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Abstract

Political campaigns influence how voters prioritize issues, which in turn impacts electoral outcomes. In this paper, we study how candidates' communication shapes which issues prevail during the campaign, through which mechanisms, and to what extent. We develop an electoral competition model with two candidates, each endowed with exogenous platforms and characteristics. Candidates allocate strategically their communication time across two issues to maximize their expected vote shares. We find that when one candidate holds similar comparative advantages on both issues, the disadvantaged candidate communicate on a single issue to saturate the campaign with one topic and then increases the randomness of the election. The advantaged candidate has the opposite incentive and communicate on both issues, creating an asymmetry in the campaign. We show that in some cases, the campaign can become entirely centered on a single issue.

Keywords: Electoral competition, Communication time, Priming.
JEL Classification: C72, D72

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1 Introduction

Electoral campaigns typically concentrate on political issues that dominate the public discourse. This focus shapes voters' priorities, an effect often referred to as *priming*.[1] Consequently, the electoral outcome may depend crucially on which issues prevail during the campaign.

Candidates play a central role in shaping the importance of issues during the campaign through their public speeches and media communication. Candidates' communications thus have a strategic dimension: they often have comparative advantages on specific issues (such as a more consensual platform or higher perceived competence) and can choose to emphasize some issues or downplay others. Consequently, electoral competition directly influences which issues dominate the campaign.

A central question is thus how candidates' strategic communication shapes which issues prevail during the campaign, through which mechanisms, and to what extent. To address this question, we develop an electoral competition model between two candidates competing in a two-dimensional policy space. Candidates are endowed with exogenous platforms and characteristics and decide how to allocate their communication time across issues to maximize their expected vote shares in a probabilistic voting model. An issue matters more for voters' choices as candidates communicate more about it.

We find that for a broad class of voters' utility functions, the comparative advantages can be summarized into two key parameters which define the types of candidates: they are either *strongly specialized* (a candidate is advantaged in one issue but disadvantaged on the other), *weakly specialized* (a candidate is advantaged on both issue, but more significantly on one) or *non-specialized* (one candidate has an advantage over his opponent on both issues, and these two advantages are comparable). We then characterize the different forms these advantages can take and show how candidates' platforms or characteristics can be endogenized.

One key finding of our analysis is that candidates spend their entire time on the topic where they have the greater advantage when they are specialized (weakly or strongly). However, when they are non-specialized, the advantaged candidate wants the campaign to be balanced between issues, because it minimizes the randomness of the election, in the absence of which he would win for sure. For the same reason, the disadvantaged candidate wants the campaign to be single-issue. To do so, the advantaged candidate splits his time between issues while the disadvantaged candidate mixes between spending all his time on one issue or the other.

This analysis provides answers to our research questions. First, unlike in much of the existing literature, candidates do not always concentrate exclusively on the issue where they have

[1]The literature emphasizes both the impact of media (see Iyengar and Kinder, 1987 for example) and candidates (Druckman, 2004).

the greatest comparative advantage. In particular, when both candidates are non-specialized, the advantaged candidate splits his communication across issues, while the disadvantaged candidate saturates one issue, creating an asymmetry in the campaign.

Second, the mechanism behind this result stems from diverging incentives. The advantaged candidate seeks to enrich the political campaign: when both issues are discussed relatively equally, the overall importance of political issues increases, making voters' decisions more strongly influenced by his advantages. The disadvantaged candidate has the opposite incentive: he aims to narrow the campaign. When the campaign becomes dominated by a single issue, the total importance of political issues decreases, thereby increasing the randomness of the election.

Finally, regarding the extent to which issues prevail, we show that when the advantaged non-specialized candidate has perfectly balanced advantages, one issue dominates three-quarters of the campaign. As candidates converge toward specialized types, the dominance of one issue can intensify, potentially leading to a campaign centered on a single issue. This outcome is particularly problematic for two reasons. An unbalanced campaign impoverishes public discourse: by being artificially narrowed to a single topic, it limits voters' exposure to arguments and information across the broader policy space. Also, our model shows that unbalanced communication increases the role of randomness in the election and harms the advantaged candidate, typically the one combining greater competence and more consensual positions.

Another interesting result is that when candidates are specialized (weakly or strongly), their expected vote share only depends on the advantages through their sum. However, when candidates are non-specialized, the expected vote share also depends on the ratio of the advantages. We show that the impact of this ratio is always detrimental to the advantaged candidate, who would therefore prefer to be specialized.

Related literature

A rich literature has emerged since Riker (1993) seminal work, which introduced the *dominance principle*. According to this principle, when a party dominates on a particular issue, it emphasizes that issue in its campaign, while its opponent strategically avoids it. While Riker leaves open what drives a party's dominance on an issue, Petrocik (1996) issue ownership theory connects this dominance to a party's reputation for greater competence in handling that issue.

Since then, several studies have modeled dominance as a form of *vertical* differentiation (Aragonès et al., 2015 Denter, 2020 Yamaguchi, 2022, Nunnari and Zápal, 2025) by introducing a common-value characteristic (such as quality of the policy proposal) that voters use to compare candidates when forming their preferences on a particular issue. Other studies have modeled dominance as a form of *horizontal* differentiation (Amorós and Puy, 2013, Dragu and Fan, 2016, Demange and Van der Straeten, 2020), introducing heterogeneity in voters' preferences over policy positions on each issue. As a result, some candidates hold more consensual positions that appeal to a larger share of voters.

An important contribution of our paper is to develop a notion of dominance that covers a wide range of possible interactions between vertical and horizontal characteristics. We show that these elements can be summarized into a single dominance parameter for each issue. The ratio of these parameters across issues determines the equilibrium structure of the campaign and whether the dominance principle holds.

A key question in this literature is whether candidates focus on similar or distinct issues, a distinction commonly referred to as *issue convergence* and *issue divergence*. In most papers, the equilibrium exhibits issue divergence: candidates do not address the same issues as they specialize on where they are dominant. This result, however, is not fully consistent with empirical evidence. In their analysis of U.S. presidential campaigns from 1960 to 2000, Sigelman and Buell Jr (2004) document a high degree of *issue convergence*: candidates tend to emphasize similar issues.[2] Our model helps reconcile theoretical predictions with empirical evidence: we derive conditions under which the equilibrium exhibits issue convergence or divergence.

A small number of papers also obtain issue convergence (Denter (2020), Demange and Van der Straeten (2020) and Amorós and Puy (2013)). In Denter (2020) campaign spending simultaneously increases the weight of issues and the perceived quality of the candidate, leading to a pure-strategy equilibrium in which candidates campaign on multiple issues. In Demange and Van der Straeten (2020), candidates's communication does not affect the importance of issue but transmit information about their platforms. The author show that candidates address all issues with positive probability but tend to focus more on issues where they are ex-ante closer to the representative voter, offering a nuanced version of Petrocik's theory. In these two papers, the issue convergence is obtained by the introduction of additional ingredients such as persuasion or information effects. In comparison, we obtain issue convergence in a model where candidates' communications only affect voters' priorities. Amorós and Puy (2013) introduce the notions of a party's absolute and comparative advantage in a setting where candidates are only horizontally differentiated. They obtain convergence when one of the parties has an absolute advantage on both issues, and when its comparative advantage is not too large.

[2] These conclusions are consistent with other empirical studies: Damore (2005), Kaplan et al. (2006) and Green and Hobolt (2008) for example.

In comparison with these papers, we introduce a general notion of dominance, which can also be endogenized. Furthermore, our probabilistic framework allows us to identify the mechanism underlying the convergence and quantify the asymmetry between issues during the campaign. Our mechanism does not rely on any additional effect of communication other than its effect on issue weights. Instead, it introduces further incentives for the candidates: beyond seeking to promote their preferred issue, they also aim to maximize or minimize the overall impact of political issues relative to the electoral randomness.

Finally, our paper contributes more broadly to the literature on electoral competition between vertically advantaged and disadvantaged candidates. This literature (Groseclose (2001), Aragones and Palfrey (2002) and Aragonès and Xefteris (2012)) emphasizes the incentives of the advantaged candidate to adopt a more consensual platform while the disadvantaged candidate avoids competition by choosing an extreme one. In contrast, our paper highlights a different strategy of the unfavored candidate to mitigate its political disadvantage: he tries to narrow the public discourse in order to increase the randomness of the election.

