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Urbanisation and the onset of modern economic growth

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Urbanisation and the onset of modern economic growth*

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Abstract: A large literature characterizes urbanisation as the result of productivity growth attracting rural workers to cities. We incorporate economic geography elements into a growth model and suggest that causation runs the other way: when rural workers move to cities, the resulting urbanisation produces technological change and productivity growth. Urban density leads to knowledge exchange and innovation, thus creating a positive feedback loop between city size and productivity that sets off sustained economic growth. The model is consistent with the fact that urbanisation rates in Western Europe, and notably in England, reached unprecedented levels by the mid-18th century, the eve of the Industrial Revolution.

Key words: Industrialization, urbanisation, innovation, long-run growth.

JEL codes: N13, O14, O41.

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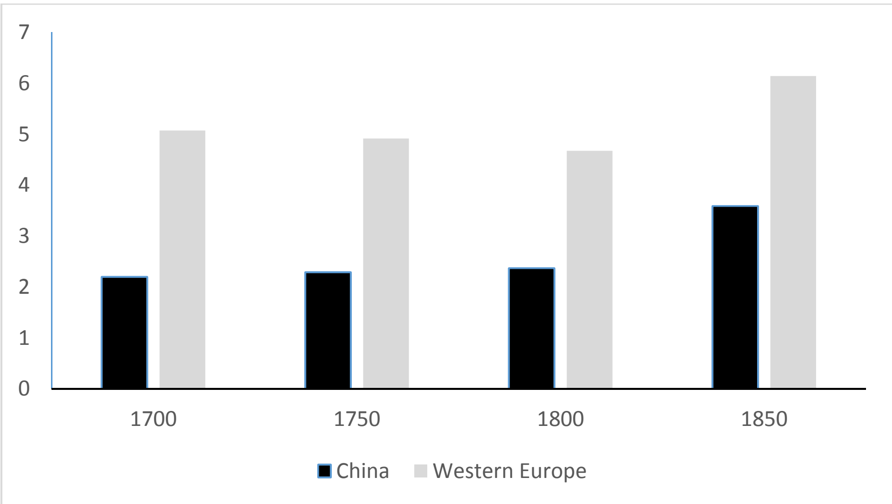
1. Introduction

Urbanisation and technological progress are closely linked. Economic historians and development economists have widely documented how workers are drawn to cities from the traditional sector in order to profit from the modern technologies that are created and adopted there.[1] Models of long-run growth thus tend to see urbanisation as a *consequence* of economic growth. At the same time, a vast literature on economic geography maintains that urbanisation itself generates productivity gains: going back to Marshall, economists have explored the idea that high density in cities results in learning – and thus in both skill upgrading and in innovation.[2] This suggests that agglomeration can *cause* economic growth through its impact on technological change. The aim of this paper is to model the two-way relationship between urbanisation and innovation in order to understand how they interacted at the onset of modern economic growth.

Our analysis is motivated by the observation that Europe was unusually urbanised before the Industrial Revolution. China is regarded by many historians as the most technologically advanced country in the Middle Ages.[3] However, despite an early upsurge in the share of population living in cities, urbanisation rates remained low throughout the modern period, as argued by Maddison (2007) and Voigtländer and Voth (2013b). By contrast, Western Europe experienced a marked increase in the share of population living in cities during the early modern period, already exhibiting an urbanisation rate of 5% by 1700, more than twice that prevailing in China, as shown in figure 1.

Figure 1. Urbanisation rates in Western Europe and China.



Source: Authors calculations, see below.

[1] See, for example, Harris and Todaro (1970), Rosenberg and Trajtenberg (2004) for an analysis of the effect of technological adoption on urbanisation in the 19th century, and Temple (2005) for a review.
[2] See Marshall (1890) as well as Glaeser (1999) and Combes, Mayer and Thisse (2008).
[3] Needham (1954) and his colleagues offered the seminal contribution here.

Interestingly, England was a significant outlier in the European urbanisation experience, with much higher rates than we would expect. Note further that the share of people living in cities had started to expand at a considerable pace well before the First Industrial Revolution. For example, the urbanisation rate had already risen threefold between 1600 and 1750, from 6 to 18% (Wrigley *et al.*, 1997). Table 1 shows the four largest cities in each of England, Scotland and Ireland, all part of the UK at that time. It indicates that the UK experienced an urban growth spurt *before* 1750, i.e. before the First Industrial Revolution. Whilst the population of London rose by 17%, the population of almost all the other cities reported doubled or tripled in the first half of the 18th century; by contrast, the English population increased by only 14% (Wrigley and Schofield, 1981).[4] We present more extensive urbanisation data for England and other countries in the next section. Yet the timing of these increases certainly raises the question of the extent to which urbanisation was a *cause* – rather than a consequence – of economic growth.

Table 1. Expansion over time of the largest UK cities (population in `000s).

City	Country	1700	1750	1800	1850
London	UK (England)	575	675	948	2 236
Liverpool	UK (England)	6	22	83	376
Manchester	UK (England)	8	18	84	303
Birmingham	UK (England)	7	24	71	233
Dublin	UK (Ireland)	60	129	200	262
Belfast	UK (Ireland)	2	9	20	87
Cork	UK (Ireland)	25	58	75	85
Limerick	UK (Ireland)	11	16	39	53
Glasgow	UK (Scotland)	13	25	70	345
Edinburgh	UK (Scotland)	36	57	83	194
Dundee	UK (Scotland)	10	12	26	79
Aberdeen	UK (Scotland)	12	15	27	72

Note: Includes the four largest cities in each country.
Source: Bairoch *et al.* (1988); De Vries (1984).

In this paper, we develop a model of growth consistent with the idea that urbanisation precedes industrialisation. The first element in our setup is a two-sector model where agriculture takes place in rural areas and manufacturing activity in cities. The manufacturing sector is modelled as a traditional artisan activity rather than modern industrial production, without the use of physical capital, and with the productivity of a worker depending only on the number of ideas that he holds. As the average number of ideas available grows, manufacturing productivity increases, and the incentives to leave the

[4] The population increase in Scotland and Ireland was around 25% (Daultry, Dixon and O'Grada, 1981).

countryside rise, leading to higher employment in manufacturing, as, for example, in Hansen and Prescott (2002). Our second element consists in modelling how ideas appear and are transmitted. We model the transmission of ideas between agents through imitation: individuals in cities may acquire an idea by meeting someone who already possess it. We also suppose that an individual can create a *novel idea* by observing the ideas or experience of others, thus endogenously inventing new ways of production, in line with Jovanovic and Rob (1989). In either case, acquiring ideas is the result of meetings between individuals. We follow Marshall and the formalization of his approach by Glaeser (1999, 2011) and suppose that the number of meetings is an increasing function of urban density. Higher density implies more meetings and hence more imitation and innovation; in contrast, the low density in rural areas implies that there are no meetings and hence no transmission or creation of ideas. Under these assumptions, the rate of urbanisation becomes the determinant of manufacturing productivity.

Two key results emerge. First, the above two elements together suffice to create a feedback mechanism between city size and technology that generates a novel set of growth dynamics. A shock to urbanisation results in knowledge creation and diffusion, higher manufacturing productivity, and a flow of labour into manufacturing and hence into cities. The resulting higher urban density further increases manufacturing productivity, thus setting off a process of increasing urbanisation and innovation that generates sustained growth. Thus our model makes urbanisation – as opposed to industrialization – the key element in a theory of development. Second, the microfounded process of knowledge generation allows us to dissociate the creation of ideas from their diffusion. Because an idea is invented by an individual, initially only the inventor holds that idea, which will then be transmitted to agents of the next generations through imitation. Those individuals who have not managed to imitate others will not be able to use the most recent technology, implying only a moderate increase in productivity. However, since now the idea is held by more individuals than just the innovator, it can be imitated by a larger number amongst the young of the next generation, further increasing the share of manufacturing workers who can use. As a result an innovation will increase productivity only slowly as imitation by subsequent generations occurs. This generates a model that can simultaneously deliver fast technological change and slow overall productivity growth in manufacturing, in line with existing estimates for the 18th and 19th centuries (see Crafts, 2004).

We calibrate our model to reproduce a number of features of the English economy over three centuries. In doing so, we consider possible reasons why England was highly urbanised. A number of aspects have been discussed in the literature, going from location fundamentals to military conflict and the Black Death.[5] While any of the above mechanisms could have been a trigger for the sustained growth model that we develop, our analysis will emphasize the role of agricultural sector triggers, as our setup implies that high urbanisation is associated with high labour productivity in agriculture, a phenomenon that was particularly important in England. As we document below, England was both more urbanised and had higher agricultural labour productivity than other Western European countries; and Western Europe was, on average, more urbanised and had higher agricultural labour productivity than China. Thus, increasing agricultural productivity is a candidate trigger for a growth take off. The period is also characterised by changes in in the share of agricultural output that went to the landlord and the taxman, i.e. in agricultural extractions. The role of agricultural extractions as an additional shock during early modern times has received little attention in the literature, yet it seems to us to have been particularly relevant in the case of England. High extractions from agriculture were the result of two phenomena. On the one hand, the established land property system in Northern Europe implied that a substantial amount of production went to the owner of the land, who was generally not the one who worked it. On the other hand, in 17th and 18th century Europe, tax revenues were rising fast – particularly in England – and the main source of this tax revenue was the agricultural sector.[6] Our model will therefore explore shocks to both the knowledge generation process and to the agricultural sector – in the form of higher productivity and higher extractions – and examine their joint implications for urbanisation and growth.

Our paper is related to several strands of literature. First is the recent development in growth theory explaining how the world moved from stagnation to growth, best represented by Unified Growth Theory (UGT); see Galor and Weil (2000) and Galor (2011). UGT is at the core of our understanding of the appearance of modern growth and of the close relationship between population dynamics and economic development. And yet, specific differences in the timing and speed of the transition are still not fully understood. A number of recent papers have focussed on the unique features of Europe, and particularly England, before the Industrial Revolution.[7] For example, Desmet and Parente (2012) emphasize the role of market size and argue that markets in England were more integrated – and thus larger – than elsewhere by the early 18th century, and this allowed firms to implement cost-reducing production technologies. Our analysis is closely related to Voigtländer and

[5] See section 5 below for further discussion.

[6] For evidence on historical fiscal patterns see Karaman and Pamuk (2010), Dincecco (2011) and Voigtläder and Voth (2013b). Karaman and Pamuk (2010), figure 5, show that, across Europe, per capita tax revenues were highest in England and the Dutch Republic in the 18th century, the Netherlands also being a highly urbanised country. For an exhaustive account of the English tax system and its reliance on agricultural sector taxes, see Dowell (1884).

[7] The literature by economic historians on the First Industrial Revolution is obviously very extensive; see Mokyr (2008, 2009) for a recent discussion.

Voth (2006, 2013a). Both of their models share with ours an approach based on the migration of workers from agriculture into manufacturing. Voigtländer and Voth (2006) identify the importance of demographic patterns in countries like France and England for the prospects of industrialization, while Voigtläder and Voth (2013a) maintain, like we do, that cities played a crucial role in early modern times. In their setup, the Black Death is the exogenous trigger that initially raises wages and results in migration towards cities, with the poor health conditions in cities then increasing average mortality rates and allowing Europe to escape from the Malthusian trap. Our analysis complements theirs by maintaining that the frequent interactions occurring in cities not only resulted in the transmission of viruses but also of ideas. Lastly, Boucekkine, Peeters and de la Croix (2007) also consider the role of population density and argue that it reduced the cost of attending and creating schools, thus increasing educational investments. Their analysis abstracts from technological change, which is the focus of our model.

The determinants of technological change have been extensively examined in the endogenous growth literature, which sees population size, production externalities or intentional R&D as sources of productivity improvement. We depart from these approaches and add to a small literature which, starting with Lucas (2009), considers the role of individually-held knowledge. Lucas maintains that the onset of modern growth is linked to the appearance of a class of individuals that "spend entire careers exchanging ideas, solving work-related problems, generating new knowledge" (Lucas, 2009, p.1), and develops a growth model in which each individual learns from all others in the same economy. This concept has been applied to different spaces for the transmission of ideas, notably firms, schooling or trade; see Alvarez et al. (2013), Buera and Lucas (2018), Caicedo et al. (2019). For example, international trade is growth-enhancing because it puts domestic producers in contact with more efficient foreign firms. We share with these works an emphasis on the contribution of human interaction to knowledge flows and the role played by random meetings. However, we consider a different source of interactions by focusing on the physical proximity provided by city life, a context that is particularly suitable when thinking about early modern growth.

Our paper is also related to the vast literature on how urban growth relates to aggregate GDP growth. Early development economists saw the shift of resources from an agricultural to an urban, modern sector as the key element in the growth process; see Lewis (1954) and Harris and Todaro (1970). These models assumed an exogenous modern, urban technology and a backward rural one. The presence of some form of friction – such as migration restrictions or wage subsidies – limited the flow of workers into the city, and delivered predictions on how these policies simultaneously affected urbanisation and the level of GDP. In the 1990s, more complex dual economy models enriched the basic framework, but shared with the early work the assumption that ongoing urbanisation results from exogenous forces, notably exogenous technological change favoring the urban sector and falling transportation costs (see, for example, Bencivega and Smith, 1997). As a result, these models are useful for examining the policies that jeopardize urbanisation and output growth, but have the drawback that there are no endogenous agglomeration forces. This question has been addressed by the economic geography literature. One strand of the literature uses core-periphery models to examine how city size is determined by the presence of scale economies in production (see Henderson, 2005, for a review), while an alternative approach has focused on the idea that agglomeration leads to knowledge diffusion and creation (Glaeser, 1999). Yet, the focus in both cases is on the relative size of cities, rather than the rural-urban divide associated with an agricultural/manufacturing dichotomy and its implications for growth. Our model combines these two approaches, endogenizing the rural-urban productivity gap by introducing the elements found in economic geography into a dual economy model.

Following the argument first put forward by Marshall (1890) and the seminal work of Jacobs (1969), numerous authors have modelled how the environment offered by cities improves the prospects for generating and diffusing new ideas. Cities affect the productivity of firms and workers as they generate technology spillovers across firms, and learning and skill-acquisition by workers. Various mechanisms can give rise to innovation and learning in cities; see Duranton and Puga (2004) for a review. One of the most influential arguments has been the idea that proximity to individuals with greater skills or knowledge facilitates the exchange and diffusion of knowledge. In this context, higher urban density results in more frequent encounters and thus leads to greater transmission of skills and ideas between workers. Our paper uses the insights of this vast literature to model the process of imitation and innovation that takes place in cities. In line with empirical findings,[8] we assume a process of knowledge creation in which innovation and productivity both increase with urban density, and by introducing these into a fully-fledged growth model we can examine the long-term implications of such learning.

The rest of the paper is organised as follows. The next section provides a more detailed discussion of the historical evidence that supports our arguments. Section 3 presents our two-sector model and examines the determinants of urbanisation. Section 4 considers the mechanism for the creation and transmission of ideas, and incorporates them into our macroeconomic model in order to

[8] See Audretsch and Feldman (2004) for a review of the empirical evidence on the advantages of cities for innovation.

obtain the joint dynamics of urbanisation and knowledge. In section 5 we discuss our model in the light of existing evidence, while section 6 provides numerical examples and calibrates the model to show that its predictions match the data available for England. We conclude in section 7.

2. The historical context

As we detail in the next section, our model links the observed spatial and temporal variation of three key variables: urbanisation levels, agricultural labour productivity and output growth. In this section we describe the salient features of two of these variables, examining urbanisation and agricultural productivity across Western Europe and China in the 18th and 19th centuries. Since a vast literature has documented the takeoff experienced in England and Europe, we refrain from discussing this aspect here.[9] We start by showing that urbanisation rates were higher in Western Europe than in China before the Industrial Revolution; and, within Europe, England was particularly urbanised. Then we will see that high urbanisation rates were accompanied by high agricultural labour productivity. While Europe's extraordinary urbanisation made it a likely candidate for an early transition to modern growth, we will see that England was unique in both dimensions by 1750, as it experienced high and growing preindustrial urbanisation, accompanied by high agricultural labour productivity.

