 14 OECD countries.

[2]Specifically, air pollution plays an important role in the development of respiratory and heart diseases (asthma, cancer, stroke...) which can be fatal.

[3]See his Resources 2020 lecture given in October 2012, http://www.rff.org/resources2020.

[4]This study deals with US population in twenty cities between 1989 and 2000. Low education corresponds to less than 8 years of schooling, while high education refers to 13 years or more.

on mortality, and to persist after controlling for other determinants such as income and employment (see among others Elo and Preston , 1996 ; Lleras-Muney , 2005 ; Cutler and Lleras-Muney , 2010 or Miech et al , 2011). For example, in 2009, the death rate of individuals with low education (less than high school diploma) was 2.7 times higher than for those with high education (with some college degree) in the United States.[5] This relationship appears to be due to several effects: more educated individuals are more likely to live and work in better socio-economic conditions, but also to enjoy better information leading to healthier behaviors and to have a better access to healthcare (see Kenkel , 1991 or Laurent et al. , 2007). Cutler and Lleras-Muney (2010) find that the education gradient, i.e. differences in health behaviors by education, is explained at 30% by income, health insurance and family background, at 30% by knowledge and cognitive ability and at 10% by social networks.

Therefore, empirical evidence suggests that life expectancy depends on pollution but also on human capital, which is unequally distributed across population. The resulting disparities in health effects of pollution observed among households, then raise the question of the role that could play an environmental policy in reducing inequalities in the economy. That is why we attempt to address the issue of health inequalities with such a policy tool in this paper. More precisely, in a framework where life expectancy is endogenously determined by the level of pollution and the individual human capital, we aim to analyze the implications of an environmental tax whose revenue is used for pollution abatement on inequality and growth.

So far, from a theoretical point of view, there has been an increasing interest for life expectancy and its interaction with human capital and/or pollution but very little consideration has been given to health inequalities. Indeed, the positive effect of human capital on life expectancy has been taken into account in few contributions as Blackburn and Cipriani (2002), Castello-Climent and Domenech (2008) or Mariani et al. (2010), which identify the risk of an underdevelopment trap where education and longevity are low. In the same way, the effect of pollution on health through the mortality channel has been studied in several papers, without consensus on the effect of an environmental policy on the economy and in particular on growth (see e.g. Pautrel , 2008 ; Jouvet et al. , 2010 ; Mariani et al. , 2010 ; Palivos and Varvarigos , 2010 ; Varvarigos , 2011 ; Varvarigos and Zakaria , 2013b or Raffin and Seegmuller , 2014).[6] However, despite the widening health disparities, none of the cited papers consider the uneven distribution of health. A notable exception

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[5]See the Supplemental tables of the report "Deaths: Final Data for 2009" on the National Center for Health Statistics website.

[6]Other contributions have also considered the morbidity aspect by considering the negative effect of pollution on productivity or on human capital accumulation (van Ewijk and van Wijnbergen , 1995 ; Aloi and Tournemaine , 2011 or Raffin , 2012).

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is Castello-Climent and Domenech (2008), who are interested in the relationship between inequality and a longevity index determined by parents' human capital. Focusing on a form of human capital depending only on an investment in time, they find that the presence of a trap, in which poor individuals have high mortality rate, is possible. Here, we extent this analysis in order to take into account a more complete relationship between health and inequalities. For that, we consider additional determinants of the stock of human capital, i.e. the intergenerational transmission and the quality of the school system (average human capital of society) as in Tamura (1991) or de la Croix and Doepke (2003), and we take into account the effects of pollution on health. Moreover, in this framework, we study the role of an environmental policy on inequalities and endogenous growth.

More closely related to our paper, Aloi and Tournemaine (2013) and Schaefer (2014) take into account the relationship between health, pollution and inequalities. In their models, households are heterogeneous in terms of human capital and suffer from different health effects of pollution. On one hand, Aloi and Tournemaine (2013) formalize a model where pollution has a direct effect on human capital accumulation and find that a tighter environmental policy always reduces income inequality, as lower-skilled are assumed to be more affected by pollution, and it can also improve growth, if the tax is not too high. On the other hand, in a framework where the health effects of pollution and human capital go through child mortality, Schaefer (2014) obtains that pollution impose an increase in inequalities and a decline of growth, through the drop in the willingness of parents to invest in education. Here, we first depart from these contributions by looking at a different health mechanism, i.e. adult mortality. Moreover, in accordance with empirical evidence mentioned above, we endogenize disparities in the effect of pollution on health, in the sense that in our model the vulnerability of an agent to pollution depends on her level of human capital and hence evolves with it.

More precisely, we formalize an overlapping generations model, where agents can live up to three periods depending on their survival probability when old. Their longevity is endogenously determined by their human capital and the level of pollution, in accordance with empirical evidence. Pollution is represented as a flow due to aggregate production, while human capital is the source of endogenous growth but also of the heterogeneity among households.

We obtain that there may exist multiple balanced growth paths. There is always a long-term equilibrium without inequality, while one or several long-term equilibria with inequalities may also occur. Through a numerical illustration, we draw two cases. First, the balanced growth path without inequality is the only one but is a saddle point, so that it delimits a huge inequality trap. Second, this long-term state without inequality is stable and coexists with a long-term equilibrium with inequalities delimiting an inequality trap

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with a smaller size than the first case. Therefore, we obtain that a trap where inequalities persistently grow always exist, but its size depends on the parameters of the model. Human capital is accumulated through intergenerational transmission, investment in education done by altruistic parents and the educational system (represented by the average human capital). The two former elements perpetuate inequalities, whereas the latter favors human capital convergence. As we consider that longevity depends on pollution and on the individual's human capital, it follows that preferences for the future and the return on investment in education evolve according to these variables. When pollution or inequalities are high in the economy, lower-skilled agents die sooner, thus they benefit only for a short period of time from their investment, and they have a lower return to educate their children. It follows that the gap in human capital increases for the future generation.

When initial levels of inequalities and/or pollution are too large, the economy is stuck in the trap where disparities worsen across time, but when these levels are sufficiently low, the economy can converge in the long run to an equilibrium where inequalities vanish among households. We find also that inequalities in terms of human capital and hence of health have a cost in terms of growth and development in addition to the human cost, as the long-term growth is always lower when there are inequalities. Therefore, the levels of inequalities and pollution are crucial to determine the long-term situation if the economy. Moreover, the fact that if the pollution intensity is high, the economy is likely to be stuck in an inequality trap raises questions about the possible redistributive power of an environmental policy and about its effect on growth.

We analyze if an environmental policy, that reduces pollution, can be sufficient to improve growth and to reduce or eliminate disparities in the economy. We emphasize that a tighter tax on pollution associated with an investment in environmental protection reduces the size of the inequality trap and thus can allow the economy to escape from the trap. It comes from the fact that lower-skilled households are more vulnerable to the negative health effects of pollution than higher-skilled agents. Therefore, the improvement in environmental quality increases more the return on investment in education of lower-skilled than the one of higher-skilled parents. However, a tighter tax on pollution may be insufficient to get out of the trap, especially for moderate tax rate. If the level of inequalities and/or pollution are too high, the level of the tax on pollution required to overcome existent disparities may be very high. Consequently, such environmental policy is an efficient tool to reduce inequalities in the short run and remove them entirely in the long-run but the government should implement it as soon as possible, before the gap among agents is too wide. Moreover, we show that a tighter environmental policy enhances the long-term growth rate of the economy, through the positive effect of the decrease in pollution on life expectancy and the resulting increase in preferences for education, which enhances human

capital accumulation.

The paper is organized as follows. In Section 2, we set up the theoretical model. Section 3 focus on the long-term equilibria of the economy. The implications of the environmental policy on the dynamics and growth are examined in Section 4. In Sections 3 and 4, we provide analytical results followed by a numerical illustration. Finally, Section 5 concludes and technical details are relegated to an Appendix.

2 The model

Consider an overlapping generations economy, with discrete time indexed by $t = 0, 1, 2,$ $..., +\infty$. Households may live three periods, childhood, adulthood, and old age depending on a longevity index. At each date t, a new generation of N heterogeneous agents is born. We assume no population growth, so we normalize the size of the population (N) to unity. Individuals are indexed by $i = u, s$, corresponding to the two groups of workers in the economy, lower-skilled (u) and higher-skilled (s), of size ξ and $1 - \xi$ respectively.[7] The two groups of agents are characterized by an inequality in initial endowment in terms of human capital, so that agents born in $t - 1$ differ only in the human capital level of their parents $(h_{t-1}^u < h_{t-1}^s)$.

2.1 Consumer's behavior

Individual of type i born in $t-1$ cares abouts her adult consumption level c_t^i, her old-age consumption level d_{t+1}^i and about the future level of human capital of her child through paternalistic altruism h_{t+1}^i. Preferences are represented by the following utility function:

$$\ln(c_t^i) + \pi_t^i \left[\beta \ln(d_{t+1}^i) + \gamma \ln(h_{t+1}^i) \right] \tag{1}$$

with γ and $\beta > 0$.

The weight π_t^i represents the agent's longevity or her survival probability in old age.[8] A higher life expectancy enhances the welfare obtained from consuming when old, but also from the future human capital of her child. Therefore, parents value more the future, for them and for their children, when they live longer. Reversely, parents with shorter life expectancy have shorter time horizons and hence put less weight on investment whose return is future (including the future human capital of their children), not necessarily

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[7]The reason why we refer to lower-skilled individuals as u and to higher-skilled individuals as s is that the first category is relatively unskilled while the second is relatively skilled.

[8]Since individual i born in $t-1$ lives $2 + \pi_t^i$, we interchangeably use the terms "life expectancy", "longevity" and "survival probability" in the paper. We will also refer to it as health, although it is just one measure of health among others.

because they are less altruistic but because they simply assess differently what is good for their children.[9] Note that, this assumption enables the longevity to affect positively investments in education, in accordance with empirical evidence (see *e.g.* Jayachandran & Lleras-Muney , 2009 or Hansen , 2013).

Longevity is an index of health status assumed to depend on individual's human capital h_t^i and pollution P_t in accordance with empirical evidence. As we mentioned in the Introduction, the effect of human capital on health has been well established in empirical studies and is explained by the fact that higher human capital mostly involves better living and working conditions, better access to healthcare and better information about health problems and prevention (see e.g. Kenkel , 1991 or Lleras-Muney , 2005). Whereas, in the same way, there is considerable evidence that pollution has a positive and significant effect on mortality, whether it goes through air, water, soil *etc.* (see e.g. Pimentel et al. , 1998; Bell and Davis , 2001; Pope et al. , 2002 ; Bell et al. , 2004 or Evans and Smith , 2005). For the sake of simplicity, we assume a functional form for the life expectancy index, which is in line with the form adopted by Blackburn and Cipriani (2002), Chakraborty (2004), Castello-Climent and Domenech (2008) or Raffin and Seegmuller (2014):

Assumption 1

$$\pi_t^i = \pi\left(\frac{h_t^i}{P_t}\right) = \frac{\sigma h_t^i / P_t}{1 + h_t^i / P_t} \tag{2}$$

with $\sigma \in (0,1]$, *the upper bound of longevity. Thus,* $\pi_t \in [0,1]$, $\pi'(h_t^i/P_t) > 0$ *and* $\pi''(h_t^i/P_t) < 0$.

During childhood, individuals are reared by her parents and do not make any decisions. When adult, they supply inelastically one unit of labor remunerated at the wage w_t per unit of human capital. They allocate this income to consumption c_t^i, savings s_t^i and education of their children.

We assume that the education of a child requires that her parent invests $e_t^i \bar{h}_t$ units of human capital, such that schooling time e_t^i corresponds to an opportunity cost for the parent of $e_t^i \bar{h}_t w_t$ (as she does not use this efficient labor in production). Therefore, the total cost of education is the same for all types of agents and is relatively more expensive for poor parents.[10] When old, agents only consume. In line with Yaari (1965), Blanchard (1985) or Chakraborty (2004), we assume a perfect annuity market to abstract from the risk associated with uncertain lifetimes. Therefore, households deposit their savings to a

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[9]Alternatively, Mariani et al. (2010) explain that parents will be relatively more affected by the success or the failure of their children if they live long enough to witness it. The same formalization is also adopted by Osang and Sarkar (2008).

[10]This assumption is perfectly equivalent to the one of de la Croix and Doepke (2003) or Schaefer (2014), where education is provided by teachers with a level of human capital equal to the average in the economy.

mutual fund, which invests these amounts in physical capital. In return, the mutual fund provides them an actuarially fair annuity during retirement, corresponding to their savings increased by the gross return adjusted from their life expectancy R_{t+1}/π_t^i.

The two budget constraints for an adult born in $t-1$ are:

$$c_t^i + e_t^i \bar{h}_t w_t + s_t^i = w_t h_t^i \tag{3}$$

$$d_{t+1}^i = \frac{s_t^i R_{t+1}}{\pi_t^i} \tag{4}$$

Human capital of her child h_{t+1}^i depends on the education e_t^i, human capital of the parents h_t^i and average human capital $\bar{h}_t$, representing the quality of the school system.

$$h_{t+1}^i = \epsilon (e_t^i)^\mu (h_t^i)^\eta (\bar{h}_t)^{1-\eta} \tag{5}$$

with $\epsilon > 0$, the efficiency of human capital accumulation. The parameters μ, η and their sum $\mu + \eta$ all $\in (0,1)$.[11] They are compatible with endogenous growth and capture respectively the efficiency of education and the intergenerational transmission of human capital within the family relatively to the transmission within the society.

The consumer program is summarized by:

$$\max_{e_t^i, s_t^i} U(c_t^i, d_{t+1}^i, h_{t+1}^i) = \ln c_t^i + \pi_t^i \left[\beta \ln(d_{t+1}^i) + \gamma \ln(h_{t+1}^i) \right] \tag{6}$$

$$s.t \quad c_t^i + e_t^i w_t \bar{h}_t + s_t^i = w_t h_t^i$$

$$d_{t+1}^i = \frac{s_t^i R_{t+1}}{\pi_t^i}$$

$$h_{t+1}^i = \epsilon (e_t^i)^\mu (h_t^i)^\eta (\bar{h}_t)^{1-\eta}$$

The maximization of this program (6) leads us to the following optimal choices in terms of education and savings.

$$e_t^i = \frac{\pi_t^i \gamma \mu}{1 + \pi_t^i (\beta + \gamma \mu)} \frac{h_t^i}{\bar{h}_t} \tag{7}$$

$$s_t^i = \frac{\pi_t^i \beta}{1 + \pi_t^i (\beta + \gamma \mu)} w_t h_t^i \tag{8}$$

Higher-skilled households invest more in savings and in children's education than lower-

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[11]We assume that $\mu + \eta < 1$ so that human capital convergence is possible. Education choice depends positively on h_t^i and negatively on $\bar{h}_t$ (representing the cost of education). Therefore, if $\mu + \eta > 1$, the return of h_t^i is always increasing and the return of $\bar{h}_t$ is negative, such that human capital convergence is impossible.

skilled households. The reason for this is twofold. First, there is a traditional income effect. The total wage of a worker depends on the wage rate w_t, equal for all agents, and on the level of human capital of this worker h_t^i. Therefore, higher-skilled parents benefit from a higher pay than lower-skilled one and can afford to spend more on education and savings. Second, longevity plays also an important role in the optimal choices for education and savings. An individual living a longer time gives more weight to the future. She saves more to consume during the extended retirement period and invests more in her child education, as she value more the future and will witness the returns of this investment. Moreover, as stated above, individuals with higher education are likely to live longer, mainly because they have better access to healthcare and are better informed, which allow them to adopt healthier behaviors and to suffer less from the negative health effects of pollution. Therefore, human capital disparities imply inequalities in life expectancy and hence in preferences for the future, which translates into larger preferences of higher-skilled households for savings and children's education. Both the longevity effect and the income effect reinforce the persistence of human capital disparities.

One can notice that the optimal choice in terms of child's education is determined by the relative human capital of parents (with respect to the average level of human capital in the economy) rather than by the absolute level. The rational for this is that education implies an opportunity cost associated with the investment in human capital agents have to do to educate their children. This investment is the same for all and is equal to the average human capital in the economy, in order to represent a standardized educational system (where a unit of schooling time is equivalent for all types of agents). Thus, it implies that the schooling time parents choose for their children depends on their relative human capital and will be relatively more expensive for the lower-skilled parents.

2.2 Production

Production of the consumption good is carried out by a single representative firm. The output is produced according to a constant returns to scale technology:

$$Y_t = A K_t^\alpha L_t^{1-\alpha} \tag{9}$$

where K_t is the aggregate stock of physical capital, L_t is the aggregate efficient labor supply to production, $A > 0$ measures the technology level, and $\alpha \in (0, 1)$ is the share of physical capital in the production. Defining $y_t \equiv \frac{Y_t}{L_t}$ as the output per unit of labor and $k_t \equiv \frac{K_t}{L_t}$ as the capital labor ratio, we have the following production function per unit of labor:

$$y_t = A k_t^\alpha$$

The government collects revenues through a tax rate $0 \leqslant \tau < 1$ on production, which is the source of pollution. The firm chooses inputs by maximizing profits $(1-\tau)Y_t - R_t K_t - w_t L_t$, such that:

$$w_t = A(1-\alpha)(1-\tau)\bar{k}_t^\alpha$$
$$R_t = A\alpha(1-\tau)\bar{k}_t^{\alpha} k_t^\alpha$$

(10)
(11)

2.3 Pollution

In this paper, we focus on the effect of pollution on health. Considering air pollution which represents the world's largest single environmental health risk according to the World Health Organization, we notice that its direct harmful effect on human health corresponds to the level before absorption, deposition or dispersion in the atmosphere. Moreover, air pollutants identified as the most significant health threats, i.e. particulate matter and ground-level ozone, remain in the atmosphere only for short periods of time (from hours to weeks). Thus, we choose to consider pollution as the flow currently emitted in the economy.[12] The same choice has been done theoretically by Pautrel (2008) and Aloi and Tournemaine (2013) among others.

