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Variational rationality: Finding the inequations of motion of a person seeking to meet his needs.

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Abstract

As physics provides the equations of motion of a body, this paper formulates, for the first time, at the conceptual and mathematical levels, the inequations of motion of an individual seeking to meet his needs and quasi needs in an adaptive (not myopic) way. Successful (failed) dynamics perform a succession of moves, which are, at once, satisficing and worthwhile (free from too many sacrifices), or not. They approach or reach desires (fall in traps). They balance the desired speed of approach to a desired end (a distal promotion goal) with the size of the required immediate sacrifices to go fast (a proximal prevention goal). Therefore, each period, need/quasi need satisfaction success requires enough self control to be able to make, in the long run, sufficient progress in need/quasi need satisfaction without enduring, in the short run, too big sacrifices. A simple example (lose or gain weight) shows that the size of successful moves must be not too small and not too long. A second paper will solve this problem, using variational principles and inexact optimizing algorithms in mathematics. This strong multidisciplinary perspective refers to a recent mathematical model to psychology: the variational rationality theory of human life stay and change dynamics.

Key words: need satisfaction, speed of progress, sacrifices, dynamical system, variational rationality.

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1 Introduction

1.1 Preliminaries.

The problem at hand is: how to meet our needs and quasi-needs including our wants, desires and aspirations defined as desired proximal or distal ends, say proximal and distal goals. That is, what are the inequations of motion of a human seeking to meet in the short run and in the long run his needs and quasi-needs. To be short, in this paper the term need will represent needs and quasi needs. Because this problem is so complex, with strong multidisciplinary aspects, we try to reformulate it as simply as possible in a way that the concepts

lead directly to simple mathematics and, later, to its resolution, with the help of much more advanced tools. This operational reformulation is: how to satisfice (= satisfy enough) our needs in the long run without sacrificing too much in the short run. The balance between the long run and the short run being far from trivial because it requires self control. To help the reader,

- A) A list of references to the (VR) variational rationality approach of stay and change human behaviors is given in https://sites.google.com/view/antoine-soubeyran.
- B) In section 2, we give a concrete example to make this paper more readable. It is a diet problem, i.e., eating less or eating more to lose or to gain weight. It accompanies the whole presentation. We urge the reader to give a look at it, to direct his attention on a few main points (this is why a simple model is useful), avoiding the risk of wandering in the middle of all the multidisciplinary aspects of the so vast need satisfaction-need frustration dynamical problem we are looking at.
- C) In the different sections, we give a step by step careful construction of the main concepts, verbally, formally and numerically.
- D) We give several figures to enrich the presentation with a geometrical description. They are given at the end of section 7 to illustrate the inequations of motions. These figures are very important for understanding the underlying analysis. They show that, with six simple figures where three straight lines intersect a parabola, the main ideas of our mathematical approach can be fully understood. They include Figure 2 for a satisficing move, Figure 3 for a worthwhile move, Figure 4 for a trap, Figure 5 for self regulation failures, and Figures 6, 7 for self regulation success.
- E) Even if it is unusual, we introduce some notation (very few) in the introduction. We do that to start going from verbal concepts to simple formula. This highlights the spatial perspective that drives our conceptual and mathematical presentation of human motivational dynamics, given that to be able to fulfill needs we must move in a space of bundles of activities. This is the heart of the concept of motivation (= movere) in motivation science.

1.2 The need satisfaction-need frustration problem.

This problem is important. It consists in becoming aware of, becoming able to fill, and finally fill a discrepancy between where you are (in an uncomfortable situation) and where you want to be (in a more comfortable situation). This is probably the most important problem in behavioral sciences because to fulfill a need is like solving a problem among the myriad of problems we have to solve in our daily life. As Einstein (1949, pp 24-28) puts it "everything that men do or think concerns the satisfaction of the needs they feel or the escape from pain".

We have plenty of needs. They can be recurrent or changing, proximal or distal, direct or indirect, active or not, explicit or hidden, The coexistence, each period, of contentment and frustration feelings is a major point. This occurs because resource constraints and a multiplicity of needs make difficult to fulfill several needs at the same time. Then, each period, some needs can be satisfied enough only at the expense of others that must wait for their sufficiently great satisfaction.

The problem at hand, i.e., how to fulfill needs, is very complicated. Because, each period, the list of our current needs changes for two reasons. Exogenous reasons are changes in the external environment (we need to drink more when the weather is hot). Endogenous reasons are changes in the internal environment. That is, changes in the list of our current motives, needs, feelings and driving goals (= how much of each of our needs we choose to fulfill, each period). This occurs endogenously because when we fill a need, the list of our activated needs changes. Moreover, doing something to fulfill our needs changes also our external environment endowed with resources like objects, persons and landscapes that help to fulfill them. We use some of these resources in a given environment (this impoverishes the old environment), and we change from an environment to a richer one to find new resources. Thus, we are amid a world where all things change.

The multidisciplinary solution we give is unifying. The joint contribution of the present paper in psychology and a companion paper in mathematics (see Soubeyran, 2022 for a preliminary version) will show the large unification that we operate between motivation science in psychology and the theory of dynamical systems in mathematics. This unification was not easy to do. But it is simple and surprising:

- i) in psychology (the present paper), our so called satisficing and worthwhile moves drive the long term and short term aspects of success in the quest for need satisfaction;
- ii) in the mathematics of dynamical systems (the second paper), famous (long term) error bound conditions and (short term) sufficient descent conditions drive the convergence of variational principles and optimizing algorithms until they approach sufficiently and reach the minimum of a function. The finding of our second paper is this: point ii) is a very rich but specific instance of point i). This means that the psychological perspective is larger than the mathematical perspective, while the mathematical perspective being more restrictive is more operational at the algorithmic level. They complement each other. Then, it appears that most of the psychological principles at the service of the need satisfaction problem in psychology/behavioral sciences and most of the variational principles and optimization algorithms in the theory of dynamical systems are the two faces of the same coin. The main intuitions being the same.

1.3 A conceptual question is: what must be done, each period, to fulfill needs sufficiently.

More precisely, the question is: what must be done to fulfill sufficiently different recurrent and changing needs, fast enough in the long run, without too many frustrations and sacrifices in the short run, when the (internal or external) environment is changing and resistance to move matters. Because self regulation is at the service of the satisfaction of needs, this question can also be a way to give a broad definition of self regulation, given that, relatively to desired and undesired outcomes, "self-regulation is the dynamic process by which people manage competing demands on their time and resources as they strive to achieve desired outcomes, while simultaneously preventing or avoiding undesired outcomes" (Neal et al., 2017). More precisely, for us, self regulation is what we do to fulfill recurrent and changing needs. That is, "self regulation is the ability to flexibly activate, monitor, inhibit, persevere, and/or adapt one's behavior" (McClelland et al., 2018).

This initial question Q.1 raises several questions about goal setting:

- Q.2. How to become aware of our motives, the related needs and their sizes. This question is about problem recognition.
- Q.3. How much of each need we choose to fulfill at least each period? This question is about the formulation of value expectancies and the new concept of driving goals, i.e., satisficing levels = sufficient level of fulfillment of each need.
- Q.4. What activities must be done to fulfill enough each need (that bring us enough satisfaction)? This question has to do with the modelization of outcome expectancies, valence expectancies and action goals.
- Q.5. Which capabilities to acquire to become able to do these activities? This question requires a reformulation of self efficacy expectancies in term of expected capabilities defined as expected means and expected abilities to use these means.

The two last questions Q.4 and Q.5 are also about:

- a) goal striving, i.e., how to become able to stop, continue and start doing activities that are at the service of satisfying enough our needs;
- b) goal achievement, i.e., how to do these activities, i.e., stop, continue and start doing these activities to effectively satisfy enough our needs.

1.4 The topic of this paper: a proposal for a conceptual and mathematical solution

This paper considers the need satisfaction-need frustration dynamical problem. Its goal is to exhibit, it seems for the first time in psychology, the inequations of motion ending in need satisfaction success (reaching desires) or in need satisfaction failure (falling in traps). As physics does when it builds the equations of motion of a body, we formulate the inequations of motion $x^{k+1} \in \Gamma_{k,k+1}(x^k), k = 1, 2, \dots$ of an individual who makes a succession of moves (x^k, x^{k+1}) going from doing x^k in period k to do x^{k+1} in period k+1 to fulfill in the current period k+1 his different recurrent and changing needs, through disengagements, reengagements and engagements in different activities. A second paper (in preparation) will solve these inequations of motion, using the modern tools of variational principles and inexact optimizing algorithms. See Soubevran (2022) for a start in this algorithmic direction. This non autonomous and set valued dynamical system (a set of non linear variational inequalities) models success and failure in need satisfaction in term of inequations instead of equations. This highlights the undeterminacy of our human lifes, given the difficulties that humans have to know their recurrent and changing needs and to know, ex ante, how to fulfill each of them. To find these inequations of motion has not been easy. But their meaning is clear. They model, each period, an essential trade off between, i) the speed at which an individual will choose to fulfill his needs, i.e., wanting to make sufficient progress in the long run (a promotion goal) and, ii) the size of the sacrifices that he can accept to do for this purpose in the short run (a prevention goal). Thus, these inequations of motion will also drive the dynamics of self regulation success and failure (Soubeyran, 2022). For the dynamics of self regulation see Fishbach et al. (2009). For promotion and prevention goals, see Higgins (1998).

In this way these two papers will provide a conceptual and mathematical model for human dynamics as one of the final outputs of the recent (VR) variational rationality approach of concrete stay and change human dynamics (Soubeyran, 2009, 2010, 2021a,b,c,d, 2022). As a consequence these two papers will respond to the request of Kruglanski et al.(2015) who urged researchers of different disciplines to propose multidisciplinary approaches serving motivation science. As they said in their conclusion: "Thus, motivation defines a general theme that cuts across diverse fields of inquiry in the social and behavioral sciences. It would seem useful to create channels of communication through which those various disciplines could interact and stimulate each other".

The VR approach: an, at once, conceptual and mathematical model for psychology. The VR approach is multidisciplinary. It has two parts: a behavioral part that has not been published yet, waiting, before submission, for a complete picture to provide a broad enough perspective that can be applicable in a lot of different disciplines including psychology, but also economics, management sciences, decision theory, sociology, game theory, the mathematics of dynamical systems.... Like in psychology, our approach has four main goals: to describe, explain, predict and to change behavior. This is a descriptive definition of psychology. And a mathematical part that has been published in many top ranking journals in mathematics (more than 53 papers)¹.

The behavioral part of the VR approach started with giving a lot of real life examples of such stop and go dynamics where different things change and other things stay, i.e., starting to do new activities, stopping to do less recent new activities, trying and preventing doing other activities, breaking bad habits, and forming good habits. That is, we showed how human life can be modelled as an interlacing of reengagements (stays = continuations) and disengagements-

¹https://sites.google.com/view/antoine-soubeyran.

engagements (changes including stops and starts). See Wrosch et al. (2003.a, 2003.b, 2007). As Lewin puts it "life is a constant interplay between completing old situations and opening up new ones" (see Psicopolis, Kurt Lewin Notes, Victor Daniels). These stop, continue and start dynamics being at the service of the satisfaction of recurrent and changing needs in recurrent or changing internal and external environments. Then, comes the question: why people do something, how and when? why do they constantly complete old situations and open up new ones. This is the very question that starts motivation theory. Our answer is: to succeed to fill recurrent and changing needs requires adaptive behaviors like disengagements, re-engagements and engagements in different activities.