Patterns of urbanisation

Measuring urbanisation is not straightforward because there is no consensus on what population threshold defines a city. For example, Bairoch *et al.* (1988) use a cutoff of 5 000 inhabitants, while De Vries (1984) reports data for all European towns and cities larger than 10 000. But most of us would probably think of cities as being larger conurbations than this, even in the Middle Ages. Moreover, Chinese data are available only for larger cities. Hence we use 200 000 inhabitants as the cutoff for the descriptive statistics that we present here, in order to ensure comparability and accuracy across space and over time. Although our model does not guide us to prefer one cutoff over another, we believe that agglomeration externalities exist in cities of less than 200 000 inhabitants and therefore we use the 10 000 threshold whenever possible, such as in our simulations for England below.

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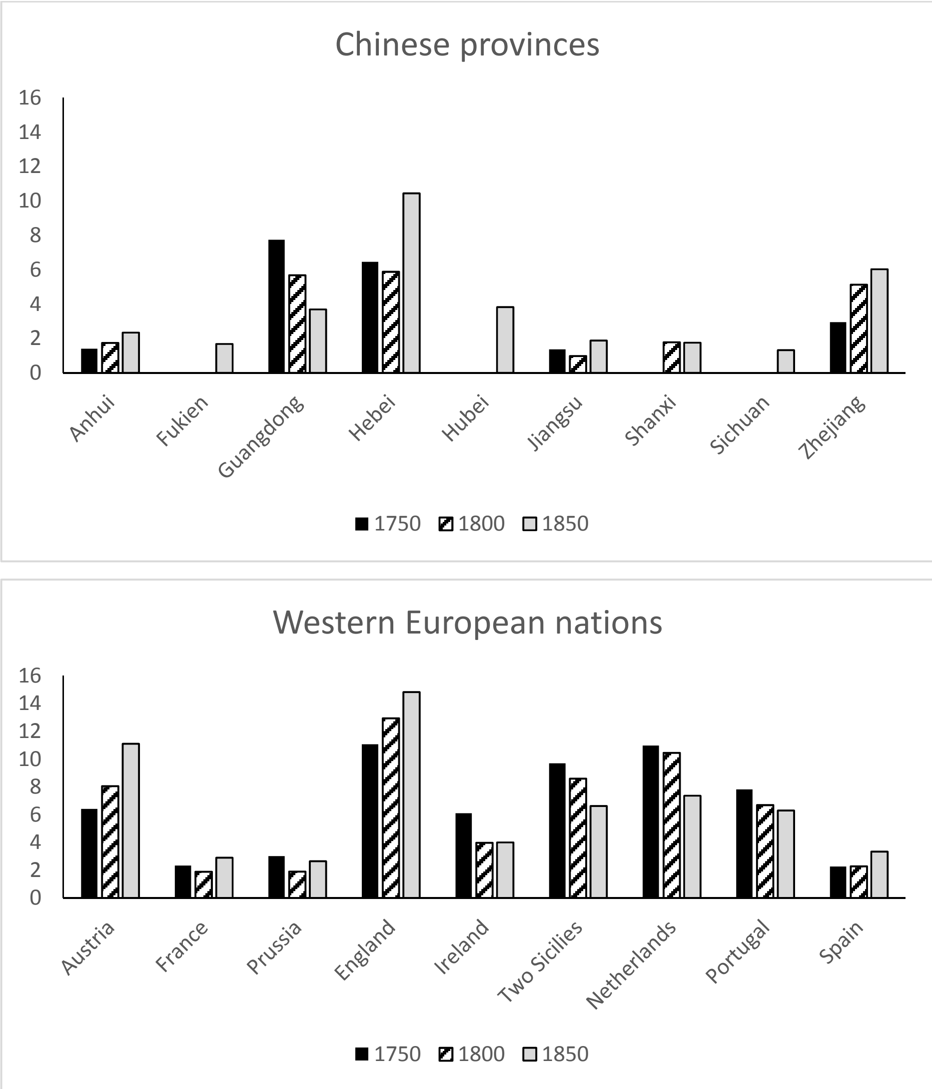
[9] See, amongst others, Crafts (2004) and Broadberry and O'Rourke (2010). We will return to output dynamics in our calibration exercise.

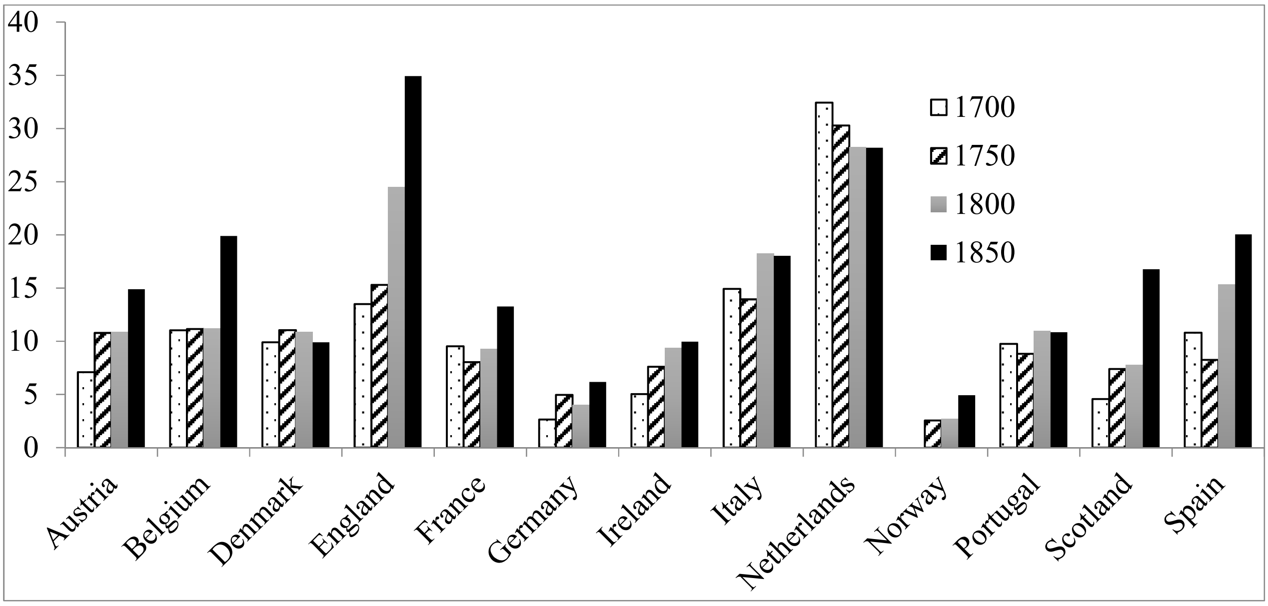
Figure 2. Shares of urban population in Chinese provinces and Western European nations.



Sources: Chandler and Fox (1974), Skinner (1977), and De Vries (1984). See Appendix A for details.

Figure 2 presents the share of population living in cities in China and Western Europe. The sizes of Chinese provinces are roughly comparable to European countries, so the units of analysis are not too disparate. We include all provinces/nations that had at least one city reaching a population of 200 000 at some point during the period of analysis, and we compute urbanisation rates as the sum of all inhabitants in such cities divided by the total population of the province/nation. Appendix A (tables A.1 to A.6) provides further details on the data, as well as the list of cities that we consider. Figure 2 indicates that, on the eve of industrialization, Western Europe had reached considerably higher levels of urbanisation than China. In 1750, the two most urbanised Chinese provinces, Guangdong (home of Canton/Guangzhou) and Hebei (home of Peking/Beijing), had urbanisation rates of 8 and 6% – well below the 11% found in England and the Netherlands. Moreover, while only two Chinese provinces had urbanisation rates above 3%, in Europe it was not only England and the Netherlands that had reached at least *twice* this rate but also Austria, the Kingdom of the Two Sicilies (the largest of the Italian states before unification), Ireland, and Portugal. In 1750, the (population-weighted) average urbanisation rate among the Chinese provinces was 2.29% while that for the European nations was twice as high, at 4.82%. By 1850, China and Western Europe reached 3.59% and 6.26%, respectively (see Appendix A).

Figure 3. Shares of urban population across Western Europe



Source: Bairoch *et al.* (1988), De Vries (1984) and Brunt and Fidalgo (2008). See Appendix A for details.

Among the Western European nations, England had the highest urbanisation rate, with 11% of the population living in large (i.e. greater than 200 000) cities, followed closely by the Netherlands and Italy. Figure 3 presents an alternative measure of urbanisation. It uses De Vries' definition of cities as agglomerations of 10 000 or more inhabitants. Using this cutoff allows us to incorporate a number of other European nations that did not have, at that time, cities above 200 000 inhabitants. Adopting a lower threshold for the definition of a city obviously counts many more people as urbanites, and thus generates urbanisation rates considerably higher than in the previous figure. When considering medium-sized cities, the Netherlands dominates all other countries by a vast margin, with England exhibiting the second highest urbanisation rate in 1750 at 15.3%. This figure is well above that of other large nations, such as France or Spain, where urbanisation was only around 8%. Urbanisation was also high in Italy and Belgium.[10] Note that Belgium was the first country in Continental Europe to industrialize, following England quite closely, while the fact that the Netherlands experienced a relatively late industrialization has been seen as a puzzle by historians (Mokyr, 1999).

So far, our discussion has focused on variations in the *level* of urbanization across space. But we also need to consider the *change* in urbanization over time. The traditional approach sees urbanisation as the result of productivity growth, which implies that urban growth should have started *after* the increased innovation that we see in the Industrial Revolution. In our model, too, higher innovation will cause urbanization because an increase in *either* agricultural or manufacturing productivity raises urbanisation. In consequence, an exogenous increase in agricultural or manufacturing TFP (such as Hargreaves' 1764 invention of the "Spinning Jenny" for producing cotton thread) would have the effect of triggering a flow of labour into the city. Alternatively, shocks that accelerate the diffusion of ideas within cities would, for given knowledge, increase productivity. For example, there might be more encouragement for innovators to meet – as with the foundation of the Royal Society for the Encouragement of Arts, Manufactures and Commerce in London in 1754. Or there might be improved access to technical literature – such as the appearance of Chamber's *Cyclopaedia: An universal dictionary of arts and sciences* (the first modern encyclopaedia, which was published in London in 1728). Mokyr (2004) offers many other examples of this type of propagation of 'useful' knowledge. But these developments occurred too late to explain English urbanisation, which had already tripled between 1600 and 1750 (from 6 to 18% of the population), indicating that something else must have prompted increased urbanization.

Location fundamentals have long been argued to be a central aspect of city growth but, as Bleakley and Lin (2015) discuss, other factors are important. The urbanisation impact of war and the Black Death has been explored by Voigtländer and Voth (2013a), and evidence on the role that military conflict played is provided by Dincecco and Onorato (2016) and Voigtländer and Voth (2013b). Interestingly, in our context, a number of recent authors have focused on the importance for city size of knowledge-enhancing changes such as the arrival of the printing press or the creation of universities; see Dittmar (2011) and Cantoni and Yuchtman (2014). Nunn and Qian (2011) argue that increases in agricultural productivity, partly due to the introduction of new crops such as the potato, were fundamental because they generated the potential to free labour from food production. This last aspect seems to have been important in the context we are considering, hence we turn next to the agricultural

[10] Note that Belgium was not present in the previous table because it had no cities greater than 200 000, although it had a substantial number of agglomerations with more than 10 000. This exemplifies the difficulty of defining 'cities' and the importance of looking at alternative thresholds.

sector.

Agricultural labour productivity

European urbanisation was accompanied by high and rising agricultural output per worker. By the beginning of the 18th century, agricultural yields were higher in Western Europe – and particularly England – than elsewhere in the world. Recent estimates of agricultural labour productivity (Brunt and Fidalgo, 2013) reveal massive differentials in output per agricultural worker around the world over the period 1700 to 1870, as reported in table 2.[11]

Table 2. Real output per worker in agriculture (England in 1870=100).

	1705	**1775**	**1845**	**1870**
England	66	89	82	100
Scotland	NA	59	129	163
Belgium	NA	33	52	78
France	27	24	39	34
Netherlands	NA	41	32	37
Prussia	10	8	25	50
Spain	15	14	14	23
China (Hubei, most productive)	273	48	39	45
China (Guizhou, least productive)	2	4	4	2
USA	19	29	45	45

Source: Brunt and Fidalgo (2013).

Table 2 highlights not only the high level of output per worker in England but also its strong growth during the 18th century, with an increase of 35% over the period 1705-1775. Alternative estimates for the growth in output per worker in English agriculture yield similar growth rates: for example, Clark (2002b, table 4) finds an increase in output per worker of 44% between the decades 1700-09 and 1770-79. Moreover, he documents that output per worker had already risen sharply in the 16th and 17th centuries, increasing fivefold between 1500 and 1650. Thus increasing agricultural productivity is an important candidate to be the trigger for the Industrial Revolution. The proposition that an agricultural revolution preceded the Industrial Revolution in England is hardly controversial – virtually every scholar agrees on this point – but the timing has been much debated. The earliest analysis (Ernle, 1912) suggested that the agricultural revolution was almost concurrent with the

[11] The sizes of the differentials may seem surprising. However, Maddison (2001) reports similar magnitudes for aggregate labour productivity and, since agriculture was by far the largest sector in all economies throughout this period, aggregate and agricultural productivity are bound to be quite similar.

Industrial Revolution, with major changes occurring in agriculture after 1760. More recent research has almost universally favoured much earlier dates: classic contributions by Jones (1974) and Allen (1992) argued for the 17th century, whilst Kerridge (1967) maintained that it started in the late 16th century. Overton (1996) pushed back towards the late 18th century. In all cases, these dates precede the increase in output and productivity growth that is documented from 1800 onwards (Crafts, 2004), indicating that the timing of the agricultural revolution in England roughly matches the increase in urbanisation.

To sum up, Europe – especially England – exhibited high and growing urbanisation prior to the Industrial Revolution, while agriculture exhibited high and rising labour productivity. These stylized facts are consistent with our model's prediction that a number of shocks that affected Europe resulted in an increase in urbanisation that predated the Industrial Revolution and contributed to knowledge generation. We formalize these possibilities in our model.

3. A model of urbanisation

We begin by setting out the components of the model. This section considers the determinants of urbanisation, taking manufacturing productivity as given. The endogenous evolution of manufacturing productivity is developed in section 4.

3.1 Population and preferences

We consider an overlapping-generations economy with workers and an elite. Agents live for two periods. At time *t*-1, $(1+\varepsilon)N_t$ individuals are born who will be adult at *t*. Of these, N_t are workers and εN_t belong to elite dynasties, comprising landlords and public officials, with $\varepsilon<1$. Workers have two possible occupations, farmer or manufacturer. We suppose that these occupations require agents to live either in the countryside or in cities, respectively, so agents' location decision is bundled with their occupational choice.[12] Young agents are born in the location chosen by their parents and make their occupation/location choice in the first period of their life; in the second period they work and have offspring. During the first period of their life, they may or may not acquire "ideas" that make them more productive, according to a process defined below.[13]

The elite receive rents from the farmers, to whom they let their land, as well as tax revenues

[12] The simplifying assumption that manufacturing occurs only in cities is consistent with evidence indicating that only limited manufacturing production occurred outside cities in early modern times; see Coleman (1983).

[13] The fact that individuals acquire skills when young, i.e. before they start working, fits well with a structure of apprenticeships, whereby young individuals chose their 'trade' at a relatively early age and acquired specific skills. See, for example, De la Croix et al. (2018).

that finance public offices. For simplicity, we suppose that the members of the elite are simultaneously landowners and public officials. This is most obviously true in England, where Parliament passed the Justices of the Peace Act in 1361 – creating the role of local magistrate, whose job was to administer local government and enforce the law but who remained unremunerated until 1835 (Magistrates Association, 2018). Since the magistrates' position required education, free time and an incentive to manage local affairs (such as collecting local property taxes and disciplining paupers, as well as pursuing criminals), only the landed elites were qualified to fulfil this role.