Environmental degradation is a by-product of the current production process ($Y_t = y_t L_t$). The government can use the revenue of the pollution tax (τ) to reduce pollution, by providing a public environmental maintenance $M_t > 0$. This public maintenance, also called pollution abatement, represents a public investment in favor of the environment. Considering air pollution, governments may implement clean air strategies to reduce the use of fossil fuels through investment in renewable energy or subsidy of green transportation (public and private) for example.

Keeping in mind that we focus mainly on air pollution, it is clear that such environmental damage is not exclusively due to production activities but also to transportation, energy consumption *etc.* Thus, in order to consider the effect of the whole economic activity on pollution, we weight the pollution flow by the total labor force ($\bar{h}_t$) over the labor used in the production of the consumption good (L_t). Therefore, we define the law of motion of pollution as:

$$P_t = (a y_t L_t - b M_t) \frac{\bar{h}_t}{L_t}$$

(12)

where the parameters $a > 0$ and $b > 0$ correspond to the rate of pollution flow and the efficiency of environmental maintenance respectively.

The government budget is balanced at each period such that the level of public environmental maintenance is equal to $M_t = \tau y_t L_t$. Then, the pollution flow at a period t can

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[12]Considering pollution as a stock rather than a flow would increase the effect of pollution on health and the importance of environmental policy, but would make the model much more complicated.

be rewritten as:

$$P_t = (a - b\tau)y_t\bar{h}_t \tag{13}$$

Thus, pollution is composed of two elements. While production per unit of efficient labor y_t (which depends on capital intensity k_t) represents an index of pollution intensity, the aggregate human capital $\bar{h}_t$ corresponds to a "scale effect". Indeed, even if human capital is not pollutant *per se*, it defines the scale of economic activities and hence of pollution emissions.

In order to ensure that human activities lead to a positive pollution flow regardless the tax level, we assume:

Assumption 2 $a > b$

3 Equilibrium

The market clearing conditions for capital and labor are given by:

$$K_{t+1} = \xi s_t^u + (1 - \xi)s_t^s \tag{14}$$

and

$$L_t = \xi[h_t^u - e_t^u\bar{h}_t] + (1 - \xi)[h_t^s - e_t^s\bar{h}_t] \tag{15}$$

The presence of $e_t^i\bar{h}_t$ illustrates the investment of parents in terms of human capital, which does not enter the production of the consumption good. The values of e_t^i and s_t^i are given by the optimal choices of consumers (7) and (8) with the wage corresponding to (10). Thus, the market clearing conditions can be rewritten as:

$$K_{t+1} = A(1-\alpha)(1-\tau)k_t^\alpha \left[\xi h_t^u \frac{\pi_t^u\beta}{1 + \pi_t^u(\beta + \gamma\mu)} + (1 - \xi)h_t^s \frac{\pi_t^s\beta}{1 + \pi_t^s(\beta + \gamma\mu)}\right] \tag{16}$$

and

$$L_t = \bar{h}_t \left[\xi x_t^u \frac{1 + \pi_t^u\beta}{1 + \pi_t^u(\beta + \gamma\mu)} + (1 - \xi)x_t^s \frac{1 + \pi_t^s\beta}{1 + \pi_t^s(\beta + \gamma\mu)}\right] \tag{17}$$

The variable $x_t^i \equiv \frac{h_t^i}{\bar{h}_t}$ corresponds to the relative human capital of an individual i in period t. Using (5), the relative human capital of her child is described by:

$$x_{t+1}^i = \epsilon \left(\frac{\pi_t^i\gamma\mu x_t^i}{1 + \pi_t^i(\beta + \gamma\mu)}\right)^\mu \frac{1}{g_t}(x_t^i)^\eta \tag{18}$$

with $g_t \equiv \frac{\bar{h}_{t+1}}{\bar{h}_t}$, the growth factor of average human capital. From the definition of the average human capital ($\bar{h}_t = \xi h_t^u + (1 - \xi)h_t^s$), we can deduce the expression of the growth

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of human capital:

$$g_t = \epsilon(\gamma\mu)^\mu \left[\xi \left(\frac{\pi_t^u}{1 + \pi_t^u(\beta + \gamma\mu)} \right)^\mu (x_t^u)^{\mu+\eta} + (1 - \xi) \left(\frac{\pi_t^s}{1 + \pi_t^s(\beta + \gamma\mu)} \right)^\mu (x_t^s)^{\mu+\eta} \right] \quad (19)$$

And the pollution flow corresponds to:

$$P_t = (a - b\tau) A k_t^\alpha \, \bar{h}_t \quad (20)$$

We can rewrite the longevity given in (2) in terms of the individual relative human capital and of the capital-labor ratio:

$$\pi_t^i = \pi \left(\frac{x_t^i}{P_t/\bar{h}_t} \right) = \frac{\sigma x_t^i}{(a - b\tau) A k_t^\alpha + x_t^i} \quad (21)$$

Equations (16), (17) and (18) characterize the dynamics of the economy.

Definition 1. *Given the initial condition $K_0 \geqslant 0$, $h_0^u \geqslant 0$ and $h_0^s \geqslant 0$, the intertemporal equilibrium is the sequence $(k_t, x_t^u, x_t^s)_{t\in\mathbb{N}}$ such that the following dynamic system is satisfied for all $t \geqslant 0$:*

$$\begin{cases} k_{t+1} &= \frac{A(1-\tau)(1-\alpha)k_t^\alpha}{g_t} \left[\xi x_t^u \frac{\pi_t^u \beta}{1 + \pi_t^u(\beta+\gamma\mu)} + (1 - \xi)x_t^s \frac{\pi_t^s \beta}{1 + \pi_t^s(\beta+\gamma\mu)} \right] \\[2mm] & \quad \left[\xi x_{t+1}^u \frac{1+\pi_{t+1}^u \beta}{1+\pi_{t+1}^u(\beta+\gamma\mu)} + (1 - \xi)x_{t+1}^s \frac{1+\pi_{t+1}^s \beta}{1+\pi_{t+1}^s(\beta+\gamma\mu)} \right]^{-1} \\[3mm] x_{t+1}^u &= \epsilon \left(\frac{\pi_t^u \gamma\mu}{1+\pi_t^u(\beta+\gamma\mu)} \right)^\mu \frac{1}{g_t} (x_t^u)^{\mu+\eta} \\[3mm] x_{t+1}^s &= \epsilon \left(\frac{\pi_t^s \gamma\mu}{1+\pi_t^s(\beta+\gamma\mu)} \right)^\mu \frac{1}{g_t} (x_t^s)^{\mu+\eta} \end{cases} \quad (22)$$

with g_t and π_t^i given by (19) and (21) respectively.

The evolution of the economy is summarized by the laws of motion of the physical to labor ratio (k), and of the relative human capital of lower-skilled agents (x^u) and higher-skilled agents (x^s). We can rewrite the dynamical system (22) by substituting the growth of the average human capital by its expression given in (19). Moreover, from the definition of average human capital, we can express the relative human capital of higher-skilled x_t^s as a function of the one of lower-skilled workers: $\frac{1-\xi x_t^u}{1-\xi}$. After some computations, it follows that the dynamical system given in Definition 1 can be simplified as a two dimensions system in terms of the capital-labor ratio in the production of the consumption good k and the relative human capital of lower-skilled agents x^u.

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$$
\begin{cases}
k_{t+1} &= \frac{A(1-\tau)(1-\alpha)k_t^\alpha}{\epsilon(\gamma\mu)^\mu}\left[\xi x_t^u \frac{\pi_t^u \beta}{1+\pi_t^u(\beta+\gamma\mu)} + (1-\xi x_t^u)\frac{\pi_t^s \beta}{1+\pi_t^s(\beta+\gamma\mu)}\right] \\[2mm]
&\quad \left[\xi x_{t+1}^u \frac{1+\pi_{t+1}^u \beta}{1+\pi_{t+1}^u(\beta+\gamma\mu)} + (1-\xi x_{t+1}^u)\frac{1+\pi_{t+1}^s \beta}{1+\pi_{t+1}^s(\beta+\gamma\mu)}\right]^{-1} \\[2mm]
&\quad \left[\xi \left(\frac{\pi_t^u}{1+\pi_t^u(\beta+\gamma\mu)}\right)^\mu (x_t^u)^{\mu+\eta} + (1-\xi)\left(\frac{\pi_t^s}{1+\pi_t^s(\beta+\gamma\mu)}\right)^\mu \left(\frac{1-\xi x_t^u}{1-\xi}\right)^{\mu+\eta}\right]^{-1} \\[2mm]
x_{t+1}^u &= (x_t^u)^{\mu+\eta}\left(\frac{1-\xi x_t^u}{1-\xi}\right)^{-\mu-\eta}\left(\frac{1-\xi x_{t+1}^u}{1-\xi}\right)\left(\frac{\pi_t^u}{1+\pi_t^u(\beta+\gamma\mu)}\right)^\mu \left(\frac{\pi_t^s}{1+\pi_t^s(\beta+\gamma\mu)}\right)^{-\mu}
\end{cases}
\tag{23}
$$

with π_t^i given by (21).

Given the definitions of the average human capital and of the relative human capital, a decrease in the relative human capital of lower-skilled individuals x^u is due to a decrease in the level of human capital of lower-skilled h^u and/or to an increase in the level of human capital of higher-skilled individuals h^s. Moreover, a decrease in the lower-skilled relative human capital corresponds to a proportional increase in the higher-skilled relative human capital. Therefore, the lower x^u is, the lower is the level of human capital of lower-skilled workers respectively to higher-skilled workers, and hence the wider are disparities. Consequently, we use the relative human capital of lower-skilled individuals x^u to approximate the level of inequalities in the economy.

In this section, we aim to analyze the long-run behavior of the economy. Thus, from Definition 1, we specify:

Definition 2. *A balanced growth path (BGP) is an equilibrium satisfying Definition 1 and where the stock of physical and human capital grow at the same and constant rate $(g - 1)$.*

At a balanced growth path, the capital-labor ratio k_t, the growth of average human capital g_t, the relative human capital x_t^i and the flow of pollution P_t are constant.

For technical reasons, the study of the existence of balanced growth path equilibria is done in two parts. First, we analyze the case where $x^u = x^s = 1$, at which there is no inequality at the BGP. Second, we look at the case where $x^u \neq x^s \neq 1$, which means reversely that inequalities exist among households at the BGP.[13]

3.1 BGP without inequality: $x^u = x^s = 1$

We examine first the existence of a BGP equilibrium where there is no inequality. In this case, all individuals have the same human capital $(h_t^u = h_t^s = \bar{h}_t)$, hence all the relative

---

[13]When $x = 1$, rewrite the system (23) as two functions of k depending on x^u requires to divide by zero in the second dynamical equation.

human capital levels x_t^i are equal to 1. The dynamics of the economy described in (23) when $x^u = x^s = 1$ reduces to:

$$k_{t+1} \frac{1 + \beta\pi_{t+1}}{1 + \pi_{t+1}(\beta + \gamma\mu)} = \frac{A(1-\tau)(1-\alpha)\beta k_t^\alpha}{\epsilon(\gamma\mu)^\mu} \left[\frac{\pi_t}{1 + \pi_t(\beta + \gamma\mu)} \right]^{1-\mu} \tag{24}$$

with $\pi_t = \frac{\sigma}{1 + (a - b\tau)Ak_t^\alpha}$.

From this dynamical equation, we obtain that:

Proposition 1 *Under Assumptions 1 and 2 and the conditions $\alpha < \frac{1}{2}$ and $\epsilon > \bar{\epsilon}$, there exists a balanced growth path without inequality $(k_E, 1, 1)$ with a positive growth rate $(g_E - 1 > 0)$. This BGP is locally stable when $\eta < \bar{\eta}(\tau)$ and corresponds to a saddle point otherwise. The thresholds $\bar{\epsilon}$ and $\bar{\eta}(\tau)$ correspond to:[14]*

$$\bar{\epsilon} \equiv \left[\frac{(a - b\tau)Ak_E^\alpha + 1 + \sigma(\beta + \gamma\mu)}{\sigma\gamma\mu} \right]^\mu \quad and \quad \bar{\eta}(\tau) \equiv 1 - \mu \frac{2(a - b\tau)Ak_E^\alpha + (1 + \sigma(\beta + \gamma\mu))}{(a - b\tau)Ak_E^\alpha + (1 + \sigma(\beta + \gamma\mu))}$$

Note that the BGP without inequality is always stable if $\mu + \eta \to 0$ and is always a saddle point if $\mu + \eta \to 1$.

Proof. See Appendix 6.1. ∎

From this proposition, we know that human capital accumulation has to be sufficiently efficient $(\epsilon > \bar{\epsilon})$ so that the growth rate is positive at the long-term state without inequality. Most importantly, we see that the economy may converge in the long run to a balanced growth path without inequality among households.[15] But it is not always the case. When the equilibrium is a saddle point $(\eta > \bar{\eta}(\tau))$, the convergence toward the long-term equilibrium without inequality is very unlikely. Thus, the economy will most likely exhibit inequalities in the long run.[16]

Several effects intervene against and in favor of the human capital convergence. The model combines channels usually found in the literature on human capital and inequality (e.g. Tamura , 1991, Glomm and Ravikumar , 1992 or de la Croix and Doepke , 2003) and the more uncommon longevity channel (see Castello-Climent and Domenech , 2008 for an exception).[17] First, the presence of parents' human capital in the production of children's

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[14]The effect of τ on $\bar{\eta}(\tau)$ is detailed in Section 4.1.

[15]Such equilibrium is possible because agents differ only in the initial level of human capital. We do not assume in this model that poor individuals are less able to acquire skills.

[16]Note that by definition $0 \leqslant x^u \leqslant 1$. Therefore, the economy can achieve the BGP where $x^u = 1$ only for few initial conditions where x_0^u is very high and k_0 is very low. We analyze the extent of inequalities when $(k_E, 1, 1)$ is unstable in Section 3.3, after determining if there are also BGP with inequalities.

[17]Castello-Climent and Domenech (2008) isolate the life expectancy channel by considering that human

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knowledge acts as a divergent force. This intergenerational spillover results directly in the transmission of inequalities and hence in their persistence over time. Then, this channel is reinforced by two other divergent forces leading to disparities in education choices among parents. The income differential among parents, due to inequalities in their human capital, implies that lower-skilled individuals can less afford the cost of educating their children. Moreover, the life expectancy of an individual determines her preference for the future and in particular for her child's education. In our model, longevity is endogenous and depends on individual human capital and pollution. It follows that lower-skilled agents have also a poor health and give less value to education. The reverse occurs for higher-skilled parents who are more able and willing to finance education. Thus, the gap among children's future human capital is widened by these divergent forces (through intergenerational transmission and education). Finally, on the opposite, the presence of average human capital in the production of human capital represents a convergent force, which is crucial to ensure that human capital convergence is possible, as Tamura (1991) shows. In other words, the quality of the school system has to be at least partly the same for all children in the economy, so that they can achieve the same level of human capital in long run.

According to these effects, it emerges that the weight of the divergent forces, i.e. of education and intergenerational transmission, in human capital accumulation has to be not too high in order to avoid that poor agents are further disadvantaged and hence that the poor-rich gap gets worse. Thus, we identify a condition in terms of the weight of intergenerational transmission in the production of human capital so that the economy can converge to an equilibrium where inequalities vanish $(\eta < \bar{\eta}(\tau))$. Moreover, if the sum of the weight of education and intergenerational transmission in human capital is at its maximum $(\mu + \eta \to 1)$, the long-term equilibrium without inequality is always a saddle point, as the convergent force is eliminated. Reversely, when it is at its minimum $(\mu + \eta \to 0)$, the BGP is always stable as divergent forces are removed.

Note that the dispersion through the longevity channel will be even larger when pollution is high. Lower-skilled households being more vulnerable to pollution than higher-skilled households, the environmental damages amplify health disparities and hence human capital inequalities. Thus, the levels of both inequalities and pollution intensity in the economy modify the return on investment in education and favor the persistence of inequalities and hence could play a key role in the determination of the long-term behavior of the economy. However, we need to know if there are other BGP to conclude precisely on this point, which is what we do in the rest of the section.

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capital formation depends only on an investment in time. Thus, they do not consider the other aforementioned effects.