The following of the paper is organized as follows. We describe the model in section 2 and discuss the resulting payoff functions in section 3. We analyze the equilibrium in section 4. We study equilibrium payoffs in section 5 and communication times at equilibrium in section 6. Proofs are postponed to section 7.

2 Model

We consider a policy space with two issues (x, y) as a compact subset $\mathcal{X}$ of $\mathbb{R}^2$. Candidates A and B decide how to allocate their communication times between the two issues.[3] Candidate $j \in \{A, B\}$ selects $\boldsymbol{t}^j := (t^j_x, t^j_y)$ under the constraint that $t^j_x + t^j_y = 1$, $t^j_x \geq 0$, $t^j_y \geq 0$.[4] The total time allocated on issue x (resp y) by both candidates is denoted $t_x := t^A_x + t^B_x$ (resp $t_y := t^A_y + t^B_y$) and influences the perception that citizens have about the importance of issue x (resp. y).

Voters are identified with their ideal policies $\boldsymbol{\theta} := (\theta_x, \theta_y)$ distributed on $\mathcal{X}$ according to the density function f. Each candidate j is endowed with an exogenous platform $\boldsymbol{p}^j :=$

[3]Some papers study "campaign spending" instead of "communication time," but this difference is merely terminological and does not affect the interpretation of our results.

[4]We assume for simplicity that both candidates have the same time constraint, but our main results hold under heterogeneity. These time budgets can be endogenized by the introduction of a cost function and a motivation to communicate (for example, they must participate in debates, maintain visibility in the media, and ensure that voters are familiar with their positions). Exploring these additional constraints goes beyond the scope of the present paper but justify our assumption that candidates spend their entire budgets.

$(p_x^j, p_y^j) \in \mathcal{X}$ and profile of characteristics $\boldsymbol{\nu}^j := (\nu_x^j, \nu_y^j) \in (\mathbb{R}_+^\star)^2$.

On issue x (similarly with issue y), a citizen $\boldsymbol{\theta} = (\theta_x, \theta_y)$ evaluates candidate A by $u(\theta_x, p_x^A, \nu_x^A)$. A typical example is $u(\theta_x, p_x^A, \nu_x^A) = \nu_x^A - (\theta_x - p_x^A)^2$. In this example, the voter's evaluation decreases with the distance between his ideal θ_x and the platform of the candidate p_x^A, and increases with the characteristic ν_x^A, which is seen as the common-value quality of the candidate on issue x. Different examples are discussed in subsection 3.1. Voter $\boldsymbol{\theta}$ aggregates his evaluation of candidate A (resp. B) on both issues using weighted issue preferences (see Krasa and Polborn (2010)):

$$\begin{cases} U_{\boldsymbol{\theta}}^A(\boldsymbol{t}^A, \boldsymbol{t}^B) = w(t_x)u(\theta_x, p_x^A, \nu_x^A) + w(t_y)u(\theta_y, p_y^A, \nu_y^A) \\ U_{\boldsymbol{\theta}}^B(\boldsymbol{t}^A, \boldsymbol{t}^B) = w(t_x)u(\theta_x, p_x^B, \nu_x^B) + w(t_y)u(\theta_y, p_y^B, \nu_y^B) \end{cases}$$

where $w : [0, 2] \to \mathbb{R}^+$ is the function that maps the aggregate time spent on an issue to its weight in the voters' evaluations. We only assume that w is strictly increasing and strictly concave.[5]

We introduce an exogenous shock capturing all unpredictable events that might impact voters' decisions, such as sudden natural disasters, international crises or scandals. We model this shock with a white noise[6] $\varepsilon \sim \mathcal{U}_{\left[-\frac{1}{2\phi}, \frac{1}{2\phi}\right]}$ with a positive parameter ϕ. A citizen $\boldsymbol{\theta}$ votes for candidate A when $U_{\boldsymbol{\theta}}^A(\boldsymbol{t}^A, \boldsymbol{t}^B) - U_{\boldsymbol{\theta}}^B(\boldsymbol{p}^A, \boldsymbol{p}^B) > \varepsilon$ and for B otherwise. For simplicity, we assume that the parameter $\phi > 0$ is sufficiently small so that the noise amplitude ensures that each candidate remains uncertain about obtaining the support of any given voter.

Note that we decide not to normalize the weights so that the the importance of the two political issues $w(t_x) + w(t_y)$ can vary compared with the random shock. This sum is higher as the total time spend on each issues are balanced, our choice thus emphasizes that a more diverse and thus richer political discourse increases overall political interest and attention. This modeling choice deepens the strategic dimension of communication: in addition to talk about issues they dominate the most, candidates have conflicting incentives regarding the overall weight of political issues.

3 Payoff functions

We define $V_A(\boldsymbol{t}^A, \boldsymbol{t}^B)$ as the expected share of citizens that vote for candidate A. We simplify the notations by denoting the expected vote shares V_A, V_B.

[5]It is well documented that media exposure increases the perceived importance of political issues. Moreover, the concavity assumption captures the saturation effect, so that additional exposure yields diminishing returns.

[6]We use therefore a standard probabilistic voting framework, see Lindbeck and Weibull (1987) and Persson and Tabellini (2002).

Proposition 1. *The expected vote shares of candidates are:*

$$V_A = \frac{1}{2} + \phi[w(t_x)\Delta_x + w(t_y)\Delta_y]$$

$$V_B = \frac{1}{2} - \phi[w(t_x)\Delta_x + w(t_y)\Delta_y]$$

where

$$\Delta_x := \int_{\mathcal{X}} [u(\theta_x, p_x^A, \nu_x^A) - u(\theta_x, p_x^B, \nu_x^B)] f(\theta) d\theta \tag{1}$$

and similarly

$$\Delta_y := \int_{\mathcal{X}} [u(\theta_y, p_y^A, \nu_y^A) - u(\theta_y, p_y^B, \nu_y^B)] f(\theta) d\theta$$

Proof. The proof is postponed to Appendix 7.1 □

The above proposition shows that candidates' payoffs depend on their platforms and characteristics only through Δ_x and Δ_y, which therefore summarize how they matter electorally. Since all equilibrium strategies and comparative statics inherit this simplification, we express our results in terms of these parameters.

This formulation provides a natural answer to the long-debated question of how to model a candidate's dominance over another on a given issue. For a general voter evaluation function u, our term Δ_x captures the comparative advantage of candidate A over candidate B on issue x. To compute this comparative advantage, it suffices to evaluate how a voter with opinion θ_x on issue x prefers candidate A over candidate B, and then take the average over $\boldsymbol{\theta}$.

The next subsection illustrates that platforms and characteristics can interact in different ways: in the simple case of additive valence, a large Δ_x reflects the additive combination of a more consensual platform and a higher perceived competence in implementing it on issue x. In general, however, these two types of advantages may interact in more complex ways.

3.1 Illustrative examples

We illustrate the comparative advantages obtained in Proposition 1 with different functional forms of u, in the simple case where $\mathcal{X} = [-1, 1]^2$ and where $\boldsymbol{\theta}$ is uniformly distributed on $\mathcal{X}$.

1. Consider first the classical model of *additive valence* (see Enelow and Hinich (1982) for example): $u(\theta_x, p_x^A, \nu_x^A) = \nu_x^A - (\theta_x - p_x^A)^2$. Using equation 1, we obtain that

$$\Delta_x = \nu_x^A - \nu_x^B - (p_x^A)^2 + (p_x^B)^2$$

The hypothesis of an additive separability in u between the ideological distance and the characteristic, here interpreted as a quality, implies that Δ_x is a simple sum of the advantage on quality $\nu_x^A - \nu_x^B$, and platforms $-(p_x^A)^2 + (p_x^B)^2$, where a candidate closer to the median opinion on issue x ($\theta_x = 0$) has an advantage. In this example, the two advantages are perfect substitutes in generating overall advantage on issue x.

2. Consider now the model of *intensity valence*, as introduced in Gouret and Rossignol (2019): $u(\theta_x, p_x^A, \nu_x^A) = \nu_x^A(K - |p_x^A - \theta_x|)$. In this example, ν_x^A represents the ability of candidate A to implement its platform on issue x. A voter can thus be either positively or negatively impacted by this characteristic, depending on whether she is ideologically close or far from the candidate's platform. We find:

$$\Delta_x = K(\nu_x^A - \nu_x^B) + \frac{\nu_x^B}{2}((p_x^B)^2 + 1) - \frac{\nu_x^A}{2}((p_x^A)^2 + 1)$$

In this example, the two advantages interact in generating the overall advantage on issue x. The cross-partial derivative $\frac{\partial^2 \Delta_x}{\partial \nu_x^A \partial p_x^A}$ is positive for platforms at the left of the median voter, indicating complementarity: a higher characteristic increases the marginal gain from moving the platform toward the median voter.

