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Abstract

We study an infinite horizon economy with a representative agent whose utility function includes consumption, real balances and leisure. Real balances enter the utility function pre-multiplied by a parameter reflecting the inverse of the degree of financial market imperfection, i.e. the inverse of the transaction costs justifying the introduction of money in the utility function. When labor is supplied elastically, indeterminacy arises through a transcritical and a flip bifurcation, for degree of financial imperfection arbitrarily close to zero. Similar results are observed when labor is supplied inelastically: indeterminacy occurs through a flip bifurcation for values of the degree of financial imperfection unbounded away from zero. We also study the existence and the multiplicity of the steady states.

Keywords: Bifurcations; Indeterminacy; Market Imperfections; Money Demand.

JEL Classification: E 31; E 32; E 40

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1 Introduction

It is well known that growth model may be locally indeterminate, i.e. may possess a steady state such that the equilibrium dynamics converge toward it for infinitely many choices for the initial conditions, when some market imperfection is accounted for. This is the case when one assumes some external effect in production not mediated by markets, when participation to the latter is restricted, as it is the case in *OLG* models, or as a consequence of the frictions due to the introduction of money as a medium of exchange. When an equilibrium is indeterminate, it is possible to construct nearby stochastic sunspot equilibria, i.e. equilibria representing optimal responses to agents' revisions in their beliefs (Woodford, 1986; Grandmont *et al.* 1998).

The extensions of the standard neoclassical growth models effectuated in order to account for the role played by money include the seminal Sidrauski (1967) and Brock (1974; 1975) models in which real balances enter into the instantaneous utility function besides to consumption good and, possibly, leisure. The inclusion of real balances in the utility function is motivated by the perception of money as one of many assets (some financial, some real) and the indirect utility money provides stemming from the liquidity allowing to carry out transactions and the possibility to save time compared to being illiquid. The money-in-the-utility function approach can be viewed indeed as a device to account for the liquidity services money provides by reducing the transaction costs (e.g. shopping time models): as it is shown in Feenstra (1986) each transaction cost formulation is equivalent to an appropriate money-in-the-utility function specification. Also the cash-in-advance constraint approach (Clower, 1967; Stockman, 1981; Abel, 1985, Svensson, 1985; Lucas and Stokey, 1987; Cooley and Hansen, 1989) which compel agents to purchase the consumption good out of money balances previously accumulated falls within a specification of the utility function in which consumption and real balances are assumed to be perfect complements. Another approach, as in Benhabib and Farmer (2000) suppose that money reduces the selling costs and can then be viewed as a productive input, besides, e.g., physical capital. They show that a sufficient condition for indeterminacy is that money is sufficiently productive. In absence of transaction or selling costs, money can be positively valued only in *OLG* models (Samuelson 1958; Gale, 1973; Azariadis, 1981; Grandmont, 1985) where merely *fiat* money is delivered from the old generation to the young one and can be therefore viewed as a speculative bubble (Tirole, 1985).

Brock (1974; 1975) is the first to discuss the possibility of indeterminacy in a model that has real balances in the utility function. He shows how self-fulfilling hyperinflationary and deflationary equilibria can occur and under which conditions to rule out such phenomena. Calvo (1979) within a money-in-the-utility function approach shows that indeterminacy is most easily obtained when changes in the stock of real balances have large effects on output, while Matsuyama (1991) proves that cyclical and even chaotic equilibria can be obtained when the rate of money growth supply is high. Farmer (1997) consider a *RBC* model with money in the utility function and calibrates the model to fit the first moments of *U.S*. by choosing a parametrization of utility for which the model admits the existence of indeterminate equilibria in order to explain the monetary propagation mechanism. The mechanism that generates indeterminacy is that a small increase in real balances must be associated with a big increase, in equilibrium, of labor allocated to production.

