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Working Papers / Documents de travail

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WP 2025 - Nr 03

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February 3, 2025

Abstract

Based on the calibration of a simple model, we decompose the decline in the labor share into four structural components: task displacement, labor rents, capital rents, and labor-capital substitution effect. Our estimation suggests that task displacement and the switch of distributed rents from labor to capital are the main drivers of the labor share decline over the past three decades. On the other hand, the neoclassical substitution effect seems not to have a long term impact on the labor share.

Keywords: labor share, task displacement, automation, labor rents, capital rents, markup, rate of return on capital, productivity.

JEL Classification: E2, E4, J3, N12, O4

1 Introduction

Since the early 1980s, the labor share in the US business sector has significantly declined. Research identifies four structural factors that could explain this trend (Bergholt et al., 2022). The first two factors involve the decomposition of value added into labor share, capital share, and profit share. Rents can be allocated to either capital owners or workers, so an increase in capital rents (Autor et al., 2020; Barkai, 2020; Kehrig and Vincent, 2021; Philippon and Gutierrez, 2023) or a decrease in labor rents (Stansbury and Summers, 2020) can reduce the labor share. The third factor is task displacement due to automation (Acemoglu and Restrepo, 2019) or offshoring (Dao et al., 2019). This shift of tasks from labor to capital raises capital income at the expense of wages, thereby lowering the labor share (Acemoglu and Restrepo, 2018, 2019, 2020). The fourth factor is the neoclassical labor-capital substitution effect. When the elasticity of substitution between labor

*We would like to thanks Raphael Giraud, Antoine Terracol, and Thibault Darcillon for their help, comments and suggestions. The project leading to this publication has received funding from the French government under the "France 2030" investment plan managed by the French National Research Agency (reference :ANR-17-EURE-0020) and from Excellence Initiative of Aix-Marseille University - A*MIDEX

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and capital is high (greater than one), the capital share increases as the relative price of capital goods falls (Karabarbounis and Neiman, 2014). Conversely, if the elasticity is low (less than one), an increase in the capital risk premium boosts the capital share (Caballero et al., 2017). Overall, changes in the relative productivity and costs of labor and capital, depending on the elasticity of substitution, can influence the labor share.[1]

While all these explanations may operate simultaneously, they also compete with one another, making it challenging to determine the contribution of each mechanism for two reasons. First, individual effects can obscure or disregard other causes. Second, the established relationships often rely on indirect measurements, making it difficult to estimate their contributions at a macro level. Therefore, this paper aims to account for all these mechanisms concurrently. To achieve this, we gather a comprehensive set of macroeconomic data and evidence from the literature and conduct an accounting exercise using simple model calibration.

Our methodology follows a structured approach. First, we adopt the methods of Barkai (2020), Philippon and Gutierrez (2023), and Reis (2022a) to derive a measure of the rate of return on capital (RRK) based on investment decisions or consumer choices. With the RRK determined, we employ a profit maximization framework using a CES production function to break down the effects into structural components: capital-labor substitution, markup changes, and task displacement. Task automation and capital-labor substitution are measured within this model, similar to the macro approach by Acemoglu and Restrepo (2019), while accounting for markups in the calculations. Finally, we further decompose profits into labor and capital rents using Stansbury and Summers' (2020) measure of labor rents, allowing us to capture the potential shift of rents from labor to capital.

Our method can be compared with Bergholt et al. (2022), who also propose a decomposition of the labor share using a VAR model. However, our estimation is more direct, as it does not rely on simulating shocks and performing variance decomposition. Instead, our primary contribution is to provide an accountability measure of the phenomenon. This approach is similar to the accounting exercise by Fahri and Gourio (2018) using DSGE model calibration. However, while their model allows to measure the impact of macroeconomic shock expectations on risk premia and replicates recent trends in key ratios (e.g., an increase in Tobin's Q, decline in investment rate), it may have some costs. Specifically, their approach uses a Cobb-Douglas production function, does not consider labor rents, and assumes the economy is at the steady state. Additionally, the choice of targeted moments can influence the results.[2] For that reason our approach should be viewed as complementary to theirs.

Our decomposition reveals that the explanation for the decline in the labor share is largely

[1]See Grossman and Oberfield (2022) or Bazot and Guerreiro (2024), for a review of the literature explaining the decline in the labor share.

[2]For instance, the use of TFP growth implies some assumption about parameters value (*e.g.* the elasticity of production to labor and capital) which may enter in contradiction with the estimation of these parameters in this very exercise.

independent of the assumptions used to measure the RRK and markups. Specifically, the reduction in tasks performed by labor consistently negatively impacts the labor share, regardless of the elasticity of substitution between labor and capital. Additionally, the increase in markups has either no effect or a negative effect, indicating that the rise in rents allocated to capital invariably reduces the labor share. Meanwhile, the substitution effect does not impact the labor share, even with an increase in the capital risk premium. Our results align with Bergholt et al. (2022), although the effect of labor rents appears more pronounced in our accountability exercise, as it directly correlates with the rise in capital rents.

In addition to addressing the value-added distribution puzzle, our results contribute to various fields of economic literature. First, our new measure of task displacement supports Acemoglu and Restrepo's (2018, 2019, and 2020) analyses on the macro-level impact of automation on the labor market. However, the effect we observed is smaller than theirs, due to the significant role of rents in estimating task displacement. Second, our measure of markups reveals two key insights. On one hand, markups remain relatively stable over time when labor rents are considered as distributed profit. On the other hand, despite some heterogeneity, all our estimations show an increasing trend in the ratio of capital rents to value added. This suggests that while the rise in capital rents correlates with the decline in labor share, it does not necessarily imply reduced overall competition. This distinction depends on whether labor rents are viewed as profit distribution or part of a fixed excess wages in our accounting exercise. Third, the evidence of a concurrent rise in capital rents and decline in labor rents helps explain the increase in Tobin's Q, due to shareholder profits increase (Greenwald, Lettau, and Ludvigson, 2019). Fourth, our analysis allows us to measure total factor productivity (TFP) without assuming specific values for the elasticity of production to labor and capital, unlike most TFP calculations (e.g., Bergeaud et al., 2016). Our results indicate a decline in TFP during the 2000s, followed by a rebound after 2010. Long-term data also suggest a negative trend in TFP since the 1960s, aligning with Philippon's (2023) additive growth hypothesis.

The paper proceeds as follows. Section 2 presents the model used to decompose the change in the labor share, with a particular focus on measuring the rate of return on capital, which is crucial for this exercise. Section 3 provides the main series used for the decomposition. Section 4 displays the results of the decomposition exercise. Section 5 includes a discussion, especially concerning the implications of the assumptions used in the calculations. Section 6 concludes.

2 The model

In this section, we propose a simple model to be used in our measurement exercise. The aim is to produce a set of fully identified equations that can be used to decompose the labor share into its structural components.

2.1 The Labor share decomposition

The labor share is usually defined as the ratio of wage bills to total income. However, this calculation tends to capture the share of profits distributed to workers. Stansbury and Summers (2020) propose a measure of rents distributed to workers based on micro data. These rents can be split into union rents—wage premiums from unionization—large firms' rents—where excess wages rise with firm size—and industry rents—industry wage differentials reflecting rent-sharing with workers. When these rents are known, the true labor share is obtained from:

$$\bar{s}_L = \frac{WL}{P_Y Y} = s_L - \pi_L \tag{1}$$

where π_L is the amount of rents distributed to labor relative to value added, and Y is the total income of the business sector after production taxes. Once we know $\bar{s_L}$, we can use labor productivity (Y/L) data to measure the (perfect competition) unit wage:

$$W = \bar{s}_L \left(\frac{Y}{L}\right) \tag{2}$$

The capital share is the ratio of capital costs to total income. Capital costs are equal to the required rate of return on capital R times the quantity of fixed capital used in production:

$$\bar{s}_K = \frac{RP_K K}{P_Y Y} \tag{3}$$

The profit share is what remains after accounting for factor shares:

$$\bar{\pi} = 1 - \bar{s}_L - \bar{s}_K \tag{4}$$

The markup is then obtained from:

$$\mu = \frac{1}{1 - \bar{\pi}} \tag{5}$$

With this in mind, we now use a constant elasticity of substitution (CES) production function to eventually decompose the labor share into a set of structural components:[3]

$$Y = \left[(1-\alpha)^{\frac{1}{\sigma}}(A_L L)^{\frac{\sigma-1}{\sigma}} + \alpha^{\frac{1}{\sigma}}(A_K K)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$$

where α is the share of tasks done with capital in the set of tasks required to produce the final good Y (Cf. Acemoglu and Autor, 2011 or Acemoglu and Restrepo, 2018), $\sigma \in [0, \infty)$ is the capital-

[3]It is worth noting that this model is a special case of the Bentolila and Saint-Paul model (2003). However, the aim of this model, based on the CES production function, is to easily account for the set of explanations used in the literature.

labor elasticity of substitution, and A_L and A_K are the technical progress associated respectively with labor and capital.