We suppose that preferences are identical for all agents and take the form

$$U(c_{at}, c_{mt}) = c_{at}^{\alpha} c_{mt}^{1-\alpha} \tag{1}$$

where c_{at} and c_{mt} are, respectively, consumption of the agricultural and the manufactured good in the second period of life, and $\alpha < 1$. This yields the demand functions

$$c_{at} = \alpha y_t, \tag{2a}$$

$$c_{mt} = (1-\alpha)\frac{y_t}{p_t}, \tag{2b}$$

where y_t is the income of the individual, and the prices of the agricultural and the manufactured good are, respectively, 1 and p_t. The resulting indirect utility function can be expressed as

$$U(y_t) = a y_t p_t^{\alpha-1}, \tag{3}$$

where $a \equiv \alpha^{\alpha}(1-\alpha)^{1-\alpha}$.

3.2 Technology and factor payments

Agriculture

Agricultural output is produced with a constant returns to scale technology using labour and land, *T*,

$$Y_{at} = T^{1-\gamma}(A_t L_{at})^{\gamma}, \tag{4}$$

where A_t denotes productivity in the agricultural sector, L_{at} is agricultural labour, and $0 < \gamma < 1$. Our period of interest experienced significant improvements in agricultural technology, as documented by a large literature (for an overview, see Overton, 1996). Hence we assume that TFP grows at an exogenous rate g, so that A_{t+1}=$A_t(1+g)$.[14]

The wage in the agricultural sector is given by the marginal product of labour, that is,

$$w_{at} = \gamma A_t^{\gamma}\left(\frac{T}{L_{at}}\right)^{1-\gamma} \tag{5}$$

[14] It would be possible to make agricultural productivity endogenous, for example by assuming that a share of new ideas are applicable to agriculture rather than to manufacturing, which obviously would make agricultural productivity growth move together with manufacturing productivity growth. Since there is no evidence that this was the case, we chose to allow for exogenous agricultural productivity growth.

with the remaining income being paid to landowners as rents. We further suppose that agricultural workers are taxed at rate τ, so that the net income of farmers is given by $(1-\tau)w_{at} = (1-\tau)A_{at}n_{at}^{\gamma-1}$, where n_{at} is the share of population employed in farming and $A_{at} \equiv \gamma A_t^{\gamma}(T/N_t)^{1-\gamma}$. Given the indirect utility function above, the utility of an individual working in the agricultural sector is simply $U_{at} = a(1-\tau)w_{at}p_t^{\alpha-1}$.

Manufacturing

Manufacturing takes place in cities and the only input is labour. Manufacturing workers acquire ideas, denoted by i, when they are young according to a random learning process described below. We suppose that ideas, i.e. skills, are sector-specific, so individuals will not move when old.

The randomness of the learning process implies that ex ante identical individuals will eventually hold a different number of ideas. As we will detail below, at time *t* the maximum number of ideas available is given by t, with $\pi_t(i)$ being the proportion of individuals that hold *i* ideas at *t*, and $I_t = \sum_{i=1}^{t} i \cdot \pi_t(i)$ being the average number of ideas. We suppose that an artisan with *i* ideas produces $(1+i)$ units of output. Summing across artisans, aggregate manufacturing output can be expressed as

$$Y_{mt} = B_t L_{mt}, \tag{6}$$

where L_{mt} is employment in manufacturing and $B_t \equiv 1 + I_t$ is average productivity in that sector, which depends on the average number of ideas I_t that will be endogenously determined in section 4. The wage of a manufacturing worker with *i* ideas is simply given by $(1+i)p_t$. Denote by w_{mt} the expected wage of working in manufacturing anticipated by the young. It is determined by the price and the (expected) distribution of ideas, that is, $w_{mt} = \sum_{i=1}^{t}(1+i)p_t\pi_t(i)$, which can be expressed as $w_{mt} = p_t B_t$ and is independent of employment in the sector at time *t*.

Consider now the (indirect) expected utility of a worker living in the city and working in manufacturing. The linearity of equation (3) in income implies that expected utility depends only on the expected wage. Moreover, we suppose that there is a disutility associated with living in cities, denoted v_t, so that expected utility can be expressed as that $EU_{mt} = aw_{mt}p_t^{\alpha-1} - av_t$. This cost creates a wedge between rural and urban utilities that, in equilibrium, will result in higher wages in cities. Evidence of a gap between rural and urban wages is provided by Clark (2001), whose data show that an urban labourer earned around 20% more than a farm labourer in the mid-18th century. There are various possible justifications for the cost. Murphy, Shleifer and Vishny (1989) argue that manufacturing work itself generates a disutility for which workers have to be compensated, while Voigtläder and Voth (2013a) emphasize the health costs of living in cities, which is consistent with falling average life expectancy in England through most of the 18th century (Davenport, 2015). Yet, technological progress is likely to have reduced it, due both to lower mortality rates stemming from improved medical know-how and reduced transportation costs due to the expansion of the canal system and railways.[15] We hence allow the disutility cost to fall over time. In order to make this decline endogenous, we suppose that the cost falls with the level of technological know-how in manufacturing, that is, $v_t = v(B_t)$, where $v(.)$ is a decreasing function and bounded below at zero, $\lim_{B_t \to \infty} v(B_t) = 0$.

As well as being consistent with the existing literature, the presence of this cost is essential for our results. We will see below that increases in manufacturing productivity, B_t, will reduce the relative price of manufactures. In the absence of v_t, Cobb-Douglas preferences and technology would imply that the price reduction exactly offsets the increase in productivity, leaving $p_t B_t$ and hence the manufacturing wage unchanged. The disutility creates a wedge in wages that implies that prices fall less than in its absence, thus resulting in an increase in the manufacturing wage.

Lastly, note that urban dwellers are assumed not to be taxed. This is a good approximation to the English context: the land tax was the main source of government revenue from the late 17th century to the First World War (typically generating around £2 million per annum, since the price level and land area were both approximately constant) and the agricultural tithe generated a similar figure; by contrast, the medieval shop tax had fallen into abeyance and urban producers remained virtually untaxed. Naturally, there were many minor changes in taxation over such a long period; an exhaustive history can be found in Dowell (1884).

3.3. Static equilibrium

Employment in agriculture and manufacturing

Workers can choose freely between agriculture and manufacturing, and will do so such that expected utility is equalized across the two sectors. The decision to migrate is taken by young individuals at time t-1 on the basis of their (rational) expectations of the income they would get at t in each of the two sectors. Note that some of the young in the city get ideas and others do not. This means that, *ex post*, those in the city who got a below-average number of ideas will have lower utility than those in agriculture. However, these individuals will not move into farming due to our assumption that skills

[15] Because of a lack of historical evidence on the cost of rural-urban migration, we turn to contemporary evidence. In rural India, cash transfers that cover the cost of regular visits to the village of origin increase considerably the probability of migrating from a rural to an urban location (Lagakos et al., 2018), indicating that in low-income countries these costs still exist. In contrast, evidence on China shows that, in recent years, rural migrants permitted to move to urban locations are willing to forgo wage income in order to locate in larger cities with more amenities, implying that the cost has disappeared (Xing and Zhang, 2017).

are sector-specific and need to be acquired when young.

Equating U_{at} to EU_{mt}, and using the expressions above for wages in agriculture and manufacturing, we obtain the following relation between the price of manufactures and employment in agriculture

$$(1-\tau)A_{at}n_{at}^{\gamma-1} = p_t B_t - v_t p_t^{1-\alpha} \tag{7}$$

This equation implies a negative relationship between n_{at} and p_t, as a higher price will increase the manufacturing wage and result in migration away from farming. Higher productivity, B_t, or higher taxation, i.e. a higher value of τ, result in a lower share of agricultural employment for any given price, since the former increases the manufacturing wage and the latter reduces the agricultural wage.

Goods market equilibrium

To obtain the goods market equilibrium, we need to consider the demands of workers and the elite. Worker demand is given by equations (2) above. The εN_t elite members receive both rents and tax revenue, i.e. all the output from the agricultural sector that is not kept by farmers, which is simply $(1-\gamma(1-\tau))Y_{at}$. Since they have the same utility function as workers, their demands are also given by (2).[16] Equating the supply and the demand for agricultural goods we have

$$Y_{at} = \alpha(1-\gamma(1-\tau))Y_{at} + \alpha N_t n_{at} w_{at} + \alpha N_t (1-n_{at}) w_{mt}. \tag{8}$$

The first term on the right-hand side is the demand from the elite, followed by that from farmers and that from urban workers. Substituting for wages and output, this equation can be expressed as

$$\frac{1-\alpha}{\alpha}\frac{A_{at}}{\gamma}\frac{n_{at}^{\gamma}}{1-n_{at}} = p_t B_t, \tag{9}$$

which gives the goods market equilibrium. This equation implies a positive relationship between agricultural employment and p_t, for a given value of technology B_t. The higher is employment in agriculture, the higher is agricultural output, which reduces the price of agricultural goods relative to manufactures.

The equilibrium allocation of labour

Equations (7) and (9) jointly determine agricultural prices and employment, so that the equilibrium allocation is determined by the system

[16] We could allow the elite to consume imported 'luxury goods' as well as agricultural goods and manufactures, so that a fraction of output is not spent domestically. This would change slightly the expression for the relative price but would have no qualitative impact on our results.

$$(1-\tau)A_{at}n_{at}^{\gamma-1} = p_tB_t - v_tp_t^{1-\alpha} \quad (10a)$$

$$\frac{1-\alpha}{\alpha}\frac{A_{at}}{\gamma}\frac{n_{at}^{\gamma}}{1-n_{at}} = p_tB_t \quad (10b)$$

Since equation (10a) implies that n_{at} is a decreasing function of p_t, while equation (10b) yields an increasing function, there will be a unique equilibrium pair of price and employment for each value of B_t, n_{at}^* and p_t^*. Moreover, differentiating the implicit functions we have $dn_{at}^*/dB_t < 0$, and $dp_t^*/dB_t < 0$. That is, higher values of B_t result in a lower n_{at}^* as higher productivity in manufacturing increases the wage in cities, inducing a shift of labour away from agriculture. This effect is partially offset by a price effect, since lower farming employment reduces agricultural output and drives up the relative price of agricultural goods. It is possible to show that the direct effect dominates and that p_tB_t increases, implying a higher urban wage. Agricultural employment is also affected by taxes and agricultural productivity, with the former reducing rural wages and hence inducing a flow of labour to cities. A higher value of A_t (i.e. a larger A_{at}), shifts both functions as higher agricultural TFP increases the marginal physical product of labour in farming but reduces the price of agricultural goods, with the price effect dominating so that $dn_{at}^*/dA_t < 0$. Lastly, a larger population (which reduces A_{at}) has the opposite impact, with the increase in the price of agricultural goods dominating and attracting more individuals into farming, thus raising n_{at}^*. Note that this scale effect depends on the supply of land being fixed, and that it can be offset by increases in either agricultural or manufacturing productivity.[17]

It is important to note at this point that the effect of productivity changes on the allocation of labour across sectors in dual economy models has been much debated; see Temple (2005) for a review. The predictions crucially depend on three aspects: the shape of the utility function, the factors used in production, and the potential wedges preventing wage equalization across sectors.[18] In particular, the shape of the utility function determines the relative importance of income and substitution effects. When the substitution effect dominates, as is the case with CES utility, an increase in agricultural TFP and in manufacturing productivity have opposite effects on urbanisation, with the sign being determined by whether the elasticity of substitution is greater or smaller than one (i.e. by how prices change in response to productivity). If the income effect dominates, for example due to the existence

[17] A possible interpretation is that scale effects are present only within a country, since the pressure on food demand as population increases can only be compensated by increases in productivity in either sector. In contrast, they do not appear across countries because population and land supply tend to be correlated.

[18] An example of the role of inputs is Hansen and Prescott (2002). In their model, agriculture uses land, labour and capital but manufacturing only the last two factors. As a result, the manufacturing sector will not be operational for low levels of manufacturing productivity, and changes in productivity in either sector have no impact on agricultural employment.

of a minimum consumption requirement in agriculture, then increases in productivity in the agricultural sector raise demand for the manufacturing good more than for the agricultural one, inducing a flow of labour into cities. The income effect may also dominate in the presence of disutility costs associated with urban life, as is the case here. Note that for $v_t = 0$, equations (10) simplify to $\alpha(1-\tau)\gamma(1-n_a) = (1-\alpha)n_a$. In that case, the allocation of labour is independent of the level of technology in either sector (as well as of the size of the population), the reason being that the price change exactly offsets the impact of any productivity change.

3.4. Per capita output

We are ultimately interested in the dynamics of per capita output. To be consistent with the evidence for the period of interest, the population is assumed to grow at a net rate of b_t so that $N_{t+1} = N_t(1+b_t)$, and we allow this rate to be endogenous. The way in which individuals choose their fertility as output and productivity change has been well studied (see, for example, Galor, 2011); we therefore abstract from fertility choices and simplify the population dynamics as much as possible, assuming that net population growth is an increasing function of per capita income, y_t, i.e. $b_t = b(y_t)$.

Per capita real output in the economy is given by

$$y_t = \frac{A_{at}n_{at}^{\gamma} + \gamma p_t B_t n_{mt}}{\gamma(1+\varepsilon)p_t^{1-\alpha}} = \frac{1}{1+\varepsilon}\left(\frac{A_t^{\gamma}T^{1-\gamma}}{\alpha n_{at}^{\gamma} N_t^{1-\gamma}}\right)^{\alpha}\left(\frac{B_t n_{mt}}{1-\alpha}\right)^{1-\alpha}, \qquad (11)$$

where equation (10b) has been used to substitute for the relative price and obtain the second equality. The first expression indicates that there are three effects of increased manufacturing productivity on aggregate output. Higher B_t directly increases output, while it reduces the price of manufactures, making the price index, $p_t^{1-\alpha}$, fall which further increases real output. Lastly, the flow of labour away from agriculture and into the (now) more productive manufacturing sector provides an additional boost. Output is also affected by the dynamics of per capita productivity in farming, which are a combination of the (exogenous) productivity growth in agriculture and the (endogenous) rate of growth of the population. The former tends to increase and the latter to reduce output.

4. Cities and the transmission of ideas

The model so far has analysed the way in which a number of shocks to the agricultural sector can affect urbanisation and industrialization for a given level of manufacturing know-how. The next step in our analysis is to consider how urbanisation, in turn, has an impact on knowledge. This will result in a virtuous circle in which a shock increases urbanisation, which in turn raises productivity in manufacturing, thus inducing a further flow of labour into cities, and so on. Before we proceed, note that a simple 'black-box' specification for knowledge creation – in which the latter is increasing in urbanisation – would deliver the feedback mechanism that we postulate. However, we are interested in understanding also what could drive the *acceleration* of growth observed in the data, as well as the gap between the paths of innovation and productivity. In order to do so, we now turn to the microfoundations of the knowledge generation process.

4.1 The creation and transmission of ideas

We suppose that, in each period of time, at most one idea can be invented following a process defined below. Ideas are ordered from i=1, 2, $\ldots t$, according to the period in which they were invented, and an individual acquires knowledge sequentially, so that in order to hold idea i, he must also hold all ideas from 1 to i-1. Our two assumptions – bounded knowledge and sequentiality – are made for analytical tractability. The mechanism that we explore does not require either and we could, in principle, allow for an unbounded number of new ideas each period and non-sequential knowledge acquisition. However, the distribution of ideas would then have more parameters and following its evolution over time would become cumbersome.