In our model there is an infinitely lived representative agent whose utility function includes consumption, real balances and leisure. Besides money, there are safe bonds whose initial endowment is zero and whose nominal interest rate is strictly positive, implying that money is dominated by them in terms of returns. Our model differ from analogous ones by the fact that in the utility function real balances do appear multiplied by a parameter included between zero and infinite. Such a parameter can be viewed as the inverse of the degree of financial imperfection: a large value of it reflects the fact that the liquidity services provided by money are very sensitive to the amount of the real balances held. A larger value of such a parameter reflects indeed underlying diminishing transaction costs and as a consequence, for a given level of real balances, a larger marginal utility of money. We make abstraction of the emergence of self-fulfilling hyperinflationary equilibria studied in Brock (1974) and concentrate the attention on the monetary stationary solutions and their stability. We show that when labor is supplied rather elastically, in contrast to Farmer (1997), indeterminacy occurs for a wide range of the degree of financial imperfection values arbitrarily close to zero. As a matter of fact, indeterminacy appears first through a transcritical bifurcation and then through a flip bifurcation: in the first case, as shown in Grandmont (2008) and Bosi and Ragot (2011), one will assist to a change in stability between two nearby steady states and, in the second case, a 2-period cycle (stable or unstable, according to the direction of the bifurcation) will arise arbitrarily close to the stationary solution. On the other hand, when labor is supplied rather inelastically, the transcritical bifurcation is ruled out, meanwhile indeterminacy is bound to prevail through a flip bifurcation for a whole range of the degree of financial imperfection unbounded away from zero.

The fact that indeterminacy arises more easily when the degree of financial imperfection is lower seems to be counter intuitive since it is usual to see that indeterminacy is more likely to occur when the market imperfection are important, and to do not shrink when the latter are reduced. However, our results confirm that of Bosi *et al.* (2005) in which a fractional cash-in-advance on consumption purchases is assumed; they show that indeterminacy (and even chaotic dynamics) occurs for a wide range of the structural parameters as soon as the share of the consumption good to be paid cash is set sufficiently low. The mechanism leading to indeterminacy is, roughly, the following. Assuming for sake of simplicity an inelastic labor supply, let us suppose that the system is at the beginning at its steady state and that agents anticipate, say, an increase in the price level of the following period. Accordingly, they will react by decreasing the desired amount of money balances held at the end of the foregoing period and by increasing the current amount of nominal balances. But, if money balances are not too much substitutable across time and if such an effect is magnified by the lower degree of financial imperfection, the investment in money balances will decrease only lightly. At the same time, the current price level will decrease sharply in order to re-establish equilibrium in money market (we assume that the money supply is constant), giving rise to a oscillatory dynamics with decreasing amplitudes and therefore converging back to the stationary solution.

2 The model

We consider an infinite horizon discrete time economy populated by a constant mass of agents whose size is normalized to one. The preferences of the representative agent are described by the following intertemporal utility function:

$$\sum_{t=0}^{+\infty} \beta^t \left[u\left(c_t, \mu \frac{M_t}{p_t}\right) - Av\left(l_t\right) \right] \tag{1}$$

where c_t is the unique consumption good, M_t the money balances, l_t the labor supply, p_t the price of consumption good, $\beta \in (0, 1)$ the discount factor and $\mu > 0$ the inverse of the degree of market imperfection: a larger value of μ reflects indeed underlying diminishing transaction costs and as a consequence, for a given level of real balances, a larger marginal utility of money. Finally, $A > 0$ is a scaling parameter that will allow the calibration of the model. Setting $m_t \equiv M_t/p_t$ the real balances, the instantaneous utility function $u(c, \mu m) - Av(l)$ satisfies the following standard Assumption:

Assumption 1. *$u(c, \mu m)$ is C^2 over $\mathbb{R}^2_+$, increasing with respect to each argument*, i.e. $u_c(c, \mu m) > 0$ and $u_m(c, \mu m) > 0$, and concave over $\mathbb{R}^2_{++}$. In addition, $\forall c > 0$, $\lim_{m\to 0} u_c(c, \mu m) = 0$, $\lim_{m\to +\infty} u_c(c, \mu m) = +\infty$, $\lim_{m\to 0} u_m(c, \mu m) = +\infty$ and $\lim_{m\to +\infty} u_m(c, \mu m) = 0$. Moreover, $v(l)$ is C^2 over $\mathbb{R}_+$, strictly increasing and weakly convex.*