The mathematical part of the VR approach contrasts two kinds of human dynamics: moving when resistance to move is strong or is weak. This helps to classify in two groups the main results (seen as mathematical tools for the psychologist) given in the modern variational analysis and in optimizing algorithms in mathematics (see the celebrated books of Aubin & Ekeland, 1984, Mordukhovich, 2006):

- moving when resistance to move is strong. Belong to this category the main variational principles including the famous Weierstrass theorem, Banach fixed point theorem, Ekeland, Caristi and Takahashi equivalent variational principles, Nash equilibrium and quasi equilibrium theorems with potential functions (game theory);
- ii) moving when resistance to move is weak. Belong to this category the main optimizing algorithms including the famous gradient, line search, descent, proximal, trust region algorithms and their accelerated versions.

1.5 The choice of a spatial perspective: should I go, should I stay.

To fulfill needs, we must move. A simple language element directs the mathematical way to deal with the need satisfaction dynamical problem. To fill a need in the current period, we must do a bundle of activities in the current period. Therefore, given that we have done some bundle of activities in the previous period, we must do a move m going from $\underline{x} =$ "having done a bundle of activities x in the previous period" to end in y = " do a bundle of activities y in the current period". It is a change $m = (\underline{x}, y)$ if $y \neq x$ and it is a stay $\sigma = (\underline{x}, x)$ if y = x. Then, doing something is like doing a move (a change or a stay).

The "should I go, should I stay" principle. A first principle drives the formulation of the inequations of motion of a need satisfaction-need frustration dynamical problem. To stop, continue or start to fulfill recurrent needs at the same levels, or not, and to start to fulfill news needs, we must, each period, balance between,

- i) change (go), i.e., do the change $m=(\underline{x},y)$ if $y\neq x$. In this way, on one side, we hope to better fulfill some needs, to benefit of improved levels of satisfaction and to endure lower levels of frustration, and on the other side, given resource constraints, we accept to wait for the fulfilment of other needs, with possible disengagements, being ready to endure (again) the same levels of frustration as before or,
- ii) stay, i.e., do the stay $\sigma = (\underline{x}, x)$ (an emerging habit) if y = x. In this way, we accept to suffer from the same levels of satisfaction and frustration as previously, waiting one more time to better fulfill all these needs.

A problem of notation. In this context the notation $m/\sigma = (\underline{x}, y)/(\underline{x}, x)$ models the "should I go-should I stay" principle. It means "to change rather than to stay". Most of the time, to simplify this notation, we will write $m/\sigma = y/x$. This means that, i) we take the status quo \underline{x} as implicit, ii) we identify \underline{x} with x, and, iii) we write a move $m = (\underline{x}, y) = (x, y)$ without any ambiguity.

1.6 Then, the main question evolves in: each period, which kind of move will lead to success or failure?

Two kinds of moves matter for need fulfilment: satisficing and worth-while moves. The original question Q.1 was: "what must be done to fulfill our needs". Given our spatial perspective, this question becomes "which kind of move will lead to success or failure in meeting needs". Our answer is: if you try to succeed, do each period a move that is, at once, satisficing and worthwhile. Such a succession of moves defines a system of inequations of motion that can end in success in meeting needs (approach or reach desires). The opposite for failures (fall in traps).

To clearly understand the significance of satisficing and worthwhile moves requires to define a lot of VR concepts. What we can say, without taking time to define each concept (this will be done later) is the following.

The significance of a satisficing move. A satisficing move makes, in the long run, enough progress in need satisfaction with respect to the aspiration gap. It is defined by a distal non negative balance that defines its desirability aspect in term of speed of moving. The idea behind this is that the more we are obliged to wait for the fulfilment of a need, the larger our unpleasant frustration feelings. In this way frustration feelings play a major role in the speed of movement because, each period, it forces you to make sufficient progress to fulfill needs fast enough to avoid being frustrated for too long. This is, for us, the main intuition that drives the goal gradient hypothesis (Hull, 1932). It leads directly to an original formulation of this famous hypothesis.

The significance of a worthwhile move. A worthwhile move is such that, in the short run, the sacrifices required to make enough progress in need satisfaction in the long run are not too big. More generally a worthwhile move requires that motivation to move (the distal desirability aspect of a move = I want) is high enough with respect to resistance to move (the proximal feasibility

aspect of a move = I can). Then, a worthwhile move is defined by a non negative worthwhile balance between the distal "I want aspect", and the proximal "I can aspect".

To become aware of our needs starts providing an unpleasant frustration feeling. The feeling that different things are missing. At this stage, we can choose to renouncer or to try to fill our needs. If we choose to try, the unpleasant frustration feeling transforms in the hope to succeed and in the anticipated pleasure to think about possible success. Moreover, to meet partially our needs provides, ex post, at once satisfaction and frustration feelings. Of course if we want to fulfill needs, this is costly, i.e., the question is: do we can fulfill them, i.e., which bundle of activities we must become able to do and, then, do. There are three kinds of sacrifices (costs) related to need fulfilment: i) frustration costs, being obliged, given resource constraints, to wait for the fulfilment of some needs at the expense of the immediate fulfilment of others, ii) capability costs to become able to do a bundle of activities and, iii) competence and execution costs to effectively do this bundle of activities in a competent way. Then, when trying to fulfill needs, there is always a balance between satisfaction and frustration.

The balance between speed of moving and sacrifices. To sum up, satisficing and worthwhile balances are about the choice between the chosen speed of moving to meet needs and the chosen size of the sacrifices that must be done. They provide, each period, the inequations of motion leading to need satisfaction successes. Failures in need satisfaction occur in the opposite case. A satisficing balance has to do with the concept of rate of sufficient progress (the TOTE model, Carver & Scheier, 1990). See also Liberman & Dar (2009) who examined carefully the normal and pathological consequences of encountering difficulties in monitoring progress toward goals and Busemeyer & Townsend (1993) about the speed of decision making. A worthwhile balance has to do with Lewin's comparison between driving and restraining forces (the force field analysis, Lewin, 1935, 1936, 1938, 1951).

1.7 Finally, our paper opens the door to a lot of applications.

Among so many emerging applications², the present paper and more generally the VR approach can help,

A) in psychology (unpublished papers), 1) to build a grand theory of motivation and emotion (Soubeyran, 2021a,b) including a theory of intentions and moving goals (Soubeyran, 2021d), 2) to meet the Lewin's dream of topological psychology (Lewin, 1935, 1936, 1938, 1951, Soubeyran, 2021c), 3) to provide a mathematical theory of self regulation (Soubeyran, 2022) with a new look at the impact of a lot of bias including vague (non smart) goals, aspiration failures, intrinsic motivation, the goal gradient hypothe-

²In progress, see https://sites.google.com/view/antoine-soubeyran.

- sis, lack of self control, loss aversion, the status quo bias, changing preferences, ego-depletion effects,, 4) to propose a general and dynamical theory of satisficing and aspirations, where a goal system contrasts promotion goals with prevention goals (Simon, 1955, Genicot & Ray, 2020).
- B) In mathematics (published papers), with the help of generalized variational principles and optimizing algorithms in asymmetric distance spaces, to start the beginning of, 5) a theory of goal systems when valences are changing (Bao et al., 2014), 6) a theory of habit forming and breaking (Soubeyran & Souza, 2020), 7) a behavioral approach of the formation of moving consideration sets (choice sets) in management science, i.e., a reformulation/resolution of local search algorithms (Attouch & Soubeyran, 2010) and, 8) a theory of variational rationality games (Attouch et al., 2010, Flores Bazan et al., 2012, Soubeyran et al., 2019).....

Summary. The introduction poses the need satisfaction/frustration dynamical problem and proposes a solution as a succession of moves, at the same time satisficing and worthwhile. Section 2 provides an example: losing or gaining weight. Section 3 provides a spatial persective to deal with the problem. Section 4 builds a hierarchy of expectancies. Section 5 lists the positive and negative aspects of moving (change or stay). Section 6 defines satisficing and worthwhile moves. Section 7 exhibits the inequations of motion ending in success (to approach or to reach desires) or ending in failures (to fall in traps). It ends with six figures The last section 8 provides extensions. The conclusion and a list of references follow.

2 A simple example: losing or gaining weight. A dynamical promotion-prevention perspective

An example of inequations of motion leading to self regulation success. The problem of need satisfaction-need frustration is so vast that giving without delay a simple illustration is a necessity. For example, consider each period, an individual who takes care of his weight. His need is to reach his ideal weight. This indirect need comes with several more basic needs: to feel good about his own body (a physiological need), to enjoy pleasant life (eating tasty food), to become attractive to loved ones (a social need) and to avoid pains, i.e., to be in a good health, avoiding health penalties. To fulfill this indirect need requires to eat less, the same or more of a given food, given that, if you eat too much (not enough), your weight will increase (decrease). Alas for this individual, this food, say chocolate, is good for taste and can be bad for health.

The entanglement between a long term balance and a short term balance. Suppose that x^k and x^{k+1} are the quantities of food (chocolate) that the consumer eats in the previous and current periods k, k+1. Then, he starts to change his food habits if $x^{k+1} \neq x^k$ and he forms a food habit if $x^{k+1} = x^k$. In the first case he does the change (x^k, x^{k+1}) and in the second instance he

does the stay (x^k, x^k) . In this dynamical context, we formalize and demonstrate how the two following inequations of motion (non negative balances) can help to approach and reach, after a succession of periods, a final need satisfaction success, i.e., eating the ideal level of chocolate x^* . That is,

- A) a first non negative balance $g(x^{k+1}) g(x^k) \ge \theta_{k+1} \left[g^* g(x^k) \right]$ deals with a promotion and distal perspective;
- B) a second non negative balance $g(x^{k+1}) g(x^k) \ge \xi_{k+1} q(x^k, x^{k+1})$ concerns a prevention and proximal perspective, with $0 < \theta < 1$ and $\xi > 0$, $k = 1, 2, \ldots$, where,
- $g(x^k)$ and $g(x^{k+1})$ define the levels of satisfaction the consumer derives from eating the quantities of chocolate x^k, x^{k+1} . They take care of the different satisfaction levels derived from the lesser, the same or greater fulfilment of each basic need;
- $g(x^{k+1}) g(x^k) \ge 0$ models the improvement in the satisfaction level, i.e., the advantage to move from x^k to x^{k+1} , eating less, the same or more of the given food;
- $g^* = g(x^*)$ is the ideal level of satisfaction derived from an ideal level of consumption x^* , not too small and not too big and,
- $q(x^k, x^{k+1})$ represents the inconvenience derived from breaking and forming food habit (this is not easy). That is, it models how much it costs to move from eating less or more than before.

In the current period k+1, the parameter θ_{k+1} models the desired rate of progress of the satisfaction level, comparing the improvement in the satisfaction level $g(x^{k+1}) - g(x^k) \ge 0$ with the aspiration gap $g^* - g(x^k) \ge 0$. The parameter ξ_{k+1} represents the importance given to the inconvenience of changing the level of consumption from x^k to x^{k+1} , i.e., $q(x^k, x^{k+1})$, compared with the advantage to change $g(x^{k+1}) - g(x^k) \ge 0$.

