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Paul Belleflamme
Wing Man Wynne Lam
Wouter Vergote

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Paul Belleflamme[†] Wing Man Wynne Lam[‡] Wouter Vergote[§]

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Abstract

Two duopolists compete in price on the market for a homogeneous product. They can use a 'profiling technology' that allows them to identify the willingness-to-pay of their consumers with some probability. If both firms have profiling technologies of the exact same precision, or if one firm cannot use any profiling technology, then the Bertrand paradox continues to prevail. Yet, if firms have technologies of different precisions, then the price equilibrium exhibits both price discrimination and price dispersion, with positive expected profits. Increasing the precision of both firms' technologies does not necessarily harm consumers.

Keywords: Price discrimination, price dispersion, Bertrand competition

JEL-Classification: D11, D18, L12, L86

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†Aix-Marseille University, CNRS, EHESS, Centrale Marseille, AMSE. Centre de la Vieille Charité - 2 rue de la Charité, 13002 Marseille, France, Paul.Belleflamme@univ-amu.fr; also affiliated with KEDGE Business School.

‡Department of Economics, University of Liège, Quartier Agora, Places des Orateurs 3 (B31), B-4000 Liege Belgium, wingmanwynne.lam@ulg.ac.be.

§CEREC, University Saint-Louis - Bruxelles, Boulevard du Jardin Botanique 43, B-1000 Brussels, Belgium, vergote@fusl.ac.be; also affiliated with CORE.

1 Introduction

Context. In September 2000, a customer of Amazon.com accused Amazon of price discrimination (a.k.a. differential pricing): this customer realized that after having deleted the cookie that identified him as a regular Amazon customer, he was offered some DVD for a much lower price than the one he observed the first time he visited the web site. The company later apologized and explained that the price difference was not the result of differential pricing but of random price testing.[1]

Both the customer's allegation and Amazon's defense look plausible. Price variations may indeed be an outcome of both price discrimination and price randomization. On the one hand, firms can use random price variation to learn more about the shape of the demand they are facing. When this is the case, consumers with the same valuation receive different prices. On the other hand, the anecdote also suggests that consumers are not always aware that other prices may be available to them, or that even if they are aware, it may be (too) costly to obtain access to a lower price. This paves the way for firms to price discriminate some consumers against others by selling the same good at different prices to different consumers. This practice becomes increasingly prevalent nowadays as advances in digital tracking facilitate more accurate consumer profiling, which in turn helps to facilitate first-degree price discrimination. Shiller (2014) provides empirical evidence that web browsing data gives firms more information on consumer willingness to pay compared to 'old school' demographic data, which increases profits by around 12.2%.[2] Firms are also more open nowadays regarding their use of differential pricing. For instance, Ant Financial Services Group (the financing unit of Alibaba) announced in 2015 the launch of Sesame Credit, a credit-scoring service that leverages big data and customer behavior analytics to calculate personalized interest rates for micro loans or personalized premiums for insurance services.[3] Finally, recent regulatory changes in the U.S. are likely to make customer data even more widely available to firms.[4]

While firms have, in data-rich environments, the opportunity to profile customers, firms often differ in their ability to do so. This could be due to different capacities to collect and analyze data (think of Amazon or Alibaba vs smaller online shops).[5] This could also be due to having obtained data products of different quality from data collecting and processing companies, so-called 'data brokers'. One such data broker is the Belgian Realdolmen, which details on its

[1]See Ramasastry (2005).

[2]Similarly, a report of McKinsey&Company (2016) shows that finer data analytics has improved customer segmentation and firm's profitability. See also Mikians et al. (2012), who empirically demonstrate the existence personalized pricing.

[3]See Business Wire (2015).

[4]In October 2016, the US Federal Communications Commission, then under Democratic majority, imposed a set of privacy rules on Internet service providers, requiring them to get opt-in consent from consumers before using, sharing, or selling their Web browsing data, app usage history and other private information. Yet, on April 3 2017, President Donald Trump signed a repeal of these rules, following actions taken by both houses of the US Congress (see, e.g., Brodkin, 2017).

[5]BenMark et al. (2017) describe the expertise that firms must develop to be effective in applying differential-pricing solutions.

website that it offers 'tailored' data services.[6]

> Storing and structuring huge amounts of data is one of our strengths. The challenge comes from correlating various data sources. Our customers need a partner who can combine data sets and understand how the whole process fits together. *There aren't any standard packages for this*; every process is different. *Realdolmen's strength comes from its ability to deliver end-to-end custom packages*.

Interestingly, Realdolmen is partially controlled by Belgian supermarket chain Colruyt.[7] One may speculate that Colruyt, given its privileged relationship with Realdolmen, may have an advantage when it comes to obtaining information about potential consumers compared to other companies, including its competitors. This advantage may translate into asymmetric abilities to profile and target consumers.

Against this backdrop, we study a model of price competition between firms in a homogenous good setting. Thanks to a clever use of big data, firms are able to identify any consumer's willingness to pay and have the opportunity to charge a personalized price to consumers they are able to 'recognize', while charging a 'uniform' price to all consumers who remain 'anonymous'. Yet, they can only do this imperfectly: there is always a positive probability that any particular consumer will remain anonymous. This may be due to the available data being not sufficiently precise, or to consumers acting to protect their privacy.[8] Firms first set a 'uniform' price, which they charge to any anonymous consumer; then, they set personalized prices for consumers whom they can identify. Consumers may observe that firms charge different prices, but it is assumed that they can only buy at the price they have been individually offered.[9]

We assume that firms have correlated (and potentially different) abilities to profile consumers (which makes sense if they both use the services of the same data broker). Specifically, this means that one firm has a better 'profiling technology' than the other firm, and when the firm with the inferior technology can profile a consumer, the firm with the superior technology can also, but not vice versa. Then, it is as if firms were competing on three distinct groups of consumers: an 'opaque' segment where consumers are anonymous (no firm profiled them), a 'transparent' segment where consumers are fully identified (both firms profiled them), and a 'translucent' segment where consumers are anonymous/identified (the 'better-informed' firm profiled them but not the other firm).

In this setting, we ask the following questions: Is being able to profile consumers enough to obtain market power through price discrimination in a duopolistic setting? When it does, does one also observe price dispersion? If profiling capacity of firms depends on the data they need

[6] `http://www.realdolmen.com/tactical-ict/ict-trends/big-data`, accessed on December 11, 2016; emphasis added.

[7] The Colruyt family owns 16% of Realdolmen and both Jef Colruyt and Wim Colruyt are members of the board of directors of Realdolmen.

[8] Consumers can use obfuscation strategies such as clearing cookies from their browsers, logging off their Google and Facebook accounts, or adopting proxy servers or ad-blockers. In our baseline model, we take such hiding as exogenous; in Section 4.3, we discuss how our results could be affected if hiding was endogenized.

[9] In our model, firms do not behave like Amazon: they do not apologize and refund customers who paid higher prices once price differences have been exposed.

to purchase from a data-broker, would this data-broker sell these data exclusively to one firm? If consumers would have the option to hide their characteristics, would they like to do so? If uniform prices were observable ('list prices') would this lead to more competition?

Main results. Before sketching our results, note that without the possibility to price discriminate, price competition in uniform prices would result in marginal cost pricing, the classical Bertrand Paradox. We first show a *partial irrelevance result:* having the ability to profile consumers does not automatically translate into market power (Proposition 1). It only does so when both firms have the ability to profile consumers but one firm is better at it than the other. The mechanism that yields market power in our model relies on the strategic effect of the less-informed firm's uniform price (set in stage 1) on both firms' personalized prices. Indeed, there will be a positive probability that the better informed firm recognizes consumers the other firm does not and this firm can thus guarantee positive profits as long as the uniform price of the less informed firm is positive. Hence, when the latter increases its uniform price, competitive pressure on 'profiled' consumers decreases. Therefore, the less-informed firm will always have an incentive to set a uniform price above its cost, yielding a price equilibrium that displays market power. This mechanism breaks down in two scenarios: when both firms always profile the same consumers (making the translucent segment disappear), and when the less informed firm can never profile any consumer (making the transparent segment disappear). In these two cases, the only equilibrium is the Bertrand paradox, where both firms charge all consumers a price equal to marginal cost (irrespective of consumers being identified or not). Hence, having the ability to price discriminate against profiled consumers does not necessarily imply market power.

Second, we contend that price variations may result from a *combination* of price discrimination and price dispersion, meaning that in the case presented above, both Amazon and its customer may have been right. We show indeed that the coexistence of price discrimination and price dispersion at equilibrium hinges on the coexistence of the three consumer segments. In this event, each firm must balance the effects of all its prices on the three segments; moreover, it has to do so without knowing exactly which consumers the competitor is able to profile on the translucent segment. Hence, both price discrimination and price dispersion arise in equilibrium when firms have imperfect and asymmetric profiling technologies. In particular, we show that equilibrium personalized prices always exhibit price dispersion (proposition 2). We also show that the dispersion of personalized prices may lead firms to randomize equilibrium uniform prices as well (proposition 3). This is so when the transparent segment is not valuable enough (i.e., when the firms' profiling technologies are not very precise); in that case, the firm with the inferior profiling technology cannot contend itself with the transparent segment and decides to compete on the opaque segment as well, which requires the randomization of uniform prices to avoid the Bertrand paradox.

In a nutshell, we establish that there may be three simultaneous explanations as to why two consumers of the same good end up paying different prices: firms may charge them either different personalized prizes (price discrimination), or randomized uniform prices (price dispersion), or randomized personalized prices (price discrimination *and* dispersion). The necessary ingredients for this result are price competition and 'imperfect consumer profiling', which refers to the idea that firms are able to identify consumers' valuations for the product but only in an imperfect way. The profiling of consumers enables price discrimination, but its imperfect nature makes firms uncertain about the competitor's pricing strategy, which triggers strategic random pricing. These results explain why, in contrast to classic competitive price discrimination models (e.g., Thisse and Vives, 1988), the Bertrand paradox does not arise when firms can set both uniform and personalized prices. As long as both firms can track consumers, asymmetric profiling technologies can generate a way out of the Bertrand paradox because they induce uncertainty about the nature of market competition.

We also provide a set of comparative statics results with respect to the precision of the profiling technology and the degree of asymmetry between firms' profiling technologies. We show that increasing the precision of profiling technologies increases both uniform and personalized prices (Corollary 1), leading to instances in which some targeted consumers are right to fear that they are being priced closer to their 'pain point' compared to the uniform price they would receive if they would be able to hide. This provides incentives to hide so as to avoid excessive prices, i.e., there is a demand for privacy. However, some consumers may gain from improved profiling, namely those consumers with a low valuation, who start purchasing (and enjoying a positive surplus) when they are no longer anonymous; as what they gain may outweigh what other consumers lose, improved profiling may lead to an overall increase in consumer surplus.

We then show, in Corollary 2 that reducing the asymmetry between firms' profiling technologies has a non-monotone effect on the uniform prices. More specifically, uniform prices are at their highest level for an intermediate level of asymmetry. This has important implications for the selling strategy of data brokers. In particular, data brokers (e.g., Colruyt's subsidiary Realdolmen) would find it optimal to provide data services of different quality to different firms. This is akin to the result of Gabszewicz and Thisse (1979), where one firm selects the high quality and the other selects the lower one (though they do not study the incentives of third party sales). The consequence of our result is that data-brokers would like to 'tailor' their data services in a non-exclusive way, not to satisfy the 'needs' of the firms that demand data services, but rather to soften price competition in the product market.We also note in Corollary 3 that for given uniform prices, personalized prices decrease as firms' profiling technologies become more symmetric, which is due to the higher intensity of competition between firms in the transparent segment. This has interesting implications for consumer surplus. For relatively high levels of symmetry between firms' profiling technologies, further increasing the symmetry benefits the consumers as it leads to both lower uniform prices and lower personalized prices. For relatively low levels of symmetry, however, increasing the symmetry leads to higher uniform prices but lower personalized prices.

Lastly, we confirm in Section 5.3 that when consumers can always purchase the product at the 'list' price, competitive pressure is higher. The intuition is simple: a firm that wants to price discriminate does not just compete against the prices of its competitor but also against its own uniform price. The effect is very stark in our model: marginal cost pricing prevails again at the subgame perfect equilibrium, bringing us back in the Bertrand Paradox. Firms thus have incentives to prevent consumers from having access to multiple prices. In addition, we show in a simplified version of our model in which consumers can only be of two types, that no other (mixed strategy) subgame perfect equilibrium exists.

Related literature. Our paper contributes to oligopoly theory in two dimensions. First, we provide a partial irrelevance result in the sense that the ability to profile consumers does not allow firms to obtain market power when consumer profiling is fully symmetric or when only one firm can profile consumers. Second, we provide a novel explanation of the *simultaneous* observation of price dispersion (including of uniform prices) and of price discrimination in the classic Bertrand model. In the existent literature, price dispersion is usually attributed to differences in search costs on the buyer's side (Varian, 1980), differences in the cost structure on the seller's side (Spulber, 1995), and uncertainty about the number of active firms on the market (Janssen and Rasmusen, 2002). However, this paper emphasizes that it is the uncertainty about the nature of price competition that may generate market power and price dispersion of both uniform and personalized prices.

