

Prevention Incentives in Long-Term Insurance Contracts

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Abstract

Long-term insurance contracts are widespread, particularly in public health and the labor market. Such contracts typically involve monthly or annual premia which are related to the insured's risk profile, where a given profile might change based on observed outcomes which depend on the insured's prevention efforts. The aim of this paper is to analyze the latter relationship. In a two-period optimal insurance contract in which the insured's risk profile is partly governed by the effort he puts on prevention, we find that both the insured's risk aversion and prudence play a crucial role. If absolute prudence is greater than twice absolute risk aversion, moral hazard justifies setting a higher premium in the first period but also greater premium discrimination in the second period. For specific utility functions, moreover, an increase in the gap between prudence and risk aversion increases the initial premium and the subsequent premium discrimination. These results provide insights on the tradeoffs between long-term insurance and the incentives for primary prevention arising from risk classification, as well as between inter- and intra-generational insurance.

Keywords Long-term insurance; Classification risk; Moral hazard; Prudence

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1 Introduction

Long term insurance contracts are common, notably in public health and the labor market (e.g. unemployment insurance). One of their main features is the adjustment of premia in response to observed outcomes and the consequent evolution of risk profiles. The aim of this paper is to analyze the effect of this on an insured's preventive efforts.

The topic of risk classification (*i.e.* the use of observable characteristics to pool together individuals with similar risk exposure) has received significant attention in the literature (see Crocker and Snow 2014 for a survey). Being analogous to price discrimination, it has raised equity issues (see e.g. Dionne and Rothschild 2014) that have recently be tackled by regulators.¹ The allocative efficiency resulting from information acquisition by insurance companies has also been looked at (see Bond and Crocker 1991 or Polborn et al. 2006).

In long-term insurance, however, risk classification is not only a response to hidden knowledge or adverse selection; it also relates to moral hazard, as it shapes incentives for prevention that can impact the insured's future risk profile. Accordingly, we study here optimal long-term insurance contracts under moral hazard, focusing on the evolution of premia and the impact of classification risk.

We do so using a two-period model of dynamic insurance with change in risk exposure during the life cycle. In the first period, agents are identically exposed to a risk and can invest in prevention. In the second period, agents can either be of high-risk or low-risk type. Prevention effort in the first period have the effect of lowering the probability of being high-risk (that is, the probability of having a high probability of falling ill, in the case of health insurance) in the second period. When effort is observable and contractible upon, long-term insurance fully covers classification risk (high-risk and low-risk agents optimally pay the same premium). On the other hand, when effort is unobservable, the insurance offered during the second period depends on the risk type (which we assume to be observable and public information).² This generates classification risk.

We then highlight two ways to deal with this risk. The first consists in transferring wealth between the two periods through the prepayment of premia (that is: intergenerational insurance). By paying a higher premium in the first period, an insured can reduce second-period premia and classification risk. Such a strategy is related to pain disaggregation and to the notion of prudence (see Eeckhoudt and Schlesinger 2006). As it allows an unequal distribution of the prepaid premium between the two types in the second period³, however, it also appears to be linked to risk aversion. The second strategy is to induce the insured to take on prevention efforts. As shown by Chiu (2000) and Eeckhoudt and Gollier (2005) in the case of self-protection, prevention can be linked to both prudence and risk aversion. This suggests that the trade-off between the two means of reducing classification risk also depends on both prudence and risk aversion.

¹Risk classification has been restricted by the Patient Protection and Affordable Care Act in 2010 in the US and the EU has banned classification based on gender in 2012.

²In other words, we focus here on moral hazard (hidden actions) rather than on adverse selection (hidden information). This last case has been extensively studied in the literature (see *e.g.* Bond and Crocker 1991 or Polborn et al. 2006).

³In this sense, prepayment of premiums appears to be more flexible than precautionary saving.

The present paper shows that the critical level of the ratio of absolute prudence (P , as introduced by Kimball 1990) to absolute risk aversion (A) is actually 2. If P is greater than two times A , in response to future uncertainty an insured prefers to transfer wealth in the second period rather than exert effort. In this case, moral hazard – through the unobservability of preventive efforts – increases the first-period premium (hence enhances intergenerational insurance). On the contrary, when $P < 2A$, the insured favors prevention (rather than inter-period transfers) and moral hazard reduces classification risk and intergenerational insurance. With CRRA (Constant Relative Risk Aversion) preferences, it appears that an increase in the difference between prudence and twice risk aversion ($P - 2A$) increases classification risk if the cost of effort is low enough. After defining a suitable utility function that satisfies the simplifying property of having a linear reciprocal derivative, we moreover show that the amount of prepaid premium increases with this difference.

From a normative point of view, considering public policy, we also investigate the beneficial effect risk classification can have on preventive efforts. This corresponds, for example, to safe behavior or a "hygieno-dietetic" regime in the case of health insurance,⁴ or to training that reduces the probability of unemployment in the future. Insights on the latter might in particular contribute to the current debate in Europe about the tradeoff between the generosity of unemployment insurance and incentives to train.

Finally, we prove that classification risk can be reduced by a decrease in the cost of prevention, whatever the insured's preferences, or by increasing the effectiveness of prevention when $P < 2A$.

To reduce this risk that could make insurance unaffordable to most agents, Pauly et al. (1995) propose guaranteed renewable insurance policies consisting in a declining schedule of premia, whereas Cochrane (1995) recommends time-consistent insurance contracts that safeguard against classification risk using severance payments. These proposals, however, rely on the assumption of a perfect market and can be ineffective if agents are too impatient or face borrowing constraints (Frick 1998, Pauly et al. 1998). Hendel and Lizzeri (2003) moreover point out that severance payments cannot be implemented in life insurance for legal reasons.

Our paper is not the first attempt to introduce moral hazard in dynamic insurance contracts. Abbring et al. (2003) use dynamic insurance contracts in their empirical study on the distinction between moral hazard and adverse selection, but they analyze moral hazard related to the probability of accident. An important difference is that, in our model, the effort exerted in the current period reduces the probability of being high-risk next period. In a two-period model similar to ours, Hendel and Lizzeri (2003) analyze to what extent the prepayment of premia (front-loading) can reduce classification risk when accounting for cream-skimming. They state that front-loading allows the reduction of both cream-skimming (low-risk agents are insured at their fair premium in the second period) and classification risk (agents of different types have the same insurance contract).⁵ On this basis, they moreover argue that the various degrees of front-loading and lapsation observed in insurance data can be explained by heterogeneous costs of front-loading (that

⁴Insurance market initiatives show how attractive this kind of prevention is to insurers, and especially mutual insurers. In 2005 and 2007 respectively, French mutual insurers AGF and MAAF began to reimburse some food products designed to lower cholesterol. A similar program was also introduced in 2005 by the Dutch insurer VGZ.

⁵Contrary to us, Hendel and Lizzeri (2003) allow for more than two risk types in the second period.

is by heterogeneous profiles of income growth). Through preventive effort we add moral hazard to this model.⁶ To be incentive-compatible, the optimal contract then necessarily specifies different insurance schemes for different types. We therefore offer an alternative explanation for the stylized fact highlighted by Hendel and Lizzeri (2003): the variety of front-loading and lapsation observed can be explained by heterogeneous behavior toward risk (risk aversion and prudence).

This is also not the first work where the relationship between P and $2A$ turns out to matter. It has already been found to play a role in various context, such as the opening of a new asset market (Gollier and Kimball 1996), when there is uncertainty on the size (Gollier et al. 2000) or the probability of losses (Gollier 2002), and under contingent auditing (Sinclair-Desgagné and Gabel 1997). Below, we provide an intuitive and new interpretation of this relationship. It is now well established that the inverse of marginal utility ($1/u'$) plays a preponderant role in models with moral hazard. Our paper underlines the influence of the degree of concavity of this function, which is captured by the difference between P and $2A$.

We present the model in the next section. The optimal dynamic contract under moral hazard is defined in Section 3 and general results of comparative statics are provided in Section 4. To take the analysis of comparative risk preferences further, we rely on specific utility functions in Section 5. We present a possible application of our model to unemployment and life insurance in Section 6. Section 7 contains concluding remarks and directions for future research.

2 The Model

To analyze the impact of moral hazard on prepayment and classification risk, we build a two-generation model with change in risk exposure during the life cycle. We model the simplest 2-period, 2-type case and assume that (homogeneous) newborn agents can affect their second-period risk status through prevention.

Consider generations (of identical size) living for two periods $t = 1, 2$. In each period, all agents receive a sure revenue R . During the first period, all (young) individuals face the same risk, that is the same probability q_1 of suffering a loss L . Let us note $K_1 \equiv q_1 L$ the expected loss, that is the expected health cost in the case of health insurance. At $t = 2$, (old) agents may be of two types. Either, with probability p , they are low-risk type and face a probability of loss q_2^l ($K_2^l \equiv q_2^l L$) or, with probability $1 - p$, they are high-risk type and suffer the loss with probability q_2^h , with $q_2^l < q_2^h$ (therefore $K_2^l < K_2^h \equiv q_2^h L$). At each period, we assume that two generations coexist (one composed of young agents and the other of older individuals).

Information about an agent's risk type is revealed at the beginning of the second period (for example through medical check-ups) and is then public information. Young agents can exert a preventive effort that reduces the probability of becoming high-risk in the second period. We assume that agents choose between two levels of prevention \underline{e} and \bar{e} ($\underline{e} < \bar{e}$) leading respectively to probabilities of being low-risk $p(\underline{e}) \equiv \underline{p}$ and $p(\bar{e}) \equiv \bar{p}$, with $\underline{p} < \bar{p}$. Let us note $\Delta p \equiv \bar{p} - \underline{p}$.