Then, the inequations of motion of this need satisfaction-need frustration dynamic are :

$$x^{k+1} \in \Gamma_{k,k+1}(x^k) = \left\{ \begin{array}{l} y \in \mathbb{R}_+, \ g(y) - g(x^k) \ge \theta_{k+1} \left[g^* - g(x^k) \right] \\ g(y) - g(x^k) \ge \xi_{k+1} q(x^k, y) \end{array} \right\}, k = 1, 2$$

Satisficing and worthwhile moves. With a promotion and long run perspective, condition A) defines a satisficing move (x^k, x^{k+1}) . It requires that the improvement in the satisfaction level makes sufficient progress, fulfilling a sufficient portion of the long run aspiration gap. With a prevention and short run perspective, condition B) defines a worthwhile move. It requires that the required short run sacrifice, i.e., the inconvenience to move must be sufficiently small relative to the improvement in the satisfaction level, i.e., the advantage to move. Taken together, conditions A) requires that the speed at which we choose to approach the ideal weight must be sufficiently high (a promotion

goal) and condition B) requires that the short run sacrifice must be sufficiently small (a prevention goal). This is, in psychology, a new and dynamical way to reformulate the famous self determination theory (Higgins, 1998). It advocates that the hedonic principle, i.e., to seek pleasure and to avoid pain, operates in two ways, with a promotion focus versus a prevention focus, making the distinction between maximal and minimal goals. This says, in mathematics, that we try to improve something in the long run under constraints in the short run.

The problem is to show when and how a succession of such moves, at the same time satisficing and worthwhile, leads to need satisfaction success (reaching desires) or to need satisfaction failures (falling in traps). This paper considers an individual. The case of, i) a group of individuals or of an organization that wants to reach different moving goals and, ii) interrelated individuals (games) that want to satisfy several needs, will be examined elsewhere.

3 Starting with a spatial perspective to fulfill needs: moving in a space of activities

3.1 Need fulfilment

The need for eating something. A need is a dissatisfied motive. That is, having or doing too much or not enough of something. Say some food, like chocolate. The size of a need is $n \in R_+$. A need can be direct (to be hungry) or indirect (wanting to eat something to fill one's hunger). A need can be proximal or distal, activated or not, recurrent or changing.

The degree of fulfilment of a need. It is $\varphi \in \mathbb{R}_+$, $\varphi \geq n$. It can be too high or too low relative to the complete level of fulfilment of this need, i.e., its size $\varphi^* = n \in \mathbb{R}_+$. The degree of non-fulfilement of a need is $n - \varphi = \varphi^* - \varphi \in \mathbb{R}_+$. The complete level of fulfilment of a need can be infinite, i.e., $n = +\infty$.

A (perhaps irrealistic) aspiration level. In term of need fulfilment level, an aspiration level is the highest degree of fulfilment on a need that we can imagine, i.e., $\varphi^* = n \in \mathbb{R}_+$.

Different lists of motives and basic needs exist. See Vansteenkiste et al., 2020) and Schwartz (2012). A prominent theory of needs is the self determination theory (Ryan & Deci, 2000a,b).

3.2 Moving

A spatial perspective in psychology. As said in the introduction, the initial main question of our VR perspective is: "what must be done to fulfill needs". Let us show how a spatial perspective leads to a new formulation, much more favorable to an ongoing mathematical approach. It is: "what kind of moves must be done to satisfy needs enough and speedy enough to escape from too many sacrifices and frustrations (waiting too much for their fulfilments)". The reason is simple. To fulfill a need requires to do something. For example, if you

are hungry (a need for food), you must eat some food. To do that, you must buy this food and cook it. Then, if you do something now, this means that you move from what you have done before in the previous period to what you will do now, in the current period. That is, doing something represents the end of a move going from having done some bundle of activities in the previous period to do some bundle of activities in the current period. This move can be a change if you want to fulfill less or more a recurrent need, or a stay it you choose to fulfill this need as much as before. It will be a change if you plan to fulfill a new need. Then, even if needs are not changing, to fulfill them requires to do a move (change or stay). Lewin is the father of a celebrated spatial perspective in psychology. See his inspiring Topological psychology (Lewin, 1936). However he did not used the simple argument we give here to show that doing something is like moving.

One activity. If the need for an individual is to reach an ideal weight, a partial fulfilment of this need requires, depending of his initial weight, to eat less or more of some food. Let $y \in \mathbb{R}_+$ be this quantity. Then, taking care of the situation, in particular, given what this individual has done before, he must do a move $m=(x,\omega,y)$, i.e., a change, $m=(x,\omega^{\neq},y)$ going from having done x in the previous period to do $y \neq x$ in the current period, or a stay, $\sigma = (x, \omega^{-}, x)$, i.e., staying at x = doing x again. The term $\omega \in \{\omega^{\neq}, \omega^{-}\}$ represents a translocation. This transition between the beginning x and the end y of the move includes disengagements, re-engagements and engagements. For example, if $x, y \in \mathbb{R}_+$ represent two quantities of chocolate, a move $m = (x, \omega, y)$ going from eating the quantity of chocolate x in the previous period, to eat the quantity y in the current period is an engagement $\omega = y - x > 0$ (eating more) if y > x, and a disengagement $\omega = x - y > 0$ (eating less) if y < x. It is a stay, i.e., a reengagement (eating the same quantity) if $\omega = 0$.

Several activities. If
$$x = \left\{x^1, x^2, ..., x^j, ..., x^{\overline{j}}\right\} \subset X$$
 and

Several activities. If $x = \left\{x^1, x^2, ..., x^j, ..., x^{\overline{j}}\right\} \subset X$ and $y = \left\{y^1, y^2, ..., y^j, ..., y^{\overline{j}}\right\} \subset X$ represent two bundles of elementary activities. ties, each of them being defined as a succession of rounds, the translocation of a move $m = (x, \omega, y)$ is $\omega = \{x \setminus y, x \cap y, y \setminus x\}$. It includes to stop, continue and start doing the bundles of elementary activities $x \setminus y$, $x \cap y$ and $y \setminus x$.

Example. If $x = (x^1, x^2)$ and $y = (y^1, y^2) \in \mathbb{R}^2_+$ represent, each day, the times spend to rest and to work, with the time constraint $x^1 + x^2 = y^1 + y^2 = r > 0$ 0, a move $m=(x,\omega,y)$ lies in the simplex $\Delta=\left\{y=\left\{y^1,y^2\right\}\in\mathbb{R}_+^2,\ y^1+y^2=r\right\}$. Thus, the translocation $\omega=y-x=(y^1-x^1,y^2-x^2)$ includes an engagement in one activity and a disengagement in the other activity because $y^1 - x^1 =$ $x^2-y^2 \ge 0$. Notice that a move in the simplex is, at once, intensive (doing more or less of each thing), and extensive (doing more or less of two different things, where a thing is an activity).

4 Build a hierarchy of expectancies (= expectancy system) to better know what can be done

4.1 Expectancies

Expectancies. They help to know, each period, what must be done to be able to fulfill enough different needs. In psychology, Bandura (1978) and others (see Maddux, et al., 1986 for a survey) defined several kinds of expectancy: value expectancy, outcome expectancy, and self efficacy expectancy. The traditional definitions has been:

- value expectancy is the importance given to the results;
- outcome-expectancy refers to the result ones anticipates from having performed a task;
- valence expectancy is another expectancy. It is rarely associated with the other ones. A valence "usually derives from the fact that the object is a means to the satisfaction of a need, or has indirectly something to do with the satisfaction of a need" (Lewin, 1935, p.78);
- self-efficacy expectancy defines one's beliefs in the ability to perform a task.

Their formulation along a chain of value. The VR approach models expectancies in a spatial perspective that emphasizes their dynamical aspects: the three first are relative to the end of a move (value, outcome and valence expectancies) and the last to the move itself (self efficacy expectancies). Expectations are defined directly or indirectly in relation to the degree of fulfilment of different needs. In this context, VR expectations define, along a chain of value,

capabilities
$$\longrightarrow$$
 activities \longrightarrow levels of need fulfilment \longrightarrow levels of satisfaction
$$C(x,y) \longrightarrow y \longrightarrow \varphi = \varphi(y) \longrightarrow g = \mathfrak{g} [\varphi]$$

Figure 1: A chain of value.

where $\mathfrak{g}[\varphi]=g(y)=\mathfrak{g}[\varphi(y)]$, and capabilities = means + abilities to use them. That is,

A) downstream, what can be expected in term of levels of satisfaction (value expectancies $\mathfrak{g}\left[\varphi\right]$) when we fulfill each need, given its level of fulfilment φ ;

B) upstream, what can be expected i) relative to the level of fulfilment of each need when we do something (outcome expectancies $\varphi = \varphi(y)$); ii) relative to the levels of satisfaction when we do something (valence expectancies $g = g(y) = \mathfrak{g}\left[\varphi(y)\right]$) and, iii) relative to the size of costs to move C(x,y) (self efficacy expectancies) when we try, first, to become able to do these things (ability expectancies) and, second, when we actually do them (competency expectancies).

4.2 Value expectancy.

Given a need whose magnitude is given, equal to $n = \varphi^* > 0$, say, eating each day the ideal quantity of chocolate n, a value expectancy is the expected level of satisfaction $\mathfrak{g} = \mathfrak{g} [\varphi] \in \mathbb{R}$ derived from the degree of fulfilment φ of this need. To simplify, we will notice $\mathfrak{g} = \mathfrak{g}_n [\varphi]$ only when the size n of this need changes.

Ideal level of satisfaction. The optimal level of satisfaction

$$\mathfrak{g}^* = \sup \{ \mathfrak{g} [\varphi], \varphi \in \mathbb{R}_+ \} < +\infty$$

derived from some degree of fulfilment φ of a need is the satisfaction level $\mathfrak{g}^* = \mathfrak{g}[n]$ derived from the exact level of fulfilment of this need $\varphi^* = n \in \mathbb{R}_+$. It represents an ideal level of satisfaction, i.e., a possibly unrealistic aspiration level if the size n of the need is high compared with the resources that an individual can use to fulfill this need. In this setting, the aspiration gap is $\mathfrak{f}(\varphi) = \mathfrak{g}^* - \mathfrak{g}[\varphi] \in \mathbb{R}_+$. It has two sides. Its other side represents the level of frustration feeling related to the level of fulfilment φ of this need, compared to its size n.

Construction of a value expectancy. We can suppose that the closer a need of size $\varphi^* = n$ is fulfilled, i.e., the lower is $|\varphi^* - \varphi|$, the higher will be the level of satisfaction $\mathfrak{g}\left[\varphi\right] \in \mathbb{R}$ derived from the degree of fulfilment φ of this need. This means that we can write the value expectancy $\mathfrak{g}\left[\varphi\right] = \mathfrak{g}^* - \delta(\varphi^* - \varphi)$ as the difference between the ideal level of satisfaction $\mathfrak{g}^* = \mathfrak{g}\left[n\right]$ and a decrement in the satisfaction level $\delta(\varphi^* - \varphi) \in \mathbb{R}_+$. This decrement function $\delta(.) : u \in R \longmapsto \delta(u) \in \mathbb{R}_+$ increases with the modulus of $u = \varphi^* - \varphi$. Furthermore $\delta(0) = 0$. This shows that the less a need is filled $(|\varphi^* - \varphi| | \text{large})$, the lower is the level of satisfaction.

Example 1: a sharp value expectancy. For example, the piecewise linear value expectancy $\mathfrak{g}\left[\varphi\right] = \left[\begin{array}{c} \mathfrak{g}^* - \delta_+(\varphi^* - \varphi) \text{ if } \varphi^* - \varphi \geq 0 \\ \mathfrak{g}^* - \delta_-(\varphi - \varphi^*) \text{ if } \varphi^* - \varphi < 0 \end{array} \right]$

with $\delta_+, \delta_{\setminus} > 0$. Notice that the linear decrement function has not the same slope on the left and on the right of u = 0. In contrast, in the symmetric case, $\mathfrak{g} [\varphi] = \mathfrak{g}^* - \delta [\varphi^* - \varphi]$ if $\delta = \delta_+ = \delta_{\setminus} > 0$.