3.2   BGP with inequalities: $x^u \neq x^s$

At an equilibrium with inequalities, the two groups of individuals (lower- and higher-skilled) have not the same human capital ($h_t^u < \bar{h}_t < h_t^s$), hence the relative human capital of each goup x_t^i differs from 1. By developing the dynamical system (22) with the explicit form of longevity given in (21), we can rewrite the system at a balanced growth path with inequalities as:

$$
\begin{cases}
k^{1-\alpha} \dfrac{\epsilon(\sigma\gamma\mu)^\mu}{A(1-\tau)(1-\alpha)\beta} \left[\dfrac{\left[(a-b\tau)Ak^\alpha + \frac{1-\xi x^u}{1-\xi}(1+\sigma\beta)\right]\left[(a-b\tau)Ak^\alpha + x^u(1+\sigma(\beta+\gamma\mu))\right] + \xi x^u(a-b\tau)Ak^\alpha \sigma\gamma\mu\left(\frac{1-\xi x^u}{1-\xi} - x^u\right)}{(a-b\tau)Ak^\alpha \sigma\left(\frac{(1-\xi x^u)^2}{1-\xi} + \xi(x^u)^2\right) + \frac{1-\xi x^u}{1-\xi}\sigma x^u(1+\sigma(\beta+\gamma\mu))} \right] \\[20pt]
\left[\dfrac{\xi(x^u)^{2\mu+\eta}}{\left[(a-b\tau)Ak^\alpha + x^u(1+\sigma(\beta+\gamma\mu))\right]^\mu} + \dfrac{(1-\xi)\left(\frac{1-\xi x^u}{1-\xi}\right)^{2\mu+\eta}}{\left[(a-b\tau)Ak^\alpha + \left(\frac{1-\xi x^u}{1-\xi}\right)(1+\sigma(\beta+\gamma\mu))\right]^\mu} \right] - 1 = 0 \\[20pt]
k = \left[\dfrac{1+\sigma(\beta+\gamma\mu)}{(1-\xi)A(a-b\tau)} \dfrac{(1-\xi)x^u(1-\xi x^u)^{\frac{2\mu+\eta-1}{\mu}} - ((1-\xi)x^u)^{\frac{2\mu+\eta-1}{\mu}}(1-\xi x^u)}{((1-\xi)x^u)^{\frac{2\mu+\eta-1}{\mu}} - (1-\xi x^u)^{\frac{2\mu+\eta-1}{\mu}}} \right]^{\frac{1}{\alpha}}
\end{cases}
\tag{25}
$$

The study of this system results in the following proposition on the existence of balanced growth paths with inequalities:

Proposition 2 *Under Assumptions 1 and 2, for $2\mu + \eta > 1$ and $\alpha < 1/2$, there exists at least one BGP with inequalities when $A < \bar{A}(\tau)$, with :*[18]

$$
\bar{A}(\tau) \equiv \left\{ \frac{\epsilon(\gamma\mu)^\mu}{(1-\tau)(1-\alpha)\beta\sigma^{1-\mu}} \left[\frac{2\mu+\eta-1}{\mu(1+\sigma(\beta+\gamma\mu))} \right]^\mu \left[\frac{1+\sigma(\beta+\gamma\mu)}{(a-b\tau)} \frac{1-\mu-\eta}{2\mu+\eta-1} \right]^{\frac{1-\alpha}{\alpha}} \right.
$$
$$
\left. \left[\frac{\sigma\gamma\mu(1-\mu-\eta) + (1+\beta\sigma)\mu}{2\mu+\eta-1} \right]^\alpha \right\}
$$

Note that there is no BGP with inequalities in the extreme case where $\mu + \eta \to 0$.

Proof. See Appendix 6.2. ∎

While there always exists a balanced growth without inequality, one or several balanced growth path(s) characterized by inequalities among households may also occur. More precisely, it is the case at least for a total factor productivity not too high. At such a long-term state, human capital and longevity of individuals diverge across lower- and higher-skilled individuals in the long run. The existence of a balanced growth path with inequalities stems from the balance between the convergent force and the divergent forces in the formation

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[18]The effect of τ on $\bar{A}(\tau)$ is detailed in Section 4.1.

of human capital, that we have mentioned in the previous section. In particular, from the condition $A < \bar{A}(\tau)$, it follows that when the weights of intergenerational spillover and dispersion in education choices, corresponding to the divergent forces, are zero $(\mu + \eta \to 0)$, there is no BGP with inequalities and the only long-term equilibrium is the BGP without inequality, which is stable in this case.[19] Thus, as we explained previously, the weight of the divergent forces has to be sufficiently high so that inequalities can persist in the economy in the long run.

More particularly, the existence of multiple balanced growth paths and in particular of an inequality trap, where disparities among households are persistent or widening across time, results from the fact that longevity is endogenous in our model. The life expectancy of an individual depends on her level of human capital and the aggregate level of pollution. Therefore, pollution and human capital affect the returns of investment in education through health. Indeed, at an individual level, more pollution or less human capital entail that she will die sooner, hence her preferences for the future are lower and she benefits less from her investment in her child education. Moreover, lower-skilled agents are more vulnerable to the negative effect of pollution on health than higher-skilled agents. Thus, the initial levels of pollution and inequalities shape the extent of disparities in terms of health and return to education, and thus the level of inequalities for the next generation. More precisely, pollution and inequalities determine if the growth of individual human capital is higher for lower-skilled or higher-skilled households. Usually in the literature on human capital and inequality, it is always higher for lower-skilled under the diminishing return of the divergent forces in human capital accumulation (equivalent to the condition $\mu + \eta < 1$ in our model), so that human capital convergence occurs (see e.g. Tamura , 1991 or Glomm and Ravikumar , 1992).[20] In our model, despite the condition $\mu + \eta < 1$, it is not always the case. When the levels of pollution and/or inequalities are too high, the growth of individual human capital can be lower or equal for lower-skilled, which implies that there exists an inequality trap.[21] Therefore, the extent of the dispersion through the longevity channel, and more generally of the divergent forces, varies according to the initial conditions of the economy in terms of pollution and inequalities, which are then determinant for its behavior in the long run. Note that if longevity would be exogenous in our model, the growth

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[19]A further analysis of the condition under which BGP with inequalities exist(s) is provided in the numerical illustration in Section 3.3.2.

[20]In Glomm and Ravikumar (1992), when education is public, human capital convergence always occurs, while when education is private, it is the case when the sum of the weights of private spending in education and of intergenerational transmission in human capital formation are lower than 1.

[21]The individual growth of human capital g^i is increasing and then decreasing in x^i. The maximum value of g^i is achieved in $x^i < 1$, while $g^i(0) = 0$ and $g^i(Max\{x^s\}) > 0$. Thus, there exists a level of x^u under (resp. above) which the individual growth of human capital is lower (resp. higher) for lower-skilled than higher-skilled agents $g^u < g^s$ (resp. $g^u > g^s$).

of individual human capital would always be higher for lower-skilled so that the economy would always converge to the long-term equilibrium without inequality (under $\mu + \eta < 1$).

Analytically, we are not able to conclude on the dynamics of the long-term equilibrium(a) with inequalities. However, it is important to identify what scenario will take place and under what conditions, that is why we will analyze numerically the number and the characteristics of the balanced growth path(s) with inequalities in the next section.

3.3 Numerical illustration

In this section, we provide a numerical analysis of the model in order to describe more precisely the long-term behaviors of the economy. After motivating the choice of the parameters value, we study in details the characteristics of the different balanced growth paths.

3.3.1 Calibrations

To solve the model numerically, we give values to the parameters of technology and preferences so that they fit empirical observations and projections of the US economy. We calibrate the model assuming that a period represents thirty years. Therefore, an individual starts working at 30, retires at 60 and may live for up to 90, according to her longevity. In the real-business-cycle literature, the quarterly psychological discount factor is estimated to 0.99 (see Cooley , 1995 or de la Croix and Michel , 2002). A period representing 30 years in our model, β is set to $0.99^{120} = 0.3$. We choose parameters to match US data on the annual long-term growth rate (i.e. around 1.7%) and the US share of education expenditure in GDP at the balanced growth path (i.e. between 5 and 8%).[22] Thus, considering that the two groups of workers have the same size ($\xi = 0.5$)[23], we set the scale parameter ϵ to 6 and the preference for child human capital γ to 0.35.

The parameter μ represents the weight of education in human capital accumulation and corresponds more precisely in our model to the elasticity of human capital with respect to education. In the literature, the return to schooling in developed countries is estimated between 8% and 16% (see Ashenfelter and Krueger , 1994; Psacharopoulos , 1994 or Krueger and Lindahl , 2001). These figures correspond to Mincerian returns, which means that they include only an opportunity cost (forgone earnings) and do not consider education expenditure. Following de la Croix and Doepke (2003), we assume that an additional year of schooling raises education expenditure by 20%. The resulting elasticity of schooling is

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[22]See the long-term projections for the US economy of OECD (2014a) on the growth rate and the Digest of Education Statistics 2012 of the US Department of Education for data on the education share.

[23]We will provide a sensitivity analysis with respect to the repartition of the population between the two groups ξ in Appendix 6.5.

from 0.4 to 0.8. Thus, we set μ to be 0.6. The weight of intergenerational transmission of human capital η being a key parameter in our analysis, we will consider all values allowed by the model, i.e. $\eta \in [0, 1 - \mu)$.

The weight of production in the pollution flow and of environmental maintenance are chosen to satisfy the condition $a > b$, ensuring that there is pollution emission in the presence of economic activities, i.e. $a = 0.6$ and $b = 0.4$. The parameter in the longevity function σ is set to 0.9, so that the values of individuals' life expectancy at the balanced growth paths corresponds to realistic values (between 60 and 87 years).[24] Finally, concerning the production technology, the share of physical capital in the production of the consumption good α is set to $1/3$ in accordance with empirical data, and the total factor productivity A is a scale parameter set to 1. Note that the value of A with those of the other parameters, allows that the condition $A < \bar{A}(\tau)$, such that there exists at least one BGP with inequality, is always satisfied for a sufficient level of the tax $\tau \in [0, 1)$.

3.3.2 Long-term behaviors of the economy

First, we identify the different long-term equilibria of the calibrated economy and we compare their characteristics. For the set of parameters considered, we obtain:[25]

Numerical result 1 (*i*) *When* $\eta > \bar{\eta}(\tau)$, *there exists a unique BGP, which is the one without inequality* $(k_E, 1, 1)$. (*ii*) *When* $\eta < \bar{\eta}(\tau)$, *there exist multiple balanced growth paths: the BGP without inequality* $(k_E, 1, 1)$ *and an additional BGP with inequalities* (k_I, x_I^u, x_I^s). *The two long-term equiliria are characterized by:*

- $x_I^s > 1 > x_I^u$

- $k_I > k_E$

- $\pi_I^s > \pi_E > \pi_I^u$ *and* $\pi_E > \bar{\pi}_I$

- $g_E > g_I$

First, it is worth noticing that the numerical analysis enables us to identify how many and when the BGP with inequalities exist(s). We find that one equilibrium with inequalities may exist, and more interestingly, that it is the case under the threshold $\bar{\eta}(\tau)$, identified in Proposition 1. Therefore, we have that, in the numerical illustration, this threshold corresponds to the value under which the equilibrium without inequality is stable and

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[24]While the lower bound is equivalent to the US life expectancy at birth in the thirties, the upper bound is close the value expected by the US Census Bureau for 2060 (84 years).

[25]All the numerical results are obtained by considering the parameters η and τ with a pitch value of 0.000001 unit.

under which the balanced growth path with inequalities appears, for all the levels of the tax.

Concerning the characteristics of the economy, the long-term equilibrium with inequalities corresponds to a state where there are persistent inequalities among households in the economy. Higher-skilled agents are characterized by a higher level of human capital than lower-skilled agents ($x_I^s > 1 > x_I^u$). Moreover, at this state, the capital labor ratio is higher ($k_I > k_E$), which implies that the pollution intensity is also larger in I. This is essentially due to the fact that average savings increase with inequalities, as they are convex in the level of relative human capital. Due to a higher dispersion of human capital and a higher pollution intensity at the BGP with inequalities, it follows that disparities in terms of health are also higher at this equilibrium. While lower-skilled agents have a lower relative human capital and suffer from the higher pollution intensity, higher-skilled workers benefit from an increase in their relative human capital, which more than offsets the increase in the pollution intensity. Therefore, at the balanced growth path I, rich individuals live much longer than poor individuals ($\pi_I^u < \pi_E < \pi_I^s$). Moreover, the average life expectancy in the economy is higher at the long-term equilibrium without inequality (π_E) than at the one with inequalities ($\bar{\pi}_I$), which means that the health cost of inequalities for poor is higher than the benefit for rich.

Concerning the long-term growth which is driven by human capital, we obtain that it is higher at the balanced growth path without inequality ($g_E > g_I$). Several effects occur. The average education is higher at the long-term equilibrium with inequalities I, as education is convex in the relative human capital. In other words, the increase in the education choice of higher-skilled parents, due to their higher longevity and income, outweighs the decrease in the education choice of lower-skilled parents, arising from lower longevity and income. However, human capital accumulation depends also on the relative human capital they transmit to their children. At the BGP with inequalities, the associated dispersion harms more lower-skilled than it benefits to higher-skilled. The net effect is that the long-term growth is higher in the absence of inequalities. Therefore, inequalities have a cost in terms of growth and development in addition to their human cost.

This result is supported by empirical evidence: OECD (2014b) emphasizes, for example, that inequalities reduce growth, by making the poorer individuals less able to invest in education. It contributes also to the wide theoretical literature on inequalities and growth, surveyed in Galor (2011) and Sauer & Zagler (2012). More particularly, this finding is in line with the modern approach of this issue, whose contributions demonstrate that inequalities may be detrimental for economic growth, through their negative effect on human capital formation.

To identify the long-term behaviors of the economy, we analyze the dynamics at the balanced growth paths and we obtain that:

Numerical result 2 (*i*) *When* $\eta > \bar{\eta}(\tau)$, *the unique BGP* $(k_E, 1, 1)$ *is a saddle point, with a stable branch* SS_E. (*ii*) *When* $\eta < \bar{\eta}(\tau)$, *the BGP without inequality* $(k_E, 1, 1)$ *is stable, while the BGP with inequalities* (k_I, x_I^u, x_I^s) *is a saddle point with a stable branch* SS_I. *Below* SS_j, *with* $j = I, E$, *the economy is stuck in an inequality trap, whereas above* SS_j *it converges to the BGP without inequality.*

The dynamics of the economy in the cases (*i*) and (*ii*) can be represented by the Figures 1 and 2 respectively.[26]

As the threshold $\bar{\eta}(\tau)$ represents the value above which the unstable BGP with inequalities vanishes but also the value above which the BGP without inequality becomes unstable, in the numerical illustration, there always exists an inequality trap, where disparities are constantly widening.

The underlying mechanism behind the coexistence of an inequality trap and a long-term equilibrium without inequality stems from the fact that the return on investment in education varies according to the levels of inequalities and pollution intensity in the economy, as we explained in Section 3.2. Indeed, longevity depends on both pollution and individual human capital, which implies that agents have not the same vulnerability to pollution. Consequently, the levels of both pollution and inequalities in human capital lead to a wider dispersion of life expectancy. It means that the period of time during which parents can benefit from their investment is very different among agents. Thus their preferences for the future and the return to educate their children are also more unequal. Due to endogenous longevity, the levels of pollution intensity and of inequalities in the economy are crucial in the balance between the convergent force (i.e. average human capital) and the divergent forces (i.e. intergenerational spillover, income dispersion and longetivity dispersion) that drive human capital accumulation.

When the set of initial conditions is such that the economy is above the stable branch SS_j, the convergent force overcomes the divergent forces, as inequalities are not too wide and the environmental quality is sufficiently high. Therefore, lower-skilled households have a higher return to education investment than higher-skilled households. The growth of individual human capital is larger for the more disadvantaged agents, which allow them to narrow existing disparities over generations and to converge to an equal equilibrium in the long run. Note that this result holds whether the unstable BGP is the one with inequalities I or the one without inequality E. The economy can converge to E even if it is a saddle

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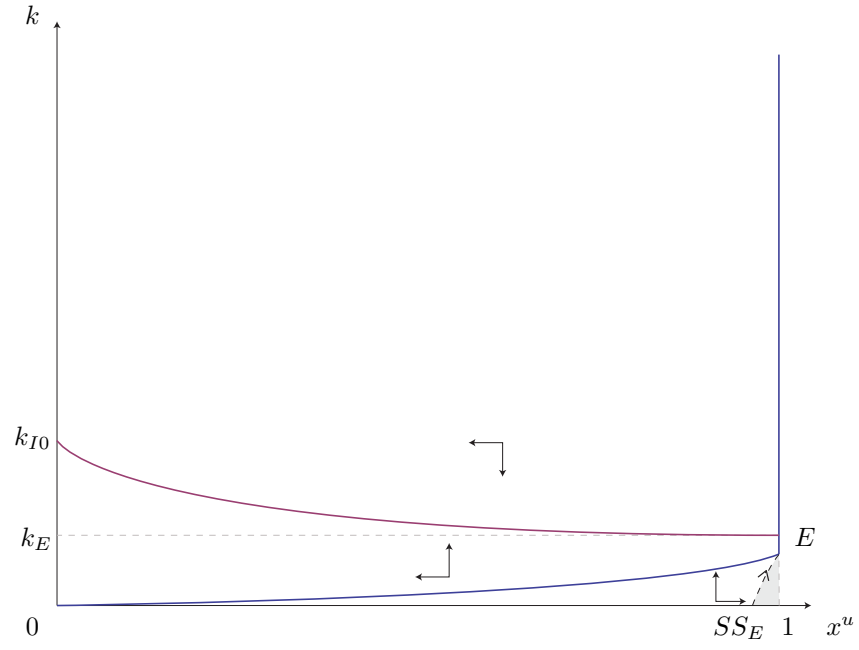[26]Note that the second dynamical equation in (23), represented by the blue curve, is discontinuous at $x^u = 1$.

