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Abstract: In this paper, we revisit the question of how domestic and foreign risks affect growth through the lens of an AK small-open economy model with risky borrowing/lending and global diversification. Wealth is allocated between domestic and foreign assets and the optimal allocation depends on both the difference in deterministic returns and the relative magnitude and correlation of domestic and foreign risks. Depending on parameters, the small-open economy may choose to either borrow from abroad, despite the fact that this is risky, or lend. In contrast to standard N-country models, whether growth is faster or slower (and whether growth is more or less volatile) compared to autarky is not entirely driven by relative risk aversion but also depends on the return and risk characteristics of domestic and foreign assets. We also show that growth volatility and mean growth have typically nonmonotonic relationships with the levels and correlation of domestic and foreign risks. We argue that these results are in line with, and lay down some theoretical foundations for explaining the conflicting empirical results regarding the impact of international financial integration on growth and in particular threshold effects.

Keywords: International Financial Integration, Endogenous Growth, Small Open Economy, Domestic and Foreign Risks

Journal of Economic Literature Classification Numbers: F34, F43, O40

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1 Introduction

Globalization means both exposure to foreign shocks and risk diversification. A much debated question is how financial globalization, in particular in the post-second world war period, has affected the average and volatility of growth rates in industrial and developing countries. Empirical studies have so far delivered conflicting results. While Van Wincoop [19] found that potential growth and welfare gains from international risk sharing arising from financial globalization are sizeable, Kose et al. [6] observed that "cross-country consumption correlations have not increased in the 1990s, precisely when financial integration would have been expected to result in better risk-sharing opportunities, especially for developing countries". Indeed, many authors have pointed at the risks conveyed by financial globalization for less advanced economies, increasing macroeconomic instability being the main argument put forward (see Stiglitz [18] or Schmukler [16]). More recently, a few relatively robust findings have emerged. One is related to equity markets liberalization (in contrast to the broad liberalization of the capital account), which is increasingly viewed as growth-enhancing (see the extensive survey of Kose et al. [5]). On the other hand, there is now a common view that financial liberalization may be beneficial or not depending on whether the countries' fundamentals are above certain threshold levels (see for example Kose et al. [9]). In particular, it is nowadays broadly argued that financial and institutional development should be above a certain level in order to reduce the risks associated with financial openness.

We revisit this issue theoretically through the lens of an AK small open-economy model in which the domestic country's portfolio is optimally split between domestic and foreign assets. All assets are supposed to be risky, for simplicity. As a result of the inherent optimal portfolio choice, mean growth and growth volatility are both endogenous and they depend on the relative magnitude of domestic and foreign risks and on their correlation. Within this framework, we examine the impact of financial globalization on growth on one hand, and we inquire into how mean growth and growth volatility are related to the levels of domestic and foreign risks and their correlation on the other. While our setting may seem basic at first glance, it does generate a rich set of implications regarding the latter issues. Because the small-open economy assumption is everything but irrelevant from an empirical standpoint, especially because the issue of financial globalization is acute in many developing countries, we do believe that our theoretical analysis is a useful complement to the related literature, that we review below.

Key ingredients of our stochastic model are endogenous growth and international risk sharing. As such, our analysis is closely related to several earlier contributions, among which Devereux and Smith [3] and Obstfeld [12] are prominent examples. Both papers study N-country models where each country selects its optimal portfolio of (domestic and international) assets. However, there is no aggregate risk in Devereux and Smith [3] since they assume away aggregate uncertainty and focus on idiosyncratic national shocks. In contrast, uncertainty does not vanish in the aggregate in Obstfeld's [12] seminal paper, and national risks may be correlated. We shall keep this essential feature in our framework and devote part of our work to uncover the role of risks correlation in financial globalization outcomes. Nonetheless, we depart from Obstfeld in the essential fact that we do not consider N optimizing countries but only one small-open economy country facing exogenous (and possibly correlated) risks arising from both domestic technology and the rest of world's through international financial markets. As argued above, the economic relevance of such an alternative framework is out of question, as it applies to many developing (and developed) countries.

With respect to the engine driving endogenous growth, while Devereux and Smith [3] use Arrowian learning-by-doing, Obstfeld [12] uses a standard Merton-like optimal portfolio model, though the main pro-liberalization argument is more in the vein of Romer's increasing number of varieties (see Romer [14]): growth is boosted when switching from autarky to globalization because more assets become available and improve international risk sharing. Mathematically however, Obstfeld's model is a stochastic AK model like ours, and it involves the same type of linear homogenous stochastic differential equations. See also Jones and Manuelli [4], Steger [17] and Boucekkine et al. [1] for other analogous stochastic AK modelings. On one hand, we simplify preferences and assume typical CRRA time-separable utility, while Obstfeld introduces Epstein-Zin-Weil preferences. On the other hand, one attractive addition of our model is that foreign borrowing is supposed to be risky, which is more in line with experiences in many countries that face uncertainty in the borrowing cost. One extreme example of such risk is the well-documented "sudden stops" in capital inflows. It turns out, as we show below, that such a realistic assumption affects in a deep way how financial globalization affects growth and volatility.

Our main findings can be summarized as follows. While whether or not financial globalization boosts average growth depends *only* on parameters governing the attitude towards risk in Obstfeld's N-country model with only risky assets, our model delivers a much more contrasted picture. In our analysis, whether growth is faster or slower (and whether growth is more or less volatile) compared to autarky is not entirely driven by relative risk aversion but it also relates, more fundamentally, to the return and risk characteristics of domestic and foreign assets. In particular, even if the small open economy is able to choose optimally its allocation of wealth between domestic and foreign assets, it is conceivable that financial integration leads to faster or slower mean growth depending both on the relative magnitude of domestic risk and foreign risk and on their correlation. A key aspect of our analysis is whether the country is a net borrower and lender towards the rest of the world, which itself depends on the risk characteristics of domestic and foreign assets. For example, it can be readily shown that if the domestic and foreign risks are equal in size (and have possibly nonzero correlation), moving from autarky to financial integration does raise mean growth if the small open economy is a net borrower but is not necessarily growth-enhancing if the economy is a net lender. On the other hand, growth variance can be larger or smaller after the economy integrates, which contrasts with Obstfeld's result that financial integration unambiguously reduces volatility. In that sense, our results are broadly consistent with the volatility-enhancing effect of globalization outlined by the authors mentioned at the outset of this introduction.

Different from Obstfeld's results, we uncover several possible parametric conditions under which financial integration may induce more unstable growth compared to autarky. These conditions amount to setting threshold values to deep parameters of the small open economy, consistently with the recent empirical literature (see again the survey of Kose et al. [5]). Therefore our results provide conditions that gives some theoretical flesh to the instability argument put forward by several authors like Stiglitz or Schmukler (cited above), who have been warning against financial liberalization in developing and medium-income countries.

Another set of findings is obtained from the inspection of the relationship between mean growth and growth volatility on one hand and the levels and correlation of domestic and foreign risks on the other. Since the seminal empirical work of Ramey and Ramey [13], the relationship between mean growth and growth volatility has been abundantly studied. More recent evidence provided by Kose et al. [7, 8] show that such relationship has been affected by both trade and financial liberalization. In theory, though, few studies have addressed the growth-volatility relationship in the context of international risk sharing and our analysis hopefully contributes to filling this gap. This void is particularly acute regarding how the correlation of risks can affect growth and volatility. In several recent empirical papers (see in particular Schmukler [16] for the case of developing countries), a meticulous account of different types of domestic and foreign risks conveyed by financial globalization is performed. We propose here a complete analytical study of how the levels and correlation of domestic and foreigns risks impact mean growth and growth volatility in the small open AK economy. Just like the results on the shape of the latter as function of financial openness, we also identify non-monotonic relationships and interpret them.