Let X_i^j be the wealth of agents of type j in period i . In the absence of insurance, the income

⁶This is partly done at the expense of the study of limited commitment. Although we discuss the issue of lapsation, we do not explicitly model the possibility for low-risk agents to exit the long-term insurance contract.

profile of a newborn agent can be schematized as follows:

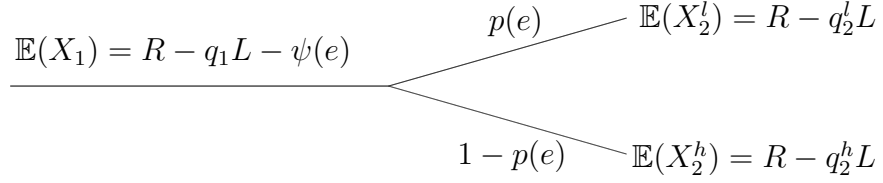


Figure 1: Income profile without insurance

During the first period, the utility function is supposed to be separable in wealth and effort and the utility-cost of exerting a high effort of prevention is noted $\varphi \equiv \psi(\bar{e}) - \psi(\underline{e})$. We moreover assume time separability of preferences (to distinguish saving and insurance behavior)⁷, and for the sake of simplicity, that utility is linear in wealth during the first period. This last assumption simplifies the analysis in several ways. First, it allows us to isolate second-period preferences concerning risk. Moreover, the agents being risk neutral during the first period, they have no reason to buy insurance at this stage, other than for the sake of prepayment.⁸ We can therefore identify a first-period premium with prepayment. Note here that our qualitative results remain robust as long as we assume that agents have different utility functions for the two periods.⁹

Let $u(\cdot)$ (with $u'(\cdot) > 0$ and $u''(\cdot) < 0$) be the utility function of both types of old agents. We assume that there is no direct utility loss due to risk status. This would anyway be detrimental to the welfare of high-risk agents and therefore argue for a lower classification risk.

To insure against this two-period risk, a mutual (or public) insurer offers young agents a long-term insurance contract, that is a contract specifying premium and coverage for both periods that depends on risk status in the second period.¹⁰ The timing of the game is described in Figure 2.

⁷Using incentive constraints prevents us from using Kreps-Porteus preferences, which would allow us to fully disentangle risk and time effects. However, the use of such non-expected utility makes the problem untractable as it greatly complicates the writing of the incentive compatible constraint.

⁸The first-period risk is therefore not crucial for our model. However, it justifies buying insurance in the first period, that is, intergenerational insurance.

⁹This can for example reflect higher risk aversion for older agents, as argued by Bakshi and Chen (1994) or Palsson (1996).

¹⁰The experienced reader will note that the following problem also fits in the case of competing insurance companies that do not seek to propose profitable one-period contracts. We briefly discuss this alternative interpretation in Section 5. However, as illustrated in Geoffard (2000), premiums of mutual insurers (in France) increase less with age than those of stock insurers. This seems to indicate that long-term contracts, as modeled here, are more likely to be offered by mutual insurers.

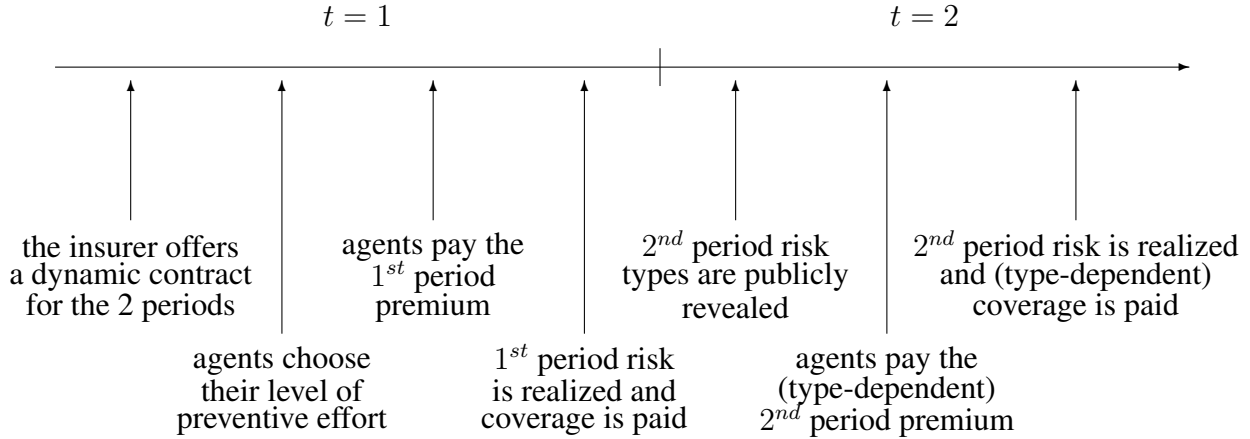


Figure 2: Timing of the game

Dynamic insurance contracts allow the insurer to use first-period premiums to decrease the premium offered to high-risk agents when old (as shown in Hendel and Lizzeri 2003). However, when effort is unobservable and not contractible upon, the incentive to exert high preventive effort may be reduced.¹¹ The aim of this paper is to analyze the trade-off resulting from a decrease in the premium for high-risk in the second period: the trade-off between an increase in insurance and a decrease in the incentive for preventive effort.¹²

Note here that prepayment of premiums can be related to precautionary saving as it corresponds to an intertemporal transfer of wealth used to deal with future uncertainty. These two mechanisms however differ in a fundamental respect that plays a important role in our setting. Whereas savings have the same return in every future state (and therefore do not have much impact on classification risk), the insurer can choose to reallocate the prepaid premiums differently across second-period types (which can have an important impact on classification risk).

3 The optimal dynamic contract

3.1 The benchmark case of observable effort

It is easy to show, using the concavity of the utility function, that the dynamic insurance contract necessarily specifies complete insurance (in the sense that it provides an agent with the same wealth whether she suffers damage or not) once risk types are known.¹³ A dynamic contract

¹¹A polar case of such a mechanism is genetic insurance – as proposed by Tabarrok (1994) – which corresponds to a full insurance of classification risk and is therefore not optimal in presence of moral hazard.

¹²Contrary to Hendel and Lizzeri (2003), we do not allow agents to exit the contract (*i.e.* to lapse) once their 2nd period risk types are revealed. However, as noted in Hendel and Lizzeri (2003), lapsation is closely related to prepayment, as early payment of premiums tends to lock agents into the contract. We discuss this issue in Section 5.

¹³This issue is more problematic in Hendel and Lizzeri (2003) who model life insurance and therefore specify a state- (alive/dead) dependent utility function.

is therefore fully defined by a triplet $(\Pi_1, \Pi_2^l, \Pi_2^h)$ of premiums corresponding respectively to the expected costs K_1 , K_2^l and K_2^h . The coverage in all cases equals the amount of the loss L . Under such a dynamic insurance contract, the income profile of a newborn agent can then be summarized as follows:

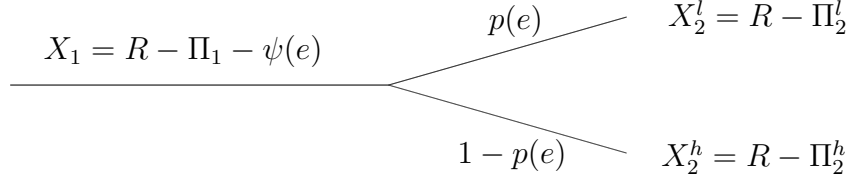


Figure 3: The income profile under the insurance contract

The risk of being classified high-risk is then measured by the difference between the second-period premiums.

Definition 1 *The classification risk is defined as the risk of being classified high-risk by one's insurer and therefore paying a higher premium. In our two-type model with complete insurance in each state, this risk is simply measured by the spread between the premiums paid by each type in the second period: $\Pi_2^h - \Pi_2^l$*

The insurer then seeks to maximize the expected utility of a young individual exerting an effort e :

$$(R - \Pi_1) - \psi(e) + p(e)u(R - \Pi_2^l) + (1 - p(e))u(R - \Pi_2^h) \quad (3.1)$$

If the insurer is large enough, it can rely on the law of large numbers, and the zero profit condition writes

$$\Pi_1 + p(e)\Pi_2^l + (1 - p(e))\Pi_2^h = K_1 + p(e)K_2^l + (1 - p(e))K_2^h \equiv \mathbb{E}(K|e). \quad (3.2)$$

This states that the sum of premiums collected (from young and old agents) allows (in expectation) for the reimbursement of health costs. Recall here that we are considering a model with identical agents (that therefore exert the same preventive effort) and generations of identical size.