Profit maximization leads to the usual equality between wages and the Return Rate of Capital (RRK hereafter) on one side and labor and capital marginal productivity on the other. We can then use the related results to establish the link between the labor share on one side and task content, markups, and capital-labor costs on the other (see appendix for calculation):

$$\bar{s_L} = \frac{1}{\mu} \frac{1}{1 + \left(\frac{\alpha}{1-\alpha}\right) \left(\frac{\frac{W}{A_L}}{\frac{R}{A_K}}\right)^{\sigma-1}}$$

It is worth noting that, following Stansbury and Summers (2020), labor rents are included in the markup. In other words, excess wages are purely due to profit distribution to workers. We then define $\lambda = \frac{W}{R}\frac{A_K}{A_L}$ and, as in Acemoglu and Restrepo (2019) apply a Taylor expansion to the former equation around $\ln S_L(\alpha_0; \lambda_0)$, with the index "0" indicating the reference year from which we want to start the estimation (see the appendix for calculation details):

$$\text{Change in task content} = \underbrace{\ln(s_{L,t}) - \ln(s_{L,t_0})}_{\text{change in labor share}} - \underbrace{\left[\ln\left(\frac{1}{\mu_t}\right) - \ln\left(\frac{1}{\mu_{t_0}}\right)\right]}_{\text{change in markup}} - \underbrace{(1-\sigma)(1-\mu_{t_0} s_{L,t_0})\left[\ln\left(\frac{\frac{W_t}{R_t}}{\frac{W_{t_0}}{R_{t_0}}}\right) - g_A\right]}_{\text{substitution effect}} \quad (6)$$

Using (6), we can now decompose the percentage change in the true labor share into its three theoretical components: the percentage change in markup, the percentage change in labor-task share, and the percentage change in labor-capital substitution. For this, we only need to use equations (1) to (5), to calculate the growth rate of the relative technical progress between labor and capital, g_A, and assign a value to σ. The only missing information now is the rate of return on capital.

2.2 The rate of return on capital

Two methods can be used to produce the RRK. The first one focuses on Jorgenson's (1963) methodology based on the link between investment behaviors and risk-adjusted rates. The second one looks at the Euler equation, making the link between intertemporal consumption choice and the RRK, as in Fahri and Gourio (2020) and Reis (2022a).

2.2.1 The RRK based on investment behaviors

According to Jorgenson (1963), the RRK is based on the link between the locative cost of capital and the risk-adjusted discount rate. Thus, the RRK is the sum of the real (risk-adjusted) interest rate (r), the depreciation rate (δ), and the expected inflation rate of capital assets $\mathrm{E}[\mathrm{d}P_K/P_K]$ (Caballero et al., 2017; Barkai, 2021; Philippon and Gutierrez, 2022):

$$R = r + \delta + (1-\delta) \times \mathrm{E}[\mathrm{d}P_K/P_K]$$

The risk-adjusted rate of return must account for the capital risk premium. Two factors must be considered. First, there is an equity premium that increases the cost of capital. This additional cost is proportional to the prime and the share of equity in total financial assets. Second, credit to enterprises is not without risk, which raises the cost of lending and interest rates. To capture both effects, we must distinguish between the cost of lending (r_L) and the cost of equity (r_E) and apply a weight coefficient according to the relative importance of both types of assets. This is why Barkai (2020) and Gutierrez and Philippon (2023) use the following calculation:

$$R = \chi . r_L + (1-\chi)(r_f + ERP) + \delta + (1-\delta) \times \mathrm{E}[\mathrm{d}P_K/P_K] \tag{7}$$

with $\chi = L/(L+E)$ the share of credit in total business-related financial assets and r_L the risk-adjusted lending rate. Here, $r_E = r_f + ERP$, with r_f the risk-free rate and ERP the equity risk premium.

2.2.2 The RRK based on consumption choice

The aim of this approach is to produce a series on the RRK based on consumer intertemporal choices. For the sake of tractability, we keep the related calculation as simple and general as possible. In this respect, we first assume that preferences are non-cumulative but discuss this hypothesis in section 5 below. So, let's consider a representative consumer, whose intertemporal utility function is defined by:

$$U = \int_0^{+\infty} u[c(t)] \exp\{-(\rho - n)t\} \, \mathrm{d}t$$

with $c(t)$ the level of consumption of non-durable goods at time t, ρ the rate of time preference, and n the growth rate of population.

The representative household intertemporal budget constraint is:

$$\dot{a}(t) = W(t) + \pi(t) + r(t)a(t) - c(t) - na(t)$$

with r the rate of return on assets a, and $na(t)$ accounting for the impact of the population growth rate on the stock of assets.

$r(t)a(t)$ is the amount transferred to investors for holding assets. Since the representative household holds all the assets, r is the return on a $1 worth portfolio composed of a weighted set of assets available in the economy. It is assumed that the representative agent is price-taker, so she has no impact on the rate of return.

It is also important to distinguish between income from assets and profit. Income from assets is paid proportionally to the amount of assets invested by the household, while profit is what remains once labor and capital costs are covered. In other words, although firms' profits can be distributed to shareholders based on their shares, the total amount of distributed profits does not depend on the volume of assets. This distinction is reflected in the budget constraint equation above. For this reason, the RRK does not include rents, assuming markups are independent of total assets.

Intertemporal maximization provides the Euler equation:

$$r = \rho + \gamma g_c$$

with $\gamma \equiv -\frac{u_{cc}[c(t),l(t)]c(t)}{u_c[c(t),l(t)]}$ the relative risk aversion (or the elasticity of intertemporal substitution) and $g_c = \frac{\dot{c}(t)}{c(t)}$ the growth rate of consumption (g_c).

As in Reis (2022a), we would certainly like to account for consumers' heterogeneity, especially because a fraction θ of hand-to-mouth households consume their entire labor income (Kaplan et al., 2014). Therefore, since these households are not sensitive to the RRK, this can lead to a biased estimation of r. Because consumption is composed of two types of households, the growth rate of consumption is now given by the following formula:

$$g_c - \theta g_{y,l} = (1-\theta)(r-\rho)\gamma^{-1}$$

with $g_{y,l}$ the average growth rate of labor income. Thus, the RRK can be easily derived as:

$$r = \rho + \frac{\gamma(g_c - \theta g_{y,l})}{1-\theta} + \delta$$

Finally, as in Reis (2022a), we distinguish between two types of households among non-hand-to-mouth households. A fraction of households invests in risky projects, the rate of return on which is $m > r$, while the other fraction invests in government bonds for a return $r_f < r$. In this respect, the rate of return obtained above corresponds to the weighted average return on both types of investments, so that: $r = \frac{am+dr_f}{a+d}$, with a the amount of risky assets invested and d the amount of safe assets.[4] So, the RRK on risky assets can be computed from:

$$R = m + \delta = r_f + \left(1 + \frac{d}{a}\right)(r - r_f) + \delta \tag{8}$$

Therefore, two measures can be used depending on whether we look at the average required rate of return or the rate of return on risky assets. Since we are interested in the business sector, which is more risky, our analysis will focus on equation (8).