The paper is organized as follows. Section 2 exposes the model and solves for the optimal consumption and wealth allocation. Section 3 studies the effects of financial globalization (that is, moving from autarky to full integration into international financial markets) on growth in the small-open economy. Section 4 shows that domestic and foreign risks have nonmonotonic effects on mean growth and growth variance. Finally, Section 5 gathers concluding remarks and some proofs and additional material are exposed in two appendices.

2 Growth in a Small-Open Economy AK Model

We consider a stochastic extension of a simple small-open economy AK model. The dynamics of domestic capital and external debt are described by the following equations:

$$\begin{cases} \mathrm{d}K(t) = [I(t) - \delta K(t)]\,\mathrm{d}t + \eta_K K(t)\,\mathrm{d}W(t) \\ \mathrm{d}D(t) = [rD(t) + I(t) + C(t) - AK(t)]\,\mathrm{d}t + \eta_D D(t)\,\mathrm{d}\tilde{W}(t) \\ \tilde{W}(t) = \omega W(t) + \sqrt{1-\omega^2}W'(t) \end{cases} \tag{1}$$

where $K(t)$, $I(t)$ and $C(t)$ represent, respectively, capital, investment in capital and aggregate consumption at time t, $D(t)$ represents the stock of debt at time t, with initial conditions given by $K(0) = K_0 > 0$ and $D(0) = D_0$ with $K_0 > D_0$. In addition, productivity A, the world interest rate r and the rate of capital depreciation $\delta \geq 0$ are deterministic parameters.

The first stochastic differential equation in (1) describes the accumulation of domestic capital, while the second equation describes the evolution of debt that the home country accumulates towards the rest of the world. Finally, the last equation in (1) describes the risks structure. We are interested in situations such that the risk associated to the domestic return may be correlated with the foreign return's risk. More specifically, the two correlated Brownian motions W and $\tilde{W}$ reflect random disturbances that can be thought of as affecting respectively the level of domestic productivity - domestic risk - and the level of foreign productivity - foreign risk. To model the possible dependence, we assume that W and W' are independent Brownian motions so that W and $\tilde{W}$ have their correlation equal to parameter ω, while parameters $\eta_K > 0$ and $\eta_D > 0$ represent the respective standard deviations of $\eta_K W$ and of $\eta_D \tilde{W}$. Although we assume away risk-free assets, for simplicity, the analysis could be interestingly extended to account for the presence of domestic and/or global safe assets.

The social planner maximizes the expected sum of discounted utility flows defined by:

$$\mathbf{E}\left[\int_0^{+\infty} e^{-\rho t}\frac{C(t)^{1-\sigma}-1}{1-\sigma}\,\mathrm{d}t\right] \tag{2}$$

where $\sigma > 0$ denotes relative risk aversion while $\rho > 0$ measures impatience. Following the portfolio approach pioneered by Merton [11] and Samuelson [15], we now simplify the problem of maximizing (2) under (1) by using the variable $x(t) \equiv K(t) - D(t)$, which is the country's total wealth. In particular, we define $\beta(t)$ as the fraction of wealth that is invested domestically, that is, $K(t) = \beta(t)x(t)$ and $D(t) = (\beta(t) - 1)x(t)$. Taking the difference of the two state equations in (1) gives the stochastic equation that describes the evolution of $x(t)$ in terms of $\beta(t)$ and $C(t)$:

$$dx(t) = [R(t)x(t) - C(t)]\ \mathrm{d}t + \eta_K\beta(t)x(t)\,\mathrm{d}W(t) + \eta_D(1-\beta(t))x(t)\,\mathrm{d}\tilde{W}(t) \tag{3}$$

where $R(t) \equiv (A-\delta)\beta(t) + r(1-\beta(t))$ is the return of the portfolio that is composed of domestic and foreign assets and is optimized upon. The initial condition is given by $x(0) = x_0 = K_0 - D_0 > 0$. Then maximizing (2) subject to (1) choosing positive $\{I(\cdot), C(\cdot)\}$ is equivalent to maximizing (2) subject to (3) choosing $\{\beta(\cdot), C(\cdot)\}$ among all couples of processes that are adapted to the filtration generated by W and $\tilde{W}$ while ensuring that $x(t)$, $\beta(t)$ and $C(t)$ are all almost surely positive for any $t \geq 0$. Choosing the latter formulation, we denote by v the value function of the problem, i.e. the maximal attainable utility given the initial condition $x_0 > 0$:

$$v(x_0) \equiv \max_{\{\beta(\cdot), C(\cdot)\}} \mathbf{E}\left[\int_0^{+\infty} e^{-\rho t} \frac{C(t)^{1-\sigma} - 1}{1-\sigma}\, \mathrm{d}t\right]. \tag{4}$$

The following proposition derives the explicit solution to the above problem, which we will extensively use to address the main questions of this paper.

Proposition 2.1 (Optimal Consumption and Wealth Allocation)

The optimal share of investment in the domestic asset is constant and given by:

$$\beta(t) = \beta_M = \frac{(A-\delta-r)/\sigma}{\eta_K^2 + \eta_D^2 - 2\omega\eta_D\eta_K} + \frac{\eta_D^2 - \omega\eta_D\eta_K}{\eta_K^2 + \eta_D^2 - 2\omega\eta_D\eta_K}. \tag{5}$$

It follows that $\beta_M > 1$, that is, the country is a net debtor if and only if:

$$\frac{A-\delta-r}{\sigma\eta_K} > \eta_K - \omega\eta_D. \tag{6}$$

The value function in terms of initial wealth $x_0 = K_0 - D_0 > 0$ is given by:

$$v(x_0) = \frac{1}{1-\sigma}\left\{\gamma\,(x_0)^{1-\sigma} - \frac{1}{\rho}\right\}$$

with

$$\gamma = \left\{\frac{1}{\sigma}\left[\rho + (\sigma-1)\Sigma_M\right]\right\}^{-\sigma} > 0 \tag{7}$$

and

$$\Sigma_M = R_M - \frac{1}{2}\sigma\left[\eta_K^2\beta_M^2 + \eta_D^2(1-\beta_M)^2 + 2\omega\beta_M(1-\beta_M)\eta_K\eta_D\right] \tag{8}$$

where $R_M \equiv (A-\delta)\beta_M + r(1-\beta_M)$ denotes the deterministic return of the optimal portfolio. At each time $t \geq 0$ the optimal consumption is a constant fraction of wealth:

$$C(t) = \gamma^{-1/\sigma} x(t) \tag{9}$$

The optimal trajectory of wealth $x(t)$ is the unique solution of the following stochastic differential equation:

$$\begin{aligned} dx(t) = & \left[(A-\delta)\beta_M + r(1-\beta_M) - \gamma^{-1/\sigma}\right] x(t)\,\mathrm{d}t \\ & + \eta_K\beta_M x(t)\,\mathrm{d}W(t) + \eta_D(1-\beta_M)x(t)\,\mathrm{d}\tilde{W}(t) \end{aligned} \tag{10}$$

The economy follows a balanced-growth path for all $t \geq 0$ such that all variables grow at the same optimal growth rate g, which is normally distributed with variance $\mathbf{V}[g]$ and mean $\mathbf{E}[g]$ given by:

$$\mathbf{V}[g] = \eta_K^2 \beta_M^2 + \eta_D^2 (1-\beta_M)^2 + 2\omega\beta_M(1-\beta_M)\eta_K\eta_D \tag{11}$$

$$\mathbf{E}[g] = \frac{(A-\delta)\beta_M + r(1-\beta_M) - \rho}{\sigma} + \frac{(\sigma-2)}{2}\mathbf{V}[g] \tag{12}$$

Proof: See Appendix A.