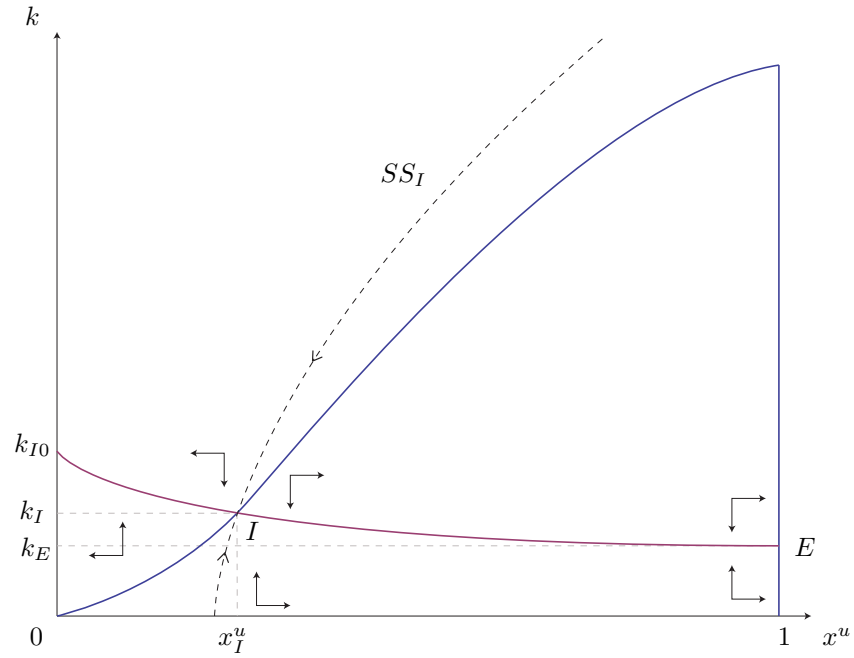Figure 1: Representation of the dynamics of the economy in the case (i)

Figure 2: Representation of the dynamics of the economy in the case (ii)

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point when its initial conditions are on the stable branch SS_E but also above it (grey area in Figure 1) as the relative human capital of lower-skilled agents x^u is limited by definition to $[0,1]$. However, it corresponds to very few cases since inequalities and pollution intensity have to be very low (x^u high and k low).

On the contrary, when the initial conditions are below the stable branch SS_j, disparities and/or pollution intensity are initially too large (low x_0^u and high k_0), thus the divergent forces outweigh the convergent force. In this case, the longevity dispersion leads to huge disparities among households education. The return on the investment in education is higher for higher-skilled agents, and the growth of individual human capital is lower for lower-skilled agents. It entails that inequalities are too wide for lower-skilled agents to be able to fill the gap and that the economy is stuck in an inequality trap where inequalities will steadily increase. In particular, when the long-term equilibrium without inequality E is a saddle point ($\eta > \bar{\eta}(\tau)$), the economy is in the trap for most of the initial conditions. Moreover, the higher is the weight of intergenerational transmission in human capital accumulation η, the heavier is the weight of divergent forces and therefore the larger is the size of the inequality trap.[27]

When the economy is in an inequality trap, it converges asymptotically to a state where the level of inequalities is maximum ($k_{I0}, x_{I0}^u, x_{I0}^s$). The human capital of lower-skilled tends to zero ($x_{I0}^u \to 0$), as well as their income and their longevity π^u. Therefore, poor agents die at the end of the second period of life, before retirement. Moreover, they are not able anymore of consuming, saving or educating their children so that the lower-skilled category collapses. On the opposite, higher-skilled households become richer, more educated and live longer. The physical capital to labor ratio and hence the pollution intensity are also higher in this extreme state ($k_{I0} > k_I$). Finally, the long-term growth rate is the lowest at this state $g_{I0} < g_I < g_E$, which confirms that inequalities have a cost in terms of growth. Note that such an extreme case, where the lower-skilled group collapses, is not achieved in reality but it illustrates the constant worsening living conditions of the more disadvantaged households in such a trap.

The study of the long-term behaviors of the economy reveals that the risk to be stuck in an inequality trap, where disparities steadily increase, is important. This result is in line with Castello-Climent and Domenech (2008) who show that an inequality trap can exist due to the transmission of inequalities through life expectancy. Here, we extent their result to the environmental dimension and emphasize that the initial levels of inequalities and pollution in the economy are both crucial to determine if the human capital convergence can occur or not. In particular, the numerical analysis points out that the economy is most likely to be stuck in the trap when the pollution intensity is high, which raises the question of

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[27]See the sensitivity analysis in Appendix 6.5 for more details on the effect of η on the results.

the role that an environmental policy could play in breaking such vicious circle. Therefore, the next section is devoted to the analysis of the consequences of an environmental policy on the behavior and the growth of the economy in the long run.

4 Environmental policy implications

In this section, we assess the effect of a tighter pollution tax associated with an increase in public maintenance on the dynamics and on the growth of the economy. In particular, we want to know whether such an environmental policy can have a redistributive power, as we emphasized the role of pollution intensity in the persistence of inequalities, and whether it can allow to enhance the long-term growth, which is driven by human capital. For each point, we provide, first, an analytical analysis of the implications of the policy, while we illustrate them numerically in a second time.

4.1 Environmental policy implications on the balanced growth paths

From Proposition 1, we know that the stability of the long-term equilibrium without inequality depends on the environmental policy, as the tax on pollution intervenes in the threshold determining its stability $\bar{\eta}(\tau)$. Examining how an increase in the pollution tax affects this threshold, we make the following proposition.

Proposition 3 *Under Assumptions 1 and 2 and for $\alpha < 1/2$, the threshold $\bar{\eta}(\tau)$ depends positively on the tax rate τ. Thus, a tighter tax on pollution increases the range of parameters such that the long-term equilibrium without inequality is stable.*

Proof. See Appendix 6.3. ∎

When the tax on pollution is tighter, it allows the associated investment in environmental maintenance to increase, which reduces the pollution flow. Thus, the longevity of all agents increases, such that their preferences for the future and their investments in education increase too. It implies that the level of human capital is higher. However, although the individual growth of human capital increases for all levels of relative human capital, the decrease in pollution affects relatively more lower-skilled households.[28] Intuitively, even if all agents suffer from the same level of environmental damages, higher-skilled individuals are more able to protect themselves from the negative effect of pollution on health through

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[28]More precisely, $\frac{\partial g^i}{\partial \tau} > 0$ but $\frac{\partial^2 g^i}{\partial \tau \partial x^i}$ is > 0 when x^i is small and < 0 when x^i is high, with a threshold equal to $\frac{(a-b\tau)Ak^\alpha(2\mu+\eta-1)}{(1+\sigma(\beta+\gamma\mu))(2-\mu-\eta)}$. Therefore, the increase in the tax reduces this threshold, so that there are more levels of x^i such that lower-skilled are more affected by the increase in environmental quality than higher-skilled.

knowledge, information, or financial means.[29] Thus, public maintenance has a larger effect on the return of investment in education of lower-skilled agents. All agents being proportionally taxed, it follows that an increase in the tax on pollution makes more likely the convergence toward the long-term equilibrium without inequality. Moreover, note that there always exists a sufficient tax rate τ such that the BGP without inequality is stable, i.e. $\eta < \bar{\eta}(\tau)$.[30]

The environmental tax affects also the existence of long-term equilibria with inequalities. The analysis of the threshold $\bar{A}(\tau)$ enables us to identify how it does.

Proposition 4 *Under Assumptions 1 and 2, for $2\mu + \eta > 1$ and $\alpha < 1/2$, the threshold $\bar{A}(\tau)$ depends positively on the tax rate τ and the condition $A < \bar{A}(\tau)$ is always satisfied when τ tends to 1. Thus, a tighter tax on pollution increases the range of parameters for which there exists at least one BGP with inequalities.*

A tighter tax makes more likely the existence of one or several balanced growth path(s) with inequalities. However, to be able to conclude on the implications of this proposition on the long-term behavior of the economy, we need to know how many such equilibria are and their dynamics and we cannot evaluate these elements analytically in our model. That is why we will analyze the implications of the environmental policy on the balanced growth paths and on inequalities in the numerical illustration (Section 4.3).

4.2 Environmental policy implications on growth

The growth factor of human and physical capital at the balanced growth path without inequality is given by:

$$g_E = \epsilon \left[\frac{\sigma \gamma \mu}{(a - b\tau)Ak_E^\alpha + 1 + \sigma(\beta + \gamma \mu)} \right]^\mu \tag{26}$$

The study of the effect of the environmental policy on this long-term growth rate reveals that:

Proposition 5 *Under Assumptions 1 and 2, a tighter tax on pollution improves the growth rate at the BGP without inequality ($g_E - 1$).*

Proof. See Appendix 6.4 ∎

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[29] As Blackburn and Cipriani (2002), we do not formalize health expenditures in this paper (neither private nor public one), but we consider that individual human capital includes the capacity of agents to spend in healthcare. For models with explicit healthcare spending, see e.g. Chakraborty (2004), Pautrel (2008), Varvarigos and Zakaria (2013a) or Raffin and Seegmuller (2014).

[30] This is due to the fact that when the tax on pollution τ tends to 1, the threshold $\bar{\eta}(\tau)$ tends to $1 - \mu$, while we assume that $\eta < 1 - \mu$.

Several effects occur. On one hand, a tighter tax on pollution implies a negative income effect. Firms report the pollution tax on the wage rates (w_t) and the returns on savings of households (R_t). On the other hand, a higher tax rate leads to more maintenance activities, which decrease the level of pollution and hence improve health. Through this channel, individuals' longevity enhances, which leads to greater preferences for future motives in the utility function, i.e. savings and children's education. Concerning savings, the negative income effect outweighs the longevity effect, such that savings decrease with a tighter tax on pollution. However, concerning education, a third effect operates. It is important to keep in mind that education is an investment in terms of human capital done by parents and corresponds to an opportunity cost associated with the fact that they do not use this efficient labor to produce, and hence are not paid for that. Therefore, the negative income effect of the tax is neutralized by its positive effect through the reduction of the opportunity cost. Education is thus only affected positively by the environmental policy through longevity. It follows that the stock of human capital improves with the tax. Human capital being the engine of growth in the economy, the long-term growth rate is also enhanced with a tighter environmental policy.

4.3 Numerical illustration

We emphasized previously that the environmental policy has severals effect on inequalities and a positive effect on growth at the long-term equilibrium without inequality. In this section, we want to complete the analytical analysis to get a comprehensive overview of the environmental policy implications.

Concerning inequalities, we pointed out in Section 4.1 that a tighter environmental policy favors simultaneously the stability of the balanced growth path without inequality and the existence of a balanced growth path with inequalities. With the numerical analysis, we want to illustrate these findings and detail more precisely how the environmental policy affects the long-term behavior of the economy for the parameters considered. Proposition 3 indicates that the tax on pollution increases the value of the threshold $\bar{\eta}(\tau)$. Furthermore, in the numerical illustration, this threshold represents the value under which the long-term equilibrium E is stable but also the value under which the long-term equilibrium with inequalities I exists. In other words, the equilibrium without inequality E becomes stable when the one with inequalities I appears. Thus, it follows that:

Numerical result 3 (*i*) *When $\eta > \bar{\eta}(0)$, for low levels of the tax on pollution, there exists only the BGP without inequality E which is a saddle point and delimits the inequality trap. However, when the level of the tax becomes sufficiently high, η becomes lower than $\bar{\eta}(\tau)$, which implies that the long-term equilibrium without inequality E becomes stable, while the*

one with inequalities I appears and is a saddle point, delimiting the new trap.[31]

(ii) When $\eta < \bar{\eta}(0)$, the condition such that the BGP without inequality E is stable and the BGP with inequalities I exists as a saddle point is satisfied for all levels of the tax on pollution.

Therefore, we deduce that, when $\eta > \bar{\eta}(0)$, a sufficient tax on pollution allows to reduce the size of the inequality trap, now delimited by the long-term equilibrium I. But beyond this case, we obtain that:

Numerical result 4 *An increase in the environmental tax decreases the size of the inequality trap. Thus, a tighter pollution tax can allow an economy to escape the trap and to converge toward the BGP without inequality.*

By decreasing the size of the trap, a tighter tax may reduce inequalities among households in short run and eradicate them in long run. The mechanism by which environmental policy favors human capital convergence is detailed in Section 4.1. To sum up, a tighter tax on pollution enables to reduce environmental damages, which improves the life expectancy of agents and hence their ability to look to the future. In this way, households pay more attention to the education of their children. This effect is even stronger for the more disadvantaged households, who are relatively more sensitive to environmental damages. Consequently, a tighter tax on pollution can allow unequal economies to escape the trap and to reduce disparities along the convergence to a long-term equilibrium without inequality.

To illustrate the Numerical Results 3 and 4, we use the Figure 3 corresponding to the case (i) where $\eta > \bar{\eta}(0)$ as it includes the different scenarios. When $\eta > \bar{\eta}(0)$, we have that $\eta > \bar{\eta}(\tau)$ for low values of the tax on pollution, as in the phase diagrams (a) and (b) of the Figure 3. Then, the only long-term equilibrium is the one without inequality, but it corresponds to a saddle point and hence delimits an inequality trap in which the economy is stuck for most of initial conditions. The economy converges to the equilibrium without inequality only if the levels of pollution intensity and inequalities are very low (on the right of the dotted line). When the tax on pollution increases, the inequality trap goes to the left. As one can see in the Figure 3, this is due to the fact that the blue curve, representing the second dynamical equation in (25), goes up. Therefore, there are more conditions such that the economy can converge to E. When the tax is sufficiently high (phase diagram (c)), the condition $\eta < \bar{\eta}(\tau)$ is satisfied such that the BGP with inequalities I appears, E becomes stable while I is a saddle. This change implies that the size of the inequality trap reduces and that there are more conditions such that the economy can escape the trap. After that,

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[31]Note that when τ tends to 1, $\bar{\eta}(\tau)$ is always greater than η.

(a) $\tau = 0$

(b) $\tau = 15\%$

(c) $\tau = 30\%$

(d) $\tau = 45\%$

(e) $\tau = 60\%$

(f) $\tau = 75\%$

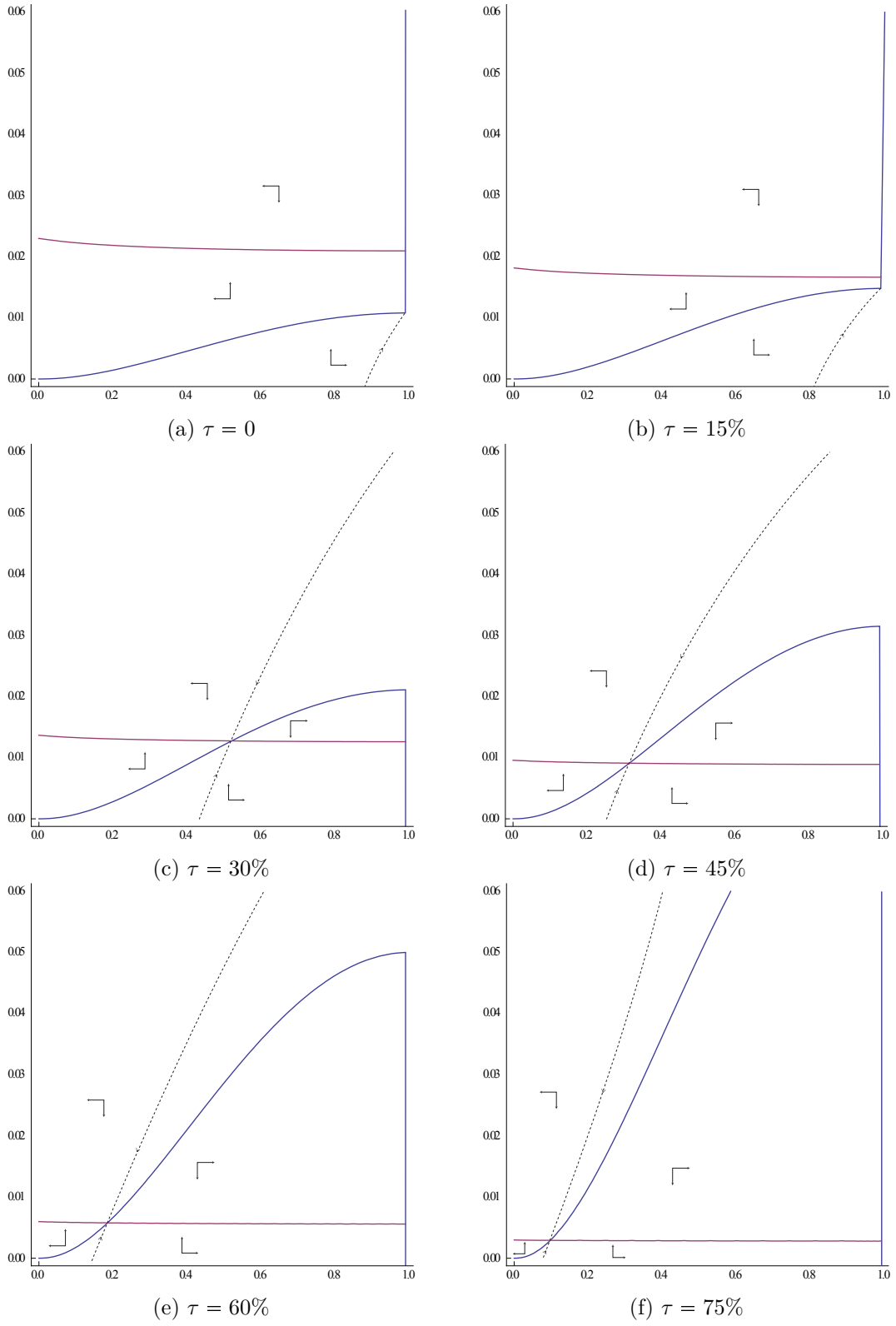Figure 3: Phase diagrams when $\eta = 0.35$, i.e. $\eta > \bar{\eta}(0)$, for different tax levels, with x^u on the X-axis and k on the Y-axis.