3 Data and main series

Our model consists of 7 variables ($\bar{s_L}, W, \bar{s_K}, \bar{\pi}, \mu, \mathrm{d}\ln\alpha$, R). To measure them, we rely on equations (1)-(6) plus (7) or (8) depending on the chosen measure of the RRK. For this, we need data for several variables and parameters. We use Stansbury and Summers (2020) to measure the share of labor rents to value added (π_l); labor productivity ($P_Y Y/L$) and capital output ratio ($P_K K/P_Y Y$) are from the BEA; σ is set to 0.8 based on Knoblach et al. (2019) meta-analysis results.[5] Lastly, as suggested by the theory (Acemoglu, 2003), g_A (the growth rate of A_L/A_K) is assumed to be equal to the growth rate of labor productivity, as in Acemoglu and Restrepo (2019).[6]

Regarding the value of R, we follow three approaches. First, we propose a direct calculation of (7) in the manner of Barkai (2020). We use the government bonds rate to measure r_f, the ERP is from Duarte and Rosa (2018), r_L is based on Moody's Baa corporate bonds yields, and χ is calculated on market capitalization (E) and total credit (L) data. Second, we directly use Gutierrez and Philippon (2022) estimation, which is also based on equation (7). The advantage of using this measure is that it utilizes industry-level data and estimates industry-level cost of equity using analyst reports, then aggregates firm-level data (COMPUSTAT) to derive an overall estimation of excess profit (π_k) from 1989 to 2015.

Third, we calibrate equation (8). We use Kaplan et al. (2014) results on the share of hand-to-mouth households in the US to set $\theta = 0.3$. The rate of time preference is assumed to be 0.02, consistent with typical calibration exercises. The rate of risk aversion, γ, is based on meta-analyses estimating the elasticity of intertemporal substitution (EIS) (Havranek et al., 2015), set at $\gamma = 2$.[7] However, since this parameter is assumed to be constant, it primarily influences the level of the RRK rather than its trend. Therefore, for our focus on changes in the variables of interest, this choice is not critical. Lastly, because consumption may vary stochastically, we employ

[4] The interested reader can refer to Reis (2022a) for a theoretical specification of this result.

[5] This hypothesis is corroborated by the most recent estimations of σ (Cf. Raval, 2019; Oberfeld and Raval, 2021)

[6] Acemoglu and Restrepo (2019) discusses this hypothesis and show that the sensitivity of the estimation is not affected by it.

[7] It is important to note that measures of IES and RRA can vary across studies and depend significantly on how the real rate of return on capital is measured.

smoothing techniques to remove this "noise" from the calculations. Figure A1 in the appendix displays the series for the RRK from 1988 onward. All three calculations exhibit a very slight negative trend, with each estimation fluctuating within a narrow band of 3 percentage points. Such a small fluctuation contrasts with the substantial decrease in interest rates observed over the period, suggesting that an increase in the capital risk premium (KRP) pushes the "true" capital share up (see Figure A2 in the appendix). However, given the simultaneous capital deepening, it remains challenging at this stage to disentangle the effect of a rising KRP from changes in task composition. This ambiguity underscores the rationale behind our decomposition exercise.

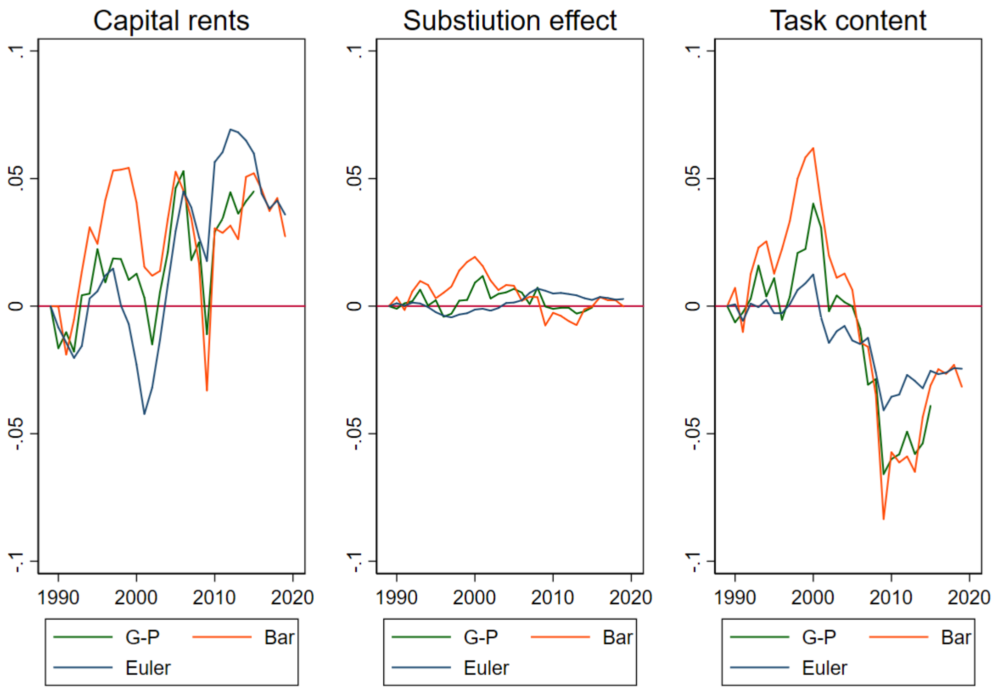
Figure 1 illustrates the evolution of key series influencing the change in the labor share from 1988 to 2019, namely: the share of capital rents, the change in task content, and the labor-capital substitution effect. Three main observations emerge. First, capital rents show a noticeable increase over the period, regardless of the RRK used for estimation, rising by approximately 4 percentage points in all scenarios. It is noteworthy that there is a strong correlation between each estimation; for example, the correlation between the capital rent estimation by Gutierrez and Philippon and the one derived from the Euler equation is 0.8. Second, despite significant capital deepening and rising risk premium, the substitution effect remains minimal. In fact, the growth rate of factors relative productivity offsets the growth rate of factors relative cost. Third, the share of labor-tasks decreases by approximately 2.5% to 3.5% over the period. However, this decline is less pronounced compared to the estimate proposed by Acemoglu and Restrepo (2019). The discrepancy arises because their estimation did not account for the shift of rents from labor to capital, which mitigates the decrease.

4 Labor share decomposition

Two aspects of the labor share decline need careful consideration. First, there has been a decrease in rents distributed to labor since 1982 (Stansbury and Summers, 2022). One possibility is that these rents have been redirected towards capital rents, thereby stabilizing the share of profits to value added. Another hypothesis is that total profits have declined, but this would contradict recent evidence on markup change.[8] Second, even after accounting for the removal of rents to labor, the "true" labor share still exhibits a negative trend. This ongoing decline can hence be attributed to the structural variables described in equation (6).

To distinguish between these aspects, particular attention must be paid to the role of markups. An increase in markups would indicate that the rise in capital rents fully offsets the decrease in labor rents while further reducing the "true" labor share. If markups stabilize, it would suggest an unchanged profit share with a shift in rent distribution from labor to capital. Declining markups

[8]See Basu (2019), for a literature review on the issue. Note that evidences based on inderect measure of markups from firms or establishments level data suggest either a rise (DeLoeeker et al., 2020) or a stagnation (Foster et al., 2022) in markup but never a decline.