3. Consider finally the model of *multiplicative valence advantage*, as introduced in Hollard and Rossignol (2008): $u(\theta_x, p_x^A, \nu_x^A) = -\frac{(\theta_x - p_x^A)^2}{\nu_x^A}$. In this example, the valence can be interpreted as the ability of a politician to attenuate the ideological distance between a voter and his platform. We find:

$$\Delta_x = \frac{1}{3}\left(\frac{1}{\nu_x^B} - \frac{1}{\nu_x^A}\right) + \left(\frac{(p_x^B)^2}{\nu_x^B} - \frac{(p_x^A)^2}{\nu_x^A}\right)$$

In this example, the two advantages also interact in generating the overall advantage on issue x, but the cross-partial derivative is negative for platforms at the left of the median voter, indicating substitutability: a higher characteristic diminishes the marginal gain from moving the platform toward the median voter.

Other examples include the case of discounted valence, and the model of perfect complement as discussed in Denter (2021)[7]. Beyond these benchmark cases, our formulation also covers more general settings where u need not be decreasing in the distance $|p - \theta|$.

[7]The discounted valence is modeled by $u(\theta_x, p_x^A, \nu_x^A) = \frac{\nu_x^A}{K + (\theta_x - p_x^A)^2}$ while the prefect complement model assumes that $u(\theta_x, p_x^A, \nu_x^A) = \min\{\nu_x^A, K - |p_x^A - \theta_x|\}$.

3.2 Endogenous advantages

Our framework allows for endogenizing the advantages of candidates, by considering a two-stage game in which candidates first choose their platforms or characteristics and then allocate their communication times.[8] The analysis of the subgame perfect Nash equilibrium (SPNE) is facilitated by the following tractable properties of our model. First, we show in Proposition 5 that the vote share of the advantaged candidate is strictly increasing with Δ_x and Δ_y. At the first stage of the game, the advantaged candidate (resp. disadvantaged candidate) simply aims at maximizing (resp. minimizing) these two parameters.
Second, it follows from equation 1 that Δ_x can be written as a difference of two terms: the first one only depends on candidate A's platform and characteristic while the second term only depends on candidate B's platform and characteristic. The choices that maximizes Δ_x are therefore independant of the choices of his opponenent. A similar argument holds for Δ_y and for candidate B: there is no strategic interaction between candidates at this stage. As a consequence the existence of a SPNE is guaranteed.[9]

Proposition 2. *We describe the set of SPNE of the two-stage game where candidates select platforms at stage* 1 *and allocate their communication time at stage* 2*. At stage* 1*, candidates select* $(\hat{\boldsymbol{p}}^A, \hat{\boldsymbol{p}}^B)$ *such that:*

$$\hat{p}_x^A \in \arg\max_{p_x^A} \int_{\mathcal{X}} u(\theta_x, p_x^A, \nu_x^A) f(\theta) d\theta$$

and

$$\hat{p}_x^B \in \arg\max_{p_x^B} \int_{\mathcal{X}} u(\theta_x, p_x^B, \nu_x^B) f(\theta) d\theta$$

and similarly for issue y. The second stage of the game is analyzed in Theorem 1 in the next section for any Δ_x, Δ_y*. The subsequent second-stage of the SPNE is the one with advantages* $\Delta_x(\hat{p}_x^A, \hat{p}_x^B, \nu_x^A, \nu_x^B)$ *and* $\Delta_y(\hat{p}_y^A, \hat{p}_y^B, \nu_y^A, \nu_y^B)$*.*

In the game where candidates select their characteristics at stage 1 *and allocate their communication time at stage* 2*, the result is similar: the maximum is taken over* ν_x^A*,* ν_x^B *on issue* x*, and* ν_y^A*,* ν_y^B *on issue* y*.*

Proof. The proof is postponed to Appendix 7.2. □

[8] In principle, candidates could choose both their platforms and characteristics at stage 1. However, because they are ex-ante perfectly symmetric, they have no reason to differentiate in the first stage, making the subsequent time-allocation stage trivial. We choose to maintain ex-ante symmetry but assume that only platforms or only characteristics are endogenous.

[9] Our result contrasts with the standard multidimensional electoral competition where a pure equilibrium generically does not exist (see Plott (1967)).

We illustrate the above proposition in the example of intensity valence mentioned in subsection 3.1. We assume that $u(\theta_x, p_x^A, \nu_x^A) = \nu_x^A(K - |p_x^A - \theta_x|)$ for a certain $K > 0$, where the characteristics $\nu_x \in [\underline{\nu}, \overline{\nu}]$ with $\underline{\nu} > 0$, and where $\boldsymbol{\theta}$ is uniformly distributed on $[-1,1]^2$. In this case we find that $\displaystyle\int_{\mathcal{X}} u(\theta_x, p_x^A, \nu_x^A) f(\theta) d\theta = K\nu_x^A - \frac{\nu_x^A}{2}\left((p_x^A)^2 + 1\right)$ and we thus obtain that:

In the game with endogenous platforms, candidates select $(\hat{p}_x^A, \hat{p}_y^A) = (\hat{p}_x^B, \hat{p}_y^B) = (0,0)$.

In the game with endogenous characteristics, candidate A selects $\begin{cases} \nu_x^A = \overline{\nu} \text{ if } (p_x^A)^2 < 2K-1 \\ \nu_x^A = \underline{\nu} \text{ if } (p_x^A)^2 > 2K-1 \\ \nu \in [\underline{\nu}, \overline{\nu}] \text{ if } (p_x^A)^2 = 2K-1 \end{cases}$
and similarly for issue y, and for candidate B.

In this illustrative example, we find that the median voter theorem applies, as both candidates select $\boldsymbol{p}^A = \boldsymbol{p}^B = (0,0)$ at the subgame perfect Nash equilibrium with endogenous platforms. This result is not surprising as we did not introduce any exogenous differentiation costs (such as anchoring, inconsistency costs or imperfect turnout for example).

When candidates can choose their characteristics, interpreted as the ability to implement their platform in the intensity valence model, we find that they prefer to show a high ability when their platforms are popular (close to the median voter on this issue, located at 0) or a low ability when their platforms are unpopular (far from 0).

4 Equilibrium analysis

We are seeking for the Nash Equilibrium (NE) of the normal form game described above, where candidates aim at maximizing their expected vote shares by selecting how they allocate their communication times.
Note that when $\Delta_x = \Delta_y = 0$, candidates' expected votes shares are equal to $\frac{1}{2}$, so every pair of strategies is an equilibrium. In the sequel of the paper, we analyze the case where $(\Delta_x, \Delta_y) \neq (0,0)$ and without loss of generality, we suppose that $\Delta_y > 0$.

As stated above, Proposition 1 highlights that the candidates' payoffs depends on the platforms $\boldsymbol{p}^j$ and characteristics $\boldsymbol{\nu}^j$ only through the two variables Δ_x, Δ_y defined in equations 1. Theorem 1 reinforces the reduction by showing that the Nash Equilibrium depends only on the ratio $\frac{\Delta_x}{\Delta_y}$. The following definitions are useful for the statement of the theorem.