Recall that $\pi_t(i)$ is the probability that an individual has exactly *i* ideas at *t* and denote by $P_t(i)$ the probability that an individual has at least *i* ideas at time *t*. Since there is a large number of individuals, these probabilities are, respectively, the fraction of the urban population that holds exactly *i* ideas and that which holds at least *i* ideas. The sequentiality of ideas also implies that $\pi_t(i) = P_t(i) - P_t(i+1)$.

There are two ways in which a young individual living in a city may increase his productivity. First, he may acquire existing ideas by meeting someone with those ideas. Second, young agents may also innovate and acquire *one* new idea during the period. We follow Glaeser (1999) and suppose that the number of meetings D_t is a function of urban density. In particular, we suppose that the number of meetings is a strictly increasing function of the number of old individuals in the city, i.e. $D(n_{mt})$, with $D(0) = 0$. That is, when nobody lives in cities, there will be no meetings. As in Glaeser (1999), we suppose that a young agent who meets someone with at least *i* ideas acquires these ideas with probability z. This means that the probability of acquiring at least *i* ideas in each meeting is $zP_t(i)$, as $P_t(i)$ is the probability that the individual of the previous generation who has been met has at least those skills. If a young person has failed to learn something across all his meetings then he will not be able to imitate. The overall probability of not acquiring at least *i* ideas is therefore $(1 - zP_t(i))^{D_t}$, so the fraction of the population that has imitated at least *i* ideas can be expressed as

$$M_{t+1}(i) = 1 - \left(1 - zP_t(i)\right)^{D(n_{mt})} \quad (12)$$

Additionally, an individual may come up with one new idea. Assuming that the probability that the individual does so after a particular meeting is δ, then the probability that he innovates during his youth is given by $q_{t+1} = 1 - (1 - \delta)^{D(n_{mt})}$. This particular assumption delivers the simplest possible setup in which innovation depends on meetings. Alternative assumptions could have been made about the probability of the individual innovating after a meeting – for example, making it depend on the average number of ideas held by the population. Such an assumption would have reinforced the dependence of innovation on past urbanisation. [19]

The fraction of individuals that have at least *i* ideas is then given by those who imitated at least *i* ideas plus those who imitated (i-1) ideas and then came up with an idea. That is,

$$P_{t+1}(i) = M_{t+1}(i) + m_{t+1}(i-1)q_{t+1}, \quad (13)$$

where $m_{t+1}(i-1)$ is the fraction of the population that has imitated exactly *i*-1 ideas at *t*+1, and

$$m_{t+1}(i-1) = M_{t+1}(i-1) - M_{t+1}(i). \quad (14)$$

Our framework has two implications for the innovation process. First, since agents can have at most one new idea during their youth, at any point in time *t*, there is an upper bound to the number of ideas held by an agent, and thus to knowledge – namely, *t* ideas. Second, although the probability of an individual innovating is independent of the distribution of knowledge in the population, the aggregate probability of a novel idea being invented is not. To understand this, note that – at the individual level – innovation may simply reinvent an existing idea, which will not add to the stock of knowledge; it is only those individuals who have imitated the existing *t*-1 ideas that may generate a novel idea, idea *t*. Consequently, the probability that a novel idea is invented in an economy with *t*-1 ideas is given by

$$P_t(t) = M_t(t-1)q_t, \quad (15)$$

i.e. it is the product of the share of individuals that have imitated all existing knowledge and the probability that an individual innovates. Since the former depends on the distribution of ideas at *t*-1, so does the probability of an innovation occurring. $P_t(t)$ may be zero, in which case idea *t* will not be invented, either because the urban population is zero (n_{mt}=0) or because the fraction of the previous generation holding idea *t*-1 was so low that nobody has imitated it (i.e. $P_{t-1}(t-1) = 0$ and $M_t(t-1)$=0).[20]

[19] This alternative specification is discussed in Appendix B.
[20] Alternative formulations are possible that would not alter the basic mechanism. For example, there may be threshold effects so that an idea is added to the stock of knowledge only if $P_t(t)$ is sufficiently large (see Appendix B).

In this context, there are two ways in which average productivity will grow: individuals copy the ideas of others that they meet – the diffusion of knowledge – and individuals observe the experiences of others and create new knowledge. The importance of these two aspects for the growth process is examined by Jovanovic and Rob (1989). In their setup, agents possess ideas of different 'quality'. When two agents meet, there is both imitation by the less informed agent and invention of novel knowledge by both agents. We simplify their setup by separating the two processes.

We can now consider the distribution of the number of ideas. Recalling that $\pi_t(i)$ is the probability that an individual has exactly *i* ideas at *t*, the average number of ideas at time *t* is simply $I_t = \pi_t(1) + 2\pi_t(2) + \ldots + (t-1)\cdot\pi_t(t-1) + t\cdot\pi_t(t)$. Moreover, since the fraction of the population with exactly *i* ideas can be written as $\pi_t(i) = P_t(i) - P_t(i+1)$, and noting that $\pi_t(t) = P_t(t)$, we can write

$$I_t = P_t(1) + P_t(2) + \ldots + P_t(t-1) + P_t(t) = \sum_{i=1}^{t} P_t(i) \qquad (16)$$

and it follows that $I_{t+1} = \sum_{i=1}^{t+1} P_{t+1}(i)$.

Using (13) and (14), the dynamic equation for the average number of ideas in the population can be rewritten as $I_{t+1} = q_{t+1} + \sum_{i=1}^{t} M_{t+1}(i)$ and, using the expressions above for q_{t+1} and $M_{t+1}(i)$, we have

$$I_{t+1} = (1+t) - (1-\delta)^{D(n_{mt})} - \sum_{i=1}^{t}\left(1 - zP_t(i)\right)^{D(n_{mt})} \qquad (17)$$

The first term captures the fact that a new idea is potentially invented each period. This aspect – the 'creation of new ideas' – is crucial because it will drive long-run growth. The second term captures the extent to which innovation is prevalent in the population. The term $(1-\delta)^{D(n_{mt})}$ is the fraction of the population that did *not* innovate, and depends negatively on urbanisation. The last term captures imitation and indicates that the share of individuals who did *not* imitate depends negatively on n_{mt}. Crucially, equation (17) implies that the current average number of ideas is increasing in the number of ideas held by the population last period and in the number of individuals living in cities.

4.2 The dynamics of urbanisation

Equations (10) and (17) – together with the labour market clearing condition, $n_{mt} = 1 - n_{at}$, and the expressions for average manufacturing productivity, $B_t = 1 + I_t$ – determine the dynamics of skills and urbanisation. The full dynamics of the model are then given by the following system:

$$I_t = \sum_{i=1}^{t} P_t(i) \tag{E.1}$$

$$v_t = v(1 + I_t) \tag{E.2}$$

$$\frac{v_t}{(1+I_t)^{1-\alpha}} = \left(\frac{1-\alpha}{\alpha} A_t^{\gamma} \left(\frac{T}{N_t}\right)^{1-\gamma} \frac{(1-n_{mt})^{\gamma}}{n_{mt}}\right)^{\alpha} \left(1 - (1-\tau)\frac{\alpha\gamma}{1-\alpha}\frac{n_{mt}}{1-n_{mt}}\right) \tag{E.3}$$

$$P_{t+1}(i) = 1 - \left(1 - zP_t(i)\right)^{D(n_{mt})} + \left(1 - (1-\delta)^{D(n_{mt})}\right)\left(\left(1 - zP_t(i)\right)^{D(n_{mt})} - \left(1 - zP_t(i-1)\right)^{D(n_{mt})}\right) \qquad \forall i = 1, \ldots, t+1 \tag{E.4}$$

$$y_t = \frac{1}{1+\varepsilon}\left(\frac{A_t^{\gamma}(T/N_t)^{1-\gamma}}{\alpha(1-n_{mt})^{\gamma}}\right)^{\alpha}\left(\frac{(1+I_t)n_{mt}}{1-\alpha}\right)^{1-\alpha} \tag{E.5}$$

$$N_{t+1} = N_t(1 + b(y_t)) \tag{E.6}$$

$$A_{t+1} = (1+g)A_t \tag{E.7}$$

The initial conditions of the economy are an initial population and level of agricultural productivity, N_t and A_t, and a distribution of ideas, $P_t(i)$. Average knowledge is given by (E.1); it depends on the distribution of ideas at *t* and in turn determines the cost of urban dwelling, as given by (E.2). Equation (E.3) has been obtained by combining equations (10a) and (10b), and it implicitly defines manufacturing employment at time t, n_{mt}, as a function of the average number of ideas I_t (and of population and agricultural productivity). Manufacturing employment, or equivalently, the rate of urbanisation, together with the current distribution of ideas determine, by the set of equations (E.4), next period's distribution of ideas. This distribution will in turn determine next period's average productivity and the level of urbanisation, n_{mt+1}, and so on. The last three equations – (E.5) to (E.7) – give current output, as well as next period's population and agricultural productivity.

To get an intuition for the dynamics of the model, let us consider a simplified version. Suppose that population, agricultural productivity, and the disutility of urban dwelling are simply constant parameters, i.e. $N_t = N, A_t = A, v_t = v$, so that the dynamics of the model are given solely by equations (E.1), (E.3) and (E.4). Note that (E.3) defines the implicit function $n_{mt} = F(I_t; \tau, A, N, v)$, which is increasing in I_t, i.e. $F' > 0$. From equation (17) we can see that equation (E.1) implies that I_{t+1} is increasing in n_{mt} since $(1-\delta) < 1$ and $\left(1 - zP_t(i)\right) < 1$. It follows that $dI_{t+1}/dI_t > 0$, implying that ideas grow over time; by equation (E.3), so does urbanisation.

Knowledge, defined as the number of ideas available in the economy, will grow without bound, inducing a flow of workers into the city. Equation (E.3) implies that, as $I_t \to \infty$, manufacturing employment tends to an upper bound given by

$$\bar{n}_m = \frac{1-\alpha}{1-\alpha+\alpha\gamma(1-\tau)} < 1. \qquad (18)$$

Both knowledge and productivity keep growing even if rural-urban migration stops. Knowledge grows because, as long as $n_{mt} > 0$, a novel idea is invented in each period with positive probability. The arrival of a novel idea and its diffusion over the urban population imply that productivity keeps rising over time. Recall from equation (11) that there are three effects of increased manufacturing productivity on aggregate output: a direct productivity effect, the reduction in the relative price of manufactures, and the flow of labour away from agriculture. In the long run, the latter effect will be absent as urbanisation converges to $\bar{n}_m$, and output growth will be driven exclusively by the direct and the price effects of productivity growth.

At this point, it is important to discuss two aspects. First, the key mechanism of the model is independent of our assumptions on population growth, agricultural productivity and change in the disutility, which are introduced to match the data. As discussed, the simpler model obtained when these are all constant generates the prediction that the feedback between urbanisation and innovation can generate a growth takeoff. Through numerical analysis of its dynamics,[21] it is possible to show that – even in the absence of these three elements – the model also delivers an acceleration of urbanisation and growth. It cannot, however, match the historical data simply because agricultural productivity growth and population growth were substantial over our period of interest.

Second, let us consider what our setup implies compared to a 'black-box' knowledge generation function. A key result is that the rate of growth of productivity is lower than the rate of growth of knowledge because not all workers acquire the latest idea. In the early stages of development, when urbanisation is low, imitation is moderate and the gap between knowledge and productivity is large. As urbanisation increases, the gap between the two narrows due to a greater diffusion of knowledge. Secondly, depending on parameter values, we may observe an *acceleration* of productivity and urbanisation. If the probability of innovation is not too high, then most individuals increase their productivity through imitation. The nature of the imitation process implies that each generation has a higher probability of imitation than the previous one because there are more individuals holding i ideas than a period earlier, thus resulting in an acceleration of knowledge

[21] See Appendix C.

diffusion, and consequently of output and urbanisation. Neither of these two features could be obtained with a simple 'black-box' specification for knowledge creation.

5. Revisiting the facts

The model in (E.1)-(E.3) has two key implications. First, it entails a feedback effect between innovation and urbanisation. It is possible to think of an initial situation in which this process is slow, resulting in negligible changes in urbanisation and knowledge, and in which anything that positively perturbs either the technology or the urbanisation rate sets a virtuous circle into motion that leads to the accumulation of knowledge and increasing living standards. Second, the model generates a wedge between manufacturing productivity and knowledge, which implies that – although the level of technology is growing rapidly – output is not because not enough individuals have learnt the new ideas. The next section presents numerical calibrations to examine in detail the implications of the model. But, before doing so, we consider how these two key results can help us interpret important features of the Industrial Revolution.

Ideally we would map the dynamic process of urbanization and subsequent economic growth across European countries. The current state of the data does not permit this, but we can offer some evidence from England. Bairoch's city data enable us to calculate county-level urban populations in 1750 and 1800; we take county-level population data from the census (1801, 1851) and Wrigley (2009, for 1751). Since we do not have county-level output data – which would be ideal – we follow other authors (such as Dittmar and Meisenzahl, 2020) and use population growth as a metric for economic growth (see Appendix A for details on the data).[22] In table 3, we regress county-level population growth between 1801 and 1851 on the urbanization rate in 1801. We also include county-level growth between 1751 and 1801 in order to control for any unobserved factors that may have made the county grow both in both periods. Between 1801 and 1851, we see faster growth in counties that were already more urbanized in 1801 (as well as those that were already growing faster in 1801). Even if not conclusive, the evidence is suggestive of the presence of the mechanism that we highlight, whereby urbanisation determines subsequent growth.

[22] Of course, workers in the secondary sector earned significantly higher wages than those in the primary sector during industrialization. Since growth in population and secondary sector employment were strongly correlated (census data implies a correlation coefficient of 0.66), population growth is a downward-biased metric of income growth. In that sense, the coefficient obtained in our regressions underestimates the impact of urbanization on economic growth.

Table 3. Explaining county-level population growth in England, 1801-51.

Dependent variable: Population growth	
Constant	40.19†
	(9.49)
Population growth, 1751-1801	0.67†
	(0.17)
Urbanisation rate, 1801	0.85†
	(0.29)
R-squared	0.53
N	42

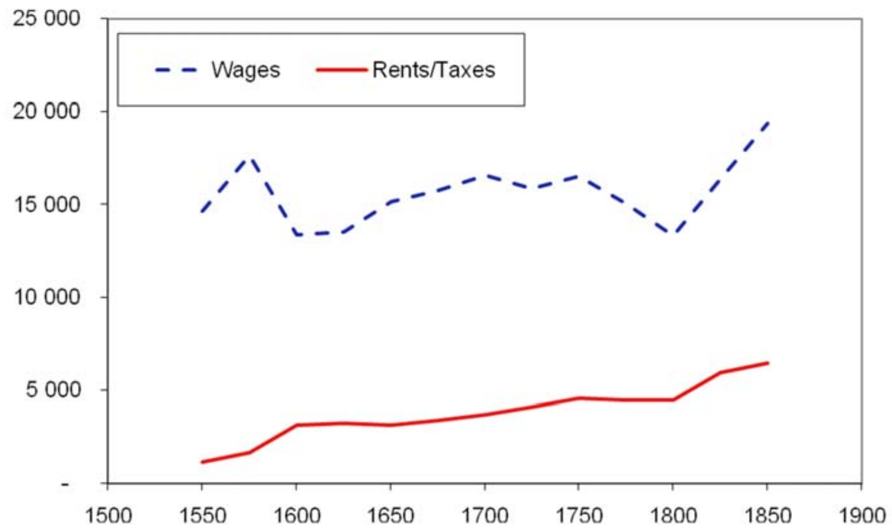
†Denotes statistically significantly different from zero at the 1% confidence level. Standard errors in parenthesis.