When maximizing (1) agents must respect the dynamic budget constraint:

$$p_t c_t + M_{t+1} + B_{t+1} = M_t + (1 + i_t) B_t + w_t l_t \tag{2}$$

where B_t denotes the safe nominal bonds issued by the government, i_t the nominal interest rate on bonds holding and w_t the nominal wage. The initial endowment of bond is zero, i.e. $B_0 = 0$, and the nominal money supply is constant and equal to $M_0 > 0$. Denoting $b_t = B_t/p_t$ the real governments bonds and $\omega_t = w_t/p_t$ the real wage, we finally get the intertemporal maximization problem of the representative agent:

$$\max_{(c_t, m_t, l_t, b_t)_{t=0}^{\infty}} \sum_{t=0}^{+\infty} \beta^t \left[u\left(c_t, \mu m_t\right) - Av\left(l_t\right) \right] \tag{3}$$

subject to the dynamic budget constraint

$$c_t + m_{t+1}\frac{p_{t+1}}{p_t} + b_{t+1}\frac{p_{t+1}}{p_t} = m_t + (1 + i_t) b_t + \omega_t l_t \tag{4}$$

Let denote λ the Lagrangian multiplier associated to the dynamic budget constraint, then the first order conditions are:

*u_c, u_m and v_l denote respectively $\partial u(c, \mu m)/\partial c$, $\partial u(c, \mu m)/\partial m$ and $\partial v(l)/\partial l$.

$$\beta^t u_c(c_t, \mu m_t) = \lambda_t \tag{5}$$

$$A\beta^t v_l(l_t) = \omega_t \lambda_t \tag{6}$$

$$\mu\beta^t u_m(c_t, \mu m_t) = \lambda_{t-1}\tfrac{p_t}{p_{t-1}} - \lambda_t \tag{7}$$

and the Fisher equation

$$-\lambda_{t-1}\tfrac{p_t}{p_{t-1}} + \lambda_t(1 + i_t) = 0 \tag{8}$$

By exploiting appropriately (5)-(8), we obtain the following equations:

$$Av_l(l_t) = \omega_t u_c(c_t, \mu m_t) \tag{9}$$

$$u_c(c_{t-1}, \mu m_{t-1}) = \beta u_c(c_t, \mu m_t)(1 + i_t)\tfrac{p_{t-1}}{p_t} \tag{10}$$

and

$$\mu u_m(c_t, \mu m_t) - \tfrac{p_t}{p_{t-1}\beta} u_c(c_{t-1}, \mu m_{t-1}) + u_c(c_t, \mu m_t) = 0 \tag{11}$$

We assume a linear technology such that one unit of labor can be used to produce one unit of the corresponding output y according to the linear production function

$$y_t = l_t \tag{12}$$

Equilibrium in the good market is $y_t = c_t = l_t$ in each period t, meanwhile equilibrium in the money market requires $p_t/p_{t-1} = m_{t-1}/m_t$ since one has in each period $m_t = M_0/p_t$. Since the technology is linear in labor, one has that the real wage is constant and equal to one, i.e. $\omega_t = 1$ for every t. Combining (11) and equilibrium of the good market, we can now define the intertemporal equilibrium of the economy in terms of the evolution of real balances and of the output.

Definition 1. *A sequence $\{m_t, y_t\}_{t=0}^{\infty}$, with $(m_t, y_t) >> 0$ for all t, is a perfect-foresight competitive equilibrium if it satisfies:*

$$\mu u_m(y_t, \mu m_t) - \tfrac{m_{t-1}}{\beta m_t} u_c(y_{t-1}, \mu m_{t-1}) + u_c(y_t, \mu m_t) = 0, \tag{13}$$

$$Av_l(y_t) = u_c(y_t, \mu m_t) \tag{14}$$

together with the transversality condition

$$\lim_{t\to+\infty} \beta^t u(c_t, \mu m_t)(m_t + b_t) = 0. \tag{15}$$

Notice that system (13)-(14) includes the temporal evolution of real balances that, being defined as the money balances/price ratio, represent a non-predetermined variable and therefore indeterminacy requires a stable steady state of system (13)-(14).

3 Steady State Analysis

A steady state of system (13)-(14) is an amount of real balances equal to zero, corresponding to an hyperinflationary equilibrium. A monetary steady state of system (13)-(14) is conversely a pair $(y, m) >> (0, 0)$ satisfying

$$\mu u_m (y, \mu m) = \left(\tfrac{1-\beta}{\beta}\right) u_c (y, \mu m) \tag{16}$$

and

$$Av_l (y) = u_c (y, \mu m) . \tag{17}$$

We will now calibrate a particular solution of system (16)-(17). To this end, let fix a arbitrary y in (16). In view of Assumption 1, the left-hand side of (16) is decreasing in m meanwhile its right-hand side is increasing in m. It follows that for each y there will exist a unique solution $m (y)$ satisfying (16). Therefore a steady state of system (13)-(14) will correspond to an output level y satisfying