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when the environmental tax continues to increase, the trap continues to decrease, as the BGP I continues to move to the left (phase diagrams (d) to (f)).

As we notice in Section 4.1, there always exists a level of the tax such that the economy can escape the trap. However, the more society is unequal and the higher is the pollution intensity before the strengthening of environmental policy, the higher is the tax rate necessary to get out of the inequality trap. In extreme cases, the required tax can be close to 100%. It entails, that for reasonable levels of pollution taxation, the environmental policy may be not sufficient to reduce inequalities. The term reasonable refers to the fact that a tax whose level is too high restricts consumption and hence harms welfare. When the economy is still in the trap after the increase in pollution taxation, it means that inequalities are initially too wide and/or pollution intensity is initially too large to ensure that the improvement in environmental quality is able to overcome the excessive initial disparities on the return on education investment. Thus, inequalities continue to grow.

Consequently, we obtain that an environmental policy, consisting in a public investment in environmental protection financed by a tax on pollution, is an efficient tool to reduce inequalities, through its positive effect on health. Nevertheless, to escape the inequality trap, the government should implement the environmental policy as soon as possible because the later it reacts, the higher is the required level of the tax on pollution.

Our results are related to those of Aloi and Tournemaine (2013) who take into account the effect of environmental policy on inequalities. In their paper, they assume that human capital accumulation of lower-skilled individuals are more affected by pollution than higher-skilled, but this difference depends only on the type of agents and not on their level of human capital. Instead, we extent this mechanism by considering that the effect of pollution on health, and in particular on longevity, is determined by the individual's human capital level, which affects her ability to protect herself through knowledge about pollution or healthy behaviors, through access to healthcare... It follows that the return on investment in education depends on the levels of inequality and pollution in our model and that the economy may be stuck in a trap with widening inequalities or may converge to a long-term equilibrium without inequality. As in Aloi and Tournemaine (2013), the environmental policy leads to a decrease in pollution and favors the more disadvantaged agents. However, in their model, a tighter tax on pollution always decreases inequalities, while it is not always the case for us. The economy may escape the trap following an increase in the tax on pollution when inequalities and pollution intensity are not too high. In this case, inequalities reduce over time until they vanish in the long run. But when these variables are too high, the increase in the tax may be insufficient, which means that the economy continues to be more and more unequal. In this case, other kinds of policy are necessary and can be combine with an environmental policy to favor both education of the less-advantaged

agents and the environment, so that the economy can escape the trap. In particular, one can think about an educational policy oriented toward the lower-skilled agents.

Concerning the effects of the environmental policy on the long-term characteristics of the economy, we analyze in the numerical illustration what happens at the two balanced growth paths E and I but also at the limit of the trap $(k_{I0}, 0, x_{I0}^s)$, where inequalities are maximum. We observe the following:

Numerical result 5 *A tighter tax $\tau \in [0, 1)$ decreases the long-term capital labor ratios (k_E, k_I and k_{I0}) and improves the long-term growth rates (g_E, g_I and g_{I0}).*

We emphasize in Section 4.2 that a tighter tax on pollution enhances the long-term growth rate at the balanced growth path without inequality E. The numerical analysis illustrates that it is also the case for an economy still in the inequality trap after the increase in the tax. The mechanism is similar than the one exposed previously. The environmental policy improves the life expectancy of all agents, which enhances their preferences for the future and hence the return on their investment in education. Therefore, the growth rate of average human capital increases. However, in this case, despite the fact that this policy favors more lower-skilled than higher-skilled individuals, it is not sufficient to makes the return to education of lower-skilled larger than the one of higher-skilled parents, so that inequalities in human capital and life expectancy continue to get worse.

Note that, the tax on pollution reduces the flow of pollution in the short run and the pollution intensity in the long run. Nevertheless, we do not know what is the effect on the long-term pollution level, as the size of the pollution depends also on average human capital which grows at the balanced growth path.

5 Conclusion

In this paper, we analyze the implications of environmental policy on an economy characterized by health disparities among its population. These inequalities stem from the fact that the life expectancy of an individual depends on the level of pollution in the economy, but also on her level of human capital. Even if everyone suffers from pollution, it is especially a problem for people with low human capital, through the lack of knowledge and information or through difficulties in accessing to healthcare.

We show that multiple balanced growth paths may exist. The economy may converge to a long-term equilibrium without inequality or be stuck in an environmental trap with steadily increasing inequalities. The reason is that endogenous longevity makes the return of the investment in education vary according to the pollution intensity and the level of inequalities in the economy. More precisely, when the levels of inequalities or pollution

increase, the life expectancy of lower-skilled agents decreases, which reduces the time they enjoy from their investments and thus their preferences for the future. Moreover, these decreases are in absolute terms but also relatively to higher-skilled agents. Therefore, the gap among households in the economy grows. When the economy is initially not too unequal and not too polluted, education is relatively more profitable for lower-skilled households, so that inequalities can reduce over time and can disappear in the long run. But when initial inequalities or pollution are too high, the return on investment in education may become lower for lower- than higher-skilled households and inequalities among agents persistently grow.

We reveal that an environmental policy, consisting in a tax on pollution and a public investment in pollution abatement, can favor both the development of the economy and the equality among households. First, a tighter environmental policy can allow the economy to escape the inequality trap. The reason is that the improvement in environmental quality increases more the return on investment in education of lower-skilled households, who are more vulnerable to pollution. However, getting out the trap is not always possible for reasonable tax rate. If the levels of inequalities or of pollution are too high initially, the decrease in pollution may be insufficient to outweigh the bad education return for lower-skilled agents. Second, we find that a tighter tax always enhances the long-term growth of the economy. This is due to the positive effect of the decrease in pollution on agents life expectancy which affects behaviors and promotes the investment for the future, as education. We conclude in favor of an environmental policy as a tool to address inequalities and enhance growth. However, it is not always efficient and the government should implement such policy as soon as possible to ensure that inequalities reduce and vanish in the long run.

6   Appendix

6.1   Proof of Proposition 1

6.1.1   Existence and uniqueness of a BGP without inequality

We study the existence and uniqueness of a BGP without inequality ($x = 1$). When there is no inequality, the dynamics is given by (24). At this BGP, we have $k_{t+1} = k_t = k$. We rewrite equation (24) as $\Omega_1 = \Omega_2$ with:

$$\Omega_1 \equiv k \frac{(a - b\tau)Ak^\alpha + 1 + \beta\sigma}{(a - b\tau)Ak^\alpha + 1 + \sigma(\beta + \gamma\mu)}$$

and
$$\Omega_2 \equiv \frac{A(1-\tau)(1-\alpha)\beta k^\alpha}{\epsilon(\gamma\mu)^\mu} \left[\frac{\sigma}{(a-b\tau)Ak^\alpha + 1 + \sigma(\beta+\gamma\mu)} \right]^{1-\mu}$$

When $\alpha < \frac{1}{2}$ and under Assumptions 1 and 2, we have:

- Ω_1 is increasing and convex in k, characterized by $\Omega_1(0) = 0$ and $\lim_{k \to +\infty} \Omega_1(k) = +\infty$.

- Ω_2 is increasing and concave in k, with $\Omega_2(0) = 0$ and $\lim_{k \to +\infty} \Omega_2(k) = +\infty$.
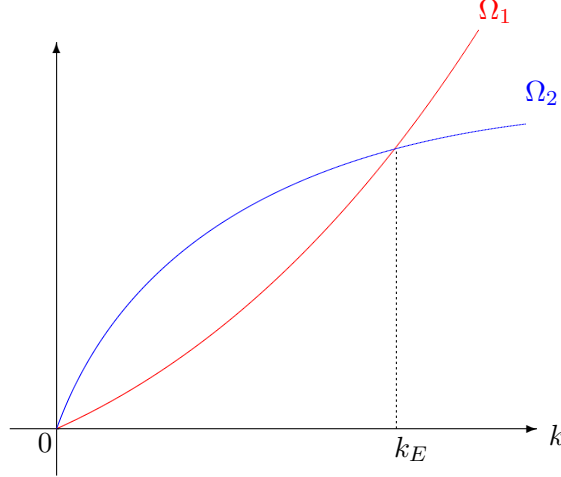

Figure 4: BGP without inequality

Moreover, $\Omega_1'(0) < \Omega_2'(0)$. Thus, the two curves cross only once and there exists a unique positive BGP without inequality $(k_E, 1, 1)$.

The growth factor at the BGP E corresponds to:

$$g_E = \epsilon \left[\frac{\sigma\gamma\mu}{(a-b\tau)Ak_E^\alpha + 1 + \sigma(\beta+\gamma\mu)} \right]^\mu \tag{27}$$

Thus, the growth rate is positive if $g_E > 1$, i.e.:

$$\epsilon > \left[\frac{(a-b\tau)Ak_E^\alpha + 1 + \sigma(\beta+\gamma\mu)}{\sigma\gamma\mu} \right]^\mu \equiv \bar{\epsilon} \tag{28}$$

6.1.2 Dynamics of the BGP without inequality

To analyze the stability of the BGP without inequality $(k_E, 1, 1)$, we compute the Jacobian matrix associated to the system (23) in E:

$$J(k_E, 1, 1) = \begin{pmatrix} \frac{\partial F_1}{\partial k_t}(k_E, 1, 1) & \frac{\partial F_1}{\partial x_t^u}(k_E, 1, 1) \\ \\ \frac{\partial F_2}{\partial k_t}(k_E, 1, 1) & \frac{\partial F_2}{\partial x_t^u}(k_E, 1, 1) \end{pmatrix} \tag{29}$$

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where F_1 and F_2 are two implicit functions given by the dynamical system (23), such that: $k_{t+1} = F_1(k_t, x_t^u)$ and $x_{t+1}^u = F_2(k_t, x_t^u)$. Therefore, we use the implicit function theorem to obtain the elements of the Jacobian matrix.

The partial derivatives of the F_2 at a BGP (k, x^u) are given by:

$$\frac{\partial F_2}{\partial k_t}(k, x^u) = \mu \left(\frac{1-\xi x^u}{1-\xi}\right)^2 \left(\frac{(1-\xi)x^u}{1-\xi x^u}\right)^{2\mu+\eta} \left(\frac{(a-b\tau)Ak^\alpha + \frac{1-\xi x^u}{1-\xi}(1+\sigma(\beta+\gamma\mu))}{(a-b\tau)Ak^\alpha + x^u(1+\sigma(\beta+\gamma\mu))}\right)^{\mu-1}$$
$$\left[\frac{(a-b\tau)A\alpha k^{\alpha-1}(1+\sigma(\beta+\gamma\mu))(x^u-1)}{((a-b\tau)Ak^\alpha + x^u(1+\sigma(\beta+\gamma\mu)))^2}\right] \tag{30}$$

$$\frac{\partial F_2}{\partial x_t^u}(k, x^u) =$$
$$\left(\frac{(1-\xi)x^u}{1-\xi x^u}\right)^{2\mu+\eta-1} \left(\frac{(a-b\tau)Ak^\alpha + \frac{1-\xi x^u}{1-\xi}(1+\sigma(\beta+\gamma\mu))}{(a-b\tau)Ak^\alpha + x^u(1+\sigma(\beta+\gamma\mu))}\right)^{\mu-1} \left(\frac{(1-\xi x^u)^2}{1-\xi}\right)$$
$$\left[(2\mu+\eta)\frac{1-\xi}{(1-\xi x^u)^2}\left(\frac{(a-b\tau)Ak^\alpha + \frac{1-\xi x^u}{1-\xi}(1+\sigma(\beta+\gamma\mu))}{(a-b\tau)Ak^\alpha + x^u(1+\sigma(\beta+\gamma\mu))}\right) - \mu \frac{x^u}{1-\xi x^u} \frac{(1+\sigma(\beta+\gamma\mu))[(a-b\tau)Ak^\alpha + 1 + \sigma(\beta+\gamma\mu)]}{((a-b\tau)Ak^\alpha + x^u(1+\sigma(\beta+\gamma\mu)))^2}\right] \tag{31}$$

The partial derivatives of the F_1 at a BGP (k, x^u) are given by:

$$\frac{\partial F_1}{\partial k_t}(k, x^u) = \frac{A(1-\tau)(1-\alpha)\beta}{\epsilon(\gamma\mu)^\mu(V_1 V_2)^2} \frac{\alpha k^{\alpha-1}V_3 V_1 V_2 + k^\alpha\left[V_3' V_1 V_2 - V_3\left(V_1 V_2' + V_2 \frac{\partial F_2}{\partial k_t}(k, x^u)W_1\right)\right]}{V_4} \tag{32}$$

with

$$V_1 \equiv \xi x^u \frac{(a-b\tau)Ak^\alpha + x^u(1+\sigma\beta)}{(a-b\tau)Ak^\alpha + x^u(1+\sigma(\beta+\gamma\mu))} + (1-\xi x^u)\frac{(a-b\tau)Ak^\alpha + \frac{1-\xi x^u}{1-\xi}(1+\sigma\beta)}{(a-b\tau)Ak^\alpha + \frac{1-\xi x^u}{1-\xi}(1+\sigma(\beta+\gamma\mu))}$$

$$V_2 \equiv \xi \frac{\sigma^\mu(x^u)^{2\mu+\eta}}{((a-b\tau)Ak^\alpha + x^u(1+\sigma(\beta+\gamma\mu)))^\mu} + (1-\xi)\frac{\sigma^\mu(\frac{1-\xi x^u}{1-\xi})^{2\mu+\eta}}{\left((a-b\tau)Ak^\alpha + \frac{1-\xi x^u}{1-\xi}(1+\sigma(\beta+\gamma\mu))\right)^\mu}$$

$$V_3 \equiv \xi \frac{\sigma(x^u)^2}{(a-b\tau)Ak^\alpha + x^u(1+\sigma(\beta+\gamma\mu))} + \frac{\sigma\frac{(1-\xi x^u)^2}{1-\xi}}{(a-b\tau)Ak^\alpha + \frac{1-\xi x^u}{1-\xi}(1+\sigma(\beta+\gamma\mu))}$$

$$V_4 \equiv \frac{A(1-\tau)(1-\alpha)\beta k^\alpha}{\epsilon(\gamma\mu)^\mu} \frac{V_3}{V_2(V_1)^2}\left(\xi x^u \frac{(a-b\tau)A\alpha k^{\alpha-1}\sigma\gamma\mu x^u}{((a-b\tau)Ak^\alpha + x^u(1+\sigma(\beta+\gamma\mu)))^2} + (1-\xi x^u)\frac{(a-b\tau)A\alpha k^{\alpha-1}\sigma\gamma\mu\frac{1-\xi x^u}{1-\xi}}{\left((a-b\tau)Ak^\alpha + \frac{1-\xi x^u}{1-\xi}(1+\sigma(\beta+\gamma\mu))\right)^2}\right) + 1$$

$$W_1 = \xi\left[\frac{(a-b\tau)Ak^\alpha + x^u(1+\sigma\beta)}{(a-b\tau)Ak^\alpha + x^u(1+\sigma(\beta+\gamma\mu))} - \frac{(a-b\tau)Ak^\alpha\sigma\gamma\mu x^u}{((a-b\tau)Ak^\alpha + x^u(1+\sigma(\beta+\gamma\mu)))^2}\right.$$

$$\left. -\frac{(a-b\tau)Ak^\alpha + \frac{1-\xi x^u}{1-\xi}(1+\sigma\beta)}{(a-b\tau)Ak^\alpha + \frac{1-\xi x^u}{1-\xi}(1+\sigma(\beta+\gamma\mu))} + \frac{(a-b\tau)Ak^\alpha\sigma\gamma\mu\frac{1-\xi x^u}{1-\xi}}{\left((a-b\tau)Ak^\alpha + \frac{1-\xi x^u}{1-\xi}(1+\sigma(\beta+\gamma\mu))\right)^2}\right]$$

$$V_2' = -\mu\sigma^\mu(a-b\tau)A\alpha k^{\alpha-1}\left[\frac{\xi(x^u)^{2\mu+\eta}}{((a-b\tau)Ak^\alpha + x^u(1+\sigma(\beta+\gamma\mu)))^{1+\mu}} + \frac{(1-\xi)\left(\frac{1-\xi x^u}{1-\xi}\right)^{2\mu+\eta}}{\left((a-b\tau)Ak^\alpha + \frac{1-\xi x^u}{1-\xi}(1+\sigma(\beta+\gamma\mu))\right)^{1+\mu}}\right]$$