Example 2: a flat value expectancy (in the small). For example, the quadratic value expectancy $\mathfrak{g}[\varphi] = n\varphi - (1/2)\varphi^2$, with $\varphi^* = n$ and $\mathfrak{g}^* = \mathfrak{g}[\varphi^*] = n^2/2$. Then,

$$\mathfrak{g}\left[\varphi\right]=n^2/2-\left[n^2/2-n\varphi+(1/2)\varphi^2\right]=\mathfrak{g}^*-\left(1/2\right)\left[\varphi^*-\varphi\right]^2.$$

All these value expectancies $\mathfrak{g}[\varphi] = \mathfrak{g}_n[\varphi]$ depend of the size $\varphi^* = n$ of the need. We will see elsewhere how a sharp value expectancy greatly helps for the construction of SMART goals.

4.3 Outcome expectancy.

An activity represents how much of a given thing you are doing. For example, if this thing is bread, eating $y \in \mathbb{R}_+$ units of bread is an activity. To fulfill a need you must do something. For example, if you are hungry, you need to eat some quantity y of bread. An outcome expectancy represents the expected level of fulfilment of a need, $\varphi(y) \in \mathbb{R}_+$, that results from doing the level of activity $y \in \mathbb{R}_+$. For example, $\varphi = \varphi(y) = \mu y \in \mathbb{R}^+$ or $\varphi = \varphi(y) = \mu \sqrt{y} \in \mathbb{R}^+$, with $\mu \in \mathbb{R}_{++}$.

4.4 Valence expectancy.

Contrasting the valence of the end of a move with the valence of this move. As Lewin said the valence of an object (or of using that object) "usually derives from the fact that the object is a means to the satisfaction of a need, or has indirectly something to do with the satisfaction of a need" (Lewin,1935, p.78). The VR approach gave a general formulation of this powerful but too vague definition. It contrasts the valence $g(y) \in \mathbb{R}$ of the end y of a move $m = (x, \omega, y)$ (the usual case in the literature) with the valence $g(m) \in \mathbb{R}$ of the move m that helps to do y, through a translocation ω . In the separable case the valence $g(m) = g_e(y) + g_t(\omega)$ of a move $m = (x, \omega, y)$ is the sum of, i) the valence $g_e(y)$ of its end y and of, ii) the valence $g_t(\omega)$ of its translocation ω . See section 8 for this important extension. It helps to model, among other aspects, intrinsic motivation.

Valence of the end of a move. Formally, a valence expectancy is the satisfaction level $g(y) = \mathfrak{g}\left[\varphi(y)\right] \in \mathbb{R}$ derived from doing some activity level y that helps to fulfill a need. It is a composite function of a value expectancy $\mathfrak{g} = \mathfrak{g}\left[\varphi\right]$ with an output expectancy $\varphi = \varphi(y)$. This mathematical construction is new.

Example 1: sharp valence expectancy. For example if, i) the value expectancy is $\mathfrak{g}\left[\varphi\right] = \mathfrak{g}^* - \delta_+(\varphi^* - \varphi)$ if $\varphi^* - \varphi \geq 0$ and $\mathfrak{g}\left[\varphi\right] = \mathfrak{g}^* - \delta_-(\varphi - \varphi^*)$ if $\varphi^* - \varphi < 0$ with $\delta_+, \delta_- > 0$, and, ii) the outcome expectancy is $\varphi = \varphi(y) = \mu y, \mu > 0, \ n = \varphi^* = \varphi(x^*) = \mu x^*$ and $g^* = g(x^*) = \mathfrak{g}^*$, the valence expectancy is,

$$g(y) = \left[\begin{array}{c} \mathfrak{g}^* - \delta_+ \mu(x^* - y) \text{ if } x^* - y \ge 0 \\ \mathfrak{g}^* - \delta_{\searrow} \mu(y - x^*) \text{ if } x^* - y < 0 \end{array} \right].$$

Example 2: flat valence expectancy. For example if, i) $\mathfrak{g}[\varphi] = n\varphi - (1/2)\varphi^2$ and, ii) $\varphi = \varphi(y) = \mu y, \mu > 0$, then, $g(y) = \mathfrak{g}[\varphi(y)] = g^* - (1/2)[n - \varphi(y)]^2$.

Remark. We will show later that the famous goal gradient hypothesis has to do with a non linear sharp valence expectancy.

4.5 Unrealistic or realistic aspiration levels. Reaching desires

This distinction has been done by Lewin et al. (1944). We go a little further.

- A) We contrast two kinds of aspiration levels in the space of level of fulfilment of a need. The first one (ideal level) is the highest level of fulfilment of a need that an individual can hope. It is the size of this need $n \in \mathbb{R}_+$. It can be unrealistic if the size of this need is too high, given resource constraints. For example, to be the goat in tennis. The second one is the highest level of fulfilment of a need that can be reached through doing a feasible bundle of activities $y \in X$. That is, $\varphi^* = \sup \{ \varphi(y), y \in X \} < +\infty$. It is more realistic. Then, $\varphi^* \leq n$.
- B) We also contrast two kinds of aspiration levels in the space of levels of satisfaction derived from the fulfilment of a need. The first one is the highest level of satisfaction that an individual can hope. That is, $\mathfrak{g}^* = \mathfrak{g}(\mathfrak{n})$. It can be unrealistic. The second one is the highest level of satisfaction derived from the fulfilment of a need that can be reached through doing a feasible bundle of activities $y \in X$. It is realistic. That is, $g^* = \sup \{g(y) = \mathfrak{g}[\varphi(y)], y \in X\} < +\infty$. Then, $g^* \leq \mathfrak{g}^*$.

Aspirations of the poors (Appadurai, 2004) provide a good example of unrealistic aspirations.

Desire. Given the aspiration level $g^* < +\infty$, the desire to do something is a bundle of activities $x^* \in X$ such that $g^* = g(x^*)$. When desires exist, they define more realistic levels of aspiration.

4.6 Self efficacy expectancy: expected cost to be able to change and, then, change

Bandura (1977, 1997) has defined self-efficacy as people's beliefs in their capabilities to exercise control over their own functioning and over events that affect their lives. This concept represents a personal judgment of "how well one can execute courses of action required to deal with prospective situations". It determines how well one can execute a plan of action. That is, it is a person's belief in their ability to succeed in a particular situation. See Vancouver (2018) for the unifying role of self efficacy expectancies.

In the simple context of one need- one activity, the VR approach models self efficacy expectancy as expected costs $C(m) = C(x,y) \in \mathbb{R}_+$ to do a move m = (x,y). This is a complicated concept. It includes expected costs to become able to move from having done $x \in X = \mathbb{R}_+$ to do $y \in X = \mathbb{R}_+$ (expected capability costs) and expected costs to do this move (expected competency costs). Given that the VR approach defines a capability as being able or becoming able to acquire means and the abilities to use them, capability costs are the sum of two costs: costs to acquire means and costs to become able to use them. Costs to acquire means refer in turn to costs to find these means, to buy them or

to build them, including repairing them. Costs to become able to use means include learning costs and training costs. All these costs are the sum of costs to become able to stop, continue and start doing things and, then, costs to actually stop, continue and start doing these things. They refer to costs of disengagement, reengagement and engagement in different activities. A simple example of expected costs to do the move $m = (x, y) \in \mathbb{R}^2_+$ is,

$$C(m) = C(x,y) = \begin{bmatrix} c_{=}x + c_{+}(y-x), & y-x \ge 0 \\ c_{=}y + c_{\sim}(x-y), & y-x < 0 \end{bmatrix},$$

where $c_{=}$, c_{+} and $c_{\sim} > 0$ represent unit cost to stop, continue and start doing an activity. Notice that expected costs to do more of a thing are different from expected costs to do less. Costs to move are not symmetrical. Such costs represent a partial quasi distance (Soubeyran, 2021c).

More generally, expected costs to do a move can be sharp or flat, linear, concave or convex,

5 The positive and negative aspects of change

5.1 Discrepancies as variations.

5.1.1 Definitions

Discrepancies are prominent concepts in self-regulation theory. For example, think about discrepancy production and discrepancy reduction (Bandura & Locke, 2003). But they has not been formalized. The VR approach fills this deficiency. For a dynamical system, to simplify say an individual, a first important problem is to compare his present state with some future possibly better state. In this context, discrepancies (variations) define differences between where this individual presently is and where he plans to be later.

Consider the previous period and the current period. A move $m=(x,y)\in X.X$ goes from $x=\underline{x}=$ having done the bundle of activities $x\in X$ in the previous period to y= doing the bundle of activities y in the current period. To better know if we prefer to change or to stay, we will compare the consequences of a change $m=(x,y), y\neq x$ with the consequences of a stay $\sigma=(x,x)$. Then, in the short run, we can list the following concepts.

Expected advantage to move. It defines, in the current period, the difference between the expected valence to move m=(x,y) and the expected valence to stay $\sigma=(x,x)$, i.e., $A(m/\sigma)=g(m)-g(\sigma)$. This is a fundamental discrepancy in psychology between where you are (at an uncomfortable status quo σ) and where you want to be, at a more desired temporary end, that is, at the end y of the move m.

Aspiration gap. It defines the highest expected advantage to do a move m=(x,y) compared with staying at the status quo $\sigma=(x,x)$. That is, $A^*(\sigma)=\sup\{A(m/\sigma), m(x,y), y\in X\}<+\infty$.

Aspiration gaps are not easy to define. This is clear in the context of poor peoples who suffer from aspiration failures when individuals have no idea about their ideal state, or when, in the present case, the aspiration gap between their present state and their ideal state (seemingly unrealistic) is too large compared with their current resources (Appadurai 2004, Ray, 2003). This pushes them to give up reaching a better position. In the diet example (losing or gaining weight), this occurs, for example, when, to lose weight, an individual must decrease too much his consumption of chocolate.

Expected inconvenience to move. It represents, in the current period, the difference between the expected cost to move and the expected cost to stay, $I(m/\sigma) = C(m) - C(\sigma)$. That is, $I(m/\sigma) = C(x,y) - C(x,x) \in \mathbb{R}_+$.

Motivation to move. It models, in the current period, the expected utility $U[A] \in \mathbb{R}_+$ of the expected advantage to move (change rather than stay) $A = A(m/\sigma) \in \mathbb{R}_+$. That is, $M = M(m/\sigma) = U[A(m/\sigma)] \in \mathbb{R}_+$. In psychology, the classic definition of motivation is any internal process that energizes, directs, and sustains behavior (Reeve, 2016). Baumeister (2016) suggested that motivation is wanting change. However, a little before the VR approach anticipates, generalizes and formalizes this definition (Soubeyran, 2009, 2010): motivation is wanting move: change or stay, i.e., change with respect to stay, stay with respect to change. See Soubeyran (2021.b) for an an in-depth study of the concept of motivation.

Resistance to move. It represents, in the current period, the expected disutility D[I] of the expected inconvenience to move $I = I(m/\sigma)$. That is, $R = R(m/\sigma) = D[I(m/\sigma)]$.

Lewin (1951) is the father of the concept of resistance to change in the context of his force-field analysis. He believes that we should think about any change situation as factors including driving forces (motivation to change) acting to change the current condition and resisting forces acting to inhibit change (resistance to change). In the VR approach these resisting forces model obstacles that pave the way from where an individual is (unmet needs) to where he wants to be (needs fulfilled). These obstacles represent lacks of physiological, physical, material, financial, cognitive, motivational, emotional and social resources (= means and the abilities to use these means to do different things).