Our paper also provides some important insights into the debate on privacy regulation when there is imperfect targeting in an oligopoly setting. In recent years, the literature on targeting and privacy has grown considerably because of advances in digital tracking. See Acquisti et al. (2016) for a survey. However, pricing with imperfect consumer recognition has rarely been studied in an oligopoly setting (see Belleflamme and Vergote (2016), and Valletti and Wu (2016) for a monopoly analysis). Although Chen and Iyer (2002) consider an oligopoly analysis, there are a number of differences between their paper and ours. First, their analysis focuses on personalized pricing whereas this paper characterizes the interaction between personalized and uniform pricing fully.[10] Second, they consider differentiated products and focus on firms having the same but uncorrelated profiling technologies, whereas this paper considers homogeneous products and also the possibility of having asymmetric and correlated profiling technologies.

Finally, this paper relates to the literature that studies the incentives of data brokers. For example, Clavorà Braulin and Valletti (2016) and Montes et al. (2015) also consider competitive targeting. However, there are two important differences. First, we consider homogeneous products, whereas they consider differentiated products. Second, we examine the mixed strategies equilibria under different 'degrees' of asymmetry in profiling technologies, but they focus on the pure strategies equilibria under perfectly asymmetric profiling technologies (i.e., one firm is unable to track). When products are differentiated vertically (Clavorà Braulin and Valletti, 2016) and horizontally (Montes et al., 2015), they find that the data broker finds it profitable to sell its data exclusively to only one firm. We, however, show that the ability to track is irrelevant to market power if firms' profiling technologies are fully symmetric or fully asymmetric, as in both cases the Bertrand paradox prevails. Because of this irrelevance result, we find that in contrast to the result of Clavorà Braulin and Valletti (2016) and Montes et al. (2015), the data broker

[10] They discuss the interaction between personalized and uniform pricing in their section *Extension and Caveats* only loosely, without clarifying the assumptions made, for instance whether the uniform prices are observed by consumers or not.

may have incentives to sell its data to all competing firms, though the data set obtained by each firm is of different qualities. Another example is Jentzsch et al. (2013), who study price discrimination in two dimensions (group and individual price discrimination) in a Hotelling model. However, they do not study the effect of imperfect profiling; that is, in their model, each firm knows its rival's information set. Therefore, price discrimination arises in equilibrium but not price dispersion, though they also find that partial information sharing is profitable for firms.

The rest of the paper is organized as follows. In Section 2, we present our modeling framework. In Section 3, we look at the two benchmark cases where either the translucent or the transparent segment vanishes, and the Bertrand paradox continues to prevail. In Section 4, we consider the case where firms are both able to profile consumers but with different abilities; we then characterize the equilibrium of the pricing game and establish our main result, namely the coexistence of price discrimination and price dispersion at equilibrium; we also derive a number of insightful comparative statics results. In Section 5, we examine the implications of our results for firms and consumers; we also extend the baseline model for the case where uniform prices are made public. Finally, in Section 6, we draw some policy implications, discuss the limitations of our setting and propose some directions for future research.

2 The model

We consider a market where two firms produce a homogeneous product at a constant marginal cost, which is set to zero for simplicity. We describe in turn the main assumptions regarding consumers and firms, as well as the timing of the game.

Consumers. There is a unit mass of consumers who vary in the valuation that they attach to the homogeneous product. The valuation of consumer x is noted $r(x)$, which is randomly and independently drawn according to the distribution function $F(.) : \mathbb{R} \to [0,1]$ with support $[0, \bar{r}]$, and with associated continuously differentiable density function $f(.) : \mathbb{R} \to \mathbb{R}$. Consumers wish to purchase at most one unit of the product and they do so from the firm that offers the lowest price (we say more on this below). Noting this price p, we can express the expected demand from consumer x at p as the probability that the consumer has a valuation that is at least as large as p, i.e. $prob(r(x) > p)$. This probability is given by the survival function $S(.) : \mathbb{R} \to [0,1]$ where $S(p) = 1 - F(p)$. Since we assume that the mass of consumers is equal to 1 and that valuations are drawn independently, the survival function also represents the aggregate expected demand (referred to hereafter as the 'demand'). We impose the additional assumption that demand is log-concave: $S'(p)/S(p) = -h(p)$ is non-increasing in p, where $h(p)$ is the hazard rate.[11]

Firms. Each firm has access to a 'profiling technology' that allows it to identify the valuation of a consumer probabilistically.[12] Without loss of generality, assume that this probability is

[11] This is equivalent to requiring that $S(p)/f(p) = (1 - F(p))/f(p) = 1/h(p)$ is non-increasing in p.

[12] For now, we take these technologies as exogenous. In Section 5, we discuss about how to endogenize the technologies by introducing a data broker in the game. We argue that such data broker would find it profitable to sell asymmetric profiling technologies to the firms.

equal to $\lambda_A = \lambda \in [0, 1]$ for firm A and $\lambda_B = \alpha\lambda$ for firm B, where $0 \leq \alpha \leq 1$. The parameter λ can be interpreted as the precision of the profiling technology of firm A. Alternatively, $1 - \lambda$ can be interpreted as the capacity of consumers to hide from firm A. We assume that the profiling technologies are correlated: when firm B can profile a consumer, so can firm A; yet, the reverse is not true: any consumer profiles by firm A can be profiled by firm B only with probability α. So, we can interpret $1 - \alpha$ as the advantage in precision that firm A enjoys over firm B when it comes to profiling consumers. We would in general expect that the profiling technologies of firms are correlated: when one firm profiles a consumer, it increases the chances that the other firm will do so as well. We study the following limit case: if the firm with the worst profiling technology profiles a consumer then the other firm will certainly do so as well. While this assumption of perfect correlation is a strong one, it makes characterizing the equilibria tractable while allowing us to transmit the message that asymmetric imperfect profiling technologies will lead to price dispersion. When a firm identifies the valuation of a consumer, it is in a position to price discriminate and charge this consumer a personalized price. For the consumers whom a firm does not profile, the firm sets a uniform price.

Timing of the game. Before the game starts, firms acquire some profiling technology, with respective precisions λ_A and λ_B. The values (λ_A, λ_B) are assumed to be common knowledge. In the first stage of the game, firms set their uniform price p_i, with $i \in \{A, B\}$, for the consumers that they are not able to profile. In the second stage, after observing the uniform prices, firms set personalized prices for the consumers that their profiling technology allows them to profile; these prices are noted $p_i(x)$, with x referring to the identity of the consumer with valuation $r(x)$. In stage 3, consumers decide whether or not to buy the product, and from which firm to buy after observing the price offered to them by each firm. We assume that consumers receive only one price offer from each firm, either a uniform or a personalized price. As a consequence, profiled consumers are not able to arbitrage between their personalized price and the uniform price. This assumption is not unreasonable if price offers are sent out electronically and if consumers have to incur some search cost to know about the uniform price. The fact that profiled consumers do not (or have a hard time to) observe uniform prices is not incompatible with the fact that firms do observe each other's uniform price. Firms may indeed infer the other firm's uniform price by using sophisticated technologies (such as robots) that are not accessible to consumers.[13] We also need to set a couple of tie-breaking rules to decide how consumers choose when firms charge them the exact same price. For this, we assume that although consumers observe only one price from a given firm, they can determine whether this price is a personalized price or a uniform price.[14] If both firms quote the same price but one price is personalized while the other is a uniform price, we assume that the consumer chooses the firm offering the personalized price.[15] If both prices are the same and of the same nature (both personalized or both uniform), then we assume that the consumer chooses to purchase from each firm with equal probability.

[13] This is a common assumption made in the literature.

[14] For instance, because personalized offers are made on a first-name basis.

[15] That is, we assume that consumers have lexicographic preferences: first over prices and then over personalized/uniform price offers.

To sum up, our model only differs from the canonical Bertrand model of price competition in that firms can detect the willingness to pay of consumers probabilistically and adjust their pricing accordingly. We now characterize the *subgame-perfect Nash equilibria* (SPNE) of this game.

3 Symmetric profiling or profiling by only one firm

We start by examining two limit cases where the possibility to target consumers and set personalized prices does not help firms to escape from the Bertrand paradox. In the first case, both firms have a profiling technology of the exact same precision ($\alpha = 1$ so that $\lambda_A = \lambda_B = \lambda$). In the second case, only firm A is able to profile consumers ($\alpha = 0$, so that $\lambda_B = 0$). The two cases have in common that they only feature one market segment besides the opaque segment: the transparent segment when $\alpha = 1$, and the translucent segment when $\alpha = 0$.[16] As we now show, the subgame-perfect equilibrium is the same in the two cases: both uniform and personalized prices are equal to marginal cost for both firms.

Symmetric profiling. First suppose that both firms use the same profiling technology ($\alpha = 1$). This means that they obtain exactly the same information about each consumer x: either they both profile consumer x (with probability λ) or none of them does. Then it is as if they compete on two fully separated markets: the opaque segment (with a mass $1 - \lambda$ of consumers) and the transparent segment (with a mass λ of consumers). In the subgames in which a firm profiles some consumer x, then it knows that the other firm also profiles this consumer. This means that 'classic' Bertrand competition prevails for any consumer on the transparent segment. It follows that equilibrium personalized prices are $(p^*_A(x), p^*_B(x)) = (0, 0)$ for all x. This equilibrium outcome is independent of any uniform price p_i that may prevail on the opaque segment; in other words, no such uniform price p_i will have an effect on the personalized prices. As a consequence, competition for consumers on the opaque segment is also equivalent to Bertrand competition, leading to regular prices that also equal marginal cost in equilibrium: $(p^*_A, p^*_B) = (0, 0)$.

Profiling by only one firm. Now suppose that only firm A can profile a consumer x (with probability λ), while firm B cannot profile any consumer ($\alpha = 0$). The two segments featured here are the opaque segment (with a mass $1 - \lambda$ of consumers) and the translucent segment (with a mass λ of consumers). In contrast with the previous case, the two segments are not separated as firm B sets its uniform price on the two segments. However, the subgame-perfect equilibrium is the same. Here is why. In the translucent segment, i.e., in subgames in which A identifies some consumer x, this consumer will have received an offer p_B from firm B. Since consumers only care about prices, firm A can undercut any offer $p_B > 0$ with a personalized price. It follows that whenever $p_B > 0$, firm B will never make any sales to consumers that are

[16] Recall that the opaque, transparent and translucent segments contain, respectively, consumers whom no firm, both firms, or only the better-informed firm manage to profile.

targeted by firm A. As such, firm B is competing only in the opaque segment, i.e., for consumers who also remain anonymous to firm A. But then Bertrand competition ensues on the opaque segment, as for any uniform price $p_B > 0$, firm A has an incentive to slightly undercut this price; this yields to a unique equilibrium $(p_A^*, p_B^*) = (0, 0)$. Now, if $p_B^* = 0$, then being able to profile consumers gives no advantage to firm A on the translucent segment, as firm A cannot do better than setting $p_A^*(x) = 0$ for all x. Again, the Bertrand paradox continues to prevail.

We record our results in the following proposition.

Proposition 1. *If firms are equally able to profile consumers or if only one firm is able to profile consumers, then profiling does not allow firms to escape from the Bertrand paradox and make positive profits.*

4 Asymmetric profiling

As Proposition 1 shows, introducing probabilistic profiling of consumers (and so, the possibility to price discriminate) is not enough, by itself, to get out of the Bertrand paradox. We now show that if firm B (the 'less-informed' firm) is able to profile consumers, but not as well as firm A (the 'better-informed' firm), i.e. if $0 < \alpha < 1$, then prices above marginal cost become sustainable. What is more, both uniform and personalized prices equal and above the 'monopoly' price will be observed with positive probability in equilibrium. Before establishing these results formally, let us discuss the intuition behind them.

The major force driving the results is the uncertainty that the two firms face regarding the pricing strategy of their competitor. In particular, when the better-informed firm (firm A) profiles some consumer, it is uncertain as to whether the other firm has profiled her as well or not. Firm A knows that with probability $(1 - \alpha)$, firm B did not profile this consumer (who belongs then to the translucent segment) and offered her its uniform price p_B; in that case, firm A can win this consumer with an appropriate personalized price and assure itself positive profits. However, with probability α, firm B did profile the consumer, which implies that firms compete for this consumer through personalized prices (we are then on the transparent segment). We show below that this uncertainty leads to equilibrium price dispersion of personalized prices: the personalized prices of both firms are always positive (i.e., above marginal cost), but are chosen randomly from some distribution, which depends on firm B's uniform price p_B. This equilibrium will yield positive expected profits from profiling for both firms. In fact, we will show that the higher p_B, the higher firm B's (and firm A's) profits from profiling. Anticipating this, firm B will never set a uniform price equal to its marginal cost. But given that there is price competition for consumers that are not identified, we will obtain in some cases that equilibrium price dispersion will also prevail for uniform prices. To understand this, note that firm B will consider two options when setting its uniform price. The first option is to forego anonymous consumers altogether. In that case, firm B focuses on the transparent segment and chooses a uniform price p_B that only matters for the translucent segment, with a view to maximize its expected profits from personalized price competition; this also implies that firm A enjoys a monopoly position on the opaque segment where it can set the monopoly price. In the second option, firm B finds it worthwhile to compete for the consumers that it does not identify; both firms are then active on the three segments, which leads to a mixed strategy equilibrium in uniform prices.