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$$V_3' = -\sigma(a-b\tau)A\alpha k^{\alpha-1}\left[\frac{\xi(x^u)^2}{((a-b\tau)Ak^\alpha+x^u(1+\sigma(\beta+\gamma\mu)))^2} + \frac{\frac{(1-\xi x^u)^2}{1-\xi}}{\left((a-b\tau)Ak^\alpha+\frac{1-\xi x^u}{1-\xi}(1+\sigma(\beta+\gamma\mu))\right)^2}\right]$$

and

$$\frac{\partial F_1}{\partial x_t^u}(k,x^u) = \frac{\frac{A(1-\tau)(1-\alpha)k^\alpha\beta}{\epsilon(\gamma\mu)^\mu(V_1V_2)^2}\left[W_3'V_1V_2 - V_3\left(V_1W_2' + V_2\frac{\partial F_2}{\partial x_t^u}(k,x^u)W_1\right)\right]}{V_4} \tag{33}$$

with

$$W_2' = \xi\sigma^\mu\left[\frac{(2\mu+\eta)(x^u)^{2\mu+\eta-1}}{((a-b\tau)Ak^\alpha+x^u(1+\sigma(\beta+\gamma\mu)))^\mu} - \frac{(x^u)^{2\mu+\eta}\mu(1+\sigma(\beta+\gamma\mu))}{((a-b\tau)Ak^\alpha+x^u(1+\sigma(\beta+\gamma\mu)))^{1+\mu}}\right.$$

$$\left. - \frac{(2\mu+\eta)(\frac{1-\xi x^u}{1-\xi})^{2\mu+\eta-1}}{\left((a-b\tau)Ak^\alpha+\frac{1-\xi x^u}{1-\xi}(1+\sigma(\beta+\gamma\mu))\right)^\mu} + \frac{\left(\frac{1-\xi x^u}{1-\xi}\right)^{2\mu+\eta}\mu(1+\sigma(\beta+\gamma\mu))}{\left((a-b\tau)Ak^\alpha+\frac{1-\xi x^u}{1-\xi}(1+\sigma(\beta+\gamma\mu))\right)^{1+\mu}}\right]$$

$$W_3' = \xi\sigma\left[\frac{2x^u(a-b\tau)Ak^\alpha+(x^u)^2(1+\sigma(\beta+\gamma\mu))}{((a-b\tau)Ak^\alpha+x^u(1+\sigma(\beta+\gamma\mu)))^2} - \frac{2\frac{1-\xi x^u}{1-\xi}(a-b\tau)Ak^\alpha+\left(\frac{1-\xi x^u}{1-\xi}\right)^2(1+\sigma(\beta+\gamma\mu))}{\left((a-b\tau)Ak^\alpha+\frac{1-\xi x^u}{1-\xi}(1+\sigma(\beta+\gamma\mu))\right)^2}\right]$$

At the BGP without inequality, $\frac{\partial F_2}{\partial k_t}(k_E,1) = \frac{\partial F_1}{\partial x_t^u}(k_E,1) = 0$, while $\frac{\partial F_1}{\partial k_t}(k_E,1)$ and $\frac{\partial F_2}{\partial x_t^u}(k_E,1)$ are greater than 0. Therefore, the two eigenvalues are given by: $\frac{\partial F_1}{\partial k_t}(k_E,1)$ and $\frac{\partial F_2}{\partial x_t^u}(k_E,1)$. Under Assumptions 1 and 2, under the condition $\alpha < 1/2$, and substituting the expression of k_E given in (24) at the BGP $(k_E,1,1)$, we have $0 < \frac{\partial F_1}{\partial k_t}(k_E,1) < 1$. Thus, the BGP E is stable iif $\frac{\partial F_2}{\partial x_t^u}(k_E,1) < 1$, which is equivalent to:

$$1 - \left(2\mu + \eta - \frac{\mu(1+\sigma(\beta+\gamma\mu))}{(a-b\tau)Ak_E^\alpha+(1+\sigma(\beta+\gamma\mu))}\right) > 0 \tag{34}$$

When the condition given in (34) is satisfied, the BGP without inequality is locally stable (a sink), otherwise it is a saddle point. This condition can be rewritten more clearly in terms of η, as $\eta < \bar{\eta}(\tau)$ with

$$\bar{\eta}(\tau) \equiv 1 - \mu\frac{2(a-b\tau)Ak_E^\alpha + (1+\sigma(\beta+\gamma\mu))}{(a-b\tau)Ak_E^\alpha + (1+\sigma(\beta+\gamma\mu))} \tag{35}$$

Thus, the BGP without inequality E is stable when $\eta < \bar{\eta}(\tau)$ and corresponds to a saddle point when $\eta > \bar{\eta}(\tau)$. Note that when $\mu + \eta \to 0$, the condition (34) is always satisfied, i.e. the BGP E is always stable, while when $\mu + \eta \to 1$, (34) is never satisfied, i.e. the BGP E is always a saddle.

$\square$

6.2 Proof of Proposition 2

We study the existence and uniqueness of a BGP with inequality ($x^u < 1 < x^s$). The dynamical system is described by (23) and depends on two variables k and x^u. After computations, the system at the BGP with inequalities, where $x_{t+1}^u = x_t^u = x^u \neq 1$ and

$k_{t+1} = k_t = k$, corresponds to:

$$\begin{cases} k^{1-\alpha}\frac{\epsilon(\sigma\gamma\mu)^{\mu}}{A(1-\tau)(1-\alpha)\beta}\left[\frac{\left[(a-b\tau)Ak^{\alpha}+\frac{1-\xi x^{u}}{1-\xi}(1+\sigma\beta)\right]\left[(a-b\tau)Ak^{\alpha}+x^{u}(1+\sigma(\beta+\gamma\mu))\right]+\xi x^{u}(a-b\tau)Ak^{\alpha}\sigma\gamma\mu\left(\frac{1-\xi x^{u}}{1-\xi}-x^{u}\right)}{(a-b\tau)Ak^{\alpha}\sigma\left(\frac{(1-\xi x^{u})^{2}}{1-\xi}+\xi(x^{u})^{2}\right)+\frac{1-\xi x^{u}}{1-\xi}\sigma x^{u}(1+\sigma(\beta+\gamma\mu))}\right] \\[2em] \left[\frac{\xi(x^{u})^{2\mu+\eta}}{[(a-b\tau)Ak^{\alpha}+x^{u}(1+\sigma(\beta+\gamma\mu))]^{\mu}}+\frac{(1-\xi)\left(\frac{1-\xi x^{u}}{1-\xi}\right)^{2\mu+\eta}}{\left[(a-b\tau)Ak^{\alpha}+\left(\frac{1-\xi x^{u}}{1-\xi}\right)(1+\sigma(\beta+\gamma\mu))\right]^{\mu}}\right]-1=0\equiv\mathcal{A}(k,x^{u}) \\[2em] k=\left[\frac{1+\sigma(\beta+\gamma\mu)}{(1-\xi)A(a-b\tau)}\frac{(1-\xi)x^{u}(1-\xi x^{u})^{\frac{2\mu+\eta-1}{\mu}}-((1-\xi)x^{u})^{\frac{2\mu+\eta-1}{\mu}}(1-\xi x^{u})}{((1-\xi)x^{u})^{\frac{2\mu+\eta-1}{\mu}}-(1-\xi x^{u})^{\frac{2\mu+\eta-1}{\mu}}}\right]^{\frac{1}{\alpha}}\equiv\Psi_{2}(x^{u}) \end{cases}$$

$$(36)$$

6.2.1 Properties of the function Ψ_2

The second equation of (25) defines $k = \Psi_2(x^u)$. Under Assumptions 1 and 2 and the conditions $2\mu + \eta > 1$ and $\alpha < 1/2$, the properties of this function are:

- $Sign(\Psi_2') = u'v - uv'$ with:

$$u = (1-\xi)x^{u}(1-\xi x^{u})^{\frac{2\mu+\eta-1}{\mu}} - ((1-\xi)x^{u})^{\frac{2\mu+\eta-1}{\mu}}(1-\xi x^{u}) < 0$$

$$v = ((1-\xi)x^{u})^{\frac{2\mu+\eta-1}{\mu}} - (1-\xi x^{u})^{\frac{2\mu+\eta-1}{\mu}} < 0$$

v' and u' are equal to:

$$v' = \frac{2\mu+\eta-1}{\mu}\left[((1-\xi)x^{u})^{\frac{\mu+\eta-1}{\mu}}(1-\xi) + (1-\xi x^{u})^{\frac{\mu+\eta-1}{\mu}}\xi\right] > 0$$

$$\begin{aligned} u' &= \xi((1-\xi)x^{u})^{\frac{2\mu+\eta-1}{\mu}} + (1-\xi)(1-\xi x^{u})^{\frac{2\mu+\eta-1}{\mu}} \\ &\quad -\frac{2\mu+\eta-1}{\mu}\left[\xi((1-\xi)x^{u})(1-\xi x^{u})^{\frac{\mu+\eta-1}{\mu}} + (1-\xi)(1-\xi x^{u})((1-\xi)x^{u})^{\frac{\mu+\eta-1}{\mu}}\right] \end{aligned}$$

We rewrite this last equation as $u' = \mathcal{I}(x^u) - \mathcal{J}(x^u)$, where $\mathcal{I}(x^u)$ corresponds to the first part (first line) of the equation and $\mathcal{J}(x^u)$ corresponds to the second one.

- $\mathcal{I}(0) = (1-\xi)$, $\mathcal{I}(1) = (1-\xi)^{\frac{2\mu+\eta-1}{\mu}} > \mathcal{I}(0)$ and $\mathcal{I}'(x^u) > 0$.
- $\mathcal{J}(0) = +\infty$, $\mathcal{J}(1) = \frac{2\mu+\eta-1}{\mu}(1-\xi)^{\frac{2\mu+\eta-1}{\mu}} < \mathcal{I}(1)$ and

$$\begin{aligned} \mathcal{J}'(x^u) &= \frac{2\mu+\eta-1}{\mu}\left[\xi(1-\xi)\left((1-\xi x^{u})^{\frac{\mu+\eta-1}{\mu}} - ((1-\xi)x^{u})^{\frac{\mu+\eta-1}{\mu}}\right)\right. \\ &\quad \left.+\frac{\mu+\eta-1}{\mu}\left((1-\xi)^{2}(1-\xi x^{u})((1-\xi)x^{u})^{\frac{\eta-1}{\mu}} - \xi^{2}((1-\xi)x^{u})(1-\xi x^{u})^{\frac{\eta-1}{\mu}}\right)\right] \end{aligned}$$

$$(37)$$

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$\mathcal{J}'(x^u)$ is an increasing function of x^u ($\mathcal{J}''(x^u) > 0$) which is always negative in $x^u = 0$ but may become positive for high x^u when $\xi > 1/2$ ($\mathcal{J}'(1) > 0$ when $\xi > 1/2$).

- u' is negative as long as $\mathcal{J}(x^u) > \mathcal{I}(x^u)$. Thus, we can define a threshold $\hat{x}^u \in (0,1)$ under which u' is negative and above which u' is positive for high level of ξ.

- The condition $u' < 0$ is sufficient to ensure that $\Psi_2' > 0$. Thus, we show that there exists a threshold $\hat{x}^u \in (0,1)$ under which Ψ_2 is an increasing function of x^u and above which Ψ_2 may become decreasing (for high level of ξ).

- Moreover, $\Psi_2 \geqslant 0 \ \forall x^u$, $\Psi_2(0) = 0$ and

$$\lim_{x^u \to 1} \Psi_2(x^u) = \left[\frac{1 + \sigma(\beta + \gamma\mu)}{A(a - b\tau)} \frac{1 - \mu - \eta}{2\mu + \eta - 1} \right]^{\frac{1}{\alpha}} > 0 \qquad (38)$$

6.2.2 Properties of the function Ψ_1

The first equation of (25), $\mathcal{A}(k, x^u) = 0$ allows to define $k = \Psi_1(x^u)$, with $\Psi_1(x^u)$, an implicit function. Under Assumptions 1 and 2 and the conditions $2\mu + \eta > 1$ and $\alpha < 1/2$, we obtain that $\Psi_1(0)$ and $\Psi_1(1)$ are equal to two positive constants.

More precisely, in $x^u = 0$ we have:

$$\mathcal{A}(k, 0) = 0 \Leftrightarrow k^{1-\alpha} \frac{\epsilon(\sigma\gamma\mu)^\mu (1-\xi)^{2-2\mu-\eta}}{A(1-\tau)(1-\alpha)\beta\sigma} \frac{(a-b\tau)Ak^\alpha + \frac{1}{1-\xi}(1+\sigma\beta)}{\left[(a-b\tau)Ak^\alpha + \left(\frac{1}{1-\xi}\right)(1+\sigma(\beta+\gamma\mu))\right]^\mu} = 1$$

$$\Leftrightarrow k^{1-\alpha} \left[(a-b\tau)Ak^\alpha + \frac{1+\sigma\beta}{1-\xi}\right] = \frac{A(1-\tau)(1-\alpha)\beta\sigma^{1-\mu}}{\epsilon(\gamma\mu)^\mu (1-\xi)^{2-2\mu-\eta}} \left[(a-b\tau)Ak^\alpha + \frac{1+\sigma(\beta+\gamma\mu)}{1-\xi}\right]^\mu$$

$$(39)$$

We analyze the properties of $\Psi_1(0)$ by studying the last equation. For that, we name the function on the left side $f_0(k)$ and the function on the right side $g_0(k)$. Their properties are:

- f_0 is increasing and concave in k, $f_0(0) = 0$ and $\lim_{k \to \infty} f_0(k) = +\infty$.

- g_0 is increasing and concave in k, $g_0(0)$ is equal to a positive constant and $\lim_{k \to \infty} g_0(k) = +\infty$.

- In $k = 0$, $g_0 > f_0$. The two curves have not cross yet, thus $\Psi_1(0) > 0$.

- When $k \to \infty$, we have $\lim_{k \to \infty} f_0 > \lim_{k \to \infty} g_0$. Thus, the two curves cross only once and for a positive and finite value of k.

Therefore, $\Psi_1(0)$ is always a finite and positive constant.

In the same way, in $x^u = 1$ we have:

$$\mathcal{A}(k,1) = 0 \Leftrightarrow k^{1-\alpha} \frac{\epsilon(\sigma\gamma\mu)^\mu}{A(1-\tau)(1-\alpha)\beta\sigma} \frac{(a-b\tau)Ak^\alpha + 1 + \sigma\beta}{[(a-b\tau)Ak^\alpha + 1 + \sigma(\beta+\gamma\mu)]^\mu} = 1$$

$$\Leftrightarrow k^{1-\alpha}\left[(a-b\tau)Ak^\alpha + 1 + \sigma\beta\right] = \frac{A(1-\tau)(1-\alpha)\beta\sigma^{1-\mu}}{\epsilon(\gamma\mu)^\mu}\left[(a-b\tau)Ak^\alpha + 1 + \sigma(\beta+\gamma\mu)\right]^\mu \tag{40}$$

As previously, we stydy the properties of $\Psi_1(1)$, by looking at the last equation. We name the function on the left side $f_1(k)$ and the function on the right side $g_1(k)$, whose properties are:

- f_1 is increasing and concave in k, $f_1(0) = 0$ and $\lim_{k\to\infty} f_1(k) = +\infty$.

- g_1 is increasing and concave in k, $g_1(0)$ is equal to a positive constant and $\lim_{k\to\infty} g_1(k) = +\infty$.

- In $k = 0$, $g_1 > f_1$, the two curves have not cross yet thus $\Psi_1(1) > 0$.

- When $k \to \infty$, we have $\lim_{k\to\infty} f_1 > \lim_{k\to\infty} g_1$. Thus, the two curves cross only once and for a positive and finite value of k.

Therefore, $\Psi_1(1)$ is equal to a finite and positive constant.

6.2.3 Comparison of Ψ_1 and Ψ_2

From the study of the properties of Ψ_1 and Ψ_2, we know that $\Psi_1(0) > 0$ and $\Psi_2(0) = 0$, it entails that $\Psi_1(0) > \Psi_2(0)$. It follows that if $\Psi_1(1) < \lim_{x^u\to 1} \Psi_2(x^u)$, there exists at least one BGP with inequalities.