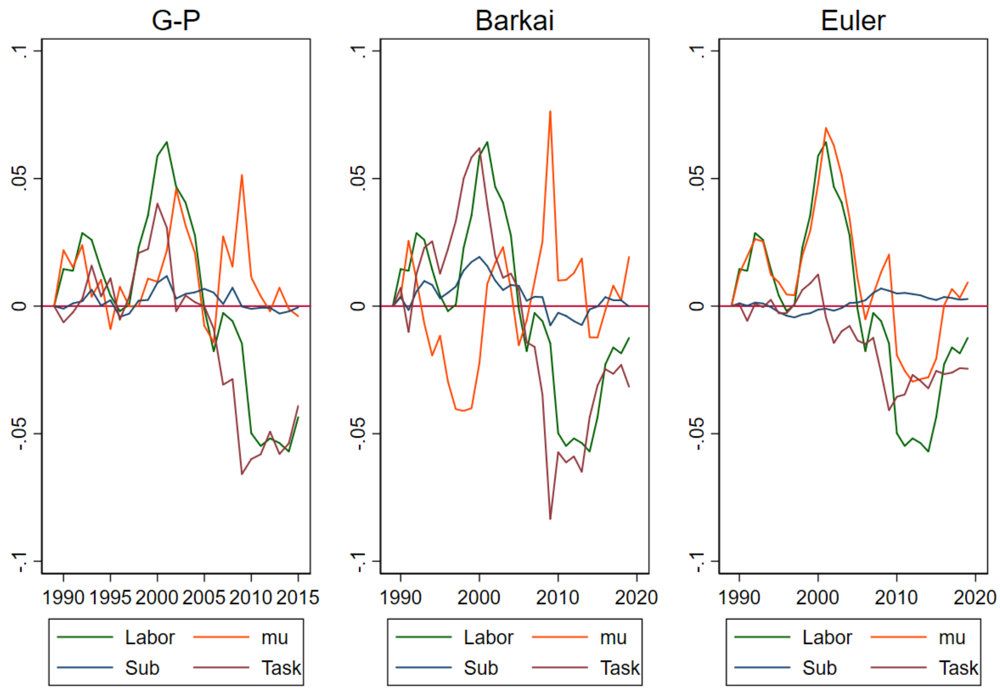
Note: "Capital rents" is the share of rents distributed to capital as a share of value added. The substitution effect and the change in task content are based on equation (6)—g_A (the growth rate of A_L/A_K) is assumed to be equal to the growth rate of labor productivity, the relative risk aversion is fixed to 2, and the elasticity of substitution between labor and capital is fixed to 0.8. "G-P" corresponds to Guitierrez and Philippon (2023) calculation of capital cost and rents; "Barkai" corresponds to estimation of the RRK in the spirit of Barkai (2020) while using Duarte and Rosa (2018) on the equity risk premium; "Euler" is the estimation based on RRK from optimal consumption choice.

Figure 1: **Labor share components**

would imply that the increase in capital rents shown in Figure 1 does not fully compensate for the decline in labor rents, thereby pushing the "true" labor share up through a decrease in the profit share.

Figure 2 illustrates the impact of structural variables on the variation of the "true" labor share. This decomposition reveals that, across all scenarios, the change in task content is the sole factor driving $\bar{s_L}$ downwards. However, while in Acemoglu and Restrepo (2019) it accounted for the entire decline in the overall labor share, here it explains only the decrease in the "true" labor share, representing approximately 40% of the reduction in the total labor share.

On the other hand, both markup and substitution effects show no discernible trend, indicating they have no influence on the "true" labor share. Regarding markups, this implies that the increase in rents distributed to capital exactly offsets the decline in rents distributed to labor. In essence, the labor share does not decrease due to rising rents but rather due to a shift in the distribution of these rents from labor to capital. This shift accounts for about 60% of the total decline in the labor share. Thus, competition does not appear to play a role in this estimation, and markup



Note: The labor share change is decomposed based on the "true" labor change from eq. (6), so, labor rents are removed from labor income. g_A (the growth rate of A_L/A_K) is assumed to be equal to the growth rate of labor productivity. The relative risk aversion is fixed to 2 while the elasticity of substitution between labor and capital is fixed to 0.8. "G-P" corresponds to Guitierrez and Philippon (2023) calculation of capital cost and rents; "Barkai" corresponds to estimation of the RRK in the spirit of Barkai (2020) while using Duarte and Rosa (2018) on the equity risk premium; "Euler" is the estimation based on RRK from optimal consumption choice.

Figure 2: **Labor share decomposition**

stabilization suggests no change in this regard.

Using consumption choices to measure the RRK offers several advantages in the present context. First, it avoids the volatility associated with interest rates and approximations of the capital risk premium (Karabarbounis and Neiman, 2019). This approach assumes that the RRK tends to be relatively stable in the medium term, yielding more robust results. Second, this method excludes rents from the RRK calculation, thereby separating capital costs from markups in the estimation process. Third, the availability of consumption data allows for the construction of time series over a longer span of time.

Given these advantages and the similarity in patterns observed in our previous estimations, we extended our calculations from 1960 onward based on the RRK derived from equation (8). However, this extension requires values for labor rents, which are only available from Stansbury and Summers (2022) starting in 1982. To address this limitation, we extrapolate the series using the coefficients obtained from an OLS regression that explains these rents based on the labor share.[9] Additionally, to explore the relationship between our findings and the estimation of labor rents, we also conducted our calculations by setting labor rents to zero, as commonly done in macroeconomic estimations (e.g., Fahri and Gourio, 2018; Barkai, 2020; Gutierrez and Philippon, 2022).

Note: The labor share change is decomposed based on the labor change whether labor rents are removed or set to zero. g_A (the growth rate of A_L/A_K) is assumed to be equal to the growth rate of labor productivity. The relative risk aversion is fixed to 2 while the elasticity of substitution between labor and capital is fixed to 0.8. "G-P" corresponds to Gutierrez and Philippon (2023) calculation of capital cost and rents; "Barkai" corresponds to estimation of the RRK in the spirit of Barkai (2020) while using Duarte and Rosa (2018) on the equity risk premium; "Euler" is the estimation based on RRK from optimal consumption choice.

Figure 3: **Decomposition from 1960**

Figure 3 confirms that the recent period is distinctive, regardless of whether labor rents are considered. In both scenarios, capital rents begin to increase after 2000 but not before. Prior to 1982, there is no observable trend in rents to labor or capital, indicating stability in markup from 1960 to 1980. As expected, setting labor rents to zero causes the markup to rise after 2000, thereby reducing the labor share (which is no longer offset by labor rents). However, this effect still accounts for 60% of the decline in the labor share after 2000, consistent with our previous estimations.

Furthermore, the estimation of the change in task content closely aligns in both calibration exercises, showing a continuous decline from 2000 onward. Thus, the change in task content

[9] The R-squared of this regression is 0.58.

appears responsible for a decline in the labor share of approximately 3% after 2000. Consequently, task displacement continues to explain 40% of the decline in the labor share after 2000. It is also noteworthy that the period before 2000 exhibits no long-term variation in the share of tasks performed by labor, suggesting that the developments of the past two decades are unprecedented.

5 Discussion

We propose three extensions to our results. First, we test the sensitivity of our conclusions using alternative estimations. Second, we question our calculation of the RRK based on eq. (8). Third, we discuss the consequence of our result on the estimation of total factor productivity.

5.1 Additional estimations

In this section, we explore two alternative approaches to our previous calculations. First, we examine the impact of the elasticity of substitution between labor and capital. We compare two different values, $\sigma = 0.8$ and $\sigma = 1.2$, to assess how this assumption influences our results.

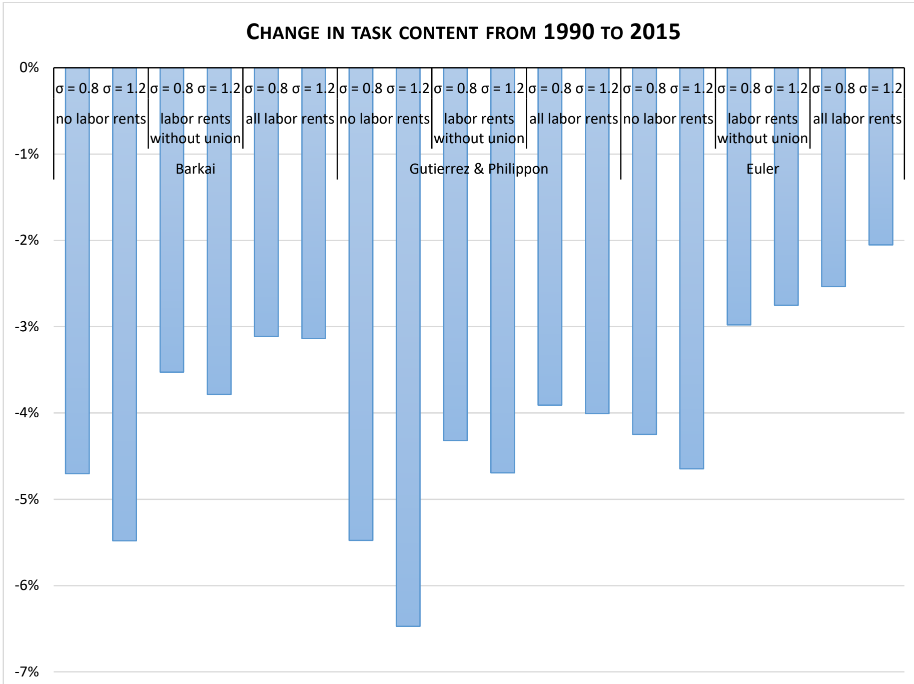
Second, to test the sensitivity of our estimations, we consider different assumptions regarding labor rents. Specifically, we compare three cases: (i) assuming no labor rents, (ii) assuming labor rents due to union power are not the result of profit distribution to labor, and (iii) assuming all labor rents come from profit. Point (ii) is informed by the notion that unions primarily negotiate wages rather than profit distribution (Blanchard and Giavazzi, 2003). Thus, markups could be overstated if "excess wages" related to union power are incorrectly included in total profits. Note that because Gutierrez and Philippon (2023) data stop in 2015, for the sake of comparability our estimations do not go beyond this point.