In short, the possibility to price discriminate together with uncertainty about the nature of price discrimination competition (did one firm recognize the consumer or did they both?), generates **strictly positive** prices and **price dispersion** of personalized prices, and possibly of uniform prices. We now characterize this subgame perfect Nash equilibrium in detail.

4.1 Stage 2: Equilibrium personalized prices

We first study the optimal personalized prices, given the observed uniform prices (p_A, p_B). That is, we focus on the competition that takes place on the translucent and transparent segments, which have respective masses of consumers equal to $(1-\alpha)\,\lambda$ and $\alpha\lambda$. In other words, firms choose their personalized prices $(p_A(x), p_B(x))$ for consumers identified by firm A and, possibly (with probability α) by firm B. As a first step, we show that any equilibrium price strategy must involve mixed strategies when firm A is able to undercut the uniform price of firm B (i.e., for $p_B > 0$). For future reference, we define $p_B^x \equiv \min\{p_B, r\,(x)\}$.

Lemma 1. *If $p_B > 0$, then no pure strategy equilibrium $(p_A(x), p_B(x))$ exists.*

Proof. Given $r(x) > 0$, firm A can always guarantee itself a positive expected profit from that consumer either by matching firm B's uniform price (if $p_B \leq r\,(x)$) or by charging this consumer her valuation (if $p_B > r\,(x)$). This strategy will work if the consumer is on the translucent segment, i.e., if firm B did not profile her, which happens with probability $(1-\alpha)$. We have thus that firm A's expected minimum profit is equal to $(1-\alpha)\,p_B^x$. It follows that any price below $(1-\alpha)p_B^x$ can never be a best reply for firm A. Hence, if a pure strategy equilibrium exists, it must be such that firm A chooses $p_A(x) \geq (1-\alpha)p_B^x$. But in the event that firm B profiles consumer x, then it will undercut $p_A(x)$; since the strategy space is continuous, there is no best response and $p_A(x)$ cannot be part of a pure strategy equilibrium. □

Note that when $p_B = 0$, the competition through personalized prices is akin to Bertrand competition: firm A cannot profitably serve consumer x if this consumer is on the translucent segment (i.e., if firm B does not identify her); if on the other hand, the consumer is on the transparent segment, then undercutting will lead to marginal cost pricing. This is summarized in the following lemma:

Lemma 2. *If $p_B = 0$, then $(p_A(x), p_B(x)) = (0, 0)$ is the unique equilibrium.*

Proof. Clearly, $p_A(x) = 0$ is a (weakly) best reply to $p_B(x) = 0$ and vice versa. Then $(p_A(x), p_B(x)) = (0, 0)$ is a Nash equilibrium. No other equilibrium in pure strategies can exist, since there would be a profitable deviation. Suppose that there exists an equilibrium in mixed strategies. The support of these strategies must contain only strictly positive prices (and be symmetric). But then, the upper limit of the support must have positive mass. If not, playing this price will yield an expected payoff of zero. Yet, for any point with positive mass in the support, there is an incentive to slightly undercut this price as to increase profit. □

A first important insight can be gained from noting that Lemmata 1 and 2 imply that while marginal cost pricing can still be a Nash Equilibrium of the two stage pricing game, it can never be a subgame perfect Nash equilibrium. That it is a Nash Equilibrium follows from Lemma 2: if the other firm's strategy is to set its uniform and personalized prices equal to marginal cost, then a best response is to do the same. That it is not a SPNE follows from Lemma 1: by setting a uniform price above marginal cost in stage 1, the less informed firm will induce above marginal cost personalized prices in stage 2. Setting its uniform price equal to marginal cost can thus never be optimal for this firm, anticipating the second stage Nash Equilibrium. We now characterize a personalized price equilibrium in mixed strategies.

Proposition 2. *For any consumer x, let $p_B^x \equiv \min\{p_B, r(x)\}$ and define*

$$G^x(p) = \begin{cases} \frac{p-(1-\alpha)p_B^x}{\alpha p} & \text{for } p \in [(1-\alpha)p_B^x, p_B^x], \\ 0 & \text{otherwise.} \end{cases}$$

If $p_B > 0$, then the following personalized strategies targeting consumer x constitute a Nash equilibrium: firm B draws $p_B(x)$ from the distribution $G^x(p)$; firm A does so as well with probability α, and sets $p_A(x) = p_B^x$ with probability $1-\alpha$. At equilibrium, both firms obtains expected profits equal to $\tilde{\pi}^x = (1-\alpha)\, p_B^x$ for each consumer x on the transparent segment.

The formal proof is relegated to Appendix 6.1. Here, we merely sketch the argument. We know from the proof of Lemma 1 that for any consumer x, firm A never sets a personalized price $p_A(x)$ below $(1-\alpha)\, p_B^x$; it is also easy clear that firm A cannot make any profit from consumer x when setting $p_A(x) > p_B^x$. That defines the relevant interval for the price distribution. The next step consists in showing that if firm B draws its price from some distribution $G^x(p)$ on this interval, then firm A achieves an expected profit of $\tilde{\pi} = (1-\alpha)\, p_B^x$ when setting its price at any of the two bounds of the interval. But $\tilde{\pi}$ must also be the profit that firm A achieves for any price p chosen within this interval. That is, we must have $(1-\alpha)\, p_B^x = (1-\alpha)\, p + \alpha(1 - G_B^x(p))p$, where the first term is firm A's profit when firm B does not profile consumer x (with probability $1-\alpha$), and the second term is firm A's profit when firm B profiles consumer x but sets a larger personalized price than p. Solving the previous equation for $G^x(p)$ gives the expression in the proposition. The rest of the proof consists in showing that firm A's best conduct is to draw its price from the same distribution if firm B profiles consumer x (i.e., with probability α) or, otherwise, to set $p_A(x) = p_B^x$ (i.e., to match firm B's uniform price if $p_B \leq r(x)$, or to charge this consumer's valuation if $p_B > r(x)$). It is interesting to note that for a given p_B^x, personalized prices become more dispersed when α increases (i.e., when the profiling technologies of the two firms become more similar). We observe indeed that the lower bound of the mixed-strategy equilibrium is decreasing in α, while $G^x(p)$ is increasing in α. However, it remains to be seen how p_B^x changes with α.

Knowing how firms will play at the second stage of the game, we can now move back to the first stage where firms set their uniform price. Note that the two stages are linked through firm B's uniform price p_B, as this price shapes the distribution $G^x(p)$, i.e., the way firms randomize their personalized prices in stage 2.

4.2 Stage 1. General price equilibrium

We learned from Lemma 2 that the less informed firm (firm B) would obtain zero profits if it were setting its uniform price at marginal cost. We also know from Proposition 2 that firm B can secure positive profits by setting its uniform price p_B above marginal cost. In fact, the higher p_B, the higher the profit firm B can expect on the transparent segment. In contrast, the better-informed firm (firm A) cannot *directly* influence profits obtained from profiling consumers (on the translucent and transparent segments) through its uniform price p_A (since these profits only depend on the uniform price of its competitor). However, it can do so indirectly, as firm B will set p_B so as to best-reply to p_A.

Coming back to firm B, we see that it faces the following trade-off when raising its uniform price: on the one hand, a higher p_B increases expected profits on the transparent segment but on the other hand, it lowers expected profits on the opaque and translucent segments (as it makes it easier for firm A to undercut B's price). Given this trade-off, one option for firm B is to focus exclusively on the transparent segment and to forgo any profit on the opaque and translucent segments (where it does not profile any consumer). To do so, firm B simply needs to set $p_B > p_A$ for any p_A. What would be firm B's expected profit under that option? From Proposition 2, we know that firm B obtains expected profits equal to $\tilde{\pi}_B^x = (1-\alpha)\, p_B^x$ for each consumer x on the transparent segment. Aggregating over all consumers and recalling that $p_B^x = \min\{p_B, r(x)\}$, we have that for any $p_B > p_A$,

$$\mathring{\pi}_B(p_B) = \int_0^{p_B} (1-\alpha)\, r f(r) dr + \int_{p_B}^{\bar{r}} (1-\alpha)\, p_B f(r) dr. \tag{1}$$

It is easily seen that the latter expression is an increasing function of p_B.[17] It follows that if firm B chooses to focus on the transparent segment, it will set the largest possible uniform price: $p_B = \bar{r}$. As the transparent segment has a mass of $\alpha\lambda$, this strategy will guarantee firm B an expected profit of (no matter the regular price p_A set by firm A)

$$\begin{aligned} \pi_B^{\min} &= \alpha\lambda\mathring{\pi}_B(\bar{r}) = \alpha\,(1-\alpha)\,\lambda\hat{r}, && (2) \\ \text{where } \hat{r} &= \int_0^{\bar{r}} r f(r) dr \text{ is the average valuation.} \end{aligned}$$

If firm B chooses that option and sets $p_B = \bar{r}$, firm A's best response is to set the optimal monopoly price, p^m, which solves $\max_p pS(p)$.[18] This is obvious because firm A is the only active firm on the opaque and translucent segments and because the choice of its uniform price bears no impact on which personalized prices firms will set afterwards. We need now to examine under which condition setting $p_B = \bar{r}$ is firm B's best response when firm A sets $p_A = p^m$. The best alternative for firm B is to set $p_B = p^m - \varepsilon$ so as to compete as well on the opaque segment without sacrificing too much profit on the transparent segment later on. What would be its expected profit in that case? With probability $\alpha\lambda$, firm B competes for consumers on the transparent segment and obtains expected profits (almost) equal to $\mathring{\pi}_B(p^m)$; with probability

[17] We have $d\mathring{\pi}_B/dp_B = (1-\alpha)\, p_B f(p_B) + (1-\alpha)\, S(p_B) - (1-\alpha)\, p_B f(p_B) > 0$.

[18] Our assumption that demand is log-concave makes sure that such price exists.

$1-\lambda$, it competes on the opaque market and, as it slightly undercuts firm A, it achieves profits (almost) equal to $\pi^m \equiv p^m S(p^m)$. In sum, firm B's expected profit when both firms set the monopoly price is competed as

$$\Pi_B^e(p^m)|_{p_A=p^m} = \alpha\lambda\mathring{\pi}_B(p^m) + (1-\lambda)\pi^m. \tag{3}$$

We have thus proved the following lemma:

Lemma 3. *In the first stage of the game, the pair $(p_A, p_B) = (p^m, \bar{r})$ is an equilibrium in pure strategies if and only if $\pi_B^{\min} \geq \Pi_B^e(p^m)|_{p_A=p^m}$, which is equivalent to*

$$\alpha(1-\alpha)\lambda\hat{r} \geq \alpha(1-\alpha)\lambda\left[\int_0^{p^m} rf(r)dr + \int_{p^m}^{\bar{r}} p^m f(r)dr\right] + (1-\lambda)\pi^m. \tag{4}$$

Condition (4) directly follows from expressions (1) to (3). When it is met, there exists thus a pure-strategy equilibrium in which firm A is a monopolist in the segments where consumers are not profiled, while firm B gives up any sales to consumers it does not profile and chooses a uniform price that maximizes its expected profits in the transparent segment.

Otherwise, if Condition (4) is not met, firm B will choose to compete on the opaque market as well; but then, firm A will have no reason to set the monopoly price. As competition ensues on the opaque segment, both firms must randomize prices, as it is the only way for them to make positive profits and, thereby, to be indifferent with the previous situation where firm B stays out of the opaque segment (and firm A monopolizes it). That is, firms draw their uniform price from some distribution, which we note $H_A(p)$ and $H_B(p)$, and they do so with respective probabilities noted ∇_A and ∇_B (meaning that with probability $1-\nabla_A$, firm A sets $p_A = p^m$ and with probability $1-\nabla_B$, firm B sets $p_B = \bar{r}$).

As we formally show in Appendix 6.2, the distributions $H_A(p)$ and $H_B(p)$ must be defined on the same connected price interval. The upper bound of this interval is clearly the monopoly price, p^m. As for the lower bound, which we denote p_l, it is implicitly defined in the following way. Let us first express firm B's expected profit of setting some price $p < p_m$ when firm A behaves as just described (i.e., with probability ∇_A, firm A draws p_A from the distribution $H_A(p)$ defined on $[p_l, p^m)$ and, with probability $1-\nabla_A$, sets $p_A = p^m$). In the spirit of expression (3), we find

$$\Pi_B^e(p) = \alpha\lambda\mathring{\pi}_B(p) + (1-\lambda)p(1-F(p))\left[(1-\nabla_A) + \nabla_A(1-H_A(p))\right].$$

As firm B must be indifferent between charging p_l and $\bar{r}$, we have that $\Pi_B^e(p_l) = \pi_B^{\min}$, which implicitly defines p_l. Firm B must also be indifferent between charging any price p in the support of H_B and $\bar{r}$, implying that $\Pi_B^e(p) = \pi_B^{\min}$ for all p in $[p_l, p^m)$. We can then solve the latter equation to derive the value of $H_A(p)$. Finally, firm B must achieve the same profit when charging p^m or $\bar{r}$; this implies that $\Pi_B^e(p^m) = \pi_B^{\min}$, from which we can derive the value of ∇_A (which is shown to be positive only if Condition 4 is violated).