With observable effort it is then optimal to set $\Pi_2^{l*} = \Pi_2^{h*} = \Pi_2^*$ such that $u'(R - \Pi_2^*) = 1$ and $\Pi_1^* = \mathbb{E}(K|e) - \Pi_2^*$. Therefore, the optimal premiums in the second stage are independent of the level of preventive effort and there is no classification risk at the optimum. However, the premium paid at the first stage is decreasing with the level of effort as $p(\bar{e}) > p(\underline{e})$ gives $\bar{K} \equiv \mathbb{E}(K|e = \bar{e}) < \mathbb{E}(K|e = \underline{e}) \equiv \underline{K}$. Therefore, without moral hazard, assuming $\underline{K} - \bar{K} > \varphi$ the optimal contract specifies:

$$\begin{cases} e = \bar{e} \\ \Pi_2^{l*} = \Pi_2^{h*} \equiv \Pi_2^* \text{ with } u'(R - \Pi_2^*) = 1 \\ \Pi_1^* = \bar{K} - \Pi_2^* \end{cases} \quad (3.3)$$

3.2 The optimal dynamic contract under moral hazard

Now, if efforts of prevention are not observable, that is under moral hazard, agents have an incentive to exert the maximum level of effort only if the insurance contract satisfies

$$u(R - \Pi_2^l) - u(R - \Pi_2^h) \geq \frac{\varphi}{\Delta p}. \quad (3.4)$$

Therefore, the optimal contract producing an incentive to exert the high level of effort¹⁴ is the solution of:

$$\begin{aligned} \max_{\Pi_1, \Pi_2^l, \Pi_2^h} \quad & (R - \Pi_1) - \psi(\bar{e}) + \bar{p}u(R - \Pi_2^l) + (1 - \bar{p})u(R - \Pi_2^h) \\ \text{s.t.} \quad & \begin{cases} \Pi_1 + \bar{p}\Pi_2^l + (1 - \bar{p})\Pi_2^h \geq \bar{K} \\ u(R - \Pi_2^l) - u(R - \Pi_2^h) \geq \frac{\varphi}{\Delta p} \end{cases} \end{aligned} \quad (3.5)$$

The contract solution of this program then represents the overall optimum if it provides agents with more expected utility than the contract $(\bar{K} - \Pi_2^*, \Pi_2^*, \Pi_2^*)$: the optimal contract with low effort. We only focus in the following on the optimal incentive compatible contract. We do not discuss the issue of the optimal level of effort, assuming that it is optimal for all agents to exert the maximum level of effort.¹⁵

As shown in the next proposition, the shape of the inverse of marginal utility plays a preponderant role in our setting when analyzing the impact of moral hazard.

Proposition 1 *If the inverse of marginal utility ($1/u'$) is concave (resp. convex) in the second period, moral hazard increases (resp. reduces) prepayment (as then $\Pi_1^{**} > \Pi_1^*$, resp. $\Pi_1^{**} < \Pi_1^*$).*

Proof: See Appendix 8.1.

The condition $1/u'$ concave can be interpreted here in terms of prudence and risk-aversion. It indeed corresponds to the index of absolute prudence $P \equiv \frac{-u'''(\cdot)}{u''(\cdot)}$, introduced by Kimball (1990), being everywhere greater than twice the index of absolute risk aversion $A \equiv \frac{-u''(\cdot)}{u'(\cdot)}$.

Since under observable effort agents have the same level of wealth in the second period whatever their type, moral hazard corresponds in our setting to increase in second period uncertainty. Proposition 1 states that it leads to a decrease in first-period consumption (through an increase in the premium paid in the first stage) if $P \geq 2A$. This condition comes from the effect prepayment has on second period premium. First, if agents are prudent ($P > 0$), the increase in uncertainty

¹⁴The generalization to continuous effort would be difficult as it would introduce marginal utility into the program through a two-step optimization. However, our model seems to be easily generalizable to a finite number of effort levels. As we focus on the incentive to exert the maximum level of effort, the contract would be incentive-compatible if for each level of effort, the benefit of exerting the highest effort outweighs the cost. The binding incentive constraint would then correspond to the level of effort with the highest cost-benefit ratio.

¹⁵In a static model, Jullien et al. (1999) give conditions under which more risk-averse agents optimally exert a higher effort of prevention.

would lead to an increase in prepayment (*i.e* a decrease in first-period consumption) because of 'precautionary motives' (pain disaggregation). This effect increases with the index of absolute prudence P (see Kimball 1990). However, through the zero profit condition, this would increase, in our setting, average wealth in the second period. Then, because of the concavity of the utility function, the optimal contract has to exhibit a higher classification risk (a higher spread between second-period premiums) to remain incentive compatible. This last effect goes against an increase in the first period premium, and dominates if agents are "too risk averse" relative to their prudence. Proposition 1 states that this will be the case if $\frac{A}{P} \geq \frac{1}{2}$.

Now turn to second-period premiums. The first order condition gives $\frac{\bar{p}}{u'(R-\Pi_2^{l**})} + \frac{1-\bar{p}}{u'(R-\Pi_2^{h**})} = \frac{1}{u'(R-\Pi_2^*)}$. Moreover, the incentive constraint implies $\Pi_2^l < \Pi_2^h$. This leads to the following proposition.

Proposition 2 *Whatever the extent of prepayment when young, the unobservability of effort improves the welfare of low-risk agents and is detrimental to the welfare of high-risk agents when old. Therefore, it increases classification risk.*

This result is mainly driven by the incentive scheme. The first order condition leads to a decreasing relationship between Π_2^l and Π_2^h and is satisfied at the first best contract ($\Pi_2^l = \Pi_2^h = \Pi_2^*$). Now, to be incentive compatible, the optimal contract necessarily specifies $\Pi_2^l < \Pi_2^h$. Therefore, at the optimum $\Pi_2^l < \Pi_2^* < \Pi_2^h$.

It is worth noting that second-period premiums are fully determined by the first order condition and the incentive constraint. The feasibility constraint then determines the premium paid when young depending on the expected second-period premium. This allows the solution to be analyzed graphically. To do so, let us recall $X_1 \equiv R - \Pi_1 - \psi(\bar{e})$, $X_2^l \equiv R - \Pi_2^l$, $X_2^h \equiv R - \Pi_2^h$ and study the optimal premiums in the plan (X_2^l, X_2^h) .

Consider first the incentive constraint $u(X_2^l) - u(X_2^h) = \frac{\varphi}{\Delta p}$. In the plan (X_2^l, X_2^h) it defines an increasing and concave curve below the 45-degree line (labeled IC in Figure 4). Moreover, the distance between the incentive constraint and the 45-degree line is increasing in X_2^l as the function $f(X_2^l) = X_2^l - u^{-1}\left(u(X_2^l) - \frac{\varphi}{\Delta p}\right)$ is increasing in X_2^l when $u'(X_2^l) < u'\left[u^{-1}\left(u(X_2^l) - \frac{\varphi}{\Delta p}\right)\right] = u'(X_2^h)$.

In the plan (X_2^l, X_2^h) the first order condition $\frac{\bar{p}}{u'(R-\Pi_2^{l**})} + \frac{1-\bar{p}}{u'(R-\Pi_2^{h**})} = 1$ (labeled FOC in Figure 4), corresponds to a decreasing curve going through point (X_2^*, X_2^*) . It is moreover tangent to the line $pX_2^l + (1-p)X_2^h = X_2^*$ at (X_2^*, X_2^*) and convex (resp. concave) when $1/u'$ is concave (resp. convex).

On the basis of these two curves, we can infer the first-period wealth using the zero-profit condition that can be written as $\mathbb{E}(X_2) = -X_1 + 2R - (\bar{K} + \psi(\bar{e}))$. This effect is represented in Figure 4 through the line $\mathbb{E}(X_2) = c$.

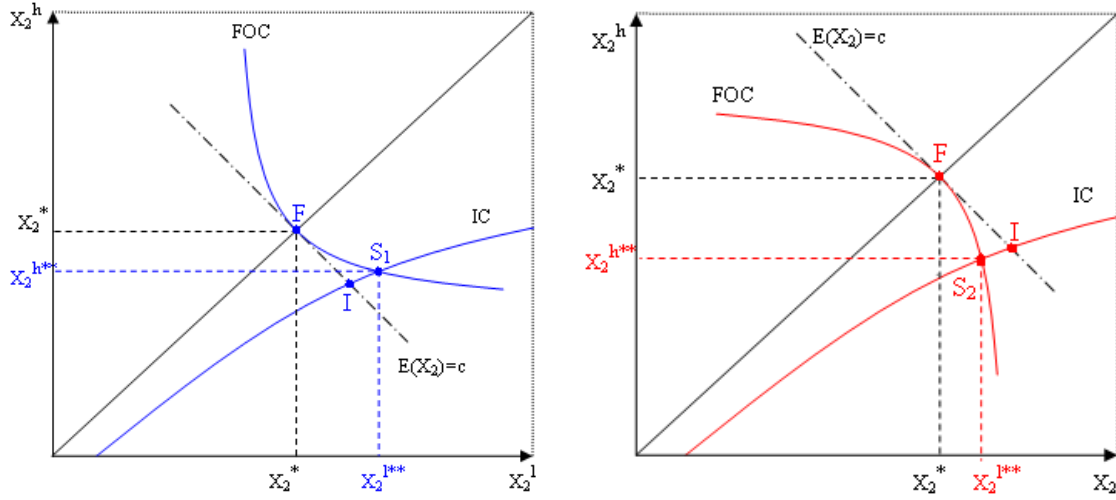


Figure 4: Second-period premiums and the concavity/convexity of $1/u'$

This graphical analysis highlights the fact that, depending on the concavity of $1/u'$, moral hazard has different effects on second-period premiums. First, whatever the concavity of $1/u'$, an "incentive" effect appears to lead to an increase in wealth for low-risk agents (decrease in Π_2^l) and a decrease in wealth for high-risk agents, relative to the complete information benchmark. Graphically, this corresponds to a move along line $\mathbb{E}(X_2) = c$ from point F to point I . This effect is combined with a "risk preferences" effect that depends on the concavity of $1/u'$. Indeed, if $1/u'$ is concave (figure on left) the "incentive" effect is coupled with a move to the north-east along the incentive constraint from point I to S_1 . Therefore, when $1/u'$ is concave (*i.e.* when $P \geq 2A$), the "risk preferences" effect corresponds to a decrease in both second-period premiums. This last effect moreover leads to the increase in prepayment described in Proposition 1, as it moves line $\mathbb{E}(X_2) = c$ upward.

The reverse effect (represented by a move from I to S_2) holds when $1/u'$ is convex (figure on right). The "risk preferences" effect then corresponds to a decrease in both X_2^l and X_2^h (leading to the increase in X_1 found in Proposition 1). However, as the first order condition is decreasing in the plan (X_2^l, X_2^h) , the "incentive" effect dominates and Proposition 2 holds.