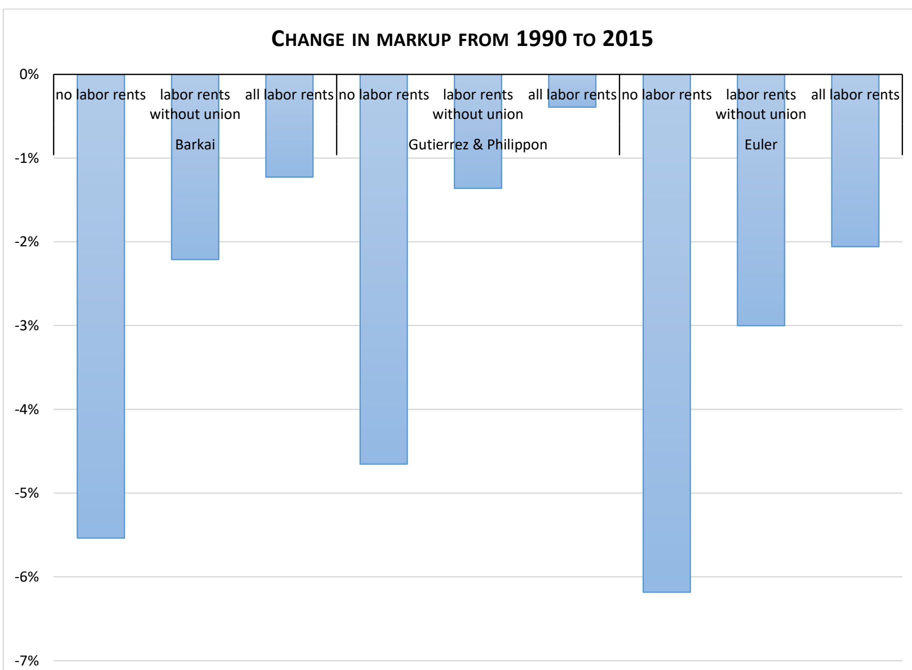
The results are summarized in Figure 4 (since the effect of the substitution effect is very low, we do not report the result for this variable here). Two main observations emerge. First, the estimation of the change in task content is minimally affected by the elasticity of substitution. In other words, assuming complementarity or substitutability between factors has little effect on the results presented thus far. Second, the share of profits distributed to labor significantly influences both the change in task content and markups. Interestingly, the effect of the change in task content increases as the share of labor rents included in total profits decreases. Therefore, the estimation of the effect of task displacement presented so far appears to be conservative.[10]

5.2 Alternative measure of the RRK

The estimation done from (8) was based on a non-recursive utility function. However, if the use of such function ease our estimation of the RRK it also comes at the cost of an oversimplification

[10]Since the estimation of the markup does not depend on the elasticity of substitution, Figure 4 does not differentiate between cases where σ is set to 0.8 or 1.2.





Note: percentage change in markup and labor-based task contents from 1990 to 2015 according to elasticity of substitution and the type of labor rents included in total profits. "G-P" corresponds to Guitierrez and Philippon (2023) calculation of capital cost and rents; "Barkai" corresponds to estimation of the RRK in the spirit of Barkai (2020) while using Duarte and Rosa (2018) on the equity risk premium; "Euler" is the estimation based on RRK from optimal consumption choice.

Figure 4: **Effect on the labor, alternative measures**

as it prevents from distinguishing between the elasticity of intertemporal substitution and the relative risk aversion. In addition, our calculation did not account for the effect of expectations, which may impact the results if g_c and r are correlated. An easy way to address those issues is to extend our calculation based on Epstein and Zin utility function:

$$U_t = \left[(1-\beta)c_t^{1-1/\psi} + \beta\left(\mathbb{E}_t U_{t+1}^{1-\gamma}\right)^{\frac{1-1/\psi}{1-\gamma}}\right]^{\frac{\psi}{\psi-1}}$$

The related Euler equation from this function is:

$$\mathbb{E}_t\left[\left(\beta\left(\frac{c_{t+1}}{c_t}\right)^{-1/\psi}\right)^{\frac{1-\gamma}{1-1/\psi}}(1+r_{t+1})^{\frac{1-\gamma}{1-1/\psi}}\right] = 1$$

We immediately see that $1/\psi = \gamma \Rightarrow \frac{1-\gamma}{1-1/\psi} = 1$ which is the usual CRRA function. In order to isolate R we use a second order Taylor approximation around $g_c = r = 0$. After some algebra, assuming $\mathbb{E}_t[r]^2 = \mathbb{E}_t[g]^2 = 0$, this gives (see the appendix for calculation):

$$\mathbb{E}_t[r] = \frac{\rho + \eta E_t[g_c] - \frac{1}{2}(\eta(\eta+1)\mathbb{V}_t[g_c]) + \nu\eta \text{Cov}[r, g_c] - \frac{1}{2}\nu(\nu-1)\mathbb{V}[r]}{\nu(1+\eta\mathbb{E}_t[g_c])} \tag{9}$$

with $\nu = \frac{1-\gamma}{1-\frac{1}{\psi}}$ and $\eta = \frac{1}{\psi}\nu$. In order to measure $\text{Cov}[r, g_c]$ and $\mathbb{V}[r]$ we use data from Jordà et al. (2017) on the rate of return on equity. We can then use common value on of the RRA used in the literature (see Elminejad et al., 2023) $\gamma = 3$, along with the EIS value we used so far ($1/\psi = 2$). Our result shows that, the use of such utility function provides very close results compared to a CRRA utility function (see appendix). In other words, the use of a CRRA function barely affect (and perhaps overestimate) the value of the RRK in our estimation.

5.3 Productivity in the long run

A fundamental question underlying the simultaneous evolution of the labor share, task automation, and rising markups is their connection to productivity and economic growth.

Up to this point, we relied on a CES production function. However, for the purpose of measuring Total Factor Productivity (TFP) through Solow decomposition, a Cobb-Douglas production function proves more practical. Despite being a simplification, recent analyses suggest that the economy may approximate this form over very long-run perspectives (Leòn-Ledesma and Satchi, 2019).