$$Av_l (y) = u_c (y, \mu m (y)) \tag{18}$$

It is then possible to calibrate the particular solution $y = 1$ by setting the scaling parameter as $A = u_c (1, \mu m(1)) / v_l (1)$. Notice in addition that, in view of (10) evaluated at the stationary solution, one has $1 = \beta (1 + i)$ and therefore the stationary nominal interest rate is

$$i = \beta^{-1} - 1 > 0$$

which implies that bonds dominate money in terms of returns. In addition, it is possible to rewrite (16) as $u_m (y, \mu m) = \left(\tfrac{1-\beta}{\mu\beta}\right) u_c (y, \mu m)$; it follows that the marginal utility of real balances holding is equal to zero when $\mu = +\infty$. In other words, under such a condition, individuals are sated with money holding. This is not surprising, once one inspects the instantaneous utility function $u (c, \mu m)$ in which μ influences the marginal utility of m and, as soon as it is set larger, the latter increases too entailing therefore a Pareto improvement. Notice that (18) defines, in a neighborhood of the fixed point, a smooth map h such that $y = h (m)$ whose deviations form the stationary solution are given by

$$dy = \frac{\mu u_{cm}}{u_{cc} + v_l} dm. \tag{19}$$

It follows that by totally differentiating with respect to m (16) becomes

$$\mu \left[u_{mc} \frac{dy}{dm} (m) + \mu u_{mm} \right] - \frac{1 - \beta}{\beta} \left[u_{cc} \frac{dy}{dm} (m) + \mu u_{cm} \right] = 0$$

which may be not monotonic and therefore may give raise to multiple stationary solutions.

4 Local Stability

We now turn to the linearization of system (13)-(14) around the calibrated stationary solution $(1, m(1))$. For the purpose of our analysis, it is more useful to define at this stage of the paper $\varepsilon_{cc} \equiv -u_{cc}c/u_c > 0$ the elasticity of the marginal utility of consumption, $\varepsilon_{mm} \equiv -u_{mm}m/u_m > 0$ the elasticity of the marginal utility of real balances, $\varepsilon_{mc} \equiv u_{mc}m/u_m > 0$ the crossed elasticity of the marginal utility of real balances and $\varepsilon_l \equiv v_{ll}y/v_l$ the inverse of the elasticity of labor supply, all evaluated at the steady state under study and all belonging to $(0, +\infty)$. To this end, notice that (14) defines in a neighborhood of the fixed point a smooth map h such that $y_t = h(m_t)$ whose deviations form the stationary solution are given by

$$dy_t = \frac{\mu u_{cm}}{u_{cc} + v_l} dm_t. \tag{20}$$

To linearize (13) at the steady state under study, one must therefore take into consideration (20) when differentiating (13). After tedious but straightforward computations one obtains the following expression for derivative dm_t/dm_{t-1} evaluated at the calibrated steady state:

$$\frac{1 + \frac{(1-\beta)\varepsilon_l \varepsilon_{cm}}{\beta(\varepsilon_{cc}+\varepsilon_l)}}{1 - \mu(1-\beta)\left[\varepsilon_{mm} - \frac{\varepsilon_{mc}^2}{\varepsilon_{cc}+\varepsilon_l}\right] + (1-\beta)\varepsilon_{cm}\left[1 + \frac{\varepsilon_{cc}}{\varepsilon_{cc}+\varepsilon_l}\right]} \tag{21}$$

Notice that the numerator of (21) is strictly positive and that its denominator is equal to zero when

$$\mu^0 = \frac{1 + (1-\beta)\varepsilon_{cm}\left[1 + \frac{\varepsilon_{cc}}{\varepsilon_{cc}+\varepsilon_l}\right]}{(1-\beta)\left[\varepsilon_{mm} - \frac{\varepsilon_{mc}^2}{\varepsilon_{cc}+\varepsilon_l}\right]}. \tag{22}$$