Propositions 1 and 2 confirm our interpretation of the condition $P \geq 2A$. As shown in Crainich and Eeckhoudt (2005), both prudence and risk aversion can be interpreted as a preference toward risk. Here we show how these two kinds of preference can be linked. In our setting, agents can use two mechanisms to reduce classification risk in the second period. Either they can exert a preventive effort that reduces the probability of becoming high-risk, or they can transfer wealth from period 1 to period 2 through prepayment of premiums. Being linked with pain disaggregation, this last mechanism is related to the notion of prudence. However, through the incentive constraint and the zero profit condition, prepayment also influences classification risk and is therefore related to risk aversion. Similarly, as shown by Chiu (2000) and Eeckhoudt and Gollier (2005), preventive effort is related to both prudence and risk aversion. Here we show that the preference for one mechanism over another is driven by the ratio of absolute prudence

to absolute risk aversion and more precisely, by the position of the ratio with respect to a critical level equal to 2. The condition $P - 2A \geq 0$ therefore comes from a trade-off between a decrease in present consumption and an increase in preventive effort, which arises from second-period uncertainty (here moral hazard generates classification risk). If $P \geq 2A$, agents prefer to transfer wealth from period 1 to period 2 rather than to exert effort when they face classification risk. In our setting, this materialized in an increase in prepayment ($\Pi_1^{**} > \Pi_1^*$) and a need for bigger incentives. It is thus necessary to specify a large spread between second-period premiums to make sure that agents exert the effort (see Figure 4). The reverse effect holds when $P < 2A$, *i.e.* when agents would rather exert preventive effort than transfer wealth. In this case, lower classification risk is therefore incentive compatible and uncertainty reduces prepayment of premiums.

It seems important to point out here that the reluctance of agents to exert effort does not come from time inconsistency (as can for example arise from beta-delta preferences *à la* Laibson 1997). Agents are perfectly time-consistent but do not choose preventive effort (preferring wealth transfers) to deal with future uncertainty. This behavior may be due to by the uncertain nature of prevention compared to the predetermined (by the insurance contract) returns on prepayment.

4 Comparative Statics: How to Reduce Classification Risk?

In this section, we analyze the effect on optimal premiums of the different parameters of the model. By defining which variables affect classification risk, we are able to formulate some policy recommendations on how to reduce this kind of risk, which produces inequalities (see Dionne and Rothschild 2014). Our model contains three classes of variables: variables regarding the income process (the sure revenue R and the expected costs K_i^j), variables regarding preventive effort (the cost of effort φ and the probabilities of being low type for both levels of effort \bar{p} and p) and variables related to behavior toward risk (prudence and risk aversion) included in the utility function.

First note that – mainly because of the hypothesis of linear first-period utility – the variables related to the income process play a minor role in the determination of optimal premiums. There is indeed no wealth effect in our model in the sense that all optimal premiums are proportional to sure revenue $\left(\frac{d\Pi_2^{**}}{dR} = \frac{d\Pi_1^{**}}{dR} = -\frac{d\Pi_1^{**}}{dR} = 1 \right)$. Moreover, the expected costs only impact (positively) the first-period premium (through the zero profit condition) and have no effect on classification risk.

4.1 Reduce the Cost of Preventive Effort

The cost of preventive effort appears to be the first lever on which policymakers (using subsidies) or insurers (as in the examples presented in the introduction) can act. Our model allows us to analyze the impact of this cost φ on optimal premiums and especially on classification risk, as summarized in the next proposition.

Proposition 3 *A decrease in the cost of prevention*

- *decreases classification risk*
- *decreases prepayment if $1/u'$ is concave (increases it otherwise).*

Proof: See Appendix 8.2.

The effect on classification risk is quite straightforward. A decrease in the cost of prevention increases the incentive to exert the effort. The insurance contract can then exhibit a lower classification risk and remain incentive compatible. Therefore, if a policymaker wants to reduce the inequality resulting from classification risk, he should work on reducing the cost of prevention. By the same mechanism as for Proposition 1, this decreases (resp. increases) prepayment if $1/u'$ is concave (resp. convex). These effects can also be displayed graphically: an increase in φ corresponds to a downward shift of the incentive curve. Note moreover that the same effects can be induced by a decrease in the probability of being low-risk when not exerting the effort: \underline{p} . These changes in \underline{p} are, however, difficult to interpret, since they involve a change in the probability of damage for the agents that don't exert the effort, keeping constant the probability for those that do.

4.2 Increase the Effectiveness of Prevention

The role of the probability of being low-risk when exerting the preventive effort is more easily understandable. An increase in \bar{p} , keeping \underline{p} constant, can be interpreted as an improvement in the effectiveness of prevention, for example by investing in research on prevention. This suggests an important role for public policy on prevention.

Proposition 4 *An increase in the probability of being low-risk type in the second period when exerting the preventive effort*

- *increases the optimal premium paid by low-risk agents in the second stage*
- *decreases classification risk if $1/u'$ is convex.*

Proof: See Appendix 8.3.

First, an increase in the probability of being low-risk for agents that exert the preventive effort (maintaining constant this probability for agents that do not exert effort) increases the benefit of effort, making it easier to provide the incentive to exert effort. The optimal contract can therefore lead to a lower welfare in the good state of nature and still be incentive compatible.

Through the incentive constraint, this implies a decrease in the second-period premium for risky agents. However, an increase in \bar{p} also decreases the weight attached to this bad state in the objective function. This leads to a decrease in optimal wealth of high-risk agents. The outcome of these two effects is ambiguous. Therefore, the effect of an increase in \bar{p} on the expected wealth in the second period is also ambiguous and we cannot conclude on the impact of this increase on first-period wealth.

However, it is possible to state that an increase in the probability of being low-risk decreases classification risk when $1/u'$ is convex, or $P < 2A$ (i.e. for agents that favor prevention rather than prepayment).

Therefore, if $P < 2A$, the policymaker can reduce classification risk by improving the effectiveness of prevention, for example through investment in medical research.

4.3 The role of risk preferences

In the previous sections we highlighted the important role of the shape of $1/u'$ on the determination of optimal premiums. Whether it is concave or not strongly impacts the consequences moral hazard has on premiums. This raises the question of the effect of changes in the degree of concavity of $1/u'$.

Remark 1 Consider two utility functions $u(\cdot)$ and $v(\cdot)$ with $1/v'(\cdot)$ a concave transformation of $1/u'(\cdot)$. Then $P_v(x) - 2A_v(x) \geq P_u(x) - 2A_u(x) \forall x$

Proof: See Appendix 8.4.

The effects of changes in the degree of concavity of $1/u'$ are hard to grasp, as this also implies changes in incentive constraint and in the first best contract. To obtain explicit results on these effects, it is necessary to impose further assumptions on preferences.

5 Explicit Examples

5.1 The case of Constant Relative Risk Aversion

The first convenient way to specify the utility function is to assume that agents' preferences are represented by CRRA (Constant Relative Risk Aversion) utility functions. CRRA utility functions indeed exhibit some interesting properties. Indeed, if

$$u(x) = \begin{cases} \frac{x^{1-\gamma}}{1-\gamma} & \text{for } \gamma > 0, \gamma \neq 1 \\ \ln(x) & \text{for } \gamma = 1 \end{cases}$$

then $A(x) = \frac{\gamma}{x}$ and $P(x) = \frac{\gamma+1}{x}$. Now consider two CRRA utility functions $u(\cdot)$ and $v(\cdot)$ with respective parameters γ_v and γ_u . From remark 1, $1/v'(\cdot)$ is "more concave than" $1/u'(\cdot)$ for every level of wealth if $\gamma_v < \gamma_u$. CRRA utility functions also exhibit the convenient feature of leading to the same first best contract $(\bar{K} - R + 1, R - 1, R - 1)$ whatever the parameter of risk aversion ($u'(X_2^*) = 1 \Leftrightarrow X_2^* = 1 \forall \gamma > 0$). These features allow us to formulate the following Proposition.

Proposition 5 If agents' preferences in the second period are represented by a Constant Relative Risk Aversion utility function with $\gamma > 1$, an increase in the difference between prudence and twice risk aversion (i.e. in the degree of concavity of $1/u'$)

- decreases the premium paid by low-risk agents

- *increases the premium paid by high-risk agents*
- *and thus increases classification risk*

in the second period, provided the cost of effort is low enough (relative to its benefits).

Proof: See Appendix 8.5.

This result can be explained by the two effects already highlighted. First, the pure "risk preferences" effect leads to a decrease in both second-period premiums and an increase in first-period premium (as all CRRA utility functions lead to the same first best allocation). Proposition 5 states that the extent of the incentive effect highly depends on the cost of effort. This mainly comes from the influence that moves in γ have on the level of utility reached for low levels of wealth. Indeed, an increase in the coefficient of relative risk aversion has a large impact on low levels of utility, whereas the change in utility for high levels of consumption is relatively small. We can moreover show that a move from a less to a more concave inverse of marginal utility leads to a counter-clockwise move of the incentive constraint. When the required spread between second-period wealth $\frac{\varphi}{\Delta p}$ is low, the incentive constraints of agents having different CRRA utility functions cross for high levels of wealth. Therefore, the first order condition of agents with preference $v(\cdot)$ crosses their incentive constraint when it is below the incentive constraint of agents with preference $u(\cdot)$ as represented in figure 5 (with $1/v'(\cdot)$ "more concave" than $1/u'(\cdot)$).