In a Cobb-Douglas production function, the share of labor equals the elasticity of output with respect to labor in conditions of perfect competition. However, with the introduction of markups, this elasticity diverges. In many TFP calculations, this elasticity is typically assumed to be equal to the labor share or set arbitrarily. Given our ability to measure markups, we can now infer the true elasticity of production to inputs and use this value to calculate TFP accurately. So, if:

$$Y = F(K, L) = AK^{\alpha}L^{1-\alpha}$$

Given that prices equal marginal productivity we come up with the elasticity of production to capital and labor based on the first order condition:

$$\alpha = \mu R \frac{K}{Y}$$

Because it is not fixed, the use of α can steadily affect the calculation of TFP. Let's inquire this in details. We know that $y = Ak^{\alpha}$ with $y \equiv Y/L$ and $k \equiv K/L$, so TFP is obtained from:

$$A = \frac{y}{k^{\alpha}} \tag{10}$$

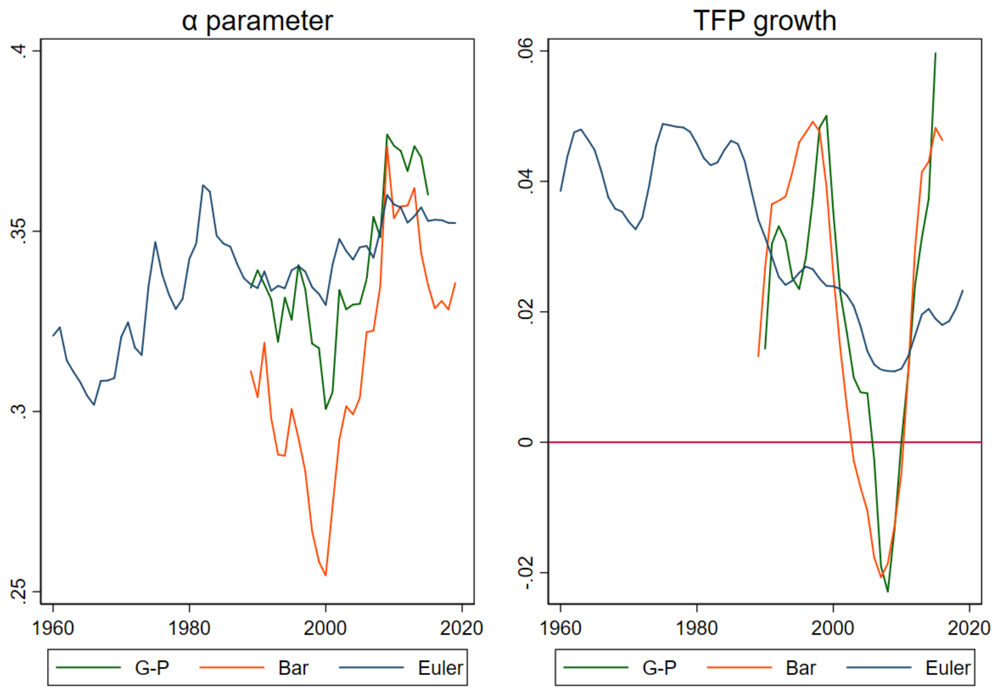
We then apply lowess smoothing techniques to compare the growth rates of each TFP series based on equation (10). Focusing on the post-1990 period, Figure 5 illustrates that all series decline by the late 1990s and rebound in the early 2010s. However, over a longer time horizon, based on the estimation of the RRK from eq. (8), we observe that TFP growth has been lower since the 1990s compared to the 1960-1990 period. This suggests a long-term decline in TFP growth, which may not return to the levels observed during the golden age. The most recent period shows lower productivity growth, aligning with the lower GDP growth observed over the past 20 years.

It is important to note that this gap is associated with the rise in capital rents and the changes in task content documented earlier. Thus, if one attributes the rise in capital rents to innovation or firm size (Autor et al., 2020; Kehrig and Vincent, 2021), it supports the notion of lower TFP growth (Baquee and Fahri, 2020). The same applies to the increasing use of digitalization, although this might be due to the lack of productivity gains from "so-so" automation (Acemoglu and Restrepo, 2018).

6 Conclusion

Knowledge about labor rents and the Rate of Return on Capital (RRK) opens up significant opportunities for measuring structural variables at the macro level, including aggregate markups, changes in task content, substitution effects, and total factor productivity. Our accountability exercise has yielded new insights that distinguish between various explanations and hypotheses regarding the shift in value-added distribution over the past two decades.

Our primary finding is that the shift of rents from labor to capital and the displacement of tasks are the main culprit for the decrease in the labor share over the past 20 years. Meanwhile, the



Note: Series are based on eq. (10) from Cobb-Douglas function calculation. A lowess smoothing technique has been applied to the series. "G-P" corresponds to Guitierrez and Philippon (2023) calculation of capital cost and rents; "Barkai" corresponds to estimation of the RRK in the spirit of Barkai (2020) while using Duarte and Rosa (2018) on the equity risk premium; "Euler" is the estimation based on RRK from optimal consumption choice.

Figure 5: **Total factor productivity estimation**

decline in TFP growth during the same period suggests that these phenomena may not necessarily contribute to improved welfare, at least in the medium term.

However, several issues warrant further investigation. Specifically, the analysis of markups should better account for how the related rents are distributed between workers and capital holders. Although our study captures the evolution of markups over time based on econometric estimates from Stansbury and Summers (2020), additional measures are still necessary. Besides, questions remain about the correct implementation of excess wages in the model—whether they are part of total profits, as assumed so far, or the result of a negotiation playing on firms' cost minimization—as this can affect the measure of markups. For that reason, the measure of markup change proposed in this paper is rather conservative.

Lastly, this study opens new avenues for research on topics where the RRK, value-added distribution, and productivity play crucial roles. These include analyzing inequality based on the $r > g$ hypothesis (Piketty, 2013; Jordà et al., 2019), assessing the sustainability of public debt given the direct impact of the RRK on debt revenue (Reis, 2022b), understanding automation and offshoring of tasks (Acemoglu and Restrepo, 2018, 2020; Dao et al., 2019), exploring inflation and monetary policy in light of changes in real interest rates affecting prices (Cochrane, 2022), and studying long-term economic growth where markups and innovation are intertwined (Aghion et al., 2019). Additionally, data on markups and the RRK can be used for macroeconomic model calibration and estimation, as demonstrated in Reis (2022b) regarding debt revenue. Thus, our inquiry contributes to further analysis on these complex and important topics.

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Appendix A: calculation

Labor share derivation

$$\min_{L,K} \quad \left\{\mathcal{C}(Y) = wL + rK\right\}$$
$$\text{s.t.} \quad Y = \left[(1-\alpha)^{\frac{1}{\sigma}}(A_L L)^{\frac{\sigma-1}{\sigma}} + \alpha^{\frac{1}{\sigma}}(A_K K)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$$

Where $\mathcal{C}(Y)$ is the aggregate total cost in the economy, and the price P is the numeraire, such that $PY = Y$. By defining the following Lagrangian function:

$$\mathcal{L}(L,K,\lambda) = (wL + rK) - \lambda\left[\left[(1-\alpha)^{\frac{1}{\sigma}}(A_L L)^{\frac{\sigma-1}{\sigma}} + \alpha^{\frac{1}{\sigma}}(A_K K)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} - Y\right]$$

where λ is the Lagrange multiplier, the first-order conditions lead to:

$$\frac{\partial \mathcal{L}(L,K,\lambda)}{\partial L} = 0 \Longleftrightarrow w = \lambda\left[(1-\alpha)^{\frac{1}{\sigma}} A_L^{\frac{\sigma-1}{\sigma}} L^{\frac{-1}{\sigma}}\left[(1-\alpha)^{\frac{1}{\sigma}}(A_L L)^{\frac{\sigma-1}{\sigma}} + \alpha^{\frac{1}{\sigma}}(A_K K)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{1}{\sigma-1}}\right]$$

$$\frac{\partial \mathcal{L}(L,K,\lambda)}{\partial K} = 0 \Longleftrightarrow r = \lambda\left[\alpha^{\frac{1}{\sigma}} A_K^{\frac{\sigma-1}{\sigma}} K^{\frac{-1}{\sigma}}\left[(1-\alpha)^{\frac{1}{\sigma}}(A_L L)^{\frac{\sigma-1}{\sigma}} + \alpha^{\frac{1}{\sigma}}(A_K K)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{1}{\sigma-1}}\right]$$

By the Envelope theorem, we have:

$$\left.\frac{\partial \mathcal{L}}{\partial Y}\right|_{\substack{L=L^*(Y)\\K=K^*(Y)\\\lambda=\lambda^*(Y)}} = \lambda^*(Y)$$