Evidence from US development is also consistent with our urbanisation-industrialization-productivity growth sequencing. It makes little sense to consider the US economy as a single entity in the 19th century because some regions were heavily urbanised and industrialized before others had even experienced significant European settlement. For example, parts of the Northeast (Massachusetts, Rhode Island) had urbanisation rates above 50% by 1850 (US Census Bureau, 2012), while they were low in California before the Gold Rush of 1849. Regional patterns of urbanisation seem to have strongly impacted industrial development. For example, the first significant industry in the US was cotton manufacture and it is interesting that all the factories were founded in Massachusetts and Rhode Island – even though raw cotton was produced 1 000 miles further south. There are several reasons for this, but it has been argued that the business and technological know-how available in urban commercial centres (particularly Providence and Boston) were key elements (Nicholas and Guilford, 2014).

As we have argued above, a number of different factors can result in city growth, going from geography to plagues. Notably, Nunn and Qian (2011) argue that increases in agricultural productivity related to new crops were fundamental because they generated the potential to free labour from food production. Such a hypothesis fits well the case of England. A large literature has documented the fact that the agricultural revolution preceded the Industrial Revolution, yet the fact that the timing of the former roughly matches the increase in urbanisation in England is rarely discussed. Our model provides a possible explanation for the coevolution of these two variables.

Agricultural labour productivity may have increased because of exogenous technological shocks related to improved know-how or, as suggested by Nunn and Qian, to new crops. Our model proposes an additional source of increased productivity: agricultural extractions. The role of agricultural extractions as an additional shock during early modern times has received little attention in the literature, yet it seems to us to have been particularly relevant in the case of England. High extractions from agriculture were the result of two phenomena. On the one hand, the established land property system in Northern Europe implied that a substantial amount of production went to the landowner, who was generally not the one who worked the land. On the other hand, agriculture was heavily taxed. In 17th and 18th century Europe, tax revenues were rising fast – particularly in England[23] – and the main source of this tax revenue was the agricultural sector.[24]

Figure 4. Wages and rents plus taxes in agriculture in England (1775 English d).



Source: Authors' calculations from Clark (2002b) and Brunt (2000); see text.

Figure 4 presents the evolution of indices for real wages, taxes and rents in the sector.[25] English agricultural extractions had four components: land rents paid to landlords, the tithe paid to the church, local taxes (predominately the tax to support the poor), and the land tax paid by the occupier of the land to the central Government; see Appendix A for further discussion. The measure of extractions includes these four payments. Assuming that wages, taxes and rents exhausted the entire agricultural

[23] See Karaman and Pamuk (2010), Dincecco (2011) and Voigtländer and Voth (2013b). Figure 5 in Karaman and Pamuk (2010) shows that, across Europe, per capita tax revenues were highest in England and the Dutch Republic in the 18th century; the was Netherlands also a highly urbanised country.
[24] For an exhaustive account of the English case, see Dowell (1884).
[25] Clark (2002b; table 1) provides an index of real wages and the real value of taxes and rents. In order to get the level of tax rates, we used data from Brunt (2000) on the value of output per worker and rents and taxes for 1775. Output was 975d/acre, which – with 20 acres per worker – yields an output per worker of 19 500d. This generates an extraction rate from gross output of 23% (=4 487/19 500) in 1775. We assume that total output is equal to wages plus extractions.

output, we can compute extractions as a share of output. In 1550, taxes and rents paid by farmers represented only 7% of total output; by 1600 their share had increased to 19%. They fluctuated between 17 and 19% during the 17th century and then increased steadily over the next hundred years, reaching 28% by 1825. Data from Brunt and Fidalgo (2010) indicate that in 1775 land rent constituted 75% of the burden, the tithe 10%, the poor rate 6% and the land tax 8%. That is, agricultural workers paid a rent of about 17% and a total tax bill of about 6%, implying a total extraction rate of 23% in 1775.

England was not the only country to experience a migration "push" from agriculture – and consequent urbanisation – prior to the onset of industrialization and sustained productivity growth. Oliver Grant (2005, especially chapter 4) notes that the exodus of agricultural workers from eastern Prussia occurred before the advent of German industrialization. Until 1850, most agricultural migrants headed to the USA. After 1850, they began to gather in German cities (where real wages were low and living conditions were famously awful). Only after 1870 did the migrants come to be absorbed by the growing heavy industries (coal, iron, etc.) and real wages begin to rise. Productivity growth then followed (Allen, 1979).

Lastly, recall that the knowledge generation process we postulate implies that the rate of growth of productivity is lower than the rate of growth of knowledge, and that an acceleration of productivity will occur over time. These implications are important because recent analyses of the Industrial Revolution have emphasized the delayed increase in productivity growth. Crafts (2004) summarizes three independent estimates of TFP growth over the period 1770 to 1870 (his own, as well as those in Feinstein, 1981, and Antràs and Voth, 2003). They all give very low estimates for initial productivity growth – around 0-0.2% per annum up to 1800, increasing to around 0.6-1% per annum for the period 1800-1830, and then settling at 0.5-0.8% per annum up to 1870. This is surprising, given the significant acceleration of patenting activity after 1757 which, in the absence of any institutional explanation, can be interpreted as evidence of a widespread increase in inventive activity; see Sullivan (1989). Separating the creation of ideas from their diffusion in a microfounded model implies that those individuals who have not managed to imitate others will not be able to use the most recent technology: this generates a model that simultaneously delivers fast technological change and slow overall productivity growth in manufacturing, in line with the quantitative estimates for the 18th century.

To conclude, contrary to the traditional approach, our model is consistent with the fact that high urbanisation preceded the Industrial Revolution, and can reconcile the fact that the increase in urbanisation occurred roughly at the same time as English agricultural productivity started to grow. One key aspect in the data is the observation that the spur of technology in the late-18th century was not accompanied by an immediate increase in manufacturing labour productivity. To illustrate this point in detail we need to resort to numerical calibrations.

6. A calibrated model of English development

6.1 Functional forms

In order to take our argument further, we provide a calibrated numerical example of the dynamics of the model. To do so, a number of functional forms need to be specified. First, we need to pin down the functional form of the relationship between urbanisation rates and the number of meetings. Since there is no evidence to guide this choice, we simply suppose that the number of meetings is a linear function of the urbanisation rate, that is,

$$D(n_{mt}) = \rho n_{mt}. \quad (19)$$

Second, the cost of living in a city, v_t, is assumed to decrease with the level of technological know-how in manufacturing according to the function $v_t = dB_t^{-\theta}$, where d and θ are positive constants. This expression implies that as technology goes to infinity, the cost of living in the city goes to zero. Concerning demographics, we need to distinguish between the entire population, which we denote Pop_t, and the labour force consisting of agricultural and manufacturing adult workers, N_t. The latter exclude, as well as the young and the elite, women and those working in services, both of which are not in our model.[26] We suppose that the ratio between the male, non-service labour force and the population is exogenous and constant over time. As a result, population will grow at the same rate as N_t, i.e. $Pop_{t+1} = Pop_t(1 + b_t)$. We assume that net population growth takes the form $b_t = by_t^{\varphi}$, which implies a linear relationship between the log of population and the log of output.

A last important assumption concerns agricultural extractions. The model assumes that agricultural workers are paid their marginal product and the literature often argues that a realistic value for the elasticity of agricultural output with respect to labour γ, and hence for the labour share, is 0.6. Yet the data for England does not seem to support either the assumption of a constant labour share in agriculture, nor a 60% labour share (as revealed in figure 4 above). In order to be consistent with the empirical evidence, we modify the model and define the net wage in agriculture as $(1\text{-}\tau_t)w_{at} = (1 - \omega_t)A_t^{\gamma}(T/N_t)^{1-\gamma}n_{at}^{\gamma-1}$, where ω_t is the overall extraction rate that includes both taxes and rents. This

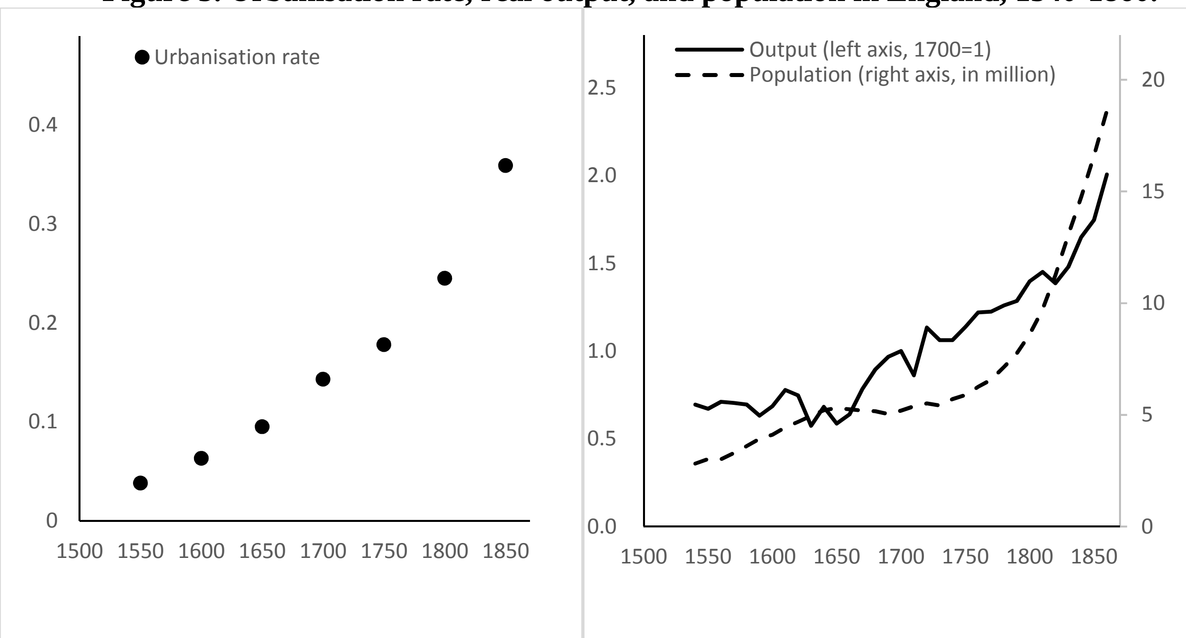
[26] Women, of course, worked but this allows us to be consistent with our measure of land per capita which is constructed using the male agricultural labour force.

extraction rate can be lower than the elasticity of agricultural output with respect to labour and can be allowed to vary considerably over time.[27]

6.2 Parameter values

The parameter values have been chosen to match the English case. We suppose that each period lasts 20 years, starting our numerical exercise in 1540 and ending in 1860, on the eve of the Second Industrial Revolution. Our calibrations target two time series: the urbanisation rate and per capita output. For the former we follow the definition used in figure 3, where urbanisation is measured by the fraction of the population living in cities of at least 10 000 people; data on output per capita are from Broadberry *et al.* (2015). The left hand panel of figure 5 shows the data on urbanisation, while the right hand one depicts that for output as well as the time series for population (also from Broadberry *et al.*, 2015). The latter will not be targeted by the model, but will be used to set certain parameters and initial conditions. The three series in figure 5 show the acceleration that occurred in urbanisation, output and population over the period.

Figure 5. Urbanisation rate, real output, and population in England, 1540-1860.



Sources: Bairoch *et al.* (1988), De Vries (1984) and Brunt and Fidalgo (2008) for urbanisation (see Appendix A), Broadberry *et al.* (2015) for output and population.

We proceed in three steps. We start by using external evidence on a number of parameters. This allows

[27] Appendix C shows that the model with a standard assumption about agricultural wages delivers the same qualitative predictions.

us to pin down preferences, the parameters of the agricultural sector, and the relationship between output and population growth. Second, we choose initial conditions in order to match the initial population and urbanization rate. There are two sets of parameters for which it is particularly difficult to find empirical counterparts: those governing the disutility of urban life and those related to knowledge generation. Hence our last step consists of choosing these so that the model tracks the three time series as closely as possible.

External parameters

External information allows us to set seven parameters, as follows.

Preferences and demographics. We suppose that the share of agricultural goods in expenditure is 0.75. This number is consistent with evidence showing that the share of food in total household expenditure in late 18th century England was 71% and 76% for tradesmen and farmers, respectively (Horrell, 1996). Estimates for developing countries in the late-20th century are similar. For example, Subramanian and Deaton (1996) find a food share in total expenditure of 67% on average, and 73% at the bottom of the consumption distribution, in India. Concerning demographics, we set the fraction of landowners (i.e. elites) to 3%, as estimated by Contamine (2006).

The agricultural sector. Three parameters define agricultural production: total factor productivity, the marginal productivity of labour, and the amount of land. The marginal productivity of labour is assumed to be $\gamma = 0.6$, as is common in much of the literature (for example, Voigtländer and Voth, 2013a). Agricultural productivity is assumed to grow by 72% over the period 1540 to 1860 – i.e. an annual growth rate of 0.17%, which is the average of the estimates provided by Crafts (2004) and Clark (2002b). This implies a growth rate of 1.03% over each 20-year period. The amount of land available is set at T=25 (million acres), chosen to fit evidence from Brunt (2000), who finds an average of 20 acres per male agricultural worker in the late 18th century.[28]

We use the data reported in figure 4 to compute ω_t, the overall extraction rate that includes both taxes and rents in agriculture. The data are available every 25 years, while the model uses 20-year periods. We therefore use a spline to approximate annual data and use the relevant average rates for each 20-year period. In our simulations we set ω_t according to the resulting values, reported in table A.7 in Appendix A. The extraction rate was only 7% in 1540 but started increasing in the late 16th century, reaching 19% by the early 17th century; after a period of stability, it rose again from the mid-18th century and was 27% by the mid-19th century.

[28] In our calibrations, the labour force in 1780 will be 1.57 million and the agricultural employment share for males 79%.

Estimating population growth. Population growth is assumed to be an increasing function of per capita income, with the net population growth rate being given by $b_t = by_t^{\varphi}$. We use the time series for output and population to estimate the two parameters, which yields b=0.08 and $\varphi = 0.3$.

Initial conditions

The model needs three initial conditions: agricultural productivity, the size of the population and the distribution of ideas. For simplicity, we suppose that the initial number of ideas is zero, so all agents hold no ideas with probability one, that is and $I_0 = 0$ and $B_0 = 1$, while initial agricultural productivity is normalized at $A_0 = 1$. Following Broadberry *et al.* (2015), we take an initial population of 2.85 million. The labour force is assumed to be 22% of the total population, once we exclude women, children, services and the elite,[29] implying an initial labour force of 630 000, so we set N_0=0.63. Note that the initial urbanisation rate is not an initial condition but a model outcome. In the first period (with $B_0 = 1$), the disutility of urban life is simply $v_t = d$ and hence d is a key determinant of the initial rate of urbanization (see equation (E.3)). We set $v_0 = 15$ so that the initial static equilibrium yields a rate of urbanisation consistent with the data (3.6% in 1540), which gives us the value of d.[30]

Calibrated parameters

Four parameters remain to be chosen: θ, ρ, z, and δ, where the first is a parameter in the disutility of urban dwelling and the other three capture various aspects of imitation and innovation. It is difficult to find measurable empirical counterparts to these parameters. Hence our last step consists of choosing them so as to fit the simulated series to the actual time series, seeking to match both the final levels of urbanisation and output and the curvature of the series. This leads us to set $\theta = 1.5$, $z = 0.56$, $\rho = 10$, and $\delta = 0.36$. Interestingly, the value of z, – the probability of imitating – fits with existing evidence on the proportion of apprentices who eventually became masters in the 16th and 17th century.[31]

It is important to note that the dynamics of urbanisation are highly sensitive to the knowledge-

[29] This may seem to be a small labour force, but as well as a 50% share for women, there was also 35% for children (both numbers taken from Wrigley and Schofield, 1981), 30% for services and 3% for the elite.

[30] The calibration of the basic model in Appendix C shows how initial urbanization depends on the disutility cost.