Turn now to firm A. When firm B draws p_B from the distribution $H_B(p)$ defined on $[p_l, p^m)$ with probability $1-\nabla_B$, and sets $p_B = \bar{r}$ with probability $1-\nabla_B$, firm A's expected profit is given by

$$\Pi_A^e(p) = \lambda \hat{\pi}_A(\nabla_B, H_B) + (1-\lambda)\, p\,(1-F(p))\,(1-\nabla_B+\nabla_B(1-H_B(p)))\,,$$

where $\hat{\pi}_A(\nabla_B, H_B)$ is firm A's expected profit on the transparent segment given firm B's uniform price. If $\nabla_A > 0$, it must be that firm A is indifferent between charging p^m and p_l, or $\Pi_A^e(p^m) = \Pi_A^e(p_l)$. Using the fact that $H_B(p^m) = 1$ and $H_B(p_l) = 0$, we can solve this equation to derive the value of ∇_B. To find the value of $H_B(p)$, we note that firm A must also be indifferent between setting any price in the interval $[p_l, p^m)$ or setting p_l, implying that $\Pi_A^e(p) = \Pi_A^e(p_l)$.

Combining Lemma 3 and the previous discussion, we can describe the equilibrium at the first stage of the game as follows.

Proposition 3. *At the subgame-perfect Nash equilibrium of the game, the firms choose their uniform price according to the following strategies. Firm A draws p_A from the distribution $H_A(p)$ with probability ∇_A, or sets $p_A = p^m$ with probability $1-\nabla_A$. Firm B draws p_B from the distribution $H_B(p)$ with probability ∇_B, or sets $p_B = \bar{r}$ with probability $1-\nabla_B$. The distributions $H_A(p)$ and $H_B(p)$ are defined on the interval $[p_l, p^m)$. If Condition (4) is met, then $\nabla_A = \nabla_B = 0$ and the pair $(p_A, p_B) = (p^m, \bar{r})$ is an equilibrium in pure strategies.*

Proof. See Appendix 6.2 for the proof and for the exact expressions of $H_A(p)$, $H_B(p)$, ∇_A, ∇_B, and p_l. □

4.3 Comparative statics

Our objective is now to understand how the equilibrium prices depend, for any log concave demand function, on the two key parameters of the model: the level of precision of the detection technology λ and the level of asymmetry in the profiling technology α. We state our main results in this section and examine their implications in the next section.

Improved profiling. How do firms modify their pricing behavior when their profiling technologies become more precise, i.e., when λ increases? Recall that we assume that the profiling technologies of the two firms are correlated, with respective precision levels $\lambda_A = \lambda$ and $\lambda_B = \alpha\lambda$. Hence, an increase in λ makes it easier for *both* firms to profile consumers. The following corollary summarizes the answer to this question.[19]

Corollary 1. *Higher precision of the detection technology increases the uniform prices in the sense of first-order stochastic dominance, and it does not affect the distribution of personalized prices. Hence, more precision leads to higher uniform and personalized prices.*

It is clear from Proposition 2 that for a given uniform price of firm B, p_B, the distribution of personalized prices is independent of λ. However, increasing λ has three effects on the uniform prices. First, it increases the lowest possible uniform price charged by both firm A and firm B, i.e. $\partial p_l / \partial \lambda > 0$. Second, it increases the probability that both firms charge their respective

[19]The proofs of all comparative statics results can be found in Appendix 6.3.

highest uniform price, i.e. $\partial\nabla_A/\partial\lambda < 0$ and $\partial\nabla_B/\partial\lambda < 0$. Finally, they tend to charge higher prices when they randomize their uniform prices, i.e. $\partial H_A(p)/\partial\lambda < 0$ and $\partial H_B(p)/\partial\lambda < 0$. In sum, all three effects push the distribution of uniform prices to the right. Since the precision of the detection technology λ has no effect on the distribution of equilibrium personalized prices, we can be sure that more precision leads, ceteris paribus, to higher prices. Intuitively, when detection becomes more precise, it is more likely that firms will be competing in the transparent segment, which relaxes competition in the opaque segment and thus raises uniform prices.

More symmetric profiling technologies. Let us now consider the effect on the pricing behavior of both firms of increasing the symmetry between the profiling technologies (measured by the parameter α). It would seem logical to expect that as the profiling technologies become more similar, competition between the two firms would get more intense and prices would go down. However, things are not as simple here, as stated in the following corollary.

Corollary 2. *The uniform prices are at their highest level in the sense of first-order stochastic dominance when the level of asymmetry is intermediate (i.e. $\alpha = 1/2$).*

As above, we can show that the lower bound of the uniform price distribution, the probability of firms charging the highest uniform price, and the probability of firms charging higher uniform prices when they randomize their uniform prices are at their highest level when $\alpha = 1/2$. This implies that more symmetry increases the uniform prices when firms' detection technologies are very different, but decreases the uniform prices when firms' technologies are slightly different. There is thus an intermediate level of asymmetry, $\alpha = 1/2$, that maximizes the profits of the two firms.

Another effect of increasing α (i.e., of making the profiling technologies of the two firms more symmetric) is to push personalized prices down for given uniform prices. This is due to the more intense competition between firms in the transparent market.[20]

Corollary 3. *For given uniform prices, the personalized prices decrease with the symmetry between firms in the sense of first-order stochastic dominance.*

We now illustrates the previous comparative statics results with a specific example.

Example 1. *Uniform distribution*

We know from Proposition 3 that the mixed-strategy equilibrium in uniform prices is as follows: with probability $1 - \nabla_A$, firm A plays $p_A = p^m$ and with probability ∇_A, it draws a price from the distribution $H_A(p)$ defined on $[p_l, p^m)$; with probability $1 - \nabla_B$, firm B plays

[20] However, this result ignores the indirect effect a higher level of symmetry has on personalized prices through the effect it has on uniform prices. The expected uniform price is:

$$Ep_B = (1 - \nabla_B)\bar{r} + \nabla_B \int_{p_l}^{p^m} p h_B(p)dp = (1 - \nabla_B)\bar{r} + \nabla_B \left[p_l + \int_{p_l}^{p^m} (1 - H_B(p))dp \right].$$

We can show that $\frac{\partial Ep_B}{\partial \alpha} > 0$ when $\alpha < \frac{1}{2}$, which means that the expected highest personalized prices are maximized when $\alpha = 1/2$.

$p_B = \bar{r} = 1$ and with probability ∇_B, it draws a price from the distribution $H_B(p)$ defined on $[p_l, p^m)$. Suppose that consumer valuations are uniformly distributed on the unit interval and, to ease the exposition, define $k \equiv \alpha(1-\alpha)$. Then, supposing that $2(1-\lambda) > \lambda k$, we find the following results:[21]

$$\begin{aligned}
\nabla_A &= \frac{2(1-\lambda)-\lambda k}{2(1-\lambda)},\ \nabla_B = \left(\frac{2(1-\lambda)-\lambda k}{2(1-\lambda)+\lambda k}\right)^2, \\
H_A(p) &= \frac{1}{p}\frac{(2(1-\lambda)+\lambda k)\,p-\lambda k}{2(1-\lambda)-\lambda k}, \\
H_B(p) &= \frac{(2(1-\lambda)(1-p)-\lambda k p)(2(1-\lambda)p-\lambda k(1-p))}{p(1-p)(2(1-\lambda)-\lambda k)^2}, \\
p_l &= \frac{\lambda k}{2(1-\lambda)+\lambda k},\ p^m = \frac{1}{2}.
\end{aligned}$$

It is readily checked that ∇_A, ∇_B, $H_A(p)$, and $H_B(p)$ are decreasing function of λ and k, while p_l is an increasing function of λ and k. This means first, as stated in Corollary 1, that the distribution of uniform prices are pushed to the right for both firms when the profiling technologies become more precise. Second, recalling that $k \equiv \alpha(1-\alpha)$ is bell-shaped in α with a maximum at $\alpha = 1/2$, we also have that the distribution of uniform prices of both firms are also pushed to the right when α comes closer to $1/2$, as indicated in Corollary 2. Third, we observe in this example that $H_B(p) > H_A(p)$ for all p, meaning that firm B always puts more weight on lower prices than firm A whenever both firms randomize. Figure 1 illustrates these results by representing $H_A(p)$ and $H_B(p)$ for different values of λ and α: going from left to right, the first locus is $H_B(p)$ for $(\lambda,\alpha) = (\frac{1}{2},\frac{2}{3})$, the second is $H_A(p)$ for $(\lambda,\alpha) = (\frac{1}{2},\frac{2}{3})$, the third is $H_A(p)$ for $(\lambda,\alpha) = (\frac{3}{4},\frac{2}{3})$, and the fourth is $H_A(p)$ for $(\lambda,\alpha) = (\frac{3}{4},\frac{1}{2})$.

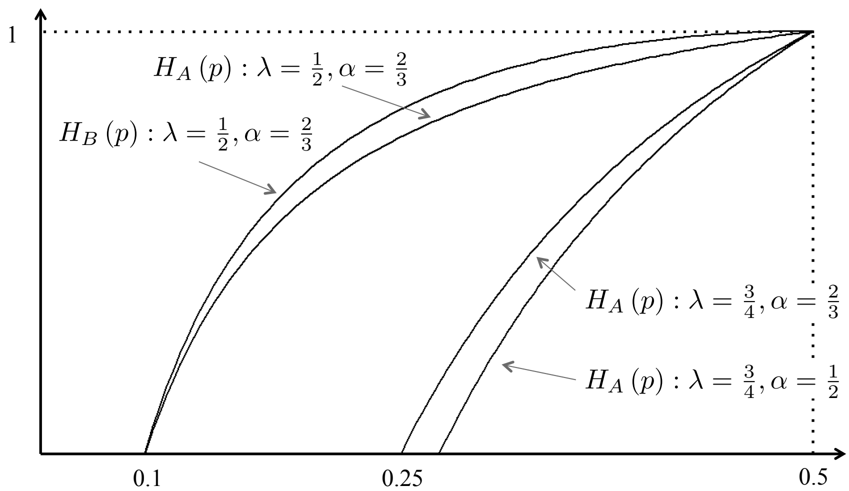

Figure 1: Impacts of improved profiling on price distributions

[21] If $2(1-\lambda) \leq \lambda k$, then $\nabla_A = \nabla_B = 0$, meaning that, with probability 1, firm A sets $p_A = 1/2$ and firm B sets $p_B = 1$. See Appendix 6.3 for the detailed computations.

5 What's at stake for firms and consumers?

In this section, we exploit the previous results to draw important implications for firms and consumers. As for firms, we examine what our results imply regarding the supply of profiling technologies. As for consumers, we investigate further how asymmetric profiling by competing firms affects their surplus; we also consider an alternative scenario where all consumers can observe the uniform prices chosen by the two firms.

5.1 Data: Happy to share but not equally

So far, we have not been very specific concerning the origin of the profiling technologies. We have simply indicated that profiling was made possible by the use of big data and that it was reasonable to assume that the two profiling technologies were correlated if firms had access to 'similar' data sets. We discuss here two potential origins of such correlated technologies: the sharing of data among firms and the provision of data by a common third party.

Data sharing. We showed in Corollary 2 that uniform prices reach a maximum (in the sense of first-order stochastic dominance) when $\alpha = 1/2$, i.e., when the less-informed firm uses a profiling technology that is just half as efficient as its rival's technology. A direct consequence of this finding is that there are circumstances in which the better informed firm has an incentive to share some of its data (i.e., its 'ability to profile') with the less informed firm. Intuitively, when firm B has a very low precision of profiling compared to firm A, it mainly competes in the opaque segment. So, improving firm B's technology enables it to also compete in the transparent segment, which in turn relaxes competition in the opaque segment (and hence the higher uniform prices). However, when firm B's technology is very similar to that of firm A, both of them mainly compete in the transparent segment. Improving firm B's profiling technology makes competition in the transparent segment even more intense, which in turn leads to more competition in the opaque segment (and hence the lower uniform prices).

Data brokerage. In the introduction, we documented the activities of data brokers, which provide firms with sets of data, allowing them to identify consumer valuations. Let us then extend our baseline model by assuming the presence of a single data broker that supplies the two firms with similar data sets (leading then to correlated profiling technologies). If we had to endogenize the behavior of such data broker, we already know that the data broker would never provide the two firms with the exact same set of data (implying $\alpha = 1$), nor would it give an exclusive access to data to one firm (implying $\alpha = 0$). We indeed know from Proposition 1 that each firms' willingness-to-pay for data is equal to zero in these two scenarios.