This equation shows that the Lagrange multiplier can be interpreted as the rate at which the optimal value of total costs ($\mathcal{C}(Y)$) changes with respect to changes in the quantities produced (Y). In other words, λ is the marginal cost. Recording that the markup is the price divided by the marginal cost:

$$\mu = \frac{P}{\mathcal{C}'(Y)}$$

setting $P = 1$, there is a direct link between the Lagrangean multiplier (λ) and the markup (μ):

$$\lambda = \frac{1}{\mu}$$

From the FOC we can now easlily derive the labor share:

$$\begin{aligned} s_L =& \frac{1}{\mu}(1-\alpha)^{\frac{1}{\sigma}} A_L^{\frac{\sigma-1}{\sigma}} \left(\frac{L}{Y}\right)^{\frac{\sigma-1}{\sigma}} \\ =& \frac{1}{\mu}(1-\alpha)^{\frac{1}{\sigma}} (A_L L)^{\frac{\sigma-1}{\sigma}} \left[(1-\alpha)^{\frac{1}{\sigma}} (A_L L)^{\frac{\sigma-1}{\sigma}} + \alpha^{\frac{1}{\sigma}} (A_K K)^{\frac{\sigma-1}{\sigma}}\right]^{-1} \\ =& \frac{1}{\mu} \frac{1}{1 + \left(\frac{\alpha}{1-\alpha}\right)^{\frac{1}{\sigma}} \left(\frac{A_K K}{A_L L}\right)^{\frac{\sigma-1}{\sigma}}} \end{aligned}$$

Then substituting by factor costs, we obtain:

$$s_L = \frac{1}{\mu} \frac{1}{1 + \left(\frac{\alpha}{1-\alpha}\right) \left(\frac{\frac{W}{A_L}}{\frac{R}{A_K}}\right)^{\sigma-1}}$$

Labor share decomposition calculation

Let's rearrange the labor share equation :

$$\bar{s} := \bar{s_L}\mu = \frac{1}{1 + \left(\frac{\alpha}{1-\alpha}\right) \left(\frac{\frac{W}{A_L}}{\frac{R}{A_K}}\right)^{\sigma-1}}$$

The markup-augmented labor share is thus a function of effective factor prices and the task content of production, $\bar{s_t} = \bar{s}(\lambda_t, \alpha_t)$, where $\lambda_t = \frac{W_t A_{K,t}}{A_{L,t} R_t}$ is the relative effective price of labor. So as to obtain the percent change in labor share, $\ln(\bar{s}_{L,t}) - \ln(\bar{s}_{L,t_0})$, we use a first-order Taylor expansion: $\ln[\bar{s}_L(\lambda_t, \alpha_t)]$ around $(\lambda_{t_0}, \alpha_{t_0})$. This gives :

$$\ln(\bar{s}_{L,t}) = \ln(\bar{s}_{L,t_0}) + (\lambda_t - \lambda_{t_0}) \frac{\partial \ln[\bar{s}_L(\lambda_{t_0}, \alpha_{t_0})]}{\partial \lambda_{t_0}} + (\alpha_t - \alpha_{t_0}) \frac{\partial \ln[\bar{s}_L(\lambda_{t_0}, \alpha_{t_0})]}{\partial \alpha_{t_0}}$$

$$\ln(s_{L,t}) - \ln(\mu_t) - \ln(s_{L,t_0}) + \ln(\mu_{t_0}) = (1-\sigma)(1-\bar{s}_{L,t_0}) \frac{\lambda_t - \lambda_{t_0}}{\lambda_{t_0}} + \left(\frac{1-\bar{s}_{L,t_0}}{1-\alpha_{t_0}}\right) \frac{\alpha_t - \alpha_{t_0}}{\alpha_{t_0}}$$

Considering small values for $\lambda_t - \lambda_{t_0}$ and $\alpha_t - \alpha_{t_0}$, it comes :

$$\ln(s_{L,t}) - \ln(s_{L,t_0}) = \ln(\mu_t) - \ln(\mu_{t_0}) + (1-\sigma)(1-\bar{s}_{L,t_0}) [\ln(\lambda_t) - \ln(\lambda_{t_0})] + \left(\frac{1-\bar{s}_{L,t_0}}{1-\alpha_{t_0}}\right) [\ln(\alpha_t -) \ln(\alpha_{t_0})]$$

$$\begin{aligned} \ln(s_{L,t}) - \ln(s_{L,t_0}) = \ln(\mu_t) - \ln(\mu_{t_0}) + (1-\sigma)(1-\mu_{t_0} s_{L,t_0}) \left[\ln\left(\frac{W_t}{W_{t_0}}\right) - \ln\left(\frac{R_t}{R_{t_0}}\right) - g_A\right] \\ + \left(\frac{1-\bar{s}_0}{1-\alpha_0}\right) [\ln(\alpha_t) - \ln(\alpha_{t_0})] \end{aligned}$$

Where $g_A = \ln\left(\frac{A_{L,t}/A_{L,t_0}}{A_{K,t}/A_{K,t_0}}\right)$, and $\left(\frac{1-\bar{s}_0}{1-\alpha_0}\right) [\ln(\alpha_t) - \ln(\alpha_{t_0})]$ is the change in task content. By rearranging, we get equation (6) :

$$\text{Change in task content} = \underbrace{\ln(s_{L,t}) - \ln(s_{L,t_0})}_{\text{change in labor share}} - \underbrace{\left[\ln\left(\frac{1}{\mu_t}\right) - \ln\left(\frac{1}{\mu_{t_0}}\right)\right]}_{\text{change in markup}} - \underbrace{(1-\sigma)(1-\mu_{t_0}s_{L,t_0})\left[\ln\left(\frac{\frac{W_t}{R_t}}{\frac{W_{t_0}}{R_{t_0}}}\right) - g_A\right]}_{\text{substitution effect}}$$

Taylor decomposition of the Epstein-Zin case

The Taylor decomposition for a two variables function (g, r) around (g_0, r_0) is given by:

$$f(g,r) = f(g_0,r_0) + f_r^{'}(g_0,r_0)(r-r_0) + f_g^{'}(g_0,r_0)(g-g_0) + \frac{1}{2}(f_r^{''}(g_0,r_0)(r-r_0)^2 + f_g^{''}(g_0,r_0)(g-g_0)^2 + 2f_{rg}^{''}(g_0,r_0)(r-r_0)(g-g_0))$$

Here the Euler euqation from the Epstein-Zin untility function:

$$\mathbb{E}_t\left[\left(\left(\frac{c_{t+1}}{c_t}\right)^{-1/\psi}\right)^{\frac{1-\gamma}{1-1/\psi}}(1+r_{t+1})^{\frac{1-\gamma}{1-1/\psi}}\right] = 1/\beta$$

Let's pose $c_{t+1}/c_t = 1 + g_{t+1}$, $\nu = \frac{1-\gamma}{1-1/\psi}$ and $\eta = \nu/\psi$, the term to be approximated is:

$$f(g_{t+1}, r_{t+1}) = (1+g_{t+1})^{-\eta}(1+r_{t+1})^{\nu}$$

Proceedings to the order 2 approximation around $r = g = 0$, gives:

$$f(g_{t+1}, r_{t+1}) = 1 + \eta g + \nu r - \frac{1}{2}\left(\eta(\eta+1)g^2 + \nu(\nu-1)r^2\right) + \nu\eta g r$$