5.1.2 The "lose or gain" weight example

Expected valence. In the ideal weight model the quantities of chocolate consumed in the previous and current periods are $x, y \in X = \mathbb{R}_+$. In this simple model, the expected valence of a move m = (x, y) is equal to the expected valence of its end y, i.e., g(m) = g(y). A quadratic formulation gives $g(y) = ny - (1/2)y^2$ where $n = x^*$ defines the ideal weight as well as the related ideal quantity of chocolate. This optimal quantity x^* maximizes g(y) as explained before.

Expected cost to move. In the ideal weight model the expected cost to be able to move and then move from eating not enough (too much) chocolate to eating more (less) is,

to eating more (less) is,
$$C(m) = C(x,y) = \left[\begin{array}{c} c_{=}x + c_{+}(y-x), \ y-x \geq 0 \\ c_{=}y + c_{\sim}(x-y), \ y-x < 0 \end{array} \right].$$

Expected advantage to move. It is, if the valence of a move is equal to the valence of its end g(m) = g(y), $A(m/\sigma) = g(y) - g(x) = f(x) - f(y)$.

In the ideal weight model $A(m/\sigma) = g(y) - g(x) = n(y-x) - (1/2)(y^2 - x^2)$ with $x^* = n$.

Expected inconvenience to move. In the leading example given before,

$$I(m/\sigma) = I(y/x) = \begin{bmatrix} \rho_+(y-x), \ y-x \ge 0 \\ \rho_{\nwarrow}(x-y), \ y-x < 0 \end{bmatrix} = q(x,y).$$
 with $\rho_+ = c_+$ and $\rho_{\nwarrow} = c_{\nwarrow} - c_=$. We will suppose $\rho_{\nwarrow} > 0$.

In this example $q(.,.):(x,y)\in X.X\longmapsto q(x,y)\in R_+$ is a quasi distance. Then, q(x,x)=0 for all $x\in X$ means that the inconvenience to stay is zero. It can be a w distance, a Bregman distance, a quasi norm,To save space, see a lot of justifications given in Soubeyran (2021.a,b,c,d).

Motivation to move. $M(m/\sigma) = U[A(y/x)] = A(y/x)^{\alpha}, \alpha > 0$. In this example motivation is strong in the small and weak in the large if $0 < \alpha \le 1$. It is weak in the small and strong in the large if $\alpha > 1$. To better see this, draw a figure of the function A^{α} for $A \in \mathbb{R}_{+}$. In the diet example where $\alpha = 1$, $M(m/\sigma) = g(y) - g(x).$

Resistance to move. $R(m/\sigma) = R(y/x) = D[I(y/x)] = I(y/x)^{\beta}, \beta > 0.$ In this example resistance is strong in the small and weak in the large if $0 < \beta \le 1$. It is weak in the small and strong in the large if $\beta > 1$. In the ideal weight example where $\beta \in \{1,2\}$ and I(y/x) = q(x,y), $R(m/\sigma) = q(x,y)^{\beta}$.

5.2Balances as equations of motion

Balances weight the positive and negative aspects of change.

5.2.1Definitions.

Two balances drive need satisfaction-need frustration dynamics. To meet needs provides satisfaction feelings but this also causes inconveniences. Given resource constraints, each period, there is a desirability feasibility trade off (I want -I can) between,

- i) I want to fulfill each need sufficiently and quickly enough and,
- ii) I must become able to meet these needs, and then, I must meet these needs at a sufficiently low cost.

This models the fundamental "I want-I can" trade off that drives the dynamic leading to success or failure in need satisfaction.

Long term satisficing balance.

It can be defined in two different spaces:

A) in the space of need fulfilment levels, a satisficing balance is the difference between the improvement in need fulfilment $\varphi(y) - \varphi(x)$ and the sufficient portion $0 < \theta \le 1$ of the aspiration gap $\varphi^* - \varphi(x) = n - \varphi(x) \in \mathbb{R}_+$, that the move from x to y is asked to fulfill, i.e., $\Pi_{\theta}(m/\sigma) = [\varphi(y) - \varphi(x)] - \theta [\varphi^* - \varphi(x)]$.

B) in the space of satisfaction levels, a satisficing balance is the difference between the advantage to move g(y)-g(x) and the sufficient portion $0 < \theta \le 1$ of the aspiration gap $g^* - g(x) \in \mathbb{R}_+$ that the move from x to y is asked to fulfill, i.e., $\Pi_{\theta}(m/\sigma) = [g(y) - g(x)] - \theta [g^* - g(x)]$. The term θ defines a sufficient rate of progress (Carver & Scheier, 1990).

The aspiration gap $g^*-g(x)=\sup\{g(y)-g(x),y\in X\}<+\infty$ measures two things: i) the highest level of the advantage to change you can get when starting from the status quo x, and, ii) the size of the frustration feeling $f(x)=g^*-g(x)$ you endure if you stay at the status quo x. From the equality between the improvement in satisfaction $g(y)-g(x)=f(x)-f(y)\geq 0$ and the reduction in frustration feeling f(x)-f(y), the satisficing balance is $\Pi_{\theta}(m/\sigma)=[f(x)-f(y)]-\theta f(x)=(1-\theta)f(x)-f(y)$.

In the more general case where g=g(m), the satisficing balance generalizes to $\Pi(m/\sigma)=g(m)-g(\sigma)-\theta\left[g^*-g(m)\right]$.

5.2.3 Short term worthwhile balance.

In the current period a worthwhile balance between motivation and resistance to move is $B_{\xi} = B_{\xi}(m/\sigma) = M(m/\sigma) - \xi R(m/\sigma)$, where the weight $\xi > 0$ represents the relative importance given to the inconvenience to move compared to the advantage.

This second balance has a lot to do with the celebrated Lewin's force field analysis and his theory of change management (Lewin, 1947, Cummings et al., 2016). It helps to compare two opposing sets of forces: driving forces that promote change and restraining forces that attempt to maintain the status quo.

Three concepts of motivation. The VR approach makes a clear distinction between three concepts of motivation. The first concept is, in the short run, "crude motivation". It is, in the current period, the utility of the advantage to move $M = M(m/\sigma) = U[A(m/\sigma)]$, that is, wanting to move. The second concept is, in the short run, the "force of motivation". It refers to the worthwhile balance $B_{\xi} = B_{\xi}(m/\sigma) = M(m/\sigma) - \xi R(m/\sigma)$. This terminology is justified because our formulation of a worthwhile balance generalizes in a lot of directions and quite surprisingly the concept of "force for a locomotion" given in Lewin (1938, page 107, equation 32). In his formula $f_{P,G} = F(Va(G)/e_{P,G})$, the force $f_{P,G}$ is a function of the valence Va(G) and of the relative position of P and P (distance between the position P of an individual and the goal P can be space, we will better show this connection elsewhere. The third concept of motivation refers, in the long run, to an aspiration gap $P(x) = P(x) \ge 0$ (Lewin et al., 1944).

5.2.4 The ideal weight example

If we summarize what has been done before, in the ideal weight example we have: $A(m/\sigma) = g(y) - g(x)$, $I(m/\sigma) = C(x,y) - C(x,x) = q(x,y)$, $M(m/\sigma) = A(m/\sigma)^{\alpha} = g(y) - g(x)$ if $\alpha = 1$ and $R(m/\sigma) = I(m/\sigma)^{\beta} = q(x,y)^{\beta}$, $\beta > 0$. All this give,

- i) the satisficing balance $\Pi_{\theta}(m/\sigma) = [g(y) g(x)] \theta [g^* g(x)] = (1 \theta)f(x) f(y);$
 - ii) the worthwhile balance $B_{\xi}(m/\sigma) = g(y) g(x) \xi q(x,y)^{\beta}$, with $\xi > 0$.

Furthermore, in the numerical case where $x^* = n$, $g(y) = x^*y - (1/2)y^2$, we have (see before) $g(y) - g(x) = x^*(y-x) - (1/2)(y^2 - x^2)$ and $f(x) = g^* - g(x) = (1/2)\left[x^* - x\right]^2$ with $q(x,y) = \begin{bmatrix} \rho_+(y-x), \ y-x \ge 0 \\ \rho_{\searrow}(x-y), \ y-x \le 0 \end{bmatrix}$. Then,

- i) the satisficing balance is $\Pi_{\theta}(m/\sigma) = (1/2) \left[(1-\theta) \left[x^* x \right]^2 \left[x^* y \right]^2 \right]$ and,
 - ii) if resistance to move is strong ($\alpha = 1, \beta = 1$), the worthwhile balance is,

$$2B_{\xi}(m/\sigma) = a_{+}(y-x) - (y-x)^{2}$$
 if $y-x \ge 0$;

$$2B_{\xi}(m/\sigma) = a_{\xi}(x-y) - (x-y)^2$$
 if $y - x < 0$,

with
$$a_{+} = 2(x^* - x - \xi \rho_{+})$$
 and $a_{\sim} = 2(x - x^* - \xi \rho_{\sim})$.

These satisficing and worthwhile balances will help to define satisficing and worthwhile moves. They represent, in the ideal weight example, the equations of motion of a need satisfaction-need frustration problem. Look at Soubeyran (2022) to see what is going on when resistance to move is weak ($\alpha = 1, \beta = 2$).

6 Satisficing and worthwhile moves

Two kinds of moves help in the success of need satisfaction dynamics: satisficing moves with a long run perspective and worthwhile moves with a short run perspective.

6.1 Satisficing moves

Definition. A satisficing move m=(x,y) is such that its satisficing balance between stay and change $\Pi_{\theta}(m/\sigma)=[g(y)-g(x)]-\theta[g^*-g(x)]\in\mathbb{R}_+$ is non negative.

If, within the current period, we choose to do the stay $\sigma=(x,x)$, the level of frustration $f(x)=g^*-g(x)\in\mathbb{R}_+$ will remain the same as in the previous period. If we choose to do the change m=(x,y), the improvement in the satisfaction level will be $g(y)-g(x)\in\mathbb{R}_+$ and the level of frustration will decrease from $f(x)=g^*-g(x)\geq 0$ to $f(y)=g^*-g(y)\geq 0$.

A satisficing move requires that the advantage $g(y) - g(x) \in \mathbb{R}_+$ to do the move m = (x, y) is high enough (larger than $0 < \theta \le 1$) relative to the aspiration gap $g^* - g(x) \in \mathbb{R}_+$. Remind that the aspiration gap is the

largest advantage to move that an individual can hope to get, when starting from the status quo x. Notice that θ defines a given reference rate of progress (= reference value or standard) that this individual wants to obtain when moving. For the meta monitoring origins of this threshold level θ see Carver & Scheier (1990). Then, a satisficing move requires a sufficient rate of progress $[g(y)-g(x)]/[g^*-g(x)]=[f(x)-f(y)]/f(x)\geq\theta$ when moving from x to y. That is, it demands a sufficient decrease in frustration feelings $0 \leq f(y)/f(x) \leq 1-\theta$ given that g(y)-g(x)=f(x)-f(y) and $\Pi_{\theta}(m/\sigma)=[f(x)-f(y)]-\theta f(x)=(1-\theta)f(x)-f(y)\geq 0$ iff $0 \leq f(y) \leq (1-\theta)f(x)$.

The importance given to frustrations. When there are several needs (as we will see later and elsewhere), a satisficing move is such that the satisfaction to meet some needs without great delay is high enough relative to the frustration of having to wait too long to meet others needs or to give up. In this way, each period, frustration pushes to speed up the rate of need fulfilment, i.e., it motivates to avoid being frustrated for too long. That is, the urge to reduce cumulated frustrations works in favor to a sufficient rate of progress. This simple observation will provide a new route to model and explain (elsewhere) the goal gradient hypothesis (Hull, 1932): the more frustrated, the more painful it is to wait, the more we accelerate.