[31] We can think of apprentices as young individuals that have met with an older individual, the master, and have the potential to imitate the ideas he holds. Whether an apprentice becomes a master can then be seen as the probability that the young individual has imitated. Although data on the share of apprentices that become masters is not available, Wallis (2008) approximates it by examining those who became a Freeman, since becoming a master was a necessary condition to become a Freeman (although not all masters did so). Wallis provides evidence on the share of apprentices who became Freemen in their city in the 16th and 17th centuries, which was 41% in London and a similar magnitude in other cities. Of the remaining apprentices, one quarter eventually became a Freeman elsewhere, yielding a lower bound for the probability of becoming a master of 56%.

generation parameters, as we discuss in Appendix B. Our choice of parameters allows us to fit the observed acceleration in urbanisation. The share of population living in cities grew by 6 percentage points between 1550 and 1650, by 8 points in the following century, and by 18 points between 1750 and 1850. The above parameter values yield increases in urbanisation that are close to these figures.[32] In contrast, when we explore alternative specifications – keeping initial and final urbanization constant – we find different paths of city growth. For example, if we set a high δ and a low z, manufacturing productivity and urbanisation grow rapidly early on because many individuals innovate; but then they slow down because there is little diffusion of knowledge. As a result, urbanization grows by 14 percentage points over the first hundred years and by 10 and 9 points in the two subsequent centuries. Our choice of a low innovation rate relative to the imitation rate provides the slow take off and subsequent acceleration observed in the data. It also seems plausible, compared to the reverse (i.e. a low imitation rate but a high innovation rate).

Table 4. Parameter values.

Parameter	**Definition**	**Value**
External parameters		
α	Share of manufacturing in consumption	0.75
γ	Coefficient on labour in agriculture	0.60
g	Productivity growth in agriculture	0.0103
T	Land endowment	0.25
ε	Share of landlords in population	0.03
ω_t	Extraction rate from agriculture	0.07 to 0.27
b	Coefficient of population growth function	0.08
φ	Elasticity of population growth with respect to income	0.30
Initial conditions – Imposed		
A_0	Productivity in agriculture	1.00
I_0	Initial number of ideas	0.00
Initial conditions – Calibrated		
Pop_0	Initial population	2.85
N_0	Initial labour force	0.63
d	Coefficient of cost of urban dwelling function (which implies an initial urbanisation rate of 3.6%)	15.0
Calibrated parameters		
ρ	Number of meetings per percentage of urbanization	10.0
z	Probability of imitation	0.56
δ	Probability of innovation	0.36
θ	Elasticity of cost of urban dwelling w.r.t. productivity	1.50

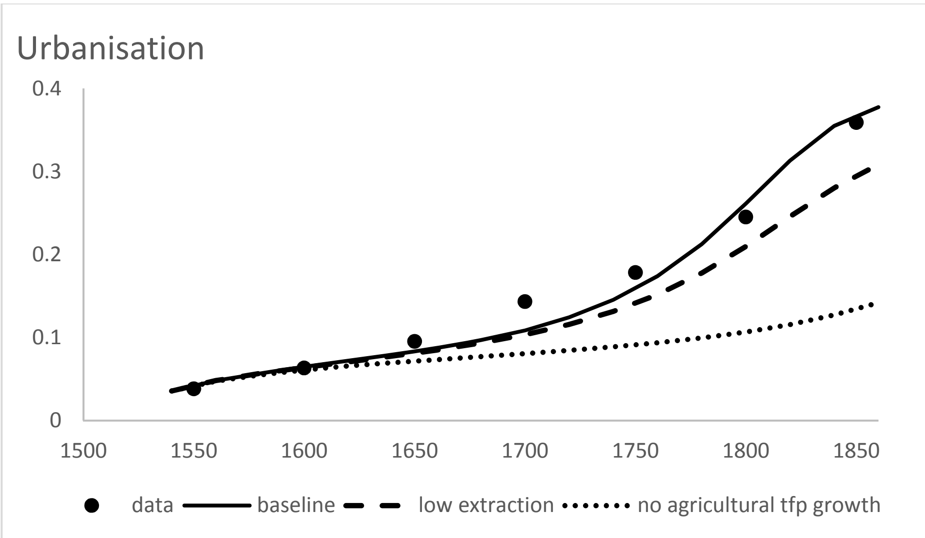
[32] The model yields increases of 4, 7 and 21 percentage points. It is possible to fit the urbanization series more closely but at the cost of getting a worse fit for output and population.

Table 4 summarizes the values given to parameters that we have just discussed, and which are used in the calibration of the English economy. Details on the extraction rates used are reported in table A.7 in the Appendix.

6.3 The evolution of key magnitudes

Figure 6 reports the evolution of urbanisation in the model, as well as that observed in the actual data.[33] Our core simulation, represented by the continuous line, is the result of two shocks. On the one hand, there is an increase in agricultural productivity; on the other hand, we allow the extraction rate to vary over time (both derived from the historical data, as described above). The model is successful at reproducing the early increase in urbanisation, as well as the acceleration over the last 100 years of our historical window, although it delivers somewhat lower urbanisation rates than those observed in the data between 1650 and 1750. Increasing agricultural extractions were an important driver of fast urbanisation. Note that, although the bulk of the tax increase occurs around 1600, urbanisation keeps rising thereafter even though tax rates change little. This is because urbanisation has reached a critical level that implies self-sustaining technological change. Once urbanisation is high, the high *level* of extraction implies that moderate changes in manufacturing productivity induce substantial flows of labour from rural to urban areas.

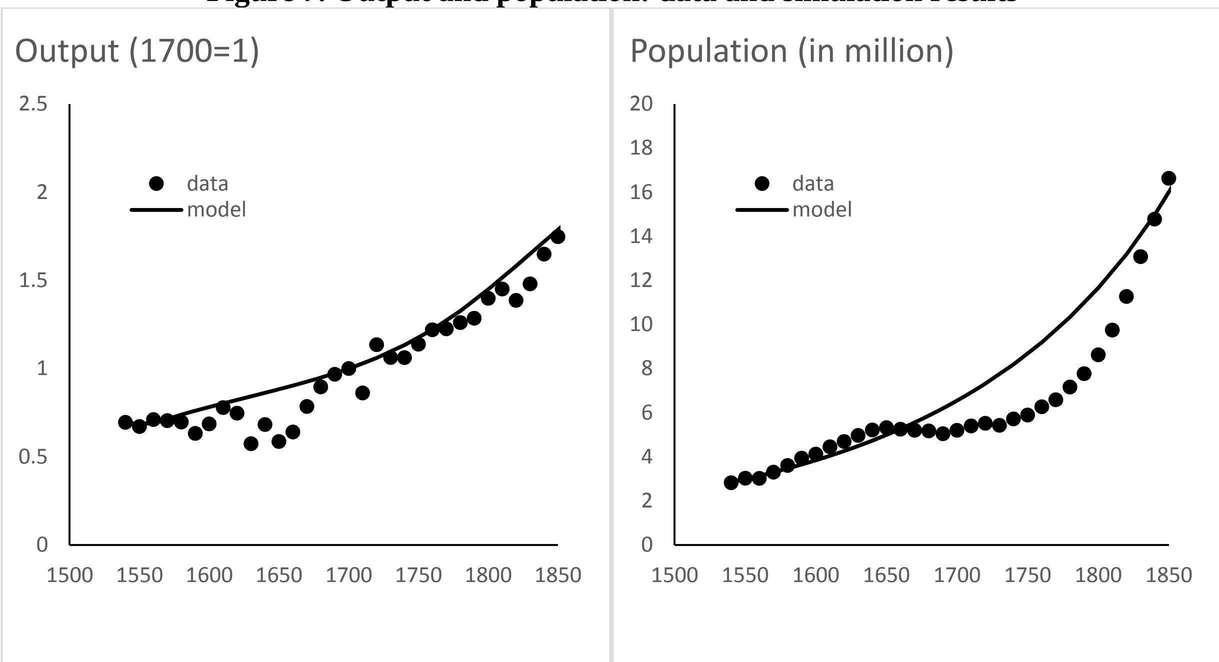
Figure 6. Urbanisation: the impact of different extraction rates.



[33] Urbanisation is defined as the fraction of the population living in cities of at least 10 000 people (from De Vries, 1984, and Bairoch *et al.*, 1988), as in figure 3. The data are available every 50 years, starting in 1550, when the rate of urbanisation was 3.8%.

Figure 7 depicts the dynamics of output and population. The left hand panel presents the simulated time series for real per capita output, as well as data on British real per capita output provided by Broadberry *et al.* (2015). It is interesting to see that, although urbanisation started increasing rapidly in the early 17th century, output growth during that period was slow and started to accelerate only in the mid-17th century. The acceleration generated in the simulation occurs somewhat earlier and more gradually than that actually observed in the data during the first half of the 17th century; the smooth process for the transmission of ideas does not allow us to generate the sharper change observed in the actual GDP data. The right hand panel of figure 7 depicts the evolution of the population. The model again follows the overall trend found in the data but misses some changes arising from a variety of idiosyncratic events – such as the population decline in the late 17th century caused by a worsening of the disease environment (notably, the spread of influenza) resulting from a succession of unusually severe winters (Wrigley and Schofield, 1981).

Figure 7. Output and population: data and simulation results



6.4 The drivers of urbanisation

In order to understand what drives the behaviour of urbanisation, figure 6 above reports two further exercises. The dashed line depicts the case in which extraction rates are kept constant at their initial level (7%). With a low and constant extraction rate, the increase in the share of individuals living in cities is considerably slower than in the data, reaching only 30.8% by 1860, rather than the 37.8% in our core model. The dotted line depicts an economy with the observed extraction rates but no (exogenous) growth of agricultural productivity. Urbanisation increases much more slowly over the period, with the population living in cities being only 14.2% by the end of the period. Agricultural productivity growth plays an important role through its contribution to the fall in the relative price of agricultural goods – by 1860, agricultural goods are 10 times dearer in the case without agricultural productivity growth – and hence it increases the incentives to move into cities. These examples highlight two potential, and complementary, drivers of the upsurge in urbanisation.

Our numerical analysis shows that the model generates a feedback effect between innovation and urbanisation and that it reproduces the slow increase and subsequent acceleration in the share of people living in cities that we observe in the data. A number of additional numerical exercises indicate that, while the latter pattern is highly sensitive to functional forms and parameter values, the key mechanism in operation – the feedback effect between innovation and urbanisation – is not. Indeed, numerical simulations show the importance of imitation for the sharp increase in the urban population that occurred towards the end of the period, as we report in Appendix B. In the absence of imitation (i.e. when $z = 0$), the fraction of the urban population that holds ideas is small, implying that average productivity in manufacturing is not growing fast enough to create a large flow of labour into industry. The predicted urbanisation rate is then only 8.1% by the mid-19th century and the model is incapable of reproducing the sharp increase observed in the urban population.

We also consider alternative values of ρ, δ and z, where the combination of parameters is chosen so as to always have the same initial and final urbanisation rate as in the baseline. The dynamics are, however, highly dependent on the choice of values. For example, when we choose a high innovation rate and a low imitation rate ($\delta = 0.70$ and z = 0.32), manufacturing productivity and urbanisation grow rapidly early on because many individuals innovate, but then slows down because the lack of imitation implies that past knowledge is not being passed on. In contrast, our baseline parametrization implies that few individuals innovate but many imitate; productivity growth is hence driven by the diffusion of ideas and follows an s-shaped path consistent with the data.

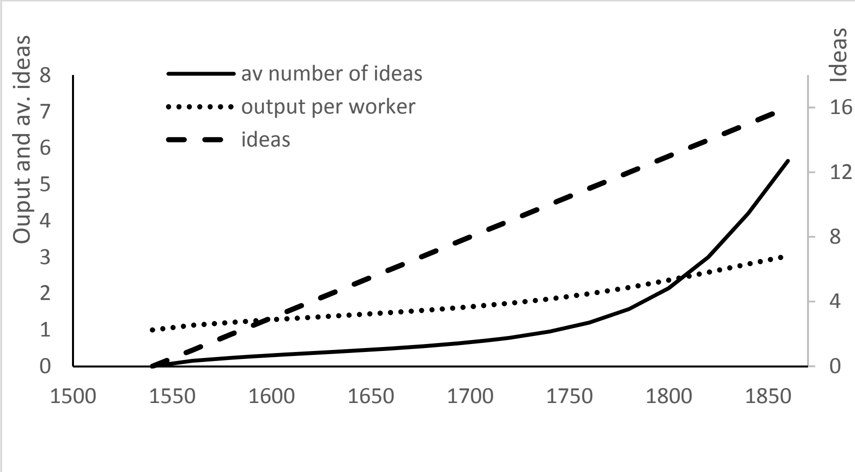
To show the robustness of the central mechanism, we also examined alternative specifications for knowledge generation. We have assumed that the probability of an individual coming up with a new idea does not depend on the amount or distribution of knowledge. In Appendix B we examine two alternatives. On the one hand, we suppose that the greater the current average number of ideas in the population, the higher the probability of innovation is. On the other, we make the stock of ideas more sensitive to their distribution by supposing that innovation results in a novel idea only if the share of innovators is sufficiently large. In the first case, we find an (excessively) early acceleration of innovation and thus of city growth, while in the second knowledge growth is sporadic and much slower than in our baseline. Yet, the mechanism driving sustained innovation and urbanisation remains.

6.5 Productivity and wages

We next examine the dynamics of two variables that we have not targeted when calibrating the model – productivity and wages.

To understand the acceleration of growth reported in figure 7, figure 8 presents the corresponding time series for the number of available ideas or knowledge (dashed line); the average number of ideas in the urban population (solid line), which is equivalent to average productivity in manufacturing; and per capita output (dotted line). The parametrization of the model yields an innovation every period, implying that knowledge (i.e. number of available ideas) increases by one idea each period. The first thing to note is the gap between knowledge (i.e. number of available ideas) and manufacturing productivity (i.e. average number of ideas held by the population), capturing the fact that available knowledge and the diffusion of innovations do not move together. Second, although the growth of knowledge is constant (one idea per period) the model delivers a productivity acceleration. This is because the low initial urban density implies a slow diffusion of innovations amongst city dwellers. As urbanisation increases, the rate of diffusion of knowledge rises, resulting in an acceleration of manufacturing productivity growth from 1750 onwards. Output follows a similar pattern, eventually lagging behind as agricultural productivity growth weighs down the rate of growth of average output per worker. Overall, output growth remains moderate despite the fact that available knowledge is increasing fast – thus making a fast pace of innovation compatible with moderate per capita output and productivity growth, in line with existing estimates for the 18th century discussed in section 5.

Figure 8. Simulated output, productivity and knowledge.



The last element that we examine is relative wages. Recall that our agents' locational choice is bound up with their occupational choice. Hence the relative wage – defined as w_{mt}/w_{at} – captures both the agricultural-manufacturing wage gap and the rural-urban wage gap. Data on these magnitudes is not often available, especially as a consistent time series. As a proxy we use those reported by Allen (2006), who provides data on labourers' daily money wages in London and Oxford. During our period of interest, Oxford was a small town (population 3 500 in 1550, and still less than 12 000 in 1801). This is not an ideal comparison because wages in a more rural location would have better captured the features of the model. Yet Oxford was a 'small town', and the comparison can shed light on the wage advantage prevalent in London.

Figure 9. Ratio of urban to rural wages: data and simulation results.

The data are presented in figure 9, where we have averaged wages for each decade to improve readability. London wages were between 25 and 43% higher than those in Oxford during the 16th century; the gap increased during the 17th century, becoming twice as large; and then it declined slowly to a differential of only 13% by the mid-19th century. The solid line in figure 9 depicts the relative wage series generated by the model. Relative wages fall smoothly from a ratio of 2.29 at the start of the period, to 1.12 by the mid of the 19th century. The model overestimates the wage gap, especially for the first hundred years in our sample, but successfully delivers a declining wage gap in line with that observed from the mid-17th century onwards.