Note that, not surprisingly, the expression of mean growth in (12) is the sum of a deterministic component, that is, $(R_M - \rho)/\sigma$, and a second term that reflects the stochastic feature of the model. One may at first sight be surprised that the latter term depends on $(\sigma - 2)$ and not on $(\sigma - 1)$ as in most papers in the literature (e.g. Jones and Manuelli [4], Steger [17]). This is because we define mean growth as the average growth rate of wealth, which is the relevant notion if the issue of stochastic stability is addressed, as it should. In contrast, the literature usually defines mean growth as the growth rate of average wealth (see Boucekkine, Pintus and Zou [2] for a discussion). It is easily shown, using Jensen's inequality, that the former definition implies a smaller mean growth rate than the latter. In the context of our model, this means that an additional term - that is, $\mathbf{V}[g]/2$ - hampers growth and alters the growth/volatility relationship compared to conventional wisdom based on a disputable definition of mean growth.

A few comments about the optimal allocation of consumption and wealth are in order. In view of its expression in (8), Σ_M can be thought of as the certainty equivalent return on the country's portfolio, that is, the difference between the portfolio's deterministic return R_M and the risk premium $\sigma\,\mathbf{V}[g]/2$. From equation (9), optimal consumption is a fixed fraction of wealth. In view of the expressions in (7)-(8), we conclude that the presence of risk, materialized by the risk premium term in (8), has an ambiguous effect on consumption and savings. In particular, the average propensity to consume $\gamma^{-1/\sigma}$ is lower, other things equal, under risk if and only if $\sigma > 1$. As is well known, this is because the (negative) income effect of the fall in return due to risk dominates the substitution effect when $\sigma > 1$.

Regarding the wealth allocation between domestic and foreign assets, the condition that $\beta(t) = \beta_M$ must be positive can now be expressed, in view of expression (5), as $A - \delta - r + \sigma(\eta_D^2 - \omega\eta_D\eta_K) > 0$. Note that when $\beta_M = 1$ then the economy is closed, that is, all wealth is invested domestically and the expression in (12) reduces to that derived in Steger [17], in a setting where growth volatility is exogenous. In other words, if the economy is in autarky, the variance of the growth rate is given by parameter η_K because there is no access to diversification strategies. In our model growth volatility is endogenous because it depends on the country's portfolio allocation. Note that if foreign risk vanishes, that is, $\eta_D = 0$, the expressions of mean growth and growth volatility collapse to those derived in Obstfeld [12] (provided, however, that mean growth is defined as the growth rate of average wealth).

In addition, the condition expressed in (6) has a simple interpretation if one defines the ratio $S \equiv (A - \delta - r)/[\sigma(\eta_K^2 - \omega\eta_K\eta_D)]$ as an adjusted Sharpe ratio, that is, the difference in the risk-adjusted returns. Fixing domestic risk η_K and foreign risk η_D, two cases occur depending on risks correlation. If $\eta_K - \omega\eta_D$ is positive, that is, if correlation is negative or positive but small enough then (6) means that the domestic economy will borrow from the rest of the world and invest more domestically provided that adjusted Sharpe ratio is larger than one. For instance, if risks are not correlated, that is, if $\omega = 0$, then the domestic economy requires a positive difference in deterministic returns to borrow from the rest of the world. If, to the contrary, $\eta_K - \omega\eta_D$ is negative, which essentially implies that risks correlation is large enough, then the domestic economy will still choose to be a net debtor even if the difference in deterministic returns is negative, provided that this difference is not too negative. In other words, because a large positive correlation improves diversification, the domestic economy will be a net borrower even if the domestic return is smaller than the foreign return. In that case, the adjusted Sharpe ratio has to be smaller than one.

Note that our definition of the adjusted Sharpe ratio $S \equiv (A - \delta - r)/[\sigma(\eta_K^2 - \omega\eta_K\eta_D)]$ accounts not only for risks correlation but also for relative risk aversion. Other things equal, the larger risk aversion the riskier domestic investment appears, from a subjective preference viewpoint. Moreover, the variance - rather than the standard deviation - of risk appears in its denominator, in contrast to the standard definition of the Sharpe ratio.

The relationship between average growth and growth volatility depicted in (12) allows to interpret the optimal portfolio decision in an intuitive way. It is not difficult to get that the optimal share of domestic investment in total wealth obtains by maximizing a mean-variance criterium. More precisely, replacing the expression of the variance from equation (11) into (12) and then finding the value of β_M that maximizes $\mathbf{E}[g] - \frac{(\sigma-1)}{2}\mathbf{V}[g]$ yields the expression of β_M in equation (5). This means that growth is driven by the optimal allocation of risk between domestic investment and international lending or borrowing.

We now investigate the two important questions outlined in the introduction: (i) in this framework, what are the conditions such that financial globalization fosters growth in a small open economy and how does volatility adjust? (ii) how domestic and foreign risks affect the shape of the relationship between mean growth and growth volatility? We start with the former topic.

3 The Growth Effects of Financial Globalization

Because both the mean and the variance of growth depend on β_M, and given the optimal portfolio characterization stated in Proposition 2.1, the study of growth and volatility effects of globalization and the associated comparative statics turn out to be quite cumbersome so our strategy unfolds in two separate steps. We first examine the case such that the domestic economy differs from the rest of the world in terms of deterministic return but faces the same level of exogenous risk, what we call the *capital deepening/exporting effect*. Second, we consider a situation such that the deterministic components of domestic and foreign returns are equal while the domestic economy is exposed to a level of risk that differs from the level of foreign risk, what we call the *pure diversification effect*. Before developing such an analysis, a preliminary remark. It is not difficult to show that, absent any frictions or market imperfections, financial globalization improves welfare in our setting, as already noticed by the literature.

3.1 The Capital Deepening/Exporting Effect on Growth when Domestic and Foreign Risks Are Equal

In this section, we focus on the capital deepening/exporting effect of financial globalization by assuming that the levels of domestic and foreign risks are equal, that is, $\eta_K = \eta_D$. This means that whether the country imports or exports capital depends on the difference between deterministic returns, that is, $A - \delta - r$, but also on the correlation ω which matters as well, as we now show. Because the adjusted Sharpe ratio simplifies to $S \equiv (A - \delta - r)/[\sigma\eta_K^2(1-\omega)]$ when $\eta_K = \eta_D$, the expression in (5) simplifies to:

$$\beta_M = \frac{1}{2}(1 + S) \tag{13}$$

which essentially says that the home country will invest more than half of its wealth in the domestic asset if and only if $S > 0$ or equivalently $A - \delta > r$, that is, when the domestic deterministic return exceeds the foreign return. The condition that $\beta_M > 0$ is now $S > -1$ and the following proposition derives the conditions such that the domestic country either lends to or borrows from the rest of the world.