The data broker's optimal conduct would thus be to provide data to both firms but with asymmetric qualities. Yet, the effect of asymmetry on prices in not univocal: increasing α will sometimes push prices up and sometimes down. This is not surprising since we know that in the two extremes ($\alpha = 1$ and $\alpha = 0$), prices equal marginal cost. There is thus an intermediate level of asymmetry that maximizes the joint profits of the two firms and this level of asymmetry would be the one chosen by a data broker who can sell datasets of different qualities to firm A and B.

In our setup it follows from Corollary 2 that the optimal level of asymmetry is to put $\alpha = 1/2$ for all log-concave demand functions. Indeed, a data broker would want to avoid too much and too little symmetry, and the profit maximizing balance is struck at the intermediate level $\alpha = 1/2$. This implies that the data broker would always wish to share its data with both firms, but in an asymmetric way: it prefers to give one firm an advantage over the other firm when it comes to detecting the willingness to pay of consumers. As we discussed in the introduction this result is in contrast to the exclusivity results of Clavorà Braulin and Valletti (2016) and Montes et al. (2015), as in our model the data broker will always have incentives to sell its data to all competing firms, though the data set obtained by each firm will be of different qualities.

We can reinterpret these results in a duopoly setting in which two data brokers can sell datasets of different quality. As long as each one of them cannot offer a menu of datasets of different qualities, they will vertically differentiate the data sets they sell at equilibrium. Ignoring investments costs, we note that the data broker who sells the highest quality data will earn more profits. This is due to the following result:

Corollary 4. *Firm A, which has the better profiling technology, earns higher expected profits than firm B.*

Proof. We know that firms are indifferent between all equilibrium uniform prices. Hence, the expected profit from any uniform price is equal to the expected profit from playing the lowest possible uniform price p_l. When setting their price equal to p_l, both firms will have the same expected profit from the opaque segment. Moreover, firm A will get additional profits from the translucent segment since she will have a lower targeted price than firm B's observed uniform price. Finally, the expected profit from the transparent market is higher for firm A than for firm B, since firm B minimizes profits from the transparent market by setting $p_B = p_l$. Firm A's expected profits on the transparent market are averaged over all possible uniform prices firm B may choose, either between p_l and p^m or equal to $\bar{r}$, and these expected profits do not depend on its own uniform price, p_A. Adding these three effects, we see that A enjoys higher expected profits. □

5.2 Consumer surplus and demand for privacy

Many observers of the data revolution fear its negative effects on consumer welfare. This is eloquently expressed by Newman (2014, emphasis added):

> "The darker version of online marketing is that it can facilitate what economists call 'price discrimination,' selling the same exact good at a variety of prices in ways unknown to the buyers. This is based on the reality that people have different maximum prices that they are willing to pay, a so-called 'pain point' after which they won't buy the product. The ideal for a seller would be to sell a product to each customer at their individual 'pain point' price *without them knowing that any other deal is available.*"

Such fears have lead the Council of Economic Advisers of the Whitehouse to release a study on big data and differential pricing in 2015, in which it is equally recognized that firms may be engaging in random price testing.[22]

How does our analysis contribute to this debate? Although it seems extremely hard to perform a general analysis of the impacts of asymmetric profiling on consumer surplus,[23] we are able to shed some light on a number of interesting issues and, thereby, to provide some useful insights. In a nutshell, we show that consumers are unequal in front of improved profiling: although many consumers may lose (and would thus be willing to protect their privacy), some other consumers may win; it may even be the case that the winners would benefit sufficiently so that they could compensate the losers, i.e., consumers as a group would welcome improved profiling.

Do consumers necessarily suffer from improved profiling? To answer formally this question, we decompose the effects that improved profiling may have on different categories of consumers. We know from Corollary 1 that an increase in λ decreases the probability that either firm will charge any uniform price below some price p. This unequivocally implies that all prices go up. It may then seem natural to conclude that the consumer surplus goes down. But there is a twist: the impact of these higher prices is only clear for consumers whose status does not change, i.e., those who remain anonymous (who face higher uniform prices) and those who remain profiled (who face higher personalized prices). Yet, the impact on consumers who switch status remains unclear. Improved profiling indeed means that some consumers who used to be anonymous are now profiled. Among them, consumers with a low valuation will start purchasing (which they did not do when they were anonymous), and will thus enjoy a larger surplus; consumers with a larger valuation, who continue to purchase, may also benefit if they are now charged a lower personalized price than the uniform price they were paying before. As the following example shows, the positive impact on 'switchers' may be sufficiently important to outweigh the negative impact on 'non switchers', so that improved profiling ends up *increasing* consumer surplus on aggregate.

Example 2. *Uniform distribution (continued)*

Suppose that $\alpha = 1/2$ and $\lambda > 8/9$, which implies that $2(1-\lambda) - \alpha\lambda(1-\alpha) < 0$, so that $\nabla_A = 0$. Under these conditions, firm A plays $p_A = p^m = 1/2$ with probability 1, whereas

[22] *Big Data and Differential Pricing.* [Washington, D.C.]: Executive Office of the President of the United States, Council of Economic Advisors, 2015.

[23] Writing down the consumer surplus supposes being able to complete the following list of tasks. First, for each consumer with value r, we need to obtain the (expected) prices that she is likely to face, taking into account that she will face (i) the minimum of the two (potentially random) uniform prices if she is not profiled by any firm, (ii) the minimum of both firms' random personalized prices if she is profiled by both firms, or (iii) the minimum of firm A's random personalized price and firm B's (potentially random) uniform price if she is profiled only by firm A. Second, for each minimum of prices, we need to calculate the densities of these minimum prices and the corresponding surplus to this consumer. Third, we need to integrate over all prices to obtain the expected surplus of this consumer. Finally, we need to integrate over all consumers to get the total consumer surplus. One understands that these operations are complex to perform, even in the case of simple distributions such as the uniform distribution. Moreover, performing comparative static exercises would add another layer of complexity.

firm B plays $p_B = 1$ with probability 1. Consumers thus face the monopoly price when they remain anonymous. For the consumers that they profile, both firms draw their personalized price from the interval $[r/2, r]$ according to the distribution function $G(p) = 2 - (r/p)$, with density $g(p) = r/p^2$. Hence, when consumers are profiled only by firm A, they face one draw from $G(p)$. When they are profiled by both firms, they face the minimum of two draws from $G(p)$, which follows the distribution $\Gamma(p) = G(p)^2$, with density $\gamma(p) = 2G(p)g(p)$. Consumer surplus is then:

$$\begin{aligned} CS(\lambda)|_{\lambda \geq 8/9} &= (1-\lambda)\int_{\frac{1}{2}}^{1} \left(r - \tfrac{1}{2}\right) dr + \frac{\lambda}{2}\int_{0}^{1}\left(r - \int_{\frac{r}{2}}^{r} pg(p)dp\right) dr \\ &+ \frac{\lambda}{2}\int_{0}^{1}\left(r - \int_{\frac{r}{2}}^{r} p\gamma(p)dp\right) dr = (1-\lambda)\frac{1}{8} + \lambda\frac{4 - 5\ln 2}{4}. \end{aligned}$$

It follows that

$$\left.\frac{\partial CS(\lambda)}{\partial \lambda}\right|_{\lambda \geq 8/9} = -\frac{1}{8} + \frac{4 - 5\ln 2}{4} = \frac{7 - 10\ln 2}{8} \approx 0,0086 > 0.$$

We observe that in aggregate, consumer surplus increases with λ in this particular case. To understand this result, let us divide the group of 'switchers' into three subgroups. We have first the 'Low value switchers' (group L), who did not purchase before precision improved; then we have the 'Middle value switchers' (group M), who would have purchased at the monopoly price but have now access, with higher precision, to a lower price (in expectation); finally, we have the 'High value switchers' (group H), who would have purchased at the monopoly price but are now faced with a targeted price that is higher than the monopoly price in expectations (these consumers would prefer to 'hide'). In the present example, the benefits to groups L and M (a market expansion effect) outweigh the losses for group H (a price discrimination effect). Moreover, the benefits for switchers with low and middle valuations also outweigh the losses for all consumers who do not switch status.[24]

But if consumers as a group benefit, does it necessarily imply that firms will have lower profits? The answers is no. In fact, in the above example *firm A's profit remain constant, while firm B's profit increases when λ increases.* For any $\lambda > 8/9$, firm A's expected profit is computed as:

$$\begin{aligned} \Pi_A(\lambda)|_{\lambda \geq 8/9} &= (1-\lambda)\int_{\frac{1}{2}}^{1} \frac{1}{2} dr + \lambda\left(\frac{1}{2}\int_{0}^{1}\int_{\frac{r}{2}}^{r} pg(p)dpdr + \frac{1}{2}\int_{0}^{1}\int_{\frac{r}{2}}^{r} p(1-G(p))g(p)dpdr\right) \\ &= (1-\lambda)\frac{1}{4} + \lambda\frac{1}{2}\int_{0}^{1}\int_{\frac{r}{2}}^{r}\frac{r^2}{p^2}dpdr = (1-\lambda)\frac{1}{4} + \lambda\left[\frac{1}{2}\int_{0}^{1} rdr\right] = \frac{1}{4}, \end{aligned}$$

[24]The benefits of market expansion are also put forward by Ant Financial Services Group about its credit-scoring system, Sesame Credit: "Sesame Credit is the first credit agency in China to use a scoring system based on online and offline data to generate individual credit scores for consumers and small business owners. These ratings provide lenders, merchants and other companies with a reliable tool for assessing their customers' creditworthiness, hence *giving more consumers access to a host of borrowing services* such as home loans, mobile-phone service contracts, car loans and other types of installment credit." (Business Wire, 2015; emphasis added).

which is invariant with λ. As for firm B, we have

$$\Pi_B(\lambda)|_{\lambda\geq 8/9} = \lambda\frac{1}{2}\int_0^1\int_{\frac{r}{2}}^r p(1-G(p))g(p)dpdr = \lambda\frac{1}{2}\int_0^1\int_{\frac{r}{2}}^r r\tfrac{r-p}{p^2}dpdr = \lambda\tfrac{1}{4}\left(1-\ln 2\right),$$

which is an increasing function of λ (as $\ln 2 < 1$).

Hence, the market expansion effect dominates for firms as well. This is no surprise for firm B, since she only obtains positive profits on the transparent segment and prices in the transparent segment are not affected by an increase in λ. For firm A the loss of profits on the opaque segment is just compensated by the gain in profits in the transparent and translucent segments. Hence, somewhat surprisingly, while consumers as a group are better off on average, this does not happen at the expense of the firms' profits. To the contrary, consumers (as a group) and firms (as a group) strictly benefit from higher precision. This is due to the market expansion effect and the corresponding reduction in the deadweight loss.

We can also study how consumers are affected when only the profiling technology of the less-informed firm improves (i.e., when α increases). In this case, we see from Corollary 3 that the impact depends of where we start from. For relatively high levels of symmetry between firms' profiling technologies ($\alpha > 1/2$), increasing further the symmetry benefits consumers as it leads to both lower uniform prices and lower personalized prices. In contrast, for relatively low levels of symmetry ($\alpha < 1/2$), increasing the symmetry leads to higher uniform prices but lower personalized prices.

Exogenous and endogenous privacy. So far, we have assumed that it is not possible for consumers to escape being identified by the firms when profiling technologies are effective. In reality, consumers may resort to obfuscation strategies that make profiling technologies inoperative; for instance, consumers may delete cookies, use tools to browse the web anonymously, or purchase ad blockers. Although a full analysis of this possibility goes beyond the scope of this paper, we can discuss under which conditions consumers may wish to 'hide' themselves.

As we saw from Proposition 3, firm B may, when the opaque segment is relatively small, have an incentive to focus only on consumers it can profile, by charging the choking price of demand as its uniform price, while firm A charges the monopoly price as its uniform price. This pushes up prices on the transparent segment, leading in some cases to a situation in which all personalized prices lie above the monopoly price (charged to consumers that a firm does not profile). In this case, some of these consumers would prefer to hide, as they will face lower prices in the opaque segment. We saw in Corollary 1 that this situation becomes more likely the higher the precision of the detection technology. This implies that the less privacy regulation there is (the higher λ), the more it will lead to a *demand for (endogenous) privacy.* As a consequence, better privacy regulation (lower λ) would tend to push prices down, increasing consumer surplus, and lower the need for consumers to hide. We conclude that the fear for consumers to face higher prices when they are 'in the open' rather than 'in the dark' does not disappear when it is not just one but several competing firms that have the ability to profile consumers.

5.3 What if uniform prices are made public?

We now study the consequences of having 'observable' uniform prices. We have in mind what is commonly known as *list prices*, i.e., (uniform) prices that any consumer can observe and can thus claim to buy at. The immediate consequence of the presence of such prices is that firms now face an upper bound on the personalized prices they can charge to the consumers they profile. Hence, a firm does not just compete against the prices of its competitor but also against its own uniform price. We expect that this would lead to more competition and lower prices in equilibrium. We provide two results that allow us to confirm this conjecture.

First, it is immediate that setting all prices equal to marginal cost is now a subgame-perfect equilibrium of the game. In contrast to the case where consumers only receive one price from each firm, no firm has an incentive to increase its uniform price at stage 1, as all consumers (profiled or not) always keep the possibility to purchase at the lowest listed uniform price. We conclude that the observability of uniform prices restores the Bertrand Paradox even when firms have asymmetric profiling technologies.