The model also predicts a Kuznets curve, with wage inequality initially increasing and then decreasing. At time zero the only source of inequality is the cost of living in the city, which implies that the urban wage is above that in the countryside. Three elements drive the subsequent dynamics of inequality. First, as knowledge is generated there will be different productivity levels in manufacturing and hence urban wage inequality emerges; as the number of ideas increases, so does the potential for wage dispersion. This effect is counterbalanced by the falling price of manufacturing goods, which limits urban inequality by reducing the wage gap between two individuals with a different number of ideas. Lastly, as we have seen in figure 9, the average gap between the urban and rural wages falls, thus providing an equalising force. Under our parameter values, a Kuznets curve appears, with the variance of relative wages increasing from 0.05 in 1540 to a peak of 0.16 in 1740 and then falling again to 0.04 (see figure A.4 in appendix B).[34]

7. Conclusions

We have formulated a model of industrialization based on the key role played by cities in creating and diffusing knowledge. Cities are *special* because they generate frequent encounters between individuals, resulting in an exchange of knowledge that leads to both imitation and innovation. We develop a two-sector model in which manufacturing takes place in cities, alongside the creation and diffusion of knowledge. In this setup, industrialization can be kick-started by any shock that increases urbanisation, such as an increase in agricultural labour productivity (which

[34] It is difficult to compare these figures with existing measures of inequality as time series on the distribution of wages for the period we consider are not available. Williamson (1980) provides a series for earnings inequality in the UK for the period 1827-1901 and finds that it first increased and then declined, peaking in the middle of the century. However, the data are not directly comparable to our model. Out of the three categories that would capture our setup – agricultural workers, general non-agricultural workers, and manual workers in commodity production – Williamson has data for the latter two only after 1881 (Williamson, 1980, table 1). His inference of increasing inequality in the early 19th century is therefore based on a comparison between agricultural labour and highly-educated workers (more specifically, professionals, clerks, government employees, and clergy), which are not the populations on which we focus.

increases the food surplus available for city dwellers). Interactions amongst city dwellers result in some of them coming up with new ideas, which then diffuse through the urban population, and do so faster when there are more meetings – i.e. when the urban population is larger. Higher urban density thereby increases productivity in manufacturing, which further induces workers to leave farming and move into the city, which further increases knowledge diffusion and productivity, and so on. As a result, growth takes off.

Growth is endogenous in our model but, in contrast to much of the literature, we suppose that innovation and imitation are not motivated by profit but rather occur as an externality resulting from high urbanisation. We view innovation as the result, not of market activities, but of social interactions, often called 'non-market interactions' to emphasize the fact that the actions of agents are not determined by the price mechanism in this context. This is consistent with evidence that early innovators were motivated more by public approbation than pecuniary benefits (Brunt *et. al*, 2012).

The central idea in our analysis – that cities are special – could have been modelled by simply assuming that productivity in manufacturing is an increasing function of the rate of urbanisation. However, unless we had made strong assumptions about the functional form of the knowledge-generating function, it is unlikely that we would have obtained an acceleration of productivity growth. In contrast, our microfounded model of interactions between agents – based on the idea that urbanisation determines the number of encounters and that the (individual) probability of imitation or innovation at each meeting is constant – implies such an acceleration. The intuition for this is that imitation depends both on the number of meetings and on the distribution of ideas in the population. This mechanism creates an acceleration of the transmission of knowledge that results in a slow initial growth rate that increases over time.

The key features of our model – high urbanisation and high agricultural labour productivity occurring *prior* to a take off in economic growth – fit well with the evidence for 18th century England. Hence we maintain that urbanisation was an important element in the unprecedented expansion of knowledge that started then and there, and thus gave rise to the First Industrial Revolution. We have calibrated the model to fit historical English data for urbanisation and per capita output. A striking feature of the First Industrial Revolution is indeed that, while knowledge grew fast from the mid-18th century, productivity and output did not. Instead, it accelerated some generations latter, a time lag that we are able to reproduce in our setup.

Our hypothesis raises the question of why the English population, and that of Europe more generally, was so concentrated in cities. Increases in agricultural productivity due to new techniques or the introduction of new crops; population shocks; and increased fiscal pressure on agriculture are all candidate factors. One of the features of our approach is that it allows for a variety of triggers, implying that – even if the causes of the increase in city size differed across European nations – high urbanisation was a common and important contributor to the growth process.

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Appendix A: Data description and sources

Urbanisation rates

European urbanisation rates: The data for figures 1, 2 and 3 come from various sources. Those for Belgium, Germany and Italy in 1800 come from de Vries (1984). All other figures are our own calculations using urban population from Bairoch *et al.* (1988) and De Vries (1984) and total population from Brunt and Fidalgo (2008). The Bairoch and De Vries datasets each have advantages and disadvantages for our purposes. Bairoch *et al.* attempt to provide figures for all towns with a population larger than 5 000 people, whereas De Vries sets the cutoff at 10 000 people. Since city size is important for the mechanism we consider, we are not interested in small towns (i.e. those of 5 000 to 10 000 inhabitants). However, De Vries' data stops at 1800, whereas Bairoch's run through to 1850. We base figures 2 and 3 on De Vries simply for convenience, since he gives national urbanisation *rates* as well as totals; but we must use Bairoch's data when considering a longer time period because he goes through to 1850. We spent some time comparing the two datasets, in particular by looking in detail at English towns that are common to both data sets (i.e. those larger than 10 000 people). The population estimates are remarkably similar and the correlation in 1800, for example, is 0.998. So we do not believe that using one or the other source will significantly affect our results.

Chinese urbanisation rates: We focus on Chinese provinces that had, at some point during our period of interest, at least one city of 200 000 inhabitants. We exclude some peripheral provinces, such as Tibet, Xinjiang and Inner Mongolia, that did not come under full Chinese control until the late 19th century (or even later) and had no significant cities. The excluded provinces are also unusually large – each of them being about the size of the *whole* of Western Europe – and very sparsely populated, unlike the rest of China. We take our urbanisation data for 1750 and 1800 from Chandler and Fox (1974) and for 1850 (actually, 1843) from Skinner (1977). More recent estimates have been presented by Cao (2001). Xu *et al.* (2018) compare the estimates of Cao and Skinner for the late 19th century (since Cao has no benchmark for 1850) and note that there is very little difference between them, despite Cao having access to more recent historical evidence and additional Chinese primary sources. Skinner put the overall rate of urbanisation at 6%, whereas Cao put it at 7%. Therefore we use data from Chandler and Fox and from Skinner, as this is the only way to get three complete cross sections in the relevant time periods. For comparison purposes, tables A.1 and A.2 list all geographical units (i.e. Western European states and Chinese provinces) containing cities with more than 200 000 inhabitants at any time between 1750 and 1850.

Table A.1. Population living in large (>200 000 people) Western European cities, 1750-1850.

	1750	1800	1850
Austria		247 000	431 000
France	570 000	550 000	1 053 000
Italy	339 000	430 000	618 000
Netherlands	210 000	217 000	225 000
Spain			501 000
England	***675 000***	***948 000***	***3 148 000***
Ireland		***200 000***	***262 000***
Scotland			***345 000***

Note: *Italy* here includes all Italian states and not just the Kingdom of the Two Sicilies.

Table A.2. Population living in large (>200 000 people) Chinese cities, by province, 1750-1850.

	1750	1800	1850
Anhui	302 000	392 000	550 000
Fukien			250 000
Guangdong	500 000	800 000	800 000
Hebei	900 000	1 100 000	2 448 000
Hubei			600 000
Jiangsu	285 000	220 000	450 000
Shanxi		224 000	275 000
Sichuan			440 000
Zhejiang	350 000	700 000	930 000

Tables A.3 and A.4 present the urbanisation rates graphed in figure 2. The last two lines in each panel of the table compute the average urbanisation rate across the group of provinces/countries, both unweighted and weighted by total population.

Table A.3. Chinese urbanisation rates.

Province	1700	1750	1800	1850
Anhui	1.04	1.4	1.74	2.34
Fukien	0	0	0	1.68
Guangdong	9.97	7.74	5.68	3.69
Hebei	9.32	6.46	5.88	10.44
Hubei	0	0	0	3.82
Jiangsu	1.3	1.36	0.98	1.88
Shanxi	0	0	1.78	1.75
Sichuan	0	0	0	1.32
Zhejiang	2.78	2.95	5.13	6.02
Mean urbanisation rates	**2.71**	**2.21**	**2.35**	**3.66**
Weighted (by population) mean urbanisation rates	**2.20**	**2.29**	**2.37**	**3.59**

Table A.4. Western European urbanisation rates.

Country	1700	1750	1800	1850
Austria	5.43	6.41	8.06	11.11
France	2.5	2.33	1.89	2.9
Prussia	3.67	3.01	1.91	2.65
England	11.53	11.07	12.94	14.82
Scotland	1.03	1.55	2.42	10.27
Ireland	3.15	6.1	3.98	4
Italy (Kingdom of the Two Sicilies)	10	9.69	8.6	6.62
Netherlands	10.5	10.97	10.45	7.36
Portugal	7.71	7.82	6.69	6.3
Spain	2.32	2.26	2.29	3.34
Mean urbanisation rates	**5.78**	**6.12**	**5.92**	**6.94**
Weighted (by population) mean urbanisation rates	**4.96**	**4.82**	**4.59**	**6.26**

Table A.5 considers the four largest cities in each of the major Western European countries in 1700 and tracks their expansion over time. The fast rate of urbanisation of the UK stands out: eight European cities doubled (or more) their population between 1700 and 1750, of which seven were in the UK. By 1850, Western Europe contained fifteen “large” cities (i.e. of more than 200 000 inhabitants) and six of them were in the British Isles, accounting for half of the urban population living in large cities in 1850. Table A.6 provides data on the four largest cities in each of the Chinese provinces we consider, as far as the sources permit. Although cities were reasonably large, urbanisation rates remained moderate in most Chinese provinces. This is because the total population of these provinces was around two times larger than the total population of the European countries that we consider (280 million in 1850, as opposed to 140 million): hence the overall Chinese urbanisation rate was around half the level observed in Europe.

Table A.5. Expansion over time of the largest European cities in 1850 (population in `000s).

City	Country	1700	1750	1800	1850
Amsterdam	Netherlands	200	210	217	225
Rotterdam	Netherlands	51	44	53	90
'S Gravenhague	Netherlands	30	38	39	72
Utrecht	Netherlands	30	25	32	48
Madrid	Spain	140	160	168	281
Barcelona	Spain	34	50	100	220
Sevilla	Spain	72	66	96	113
Malaga	Spain	30	36	49	93
Berlin	Germany	55	113	172	437
Hamburg	Germany	70	90	130	149
Konigsberg	Germany	35	60	60	76
Breslau	Germany	40	55	60	114
Lisbon	Portugal	180	185	195	240
Porto	Portugal	25	30	43	74
Milan	Italy	125	124	135	209
Naples	Italy	300	339	430	409
Rome	Italy	135	158	153	175
Palermo	Italy	100	124	139	168
Paris	France	500	570	550	1 053
Marseille	France	90	68	101	195
Lyon	France	97	114	109	177
Bordeaux	France	45	62	96	130
Vienna	Austria	114	175	247	431
Graz	Austria	22	20	31	55
Linz	Austria	10	10	17	27
Salzburg	Austria	13	15	16	17
London	*UK (England)*	***575***	***675***	***948***	***2 236***
Liverpool	*UK (England)*	***6***	***22***	***83***	***376***
Manchester	*UK (England)*	***8***	***18***	***84***	***303***
Birmingham	*UK (England)*	***7***	***24***	***71***	***233***
Dublin	*UK (Ireland)*	***60***	***129***	***200***	***262***
Belfast	*UK (Ireland)*	***2***	***9***	***20***	***87***
Cork	*UK (Ireland)*	***25***	***58***	***75***	***85***
Limerick	*UK (Ireland)*	***11***	***16***	***39***	***53***
Glasgow	*UK (Scotland)*	***13***	***25***	***70***	***345***
Edinburgh	*UK (Scotland)*	***36***	***57***	***83***	***194***
Dundee	*UK (Scotland)*	***10***	***12***	***26***	***79***
Aberdeen	*UK (Scotland)*	***12***	***15***	***27***	***72***

Note: Includes the four largest cities in each country and all Western European cities with population >200 000 in 1850.
Source: Bairoch *et al.* (1988); De Vries (1984).

Table A.6. Expansion over time of the largest Chinese cities in 1850 (population in `000s).

City	Province	1700	1750	1800	1850
Suzhou	Anhui	245	302	392	550
Fuzhou	Fukien				250
Xiamen	Fukien				85
Lanzhou	Gansu		130	150	170
Foshan	Guangdong	90	130	175	175
Guangzhou	Guangdong	300	500	800	800
Kaifeng	Henan	60	78	80	95
Beijing	Hebei	650	900	1 100	1 648
Chengde	Hebei			68	70
Tianjin	Hebei	80	92		200
Zhangjiakou	Hebei				70
Wuhan	Hupei	150	165		600
Ganzhou	Jiangxi		50		62
Jiujiang	Jiangxi				50
Nanjing	Jiangsu	300	285	220	200
Shanghai	Jiangsu	45	60	100	250
Yangzhou	Jiangsu	135	158	153	170
Zhenjiang	Jiangsu	60			168
Liaoyang	Liaoning				80
Shenyang	Liaoning				180
Jinan	Shandong	55			70
Taiyuan	Shanxi	32		40	45
Xi'an	Shanxi	167	195	224	275
Chengdu	Szechuan			110	240
Chongqing	Szechuan		17		200
Dali	Yunnan				50
Kunming	Yunnan	39			50
Hangzhou	Zhejiang	292	350	500	700
Ningbo	Zhejiang	88	144	200	230

Sources: Chandler and Fox (1974) and Skinner (1977).

The English tax system and agricultural extractions

The main source of Government revenue was the taxation of the agricultural sector. Tariffs on imported luxury goods (tea, coffee, sugar, etc.) also contributed modestly to public finances, whilst domestic manufacturing was generally exempt from taxes; for an exhaustive account, see Dowell (1884). The agricultural sector hence bore a very substantial share of the tax burden.

English agricultural extractions had four components: the tithe (i.e. the 10% of gross output taken by the church); the poor tax (instituted in 1572, after Henry VIII's Dissolution of the Monasteries removed the traditional source of poor relief); the land tax (instituted in 1692 in order to finance the war with France); and land rents. The rent was a private agreement between a landowner and a tenant farmer, with the level being determined by the free market. The land tax was based on a land assessment undertaken in the 1690s: the owner and occupier of every parcel were recorded in a local cadaster and the occupier was obliged to pay a tax – at a rate determined by the central Government – based on the value of the land (Ginter, 1992). This rate was low in peacetime (generally one shilling per pound of assessed land value) but increased in wartime (up to four shillings per pound); as England spent increasing amounts of time at war in the 18th century, so the rate went up higher and stayed up for longer. The poor tax was a local tax, with the level determined by the needs of the local parish and the total was divided between the local occupiers of land based on the land tax cadaster. The poor tax also rose over time – so much so that there were major reforms to the Poor Law in the 1790s and a complete overhaul in 1834 to try to reduce costs.

Figure 4 reports the time series for agricultural extractions and wages, yielding extraction rates between 7 and 28%. The data are available every 25 years, while the calibrated model uses 20-year periods; hence we use a spline to approximate annual data and use the relevant average rates for each 20-year period. The resulting rates are reported in table A.7.

Table A.7. Aggregate magnitudes and extraction rates used in the calibration.

1540	*1560*	*1580*	*1600*	*1620*	*1640*	*1660*	*1680*	*1700*
0.07	0.07	0.11	0.19	0.19	0.17	0.17	0.18	0.18
1720	*1740*	*1760*	*1780*	*1800*	*1820*	*1840*	*1860*	
0.20	0.22	0.22	0.23	0.25	0.27	0.27	0.26	

County-level data

Table A.8 presents descriptive statistics for the county-level data used in table 3. This includes 42 English counties (where Monmouthshire is treated as part of Wales, and London is defined as a county and subtracted from the rest of Middlesex/Kent/Surrey).