Definition 1. *We say that candidates are:*

- ***strictly-specialized*** *when* $\frac{\Delta_x}{\Delta_y} \leq 0$*. This situation corresponds to the case where each candidate has an advantage on one issue and a disadvantage on the other.*

- ***weakly-specialized*** *when* $0 < \frac{\Delta_x}{\Delta_y} < \frac{w(2)-w(1)}{w(1)-w(0)}$ *or* $\frac{w(1)-w(0)}{w(2)-w(1)} < \frac{\Delta_x}{\Delta_y}$*. This situation corresponds to the case where a candidate has an advantage on both issues, and one advantage is sufficiently large relative to the other.*

- ***non-specialized*** *when* $\frac{w(2)-w(1)}{w(1)-w(0)} \leq \frac{\Delta_x}{\Delta_y} \leq \frac{w(1)-w(0)}{w(2)-w(1)}$*. This situation corresponds to the case where one candidate has a comparable advantage on both issues.*

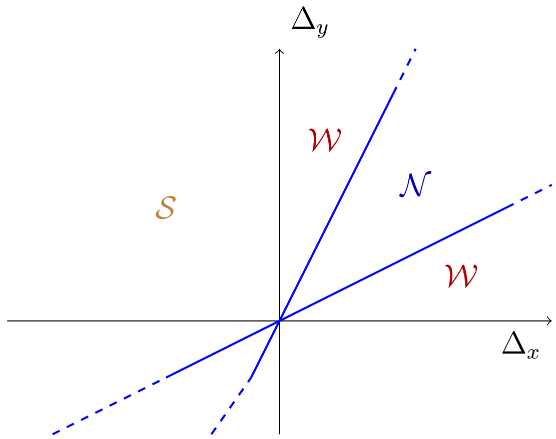

Figure 1: The above figure illustrates the different regions described in definition 1 (and that characterize the equilibrium behavior of candidates in Theorem 1 below) with respect to Δ_x and Δ_y. By symmetry, we restrict our analysis to the case where $\Delta_y > 0$.

We now characterize the NE of the game, based on the profiles of candidates.

Theorem 1. *Remember that we assumed, without loss of generality, that* $\Delta_y > 0$*.*

- *If candidates are strictly-specialized, the unique NE is* $(\boldsymbol{t}^{A*}, \boldsymbol{t}^{B*}) = ((0,1),(1,0))$*.*

- *If candidates are weakly-specialized: the unique NE is* $(\boldsymbol{t}^{A*}, \boldsymbol{t}^{B*}) = ((0,1),(1,0))$ *when* $0 < \frac{\Delta_x}{\Delta_y} < \frac{w(2)-w(1)}{w(1)-w(0)}$*, and* $(\boldsymbol{t}^{A*}, \boldsymbol{t}^{B*}) = ((1,0),(0,1))$ *when* $\frac{w(1)-w(0)}{w(2)-w(1)} < \frac{\Delta_x}{\Delta_y}$*.*

- *If candidates are non-specialized: the unique NE when* $\frac{w(2)-w(1)}{w(1)-w(0)} < \frac{\Delta_x}{\Delta_y} < \frac{w(1)-w(0)}{w(2)-w(1)}$*, is the following:*
  Candidate A plays $\boldsymbol{t}^{A*} = (\hat{t}, 1-\hat{t})$*, with* $\hat{t} \in (0,1)$ *being the unique solution to the equation*

$$\frac{\Delta_x}{\Delta_y} = \frac{w(2-\hat{t}) - w(1-\hat{t})}{w(1+\hat{t}) - w(\hat{t})} \tag{2}$$

Candidate B plays $\boldsymbol{t}^{B*} = (1,0)$ with probability π^* and $\boldsymbol{t}^{B*} = (0,1)$ with probability $1-\pi^*$ where

$$\pi^*(\hat{t}) := \frac{\frac{\Delta_x}{\Delta_y} w'(\hat{t}) - w'(2-\hat{t})}{\Big(w'(1-\hat{t}) - w'(2-\hat{t})\Big) + \frac{\Delta_x}{\Delta_y}\Big(w'(\hat{t}) - w'(1+\hat{t})\Big)}. \tag{3}$$

Note that if $\frac{\Delta_x}{\Delta_y} = \frac{w(2)-w(1)}{w(1)-w(0)}$ both the mixed equilibrium and the pure equilibrium $(\boldsymbol{t}^{A*}, \boldsymbol{t}^{B*}) = ((0,1),(1,0))$ exist, and similarly at the symmetric bound where $\frac{\Delta_x}{\Delta_y} = \frac{w(1)-w(0)}{w(2)-w(1)}$.

Proof. The proof of Theorem 1 is postponed to Appendix 7.3. □

Theorem 1 characterizes candidates' behaviors across different regions of the parameters Δ_x and Δ_y. We now discuss the underlying intuition for each region.

- in the region $\mathcal{S}$, candidates are *strictly-specialized*. Candidate A has an advantage on issue y and a disadvantage on issue x. Consequently, at the unique pure NE, he allocates all his communication time to issue y, and symmetrically for candidate B. While the two issues are equally discussed during the campaign, each candidate aims at maximizing the weight of the unique issue on which he has an advantage, and therefore focuses on this issue.

- in $\mathcal{W}$, candidates are *weakly-specialized*. While candidate A is advantaged on both issues, one advantage is sufficiently large relative to the other. Theorem 1 describes a NE where candidates still allocate their entire communication time to the issue on which they are the most advantaged, or the least disadvantaged.

In these two regions ($\mathcal{S}$ and $\mathcal{W}$), the objective of each candidate is to maximize the weights on the issue where he is more advantaged, or less disadvantaged. Each candidate has therefore a dominant strategy and there is no real strategic interaction between candidates.

- in $\mathcal{N}$, candidates are *non-specialized*. Candidate A is indeed advantaged on both issues with a comparative magnitude. In this case[10], candidate A plays a pure strategy where he splits his communication time between the two issues, while the disadvantaged candidate B plays a mixed strategy where he spends his entire communication time on one of the two issues with a certain probability.

[10]For clarity of exposition, we only consider here and in the following of the paper, the mixed equilibrium in $\mathcal{N}$ and omit that at its frontier, the pure equilibrium also exists.

We now explain the economic intuition underlying the mixed equilibrium in region $\mathcal{N}$.

In region $\mathcal{N}$, the advantaged candidate has a comparable advantage on both issues. Thus, his main objective is not to maximize the communication time on a particular issue but rather to maximize the total weight of the two dimensions. By doing so, he minimizes the randomness of the election, in the absence of which he would win for sure. This reasoning becomes particularly clear in the symmetric case where $\Delta_x = \Delta_y$: the expected vote shares of the advantaged candidate A simplifies to $\frac{1}{2} + \phi\Delta_x[w(t_x) + w(t_y)]$, so that his unique objective is to maximize the sum of the weights $w(t_x) + w(t_y)$. By increasing the total weight on the political issues x and y, the decision of voters becomes more deterministic and less sensitive to the noise ε, which favors the advantaged candidate. Conversely, the disadvantaged candidate B aims at minimizing the sum, as doing so maximizes the randomness of the election. These contradictory incentives lead to the appearance of a mixed strategy for the disadvantaged candidate, as we now explain.

Because w is concave and because the total communication time is fixed, candidate A aims at equalizing the two weights, while candidate B's objective is to polarize the weights. The equilibrium must therefore be in mixed strategy: the advantaged candidate splits his communication time equally among issues, leaving the disadvantaged candidate indifferent between spending all his communication time on one or the other issue. Reciprocally, if the disadvantaged candidate spends his entire communication time on each issue with probability $\frac{1}{2}$, we find that the advantaged candidate's best response is to spend equally his communication time.

When we relax the hypothesis that $\Delta_x = \Delta_x$ but remain in the region $\mathcal{N}$, candidates objectives are mixed between the willingness to favor the issue on which they are the most advantaged (resp. least disadvantaged) and the willingness to maximize (resp. minimize) the randomness of the election. Finally, in region $\mathcal{W}$, the first objective entirely dominates the second one.

We have found that the advantaged non-specialized candidate splits his communication times between the two issues at equilibrium. Lemma 1 below shows that $\hat{t}$, the time spent on issue x, increases with $\frac{\Delta_x}{\Delta_y} \in \mathcal{N}$, ranges from $\hat{t} = 0$ to $\hat{t} = 1$ at the boundaries of the region[11], and is balanced $\hat{t} = \frac{1}{2}$ when $\frac{\Delta_x}{\Delta_y} = 1$.

Lemma 1.
$\hat{t}$ is strictly increasing with $\frac{\Delta_x}{\Delta_y}$.

[11]The fact that $\hat{t}$ ranges from 0 to 1 at the boundaries of the region $\mathcal{N}$ ensures that the strategy of the advantaged candidate, and thus the equilibrium payoffs are continuous even at the border between regions, as stated in Remark 1 below.