It follows that μ^0 is positive if and only if the denominator of (22) is positive. The transcritical bifurcation occurs when dm_t/dm_{t-1}, evaluated at the stationary solution under study, is equal to 1, namely for a μ^t whose expression is the following:

$$\mu^t = \frac{\varepsilon_{cm}\left[1 + \frac{\varepsilon_{cc}}{\varepsilon_{cc}+\varepsilon_l} - \frac{\beta^{-1}\varepsilon_l}{\varepsilon_{cc}+\varepsilon_l}\right]}{\varepsilon_{mm} - \frac{\varepsilon_{mc}^2}{\varepsilon_{cc}+\varepsilon_l}}. \tag{23}$$

At the same time, in order to get a flip bifurcation, the expression in (21) must be equal to -1, which is true when

$$\mu^f = \frac{2 + \frac{(1-\beta)\varepsilon_l \varepsilon_{cm}}{\beta(\varepsilon_{cc}+\varepsilon_l)} + (1-\beta)\varepsilon_{cm}\left(1 + \frac{\varepsilon cc}{\varepsilon_{cc}+\varepsilon_l}\right)}{(1-\beta)\left[\varepsilon_{mm} - \frac{\varepsilon_{mc}^2}{\varepsilon_{cc}+\varepsilon_l}\right]} \tag{24}$$

Recall to mind that, when μ goes through μ^f a flip bifurcation generically occurs, a stable or unstable 2-period cycle (according to the direction of the bifurcation) arises near the stationary solution meanwhile when μ goes through μ^t a transcritical bifurcation generically occurs, one assists to a change in stability between two nearby steady states. In order to carry out the stability analysis, notice first that the numerators of (22) and (24) are always positive. The immediate consequence is that μ^0 and μ^f will be positive if and only if the (common) denominator of they relative expressions is positive, i.e. if and only if

$$\varepsilon_{mm} - \frac{\varepsilon_{mc}^2}{\varepsilon_{cc} + \varepsilon_l} > 0 \tag{25}$$

It is then immediate to verify that the existence of a positive μ^t, under the domain of inequality (25), implies the existence of positive μ_0 and μ^f satisfying $0 < \mu^t < \mu^0 < \mu^f$. The existence of a positive μ^t , under condition (25), requires

$$1 + \frac{\varepsilon_{cc}}{\varepsilon_{cc} + \varepsilon_l} - \frac{\beta^{-1}\varepsilon_l}{\varepsilon_{cc} + \varepsilon_l} > 0. \tag{26}$$

which is satisfied when ε_l is close enough to zero. Therefore, under inequalities (25) and (26), the expression (21) will be included in $(0, 1)$ for $\mu \in (0, \mu^t)$ (and the steady state will be locally indeterminate), will be larger than one for $\mu \in \left(\mu^t, \mu^0\right)$ and the steady state will be locally determinate), will be lower than -1 for $\mu \in \left(\mu^0, \mu^f\right)$ (the steady state is thus locally determinate) and will be included in $(-1, 0)$ for $\mu > \mu^f$ (and the stationary solution will be locally indeterminate). If, on the other hand, (26) does not hold, the transcritical bifurcation μ^t is ruled out, meanwhile μ^0 and μ^f are still positive. It follows that the expression (21) will be larger than one for $\mu \in \left(0, \mu^0\right)$ (and the steady state will be locally determinate), lower than -1 for $\mu \in \left(\mu^0, \mu^f\right)$ (and the steady state will be locally determinate), and included in $(-1, 0)$ for $\mu > \mu^f$ (and the stationary solution will be locally indeterminate).

Suppose now that (25) is not satisfied; the transcritical bifurcation will then occur if and only if (26) is not satisfied, feature requiring a high enough inverse ε_l of the elasticity of the labor supply, meanwhile μ_0 and μ^f are negative. It follows that for $\mu < \mu^t$ the expression (21) will be larger than one (and the steady state will be locally determinate) and for $\mu > \mu^t$ it will be included in $(0, 1)$ (and the steady state will be locally determinate). Eventually, if (25) is not satisfied and (26) is satisfied, the expression (21) will be included in $(0, 1)$ for all μ (and the steady state will be locally indeterminate). We have therefore the following Proposition.

Proposition 1. *Under Assumption 1. There exist μ^0, μ^f and μ^t such that the following results generically prevail:*

1) Let (25) and (26) holds. Then

i) For $\mu \in (0, \mu^t)$, $dm_t/dm_{t-1} \in (0, 1)$ and the steady state is locally indeterminate;

ii) For $\mu \in \left(\mu^t, \mu^0\right)$, $dm_t/dm_{t-1} > 1$ and the steady state is locally determinate;

iii) For $\mu \in \left(\mu^0, \mu^f\right)$, $dm_t/dm_{t-1} < -1$ and the steady state is locally determinate;

iv) For $\mu > \mu^f$, $dm_t/dm_{t-1} \in (-1, 0)$ and the steady state is locally indeterminate.