Proposition 4. *If all consumers can purchase a good at the (observed) uniform price, then marginal cost pricing is a subgame perfect Nash equilibrium of the pricing game.*

It remains to show, however, that this is the only equilibrium. Our second result gives an indicative answer. We show that, in a simplified version of our model where consumers can have valuations of only two types, high and low, no other equilibrium can exist.

Example 3. *Two types of consumers*

Assume that there is a unit mass of consumers and that any consumer x can be one of two types. With probability $\varphi \in (0,1)$, they have a high valuation for the homogeneous product: $r(x) = 1$; with probability $1-\varphi$, they have a low valuation for the homogeneous product: $r(x) = \zeta < 1$. We also assume that if the market was served by a (non discriminating) monopolist, this firm would set a uniform price equal to ζ and so, serve the whole market; this happens when $\varphi < \zeta$. In this setting, we can show (see Appendix 6.4) that there cannot exist a subgame-perfect Nash equilibrium of the game, in which both firms draw from a connected interval $\left[\underline{p}, \zeta\right)$ in which firm A draws p_A according to the distribution $H_A(p)$ with probability ∇_A, or sets $p_A = \zeta$ with probability $1-\nabla_A$ and in which Firm B draws p_B from the distribution $H_B(p)$ with probability ∇_B, or sets $p_B = \zeta$ with probability $1-\nabla_B$.

Given the previous two findings, there are good reasons to believe that list prices exert downward pressure on equilibrium prices in our model.

6 Discussion and concluding remarks

We have examined price competition in a homogenous goods setting in which firms can imperfectly profile consumers. When they do, they can charge these consumers a personalized price, but firms (potentially) differ in their ability to profile. This asymmetry between the firms' detection technologies leads to uncertainty about the nature of price competition, generating market power. We establish that there may be three simultaneous explanations as to why two consumers end up paying different prices: firms may charge them either different personalized prizes (price discrimination), or randomized uniform prices (price dispersion), or randomized personalized prices (price discrimination *and* dispersion). The necessary ingredients for this result are price competition and 'imperfect consumer profiling', which refers to the idea that firms are able to identify consumers' valuations for the product but only in an imperfect way. The profiling of consumers enables price discrimination, but its imperfect nature makes firms uncertain about the competitor's pricing strategy, which triggers strategic random pricing. In sum, as long as both firms can track consumers, asymmetric profiling technologies can generate a way out of the Bertrand paradox. Our results also imply that when only one firm can profile consumers, it does not manage to obtain any market power: a partial irrelevant result arises. As a consequence, upstream data owners/brokers have incentives to provide both firms with data as long as the quality of the data is different; that is, vertical data differentiation arises.

Our analysis allows us to draw the following policy implications. First, making privacy rules less protective (as decided in April 2017 in the US) has ambiguous impacts on consumers: on the one hand, some consumers will pay higher prices because firms can profile them more easily; but on the other hand, some consumers who used to be anonymous will now be profiled and among them, those with a low valuation will start purchasing and will thus enjoy a larger surplus. We even show the possibility that the positive impact on the latter consumers may be sufficiently important to outweigh the negative impact on the former ones, so that improved profiling (made possible by more lenient privacy rules) ends up increasing consumer surplus on aggregate. Yet, we also demonstrate that all consumers should be better off if firms had the obligation to make their uniform prices public. With such observable ?list prices?, firms would face an upper bound on the personalized prices they can charge to the consumers they profile, which is likely to drive prices down (possibly back to marginal costs). Finally, we also show that exclusivity contracts offered by data brokers do not necessarily harm consumers. Indeed, in our model exclusivity leads to more and not to less competition.

It is important to highlight some of the limitations of our model. We assume that profiling technologies are correlated, implying that one firm is always better informed about consumers' willingness to pay. We speculate that relaxing this assumption by introducing imperfectly correlated profiling technologies will not change the key insights we have obtained. More importantly, it would be interesting to analyze the impact of endogenous privacy choices by consumers. We have established that high value consumers would have an incentive to hide and become anonymous. Such hiding behavior would affect the firms' pricing strategies and this, in turn, will affect hiding behavior. Once established an equilibrium with endogenous hiding, one could study the effect of consumer hiding on firms' profits and consumer surplus.

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Appendix

6.1 Proof of Proposition 2

Denote by $\tilde{\pi}_i(p)$ firm i's expected profit obtained from consumer x when pricing $p_i(x) = p$ at stage 2.[25] Supposing that firm B draws its price $p_B(x)$ from some distribution $G_B^x(p)$ that is defined on the interval $[(1-\alpha)p_B^x, p_B^x]$, let us show that firm A achieves the same expected profit when playing $p_A(x) = (1-\alpha)\,p_B^x$ or $p_A(x) = p_B^x$. As $G_B^x((1-\alpha)\,p_B^x) = 0$ and $G_B^x(p_B^x) = 1$, we have indeed that

$$\begin{aligned}
\tilde{\pi}_A((1-\alpha)\,p_B^x) &= (1-\alpha)^2\,p_B^x + \alpha\,(1 - G_B^x((1-\alpha)\,p_B^x))\,(1-\alpha)\,p_B^x \\
&= (1-\alpha)\,p_B^x, \\
\tilde{\pi}_A(p_B^x) &= (1-\alpha)\,p_B^x + \alpha\,(1 - G_B^x(p_B^x))\,p_B^x = (1-\alpha)\,p_B^x.
\end{aligned}$$

For firm A to be indifferent among any price $p \in [(1-\alpha)p_B^x, p_B^x]$, its expected profit must always be equal to $(1-\alpha)\,p_B^x$; that is

$$\tilde{\pi}_A(p) = (1-\alpha)\,p + \alpha(1 - G_B^x(p))p = (1-\alpha)\,p_B^x,$$

from which we obtain

$$G_B^x(p) = \frac{p - (1-\alpha)\,p_B^x}{\alpha p} = G^x(p).$$

Suppose now that firm A plays the following strategy for any consumer x: with probability Δ_A, it draws $p_a(x)$ from some distribution $G_A^x(p)$ that is defined on the interval $[(1-\alpha)p_B^x, p_B^x]$,

[25] These profits are conditional on being on markets where personalized prices are relevant (i.e., the translucent and transparent segments for firm A, with probability λ, and the transparent segment for firm B, with probability $\alpha\lambda$).

while with probability $1-\Delta_A$, it sets $p_a(x)=p_B^x$. As $G_A^x((1-\alpha)p_B^x)=0$, firm B's expected profit from playing $p_B(x)=(1-\alpha)p_B^x$ is equal to

$$\begin{aligned}\tilde{\pi}_B((1-\alpha)p_B^x) &= \Delta_A(1-G_A^x((1-\alpha)p_B^x))(1-\alpha)p_B^x+(1-\Delta_A)(1-\alpha)p_B^x \\ &= (1-\alpha)p_B^x.\end{aligned}$$

By taking limits, we also obtain that firm B's expected profit from playing $p_B(x)=p_B^x$ is given by

$$\lim_{p\to p_B^x}\tilde{\pi}_B(p)=\lim_{p\to p_B^x}(\Delta_A(1-G_A^x(p))p+(1-\Delta_A)p)=(1-\Delta_A)p_B^x.$$

For firm B to be indifferent between playing $p_B(x)=(1-\alpha)p_B^x$ or $p_B(x)=p_B^x$, firm A must choose Δ_A such that $(1-\alpha)p_B^x=(1-\Delta_A)p_B^x$, or $\Delta_A=\alpha$. In addition, firm B must also be indifferent between any $p\in[(1-\alpha)p_B^x,p_B^x]$, which requires $(1-\alpha)p+\alpha(1-G_A^x(p))p=(1-\alpha)p_B^x$, which is equivalent to

$$G_A^x(p)=\frac{p-(1-\alpha)p_B^x}{\alpha p}=G^x(p).$$

Given that $p_B>0$, it follows that for any x such that $r(x)>0$, both firms make positive expected profits equal to $(1-\alpha)p_B^x=(1-\alpha)\min\{p_B,r(x)\}$.

6.2 Proof of Proposition 3

In any mixed strategy equilibrium, firm B must be indifferent between charging any price p in the support of H_B, p_l, $\lim_{p\to p^m}$, and $\bar{r}$. Recall that by choosing $\bar{r}$, firm B can secure a profit of $\pi_B^{\min}=\alpha(1-\alpha)\lambda\hat{r}$. If firm B chooses some price p, then with probability $\alpha\lambda$, firm B competes for consumers on the transparent segment and will obtain expected profits of $\mathring{\pi}_B(p)$; with probability $1-\alpha\lambda$, firm B does not recognize consumers and will only obtain positive profits if firm A does not recognize consumers either (the segment is opaque), which happens with probability $(1-\lambda)/(1-\alpha\lambda)$ (conditional on the event in which firm B does not recognize a consumer). Therefore, the probability that firm B is active on the opaque segment is $(1-\lambda)$. Given that with probability ∇_A, firm A draws p_A from the distribution $H_A(p)$ defined on $[p_l,p^m]$ and, with probability $1-\nabla_A$, sets $p_A=p^m$, we can compute firm B's expected profit from choosing price p as

$$\Pi_B^e(p)=\alpha\lambda\mathring{\pi}_B(p)+(1-\lambda)p(1-F(p))\left[(1-\nabla_A)+\nabla_A(1-H_A(p))\right].$$

As firm B must be indifferent between charging any price p in the support of H_B and $\bar{r}$, we must have that $\Pi_B^e(p)=\pi_B^{\min}$, from which we obtain

$$\begin{aligned}H_A(p) &= \frac{\alpha\lambda(\mathring{\pi}_B(p)-(1-\alpha)\hat{r})+(1-\lambda)p(1-F(p))}{(1-\lambda)p(1-F(p))\nabla_A} \\ &= 1-\frac{\alpha\lambda((1-\alpha)\hat{r}-\mathring{\pi}_B(p))-(1-\lambda)(1-\nabla_A)p(1-F(p))}{(1-\lambda)p(1-F(p))\nabla_A}.\end{aligned}$$

As $H_A(p_l) = 0$, firm B achieves the following expected profit when charging p_l:

$$\begin{aligned}\Pi^e_B(p_l) &= \alpha\lambda \int_0^{p_l} (1-\alpha) r f(r) dr + \alpha\lambda \int_{p_l}^{\bar{r}} (1-\alpha) p_l f(r) dr + (1-\lambda) p_l (1 - F(p_l)) \\ &= \alpha\lambda \int_0^{p_l} (1-\alpha) r f(r) dr + (\alpha\lambda (1-\alpha) + 1 - \lambda) p_l (1 - F(p_l)).\end{aligned}$$

As firm B must be indifferent between charging p_l and $\bar{r}$, we have that $\Pi^e_B(p_l) = \alpha\lambda(1-\alpha)\hat{r}$, which implicitly defines p_l as a function of α, λ and F:

$$p_l = \frac{\alpha\lambda(1-\alpha)\left(\hat{r} - \int_0^{p_l} r f(r) dr\right)}{(\alpha\lambda(1-\alpha) + 1 - \lambda)(1 - F(p_l))}.$$

As $H_A(p^m) = 1$, firm B's expected profit from charging $p^m$is obtained as follows

$$\Pi^e_B(p^m) = \alpha\lambda\mathring{\pi}_B(p^m) + (1-\lambda)p^m (1 - F(p^m))(1 - \nabla_A),$$

which must be equal to $\pi_B^{\min} = \alpha(1-\alpha)\lambda\hat{r}$. We obtain thus

$$1 - \nabla_A = \frac{\alpha\lambda(1-\alpha)\left(\hat{r} - \int_0^{p^m} r f(r) dr - \pi^m\right)}{(1-\lambda)\pi^m} > 0.$$

It must also be that $\nabla_A > 0$. This is satisfied when

$$(1-\lambda)\pi^m \geq \alpha\lambda(1-\alpha)\left(\hat{r} - \int_0^{p^m} r f(r) dr - \pi^m\right).$$

When this condition is not satisfied, then $\nabla_A = 0$ and firm A plays $p_A = p^m$ with probability 1. In this case, firm B's optimal response is to play $p_B = \bar{r}$ with probability 1. In other words, we obtain an equilibrium in pure strategies in which firm A will be a monopolist in markets for consumers which are not identified, while firm B gives up any sales to consumers it does not identify and chooses a price that maximizes its expected profits in the transparent segment.