$\frac{\Delta_x}{\Delta_y} = \frac{w(2)-w(1)}{w(1)-w(0)} \Leftrightarrow \hat{t} = 0.$
$\frac{\Delta_x}{\Delta_y} = 1 \Leftrightarrow \hat{t} = \frac{1}{2}.$
$\frac{\Delta_x}{\Delta_y} = \frac{w(1)-w(0)}{w(2)-w(1)} \Leftrightarrow \hat{t} = 1.$

Proof. The proof is postponed to Appendix 7.4. □

The fact that $\hat{t}$ strictly increases with $\frac{\Delta_x}{\Delta_y}$ is not obvious. One the one hand, the advantaged candidate would like the weight of issue x to be higher when $\frac{\Delta_x}{\Delta_y}$ is larger, as his comparative advantage on this issue has increased. But on the other hand, if issue x already has a higher weight, increasing his communication on issue x has the disadvantage of increasing the unbalance between issues, and therefore the total weights of political issues. As discussed above, this asymmetry of the communication times increases the impact of the randomness, which is to his disadvantage.

Example 1. *We illustrate Theorem 1 in the particular case where w is CARA: $w(x) = 1 - e^{-\alpha x}$, where $\alpha > 0$ captures the degree of concavity of the function w.*
First, we obtain $\mathcal{N} = \left\{\frac{\Delta_x}{\Delta_y} \in [e^{-\alpha}, e^{\alpha}]\right\}$. We observe that the region $\mathcal{N}$ increases with the concavity parameter α. When $\alpha \to 0$, $\mathcal{N} = \emptyset$ and when $\alpha \to +\infty$, the region $\mathcal{N}$ is equal to the entire quadrant where $\mathcal{W}$ has disappeared.
We obtain $\hat{t}\left(\frac{\Delta_x}{\Delta_y}\right) = \frac{1}{2} + \frac{1}{2\alpha}\ln\left(\frac{\Delta_x}{\Delta_y}\right)$ and $\pi^*\left(\frac{\Delta_x}{\Delta_y}\right) = \frac{1}{2}$.

5 Payoffs at equilibrium

In this section, we analyze the candidates' equilibrium payoffs across regions. While the payoff of specialized candidates increases linearly with their advantages, we find that the payoff of non-specialized candidates also depends in a more intricate way on their comparative advantage, namely through the ratio $\frac{\Delta_x}{\Delta_y}$.

Proposition 3. *For $\frac{\Delta_x}{\Delta_y} \in \mathcal{S} \cup \mathcal{W}$, the equilibrium payoff can be written:*

$$V_A^{\mathcal{S}} = V_A^{\mathcal{W}} = \frac{1}{2} + \phi w(1)(\Delta_x + \Delta_y)$$

Proof. The proof is postponed to Appendix 7.5 □

In this case, we find that the electoral surplus of candidate A, $V_A^{\mathcal{S}} - \frac{1}{2}$ can be written as a linear function of three terms: the first term ϕ is small when the noise of the election is large. Indeed, as the election becomes very noisy, the election tends to a simple random game where each candidate wins with probability $\frac{1}{2}$. The second term, $w(1)$, represents the political weight of both issues, which are balanced in this region. Note that $w(1)$ and ϕ play a similar role of reducing the randomness of the electoral competition. The last term highlights that the electoral advantage is a linear increasing function of the sum of the advantages of candidate A.

In this region, there is no interaction between the two advantages Δ_x and Δ_y that are simply summed up in the payoff function.

Proposition 4. *For $\frac{\Delta_x}{\Delta_y} \in \mathcal{N}$, we have:*

$$\begin{aligned} V_A^{\mathcal{N}} &= \frac{1}{2} + \phi[w(\hat{t})\Delta_x + w(2-\hat{t})\Delta_y] \\ &= \frac{1}{2} + \phi\Theta\left(\frac{\Delta_x}{\Delta_y}\right)(\Delta_x + \Delta_y) \end{aligned}$$

where $\Theta(\frac{\Delta_x}{\Delta_y}) = \frac{w(2-\hat{t})w(1+\hat{t})-w(\hat{t})w(1-\hat{t})}{w(2-\hat{t})-w(1-\hat{t})+w(1\hat{+}t)-w(\hat{t})}$ and where $\hat{t}\left(\frac{\Delta_x}{\Delta_y}\right)$ is defined in (2).

Proof. The proof is postponed to Appendix 7.6. □

In region $\mathcal{N}$, we find that the electoral surplus of candidate A, $V_A^{\mathcal{N}} - \frac{1}{2}$ can again be written as a linear function of three terms, where $w(1)$ is now replaced by the function $\Theta(\frac{\Delta_x}{\Delta_y})$. This function illustrates that the electoral surplus depends on the ratio between Δ_x and Δ_y and not only on their sum. We find that $\Theta\left(\frac{\Delta_x}{\Delta_y}\right) \leq w(1)$ for any $\frac{\Delta_x}{\Delta_y} \in \mathcal{N}$, so that for a fixed sum $\Delta_x + \Delta_y$, the advantaged candidate prefers to be in region $\mathcal{S}$ than in region $\mathcal{N}$.

Proposition 5. *The equilibrium payoff of the advantaged candidate A depends on Δ_x and Δ_y as follows:*

1. *$V_A^{\mathcal{S}}$ and $V_A^{\mathcal{N}}$ are strictly increasing with Δ_x and Δ_y.*
2. *Suppose that $\Delta_x + \Delta_y = c > 0$, $V_A^{\mathcal{S}} \geq V_A^{\mathcal{N}}$.*

Proof. The proof is postponed to Appendix 7.7 □

In the region $\mathcal{S}$, the advantages Δ_x and Δ_y have no impact on the equilibrium and thus do not modify how issues are weighted. The variations of $V_A^{\mathcal{S}}$ with respect to Δ_x and Δ_y are therefore straightforward.

However, in the region $\mathcal{N}$, these variations are not trivial: while the advantaged candidates benefits from having larger advantages, a change in Δ_x, Δ_y also impacts $\hat{t}$ and π^*, which might in turn impact negatively the payoff. The above proposition highlights that the direct benefit of increasing Δ_x and Δ_y dominates the indirect effect.

The second point raised by Proposition 5 illustrates that for a fixed sum of advantages, the advantaged candidate prefers to be specialized (weakly or strictly) while the disadvantaged candidate prefers to be non-specialized. In other words, the advantaged candidate prefers having a large advantage on one issue and being disadvantaged on the other than having a balanced advantage on both issues.

Example 2. *Suppose that w is CARA, $w(x) = 1 - e^{-\alpha x}$ for $\alpha > 0$. We find that, among pair of advantages with fixed sum, $\Delta_x + \Delta_y = c$, the vote share of the advantaged candidate is minimal when candidates are specialized and when $\Delta_x = \Delta_y = \frac{c}{2}$. It is then equal to $\frac{1}{2} + \phi c \frac{w(1/2)+w(3/2)}{2}$. The fraction of electoral surplus that is lost by the advantaged candidate, in comparison with the case where candidates are non-specialized, is measured by the ratio between $w(1)$ and $\frac{w(1/2)+w(3/2)}{2}$, which is related to the concavity of the function w.*

Remark 1. *Payoffs are continuous with respect to the parameters (Δ_x, Δ_y), even at the borders between regions, and so is the equilibrium strategy of the advantaged candidate, as $\hat{t} = 0$ when $\frac{\Delta_x}{\Delta_y} = \frac{w(2)-w(1)}{w(1)-w(0)}$ and $\hat{t} = 1$ when $\frac{\Delta_x}{\Delta_y} = \frac{w(1)-w(0)}{w(2)-w(1)}$. However, we find that the mixed equilibrium of the disadvantaged candidate is, in general, discontinuous. In the case where w is CARA for example, $\pi^* = \frac{1}{2}$ in the open set $\mathcal{N}$ while the candidate plays a pure action in $\mathcal{W}$. This discontinuity does not impact the continuity of the payoffs, as both candidate are indifferent between the possible actions of the disadvantaged candidate at equilibrium (it is the case for the disadvantaged candidate by definition of $\hat{t}$, but also for the advantaged candidate as we study a constant-sum game).*

6 The communication times at equilibrium

We now study how much time is spent on each issue at equilibrium, and whether the two topics receive equal attention. As discussed in the introduction, there are two main reasons why a balanced campaign is desirable. First, from an exogenous standpoint, a balanced campaign sustains a richer public discourse: it is not artificially reduced to a single topic, and voters are exposed to arguments and information across the broader policy space. Second, within our model, a balanced campaign reduces the influence of randomness in electoral outcomes and benefits the advantaged candidate, typically the more consensual and competent candidate, as illustrated in subsection 3.1.