2) Let (25) holds and (26) does not hold. Then:

i) For $\mu \in \left(0, \mu^0\right)$, $dm_t/dm_{t-1} > 1$ *and the steady state is locally determinate;*

ii) For $\mu \in \left(\mu^0, \mu^f\right)$, $dm_t/dm_{t-1} < -1$ *and the steady state is locally determinate;*

iii) For $\mu > \mu^f$, $dm_t/dm_{t-1} \in (-1, 0)$ *and the steady state is locally indeterminate.*

3) Let (25) does not hold and (26) holds. Then, for all $\mu > 0$, $dm_t/dm_{t-1} \in (0, 1)$ *and the steady state is locally indeterminate.*

4) Let both (25) and (26) do not hold. Then:

i) For $\mu \in (0, \mu^t)$ *it is* $dm_t/dm_{t-1} > 1$ *and the steady state is locally determinate;*

ii) For $\mu > \mu^t$ *it is* $dm_t/dm_{t-1} \in (0, 1)$ *and the steady state is locally indeterminate.*

In addition, when μ *goes through* μ^f *a flip bifurcation generically occurs and a stable or unstable cycle (according to the direction of the bifurcation) arises near the calibrated stationary solution meanwhile when* μ *goes through* μ^t *a transcritical generically occurs and one assists to a change in stability between two nearby steady states.*

Notice that when labor is supplied inelastically, i.e. $\varepsilon_l = +\infty$, one has $\mu^t = \frac{(1-\beta)\varepsilon_{cm}(\beta-1)/\beta}{(1-\beta)\varepsilon_{mm}} < 0$ so the transcritical bifurcation does not occurs , meanwhile $\mu^f = \frac{2+(1-\beta)(1+\beta)\varepsilon_{cm}/\beta}{(1-\beta)\varepsilon_{mm}} > 0$ and therefore the flip bifurcation does occur for all the parameter values, provided μ is set appropriately. In addition, for $\mu > \mu^f$, one has $dm_t/dm_{t-1} \in (-1, 0)$ and the steady state is locally indeterminate

5 Conclusion

In this paper we have studied the behavior of an infinite horizon economy populated by a representative agent whose utility function is defined over consumption, real balances and leisure. We have assumed that real balances enter in the utility function pre-multiplied by a parameter which is to be interpreted as the inverse of the degree of financial imperfection of the economy. In other words, a larger value for such a parameter is to be seen as a lower underlying transaction cost as it is reflected in the utility function. We have shown that under the hypothesis of an elasticity of labor supply large enough, indeterminacy and sunspot fluctuations occur for a wide range of the degree of financial imperfection values which turn out to be arbitrarily close to zero. More in details, indeterminacy appears first through a transcritical bifurcation and then through a flip bifurcation. As a consequence, as shown in Grandmont (2008) and Bosi and Ragot (2011), one will assist in the first case to a change in stability between two nearby steady states and in the second case to the emergence of a 2-period cycle (stable or unstable, according to the direction of the bifurcation) arbitrarily close to the stationary solution. Under the assumption of a rather inelastic labor supply, if on the one hand the transcritical bifurcation is ruled out, on the other one indeterminacy is bound to prevail through a flip bifurcation for a whole range of the degree of financial imperfection unbounded away from zero. The mechanism at the root of indeterminacy is to be found in the reaction of the present price level with respect to an higher future expected one which, in the presence of a degree of market imperfection low enough, entails a sharp contraction in the present price level, giving thus rise to a oscillatory dynamics with decreasing amplitudes and therefore converging to the stationary solution.

A suitable extension of the model should take into account capital accumulation, as in Sidrauski (1967) and Farmer (1997): such a departure should better emphasize the arbitrages intervening between physical assets and money balances and to appraise the dynamics of the economy in terms of the behavior of the nominal interest rate, by exploiting the Fisher equation function as well as to assess the impulse response functions on *GDP* of the sunspot shocks. Following Matsuyama (1991), also accounting for a positive rate of growth of the money supply could improve the understanding of the equilibrium dynamics in terms of its cyclical, and may be also chaotic, properties.

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