Emotional aspects linked to do a satisficing move. Satisficing gives rise to several kinds of emotions, relative to the consideration of where you are and to where you want to be:

- before satisficing: i) an initial frustration feeling of failing to eat, in the previous period, the ideal quantity of chocolate and, ii) the hope, in the current period, to make sufficient progress in this direction;
- after satisficing: i) an ex post frustration feeling of failing to eat the ideal quantity of chocolate in the current period as well as, ii) a frustration feeling of not having made sufficient progress in this direction in the current period.

A set of ends of satisficing moves. It is $S_{\theta}(x) = \{y \in X, \ \Pi_{\theta}(m/\sigma) \geq 0\}$, with $\Pi_{\theta}(m/\sigma) = [g(y) - g(x)] - \theta [g^* - g(x)], \ m = (x, y) \text{ and } \sigma = (x, x).$

6.2 Worthwhile moves

Definition. A worthwhile move $m = (x, y) \in X.X$ is such that its worthwhile balance is non negative, i.e., $B_{\xi}(m/\sigma) = M(m/\sigma) - \xi R(m/\sigma) \in \mathbb{R}_+$. It requires that motivation to move is high enough compared with resistance to move, i.e., $M(m/\sigma)/R(m/\sigma) \ge \xi > 0$. Then, it demands that the advantage is high enough compared with the inconvenience. That is, a move is worthwhile if the improved satisfaction level coming from the fulfilment of some needs compensates enough the added sacrifice required to become able to fulfill them, and to, finally, fulfill them. In this way a worthwhile move helps to avoid excessive sacrifices, i.e., the disutility of the inconvenience to move $R(m/\sigma)$, compared with the utility of a

sufficient improvement in satisfaction feelings $M(m/\sigma)$ in order to fulfill needs quickly enough.

The concept of worthwhile move unifies the theory of intentions. The most basic question that human dynamics can pose is the following: should I stay, or should I change?. This is an indecision problem where an individual hesitates between stay or change. In this setting, coexist, within an individual, ambivalent (positive and negative) feelings toward the same thing (person, object, or action), simultaneously drawing him or her in opposite directions. Here the thing is a move (stay or change) and the confrontational feelings are motivation and resistance to move. In this context, the VR concept of worthwhile move provides a unifying and simple answer to such an indecision problem (Soubeyran, 2021.d). A non negative worthwhile balance means that if a move is worthwhile, it cannot be rejected without a careful examination, until you find a better one, with a largest worthwhile balance. An intention refers to an intention to do something; i.e., an intention to move, from having done something to do another or the same thing. Bandura & Simon (1977) defined an intention as an action goal. However, despite its great merits, this definition remains too vague. For us, an intention represents, at the same time the end y of a worthwhile move $m = (x, \omega, y)$ and its translocation ω , i.e., the desired end and the way to this end. Having in mind that a goal must be desirable and feasible enough (Oettingen & Gollwitzer, 2015), this means that the end y must be desirable enough and that the translocation ω leading to this end must be feasible enough. Then, if a move is worthwhile, the expected improvement in satisfaction is high enough compared with the expected sacrifice. Thus, the size h>0 of a worthwhile balance $B_{\xi}(m/\sigma)\geq h$ defines the force of an intention. In other words, an individual will seriously consider changing when driving forces are high enough relative to resisting forces, thus shitting the equilibrium (Lewin, 1935,1936, 1938, 1951).

A set of ends of worthwhile moves. It is $W_{\xi}(x) = \{y \in X, B_{\xi}(m/\sigma) \geq 0\}$ with m = (x, y) and $\sigma = (x, x)$.

6.3 Examples of satisficing moves and worthwhile moves.

Consider the numerical ideal weight example given before. To save space, we will consider the case $\alpha=\beta=1$ with strong resistance. For the weak resistance event $\alpha=1,\beta=2$, see Soubeyran (2022). In this first case, if m=(x,y) and $\sigma=(x,x)$, a non negative satisficing balance (A) and a non negative worthwhile balance (B) are, with x given,

(A)
$$2\Pi_{\theta}(m/\sigma) = (1-\theta) [x^* - x]^2 - [x^* - y]^2 \ge 0;$$

(B) $2B_{\xi}(m/\sigma) = \begin{bmatrix} a_+(y-x) - (y-x)^2 \ge 0 & \text{if } y-x \ge 0 \\ a_{\sim}(x-y) - (x-y)^2 \ge 0 & \text{if } y-x < 0 \end{bmatrix}.$
Then,
 $S_{\theta}(x) = \{ y \in X = \mathbb{R}_+, |x^* - y| \le (1-\theta)^{1/2} |x^* - x| \}$ and $W_{\xi}(x) = \{ y \in X = \mathbb{R}_+, y-x \le a_+ & \text{if } y-x \ge 0 \\ x-y \le a_{\sim} & \text{if } y-x < 0 \},$
That is,

$$S_{\theta}(x) = \{ y \in X = \mathbb{R}_{+}, \ 0 < x_{\setminus} \leq y \leq x_{+} \ \},$$
 with $x_{\setminus} = x^{*} - (1 - \theta)^{1/2} | x^{*} - x |$ and $x_{+} = x^{*} + (1 - \theta)^{1/2} | x^{*} - x |$.
$$W_{\xi}(x) = \left\{ \begin{array}{c} y \in X = \mathbb{R}_{+}, \ x \leq y \leq \overline{x} = x + a_{+} \text{ if } x \leq x^{*} \\ \underline{x} = x - a_{\setminus} \leq y < x \text{ if } x > x^{*} \end{array} \right\},$$
 with $a_{+} = 2(x^{*} - x - \xi \rho_{+})$ and $a_{\setminus} = 2(x - x^{*} - \xi \rho_{\setminus})$.

Then, in the ideal weight example, these two sets represent intervals. Success (failure) in need satisfaction requires that these two intervals intersect (do not).

7 Finding the inequations of motion leading to success or failure in meeting needs

At this stage of our presentation we are in a good position to address the main question that will help to find the inequations of motion of a need satisfaction-need frustration problem: what kind of move should we make, each period, to become able to fulfill our needs sufficiently and quickly enough in the long run, without too large sacrifices in the short run.

7.1 Preferring to change within the current period

7.1.1 Making, in the current period a move, at the same time satisficing and worthwhile

At the beginning of the current period k+1, an individual will prefer to change rather than to stay if he can find and if he can do a move (x^k, x^{k+1}) ,

i) making sufficient progress in need satisfaction in the long run, i.e., such that,

$$x^{k+1} \in S_{\theta_{k+1}}(x^k) = \left\{ y \in X, \ g(y) - g(x^k) \ge \theta_{k+1} \left[g^* - g(x^k) \right] \right\}, \text{ with } 0 < \theta_{k+1} < 1;$$

ii) without enduring too big sacrifices in the short run, i.e., such that, $x^{k+1} \in W_{\xi_{k+1}}(x^k) = \{y \in X, \ g(y) - g(x^k) \ge \xi_{k+1} q(x^k, y)^{\beta} \}$, with $\xi_{k+1} > \beta > 0$.

That is, he must make a move at the same time satisficing and worthwhile. This defines the satisficing without too many sacrifices dynamical system $x^{k+1} \in \Gamma_{k,k+1}(x^k) = S_{\theta_{k+1}}(x^k) \cap W_{\xi_{k+1}}(x^k) \neq \phi$.

7.1.2 Success in making enough progress in gaining weight, without too much sacrifices.

To be concrete, consider first the ideal weight example where the initial weight is too low with respect to the ideal weight. This occurs when you do not eat enough relative to the ideal level of food x^* , i.e., if $0 < x^k < x^*$. Then, starting from eating the quantity x^k of food, the problem is to know when eating more, i.e., $x^{k+1} - x^k \ge 0$ is the end of a satisficing and a worthwhile move (x^k, x^{k+1}) . The set of ends y of all worthwhile and satisficing moves (x^k, y) starting from x^k is the intersection set $\mathcal{W}^+_{\xi_{k+1}}(x^k) \cap \$^+_{\theta_{k+1}}(x^k)$, where

 $\mathcal{W}^+_{\xi_{k+1}}(x^k) = \left\{ y \in \mathbb{R}_+, \ x^k \le y \le \overline{x}^k \right\}, \ \$^+_{\theta_{k+1}}(x^k) = \left\{ y \in \mathbb{R}_+, \ x^k \le y \le x^k_+ \right\}$ with $\overline{x}^k = x^k + 2(x^* - x^k - \xi_{k+1}\rho_+) \ge x^k, \ x^k_{\searrow} = x^* - (1 - \theta_{k+1})^{1/2}(x^* - x^k)$ and $x^+_+ = x^* + (1 - \theta_{k+1})^{1/2}(x^* - x^k).$

Let $s_+^k = (x^* - x^k)/\rho_+ > 0$ and $\pi(\theta) = (1/2) [1 + (1 - \theta)^{1/2}] \in [1/2, 1]$, with $\pi(0) = 1, \pi(1) = 1/2$ and $\pi(.)$ strictly decreasing.

Result.1. Self-regulation success (= to gain enough weight) occurs when there exists a worthwhile move that provides a given satisficing rate of progress θ_{k+1} . That is, when the intersection $\mathcal{W}^+_{\xi_{k+1}}(x^k) \cap \$^+_{\theta_{k+1}}(x^k)$ is not empty, i.e., iff $\xi_{k+1} < s^k_+ \pi(\theta_{k+1})$ (\star).

iff $\xi_{k+1} \leq s_+^k \pi(\theta_{k+1})$ (\star). Proof: $\$_{\theta_{k+1}}^+(x^k)$ and $\mathcal{W}_{\xi_{k+1}}^+(x^k)$ are not empty. Then $\mathcal{W}_{\xi_{k+1}}^+(x^k) \cap \$_{\theta_{k+1}}^+(x^k) \neq \phi$ iff the intersection of the two following intervals

 $\{y \in R_+, x^k \le y \le \overline{x}^k\} \cap \{y \in R_+, x^k \le y \le x_+^k\} = \{y \in R_+, x^k \le y \le \overline{x}^k\} \ne \phi$ is not empty. That is, if $\overline{x}^k \ge x^k$. That is, after some manipulations, iff $\xi_{k+1} \le s_+^k \pi(\theta_{k+1})$.

To save space, comments about the condition (\star) will be made in the diet example, when the problem is to lose weight.

7.1.3 Success in making enough progress in losing weight, without too big sacrifices

In this second case the initial weight is too high relative to the ideal weight, i.e., the individual eats too much, relative to the ideal level of food, i.e., $x^k > x^*$. Then, $\$_{h+1}^*(x^k) = \{y \in \mathbb{R}_+, \ 0 < x_+^k \le y \le x_+^k\}$ and

Then,
$$\$_{\theta_{k+1}}^{\sim}(x^k) = \{y \in \mathbb{R}_+, \ 0 < x_-^k \le y \le x_+^k\}$$
 and $\mathcal{W}_{\xi_{k+1}}^{\sim}(x^k) = \{y \in \mathbb{R}_+, \ \underline{x}^k \le y \le x_+^k\}$, with $x_-^k = x^* - (1 - \theta_{k+1})^{1/2}(x^k - x^*)$, $x_+^k = x^* + (1 - \theta_{k+1})^{1/2}(x^k - x^*)$, and $\underline{x}^k = x^k - 2(x^k - x^* - \xi_{k+1}\rho_{\sim})$. Let $s_-^k = (x^k - x^*)/\rho_{\sim} > 0$.