Proposition 3.1 (External Position and Capital Deepening/Exporting)

Assume that $\eta_K = \eta_D$, that is, domestic and foreign assets bear the same risk so that the adjusted Sharpe ratio is $S = (A - \delta - r)/[\sigma\eta_K^2(1-\omega)]$. In addition, assume that $S > -1$. Then $\beta_M > 0$, the fraction of wealth invested domestically, is an increasing function of

risks correlation ω and a decreasing function of risk η_K if and only if $S > 0$ or equivalently $A - \delta > r$, that is, if and only if the domestic deterministic return exceeds the foreign deterministic return.
In addition, the following holds.
(i) if $S > 1$ then $\beta_M > 1$, that is, the domestic country is a net borrower to the rest of the world.
(ii) if $S = 1$ then $\beta_M = 1$, that is, the domestic country neither lends nor borrows.
(iii) if $S < 1$ then the domestic country is a net lender.

The comparative statics of β_M with respect to common risk η_K (and to σ for that matter) is rather straightforward: the smaller risk, the larger the fraction of wealth invested domestically whenever the domestic asset's return dominates. As for risks correlation ω, here again, the intuition is that the larger the correlation between domestic and foreign risks, the smaller the threshold value of the difference in deterministic returns above which the country to become a net debtor.

We are now in a position to answer the question of how financial integration affects the mean and variance of the growth rate. We first focus on the conventional case with $\sigma \geq 1$. Autarky obtains when $\beta_M = 1$, which implies that mean growth and growth variance are given respectively by $\mathbf{E}[g^a] = \frac{A-\delta-\rho}{\sigma} + \frac{(\sigma-2)}{2}\mathbf{V}[g^a]$ with of course $\mathbf{V}[g^a] = \eta_K^2$.

Proposition 3.2 (Growth Effect of Global Diversification)

Under the assumptions of Proposition 3.1, suppose that $\sigma \geq 1$. Then the following holds.
(i) The variance of the growth rate $\mathbf{V}[g]$ attained under global diversification is larger than the variance of the autarkic growth rate if and only if the country is a net borrower, that is, if and only if $\beta_M > 1$.
(ii) The mean growth rate $\mathbf{E}[g]$ attained under global diversification is larger than the autarkic growth rate if $\beta_M > 1$, that is, if the country is a net borrower. On the other hand, if $1 > \beta_M > 0$, that is, if the country is a net lender, then the mean growth rate under financial globalization is, compared to autarky, smaller if and only if $1 > S > 2/\sigma - 1$ and larger if and only if $S < 2/\sigma - 1$.

Proposition 3.2 clearly shows that even if the home country is able to choose optimally its allocation of wealth between domestic and foreign assets, it is conceivable that financial integration has ambiguous effects on mean growth and growth variability. More precisely, the capital deepening/exporting effect tends to increase mean growth and growth volatility compared to autarky for countries that are net borrowers towards the rest of the world. On the other hand, while creditor countries enjoys a lower growth variance when opening up, they can at the same time benefits from faster mean growth or suffers from slower mean growth depending on whether the adjusted Sharpe ratio is smaller or larger than a threshold value. With a value for relative risk aversion set to the conventional value of 2, this threshold equals 0. Therefore, this condition tells us that the domestic country that is optimally integrated and is a net creditor would experience slower growth compared to autarky if the difference in deterministic returns is smaller than 1 times adjusted risk but positive. When the difference in deterministic returns becomes negative (but still is larger than -1 times adjusted risk as required for β_M to be positive), then the open domestic economy benefits from faster mean growth compared to autarky, just like the borrowing country.

This result is broadly consistent with empirical studies stressing threshold effects of financial globalization (see for example Kose et al. [9]). Indeed, in our AK setting, domestic returns (which depends mainly on parameters A) capture not only the technological state of the economy but also all other determinants of productivity, including institutional arrangements. Our study delivers a threshold value on these returns, which depends on the risk characteristics of the environment faced by the small-open economy, a quite natural outcome. In contrast, Devereux and Smith [3] and Obstfeld [12] show that whether or not financial globalization boosts growth depends only on preferences through σ. For instance, a typical result in those papers is that growth goes down when $\sigma > 1$ when, as in our setting, only risky assets are held. Proposition 3.2 delivers for the small-open economy a much more contrasted picture since even when σ is larger than one, mean growth and growth volatility can go up or down following financial integration. We believe our results may be seen as a first step towards providing some theoretical foundations for the empirically documented fact that how globalization affects growth is subject to threshold effect.

To summarize, the effect of financial globalization on mean growth is highly nonmonotonic under the pure capital deepening/exporting effect. In contrast, growth volatility moves in a monotonic way with the domestic country's external position so that debtor countries face a larger variance of the growth rate while creditor countries benefit from lower growth volatility, compared to autarky. Similarly, the case with $1 > \sigma$ also leads to a nonmonotonic relationship between openness and growth. However, we do not report results arising in that case, for sake of brevity, and instead turn to a more detailed comparison of our results with those in the existing literature.

3.2 The Growth Effect of Financial Globalization: Small Open Economy vs N-Country World

The purpose of this section is to compare the results presented in Section 3.1, that pertain to a small-open economy, with the growth effects of financial globalization that Obstfeld [12] derives from a N-country model. As stated in the introduction, besides the obvious fact that the world interest rate is independent of what happens in the small open economy, another key difference is that foreign borrowing is risky in our economy because the world interest rate is subject to random disturbances. This is an arguably reasonable assumption for many small countries that have to cope with variations in the cost of foreign borrowing. In contrast the setting developed in Obstfeld [12] assumes that countries lend to each-other only through *risk-free* bonds.

Obstfeld [12] develops two examples with two symmetric countries that face the same level of uncertainty. This is exactly the assumption regarding risks that we use in Section 3.1 so we now present examples along the same lines in order to illustrate the differences between our model and Obstfeld's.[1] As argued by Obstfeld, example 1 relies on a risk level that lies within the range of values typically experienced by developing countries, while example 2 assumes a much lower risk that corresponds to the historical experience of richer countries. In addition, we compare outcomes arising when relative risk aversion is smaller or larger than unity.

Example 1 (high risk/developing country): risk aversion is set at $\sigma = 1.5$ while $A - \delta = 0.05$ and $r = \rho = 0.02$. In addition, $\eta_K = \eta_D = 0.1$, so that growth variance is 1% under autarky. This we take to be our high risk/developing country example. How much growth and volatility change when the economy integrates is reported in Table 1a.

Table 1a. Average Growth (Top Panel) and Growth Variance (Bottom Panel) in Small-Open Economy and N-Country Models - Example 1a

$\mathbf{E}[g]$	Obstfeld model	Our model
Autarky	2.25%	2.25%
Integration	2.12%	3.63%

$\mathbf{V}[g]$	Obstfeld model	Our model
Autarky	1.0%	1.0%
Integration	0.5%	2.5%

Table 1a shows that, in example 1a, both models have very different implications. Obstfeld's setting produces a slight reduction in the mean growth rate after integration. Our

[1] More precisely, our examples are small variations of examples 1 and 2 in Obstfeld [12, p. 1318-19], in which we ensure that only risky assets are held both in autarky and under financial integration, just as in our analysis. Since Obstfeld uses Epstein-Zin-Weil preferences, we impose that relative risk aversion coincides with the inverse of the intertemporal substitution elasticity to make results comparable. Unlike in the other parts of this paper, to ease comparison we of course use the same definition of mean growth as Obstfeld's. See Boucekkine, Pintus and Zou [2] for an alternative and a thorough discussion in the context of Obstfeld's model.

small-open economy model, in sharp contrast, predicts that financial integration originates large growth gains: compared to Obstfeld's, the growth rate is 1.52 percentage points higher. This is because in our model the globally integrated economy has access to foreign borrowing - and not just investment in foreign assets. Given that the domestic return is larger than the borrowing cost and that both the asset and the liability have the same risk, the integrated economy actually borrows a significant amount, which boosts investment in domestic capital and thereby growth. Example 1a implies that $\beta_M = 1.5$, which translates into a debt-to-capital ratio of 33%. Differently, in Obstfeld's example with two symmetric countries holding identical shares of the global mutual fund, there is no net borrowing between countries. In addition, we should stress that the difference in predictions are not due to the world - safe - interest rate going up by a large margin in Obstfeld's model. In example 1a, this increase turns out not to be large, from 3.5% to 4.2%.