The communication times t_x, t_y spent on each issue at equilibrium depend on the characteristics $\boldsymbol{\nu}^j, \boldsymbol{p}^j$ only through the ratio $\frac{\Delta_x}{\Delta_y}$ and involves $\hat{t}\left(\frac{\Delta_x}{\Delta_y}\right)$ and $\pi^*\left(\frac{\Delta_x}{\Delta_y}\right)$ as defined in Theorem 1. For readability, we denote these quantities $\hat{t}$ and π^*.

Proposition 6. *At equilibrium, the communication times t_x, t_y satisfy:*

- *For $\frac{\Delta_x}{\Delta_y} \in \mathcal{W} \cup \mathcal{S}$, we have $|t_x - t_y| = 0$,*
- *For $\frac{\Delta_x}{\Delta_y} \in \mathcal{N}$ we have $|t_x - t_y| = \begin{cases} 2\hat{t} \text{ with probability } \pi^* \\ 2 - 2\hat{t} \text{ with probability } 1 - \pi^* \end{cases}$.*

Proof. The proof is postponed to Appendix 7.8 □

We obtain that the campaign is balanced in the region $\mathcal{W} \cup \mathcal{S}$. In contrast, the campaign is generally unbalanced in region $\mathcal{N}$, when candidates are non-specialized, with the possibility to be single-issued.

More precisely, in the case where $\Delta_x = \Delta_y$, we have $\hat{t} = \frac{1}{2}$ so that $|t_x - t_y| = 1$. Thus, $(t_x, t_y) = (\frac{1}{2}, \frac{3}{2})$ with probability $\pi^* = \frac{1}{2}$, and $(t_x, t_y) = (\frac{3}{2}, \frac{1}{2})$ otherwise. The unbalanced between issues is deterministic, and one topic is discussed three times as much as the other. When $\frac{\Delta_x}{\Delta_y}$ departs from 1 (while remaining in $\mathcal{N}$), there are two possible outcomes depending on the mixed strategy's realization. The distance between these two outcomes is maximized when $\frac{\Delta_x}{\Delta_y}$ is at the boundaries of $\mathcal{N}$ (which correspond to the cases where $\hat{t} = 0$ or 1). In this case, we find that $|t_x - t_y|$ is either equal to 0 or to 2. The campaign is therefore balanced or single-issued.

Note that $|t_x - t_y| = 2$ occurs with a strictly positive probability as long as $w'(0) < +\infty$ (as proved in Claim 1 in the Appendix 7.9). For example, in the case where w is CARA, we have $\pi^* \left(\frac{\Delta_x}{\Delta_y}\right) = \frac{1}{2}$ for every $\frac{\Delta_x}{\Delta_y} \in \mathcal{N}$, so that both the balanced campaign and the single-issued campaign occur with probability $\frac{1}{2}$. This result is relatively surprising as it implies both candidates to spend all their communication time on the same issue, while they compete in a constant-sum election game.

7 Appendix

7.1 Proof of proposition 1

Proof. We first compute the probability that a citizen $\boldsymbol{\theta}$ votes for candidate A:

$$\begin{aligned}\mathbb{P}(\boldsymbol{\theta} \text{ votes for A}) &= \mathbb{P}(\varepsilon < U^A_{\boldsymbol{\theta}} - U^B_{\boldsymbol{\theta}}) \\ &= \frac{1}{2} + \phi\Big[w(t_x)[u(\theta_x, p^A_x, \nu^A_x) - u(\theta_x, p^B_x, \nu^B_x)] + w(t_y)[u(\theta_y, p^A_y, \nu^A_y) - u(\theta_y, p^B_y, \nu^B_y)]\Big]\end{aligned}$$

We can now compute the expected vote share of candidate A. Note that our previous assumption on ϕ guarantees that the above probability and thus the vote share belong to $[0, 1]$.

$$V_A = \mathbb{E}\left[\int_{\mathcal{X}} \mathbb{1}_{\{\theta \text{votes for } A\}} f(\theta) d\theta\right] = \int_{\mathcal{X}} \mathbb{E}\left[\mathbb{1}_{\{\theta \text{votes for } A\}}\right] f(\theta) d\theta = \int_{\mathcal{X}} \mathbb{P}(\theta \text{ votes for } A) f(\theta) d\theta$$
$$= \frac{1}{2} + \int_{\mathcal{X}} \phi\Big[w(t_x)[u(\theta_x, p_x^A, \nu_x^A) - u(\theta_x, p_x^B, \nu_x^B)] + w(t_y)[u(\theta_y, p_y^A, \nu_y^A) - u(\theta_y, p_y^B, \nu_y^B)]\Big] f(\theta) d\theta$$
$$= \frac{1}{2} + \phi[w(t_x)\Delta_x + w(t_y)\Delta_y]$$

By definition, we also have $V_B = 1 - V_A$. □

7.2 Proof of Proposition 2.

Proof. We proceed a backward induction where the second stage is analyzed in Theorem 1. In first stage, candidates select their platforms and characteristics. Equilibrium payoffs are proved to be strictly increasing with respect to Δ_x, Δ_y (see Proposition 5). Using equation 1, we find that

$$\Delta_x = \int_{\mathcal{X}} [u(\theta_x, p_x^A, \nu_x^A) - u(\theta_x, p_x^B, \nu_x^B)] f(\theta) d\theta$$
$$= \int_{\mathcal{X}} u(\theta_x, p_x^A, \nu_x^A) f(\theta) d\theta - \int_{\mathcal{X}} u(\theta_x, p_x^B, \nu_x^B) f(\theta) d\theta$$

We observe that candidates' maximization problems do not interact: candidate A selects p^A (or ν_x^A) to maximize the first term of the difference above and candidate B selects p^B (or ν_x^B) to minimize the second term. □

7.3 Proof of Theorem 1

Proof. Using the expression of the vote shares provided by Proposition 1, the first-order condition for candidate A writes

$$\frac{\partial V_A}{\partial t_x^A} = \phi\Big[w'(t_x)\Delta_x - w'(2 - t_x)\Delta_y\Big] = 0$$

In the case where $\Delta_x \leq 0$, we have that $\frac{\partial V_A}{\partial t_x^A} < 0$, candidates A therefore selects $t_x^A = 0$ as a dominant action. A symmetric argument proves that candidates B therefore selects $t_x^B = 1$ so that $(\mathbf{t}^{A*}, \mathbf{t}^{B*}) = ((0, 1), (1, 0))$ is the unique equilibrium.

We now consider the case where $\Delta_x > 0$. Note that the vote share of candidate A is a strictly concave function of t_x^A: $\frac{\partial^2 V_A}{\partial (t_x^A)^2} = \phi\left(w''(t_x)\Delta_x + w''(2-t_x)\Delta_y\right) < 0$.

We define t_x^{A*} as the solution of the first order condition, that is the unique solution to the equation $\Delta_x w'(t_x^{A*} + t_x^B) = \Delta_y w'(2 - t_x^{A*} - t_x^B)$. Whether this solution belongs to the interval $(0,1)$ or not characterize the best response of candidate A against t_x^B, denoted $BR_A(t_x^B)$, which can be written as follows:

$$BR_A(t_x^B) = \begin{cases} 0 & \text{if } \Delta_x w'(t_x^B) - \Delta_y w'(2 - t_x^B) \leq 0, \\ 1 & \text{if } \Delta_x w'(1 + t_x^B) - \Delta_y w'(1 - t_x^B) \geq 0, \\ t_x^{A*} & \text{otherwise.} \end{cases}$$

For candidate B, we find that $\frac{\partial^2 V_B}{\partial (t_x^B)^2} = -\phi\left(w''(t_x)\Delta_x + w''(2-t_x)\Delta_y\right) > 0$ so that the vote share of candidate B is a strictly convex function of t_x^B. The best response of candidate B against t_x^A, that we denote $BR_B(t_x^A)$, can be written as follows:

$$BR_B(t_x^A) = \begin{cases} 0 & \text{if } \Delta_x[w(1+t_x^A) - w(t_x^A)] > \Delta_y[w(2-t_x^A) - w(1-t_x^A)], \\ 1 & \text{if } \Delta_x[w(1+t_x^A) - w(t_x^A)] < \Delta_y[w(2-t_x^A) - w(1-t_x^A)], \\ \{0,1\} & \text{if } \Delta_x[w(1+t_x^A) - w(t_x^A)] = \Delta_y[w(2-t_x^A) - w(1-t_x^A)]. \end{cases}$$

Combining these best-responses together, we find on the one hand that $((0,1),(0,1))$ and $((1,0),(1,0))$ are never a NE, as the equilibrium conditions violate the concavity of the function w. On the other hand, we find that $((0,1),(1,0))$ is a NE if and only if $\frac{\Delta_x}{\Delta_y} \leq \frac{w(2)-w(1)}{w(1)-w(0)}$ and that $((1,0),(0,1))$ is a NE if and only if $\frac{\Delta_x}{\Delta_y} \geq \frac{w(1)-w(0)}{w(2)-w(1)}$.