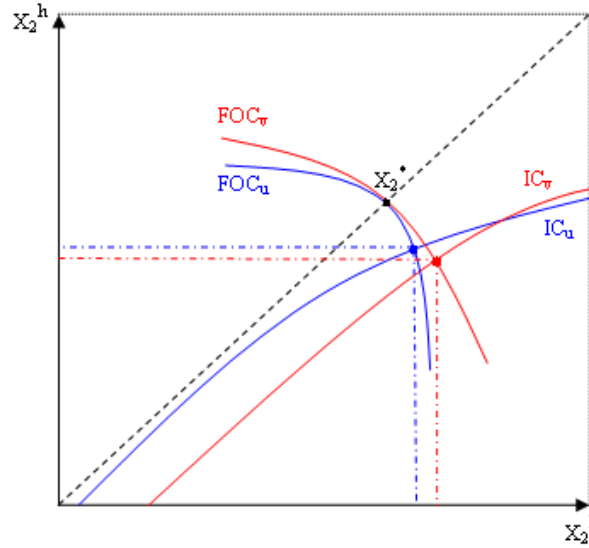


Figure 5: Changes in preferences in the case of CRRA utility functions

Thus, the combination of the two effects leads to an increase in wealth of low-risk agents and a decrease in wealth of high-risk agents. However, when $\frac{\varphi}{\Delta p}$ is too high, the incentive constraints of two agents may cross before crossing the first order condition, leading to the reverse effects (a decrease in X_2^l and an increase in X_2^h).

The restriction $\gamma > 1$ may seem awkward, but is pretty standard in the case of CRRA utility functions. For example, Gollier (2001) argues that this condition holds for most households in the

real economy. This moreover corresponds to the necessary condition for utility to be unbounded below, which is a standard assumption in principal-agent models (see for example Grossman and Hart 1983). It notably ensures that no non-negativity constraints on income bind at the optimum.

Proposition 5 moreover has implications on the first-period premium. From the zero profit condition, we have $\frac{d\Pi_1}{d\gamma} = -\bar{p}\frac{d\Pi_2^l}{d\gamma} - (1-\bar{p})\frac{d\Pi_2^h}{d\gamma}$. Therefore, for high values of \bar{p} , the effect of the low-risk premium dominates. So, under the assumptions of Proposition 5, the first-stage premium increases in γ (that is in the difference $P - 2A$). This is confirmed and generalized by a study of the system

$$\begin{cases} \frac{(X_2^l)^{1-\gamma}}{1-\gamma} - \frac{(X_2^h)^{1-\gamma}}{1-\gamma} = \frac{\varphi}{\Delta p} \\ \bar{p}(X_2^l)^\gamma + (1-\bar{p})(X_2^h)^\gamma = 1 \\ X_1 + \bar{p}X_2^l + (1-\bar{p})X_2^h = 2R - \bar{K} \end{cases}$$

that gives

$$\begin{aligned} \frac{dX_1}{d\gamma} = & \left\{ (1-\bar{p})^2 (X_2^h)^{2\gamma} \ln(X_2^h) + \bar{p}^2 (X_2^l)^{2\gamma} \ln(X_2^l) + \bar{p}(1-\bar{p}) \frac{\gamma}{(1-\gamma)^2} \left[(X_2^h)^{\gamma-\frac{1}{2}} (X_2^l)^{\frac{1}{2}} \right. \right. \\ & \left. \left. - (X_2^l)^{\gamma-\frac{1}{2}} (X_2^h)^{\frac{1}{2}} \right]^2 + \frac{\bar{p}(1-\bar{p})}{1-\gamma} \left(\ln(X_2^h) X_2^h (X_2^l)^\gamma \left[(X_2^h)^{\gamma-1} - (X_2^l)^{\gamma-1} \right] \right. \right. \\ & \left. \left. + \ln(X_2^l) X_2^l (X_2^h)^\gamma \left[(X_2^l)^{\gamma-1} - (X_2^h)^{\gamma-1} \right] \right) \right\} \left[\gamma \left(\bar{p} (X_2^l)^{2\gamma-1} + (1-\bar{p}) (X_2^h)^{2\gamma-1} \right) \right]^{-1} \end{aligned}$$

The first term being the only negative one, it turns out that, when the probability of being high-risk type $(1-\bar{p})$ is low enough, an increase in γ increases the first-period premium. Simulations with CRRA preferences (see Appendix 8.6) moreover indicate this to be the case whatever the probability of being high-risk. The intuitive effects driving Proposition 1 therefore seem to be generalizable to changes in the degree of concavity of $1/u'$.

We are unable to determine analytically the limit cost of effort in Proposition 5. However, this can be done when analyzing welfare in the good state of nature.

Proposition 6 *If agents' preferences in the second period are represented by a Constant Relative Risk Aversion utility function with $\gamma > 1$, an increase in the difference between prudence and twice risk aversion increases the welfare of low-risk agents in the second period, if $\frac{\varphi}{\Delta p} \leq \frac{e^\gamma - 1}{1-\gamma}$.*

Proof: See Appendix 8.7.

We cannot however, infer results on level of wealth (and thus on premiums) from this proposition, as the coefficient of relative risk aversion does not have a monotonic impact on utility derived from a given level of wealth.

5.2 A suitable utility function with linear reciprocal derivative

To take the analysis of the impact of the difference $P - 2A$ on premiums further and get clearer results, it is convenient to build an appropriate utility function that satisfies additional simplifying properties.

From the proof of proposition 6 (equation (8.3)), it turns out to be accommodating first to specify linear $f'(y)$, that is $f'(y) = \theta y$ where θ depicts the behavior toward risk. As $f \equiv u^{-1}$, this corresponds to $u(x) = \frac{1}{\theta} \sqrt{2\theta(x-a)}$ where a represents the constant of integration. Then, to isolate the effect $P - 2A$, it is useful to consider a class of utility function for which (as in the CRRA case) the first best premium does not depend on behavior toward risk parameters. To do so we specify the constant a such that $u'(1) = 1 \forall \theta$ that is $a = 1 - \frac{1}{2\theta}$. Let us therefore consider the class of utility function:

$$u(x) = \frac{1}{\theta} \sqrt{2\theta(x-1)+1} \quad , \quad \theta > 0 \quad (5.1)$$

which is twice continuously differentiable, increasing and concave above some (subsistence) level¹⁶ $\underline{x} \equiv 1 - \frac{1}{2\theta}$ and satisfies the Inada conditions $\lim_{x \rightarrow \underline{x}} u'(x) = +\infty$ and $\lim_{x \rightarrow +\infty} u'(x) = 0$.

An agent whose preferences are described by (5.1) is risk averse $\left(A = \frac{\theta}{2\theta(x-1)+1} > 0 \right)$ and prudent $\left(P = \frac{3\theta}{2\theta(x-1)+1} > 0 \right)$, with $P - 2A \geq 0$ for any level of θ (positive). Moreover, if we consider two agents u and v having such preferences, $P_v - 2A_v > P_u - 2A_u \forall x > \underline{x}$ if and only if $\theta_v > \theta_u$.

The levels of wealth reached under the optimal incentive contract $\left(\text{that exists only if } \frac{\varphi}{\Delta p} < \frac{1}{\bar{p}\theta} \right)$ then writes:

$$\begin{cases} X_2^{l**} &= 1 + (1 - \bar{p}) \frac{\varphi}{\Delta p} + \frac{1}{2} \left((1 - \bar{p}) \frac{\varphi}{\Delta p} \right)^2 \theta \\ X_2^{h**} &= 1 - \bar{p} \frac{\varphi}{\Delta p} + \frac{1}{2} \left(\bar{p} \frac{\varphi}{\Delta p} \right)^2 \theta \\ X_1^{**} - X_1^* &= 1 - \mathbb{E}(X_2^{**}) = -\frac{1}{2} \bar{p} (1 - \bar{p}) \left(\frac{\varphi}{\Delta p} \right)^2 \theta. \end{cases}$$

Therefore,

Proposition 7 *If agents' preferences are defined by $u(x) = \frac{1}{\theta} \sqrt{2\theta(x-1)+1}$, $\theta > 0$, an increase in the difference $P - 2A$*

- (i) *increases the first-period premium (i.e. intergenerational insurance)*
- (ii) *decreases the second-period premiums*
- (iii) *increases classification risk if the good state is the more likely ($\bar{p} > 1/2$)*

¹⁶In this sense, this class of utility functions can be related to the Stone-Geary class.

The use of a utility function whose reciprocal has a linear derivative allows us to better isolate the impact of preferences. We can indeed link the results of Proposition 7 with the effect highlighted in Proposition 1. An increase in the difference $P - 2A$ (that is in the degree of concavity of $1/u'$) increases the first-period premium for reasons of pain disaggregation (increase in P) and/or because agents are less sensitive to the increase in classification risk it may cause (decrease in A). The incentive effect of such an increase does not prevent – for this class of utility function – a decrease in both second-period premiums.

Moreover, the increase in wealth is higher in the most probable state and therefore, the increase in the first period may not be coupled with an increase in classification risk if the bad state is highly probable. The optimal incentive contract moreover confirms the result of Proposition 3 and 4: (i) an increase in the cost of effort increases the first-period premium and the classification risk (when the optimal contract exists, that is when $\frac{\varphi}{\Delta p} < \frac{1}{\bar{p}\theta}$) and (ii) an increase in the probability of being low-risk when exerting the effort \bar{p} decreases the classification risk and the optimal wealth in the good state.

Hendel and Lizzeri (2003) pointed out that, in the absence of moral hazard, dynamic insurance contracts are subject to lapsation in the second period. Let us now analyze what would happen if we allowed the older agents to leave their insurer. In the absence of severance payments (proposed by Cochrane 1995), healthier agents may then leave the dynamic contract to go to a competing short-term (spot) insurer that offers actuarially fair premiums. This will be the case if the optimal incentive premium for low-risk agents is higher than their expected costs ($\Pi_2^{l**} > K_2^l$). The second-period optimal contract presented above, however, does not depend on expected costs (the expected costs over the two periods only influence the first-period premium).