Further unreported results show that both models share predictions regarding how financial integration affects both the consumption-to-wealth ratio and the correlation between average growth and growth volatility. In example 1a, $\sigma = 1.5$ implies that the consumption-to-wealth goes up after integration, for the usual reason that the substitution effect is dominated by the income effect. In addition, both models predict that, in example 1a, average growth and growth volatility go hand in hand, which is not inconsistent with evidence for more financially integrated countries ("emerging countries", see Kose et al. [8]). However, while integration decreases mean growth and growth volatility in Obstfeld's N-country model, our small-open economy predicts that, in sharp contrast, globalization pushes up growth and volatility: because borrowing is risky, relying more on foreign credit is good for average growth but also inevitably triggers larger growth volatility, in line with the experience of emerging markets.

What happens if relative risk aversion is smaller than one? Table 1b reports how growth changes when $\sigma = 0.75$ while all other parameters are unchanged.

Table 1b. Average Growth (Top Panel) and Growth Variance (Bottom Panel) in Small-Open Economy and N-Country Models - Example 1b

$\mathbf{E}[g]$	Obstfeld model	Our model
Autarky	3.87%	3.87%
Integration	4.62%	8.93%

$\mathbf{V}[g]$	Obstfeld model	Our model
Autarky	1.0%	1.0%
Integration	0.5%	8.5%

Both models now predict that growth goes up after integration, but the growth rate increase is much larger in our model. More importantly, while growth and volatility move in opposite directions due to integration in Obstfeld's setting, our model delivers a positive correlation, which is more in line with empirical evidence about emerging markets.[2] Note that in both models, the consumption-to-wealth ratio goes down when $\sigma < 1$ since the substitution effect then dominates, which explains why the growth rate goes up in Obtfeld's setting. In example 1b we get $\beta_M = 2.5$, which means that the debt-to-capital ratio equals 60%. We now turn to our low-risk/developed country example.

Example 2 (low risk/developed country): when risk aversion is $\sigma = 1.5$, $A - \delta = 0.025$, $r = \rho = 0.02$, and $\eta_K = \eta_D = 0.02$, the growth effects of financial globalization are as reported in Table 2a.

Table 2a. Average Growth (Top Panel) and Growth Variance (Bottom Panel) in Small-Open Economy and N-Country Models - Example 2a

$\mathbf{E}[g]$	Obstfeld model	Our model
Autarky	0.34%	0.34%
Integration	0.33%	1.91%

$\mathbf{V}[g]$	Obstfeld model	Our model
Autarky	0.04%	0.04%
Integration	0.02%	1.41%

Here also, Table 2a shows that while Obstfeld's model predicts a modest decline in mean growth, our model delivers a large increase in the growth rate. This is again because it is optimal for the small open economy to be a net borrower in example 2, just as in example 1. Both models still predict a positive growth/volatility relationship, in line with the experience of OECD countries, though growth and volatility go up in our model and down in Obstfeld's.

To complete our comparison, we report in Table 2b what happens when $\sigma = 0.75$ while other parameters do not change.

[2]Although one would think that a negative correlation is in line with empirical evidence, as reported e.g. in Ramey and Ramey [13], one should keep in mind that a breakdown of estimates shows a positive correlation for industrialized countries and even a reversal of negative correlation after globalization for some developing countries - those that are more financially integrated, see Kose et al. [7, 8].

Table 2b. Average Growth (Top Panel) and Growth Variance (Bottom Panel) in Small-Open Economy and N-Country Models - Example 2b

$\mathbf{E}[g]$	Obstfeld model	Our model
Autarky	0.66%	0.66%
Integration	0.67%	5.19%

$\mathbf{V}[g]$	Obstfeld model	Our model
Autarky	0.04%	0.04%
Integration	0.02%	5.57%

Table 2b delivers conclusions that are similar to those in Table 1b: similarly, growth increases by a significant margin in our model, which also predicts a positive growth/volatility relationship, unlike Obstfeld's. Here again, differences between both model's predictions are not driven by a large increase in the world interest rate that occurs after integration in Obstfeld's model. In example 2, this increase is at most 3 basis point. What makes a difference is that our model economy borrows a lot, and even more in example 2 compared to example 1: debt-to-capital ratios are equal to 79% and 89% under the parameter values that determine Tables 2a and 2b, respectively.

The main lesson from both examples is that our small-open economy model predicts that financial globalization has different effects on growth and volatility, both in direction and magnitude, compared to the seminal N-country model developed by Obstfeld [12]. Different predictions arise, as illustrated by examples 1 and 2, because of an arguably attractive feature of the model: a globally integrated economy is likely to resort to foreign borrowing, even if the latter is risky, so as to finance investment and boost growth. Though globalization leads to both faster and more volatile growth, it is conceivable that such a combination is an optimal choice. Many - small - real-world economies that are indeed net debtor towards the rest of the world have been through large swings under the spell of volatile capital inflows.

3.3 The Pure Diversification Effect on Growth when Domestic and Foreign Deterministic Returns Are Equal

In this section we assume that investing domestically and internationally earn the same deterministic return, that is, $A - \delta = r$. This allows us to focus on how the relative strength of domestic risk relative to foreign risk impacts the mean and volatility of growth, as opposed to the capital deepening/exporting effect of financial globalization. Although the deterministic returns are supposed to be equalized, therefore, the level of domestic risk can differ from that of foreign risk and both risks may still be correlated, positively or negatively.

When $A - \delta = r$, (5) simplifies to:

$$\beta_M = \frac{\lambda(\lambda - \omega)}{1 + \lambda^2 - 2\omega\lambda}. \tag{14}$$

where we define $\lambda \equiv \eta_D/\eta_K$ as the ratio between foreign and domestic risks.

An interesting benchmark case occurs when domestic and foreign risks are equal, that is, when $\lambda = 1$. In that case, as can be seen by looking at the expression in (14), one has that $\beta_M = 1/2$: the domestic country allocates half of its wealth in domestic asset and half in foreign asset, independent of the correlation between foreign and domestic risk. However, correlation matters when domestic risk and foreign risk are not one and the same thing, as we now show.