Finally,$((t_x^{A*}, 1-t_x^{A*}),(0,1))$ is a NE if and only if $\frac{\Delta_x}{\Delta_y} = \frac{w'(2-(t_x^{A*}))}{w'(t_x^{A*}}$ and $\frac{\Delta_x}{\Delta_y} \geq \frac{w(2-(t_x^{A*}))-w(1-(t_x^{A*}))}{w(1+(t_x^{A*}))-w(t_x^{A*})}$, which can not hold simultaneously due to the concavity of the function w. A similar argument proves that $((t_x^{A*}, 1-t_x^{A*}),(1,0))$ can not be a NE.

In conclusion, we have found a pure NE in the regions where $\frac{\Delta_x}{\Delta_y} \leq \frac{w(2)-w(1)}{w(1)-w(0)}$ or $\frac{\Delta_x}{\Delta_y} \geq \frac{w(1)-w(0)}{w(2)-w(1)}$ but no pure NE in the region where $\frac{\Delta_x}{\Delta_y} \in \left(\frac{w(2)-w(1)}{w(1)-w(0)} \frac{w(1)-w(0)}{w(2)-w(1)}\right)$.

We now investigate whether there exists a mixed NE. As candidate A's payoff is a strictly concave function of t_x^A, even against a mixed strategy of candidate B, his best response is uniquely defined and he can not play a mixed strategy at equilibrium. Therefore, only candidate B can play a mixed strategy. Due to the convexity of his payoff function, he mixes between playing $t^B = (1,0)$ and $t^B = (0,1)$.

Suppose that candidate B plays $t^B = (1,0)$ with probability π^* and $t^B = (0,1)$ with probability $1-\pi^*$. The indifference condition requires that candidate A plays $\hat{t}$ such that $V_B((\hat{t}, 1-\hat{t}),(0,1)) = V_B((\hat{t}, 1-\hat{t}),(1,0))$. This condition writes $\psi(\hat{t}) = \frac{\Delta_x}{\Delta_y}$ with $\psi(t) := \frac{w(2-t)-w(1-t)}{w(1+t)-w(t)}$. We find that ψ is a strictly increasing function, that $\psi(0) = \frac{w(2)-w(1)}{w(1)-w(0)}$ and $\psi(1) = \frac{w(1)-w(0)}{w(2)-w(1)}$. The equation $\psi(\hat{t}) = \frac{\Delta_x}{\Delta_y}$ admits a (unique) solution in $[0,1]$ if and only if $\frac{\Delta_x}{\Delta_y} \in \left[\frac{w(2)-w(1)}{w(1)-w(0)}, \frac{w(1)-w(0)}{w(2)-w(1)}\right]$.

We now check under which condition does $(\hat{t}, 1-\hat{t})$ maximizes candidate A's payoff against candidate B's mixed strategy to play $t^B = (1,0)$ with probability π^* and $t^B = (0,1)$ with probability $1-\pi^*$. It is the case if and only if $\hat{t} \in \arg\max_t \{\pi^* V_A((t,1-t),(1,0)) + (1-\pi^*)V_A((t,1-t),(0,1))\}$ and, following the first and second order conditions, if and only if

$$\varphi(\pi^*) := \pi^*\phi[w'(1+\hat{t})\Delta_x - w'(1-\hat{t})\Delta_y] + (1-\pi^*)\phi[w'(\hat{t})\Delta_x - w'(2-\hat{t})\Delta_y] = 0$$

Isolating π^* we thus obtain:

$$\pi^*(\hat{t}) := \frac{\frac{\Delta_x}{\Delta_y} w'(\hat{t}) - w'(2-\hat{t})}{\left(w'(1-\hat{t}) - w'(2-\hat{t})\right) + \frac{\Delta_x}{\Delta_y}\left(w'(\hat{t}) - w'(1+\hat{t})\right)}.$$

We are left to prove that $\pi^*(\hat{t}) \in (0,1)$. By definition, $\pi^*(\hat{t})$ is the unique solution to the linear equation $\varphi(\pi^*(\hat{t})) = 0$. We only need to prove that $\varphi(1) < 0 < \varphi(0)$. By definition, $\varphi(0) = \phi[w'(\hat{t})\Delta_x - w'(2-\hat{t})\Delta_y] > 0$ if and only if $\frac{w'(2-\hat{t})}{w'(\hat{t})} < \frac{\Delta_x}{\Delta_y}$, which can also be written $\frac{w'(2-\hat{t})}{w'(\hat{t})} < \frac{w(2-\hat{t})-w(1-\hat{t})}{w(1+\hat{t})-w(\hat{t})}$ by definition of $\hat{t}$. The previous inequality holds by concavity of w. A similar argument proves that $\varphi(1) < 0$ and therefore that $\pi^*(\hat{t}) \in (0,1)$.

Note that our model does not exclude functions w such that $\lim_{x\to 0} w'(0) = +\infty$. In this particular case, the previous argument only holds in the interior of the region $\mathcal{N}$, while we can show that $\pi^*(0) = 1$ and $\pi^*(1) = 0$ (see claim 1 below). □

7.4 Proof of Lemma 1

Proof. We show that $\hat{t}$ is increasing with Δ_x and decreasing with Δ_y.
Indeed, $\hat{t}$ is the unique solution to the equation: **(E)** $\frac{\Delta_x}{\Delta_y} = \frac{w(2-\hat{t})-w(1-\hat{t})}{w(1+\hat{t})-w(\hat{t})}$. If we denote $f(\hat{t}) = \frac{w(2-\hat{t})-w(1-\hat{t})}{w(1+\hat{t})-w(\hat{t})}$, we find that $f'(\hat{t}) = (-w'(2-\hat{t}) + w'(1-\hat{t}))(w(1+\hat{t}) - w(\hat{t})) - (w(2-\hat{t}) - w(1-\hat{t}))(w'(1+\hat{t}) - w'(\hat{t})) \times \frac{1}{(w(1+\hat{t})-w(\hat{t}))^2} > 0$. Therefore, an increase in the LHS of (E) (because either Δ_x increases or because Δ_y decreases) implies an increase of the RHS and

therefore a larger $\hat{t}$.

Because f in strictly increasing, we further have that $\hat{t} = 0$ (resp. $\hat{t} = 1$) is the unique solution to $\frac{w(2-\hat{t})-w(1-\hat{t})}{w(1+\hat{t})-w(\hat{t})} = \frac{w(2)-w(1)}{w(1)-w(0)}$ (resp. $\frac{w(1)-w(0)}{w(2)-w(1)}$) that characterizes $\hat{t}$.

On the other hand, $\hat{t} = \frac{1}{2}$ is the unique solution to the equation $w(2-\hat{t}) - w(1-\hat{t}) = w(1+\hat{t}) - w(\hat{t})$. □

7.5 Proof of Proposition 3

Proof. In this region, $t_x = t_y = 1$ so that the expected vote share of candidate A can be written as $V_A = \frac{1}{2} + \phi[w(t_x)\Delta_x + w(t_y)\Delta_y] = \frac{1}{2} + \phi w(1)(\Delta_x + \Delta_y)$. □

7.6 Proof of Proposition 4

Proof. First note that a straightforward computation shows that:

$$\frac{w(2-\hat{t}) - w(1-\hat{t})}{w(1+\hat{t}) - w(\hat{t})} = \frac{\Delta_x}{c - \Delta_x} \Leftrightarrow \Delta_x = c\frac{w(2-\hat{t}) - w(1-\hat{t})}{w(2-\hat{t}) - w(1-\hat{t}) + w(1\hat{+}t) - w(\hat{t})}$$

and similarly $\Delta_y = c - \Delta_x = c\dfrac{w(1+\hat{t}) - w(\hat{t})}{w(2-\hat{t}) - w(1-\hat{t}) + w(1+\hat{t}) - w(\hat{t})}$.