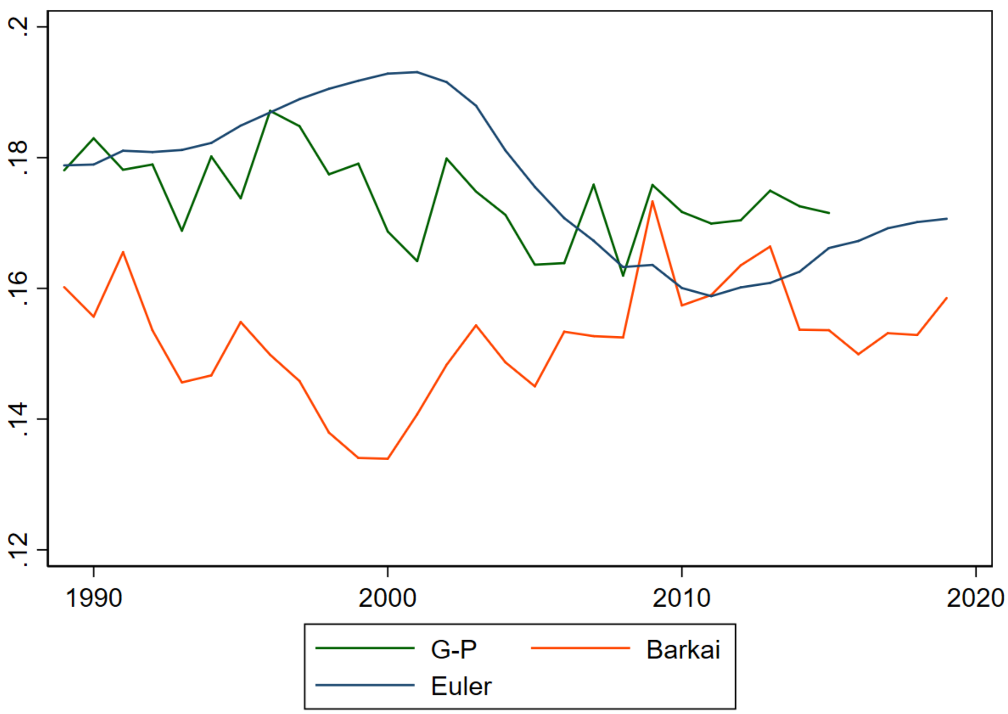
The Euler equation after approximation is given by:

$$\mathbb{E}_t\left[1 + \eta g + \nu r - \frac{1}{2}\left(\eta(\eta+1)g^2 + \nu(\nu-1)r^2\right) + \nu\eta g r\right] = 1/\beta$$

After some algebra, assuming $\mathbb{E}_t[r]^2 = \mathbb{E}_t[g]^2 = 0$, we obtain:

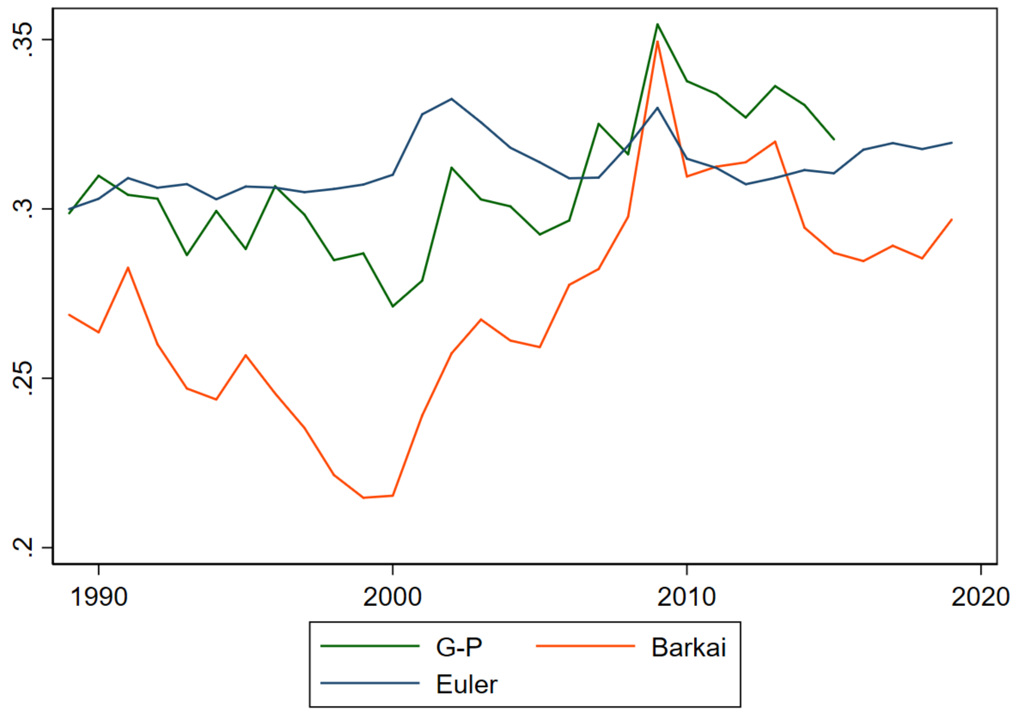
$$\mathbb{E}_t[r] = \frac{\rho + \eta\mathbb{E}_t[g_c] - \frac{1}{2}(\eta(\eta+1)\mathbb{V}_t[g_c]) + \nu\eta\text{Cov}[r, g_c] - \frac{1}{2}\nu(\nu-1)\mathbb{V}[r]}{\nu(1+\eta\mathbb{E}_t[g_c])}$$

Appendix B: additional figures



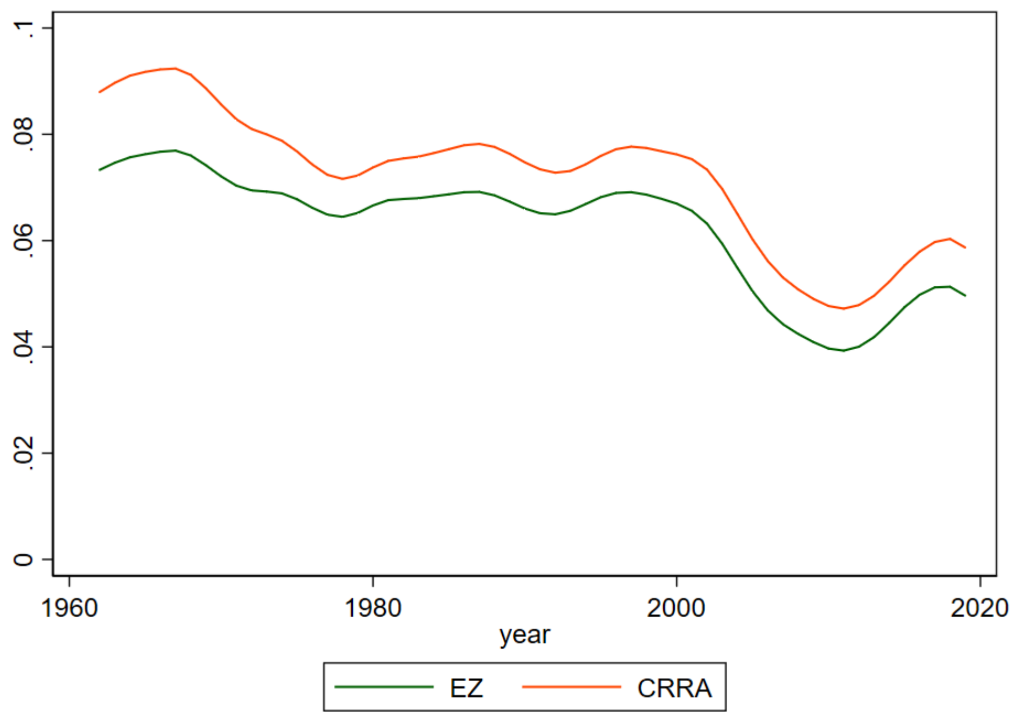
Note: the Rate of return to capital is based on eq. (7) and (8). "G-P" corresponds to Guitierrez and Philippon (2023) calculation of capital cost and rents; "Barkai" corresponds to estimation of the RRK in the spirit of Barkai (2020) while using Duarte and Rosa (2018) on the equity risk premium; "Euler" is the estimation based on RRK from optimal consumption choice.

Figure 6: **Figure A1: Rate of return on capital**



Note: the Rate of return to capital is based on eq. (8). "G-P" corresponds to Guitierrez and Philippon (2023) calculation of capital cost and rents; "Barkai" corresponds to estimation of the RRK in the spirit of Barkai (2020) while using Duarte and Rosa (2018) on the equity risk premium; "Euler" is the estimation based on RRK from optimal consumption choice.

Figure 7: **Figure A2: Capital share in value added**



Note: the Rate of return on capital is based on eq. (9). For the sake of comparison the IES is set to 0.5 and the RRA is set to 3.

Figure 8: **Figure A3: Epstein Zin and CRRA estimation of the RRK**