We still do not allow short positions on the domestic asset, hence we assume that $\lambda > \omega$ so that $\beta_M > 0$. The interpretation of this condition is here again that external debt cannot be larger than the stock of domestic capital. We get the following result:

Proposition 3.3 (External Position and Pure Diversification)

Assume that $A - \delta = r$, that is, domestic and international assets earn the same deterministic return, and $\lambda \equiv \eta_D/\eta_K > \omega$, that is, external debt is smaller than domestic capital so that $\beta_M > 0$. Then the following holds.
(i) if $1 > \lambda$, that is, if domestic risk is larger than foreign risk, then β_M is a decreasing function of risks correlation ω and $1 > \beta_M$, that is, the domestic country is a net lender to the rest of the world, independent of ω.
(ii) if $\lambda > 1$, that is, if domestic risk is smaller than foreign risk, then β_M is an increasing function of risks correlation ω and:

(a) if $1 \geq \omega > 1/\lambda$ then $\beta_M > 1$, that is, the domestic country is a net borrower when risks correlation is large enough,

(b) if $\omega = 1/\lambda$ then $\beta_M = 1$, that is, the domestic country neither lends nor borrows,

(c) if $1/\lambda > \omega \geq -1$ then $\beta_M < 1$, that is, the domestic country is a net lender when the risks correlation is small enough.

Case (i) in Proposition 3.3 has a trivial interpretation: given that domestic and foreign assets have equal returns, the small-open economy optimally chooses to be a net lender whenever domestic risk is larger than foreign risk.

The interpretation of case (ii) in Proposition 3.3 is that, when $\lambda > 1$, the domestic country borrows from the rest of the world only if the correlation between domestic risk and foreign risk is positive and large enough. In that case, for example, a fall in domestic

TFP is associated with a decrease in the world interest rate. But then the loss of resources generated by the productivity slowdown are compensated by a fall in debt service, which helps to smooth consumption. Conversely, when domestic and foreign risks are negatively correlated or have a weak positive correlation, the country has incentives to lend to the rest of the world. Case (*ii*) of Proposition 3.3 shows that the threshold value for ω above which the domestic country is a net debtor is given by η_K/η_D. If domestic risk is shut down, that is, $\eta_K = 0$, then the economy becomes autarkic, that is, $\beta_M = 1$. Now if domestic risk is slightly positive but still smaller than foreign risk, it makes sense for the country to lend if correlation is negative and to borrow if correlation is positive. The larger domestic risk compared to foreign risk, the more correlated the risks should be for the domestic country to be willing to borrow. Finally, because β_M is a ratio of second-order polynomials in η_K, how the fraction invested in the domestic asset depends on foreign risk depends in a a nonmonotonic way on all parameters. Instead of reporting results along those lines, we simply mention that β_M is either a hump-shaped, a U-shaped or a S-shaped function of η_K.

The next proposition establishes how financial globalization affects mean growth and growth volatility, with the expression $\mathbf{E}[g] = \frac{r-\rho}{\sigma} + \frac{(\sigma-2)}{2}\mathbf{V}[g]$ obtained from equation (12) when $A-\delta = r$. In addition, plugging the expression of β_M from equation (14) into equation (11), one gets a simple expression for the growth rate variance:

$$\mathbf{V}[g] = \frac{(1-\omega^2)\eta_D^2\eta_K^2}{\eta_K^2 + \eta_D^2 - 2\omega\eta_D\eta_K} \tag{15}$$

Note that, rather trivially, growth volatility vanishes in the extreme cases with perfect correlation, that is, either $\omega = 1$ or $\omega = 1$. This is because perfect hedging against risk is possible in those two corner cases, given that domestic and foreign deterministic returns are supposed to be equal.

Proposition 3.4 (Growth Effect of Global Diversification)

Under the assumptions of Proposition 3.3, the following holds.
(*i*) *The variance of the growth rate* $\mathbf{V}[g]$ *is, under financial globalization with optimal diversification, smaller than the variance of the autarkic growth rate.*
(*ii*) *The mean growth rate* $\mathbf{E}[g]$ *is, under financial globalization with optimal diversification, smaller than the autarkic growth rate if and only of* $\sigma > 2$.

In accord with intuition, Proposition 3.4 states that, under equal deterministic returns from domestic and foreign assets, global diversification helps reducing variance. This is the pure diversification effect. Therefore, under the condition that the income effect triggered by risk strongly dominates the substitution effect - that is, when $\sigma > 2$ - mean growth moves in the same direction as growth volatility.

4 The Non-Monotonic Effects of Domestic and Foreign Risks on the Average and Volatility of Growth

We now focus on how the structure of risks affect growth and volatility. As in the previous section, we may study the effects of domestic and foreign risks on growth in two steps. We report here the results corresponding to the pure diversification case, that is, when the domestic and foreign risks are of different size, which is one interesting case from the economic point of view. The case of the capital deepening/exporting effect is reported in Appendix B. Both cases deliver the same qualitative results (hump-shaped relationships between mean growth and volatility and the risk parameters) and rely on similar economic interpretations.

Let us thus examine what are the growth effects of domestic and foreign risks when $A - \delta = r$, which amounts to isolating the pure diversification effect. As explained in Section 3.2, one gets that $\beta_M = \frac{\lambda(\lambda-\omega)}{1+\lambda^2-2\omega\lambda}$, where $\lambda \equiv \eta_D/\eta_K$. It turns out that $\mathbf{V}[g]$ is a hump-shaped function of risks correlation when $\eta_D > \eta_K$, as illustrated in the Figure 1.

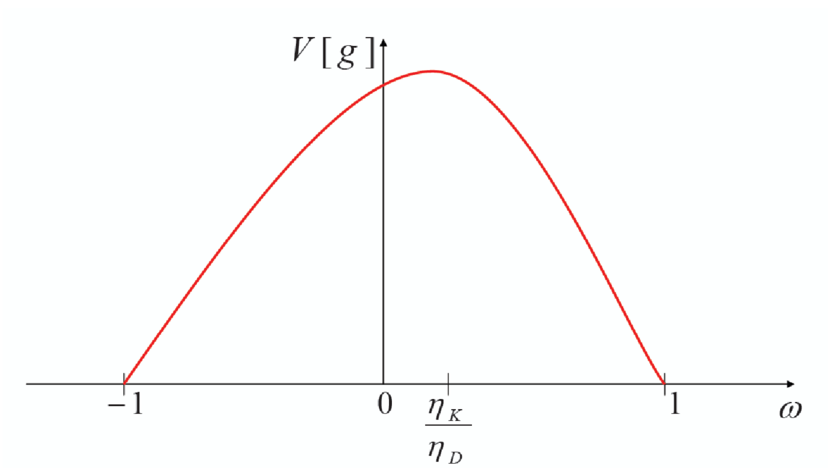

Figure 1: Variance of Growth Rate $\mathbf{V}[g]$ against Risks Correlation ω
Case (ii) of Proposition 3.3

The intuition behind Figure 1 is the following. With perfect, negative correlation $\omega = -1$, the country can lend just enough to diversify risks away. Moving away from perfect, negative correlation increases variance up to the point where the country becomes a net debtor, in which case variance goes down until, in the limit, risks are again diversified away when they are perfectly, positively correlated ($\omega = 1$). In addition, the expression of growth variance in (15) enables us to show that $\mathbf{V}[g]$ is also a hump-shaped function of foreign risk η_D when $\eta_D > \eta_K$ and a similar intuition holds.

Summarizing the results on growth variance and mean growth, one gets the following proposition.

Proposition 4.1 (Growth Effect of Domestic and Foreign Risks)

(i) Under the assumptions of case (i) in Proposition 3.3, the variance of the growth rate $\mathbf{V}[g]$ is an increasing function of both risks correlation ω and of foreign risk η_D. It follows that:

(a) if $\sigma > 2$, mean growth $\mathbf{E}[g]$ is also an increasing function of ω and of η_D,

(b) if $\sigma = 2$, $\mathbf{E}[g]$ is independent of ω and of η_D,

(c) if $\sigma < 2$, $\mathbf{E}[g]$ is a decreasing function of ω and of η_D.