Now assume that $\nabla_A > 0$, it must be that firm A is indifferent between charging p^m and p_l. Denote by $\hat{\pi}_A(\nabla_B, H_B)$ the expected profit of firm A given firm B's regular price strategy. We then have:

$$\Pi^e_A(p) = \lambda\hat{\pi}_A(\nabla_B, H_B) + (1-\lambda) p (1 - F(p))(1 - \nabla_B + \nabla_B (1 - H_B(p))).$$

As $H_B(p^m) = 1$ and $H_B(p_l) = 0$, it follows that

$$\begin{aligned}\Pi^e_A(p^m) &= \lambda\hat{\pi}_A(\nabla_B, H_B) + (1-\lambda) p^m (1 - F(p^m))(1 - \nabla_B), \\ \Pi^e_A(p_l) &= \lambda\hat{\pi}_A(\nabla_B, H_B) + (1-\lambda) p_l (1 - F(p_l)).\end{aligned}$$

For firm A to be indifferent, we need $\Pi^e_A(p^m) = \Pi^e_A(p_l)$, which is equivalent to

$$1 - \nabla_B = \frac{p_l(1 - F(p_l))}{p^m(1 - F(p^m))} = \frac{\pi_l}{\pi^m}.$$

Since $\pi_l \leq \pi^m$, it must be that $\nabla_B \in [0,1]$. It must also be the case that $\Pi^e_A(p) = \Pi^e_A(p_l)$, from which we obtain

$$\begin{aligned} H_B(p) &= 1 - \frac{\pi_l - p(1-F(p))(1-\nabla_B)}{p(1-F(p))\nabla_B} \\ &= 1 - \frac{\pi_l - p(1-F(p))\frac{\pi_l}{\pi^m}}{p(1-F(p))\left(1-\frac{\pi_l}{\pi^m}\right)} \\ &= 1 - \frac{\pi_l}{\pi^m - \pi_l}\left(\frac{\pi^m}{p(1-F(p))} - 1\right). \end{aligned}$$

This completes the proof.

6.3 Comparative statics results

6.3.1 Proof of Corollary 1

1. Let us show that $\frac{\partial p_l}{\partial \lambda} > 0$. We have that

$$p_l(\alpha\lambda(1-\alpha) + 1 - \lambda)(1 - F(p_l)) = \alpha\lambda(1-\alpha)\left(\hat{r} - \int_0^{p_l} rf(r)dr\right).$$

Totally deriving with respect to λ we obtain

$$\begin{aligned} &\frac{\partial p_l}{\partial \lambda}\left[(\alpha\lambda(1-\alpha) + 1 - \lambda)(1 - F(p_l) - p_l f(p_l))\right] + p_l(\alpha(1-\alpha) - 1)(1 - F(p_l)) \\ = \;& \alpha(1-\alpha)\left(\hat{r} - \int_0^{p_l} rf(r)dr\right) - \frac{\partial p_l}{\partial \lambda}\alpha\lambda(1-\alpha)\int_0^{p_l} rf(r)dr. \end{aligned}$$

or

$$\frac{\partial p_l}{\partial \lambda}\underbrace{\left[(\alpha\lambda(1-\alpha) + 1 - \lambda)(1 - F(p_l) - p_l f(p_l)) + \alpha\lambda(1-\alpha)\int_0^{p_l} rf(r)dr\right]}_{=A}$$

$$= \alpha(1-\alpha)\left(\underbrace{\hat{r} - \int_0^{p_l} rf(r)dr - p_l(1 - F(p_l))}_{=B}\right) + p_l(1 - F(p_l))$$

Since $p_l < p^m$ we have that $1 - F(p_l) - p_l f(p_l) > 1 - F(p^m) - p^m f(p^m) = \left.\frac{\partial \pi}{\partial p}\right|_{p=p^m} = 0$, from which we obtain that $A > 0$. Also note that $(1-\alpha)\hat{r} = \mathring{\pi}_B(\bar{r}) > \mathring{\pi}_B(p_l) = (1-\alpha)\left(\int_0^{p_l} rf(r)dr + p_l(1 - F(p_l))\right)$ and hence $B > 0$. From this, it follows that $\frac{\partial p_l}{\partial \lambda} > 0$.

2. Let us show that $\frac{\partial \nabla_A}{\partial \lambda} < 0$. Observe that

$$\nabla_A = 1 - \frac{\alpha(1-\alpha)\left(\hat{r} - \int_0^{p^m} rf(r)dr - \pi^m\right)}{\pi^m} \times \frac{\lambda}{1-\lambda}$$

and hence

$$\frac{\partial \nabla_A}{\partial \lambda} = -\frac{\alpha(1-\alpha)\left(\hat{r} - \int_0^{p^m} rf(r)dr - \pi^m\right)}{\pi^m} \times \frac{1}{(1-\lambda)^2} < 0$$

Which can be rewritten as:

$$\frac{\partial \nabla_A}{\partial \lambda} = -\eta_A \times \frac{1}{\lambda(1-\lambda)} < 0$$

3. Let us show that $\frac{\partial H_A(p)}{\partial \lambda} > 0$. Recall that:

$$H_A(p) = 1 - \frac{\alpha(1-\alpha)\left(\hat{r} - \int_0^p rf(r)dr - p(1-F(p))\right)}{p(1-F(p))} \times \frac{\lambda}{(1-\lambda)\nabla_A} + \frac{1-\nabla_A}{\nabla_A};$$

$$\begin{aligned}\frac{\partial H_A(p)}{\partial \lambda} &= -\frac{\alpha(1-\alpha)\left(\hat{r} - \int_0^p rf(r)dr - p(1-F(p))\right)}{p(1-F(p))} \times \frac{(1-\lambda)\nabla_A - \lambda\left(\frac{\partial \nabla_A}{\partial \lambda}(1-\lambda) - \nabla_A\right)}{\left((1-\lambda)\nabla_A\right)^2} \\ &\quad + \frac{-\frac{\partial \nabla_A}{\partial \lambda}\nabla_A - \frac{\partial \nabla_A}{\partial \lambda}(1-\nabla_A)}{(\nabla_A)^2};\end{aligned}$$

Grouping terms, we obtain

$$\frac{\partial H_A(p)}{\partial \lambda} = -\frac{\alpha(1-\alpha)\left(\hat{r} - \int_0^p rf(r)dr - p(1-F(p))\right)}{p(1-F(p))} \times \frac{\nabla_A - \frac{\partial \nabla_A}{\partial \lambda}\lambda(1-\lambda)}{\left((1-\lambda)\nabla_A\right)^2} - \frac{\frac{\partial \nabla_A}{\partial \lambda}}{(\nabla_A)^2}.$$

Note that $\nabla_A - \frac{\partial \nabla_A}{\partial \lambda}\lambda(1-\lambda) = 1 - \eta_A + \eta_A = 1$ and hence we have

$$\frac{\partial H_A(p)}{\partial \lambda} = -\frac{\alpha(1-\alpha)\left(\hat{r} - \int_0^p rf(r)dr - p(1-F(p))\right)}{p(1-F(p))} \times \frac{1}{\left((1-\lambda)\nabla_A\right)^2} + \frac{\eta_A}{\lambda(1-\lambda)(\nabla_A)^2}.$$

Since

$$\begin{aligned}&\frac{\alpha(1-\alpha)\left(\hat{r} - \int_0^p rf(r)dr - p(1-F(p))\right)}{p(1-F(p))} \\ &= \frac{\alpha(1-\alpha)\lambda(\hat{r} - \int_0^p rf(r)dr - p(1-F(p)))}{(1-\lambda)p(1-F(p))} \times \frac{1-\lambda}{\lambda} > \eta_A\frac{1-\lambda}{\lambda},\end{aligned}$$

we have that

$$\frac{\partial H_A(p)}{\partial \lambda} < -\eta_A\frac{1-\lambda}{\lambda} \times \frac{1}{\left((1-\lambda)\nabla_A\right)^2} + \frac{\eta_A}{\lambda(1-\lambda)(\nabla_A)^2}$$

$$\text{or } \frac{\partial H_A(p)}{\partial \lambda} < -\frac{\eta_A}{\lambda(1-\lambda)(\nabla_A)^2} + \frac{\eta_A}{\lambda(1-\lambda)(\nabla_A)^2} = 0.$$

4. Let us show that $\frac{\partial \nabla_B}{\partial \lambda} < 0$. Recall that $\nabla_B = 1 - \frac{p_l(1-F(p_l)}{\pi^m}$. Since $\frac{\partial p_l}{\partial \lambda} > 0$ and $p_l < p^m$, we have that:

$$\frac{\partial \nabla_B}{\partial \lambda} = -\frac{\frac{\partial p_l}{\partial \lambda}(1 - F(p_l) - p_l f(p_l))}{\pi^m} < 0.$$

5. Let us show that $\frac{\partial H_B(p)}{\partial \lambda} < 0$. Recalling that

$$H_B(p) = 1 - \frac{1-\nabla_B}{\nabla_B} \times \left(\frac{\pi^m}{p(1-F(p))} - 1\right),$$

we observe that

$$\frac{\partial H_B(p)}{\partial \lambda} = \left(\frac{\pi^m}{p(1-F(p))} - 1\right)\frac{\frac{\partial \nabla_B(p)}{\partial \lambda}}{(\nabla_B)^2}$$

and hence $\frac{\partial H_B(p)}{\partial \lambda} < 0$ since $\frac{\partial \nabla_B(p)}{\partial \lambda} < 0$.

6.3.2 Proof of Corollary 2

We demonstrate here that for high levels of asymmetry ($\alpha < \frac{1}{2}$), increasing α increases the uniform prices, whereas the opposite is true when the level of symmetry is high ($\alpha > \frac{1}{2}$). That is, we show

$$\frac{\partial p_l}{\partial \alpha} \lesseqqgtr 0 \Leftrightarrow \alpha \gtreqqless \frac{1}{2} \text{ and } \frac{\partial \nabla_A}{\partial \alpha}, \frac{\partial \nabla_B}{\partial \alpha}, \frac{\partial H_A(p)}{\partial \alpha} \frac{\partial H_B(p)}{\partial \alpha} \lesseqqgtr 0 \Leftrightarrow \alpha \lesseqqgtr \frac{1}{2}.$$

1. Let us show that $\frac{\partial p_l}{\partial \alpha} > 0 \Longleftrightarrow \alpha < \frac{1}{2}$. We have that

$$p_l \left(\alpha\lambda \left(1-\alpha\right) + 1 - \lambda\right) \left(1 - F(p_l)\right) = \alpha\lambda \left(1-\alpha\right) \left(\hat{r} - \int_0^{p_l} rf(r)dr\right).$$

Totally deriving with respect to α we obtain

$$\begin{aligned} & \frac{\partial p_l}{\partial \alpha} \left[\left(\alpha\lambda \left(1-\alpha\right) + 1 - \lambda\right) \left(1 - F(p_l) - p_l f(p_l)\right)\right] + p_l \lambda \left(1 - 2\alpha\right) \left(1 - F(p_l)\right) \\ = \quad & \lambda \left(1 - 2\alpha\right) \left(\hat{r} - \int_0^{p_l} rf(r)dr\right) - \frac{\partial p_l}{\partial \alpha} \alpha\lambda \left(1-\alpha\right) \int_0^{p_l} rf(r)dr. \end{aligned}$$

or

$$\frac{\partial p_l}{\partial \lambda} \underbrace{\left[\left(\alpha\lambda \left(1-\alpha\right) + 1 - \lambda\right) \left(1 - F(p_l) - p_l f(p_l)\right) + \alpha\lambda \left(1-\alpha\right) \int_0^{p_l} rf(r)dr\right]}_{=A}$$

$$= \lambda \left(1 - 2\alpha\right) \left(\underbrace{\hat{r} - \int_0^{p_l} rf(r)dr - p_l(1 - F(p_l))}_{=B}\right)$$

Since $p_l < p^m$we have that $1 - F(p_l) - p_l f(p_l) > 1 - F(p^m) - p^m f(p^m) = \left.\frac{\partial \pi}{\partial p}\right|_{p=p^m} = 0$, from which we obtain that $A > 0$. Also note that $(1-\alpha)\,\hat{r} = \mathring{\pi}_B(\bar{r}) > \mathring{\pi}_B(p_l)) = (1-\alpha)\left(\int_0^{p_l} rf(r)dr + p_l(1 - F(p_l))\right)$ and hence $B > 0$. From this, it follows that $\frac{\partial p_l}{\partial \alpha} \lesseqqgtr 0 \Longleftrightarrow \alpha \gtreqqless \frac{1}{2}$.