For $(\Delta_x, c - \Delta_x) \in \mathcal{N}$, we have $V_A^{\mathcal{N}} = \frac{1}{2} + \phi(w[\hat{t}]\Delta_x + w[2-\hat{t}](c - \Delta_x))$. Using the above computation, we find:

$$V_A^{\mathcal{N}} = \frac{1}{2} + \phi(w[\hat{t}]\Delta_x + w[2-\hat{t}](c - \Delta_x))$$

Using that $\Delta_x = c\frac{w(2-\hat{t})-w(1-\hat{t})}{w(2-\hat{t})-w(1-\hat{t})+w(1\hat{+}t)-w(\hat{t})}$ and that $c - \Delta_y = c\frac{w(1+\hat{t})-w(\hat{t})}{w(2-\hat{t})-w(1-\hat{t})+w(1+\hat{t})-w(\hat{t})}$ we obtain:

$$V_A^{\mathcal{N}} = \frac{1}{2} + \phi c\frac{w(2-\hat{t})w(1+\hat{t}) - w(\hat{t})w(1-\hat{t})}{w(2-\hat{t}) - w(1-\hat{t}) + w(1\hat{+}t) - w(\hat{t})}$$

□

7.7 Proof of Proposition 5

Proof. 1. We first consider the case where $\frac{\Delta_x}{\Delta_y} \notin \mathcal{N}$. In this case, we have that $V_A^{\mathcal{S}} = \frac{1}{2} + \phi[w(1)\Delta_x + w(1)\Delta_y]$ is strictly increasing with Δ_x and Δ_y.

We now consider the case where $\frac{\Delta_x}{\Delta_y} \in \mathcal{N}$. In this case $V_A^{\mathcal{N}} = \frac{1}{2} + \phi[\pi^*(w(\hat{t}+1)\Delta_x + w(1-\hat{t})\Delta_y) + (1-\pi^*)(w(\hat{t})\Delta_x + w(2-\hat{t})\Delta_y)]$. However, by definition of $\hat{t}$, candidate

B (and thus candidate A) is indifferent between candidate B playing 1 or 0 and the above expression simplifies to $V_A^{\mathcal{N}} = \frac{1}{2} + \phi[w(\hat{t})\Delta_x + w(2-\hat{t})\Delta_y]$.

We now prove that $V_A^{\mathcal{N}}$ strictly increases with Δ_x:

$$\begin{aligned}\frac{\partial V_A^{\mathcal{N}}}{\partial \Delta_x} &= \phi\left(w'(\hat{t}).\frac{\partial \hat{t}(\Delta_x,\Delta_y)}{\partial \Delta_x}.\Delta_x + w(\hat{t}) + w'(2-\hat{t}).(-1).\frac{\partial \hat{t}(\Delta_x,\Delta_y)}{\partial \Delta_x}.\Delta_y\right) \\ &> \phi\frac{\partial \hat{t}(\Delta_x,\Delta_y)}{\partial \Delta_x}\left[w'(\hat{t})\Delta_x - w'(2-\hat{t})\Delta_y\right]\end{aligned}$$

The lemma below guarantees that $\frac{\partial \hat{t}(\Delta_x,\Delta_y)}{\partial \Delta_x} > 0$, it is therefore sufficient to prove that $w'(\hat{t})\Delta_x > w'(2-\hat{t})\Delta_y$, which is equivalent to $\frac{\Delta_x}{\Delta_y} > \frac{w'(2-\hat{t})}{w'(\hat{t})}$. By definition of $\hat{t}$, we have $\frac{\Delta_x}{\Delta_y} = \frac{w(2-\hat{t})-w(1-\hat{t})}{w(1+\hat{t})-w(\hat{t})}$. By strict concavity of w, we indeed have that $w(2-\hat{t}) - w(1-\hat{t}) > w'(2-\hat{t})$ and that $w(1+\hat{t}) - w(\hat{t}) < w'(\hat{t})$. This proves that $V_A^{\mathcal{N}}$ is strictly increasing with Δ_x. A similar argument shows that it is also strictly increasing with Δ_y.

2. First, we have that $V_A^{\mathcal{S}} = w(1)\Delta_x + w(1)(c-\Delta_x)$. Also, by definition of $\hat{t}$, we can write: $V_A^{\mathcal{N}} = w(\hat{t})\Delta_x + w(2-\hat{t})(c-\Delta_x) = w(1+\hat{t})\Delta_x + w(1-\hat{t})(c-\Delta_x)$.

   We first prove that $V_A^{\mathcal{S}} \geq V_A^{\mathcal{N}}$ for $\Delta_x \geq \Delta_y \Leftrightarrow \Delta_x \geq c-\Delta_x$. We want to show that $w(1)\Delta_x + w(1)(c-\Delta_x) \geq w(\hat{t})\Delta_x + w(2-\hat{t})(c-\Delta_x)$. This can be written as $w(1)\frac{\Delta_x}{c-\Delta_x} + w(1) \geq w(\hat{t})\frac{\Delta_x}{c-\Delta_x} + w(2-\hat{t})$. Rearranging the terms, we have $\frac{\Delta_x}{c-\Delta_x} \geq \frac{w(2-\hat{t})-w(1)}{w(1)-w(\hat{t})}$. It follows from $\Delta_x \geq c-\Delta_x$ that the LHS is greater or equal to 1, and it follows from the concavity of $w(.)$ that the RHS is lower than 1. We can conclude that $V_A^{\mathcal{S}} \geq V_A^{\mathcal{N}}$ for $\Delta_x \geq c-\Delta_x$.

   We now prove that $V_A^{\mathcal{S}} \geq V_A^{\mathcal{N}}$ for $\Delta_x \leq \Delta_y \Leftrightarrow \Delta_x \leq c-\Delta_x$. We want to show that $w(1)\Delta_x + w(1)(c-\Delta_x) \geq w(1+\hat{t})\Delta_x + w(1-\hat{t})(c-\Delta_x)$. This can be written as $w(1)\frac{\Delta_x}{c-\Delta_x} + w(1) \geq w(1+\hat{t})\frac{\Delta_x}{c-\Delta_x} + w(1-\hat{t})$. Rearranging the terms, we have $\frac{\Delta_x}{c-\Delta_x} \leq \frac{w(1)-w(1-\hat{t})}{w(1+\hat{t})-w(1)}$. It follows from $\Delta_x \leq c-\Delta_x$ that the LHS is lower or equal to 1, and it follows from the concavity of $w(.)$ that the RHS is greater than 1. We can conclude that $V_A^{\mathcal{S}} \geq V_A^{\mathcal{N}}$ for $\Delta_x \leq c-\Delta_x$.

□

7.8 Proof of Proposition 6

Proof. If follows from Theorem 1 that, when $\frac{\Delta_x}{\Delta_y} \in \mathcal{S} \cup \mathcal{W}$, $t^j = (1,0)$ and $t^{-j} = (0,1)$ so that $t_x = t_y = 1$.
When $\frac{\Delta_x}{\Delta_y} \in \mathcal{S} \cup \mathcal{W}$, $\boldsymbol{t}^A = (\hat{t}, 1-\hat{t})$ and $\boldsymbol{t}^B = (1,0)$ with probability π^* and $(0,1)$ with probability $1-\pi^*$. □

7.9 Proof of Claim 1

Claim 1. $\pi^*(1) > 0 \Leftrightarrow \lim_{x\to 0} w'(x) = +\infty$.
In this case, we also have $\pi^*(0) < 1$.

Proof. We find that $\pi^*(1) = \frac{\frac{w(1)-w(0)}{w(2)-w(1)}w'(1)-w'(1)}{w'(0)-w'(1)+\frac{w(1)-w(0)}{w(2)-w(1)}(w'(1)-w'(2))}$. The numerator of this fraction is strictly positive. As long as w' has a finite derivative in $x = 0$, the denominator is finite and the fraction is strictly positive.
The second statement follows from the fact that $\pi^*(1-\hat{t}) = 1 - \pi^*(\hat{t})$. □

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