(ii) Under the assumptions of case (ii) in Proposition 3.3, the variance of the growth rate $\mathbf{V}[g]$ is a hump-shaped function of both risks correlation ω and of foreign risk η_D, as illustrated in Figure 1.

It follows that:

(a) if $\sigma > 2$, mean growth $\mathbf{E}[g]$ is also a hump-shaped function of ω and of η_D,

(b) if $\sigma = 2$, $\mathbf{E}[g]$ is independent of ω and of η_D,

(c) if $\sigma < 2$, $\mathbf{E}[g]$ is a U-shaped function of ω and of η_D.

It is clear from Proposition 4.1 that an increase in either foreign risk or risks correlation has opposite effects on the variance and the mean of the growth rate only if $\sigma < 2$. In that case, for example, an increasing correlation between domestic and foreign risks - that can be interpreted as associated to a wave of deeper globalization that leads to more synchronized fluctuations across countries - leads to two opposite phases: initially, mean growth goes down while growth volatility increases until this pattern reverses, with growth volatility going down and mean growth going up. In contrast, when $\sigma > 2$, mean growth and growth variance move together, first up when correlation increases from -1 then down when correlation crosses a positive threshold.

On the other hand, case (i) of Proposition 3.3 shows that the country is a net lender whenever $\eta_K > \eta_D$. The fact that the country does not borrow comes from our assumption that ω cannot be larger that η_D/η_K, for if that would be true it is easy to show that β_M would be negative and the country would choose to go short on its domestic capital stock. If $\eta_D/\eta_K > \omega$ then, it follows that whenever domestic risk is larger than foreign risk, both mean growth and growth volatility are monotonic functions of risks correlation. Of course, whether mean growth and growth volatility move together or not depends on whether σ is larger or smaller than 2.

As already noticed, in qualitative terms similar nonmonotonic relationships arise under the capital deepening/exporting effect, as shown in Appendix B. However, direct comparison of Propositions 4.1 and B.1 reveals that the directions of those relationships may or may not be aligned under the pure diversification effect and under the capital deepening/exporting effect. This means that those two forces could, when combined, result in stronger nonlinearities. In other words, the nonmonotonicities arising in the full model of Section 2 are expected to be even stronger than those reported in this section and Appendix B.

5 Conclusion

In this paper, we have revisited theoretically the growth consequences of financial openness through the lens of an AK small-open economy model. We believe that this is a useful complement to the growth literature which connects to global diversification and international risk sharing. In particular, an attractive feature of our model is that it allows, quite realistically, foreign borrowing to be risky. Different from the typical wisdom that has been derived from standard N-country models of global diversification *à la* Devereux-Smith-Obstfeld, we have shown that how financial globalization affects growth and volatility depends on *both* preferences and the structure of risk in the small open economy case: integration may either slow down or stimulate growth depending on the interaction between relative risk aversion, on the one hand, and the return and risk characteristics of domestic and foreign assets, on the other hand. Last but not least, we have also provided with a full theoretical characterization of the shapes of the relationships between growth volatility and mean growth and the levels and correlation of domestic and foreign risks, which is to a certain extent novel. We believe that our results are broadly consistent with available empirical studies that stress threshold effects, especially for developing countries.

Although a careful calibration and empirical test of the model are beyond the scope of this paper, we would like to point at the fact that our setting delivers a number of testable implications. For example, Kose et al. [7] document how a measure of correlation that is not too far from our model's notion has been time-varying since the 1960's. Relatedly, our model predicts that variations in risks correlation could account for how growth and volatility - and their correlation - have changed over time over the last half-century, in particular after economies have opted for financial integration. Similarly, the evolution through time of what is "exogenous" risk in our model has also a direct impact on average growth and growth variance. To take just one example, actual changes in economic policies and other relevant institutions that affect countries' productivities are expected to affect growth in our model that has testable implications in that respect. An exploration along those lines seems promising to us and should be the topic of further research. In addition, although our model assumes that the demand for foreign borrowing is always satisfied in the small-open economy, preliminary and unreported results show that it could easily be amended to account for credit market frictions that put some limitation on available credit. Such an extension would allow not only to investigate how growth and volatility, on one hand, and welfare, on the other, are affected under financial globalization but also to revisit the issue of "sudden stops" in an endogenous growth setting, thus completing the literature that has largely focused on business-cycle models.

Finally, at a more theoretical level, our portfolio approach to the effect of financial globalization would be enriched by the addition of mechanisms that endogenize risks correlation, for example along the lines of Mastuyama et al. [10] who show how trade integration leads to more synchronized movements of productivities in open economies (see also Kose et al. [6] for some related empirics). Because trade integration has typically preceded financial integration in many countries, such a combination would help explain how actual - and counterfactual - globalization sequences affect growth and volatility and why such an impact has no reason to be uniform across countries, as documented by numerous empirical studies. This also calls, in our view, for further research.

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A Proof of Proposition 2.1

The current value Hamiltonian of the system is given by

$$\begin{aligned}
H_{CV}(x,C,\beta,p,q) &= [(A-\delta)\beta x + r(1-\beta)x - C]\,p + \tfrac{C^{1-\sigma}-1}{1-\sigma} \\
&\quad + \tfrac{1}{2}qx^2(\eta_K^2\beta^2 + \eta_D^2(1-\beta)^2 + 2\omega\eta_K\eta_D\beta(1-\beta)) \\
&= H_{CV}^C(x,C,p,q) + H_{CV}^\beta(x,\beta,p,q) \\
&= \left\{-Cp + \tfrac{C^{1-\sigma}-1}{1-\sigma}\right\} + [(A-\delta)\beta x + r(1-\beta)x]p + \tfrac{1}{2}qx^2[\eta_K^2\beta^2 + \eta_D^2(1-\beta)^2 + 2\omega\eta_K\eta_D\beta(1-\beta)]
\end{aligned} \tag{16}$$

We define the maximum value Hamiltonian as:

$$H(x,p,q) := \max_{C\geq 0,\beta\in R} H_{CV}(x,C,\beta,p,q).$$

We look for a function $v: R \to R$ that solves the Hamilton-Jacobi-Bellman equation of the system, that is

$$\rho v(x) - H(x, v'(x), v''(x)) = 0. \tag{17}$$

We look for a solution of the form

$$v(x) = \frac{\gamma x^{1-\sigma} - \frac{1}{\rho}}{1-\sigma} \tag{18}$$

for some positive γ (γ positive even if $\sigma > 1$, in that case the term $1-\sigma$ is negative and v is negative). In this case we have

$$v'(x) = \gamma x^{-\sigma}$$

and

$$v''(x) = -\sigma\gamma x^{-1-\sigma}.$$

Since C and β appear in the current value Hamiltonian in the distinct terms H_{CV}^C and H_{CV}^β (as described in (16)) we can easily find the C and the β that maximize H_{CV}. The value of the maximizing C is given by

$$C_M := \arg\max_{C \geq 0} H_{CV}^C(x, C, v'(x), v''(x)) = (v'(x))^{-1/\sigma} = \gamma^{-1/\sigma} x \tag{19}$$

so that

$$\begin{aligned} &H_{CV}^C(x, C_M, \gamma x^{-\sigma}, -\sigma\gamma x^{-1-\sigma}) = \\ &\tfrac{C_M^{1-\sigma}-1}{1-\sigma} - C_M \gamma x^{-\sigma} = \tfrac{\sigma}{1-\sigma}\gamma^{1-\frac{1}{\sigma}} x^{1-\sigma} - \tfrac{1}{1-\sigma}. \end{aligned} \tag{20}$$