2. Let us evaluate $\frac{\partial \nabla_A}{\partial \alpha}$. Recall that

$$\nabla_A = 1 - \alpha \left(1-\alpha\right) \left(\frac{\lambda}{(1-\lambda)\pi^m} \left(\hat{r} - \int_0^{p^m} rf(r)dr - \pi^m\right)\right), \text{ so that}$$

$$\frac{\partial \nabla_A}{\partial \alpha} = -\left(1 - 2\alpha\right) \left(\frac{\lambda}{(1-\lambda)\pi^m} \left(\hat{r} - \int_0^{p^m} rf(r)dr - \pi^m\right)\right).$$

We thus have that

$$\frac{\partial \nabla_A}{\partial \alpha} = -\frac{\partial \eta_A}{\partial \alpha} = -\eta_A \times \frac{1 - 2\alpha}{\alpha\left(1-\alpha\right)} \lesseqqgtr 0 \Longleftrightarrow \alpha \lesseqqgtr \frac{1}{2}.$$

3. We now turn to $\frac{\partial H_A(p)}{\partial \alpha}$. Recall that:

$$H_A(p) = 1 - \frac{\alpha\left(1-\alpha\right)}{\nabla_A} \frac{\lambda(\hat{r} - \int_0^p rf(r)dr - p(1 - F(p)))}{(1-\lambda)p(1 - F(p))} + \frac{1 - \nabla_A}{\nabla_A}, \text{ so that}$$

$$\begin{aligned}
\frac{\partial H_A(p)}{\partial \lambda} &= -\frac{\lambda(\hat{r} - \int_0^p rf(r)dr - p(1-F(p)))}{(1-\lambda)p(1-F(p))} \times \frac{(1-2\alpha)\nabla_A - \left(\frac{\partial \nabla_A}{\partial \alpha}\alpha\,(1-\alpha)\right)}{(\nabla_A)^2} - \frac{\frac{\partial \nabla_A}{\partial \alpha}}{(\nabla_A)^2} \\
&= -\frac{\lambda(\hat{r} - \int_0^p rf(r)dr - p(1-F(p)))}{(1-\lambda)p(1-F(p))} \times \frac{(1-2\alpha)\nabla_A + (1-\nabla_A)\,(1-2\alpha)}{(\nabla_A)^2} \\
&\quad + \frac{(1-\nabla_A)\,(1-2\alpha)}{\alpha(1-\alpha)\,(\nabla_A)^2} \\
&= \frac{1-2\alpha}{(\nabla_A)^2}\left[\underbrace{\frac{1-\nabla_A}{\alpha(1-\alpha)} - \frac{\lambda(\hat{r} - \int_0^p rf(r)dr - p(1-F(p)))}{(1-\lambda)p(1-F(p))}}_{C}\right].
\end{aligned}$$

Note that, since $p \leq p^m$, we have

$$\frac{\lambda(\hat{r} - \int_0^p rf(r)dr - p(1-F(p)))}{(1-\lambda)p(1-F(p))} \geq$$

$$\frac{\lambda(\hat{r} - \int_0^{p^m} rf(r)dr - p^m(1-F(p^m)))}{(1-\lambda)p^m(1-F(p^m))} = \frac{1-\nabla_A}{\alpha\,(1-\alpha)}.$$

Hence we obtain that $C \leq 0$, which implies that $\frac{\partial H_A(p)}{\partial \alpha} \lesseqqgtr 0 \Longleftrightarrow \alpha \lesseqqgtr \frac{1}{2}$. In other words, when α is low, increasing α increases prices on the opaque segment, as firms avoid cut-throat competition. In other words, when α is relatively high, increasing α make the firms even more symmetric, thereby increasing competition and lowering prices.

4. Now we consider $\frac{\partial \nabla_B}{\partial \alpha}$. Recall that $\nabla_B = 1 - \frac{p_l(1-F(p_l)}{\pi^m}$. We then obtain, since $\frac{\partial p_l}{\partial \alpha} > 0$ and $p_l < p^m$, that:

$$\frac{\partial \nabla_B}{\partial \alpha} = -\frac{\frac{\partial p_l}{\partial \alpha}\,(1-F(p_l) - p_l f(p_l))}{\pi^m} \lesseqqgtr 0 \Longleftrightarrow \alpha \lesseqqgtr \frac{1}{2}.$$

5. We finally discuss the sign of $\frac{\partial H_B(p)}{\partial \alpha}$. As

$$H_B(p) = 1 - \frac{1-\nabla_B}{\nabla_B} \times \left(\frac{\pi^m}{p(1-F(p))} - 1\right),$$

we observe that

$$\frac{\partial H_B(p)}{\partial \alpha} = \left(\frac{\pi^m}{p(1-F(p))} - 1\right)\frac{\frac{\partial \nabla_B(p)}{\partial \alpha}}{(\nabla_B)^2}$$

and hence

$$\frac{\partial H_B(p)}{\partial \alpha} \lesseqqgtr 0 \Longleftrightarrow \alpha \lesseqqgtr \frac{1}{2}.$$

6.3.3 Proof of Corollary 3

Let us consider $\frac{\partial G^x(p)}{\partial \alpha}$ for $p \in [(1-\alpha)p_B^x, p_B^x]$.

$$\frac{\partial G^x(p)}{\partial \alpha} = \frac{\partial}{\partial \alpha}\left(\frac{1}{\alpha} - \frac{p_B^x}{p}\frac{1-\alpha}{\alpha}\right) = \frac{p_B^x - p}{\alpha^2 p} > 0 \text{ for any } p < p_B^x.$$

6.3.4 Uniform distribution

Let $F(.)$ be the uniform distribution on $[0,1]$. It follows that the average valuation ($\hat{r}$) and the monopoly price (p^m) are both equal to $1/2$; the monopoly profit is computed as $\pi^m = 1/4$. Furthermore, we have

$$\mathring{\pi}_B(p) = \int_0^p (1-\alpha)rf(r)dr + (1-\alpha)p(1-F(p)) = (1-\alpha)(p - \tfrac{1}{2}p^2),$$

and

$$\pi_B^{min} = \tfrac{1}{2}\lambda\alpha(1-\alpha).$$

The lower bound of uniform price p_l is then defined by

$$p_l = \frac{\alpha\lambda(1-\alpha)(\hat{r} - \int_0^{p_l} rf(r)dr)}{(\alpha\lambda(1-\alpha)+1-\lambda)(1-F(p_l))} \Leftrightarrow p_l = \frac{\alpha\lambda(1-\alpha)}{2(1-\lambda)+\alpha\lambda(1-\alpha)}.$$

We observe that p_l is increasing in λ, increasing in α if $\alpha \le 1/2$, and decreasing in α if $\alpha > 1/2$. This implies that the uniform price is less dispersed as tracking technologies become more precise and the difference in profiling technologies between the two firms are neither too large nor too small (i.e., p_l is at the highest level when $\alpha = 1/2$).

The probability of firm A randomizing its uniform price is defined by

$$\nabla_A = 1 - \frac{\alpha\lambda(1-\alpha)}{(1-\lambda)\pi^m}\left(\hat{r} - \int_0^{p^m} rf(r)dr - \pi^m\right) = 1 - \frac{\alpha\lambda(1-\alpha)}{2(1-\lambda)}.$$

This probability is decreasing in λ, decreasing in α if $\alpha \le 1/2$, and increasing in α if $\alpha > 1/2$.

When $2(1-\lambda) \le \alpha\lambda(1-\alpha)$, we have $\nabla_A = 0$, so that firm A plays $p_A = p^m = 1/2$ with probability 1, whereas firm B plays $p_B = 1$ with probability 1. When $2(1-\lambda) > \alpha\lambda(1-\alpha)$, we have $\nabla_A > 0$; the probability of firm B randomizing its uniform is then defined by

$$\nabla_B = 1 - \frac{p_l(1-F(p_l))}{p^m(1-F(p^m))} = \left(\frac{2(1-\lambda)-\alpha\lambda(1-\alpha)}{2(1-\lambda)+\alpha\lambda(1-\alpha)}\right)^2.$$

The latter probability is decreasing in λ, decreasing in α if $\alpha \le 1/2$, and increasing in α if $\alpha > 1/2$.

Thus, firm B plays $p_B = 1$ with probability $1 - \nabla_B$, and draws a price from $[p_l, 1/2]$ with probability ∇_B and with a distribution

$$H_B(p) = 1 - \frac{1-\nabla_B}{\nabla_B}\left(\frac{1}{4p(1-p)} - 1\right),$$

which is increasing in ∇_B.

Firm A plays $p_A = 1/2$ with probability $1 - \nabla_A$, and draws a price from $[p_l, 1/2]$ with probability ∇_A and with a distribution

$$H_A(p) = \frac{1}{\nabla_A}\left(1 - \frac{\lambda\alpha(1-\alpha)(1-p)}{2(1-\lambda)p}\right).$$

6.4 Proof of the result in Example3

1. Personalized prices

Note first that the second stage equilibrium prices are still characterized by proposition 2:

$$G^{x}(p)=\begin{cases}\frac{p-(1-\alpha)p_B'^{x}}{\alpha p} & \text{for } p\in\left[(1-\alpha)p_B'^{x},p_B'^{x}\right],\\ 0 & \text{otherwise.}\end{cases}$$

Where $p_B'^{x}\equiv\min\{p_A,p_B,r(x)\}$. Given any uniform prices p_A and p_B, the expected payoff from personalized pricing is equal to $\tilde{\pi}=(1-\alpha)\,p_B'^{x}$. Note that we assume that in stage one, no firm will charge more than ζ and hence that $p_B'^{x}\equiv\min\{p_A,p_B,r(x)\}=\min\{p_A,p_B,\}$.

2. Uniform prices

Note that in any subgame perfect 'mixed strategy' Nash equilibrium players must choose prices from the same interval and only the upper bound of the interval can be assigned positive mass by at most one firm. No firm wants to charge a price below the lower bound of the interval since this decreases both first stage and second stage expected profits. No firms wants to charge above the upper bound of the interval since this will guarantee zero profits on anonymous consumers and does not affect expected profits obtained from personalized pricing. Now take any price p in the interval different from the upper bound and suppose that at least one firm, say firm A, plays this price with positive probability. Firm B should be indifferent between playing price p and any other price p' in the interval. However, since firm A plays p with positive probability, firm B will always find a price $p'=p-\epsilon$ that will guarantee higher profits: instead of sharing the consumers at this price p it can sell to the consumer for sure by slightly undercutting price p. However, it cannot be firm B that plays the upper bound, $p=\zeta$ with positive probability. The expected payoff to firms A and B from setting a uniform price equal to p is equal to:

$$\begin{aligned}\Pi_A^e(p) &= \lambda\left\{\nabla_B\left(\int_{\underline{p}}^{p}(1-\alpha)ph_B(\rho)d\rho+(1-\alpha)p(1-H_B(p))\right)+(1-\nabla_B)(1-\alpha)p\right\}\\ &\quad+(1-\lambda)p\left[1-\nabla_B+\nabla_B(1-H_B(p))\right]\end{aligned}$$

$$\begin{aligned}\Pi_B^e(p) &= \alpha\lambda\left\{\nabla_A\left(\int_{\underline{p}}^{p}(1-\alpha)ph_A(\rho)d\rho+(1-\alpha)p(1-H_A(p))\right)+(1-\nabla_A)(1-\alpha)p\right\}\\ &\quad+(1-\lambda)p\left[1-\nabla_A+\nabla_A(1-H_A(p))\right]\end{aligned}$$

The first term is the expected payoff from personalized pricing given that the other firm follows the strategies as defined in the proposed mixed strategy equilibrium. The second terms is the expected profits to be obtained from consumers it will not recognize given that the other follows the strategies as defined in the proposed mixed strategy equilibrium.

Suppose first that $\nabla_A<1$ and hence that $\nabla_B=1$. Now consider the expected payoff to firm A from setting a uniform price equal to $p\in\left[\underline{p},\zeta\right)$ is equal to:

$$\Pi_A^e(p) = \lambda \left\{ \int_{\underline{p}}^{p} (1-\alpha) p h_B(\rho) d\rho + (1-\alpha) p (1 - H_B(p)) \right\} + (1-\lambda) p \left[1 - H_B(p)\right]$$

It must be that firm A is indifferent between any p in the interval $\left[\underline{p}, \zeta\right)$. We hence have that $\frac{\partial \Pi_A^e(p)}{\partial p} = 0$ for all $p \in \left[\underline{p}, \zeta\right)$:

$$(1-\lambda) p h_B(p) - (1 - H_B(p))(1 - \lambda + (1-\alpha)\lambda) = 0$$

The explicit solution of this differential equation is $H_B(p) = cp^{-\frac{1-\lambda\alpha}{1-\lambda}} + 1$. But then it must be that $H_B(\zeta) = 1 = c\zeta^{-\frac{1-\lambda\alpha}{1-\lambda}} + 1$ and hence that $c = 0$, a contradiction.

Suppose now that $\nabla_B < 1$ and hence that $\nabla_A = 1$. Now consider the expected payoff to firm B from setting a uniform price equal to $p \in \left[\underline{p}, \zeta\right)$ is equal to:

$$\Pi_B^e(p) = \alpha\lambda \left\{ \int_{\underline{p}}^{p} (1-\alpha) p h_A(\rho) d\rho + (1-\alpha) p (1 - H_A(p)) \right\} + (1-\lambda) p \left[1 - H_A(p)\right]$$

It must be that firm B is indifferent between any p in the interval $\left[\underline{p}, \zeta\right)$. We hence have that $\frac{\partial \Pi_B^e(p)}{\partial p} = 0$ for all $p \in \left[\underline{p}, \zeta\right)$:

$$(1-\lambda) p h_A(p) - (1 - H_A(p))(1 - \lambda + (1-\alpha)\alpha\lambda) = 0$$

The explicit solution of this differential equation is $H_A(p) = cp^{-\frac{1-\lambda+(1-\alpha)\alpha\lambda}{1-\lambda}} + 1$. But then it must be that $H_A(\zeta) = 1 = c\zeta^{-\frac{1-\lambda+(1-\alpha)\alpha\lambda}{1-\lambda}} + 1$ and hence that $c = 0$, again a contradiction.