We can still infer that, for a given expected cost schedule, the lower the premium offered by the dynamic insurance, the lower the incentive for healthy agents to lapse (and turn to the spot insurer). From Proposition 5 and 7 it therefore appears that our dynamic insurance is more likely to withstand competition from companies offering spot (short-term) contracts if it insures agents with a greater difference between absolute prudence and twice absolute risk aversion, when agents' preferences

- are described by $u(x) = \frac{1}{\theta} \sqrt{2\theta(x-1) + 1}$, $\theta > 0$
- are CRRA, provided the cost of preventive effort is low enough.

Our work also offers an alternative explanation for the varying degrees of front-loading and of lapsation observed in dynamic insurance contracts. In the empirical part of their work, Hendel and Lizzeri (2003) show that in life insurance, more front-loading is associated with lower lapsation. They then argue that this phenomenon can be explained by heterogeneity in agents' income growth (that is in the cost of front-loading). It appears from our model that this can also be explained by heterogeneity in preferences. Program (3.5) indeed also model the problem of a competitive insurance company that does not discount the future (in the zero profit condition (3.2)). In this case, it appears from our study that the co-existence of dynamic contracts and spot contracts, and the varying degrees of front-loading, may be explained by moral hazard and heterogeneous preferences. Moreover, the contracts of agents with a greater difference $P - 2A$ may exhibit more front-loading and less lapsation if the reciprocal of agents' utility function has a linear derivative ($u(x) = \frac{1}{\theta} \sqrt{2\theta(x-1) + 1}$, $\theta > 0$). Simulations with CRRA utility functions

(see Appendix) suggest that this may also be the case when agents have CRRA preferences if the cost of preventive effort is low enough.

6 Applications to other insurance markets

The model presented above in the case of health insurance appears to be applicable to other insurance markets, with slight modifications.

6.1 Life insurance

One application is life insurance, where the insurer offers protection against the risk of death and agents can reduce the probability of having a high probability of death in the second period by exerting preventive effort. If we assume ad-hoc altruism (in the sense that the indemnity paid to the beneficiary directly enters the insured's utility function), optimal insurance is complete in each state and is the solution of a program similar to (3.5). However, we need to include in this extension the fact that agents can die in the first period (that is the survival probability). As q_1 represents here the risk of death in period 1, only a portion $(1 - q_1)$ of a generation is still alive in period 2. This effect has an incidence on the objective, the incentive constraint and the zero profit condition such that the program becomes

$$\begin{aligned} \max_{\Pi_1, \Pi_2^l, \Pi_2^h} \quad & (R - \Pi_1) - \psi(\bar{e}) + (1 - q_1) [\bar{p}u(R - \Pi_2^l) + (1 - \bar{p})u(R - \Pi_2^h)] \\ \text{s.t.} \quad & \begin{cases} \Pi_1 + (1 - q_1) [\bar{p}\Pi_2^l + (1 - \bar{p})\Pi_2^h] \geq \bar{K} = q_1R + (1 - q_1) [\bar{p}q_2^lR + (1 - \bar{p})q_2^hR] \\ u(R - \Pi_2^l) - u(R - \Pi_2^h) \geq \frac{\varphi}{(1 - q_1)\Delta p}. \end{cases} \end{aligned} \quad (6.1)$$

The solution of this new program, very similar to the original, is given by:

$$\begin{cases} u(R - \Pi_2^{l**}) - u(R - \Pi_2^{h**}) = \frac{\varphi}{(1 - q_1)\Delta p} \\ \frac{\bar{p}}{u'(R - \Pi_2^{l**})} + \frac{1 - \bar{p}}{u'(R - \Pi_2^{h**})} = \frac{1}{1 - q_1} \\ \Pi_1^{**} = \bar{K} - \bar{p}\Pi_2^{l**} - (1 - \bar{p})\Pi_2^{h**} = \bar{K} - \mathbb{E}(\Pi_2^{**}). \end{cases} \quad (6.2)$$

All the above properties hold in the case of life insurance with (ad-hoc) altruistic agents. Interestingly, long-term life insurance appears more likely to withstand competition from spot insurance when it insures agents with a greater difference between prudence and twice risk aversion. Moreover, the effect of q_1 turns out to be ambiguous. While the probability of dying during the first period decreases the relative valuation of the second period (and therefore tends to decrease intergenerational insurance), it also reduces the proportion of agents sharing the prepaid premiums in the second period (and thus leads to an increase in second-period wealth).

6.2 Unemployment insurance

With more amendments, our model also seems to be applicable to unemployment insurance.

Consider unemployment insurance in our simple overlapping generation model. In their early life, all agents face the same probability of being unemployed (or have the same expected length of unemployment) and can invest in training effort e . Partially based on this effort, agents can then either be employed as a "skilled" (executive) or "unskilled" (non executive) worker in the second period. We moreover assume (as seems to be the case in real economies) that the risk of unemployment is higher among unskilled than among skilled workers. Modeling the fact that the three types of agents also differ with respect to wages, the income profile without insurance can be summarized as follows:

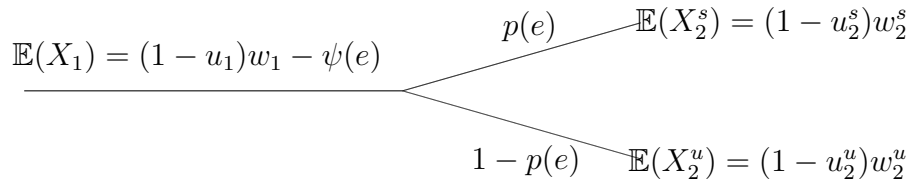


Figure 6: Income profile without unemployment insurance

with u_1 , u_2^S and u_2^U the respective probabilities of being unemployed for young, skilled and unskilled workers ; and w_1 , w_2^S and w_2^U the respective wages of young, skilled and unskilled workers.

As in our baseline model, it is then optimal for the mutual (social) insurer to provide risk-averse agents with complete insurance in each state, that is with a triplet of sure consumption profiles C_1 , C_2^S and C_2^U which solves

$$\begin{aligned} \max_{C_1, C_2^S, C_2^U} \quad & (R - C_1) - \psi(\bar{e}) + \bar{p}u(C_2^S) + (1 - \bar{p})u(C_2^U) \\ \text{s.t.} \quad & \begin{cases} C_1 + \bar{p}C_2^S + (1 - \bar{p})C_2^U \geq (1 - u_1)w_1 + \bar{p}(1 - u_2^S)w_2^S + (1 - \bar{p})(1 - u_2^U)w_2^U \\ u(C_2^S) - u(C_2^U) \geq \frac{\varphi}{(1 - q_1)\Delta p} \end{cases} \end{aligned} \quad (6.3)$$

where φ represents the cost of exerting high effort, \bar{p} and \underline{p} being the respective probability of becoming a skilled worker when exerting and not exerting the training effort. $\Delta p \equiv \bar{p} - \underline{p}$.

The model makes it possible to determine the extent of intra- and inter-generational employment insurance. This supports empirical evidence of both types of insurance, in France, for example. Employment benefit, in France, amounts to 75% of the last gross wage for the lowest wage bracket and about 57% for the highest, consistent with intragenerational insurance between skilled and unskilled workers. As these unemployment benefits are higher for workers above 50 years old, this intragenerational insurance moreover seems to be combined with an intergenerational insurance.

In this setting, our model highlights a tradeoff between training effort and intergenerational insurance. If $P \geq 2A$, agents prefer to rely on intergenerational insurance rather than on training to deal with the risk of being unemployed longer in the second period. This application therefore seems to be linked to the current debate on unemployment insurance (in particular in France) about the tradeoff between generous unemployment benefits and training subsidization. Moreover, it appears through the preceding analysis that a decrease in the cost of training or an increase in its efficiency (when $P < 2A$) enhances the optimal incentive compatible intragenerational insurance (by decreasing the spread between C_2^S and C_2^U). Finally, our work suggests that an increase in $P - 2A$ weakens the redistributive pattern of unemployment insurance between skilled and unskilled workers.

7 Conclusion

We highlight in this paper the role of prudence and risk aversion in long-term insurance contracts. Adding to the usual models an effort of prevention, we show that the concavity of the inverse of marginal utility (*i.e.* the difference between prudence, P , and twice risk aversion, A) plays a central role in defining the optimal level of prepayment (which can be understood as intergenerational insurance) and the optimal incentive compatible classification risk. First, our analysis indicates that moral hazard always increases classification risk (relative to the complete information benchmark) and increases the first-period premium if $P \geq 2A$. This reveals the tradeoff between prevention and (intergenerational) insurance that arises from future uncertainty.

It moreover appears that classification risk can be reduced by decreasing the cost of prevention or by increasing the effectiveness of prevention (when $P < 2A$). Therefore, if the objective is to make insurance more affordable to high-risk agents, the policy maker should try to reduce the cost of prevention and to increase its efficiency.

Specifying CRRA (Constant Relative Risk Aversion) preferences, we moreover show that an increase in the difference between prudence and twice risk aversion decreases the premium offered to low-risk agents in the second period, if the cost of preventive effort is low enough. To take the analysis of comparative risk preferences further, we specify a utility function that exhibits the appropriate property of having a linear reciprocal derivative. With such preferences, it appears that an increase in $P - 2A$ optimally increases intergenerational insurance and decreases second-period premiums.

It is left for future research to analyze the impact of heterogeneous preferences. Preliminary work on this refinement points out the importance of relative time preferences. It indeed appears that cross-subsidization increases prepayment for agents with lower preference for the future and is therefore likely to stabilize the dynamic contract. Future work should therefore analyze the impact of heterogeneous preferences on this cross-subsidization. Our analysis suggests that agents with a low level of $P - 2A$ subsidize others. However, this effect being coupled with the time preference effect just discussed, the effect of cross-subsidization is ambiguous. It therefore seems that the study of adverse selection in dynamic contracts calls for further assumptions and probably for a full specification of the utility functions.