The expressions of the β that maximizes the current value Hamiltonian is given by

$$\beta_M := \arg\max_{\beta \in R} H_{CV}^\beta(x, \beta, v'(x), v''(x)).$$

Since the expression of $H_{CV}^\beta(x, \beta, p, q)$ as a function of β is simply a parabola one can easily see, taking the derivative in β of H_{CV}^β, that β_M needs to satisfy

$$0 = (-\delta - r + A)\gamma x^{1-\sigma} + \frac{1}{2} x^{1-\sigma}(-\sigma\gamma)\Big[2\eta_K^2 \beta_M + 2(\beta_M - 1)\eta_D^2 - 2\omega\eta_K\eta_D\beta_M + 2\omega\eta_D\eta_K\Big]$$

that is

$$\beta_M = \frac{1}{\sigma}\frac{(A - \delta - r) + \sigma(\eta_D^2 - \omega\eta_K\eta_D)}{\eta_K^2 + \eta_D^2 - 2\omega\eta_K\eta_D}. \tag{21}$$

Putting everything together we have that an expression of the form given by (18) is a solution for (17) if and only if

$$\begin{aligned} &\rho\tfrac{\gamma x^{1-\sigma} - \frac{1}{\rho}}{1-\sigma} - H(x, \gamma x^{-\sigma}, -\sigma\gamma x^{-1-\sigma}) \\ &= \rho\tfrac{\gamma x^{1-\sigma} - \frac{1}{\rho}}{1-\sigma} - H_{CV}^C(x, C_M, \gamma x^{-\sigma}, -\sigma\gamma x^{-1-\sigma}) - \\ &H_{CV}^\beta(x, \beta_M, \gamma x^{-\sigma}, -\sigma\gamma x^{-1-\sigma}) = 0 \end{aligned} \tag{22}$$

i.e.

$$\begin{array}{l}\rho\frac{\gamma x^{1-\sigma}-\frac{1}{\rho}}{1-\sigma}-\left[\frac{\sigma}{1-\sigma}\gamma^{1-\frac{1}{\sigma}}x^{1-\sigma}-\frac{1}{1-\sigma}\right]\\ -\left(A\beta_M-\delta\beta_M-r(\beta_M-1)\right)\gamma x^{1-\sigma}\\ -\frac{1}{2}\left(\sigma\gamma x^{1-\sigma}\right)\left[\eta_K^2\beta_M^2+\eta_D^2(1-\beta_M)^2+2\omega\eta_K\eta_D\beta_M(1-\beta_M)\right]=0\end{array} \tag{23}$$

where β_M is given by (21). If we denote by

$$\begin{array}{l}\Sigma_M:=\{(A\beta_M-\delta\beta_M-r(\beta_M-1))\\ -\frac{1}{2}\sigma\left[\eta_K^2\beta_M^2+\eta_D^2(1-\beta_M)^2+2\omega\eta_K\eta_D\beta_M(1-\beta_M)\right]\},\end{array} \tag{24}$$

then (23) is verified for any x is and only if

$$\frac{\rho}{1-\sigma}-\frac{\sigma}{1-\sigma}\gamma^{-1/\sigma}-\Sigma_M=0$$

and then

$$\gamma=\left(\frac{\rho}{\sigma}-\frac{1-\sigma}{\sigma}\Sigma_M\right)^{-\sigma}. \tag{25}$$

Then the function $v(x)=\frac{\gamma x^{1-\sigma}-\frac{1}{\rho}}{1-\sigma}$ is a smooth solution of the HJB equation associated to the optimal control problem. One can then conclude using the general theory (see for example Chapter 5 and in particular Theorem 5.1 page 268 of Yong and Zhou, 1999) that $v(x)$ is indeed the value function of the problem and that the feedback induced by v

$$\begin{array}{l}C,\beta=\left(\arg\max_{C\geq 0}H_{CV}^{C}(x,C,v'(x),v''(x)),\arg\max_{\beta\in R}H_{CV}^{\beta}(x,\beta,v'(x),v''(x))\right)\\ =\left(\gamma^{-1/\sigma}x,\frac{1}{\sigma}\frac{(A-\delta-r)+\sigma(\eta_D^2-\omega\eta_K\eta_D)}{\eta_K^2+\eta_D^2-2\omega\eta_K\eta_D}\right)\end{array} \tag{26}$$

is in fact optimal. Equation (10) is a straightforward corollary of this fact.
Finally, from (10) it follows that

$$\mathbf{V}[g]=\eta_K^2\beta_M^2+\eta_D^2(1-\beta_M)^2+2\omega\beta_M(1-\beta_M)\eta_K\eta_D \tag{27}$$

$$\mathbf{E}[g]=(A-\delta)\beta_M+r(1-\beta_M)-\gamma^{-1/\sigma}-\frac{1}{2}\mathbf{V}[g] \tag{28}$$

Plugging the expressions of γ appearing in (25) in terms of the parameters and β_M we get that the mean and variance of growth are related through the following equation:

$$\mathbf{E}[g]=\frac{(A-\delta)\beta_M+r(1-\beta_M)-\rho}{\sigma}+\frac{(\sigma-2)}{2}\mathbf{V}[g] \tag{29}$$

□

B Nonmonotonic Growth Effects when Domestic and Foreign Risks Are Equal

As in Section 3.1 assume that the levels of domestic and foreign risks are equal, that is, $\eta_K = \eta_D$, which amounts to isolating the capital deepening effect. In such a case, one gets $\beta_M = \frac{1}{2}(1+S)$, and the positivity of the adjusted Sharpe ratio is equivalent to the domestic deterministic return exceeding the foreign return as explained in Section 3.1. Replacing β_M by the expression above in the optimal expression of growth volatility and mean growth given by (11) and (12) under $\eta_K = \eta_D$, one can readily establish how the levels of risk and correlation impact growth.

Proposition B.1 (Growth Effect of Domestic and Foreign Risks)

Under the assumptions of Proposition 3.1, the following holds.
(i) $\mathbf{V}[g]$ *is an increasing function of risks correlation* ω*, that is, the more correlated domestic and foreign risks, the larger growth volatility. On the other hand,* $\mathbf{E}[g]$ *is an increasing function of risks correlation* ω *if and only if* $S^2 > 2/\sigma - 1$*, which holds if* $\sigma > 2$*.*
(ii) $\mathbf{E}[g]$ *and* $\mathbf{V}[g]$ *are U-shaped functions of the level of risk* η_K*. More precisely,* $\mathbf{E}[g]$ *goes up with* η_K *if and only if:*

$$S^2 > \left(\frac{\sigma - 2}{1 + \sigma}\right)\left(\frac{1 + \omega}{1 - \omega}\right)$$

which is satisfied when $\sigma < 2$*, while* $\mathbf{V}[g]$ *goes up with* η_K *if and only if:*

$$S^2 > \frac{1 + \omega}{1 - \omega}$$

Therefore, mean growth and growth volatility decrease with increasing risk when risk is low. In contrast, when risk is large enough, mean growth and growth volatility increase with increasing risk.