From Appendix 6.2.2, the condition $\Psi_1(1) < \lim_{x^u\to 1} \Psi_2(x^u)$ is equivalent to $f_1(k) > g_1(k)$ in $k = \lim_{x^u\to 1} \Psi_2(x^u)$ given in (38). We obtain that $\Psi_1(1) < \lim_{x^u\to 1} \Psi_2(x^u)$ if $A < \bar{A}(\tau)$ with:

$$
\begin{aligned}
\bar{A}(\tau) \equiv & \left\{ \frac{\epsilon(\gamma\mu)^\mu}{(1-\tau)(1-\alpha)\beta\sigma^{1-\mu}}\left[\frac{2\mu+\eta-1}{\mu(1+\sigma(\beta+\gamma\mu))}\right]^\mu \left[\frac{1+\sigma(\beta+\gamma\mu)}{(a-b\tau)}\frac{1-\mu-\eta}{2\mu+\eta-1}\right]^{\frac{1-\alpha}{\alpha}} \right. \\
& \left. \left[\frac{\sigma\gamma\mu(1-\mu-\eta)+(1+\beta\sigma)\mu}{2\mu+\eta-1}\right]\right\}^\alpha
\end{aligned}
\tag{41}
$$

Thus, under Assumptions 1 and 2 and the conditions $2\mu+\eta > 1$ and $\alpha < 1/2$, the condition $A < \bar{A}(\tau)$ is sufficient so that there exists at least one BGP with inequalities.

Note that when $\mu + \eta \to 0$, Ψ_2 corresponds to strictly negative values of k $\forall x^u$, so that there is no BGP with inequalities in this case.
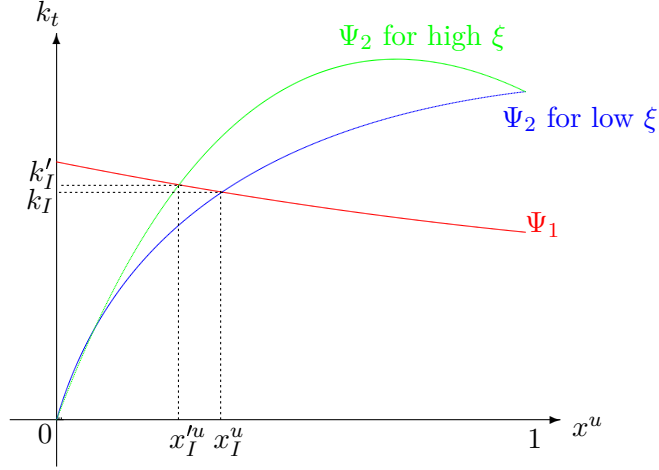
Figure 5: A representation of the dynamics when $x^u \neq 1$ (with Ψ_1 decreasing in x^u)

Note that the condition $2\mu + \eta > 1$ does not always imply that the condition $\eta < \bar{\eta}$ of Proposition 1 is not verified. Therefore, it is possible to have these two conditions verified simultaneously, and hence to have both that the BGP with inequalities exists and that the BGP without inequality is stable.

6.3   Proof of Proposition 3

The threshold under which the BGP E is stable $\bar{\eta}(\tau)$ is given by (35) in Appendix 6.1.2. To analyze the effect of τ on the dynamics of E, we compute $\frac{\partial \bar{\eta}(\tau)}{\partial \tau}$:

$$\frac{\partial \bar{\eta}(\tau)}{\partial \tau} = \frac{\mu(1 + \sigma(\beta + \gamma\mu))Ak_E^{\alpha-1}}{[(a - b\tau)Ak_E^\alpha + 1 + \sigma(\beta + \gamma\mu)]^2}\left(bk_E - (a - b\tau)\alpha\frac{\partial k_E}{\partial \tau}\right) \qquad (42)$$

The effect of the pollution tax on the dynamics at the BGP E depends on $\frac{\partial k_E}{\partial \tau}$. To compute this derivative, we use the dynamical equation (24) at the BGP:

$$\Phi(k, \tau) \equiv k\frac{(a - b\tau)Ak^\alpha + 1 + \beta\sigma}{(a - b\tau)Ak^\alpha + 1 + \sigma(\beta + \gamma\mu)} - \frac{A(1 - \tau)(1 - \alpha)\beta k^\alpha}{\epsilon(\gamma\mu)^\mu}\left[\frac{\sigma}{(a - b\tau)Ak^\alpha + 1 + \sigma(\beta + \gamma\mu)}\right]^{1-\mu} = 0$$

The effect of τ on k in E is given by the implicit function theorem:

$$\frac{\partial k}{\partial \tau}(k_E, 1) = -\frac{\frac{\partial \Phi}{\partial \tau}}{\frac{\partial \Phi}{\partial k_E}}$$

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After computations, we obtain the two partial derivatives:

$$\frac{\partial \Phi}{\partial \tau} = \left(- bAk^{1+\alpha}\sigma\gamma\mu + \frac{A(1-\alpha)\beta k^\alpha \sigma^{1-\mu}}{\epsilon(\gamma\mu)^\mu}\left[Ak^\alpha(a - b(1 - \mu(1 - \tau))) + 1 + \sigma(\beta + \gamma\mu)\right] \right.$$
$$\left. \left[(a - b\tau)Ak^\alpha + 1 + \sigma(\beta + \gamma\mu)\right]^\mu \right) \left[(a - b\tau)Ak^\alpha + 1 + \sigma(\beta + \gamma\mu)\right]^{-2} \tag{43}$$

$$\frac{\partial \Phi}{\partial k} = \left[((a - b\tau)Ak^\alpha)^2 + (a - b\tau)Ak^\alpha[2(1 + \beta\sigma) + \sigma\gamma\mu(1 + \alpha)] + (1 + \sigma(\beta + \gamma\mu))(1 + \sigma\beta) \right.$$
$$\left. - \frac{A(1-\alpha)(1-\tau)\beta\alpha k^{\alpha-1}\sigma^{1-\mu}}{\epsilon(\gamma\mu)^\mu}\left[(a - b\tau)Ak^\alpha + 1 + \sigma(\beta + \gamma\mu)\right]^\mu \left[Ak^\alpha\mu(a - b\tau) + 1 + \sigma(\beta + \gamma\mu)\right] \right]$$
$$\left[(a - b\tau)Ak^\alpha + 1 + \sigma(\beta + \gamma\mu)\right]^{-2} \tag{44}$$

And, we have:
$$Sign\{\frac{\partial\bar{\eta}(\tau)}{\partial\tau}\} = Sign\{bk_E - (a - b\tau)\alpha\frac{\partial k_E}{\partial\tau}\}$$

Thus, $\frac{\partial\bar{\eta}(\tau)}{\partial\tau} > 0$ iif:

$$bk_E\left[((a - b\tau)Ak_E^\alpha)^2 + (a - b\tau)Ak_E^\alpha[2(1 + \beta\sigma) + \sigma\gamma\mu(1 + \alpha)] + (1 + \sigma(\beta + \gamma\mu))(1 + \sigma\beta)\right]$$
$$- \frac{A(1-\alpha)(1-\tau)\beta\alpha k_E^\alpha\sigma^{1-\mu}b}{\epsilon(\gamma\mu)^\mu}\left[(a - b\tau)Ak_E^\alpha + 1 + \sigma(\beta + \gamma\mu)\right]^\mu \left[Ak_E^\alpha\mu(a - b\tau) + 1 + \sigma(\beta + \gamma\mu)\right]$$
$$+ (a - b\tau)\alpha\left(- bAk_E^{1+\alpha}\sigma\gamma\mu + \frac{A(1-\alpha)\beta k_E^\alpha\sigma^{1-\mu}}{\epsilon(\gamma\mu)^\mu}\left[Ak_E^\alpha(a - b(1 - \mu(1 - \tau))) + 1 + \sigma(\beta + \gamma\mu)\right]\right.$$
$$\left. \left[(a - b\tau)Ak_E^\alpha + 1 + \sigma(\beta + \gamma\mu)\right]^\mu \right) > 0 \tag{45}$$

It can be rewritten as:

$$bk_E\left[((a - b\tau)Ak_E^\alpha)^2 + (a - b\tau)Ak_E^\alpha[2(1 + \beta\sigma) + \sigma\gamma\mu] + (1 + \sigma(\beta + \gamma\mu))(1 + \sigma\beta)\right]$$
$$+ (a - b\tau)\alpha\left(\frac{A(1-\alpha)\beta k_E^\alpha\sigma^{1-\mu}}{\epsilon(\gamma\mu)^\mu}Ak_E^\alpha(a - b)\left[(a - b\tau)Ak_E^\alpha + 1 + \sigma(\beta + \gamma\mu)\right]^\mu\right) \tag{46}$$
$$+ (1 + \sigma(\beta + \gamma\mu))\frac{A(1-\alpha)\beta\alpha k_E^\alpha\sigma^{1-\mu}}{\epsilon(\gamma\mu)^\mu}\left[(a - b\tau)Ak_E^\alpha + 1 + \sigma(\beta + \gamma\mu)\right]^\mu (a - b) > 0$$

Under Assumption 2, the condition $bk_E - (a - b\tau)\alpha\frac{\partial k_E}{\partial\tau} > 0$ is always verified. Therefore, under Assumptions 1 and 2 and for $\alpha < 1/2$, the threshold $\bar{\eta}(\tau)$ depends positively on the tax rate τ.

Moreover, under Assumptions 1 and 2, for $2\mu + \eta > 1$ and $\alpha < 1/2$, the threshold $\bar{A}(\tau)$ depends positively on the tax rate τ and the condition $A < \bar{A}(\tau)$ is always satisfied when τ tends to 1. Thus, a tighter tax on pollution increases the range of parameters for which there exists at least one BGP with inequalities.

$\square$

6.4 Proof of Proposition 5

We analyze the effect of the tax rate on the growth factor at the BGP without inequality g_E, given by (26). Its derivative with respect to τ is:

$$\frac{\partial g_E}{\partial \tau} = \epsilon(\sigma\gamma\mu)^\mu \mu[(a - b\tau)Ak_E^\alpha + 1 + \sigma(\beta + \gamma\mu)]^{-\mu-1}k_E^{\alpha-1}\left\{bAk_E - (a - b\tau)A\alpha\frac{\partial k_E}{\partial \tau}\right\}$$

The effect of the pollution tax on the growth at the BGP E depends on $\frac{\partial k_E}{\partial \tau}$ and more precisely we have:

$$Sign\left\{\frac{\partial g_E}{\partial \tau}\right\} = Sign\left\{bk_E - (a - b\tau)\alpha\frac{\partial k_E}{\partial \tau}\right\}$$

From Appendix 6.3, we know that under Assumption 2, $bk_E - (a - b\tau)\alpha\frac{\partial k_E}{\partial \tau} > 0$. Therefore, under Assumption 2, we have that $\frac{\partial g_E}{\partial \tau} > 0$: the growth rate at the BGP without inequality g_E increases following an increase in the pollution tax.

6.5 Sensitivity Analysis

In this section, we analyze the robustness of our results with respect to two key parameters: the share of lower-skilled individuals in the economy ξ and the weight of intergenerational transmission in human capital accumulation η. Note that this two sensitivity analysis are done for a particular tax rate τ, i.e. $\tau = 0$ We have performed the analysis for other values and results are similar but to save place, we have only reported here the results for $\tau = 0$. Moreover, we show the impact of the tax on the economy in Table 3.

The effect of ξ, representing the repartition of the two types of individuals in the population, is illustrated in Figure 6 and Table 1. The share of lower-skilled individuals in the population does not affect the value of the long-term equilibrium without inequality but modifies the long-term equilibrium with inequalities. The higher the share of poor individuals in the population, the higher is the capital-labor ratio and the lower is the relative human capital of lower-skilled at the balanced growth path with inequality. It entails that, at this equilibrium, inequalities will be wider and that the growth will be lower. However, the dynamics of both balanced growth paths remains the same. Thus, the threshold in terms of initial inequalities under which the economy is in the inequality trap is lower. The fact that a higher share of population is lower-skilled, for a same average level of human capital, implies that the relative disadvantage of lower-skilled agents with respect to the rest of the population is lower, which makes the human capital convergence easier.

Figure 7 and Table 2 illustrate the evolution of the two long-term equilibria with respect to η, the weight of intergenerational transmission in human capital accumulation. As for the share of lower-skilled households in the economy, an increase in the weight of inter-

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Table 1: Sensitivity Analysis with respect to ξ when $\tau = 0$

ξ	k_I	x_I^u	g_I	$CAGR_I$	π_I^u	π_I^s	LE_I^u	LE_I^s	$AverageLE_I$
0.50	0.0241	0.0955	1.5413	1.452%	0.3197	0.8250	69.5919	84.7485	77.1702
0.70	0.0271	0.0738	1.4473	1.240%	0.2615	0.8515	67.8435	85.5435	73.1535
0.90	0.0355	0.0480	1.2439	0.730%	0.1762	0.8818	65.2848	86.4547	67.4018

ξ	k_E	g_E	$CAGR_E$	π_E	LE_E	$Eigenvalues_E$	$Eigenvalues_I$
0.50	0.0209	1.6506	1.684%	0.7724	83.1706	$\{0.911051; 0.318788\}$	$\{1.17236; 0.321711\}$
0.70	0.0209	1.6506	1.684%	0.7724	83.1706	$\{0.911051; 0.318788\}$	$\{1.20855; 0.324857\}$
0.90	0.0209	1.6506	1.684%	0.7724	83.1706	$\{0.911051; 0.318788\}$	$\{1.27981; 0.328051\}$

Notes: $CAGR_j$ represents the compound annual growth rate at the balanced growth path $j = E, I$, while LE_j^i corresponds to the life expectancy in years of the individual i at the BGP j.
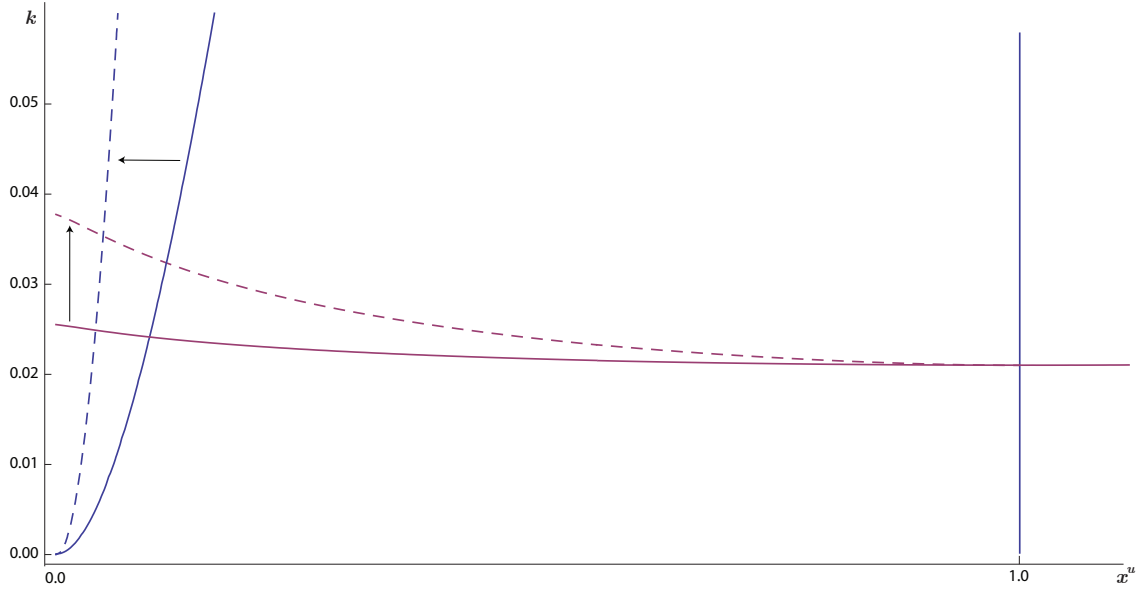

Figure 6: Sensitivity analysis with respect to ξ with $\tau = 0$, where the solid lines capture the cases $\xi = 0.5$, and the dashed lines capture the case $\xi = 0.9$.

generational spillover has no effect on the values of the variables at balanced growth path without inequality E. On the contrary, at the long-term equilibrium with inequalities, it reduces the capital labor ratio k_I and increases the relative human capital of lower-skilled x_I^u. It entails that the growth rate at the long-term equilibrium I is higher while the level of inequality is lower. Lower-skilled individuals have a higher level of human capital and live longer at this state. As we detail in Numerical Result 2, the dynamics depends on η. For $\eta < \bar{\eta}(\tau)$, the long-term equilibrium without inequality E is stable, while the one with inequalities I is an unstable saddle point.[32] Whereas for $\eta > \bar{\eta}(\tau)$, the unique BGP E is a saddle. Thus, $\bar{\eta}(\tau)$ represents not only the value above which the BGP E becomes unstable but also the value above which the BGP I disappears. The higher is the weight of intergenerational transmission in human capital accumulation, the larger is the size of the inequality trap. Indeed, up to the threshold $\bar{\eta}(\tau)$, we have that the higher η, the larger is the size of the inequality trap and the more likely the economy will converge to a situation where the lower-skilled category collapses (see Figure 7 and Table 2 for some examples). Above this threshold, the economy converges to the equilibrium without inequality only if the initial levels of inequalities and of the capital to labor ratio are very low. Therefore, for most of initial conditions the economy is stuck in an inequality trap.