Result.2. Self-regulation success (= to lose enough weight) occurs when the intersection set $\S_{\theta_{k+1}}^{\sim}(x^k) \cap \mathcal{W}_{\xi_{k+1}}^{\sim}(x^k)$ is not empty, i.e., iff $\xi_{k+1} \leq s_{k+1}^{\sim}(\theta_{k+1})$

Proof: $\$_{\theta_{k+1}}^{\sim}(x^k) \cap \mathcal{W}_{\xi_{k+1}}^{\sim}(x^k) \neq \phi$ iff $x_+^k \geq \underline{x}^k$. That is, after some manipulations, iff $\xi_{k+1} \leq s_+^k \pi(\theta_{k+1})$.

Comment 1. If $x^k > x^*$, the condition given above $(\star\star)$, i.e., $x^k - x^* \ge \rho \setminus \xi_{k+1} / \pi(\theta)$, proves that need satisfaction is a success because we can approach sufficiently our ideal weight if the aspiration gap (= the difference $x^k - x^* > 0$ between our current weight x^k and our ideal weight x^*) is high enough, the unit cost of becoming able to lose weight and to do it $\rho \setminus$ is low enough, the importance given to resistance to move ξ_{k+1} is low enough and the desired rate of progress $0 < \theta_{k+1} \le 1$ is low enough, given that the function $\pi(.)$ is decreasing.

Comment 2. Self-regulation failure to lose enough weight occurs if the opposite condition $\xi_{k+1} > s_{\cdot}^k \pi(\theta_{k+1})$ holds. In this case a worthwhile move is too small compared with the size of a satisficing move. This shows that to succeed a move must be not too small and not too big.

7.1.4 Other cases

They are:

- a) Reaching desires within the current period, i.e., succeeding in reaching the ideal weight without too much sacrifice. If an individual eats initially too much chocolate, i.e., $x^k > x^*$ (hence if his initial weight is too big), this case occurs when $\underline{x}^k \leq x^* \iff x^k x^* \geq 2\xi_{k+1}\rho_{\sim}$. This happens when the aspiration gap $x^k x^*$ is large enough compared with the importance ξ_{k+1} given to sacrifices and to their size ρ_{\sim} . This surprisingly refers to the very important error bound hypothesis in optimizing algorithms (Karimi et al., 2016). We will examine this case in the compagnon paper.
- b) Failing to satisfy needs sufficiently, like moving without doing enough progress or with enduring too big sacrifices. These situations will be examined elsewhere in the context of variational principles and optimizing algorithms.
- c) If we consider several periods, a way to succeed in need satisfaction is to spread the sacrifices over several periods. But the speed of progress will be lower.

7.2 Preferring to stay within the current period

7.2.1 Being happily stuck in a desired end

As seen in section 3, given the aspiration level $g^* = \sup\{g(y), y \in X\} < +\infty$, a desire to do something is a bundle of activities $x^* \in X$ such that $g^* = g(x^*)$. That is $x^* \in X$ is a desired end because when staying at this desired end "you have what you want" and "you want what you have". Your needs are yet fulfilled. Let $x^k \in X$ be the status quo and let the satisficing set be $S_{\theta_{k+1}}(x^k) = \{y \in X, \ g(y) - g(x^k) \ge \theta_{k+1} \left[g^* - g(x^k)\right]\}$. Then, x^k is a desired end (a desire) iff $x^k \in S_{\theta_{k+1}}(x^k)$ because $y = x^k \in S_{\theta_{k+1}}(x^k)$ iff $0 \ge \theta_{k+1} \left[g^* - g(x^k)\right] \ge 0$. Then, $\theta_{k+1} > 0$ gives $g^* = g(x^k)$. This shows precisely why "you want what you have" and "you have what you want". There is not way to do a satisficing change. This shows that a desire is a fixed point of a satisficing map. This is a very important observation at the mathematical level for the convergence of the process.

7.2.2 Being unhappily stuck in a trap (undesired end)

Definition. In the current period k+1 a stationary trap $x_* \in X$ is such that there is no way, in this current period, to make a worthwhile change, starting from the current status quo $x^k = x_* \neq x^*$, i.e., $W_{\xi_{k+1}}(x_*) = \{x_*\}$. This means that if the status quo is different from the ideal position x^* , there is no way in the current period to do a worthwhile change $m = (x^k = x_*, y), y \neq x^k$, that can help to approach x^* , i.e., there is no way to do a change without too big

sacrifices. Then, motivation to change is too low compared with resistance to change.

Comment. Traps and desires are key concepts of the VR approach. Being stuck in a trap means that even if we want to change, we will prefer to stay and to remain frustrated. In contrast, we will not want to change from a desired end (desire) free of any frustration.

Example. If, in the current period k+1, because of eating too much, i.e., $x^k > x^*$, an individual has gained too much weight, the set of ends of all worthwhile moves starting from x^k is $\mathcal{W}_{\xi_{k+1}}(x^k) = \{y \in \mathbb{R}_+, \underline{x}^k \leq y \leq x^k\}$. In this setting the status quo x^k is a trap if $\underline{x}^k = x^k - 2(x^k - x^* - \xi_{k+1}\rho_{\sim}) \geq x^k$. That is, if $0 < x^k - x^* \leq \xi_{k+1}\rho_{\sim}$. Then, $\mathcal{W}_{\xi_{k+1}}(x^k) = \{x^k\}$. This occurs if, eating initially too much, i.e., $x^k > x^*$, the excess of food $x^k - x^* > 0$ is not too large compared to $\xi_{k+1}\rho_{\sim}$. This is true if the importance $\xi_{k+1} > 0$ given to resistance to move is high and if the unit cost to be able to change and, then, change $\rho_{\sim} > 0$ is high. This condition requires not enough motivation compared to too big sacrifices. A too high unit cost to follow a diet can derive from ego-depletion (Baumeister & Vohs, 2007).

The Lewin's unfreezing, movement, and refreezing model of change. To break the trap of having too much weight, you must lower the importance ξ_{k+1} given to resistance to change and you must lower resistance to change, i.e., the unit cost to ρ_{\sim} to eat less. The Lewin's (Lewin, 1947, 1951, Cummings et al., 2016) three phases model of change maintained that for change to occur, the balance of driving and restraining forces must be altered. The driving forces must be increased or the restraining forces must be decreased. Thus, in order for any change to occur (to break a trap), the driving forces must exceed the restraining forces, thus shifting the equilibrium. This what our formula shows.

7.3 Setting a goal system with promotion and prevention goals to build satisficing and worthwhile moves

To build a need satisfaction dynamic ending in success, the problem is to build, each step, a move that can be, at the same time, a satisficing move and a worthwhile move. But, how to do this? Our answer is that this can be done with the help of setting promotion and prevention goals (Higgins, 1998). Then, promotion and prevention goals will play a major role in our approach.

Building a goal system with promotion and prevention goals. Define, each period, a distal promotion goal and two proximal prevention goals that form a goal system. To save notations we will note $x=x^k, \ y=x^{k+1}, \theta=\theta_{k+1}$ and $\xi=\xi_{k+1}$. In this context,

A) A long term promotion goal $g_{\theta}(x)$ represents a satisficing level (= a driving goal), i.e., a high enough level of satisfaction compared to the level of satisfaction g(x) at the status quo x. It is $g_{\theta}(x) = g(x) + \theta \left[g^* - g(x)\right], 0 \le \theta \le 1$. Then, $y \in S_{\theta}(x) =$

$$\{y \in X, \ g(y) - g(x) \ge \theta [g^* - g(x)]\} \text{ iff } g(y) \ge g_{\theta}(x).$$

The condition $g(y) \ge g_{\theta}(x)$ means that doing a bundle of activities $y \in X$ is acceptable iff the level of satisfaction g(y) at y is large enough compared to the level of satisfaction at the status quo x. This point of view provides a dynamic theory of satisficing (= to satisfy enough) that generalizes the famous satisficing theory (Simon, 1955).

- B) Two short term prevention goals define two threshold levels a,b>0 such that, in the short run,
- i) motivation to move must be not too low: $M[A(m/\sigma)] = A(m/\sigma) = g(y) g(x) \ge a > 0$. The threshold a can be such that $a \ge \theta[g^* g(x)]$;
- ii) resistance to move must be small enough: $0 \le R(m/\sigma) = D[I(m/\sigma)] = q(x,y)^{\beta} \le b$. This condition requires not too large short term sacrifices.

Then, $\xi > 0$ being given, we must adjust the two threshold levels a, b > 0 such that $a \ge \xi b$. Given this choice, an individual will be sufficiently motivated (a high enough), without having to endure too big sacrifices (b low enough). These conditions help to define a worthwhile move such that $g(y) - g(x) \ge a \ge \theta \left[g^* - g(x)\right] \ge \xi b \ge \xi q(x,y)^{\beta}$.

Choose the adequate size of a move, not to small, and not too large. To meet success in need satisfaction, we can,

- i) decrease $\xi > 0$ to increase the size of a worthwhile move. That is, we must accept to do more sacrifices, and
- ii) decrease $0 < \theta \le 1$ to decrease the size of a satisficing move, i.e., we must accept a smaller increase in need satisfaction. This requires to decrease the satisficing level $g_{\theta}(x)$.

The search for the adequate size of a move is strongly related to the Armijo rule in mathematics. See the companion paper and Nocedal & Wright (2006).

Balancing the speed of moving with the size of the sacrifices. The choice of the adequate step size requires to make a succession of trade off between the rate of sacrifice $\xi = \xi_{k+1}$ against the rate of progress $\theta = \theta_{k+1}$. That is, between the rate of sacrifice ξ against the speed of moving, when the outcome expectancy has the adequate shape (flat or sharp, see the companion paper). This is the exact problem of a champion who can be obliged to do a lot of sacrifices when being young, with the hope to become a champion later. This has to do with the famous exploration-exploitation trade off in biology and management science (March, 1991) that will be examined elsewhere.

If an individual wants to lose weight, the inequality $\xi_{k+1} \leq s_{\cdot}^{k} \pi(\theta_{k+1})$ models this trade off. Given that the function $\pi(.): \theta \in [0,1] \longmapsto \pi(\theta) \in \mathbb{R}_{+}$ is strictly decreasing, with $\pi(0) = 1$ and $\pi(1) = 1/2$, the larger the rate of improvement θ (speed of moving) the smaller is ξ , that is, the larger must be the sacrifice to guaranty success in need satisfaction.

Remark 1. Another way to reach success can be to accept temporary sacrifices over one period and to compensate over several periods. That is,

what must be worthwhile is not a move, but a succession of moves. We can also alternate between doing satisficing and worthwhile moves.

Remark 2. The condition $g_{\xi}(x) = \sup \{g(y), y \in W_{\xi}(x)\} \ge g_{\theta}(x)$ tells us that if the highest aspiration level $g_{\xi}(x)$ we can hope to approach and reach when doing a worthwhile move is higher than the satisficing level $g_{\theta}(x)$, then, we can find a worthwhile move that is also a satisficing move.

7.4 Figures

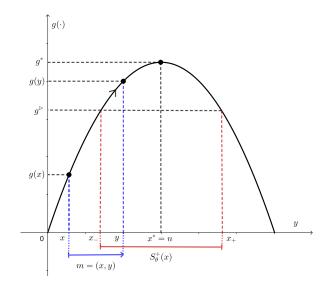


Figure 2: Satisficing move.

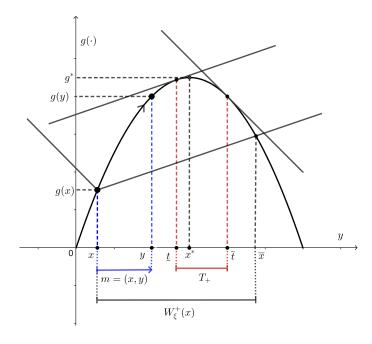


Figure 3: Worthwhile move.