Table A.8. English county-level variables on urbanization and population growth.

Variable	Mean	St Dev	Min	Max
Urban population in 1801 (% of total population)	31.80	16.51	8.90	100.00
Population growth 1751-1801 (%)	21.32	28.37	-82.65	123.20
Population growth 1801-1851 (%)	82.46	38.87	29.49	201.94

Appendix B: Additional calibration results

In this appendix we consider alternative calibration results. We first examine the impact of parameter values and specifications in the innovation function in order to understand how the knowledge generation process affects the dynamics of urbanisation. Our last exercise consists in looking at wage inequality.

Figure A.1 presents our core simulation (the continuous line) as well as economies with no innovation and no imitation. The case of $z = 0$ is depicted by the dashed line and that of $\delta = 0$ by the dotted line. The model without imitation predicts an urbanisation rate of only 7.8% by the mid-19th century, rather than the 37.7% obtained in the benchmark case. In the absence of innovation (z=0) the economy rapidly stagnates; urbanisation rises only from 3.5% to 4.8%.

Figure A.1. Urbanisation: economies with and without imitation and innovation.

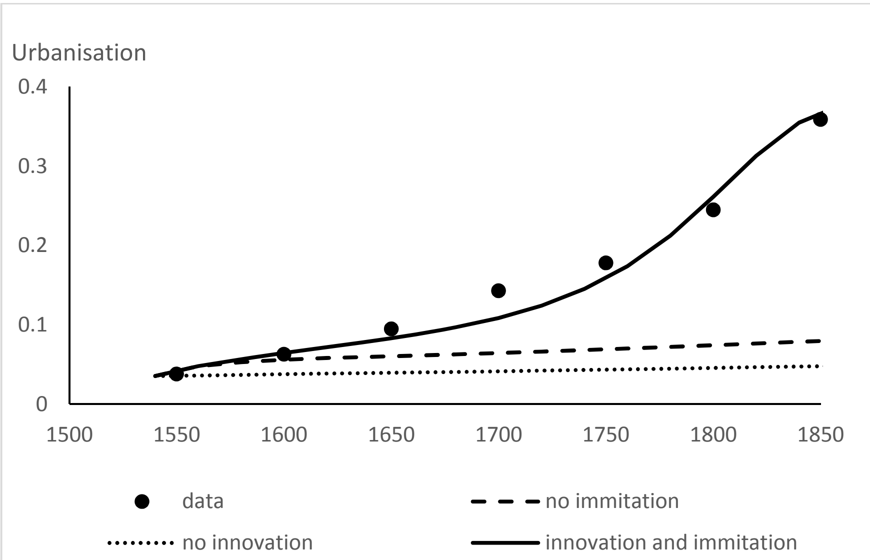
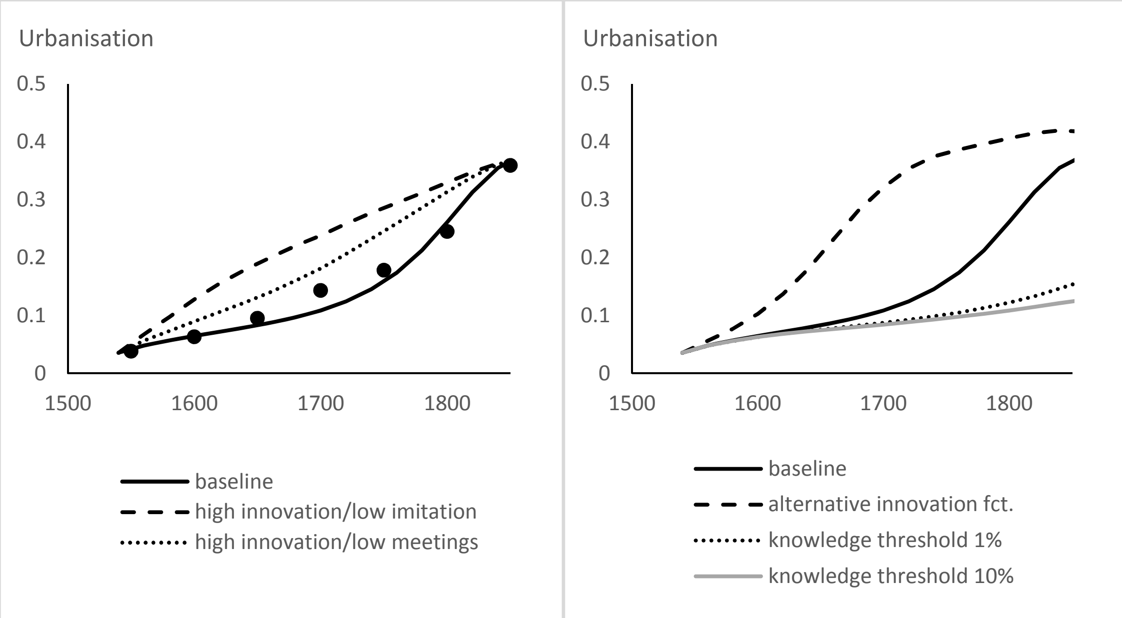


Figure A.2 further examines the role played by knowledge generation. The dashed and dotted lines in the left panel examine alternative parameter values. In both cases we double the rate of innovation to $\delta = 72$ and choose the values of ρ and z so as to have the same final urbanisation rate as in the baseline. The dashed line depicts the case $z = 0.32$ and $\delta = 0.72$, while the dotted line depicts the case $\delta = 0.72$ and $\rho = 6.2$. In both cases urbanisation increases rapidly in the beginning and then slows down, growing excessively fast early on and not fast enough in later periods.

Figure A.2. Urbanisation: Knowledge generation – parameters and functional forms.



The right panel in figure A.2 considers alternative forms for the innovation function. First, we suppose that the probability that, at time t, a meeting leads to innovation is given by $\delta(2-(1+I_t)^{-1})$, i.e. that the current stock of knowledge positively affects the probability of innovation. The results are depicted by the dashed line in the right panel of figure A.2, and show an early acceleration of urbanisation as the innovation rate grows from one period to the next, while its growth rate slows down later on because imitation plays a (relatively) less important role. After one century, the average number of ideas in the population is 0.94 (compared to 0.35 in our baseline model) and by 1860 it is 15 (rather than 6). The urbanisation rate is 42% and output is 3.7 times higher than in the initial period. Our second exercise considers the arrival of a novel idea. In order to make the stock of ideas more sensitive to their distribution, we suppose that innovation results in a novel idea only if the share of innovators is sufficiently large. That is, we require the probability of coming up with a novel idea to be above a threshold for this idea to be added to the social stock of knowledge. As a result, if the distribution of ideas is such that few people hold the most recent ideas, imitation will be low, and $P_{t+1}(t+1)$ will also be low, implying that not enough people have come up with the new idea for it to be added to the stock of knowledge. The grey and dotted lines in the right panel of figure A.3 depict the cases with population thresholds of 1% and 10%. Knowledge growth is sporadic and much slower, with the final stock of knowledge being, respectively, 7 and 3 ideas, rather than 16 as in our baseline.

City growth is thus much slower than in the baseline.

Figure A.3. Inequality: Relative wage variance in manufacturing and the entire economy.

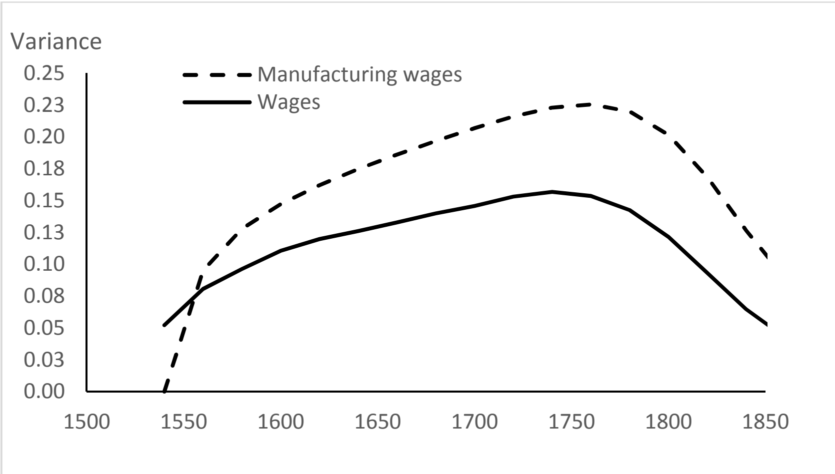


Figure A.3 reports the evolution over time of wage inequality. Our measure of inequality is simply the variance of relative wages, either for the entire population (variance of wages) or for the manufacturing sector (manufacturing wages). To obtain these measures we compute both the wage in agriculture and the wages in manufacturing corresponding to any possible number of ideas available at time *t*, as well as the share of population in each category (agricultural worker, manufacturing worker with 0 ideas, manufacturing worker with 1 idea, etc.). Both types of inequality first increase and subsequently fall, with manufacturing-wage inequality being higher than for the entire distribution of wages since there is no wage dispersion within the agricultural sector. The only instance where inequality is lower in manufacturing is the initial period, when there is no wage dispersion in the manufacturing sector because there are no ideas.

Appendix C: Calibration of a basic model

Because our calibrations try to fit the data, a number of assumptions have been made that are not needed to deliver the key result of our model. This appendix strips the model of all unnecessary elements in order to show how a simpler model can generate the key prediction that the feedback between urbanisation and innovation can result in a growth takeoff. To simplify the model we remove four elements:

Population: We suppose no population growth, implying a constant labour force at N.

Agricultural productivity: We suppose g=0, so that agricultural productivity A_t is constant.

Agricultural extractions: We suppose that agricultural workers are paid their marginal product and taxed at a constant rate τ.

Disutility of urban life: The cost of living in a city, v_t, is assumed to be constant.

The resulting dynamic model is given by

$$I_t = \sum_{i=1}^{t} P_t(i) \quad \text{(A.1)}$$

$$\frac{v}{(1+I_t)^{1-\alpha}} = \left(\gamma A^{\gamma}\left(\frac{T}{N}\right)^{1-\gamma}\frac{(1-\alpha)}{\alpha\gamma}\frac{(1-n_{mt})^{\gamma}}{n_{mt}}\right)^{\alpha}\left(1-(1-\tau)\frac{\alpha\gamma}{1-\alpha}\frac{n_{mt}}{1-n_{mt}}\right) \quad \text{(A.2)}$$

$$P_{t+1}(i) = 1-\left(1-zP_t(i)\right)^{D(n_{mt})} + \left(1-(1-\delta)^{D(n_{mt})}\right)\left(\left(1-zP_t(i)\right)^{D(n_{mt})} - \left(1-zP_t(i-1)\right)^{D(n_{mt})}\right) \qquad \forall i = 1, \ldots, t+1 \quad \text{(A.3)}$$

As in our core simulation, we assume that the relationship between urbanisation and meetings is given by $D(n_{mt}) = \rho n_{mt}$. The parameter values we use are given in table A.9. Most take the values used in our core simulation but a few differ: v is constant and equal to 15, chosen to match the initial urbanisation rate, while we set $\delta = 0.5$, $z = 0.75$ and $\rho = 100$.

Table A.9. Parameter values.

Parameters	$\alpha = 0.75,\ \gamma = 0.6,\ T = 25,\ A = 1,\ N = 0.63,\ \varepsilon = 0.03,$ $I_0 = 0,\ z = 0.75, \delta = 0.50,\ \rho = 100, v = 15.$

Figure A.4 presents several simulation exercises. The continuous black line in both panels depicts the dynamics of urbanisation using the parameters in table A.9. The economy endogenously generates a process of knowledge creation, output growth and urbanisation, with the urbanisation rate rising from 3.5% to 7.5% and output increasing by 20%. The left panel considers changes in $\rho, N,$ and v. The dashed line depicts urbanisation for $\rho = 35$. Meetings for a given urbanisation rate are so rare that innovation is almost non-existent, resulting in a negligible increase in urbanisation (from 3.5 to 3.9%). The dashed-dotted line presents an economy that loses a third of its population in period 6 (a shock comparable to the Black Death in the late 14th century). The resulting reduction in the demand for food lowers the wage in agriculture, leading to an increased flow of labour into cities and hence higher urbanisation. Our third exercise (dotted line) consists of reducing v by 15% from period 6 (a shock comparable to the decline in urban mortality rates observed in the late 19th century). The urban population by period 17 is 21% higher than without the shock.

The right panel of figure A.4 depicts changes in agricultural productivity and taxation. The dashed line depicts a situation in which there is exogenous productivity growth in agriculture, with A growing at a rate of 2.5% per period. The dashed-dotted line reports the results for a once-and-for-all productivity shock (an increase of 33% in period 6). Both accelerate growth in the rate of urbanisation. The dotted line shows the dynamics of urbanisation when agricultural wages are taxed at a rate of $\tau = 0.33$. It delivers a (slightly) higher initial urbanisation rate (3.6% instead of 3.5%) and a final rate that is 8% higher than in the absence of taxation.

Figure A.4. Urbanisation: The basic model.

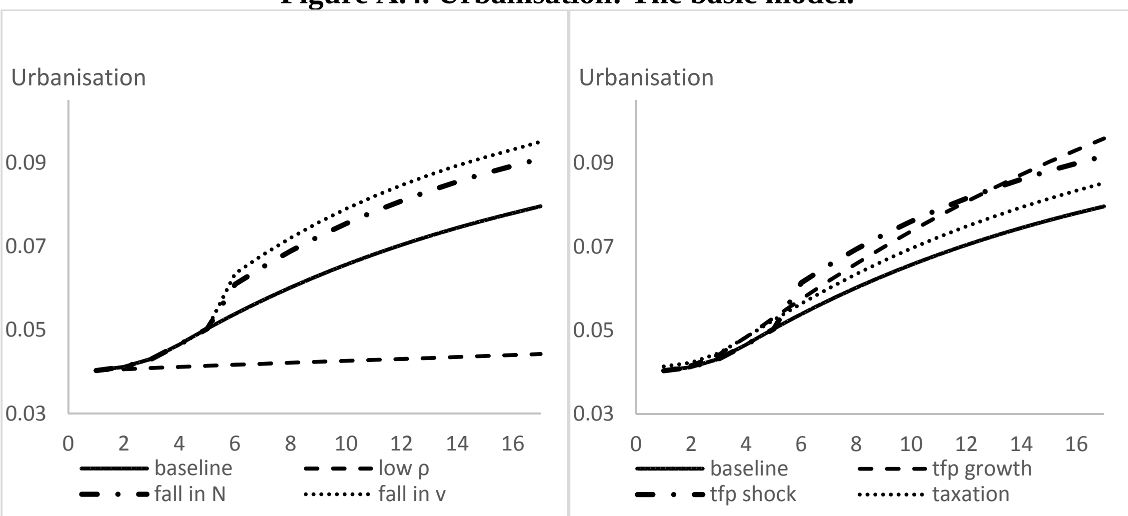


Table A.10 presents initial and final urbanisation rates for different parameter values. Together with figure A.4, it allows us to draw two conclusions from our analysis. First, depending on parameter values, the basic model results in stagnant urbanisation or in growth in the share of population living in cities. The knowledge generation parameters, z and δ, are key. But so is the probability of having a meeting, ρ, because sufficiently low values of any of these can result in stagnation. Second, several factors can act as a trigger that sets the economy on a virtuous circle of increasing urbanisation and technological know-how – including a population reduction, an agricultural TFP shock, increased extraction from agriculture, or changes in the disutility of urban life.

Table A.10. Urbanisation rate: Effect of changes in parameter values.

		ρ		z		δ		v	
Parameter values	**Baseline**	**10**	**200**	**0.4**	**0.95**	**0.25**	**0.75**	**10**	**20**
Initial urbanisation	3.5	3.5	3.5	3.5	3.5	3.5	3.5	5.6	2.5
Final urbanisation	7.5	3.5	7.7	6.5	7.5	6.7	7.7	11.0	5.0