It would also appear interesting to study the effect of preferences on the optimal level of

effort. In future work, it would be worth investigating whether agents with greater a $P - 2A$ difference exert more preventive effort in line with the work of Jullien et al. (1999) on the link between risk aversion and prevention. Our work may also open new perspectives regarding the role of information. Information on risk types plays an essential role in the writing of dynamic insurance contracts. It would therefore seem natural to study the impact of genetic testing and medical checkups in this area. Following Barigozzi and Henri  t (2011), policyholders would first choose whether or not to undertake the tests and then decide whether or not to reveal the results to their insurer. The objective of this future work would thus be to study the impact of these choices on the optimal level of effort and on the optimal dynamic contract. Useful insights might be gained by exploring the value of information through the trade-off between the resulting increases in effort and classification risk.

It would lastly be interesting to infer the value of $P - 2A$ for economic agents. The specification of the usual utility functions leads to conflicting results. It may therefore be necessary to use simple lotteries or resort to experiments to determine whether/when $P \geq 2A$. Applying the notion of "risk apportionment" (used in the case of additive risks by Eeckhoudt and Schlesinger, 2006) to multiplicative risks in simple lotteries, Eeckhoudt et al. (2009) provide a sufficient condition for P to be greater than $2A$. They offer simple conditions on preferences among lotteries for which the index of relative prudence is higher than 2 and the index of relative risk aversion is lower than 1.

8 Appendix

8.1 Proof of Proposition 1

The solution of program (3.5) is defined by¹⁷:

$$\begin{cases} u(R - \Pi_2^{l**}) - u(R - \Pi_2^{h**}) = \frac{\varphi}{\Delta p} \\ \frac{\bar{p}}{u'(R - \Pi_2^{l**})} + \frac{1 - \bar{p}}{u'(R - \Pi_2^{h**})} = 1 \\ \Pi_1^{**} = \bar{K} - \bar{p}\Pi_2^{l**} - (1 - \bar{p})\Pi_2^{h**} = \bar{K} - \mathbb{E}(\Pi_2^{**}) \end{cases} \quad (8.1)$$

Therefore, if $(1/u')$ is concave (resp. convex), the optimal incentive contract under moral hazard satisfies $\frac{1}{u'(R - \bar{K} + \Pi_1^{**})} = \frac{1}{u'(R - \mathbb{E}(\Pi_2^{**}))} > \frac{\bar{p}}{u'(R - \Pi_2^{l**})} + \frac{1 - \bar{p}}{u'(R - \Pi_2^{h**})} = 1$ (resp. $\frac{1}{u'(R - \bar{K} + \Pi_1^{**})} < 1$).

As, under observable effort we have $\frac{1}{u'(R - \bar{K} + \Pi_1^*)} = \frac{1}{u'(R - \Pi_2^*)} = 1$, Proposition 1 holds.

¹⁷Note here that the first order condition of the program (3.5) is the same as in the standard moral hazard model (see Laffont and Martimort 2002, Chapter 4). Our model can indeed be viewed as a dynamic extension of the classic moral hazard problem.

8.2 Proof of Proposition 3

Differentiating the solution system (8.1) with respect to Π_1 , Π_2^l , Π_2^h and φ gives:

$$\frac{d\Pi_2^h}{d\varphi} = \Delta p \frac{\bar{p} u''(R - \Pi_2^l) [u'(R - \Pi_2^h)]^2}{(1 - \bar{p}) u''(R - \Pi_2^h) [u'(R - \Pi_2^l)]^3 + \bar{p} u''(R - \Pi_2^l) [u'(R - \Pi_2^h)]^3} > 0$$

$$\frac{d\Pi_2^l}{d\varphi} = -\Delta p \frac{(1 - \bar{p}) u''(R - \Pi_2^h) [u'(R - \Pi_2^l)]^2}{(1 - \bar{p}) u''(R - \Pi_2^h) [u'(R - \Pi_2^l)]^3 + \bar{p} u''(R - \Pi_2^l) [u'(R - \Pi_2^h)]^3} < 0$$

and

$$\frac{d\Pi_1}{d\varphi} = \bar{p}(1 - \bar{p}) \Delta p \frac{u''(R - \Pi_2^h) [u'(R - \Pi_2^l)]^2 - u''(R - \Pi_2^l) [u'(R - \Pi_2^h)]^2}{(1 - \bar{p}) u''(R - \Pi_2^h) [u'(R - \Pi_2^l)]^3 + \bar{p} u''(R - \Pi_2^l) [u'(R - \Pi_2^h)]^3}$$

Therefore, $\frac{d\Pi_1}{d\varphi}$ is positive if $(1/u')$ is concave – as the ratio $u''(\cdot)/[u'(\cdot)]^2$ is then increasing – Proposition 3 holds.

8.3 Proof of Proposition 4

Using (8.1), comparative statics on changes in \bar{p} gives:

$$\left\{ \begin{array}{l} \frac{d\Pi_2^l}{d\bar{p}} = - \frac{\frac{1}{u'(R - \Pi_2^l)} - \frac{1}{u'(R - \Pi_2^h)} - (1 - \bar{p}) \frac{\varphi}{(\Delta p)^2} \frac{u''(R - \Pi_2^h)}{[u'(R - \Pi_2^h)]^3}}{u'(R - \Pi_2^l) \left[\bar{p} \frac{u''(R - \Pi_2^l)}{[u'(R - \Pi_2^l)]^3} + (1 - \bar{p}) \frac{u''(R - \Pi_2^h)}{[u'(R - \Pi_2^h)]^3} \right]} > 0 \\ \frac{d\Pi_2^h}{d\bar{p}} = - \frac{\frac{1}{u'(R - \Pi_2^l)} - \frac{1}{u'(R - \Pi_2^h)} + \bar{p} \frac{\varphi}{(\Delta p)^2} \frac{u''(R - \Pi_2^l)}{[u'(R - \Pi_2^l)]^3}}{u'(R - \Pi_2^h) \left[\bar{p} \frac{u''(R - \Pi_2^l)}{[u'(R - \Pi_2^l)]^3} + (1 - \bar{p}) \frac{u''(R - \Pi_2^h)}{[u'(R - \Pi_2^h)]^3} \right]} \end{array} \right.$$

which can be written as,

$$\frac{d(X_2^l - X_2^h)}{d\bar{p}} = \frac{\left[\frac{1}{u'(X_2^l)} - \frac{1}{u'(X_2^h)} \right]^2}{\bar{p} \frac{u''(X_2^l)}{[u'(X_2^l)]^3} + (1 - \bar{p}) \frac{u''(X_2^h)}{[u'(X_2^h)]^3}} + \frac{\bar{p}}{u'(X_2^l) u'(X_2^h)} \frac{\varphi}{(\Delta p)^2} \left[\frac{u''(X_2^h)}{[u'(X_2^h)]^2} - \frac{u''(X_2^l)}{[u'(X_2^l)]^2} \right] - \frac{\varphi}{(\Delta p)^2} \frac{u''(X_2^h)}{[u'(X_2^h)]^3 u'(X_2^l)}$$

It then turns out that $\frac{d(X_2^l - X_2^h)}{d\bar{p}} < 0$ if $\frac{u''(X_2^h)}{[u'(X_2^h)]^2} - \frac{u''(X_2^l)}{[u'(X_2^l)]^2} > 0$ that is if $1/u'$ is convex.

8.4 Proof of Remark 1

Let us define $g(t) \equiv \frac{1}{v'} \left(\left(\frac{1}{u'} \right)^{-1} (t) \right)$. $1/v'$ is a concave transformation of $1/u'$ if and only if g is concave. Now as,

$$g'(t) = \frac{v'' \left(\left(\frac{1}{u'} \right)^{-1} (t) \right)}{\left[v' \left(\left(\frac{1}{u'} \right)^{-1} (t) \right) \right]^2} \cdot \frac{\left[u' \left(\left(\frac{1}{u'} \right)^{-1} (t) \right) \right]^2}{u'' \left(\left(\frac{1}{u'} \right)^{-1} (t) \right)} = \frac{A_v \left(\left(\frac{1}{u_1'} \right)^{-1} (t) \right) u' \left(\left(\frac{1}{u'} \right)^{-1} (t) \right)}{A_u \left(\left(\frac{1}{u_1'} \right)^{-1} (t) \right) v' \left(\left(\frac{1}{u'} \right)^{-1} (t) \right)}$$

we have that

$$\begin{aligned} & 1/v' \text{ is a concave transformation of } 1/u' \\ \Leftrightarrow & \frac{d}{dx} \log \left(\frac{A_v(x)u'(x)}{A_u(x)v'(x)} \right) \leq 0 \\ \Leftrightarrow & \frac{A_v'(x)}{A_v(x)} + \frac{u''(x)}{u'(x)} - \frac{A_u'(x)}{A_u(x)} - \frac{v''(x)}{v'(x)} \leq 0 \\ \Leftrightarrow & P_v(x) - 2A_v(x) \geq P_u(x) - 2A_u(x) \end{aligned}$$

8.5 Proof of Proposition 5

With CRRA preferences, the optimal second-period contract is described by the system

$$\begin{cases} \frac{(X_2^l)^{1-\gamma}}{1-\gamma} - \frac{(X_2^h)^{1-\gamma}}{1-\gamma} = \frac{\varphi}{\Delta p} \\ \bar{p} (X_2^l)^\gamma + (1-\bar{p}) (X_2^h)^\gamma = 1 \end{cases} \quad (8.2)$$

Therefore,

$$\begin{cases} \frac{dX_2^l}{d\gamma} = - \frac{\frac{\varphi}{\Delta p(1-\gamma)} - \frac{1}{1-\gamma} \ln(X_2^l)(X_2^l)^{1-\gamma} + \frac{1}{\gamma(1-\gamma)} \ln(X_2^h)(X_2^h)^{1-\gamma} + \frac{\bar{p}}{1-\bar{p}} \frac{1}{\gamma} \ln(X_2^l)(X_2^l)^\gamma (X_2^h)^{1-2\gamma}}{\frac{1}{(1-\bar{p})} (X_2^l)^{\gamma-1} \left[(1-\bar{p})(X_2^l)^{1-2\gamma} + \bar{p}(X_2^h)^{1-2\gamma} \right]} \\ \frac{dX_2^h}{d\gamma} = \frac{\frac{\varphi}{\Delta p(1-\gamma)} - \frac{1}{\gamma(1-\gamma)} \ln(X_2^l)(X_2^l)^{1-\gamma} + \frac{1}{1-\gamma} \ln(X_2^h)(X_2^h)^{1-\gamma} - \frac{1-\bar{p}}{\bar{p}} \frac{1}{\gamma} \ln(X_2^l)(X_2^l)^\gamma (X_2^h)^{1-2\gamma}}{\frac{1}{\bar{p}} (X_2^h)^{\gamma-1} \left[(1-\bar{p})(X_2^l)^{1-2\gamma} + \bar{p}(X_2^h)^{1-2\gamma} \right]} \end{cases}$$

As $X_2^* = 1 \forall \gamma > 0$, from Proposition 2, $\ln(X_2^l) > 0$ and $\ln(X_2^h) < 0$. Therefore, if $\gamma > 1$, the three last terms of both numerators are positive and Proposition 5 holds.