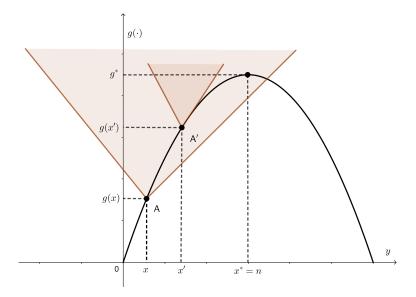


Figure 4: A trap.

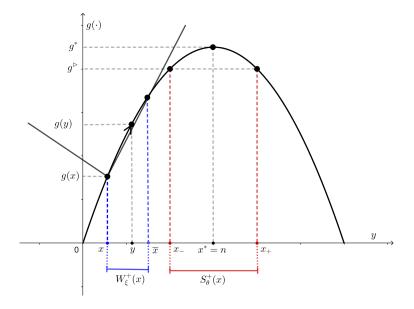


Figure 5: Self regulation failures.

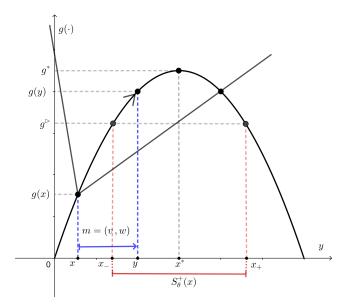


Figure 6: Self regulation success.

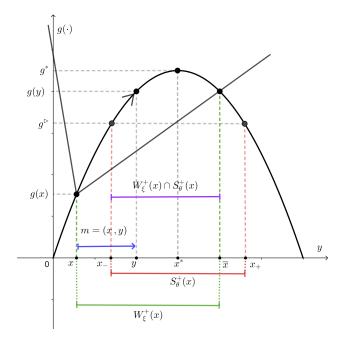


Figure 7: Self regulation success.

8 Some extensions

8.1 Generalized expectancies change with changing internal and external environments

8.1.1 Two first generalizations

Two generalizations concern valence and self efficacy expectancies: we move from the simplest expectancies (g(y), C(x, y)) to generalized expectancies $(g_s(m), C(x, y))$

 $C_s(m)$). Then, the first generalization advocates that a valence depends on a move $m = (x, \omega, y)$. That is, not only of the end y of the move, but also of the translocation ω that this move operates. The second generalization highlights that expectancies change with the internal and external environments s of an individual. This is in accordance with the Lewin's point of view (Lewin, 1936, 1938) relative to the importance of the environment. Then, the two following generalizations A) and B).

- A) Expectancies depend on the changing internal and external situation *s* of an individual. They change with it. For an individual,
- a) his internal situation/environment includes the previous levels of fulfilment of his different needs, his current aspiration levels defined as ideal levels of fulfilment of these needs, satisficing levels that represent the satisfactions derived from sufficient levels of fulfilment of his needs and his capabilities (to be able to acquire means and to use them to do different things).
- b) his external situation/environment plenty, or not, of means that can help to satisfy enough his needs. These means include objects, persons and land-scapes.
- B) Expectancies take care of intrinsic motivation. One of the main lesson of the VR approach is that the valence of a thing (an object) depends not only of the attractiveness of that thing (how much it helps to fulfill needs), but also of the translocation required to catch this thing, depending of the changing situation. More precisely, we have an extrinsic motivation to something, for example, to build an object and to use it, because it helps to fulfill a given need. But doing this thing (build and use an object) can be pleasant for its own sake, providing some intrinsic motivation. This complication shows how expectancies depend of the move $m = (x, \omega, y)$ that starts from the status quo x, follows the translocation ω and finally ends in using this object or in doing this action. In this setting expectancies depend not only of the end of the move y (i.e., $g_s(y)$ as usual), but also of the translocation (transition, path) ω , through an added valence term that changes with ω . In this way, the VR approach is able to model extrinsic and intrinsic motivation (Ryan & Deci, 2000a,b) in the setting of a general model of need satisfaction-need frustration.

8.1.2 Examples

A leading example is $(g_x(y), C_x(x, y))$ where s = x and m = (x, y). It groups a lot of important situations.

Example 1: the separable case. In this situation, the valence relative to the satisfaction of a need $g_s(m) = g(y) + \alpha v(\omega) = g(y) + \alpha v(y - x) \in \mathbb{R}$ is the sum of two terms: the valence $g(y) \in \mathbb{R}$ of the end of the move $m = (x, \omega, y)$ and the valence $v(\omega) \in \mathbb{R}$ of the translocation $\omega = y - x$. The term $\alpha > 0$ is a weight. Then, the valence of the stay $\sigma = (x, 0, x)$ is $g_s(\sigma) = g(x) + \alpha v(0)$.

Example 2: the status quo bias and the curiosity effect. A more general formulation can be $g_s(m) = \mathfrak{g}[\varphi(y)] + \alpha v [\varphi(y) - \varphi(x)]) \in \mathbb{R}$, where $\varphi(x) \in \mathbb{R}_+$ and $\varphi(y) \in \mathbb{R}_+$ are the levels of fulfilment of a need, these levels being derived from doing the bundles of activities x and y. In this way, our VR perspective helps to model several famous bias in psychology:

- a) the status quo bias where, in the first formulation $g_s(m) = g(y) + \alpha v(\omega)$, the visualization of a forthcoming translocation ω (doing different things than before) provides some dissatisfactions and stress $v(\omega) \leq 0$. In this setting an individual does not like to change. Much more can be said about this important bias:
- b) the curiosity effect (loving novelty and discovery) where, in the first formulation, the visualization of a forthcoming translocation ω provides an ex ante satisfaction level $v(\omega) \geq 0$;

Example 3: reference dependent utility functions Kőszegi & Rabin (2006, pp 1138, 1139) used a version of the last formula $g_s(m) = g(y) + \alpha v(\omega)$. That is, $g_s(m) = g_x(y) = \mathfrak{g}\left[\varphi(y)\right] + \alpha v\left[\varphi(y) - \varphi(x)\right] \in \mathbb{R}$, where the numbers $\varphi(x) \in \mathbb{R}$ and $\varphi(y) \in \mathbb{R}$ represent agregate levels of utility derived from the bundles of consumption levels x,y relative to different goods. In this formulation the situation s = x is the status quo and the move $m = (x, \omega, y)$ is limited to its end y:

Example 4. The loss aversion bias. In the second formula $v[\varphi(y) - \varphi(x)]$ is a generalized gain-loss utility that depends on the variation $\varphi(y) - \varphi(x) \in \mathbb{R}$ of the levels of fulfilment of a need. In this VR formulation the move $\mathfrak{m} = (\varphi(x), \omega = \varphi(y) - \varphi(x), \varphi(y))$ is located in the space of levels of fulfilment.

Example 5: a set valued valence expectancy. Suppose that, each period, there are several ways $\omega \in \Omega_s(x,y) \subset \Omega$ to go from having done x to do y. Then, $g_s(x,y) = \{g_s(x,\omega,y), \omega \in \Omega_s(x,y)\} \subset \mathbb{R}^{\bar{j}}$ is a set valued formulation of the valence of the bundle of moves going from x to y. A set valued valence greatly helps to model the presence of uncertainties, without using probabilities. See Qiu et al. (2020).

Example 6: a non cooperative game aspect. In the case of several players $i \in \mathbb{I}$, the expected valence $g^i(x^i, x^{-i}) \in \mathbb{R}$ of player i depends not only of his own action $x^i \in X^i$, but also of the actions $x^{-i} \in X^{-i}$ of the other players.

8.1.3 Generalized satisficing and worthwhile moves

In this changing context, the advantage and the inconvenience to move are, $A_s(m/\sigma) = g_s(m) - g_s(\sigma)$ and $I_s(m/\sigma) = C_s(m) - C_s(\sigma)$. Motivation and resistance to move are, $M_s(m/\sigma) = U_s\left[A_s(m/\sigma)\right]$ and $R_s(m/\sigma) = D_s\left[I_s(m/\sigma)\right]$; A satisficing balance is $\Pi_s(m/\sigma) = g_s(m) - g_s(\sigma) - \theta\left[g_s^*(\sigma) - g_s(\sigma)\right]$; A worthwhile balance is $B_s(m/\sigma) = M_s(m/\sigma) - \xi R_s(m/\sigma)$; The definition of satisficing moves and worthwhile moves follow.

8.2 Generalization to several needs-several activities

Conflicts between driving goals and limited resources. A driving goal (= satisficing level of need fulfilment) defines how much of a need an individual wants and then chooses to fulfill (at least), each period. Then, when several needs are activated, because of resource constraints, there are conflicts between the different driving goals and the activities that must be done to fulfill these needs. Setting driving goals starts the definition of a goal system. The problem being that to fulfill enough some needs (to satisfice) requires to accept not to fulfill enough other needs (to sacrifice). In this context, consider,

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i) several needs i \in \mathbb{I} = \left\{1,2,...,\bar{i}\right\} with their related sizes n = \left\{n^1,n^2,...,n^i,...,\bar{i}\right\} \in \mathbb{R}^{\bar{i}}_+ and ii) several activities j \in \mathbb{J} = \left\{1,2,...,\bar{j}\right\} with their related activity levels y = (y^1,y^2,...,y^j,...y^{\bar{j}}) \in X = \mathbb{R}^{\bar{j}}_+ and some resources constraints y \in K \subset X.
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Different configurations for a goal system. To save space, we consider the unifinal configuration with two needs $i \in \mathbb{I} = \{1,2\}$ and two activities $j \in \mathbb{J} = \{1,2\}$ where activity j is only at the service of the fulfilment of need i=j. The multifinal configuration where one activity serves at the fulfilment of two (several) needs and the equifinal configuration where two (several) activities serve at the fulfilment of one need will be examined in the companion paper.

The interconnected inequations of motion. Then, this (separable) unifinal configuration results in two sets of inequations of motion. Each set serves at the satisfaction of one need, different from the other. These two sets of inequations are connected by a resource constraint. That is,

Aⁱ⁾
$$y^i \in S_{\theta^i}(x^i)$$
, i.e., $\Pi^i(y^i/x^i) = \left[g^i(y^i) - g^i(x^i)\right] - \theta^i \left[g^{i*} - g^i(x^i)\right] \ge 0$;
Bⁱ⁾ $y^i \in W_{\xi^i}(x^i)$, i.e., $B^i_{\xi^i}(y^i/x^i) = \left[g^i(y^i) - g^i(x^i)\right] - \xi^i q^i(x^i, y^i)^{\beta^i} \ge 0$, for $i = 1, 2$.
where $0 < \theta^i \le 1, \beta^i > 0$, $\xi^i > 0$, $i = 1, 2$, and $x = (x^1, x^2), y = (y^1, y^2) \in Y - \mathbb{P}^2$.

with the resource constraint $x^1 + x^2 = y^1 + y^2 = r > 0$.

The resource constraint can forbidd to end in need satisfaction for at least one need. This is a crude example of conflicts between goals. We will examine this point in the second paper.

9 Conclusion

This first paper being done, it remains to end writing the second paper dealing with several needs and the resolution of the inequations of motion of a need satisfaction-need frustration dynamical problem, using generalized variational principles and inexact optimizing algorithms in mathematics. Given the

construction of a valence as a composite function (a value function time an outcome function), algorithms will deal with the very important class of optimization problems with composite objectives. We also consider the case where the aspiration level being unknown, it must be approximated, locally, through the supremum of an approximation function of the objective.

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