Table 2: Sensitivity Analysis with respect to η when $\tau = 0$

η	k_I	x_I^u	g_I	$CAGR_I$	π_I^u	π_I^s	LE_I^u	LE_I^s	$AverageLE_I$
0.20	0.0259	0.0444	1.4847	1.326%	0.1801	0.8251	65.4019	84.7530	75.0775
0.25	0.0241	0.0955	1.5413	1.452%	0.3197	0.8250	69.5919	84.7485	77.1702
0.30	0.0223	0.2365	1.6010	1.581%	0.5251	0.8213	75.7518	84.6403	80.1960
0.35	$\varnothing$	$\varnothing$							
0.39	$\varnothing$	$\varnothing$							

η	k_E	g_E	$CAGR_E$	π_E	LE_E	$Eigenvalues_E$	$Eigenvalues_I$
0.20	0.0209	1.6506	1.684%	0.7724	83.1706	$\{0.861051; 0.318788\}$	$\{1.2362; 0.32274\}$
0.25	0.0209	1.6506	1.684%	0.7724	83.1706	$\{0.911051; 0.318788\}$	$\{1.17236; 0.321711\}$
0.30	0.0209	1.6506	1.684%	0.7724	83.1706	$\{0.961051; 0.318788\}$	$\{1.08036; 0.320204\}$
0.35	0.0209	1.6506	1.684%	0.7724	83.1706	$\{1.01105; 0.318788\}$	
0.39	0.0209	1.6506	1.684%	0.7724	83.1706	$\{1.05105; 0.318788\}$	

Notes: $CAGR_j$ represents the compound annual growth rate at the balanced growth path $j = E, I$, while LE_j^i corresponds to the life expectancy in years of the individual i at the BGP j.

Concerning the effect on environmental policy implications, we obtain that Numerical Results 3, 4 and 5 hold for all the values of the share of lower-skilled households in the population and of the intergenerational spillover.[33] In other words, a sufficient increase in

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[32]$\bar{\eta}(0) = 0.34$ and $\bar{\eta}(1) = 0.4$.

[33]It is always true considering the calibrated values for the other parameters and for all $\eta \in [0; 1 - \mu)$, i.e.
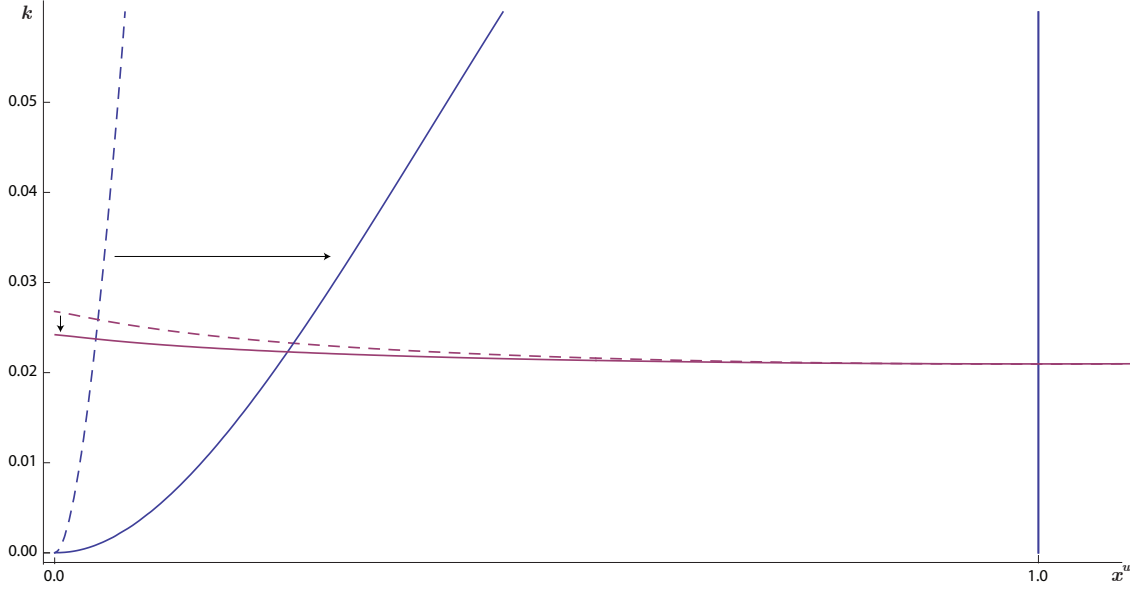
Figure 7: Sensitivity analysis with respect to η with $\tau = 0$, where the dashed lines capture the cases $\eta = 0.2$, and the solid lines refer to the case $\eta = 0.3$.

the tax on pollution can allow the economy to escape the "inequality trap". Nevertheless, when the weight of intergenerational transmission of human capital is very high $(\eta > \bar{\eta}(0))$, the effect is a slightly more complicated as the BGP with inequalities does not exist for low values of the pollution tax. We illustrate this case in Table 3. As we have noticed, above the threshold of intergenerational spillover $\bar{\eta}(0)$, the economy is most likely to be stuck in the inequality trap when no environmental policy is implemented or when the tax is too low. As $\bar{\eta}(\tau)$ is increasing in τ, when τ is sufficiently high, η becomes lower than $\bar{\eta}(\tau)$. Thus, the BGP with inequalities appears as a saddle point, while the BGP without inequality becomes stable. In this way, the environmental policy makes that the economy is more likely to converge toward the equilibrium E. For low level of taxation, the size of inequality trap remains very important, but the tighter environmental policy, the more there exist initial conditions such that inequalities among households disappear in the long run. Finally, it should be specified that the required level of tax on pollution for that can be close to 1, when inequalities are initially too wide and/or pollution is initially very high.

the values considered in the model.

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Table 3: Effect of a tighter environmental policy when $\eta = 0.35$

τ	k_I	x_I^u	$Eigenvalues_I$	g_I	π_I^u	π_I^s
0%	$\varnothing$	$\varnothing$				
5%	$\varnothing$	$\varnothing$				
10%	$\varnothing$	$\varnothing$				
15%	$\varnothing$	$\varnothing$				
20%	0.0153	0.7955	$\{1.00255; 0,32123\}$	1.6714	0.7743	0.8129
25%	0.0140	0.6280	$\{1.00873; 0.322303\}$	1.6730	0.7551	0.8273
30%	0.0127	0.5195	$\{1.01515; 0.323757\}$	1.6746	0.7404	0.8367
35%	0.0115	0.4384	$\{1.02153; 0.323801\}$	1.6761	0.7277	0.8439
40%	0.0099	0.3705	$\{1.02825; 0.333258\}$	1.6784	0.7172	0.8507
45%	0.0091	0.3132	$\{1.03507; 0.325562\}$	1.6793	0.7032	0.8555
50%	0.0079	0.2647	$\{1.04206; 0.326975\}$	1.6809	0.6917	0.8605
τ	k_E	x_E^u	$Eigenvalues_E$	g_E	π_E	
0%	0.0209	1	$\{1.01105; 0.318788\}$	1.6506	0.7724	
5%	0.0195	1	$\{1.00799; 0.318873\}$	1.6562	0.7785	
10%	0.0181	1	$\{1.00493; 0.319171\}$	1.6618	0.7846	
15%	0.0170	1	$\{1.00214; 0.315846\}$	1.6669	0.7903	
20%	0.0153	1	$\{0.998773; 0.320744\}$	1.6730	0.7971	
25%	0.0140	1	$\{0.995776; 0.320691\}$	1.6785	0.8032	
30%	0.0126	1	$\{0.992667; 0.322809\}$	1.6841	0.8096	
35%	0.0113	1	$\{0.989678; 0.323407\}$	1.6895	0.8157	
40%	0.0101	1	$\{0.986708; 0.324157\}$	1.6949	0.8219	
45%	0.0089	1	$\{0.983779; 0.324694\}$	1.7002	0.8279	
50%	0.0078	1	$\{0.98089; 0.325017\}$	1.7054	0.8340	

Notes: The value $\eta = 0.35$ corresponds to the case where $\eta > \bar{\eta}(\tau)$ in $\tau = 0$ and $\eta < 1 - \mu$ (its maximum value in the model).

References

Aloi, M. and Tournemaine, F. (2011), "Growth effects of environmental policy when pollution affects health", *Economic Modelling* 28, 1683-1695.

Aloi, M. and Tournemaine, F. (2013), "Inequality, Growth and Environmental Quality Trade-offs in a Model with Human Capital Accumulation", *Canadian Journal of Economics* 46(3), 1123-1155.

Ashenfelter, O. and Krueger, A. (1994). "Estimates of the Economic Return to Schooling from a New Sample of Twins", *American Economic Review* 84(5), 1157-1173.

Bell, M.L. and Davis, D.L. (2001), "Reassessment of the Lethal London Fog of 1952: Novel Indicators of Acute and Chronic Consequences of Acute Exposure to Air Pollution", *Environmental Health Perspectives* 109(3), 389-394 .

Bell, M.L. ; McDermott, A. ; Zeger, S.L. ; Samet, J.M. and Dominici, F. (2004). "Ozone and Short-term Mortality in 95 US Urban Communities, 1987-2000", *Journal of the American Medical Association* 292(19), 2372-2378.

Blackburn, K. and Cipriani, G. P. (2002), "A Model of Longevity, Fertility and Growth", *Journal of Economic Dynamics and Control* 26(2), 187-204.

Blanchard, O.J. (1985), "Debt, Deficits and Finite Horizons", *Journal of Political Economy* 93, 223–247.

Cakmak, S., Dales, R.E., Angelica Rubio, M. and Blanco Vidal, C. (2011), "The Risk of Dying on Days of Higher Air Pollution among the Socially Disadvantaged Elderly", *Environmental Research* 111, 388-393.

Castello-Climent, A. and Domenech, R. (2008), "Human Capital Inequality, Life Expectancy and Economic Growth", *The Economic Journal* 118 (528), 653-677.

Chakraborty, S. (2004), "Endogenous Lifetime and Economic Growth", *Journal of Economic Theory*, 116(1), 119-137.

Cifuentes, L., Vega, J. and Lave, L. (1999), "Daily Mortality by Cause and Socio-economic Status in Santagio, Chile 1988-1996", *Epidemiology* 10, S45.

Cooley T. (1995), *Economic Growth and Business Cycles.* Princeton: Princeton University Press.

Cutler, D.M. and Lleras-Muney, A. (2010), "Understanding Differences in Health Behaviors by Education", *Journal of Health Economics* 29(1), 1-28.

de La Croix, D. and Michel, P. (2002), *A Theory of Economic Growth: Dynamics and Policy in Overlapping Generations.* Cambridge: Cambridge University Press.

de La Croix, D. and Doepke, M. (2003), "Inequality and Growth: Why Differential Fertility Matters", *American Economic Review* 93, 1091-1113.

Elo, I.T. and Preston, S.H. (1996), "Educational Differentials in Mortality: United States, 1979–85", *Social Science and Medicine* 42(1), 47-57.

Evans, M. and Smith, K. (2005), "Do New Health Conditions Support Mortality-Air Pollution Effects?", *Journal of Environmental Economics and Management* 50, 496-518.

Galor (2011), "Inequality, Human Capital Formation and the Process of Development", *In:* Hanushek, E. and Hanushek, E.A., Machin, S.J. and Woessmann, L. (Eds.) *Handbook of the Economics of Education* (Vol. 4). Elsevier.

Glomm, G. and Ravikumar, B. (1992), "Public versus Private Investment in Human Capital: Endogenous Growth and Income Inequality", *Journal of Political Economy* 100(4), 818-834.

Hansen, C.W. (2013), "Life Expectancy and Human Capital: Evidence from the International Epidemiological Transition", *Journal of Health Economics* 32(6), 1142-1152.

Health Effects Institute (2000), Reanalysis of the Harvard Six Cities Study and the American Cancer Society Study of Particulate Air Pollution and Mortality, 1-285.

Jayachandran, S. and Lleras-Muney, A. (2009), "Life Expectancy and Human Capital Investments: Evidence from Maternal Mortality Declines", *Quarterly Journal of Economics* 124(1), 349–397.

Jouvet, P.A., Pestieau, P. and Ponthiere, G. (2010), "Longevity and Environmental Quality in an OLG Model", *Journal of Economics* 100, 191-216.

Kenkel, D. (1991),"Health Behavior, Health Knowledge and Schooling", *Journal of Political Economy* 99 (2), 287-305.

Krueger, A. B. and Lindahl, M. (2001), "Education for Growth: Why and For Whom?", *Journal of Economic Literature* 39, 1101-1136.

Laurent, O., Bard., D., Filleul, L. and Segala, C. (2007), "Effect of Socioeconomic Status on the Relationship between Atmospheric Pollution and Mortality", *Journal of Epidemiological Community Health* 61, 665-675.

Lleras-Muney A., (2005),"The Relationship between Education and Adult Mortality in the United States", *Review of Economic Studies* 72, 189-221.

Mackenbach, J.P. (2006), Health inequalities: Europe in profile. An independent, expert report commissioned by the UK Presidency of the EU. Rotterdam: Erasmus MC University Medical Center Rotterdam.

Mariani, F., Perez-Barahona, A. and Raffin, N. (2010), "Life Expectancy and the Environment", *Journal of Economic Dynamics and Control* 34(4), 798-815.

Miech, R., Pampel, F., Kim, J. and Rogers, R.G. (2011), "The Enduring Association between Education and Mortality: The Role of Widening and Narrowing Disparities", *American Sociological Review* 76(6), 913–934.

OECD (2013), *Health at a Glance 2013: OECD Indicators*, OECD Publishing.

OECD (2014), *OECD Economic Outlook*, Vol. 2014/1, OECD Publishing.

OECD (2014), *Focus on Inequality and Growth - December 2014*, OECD Publishing.

O'Neill, M.S., Jerrett, M., Kawachi, I., Levy, J.I., Cohen, A.J., Gouveia, N., Wilkinson, P., Fletcher, T., Cifuentes, L. and Schwartz, J. (2003), "Workshop on Air Pollution and Socieconomic Conditions. Health, Wealth and Air Pollution: Advancing Theory and Methods", *Environmental Health Perspective* 111, 1861-1870.

Osang, T. and Sarkar, J. (2008), "Endogenous Mortality, Human capital and Endogenous Growth", *Journal of Macroeconomics* 30, 1423–1445.

Palivos, T. and Varvarigos, D. (2010) "Pollution Abatement as a Source of Stabilisation and Long-Run Growth", *Discussion Papers in Economics* 11/04, Department of Economics, University of Leicester.

Pautrel, X. (2008), "Reconsidering the Impact of Pollution on Long-Run Growth when Pollution Influences Health and Agents have a Finite Lifetime", *Environmental and Resource Economics* 40, 37-52.

Pimentel, D., Tort, M., D'Anna, L., Krawic, A., Berger, J., Rossman, J., Mugo, F., Doon, N., Shriberg, M., Howard, E., Lee, S. and Talbot, J. (1998), "Ecology of Increasing Disease", *BioScience* 48 (10) 817-826.

Pope III, C.A., Burnett, R.T., Thun, M.J., Calle, E.E., Krewski, D., Ito, K. and Thurston, G.D. (2002), "Lung Cancer, Cardiopulmonary Mortality, and Long-term Exposure to Fine Particulate Air Pollution", *The Journal of the American Medical Association* 287(9), 1132-1141.

Psacharopoulos, G. (1994), "Returns to Investment in Education: A Global Update", *World development* 22(9), 1325-1343.

Raffin, N. (2012), "Children's Environmental Health, Education and Economic Development", *Canadian Journal of Economics* 45(3), 996-1022.

Raffin, N. and Seegmuller, T. (2014), "Longevity, Pollution and Growth", *Mathematical Social Sciences* forthcoming.

Sauer, P. and Zagler, M. (2012), "Economic Growth and the Quantity and Distribution of Education: A Survey", *Journal of Economic Surveys* 26(5), 933-951.

Schaefer, A. (2014), "Child Mortality and the Growth Drag of Pollution", *Working Paper.*

Singh, G.K. and Siahpush, M. (2006), "Widening Socioeconomic Inequalities in US life expectancy, 1980-2000", *International Journal of Epidemiology* 35, 969–979.

Tamura, R. (1991), "Income Convergence in an Endogenous Growth Model", *Journal of Political Economy* 99(3), 522-540.

U.K. Department of Health (2003), *Tackling Health Inequalities: A Programme for Action* London: Department of Health.

U.S. Department of Health and Human Services (2000), *Healthy People 2010: Understanding and Improving Health.* 2nd ed. Washington, DC: U.S. Government Printing Office.

van Ewijk, C. and van Wijnbergen, S. (1995) "Can Abatement Overcome the Conflict between Environment and Economic Growth", *De Economist* 143 (2), 197-216.

Varvarigos D. (2011), "Environmental Dynamics and the Links between Growth, Volatility and Mortality", *Bulletin of Economic Research* 65(4), 314-331.

Varvarigos D. and Zakaria I.Z. (2013a), "Endogenous Fertility in a Growth Model with Public and Private Health Expenditures", *Journal of Population Economics* 26, 67-85.

Varvarigos D. and Zakaria I.Z. (2013b), "Growth and Demographic Change: Do Environmental Factors Matter?", Working Paper.

World Health Organization (2014), "7 Million Premature Deaths Annually Linked to Air Pollution", *Media Centre news release*, Geneva: http://www.who.int/mediacentre/news/releases/2014/air-pollution/en/ (accessed May 3rd 2015).

Yaari, M. (1965), "Uncertain Lifetime, Life Insurance, and the Theory of the Consumer", *Review of Economic Studies* 32, 137–150.

Zeka, A., Zanobetti, A. and Schwartz, J. (2006), "Individual-Level Modifiers of the Effects of Particulate Matter on Daily Mortality", *American Journal of Epidemiology* 163(9), 849-859.