Indeed, when $(1-\gamma) < 0$, provided the first term of the numerators is low enough relative to the other terms (that is $\frac{\varphi}{\Delta p}$ low enough), the optimal wealth in the second stage decreases with the coefficient of risk aversion γ when the agent turns out to be low-risk $\left(\frac{dX_2^l}{d\gamma} > 0 \right)$ and increases with γ for high-risk agents $\left(\frac{dX_2^h}{d\gamma} < 0 \right)$. In the case of CRRA utility functions, the difference $P - 2A$ increases when γ decreases and Proposition 5 follows.

8.6 Simulations with CRRA preferences - The impact of γ on the first-period premium

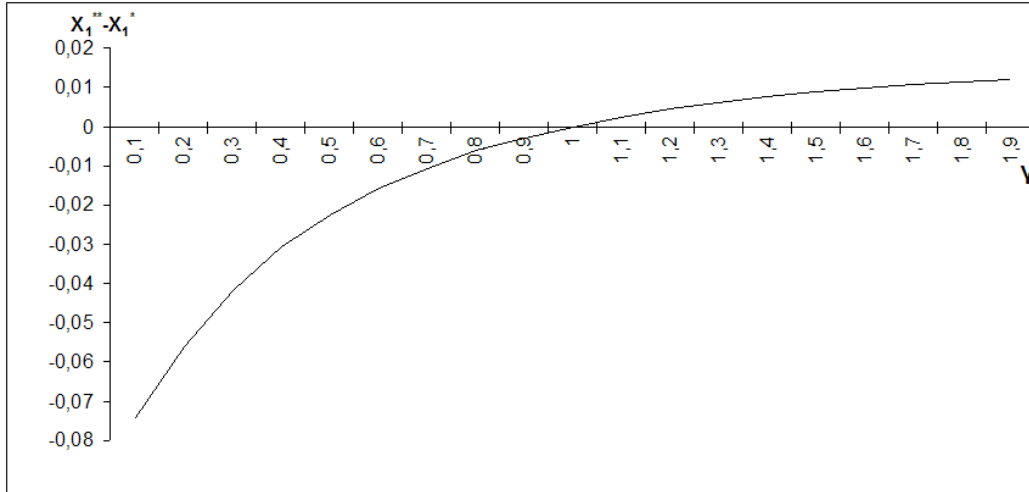


Figure 7: $\frac{\varphi}{\Delta p} = 1, \bar{p} = 0,9$

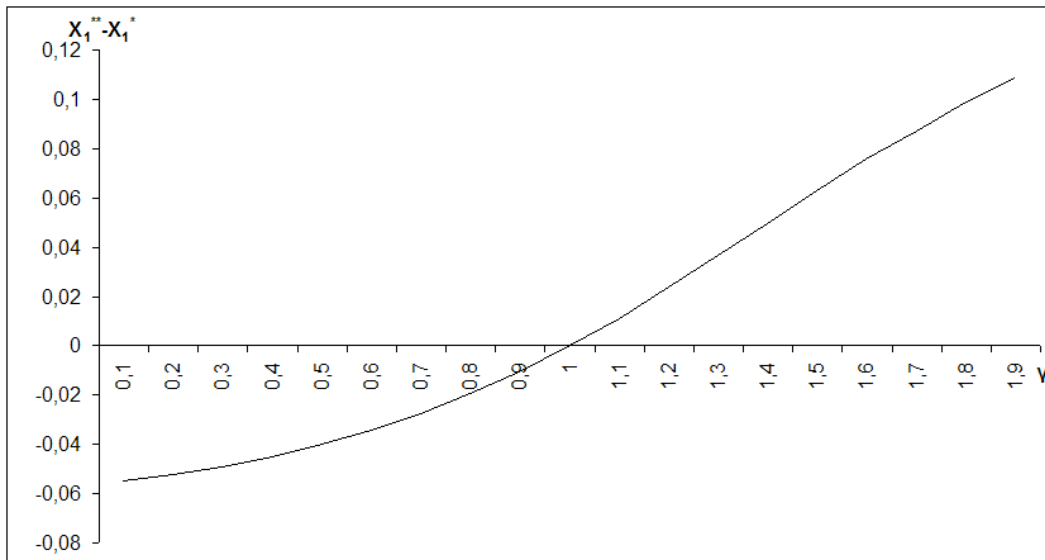


Figure 8: $\frac{\varphi}{\Delta p} = 1, \bar{p} = 0,2$

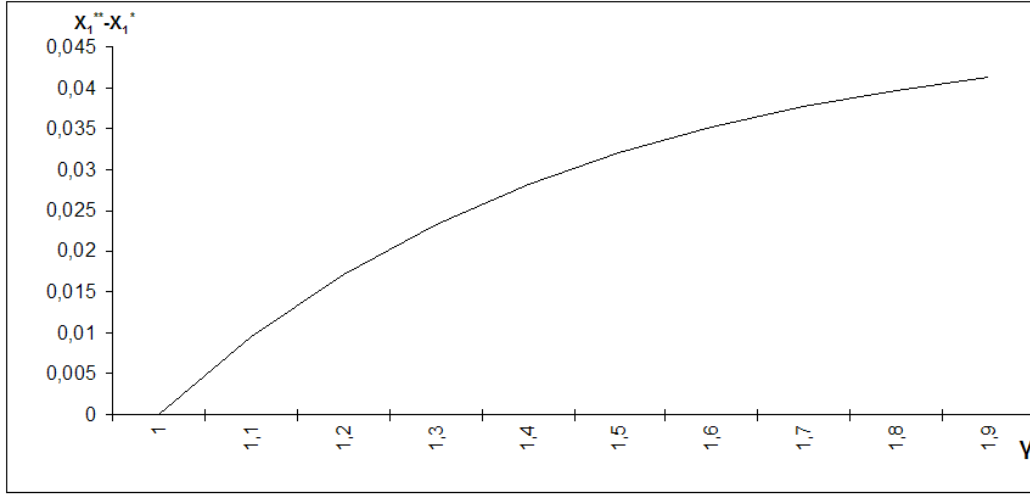


Figure 9: $\frac{\varphi}{\Delta p} = 10, \bar{p} = 0,9$

8.7 Proof of Proposition 6

Let us consider the following change of variables. If we define $Y_2^l \equiv u(X_2^l)$ and $f \equiv u^{-1}$, we have $u(X_2^h) = Y_2^l - \frac{\varphi}{\Delta p}$ and the system defining the second-period premiums simplifies in

$$H(Y_2^l) \equiv \bar{p}f'(Y_2^l) + (1 - \bar{p})f'\left(Y_2^l - \frac{\varphi}{\Delta p}\right) = 1 \quad (8.3)$$

In the case of CRRA preferences,

$$f(Y) = \exp\left[\frac{1}{1-\gamma} \ln((1-\gamma)Y)\right] \text{ and } f'(Y) = \exp\left[\frac{\gamma}{1-\gamma} \ln((1-\gamma)Y)\right]$$

(f') being increasing, it follows from (8.3) that

$$\text{sgn}\left(\frac{\partial Y_2^l}{\partial \gamma}\right) = -\text{sgn}\left(\bar{p}\frac{\partial f'}{\partial \gamma}(Y_2^l) + (1 - \bar{p})\frac{\partial f'}{\partial \gamma}\left(Y_2^l - \frac{\varphi}{\Delta p}\right)\right)$$

Then, as $\text{sgn}\left(\frac{\partial f'}{\partial \gamma}(Y)\right) = \text{sgn}(\ln((1-\gamma)Y) - \gamma)$, one gets

$$\frac{\partial f'}{\partial \gamma}(Y) \geq 0 \Leftrightarrow Y \leq Y_\gamma \equiv \frac{e^\gamma}{1-\gamma} \text{ when } \gamma > 1$$

Therefore, a sufficient condition for the welfare in low state to be increasing with γ : $\left(\frac{\partial Y_2^l}{\partial \gamma} \leq 0\right)$, is Y_2^l (and thus $Y_2^l - \frac{\varphi}{\Delta p}$) to be lower than Y_γ . As $H(Y_2^l)$ is increasing, this condition is equivalent to $H(Y_\gamma) \geq 1$, for which a sufficient condition is $f'\left(Y_\gamma - \frac{\varphi}{\Delta p}\right) \geq 1$ that is $\frac{\varphi}{\Delta p} \leq \frac{e^\gamma - 1}{1-\gamma}$.

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