amse
école d'économie d'aix-marseille
aix-marseille school of economics

Working Papers / Documents de travail

Optimal electricity consumption and storage under short-term renewable supply variability

Martin Dhaussy
Nandeeta Neerunjun
Hubert Stahn

WP 2025 - Nr 25

Optimal electricity consumption and storage under short-term renewable supply variability*

Martin DHAUSSY[a,1], Nandeeta NEERUNJUN[b,2,*], Hubert STAHN[c,1]

[a] *Aix-Marseille Univ, CNRS, AMSE, Marseille, France*
[b] *Univ. Grenoble Alpes, CNRS, INRAE, Grenoble INP, GAEL, 38000 Grenoble, France*
[c] *Aix-Marseille Univ, CNRS, AMSE, Marseille, France*

Abstract

The expansion of intermittent electricity increases supply variability and requires greater flexibility from consumers. This results in welfare losses for these agents, which can nevertheless be mitigated by energy storage. Our model analyzes these welfare consequences in the context of short-term variability in renewable energy given fixed dispatchable and storage capacities. We explore an optimal control problem that determines a welfare-maximizing electricity consumption path by adjusting dispatchable and stored energy throughout the short-term production cycle of renewables. This optimization problem identifies three regimes (no storage and active storage, with or without capacity constraints) and provides the associated consumer welfare over this cycle. Under all three regimes, a certain degree of consumer flexibility is part of the optimal solution and entails welfare losses. Active storage reduces these losses but cannot eliminate them completely due to the energy conversion losses induced by this activity. However, when storage capacity is constrained, a proactive adjustment of this capacity can offset the losses.

Keywords: intermittent renewable, energy storage, electricity consumption, welfare analysis, optimal control

JEL classification: D61, Q40, Q42

*This project has received funding from the French government under the France 2030 investment plan managed by the French National Research Agency (reference: ANR-17-EURE-0020) and from Excellence Initiative of Aix-Marseille University - A*MIDEX.

*Corresponding author

Email addresses: martin.dhaussy@univ-amu.fr (Martin DHAUSSY), nandeeta.neerunjun-demaiziere@univ-grenoble-alpes.fr (Nandeeta NEERUNJUN), hubert.stahn@univ-amu.fr (Hubert STAHN)

[1] P-mail address: AMSE, La MEGA, 424 Chemin du Viaduc, 13080 Aix-en-Provence, France

[2] P-mail address: GAEL - CS 40700 - 38058 Grenoble CEDEX 9, France

1. Introduction

The ambition of achieving net-zero greenhouse gas emissions by 2050, as pledged by the European Union and other global actors, has accelerated the deployment of renewable energy sources (IEA 2024). Yet, the intermittency of these sources, particularly in power generation, leads to variability in energy supply. Such imbalances between supply and demand are often managed through flexible generation, typically provided by dispatchable fossil-fueled plants (IEA 2020), though this reliance directly conflicts with decarbonization goals. Another way to maintain electricity balance is on the demand side through dynamic pricing mechanisms such as real-time pricing, which encourage consumers to adjust their consumption in response to supply variability (IRENA 2019). However, this raises a fundamental question: Should the burden of adjusting to supply variability rest primarily on consumers?

Alongside demand-side adjustments, energy storage can play a key role in smoothing the variability of renewable generation, thereby reducing the pressure on consumers to respond to such fluctuations (Jiang et al. 2014, Denholm 2015, Lund et al. 2015, Maghami et al. 2024). This role is particularly relevant in a context where consumers are increasingly expected to adjust their consumption patterns instantaneously to short-term variations in supply (Cabot and Villavicencio 2024). Building on this, the present work starts from the premise that storage of surplus renewable energy can act as an alternative to dispatchable generation and limit the need for short-term demand-side adjustments. Hence, our analysis adopts a short-term perspective, providing a framework to analyze flexibility and efficiency in balancing supply and demand under renewable variability.

Within this short-term focus, we examine a single production cycle driven by intermittent renewables. We therefore abstract from long-term energy transition issues, such as investment and capacity expansion decisions aimed at overcoming renewable intermittency, as explored by Ambec and Crampes (2019), Helm and Mier (2021), and Pommeret and Schubert (2022). Instead, we conduct our analysis within a fixed-capacity framework to study how storage and demand-side adjustments jointly mitigate supply variability at each instant of the cycle. In this context, flexibility is understood as the coordination of consumption with electricity generation and storage facilities, rather than shifts in demand in response to price signals.[1] As in Pommeret and Schubert (2022), we adopt the perspective of a social planner but rather take a short-term view and study the optimal allocation of electricity for consumption under fixed generation and storage capacities. This approach allows us to value storage in terms of consumer welfare rather than solely for its role in cost-minimizing generation, which has been the primary focus of the broader literature (e.g., Crampes and Moreaux 2010, Carson and Novan 2013, Steffen and Weber 2013, Sioshansi 2014, Zerrahn et al. 2018).

Our main objective is to study how different sources of electricity are valued by con-

[1] For an overview of the literature on demand-side response to price signals, see, for instance, Borenstein 2005, Borenstein and Holland, 2005, Joskow and Wolfram 2012, Puller and West 2013, Ambec and Crampes 2021, and Schittekatte et al. 2024.

sumers and to highlight the welfare effects attributable to energy storage. In particular, we find that substituting dispatchable energy with renewables–while maintaining the total energy supply unchanged–reduces consumer welfare. This reduction arises because the variability of renewable generation translates into greater fluctuations in the optimal consumption path, which consumers must accommodate. However, active storage mitigates this effect by allowing surplus energy to be stored and dispatched when needed, thereby smoothing consumption and reducing welfare losses. Furthermore, when storage capacity is binding, a moderate expansion can fully offset the negative welfare effect, underscoring its role in supporting the integration of intermittent renewables from a consumer welfare perspective.

We formalize these results through the resolution of a theoretical model in which the social planner faces a constrained intertemporal allocation problem: how to distribute limited energy resources–dispatchable generation, intermittent renewable production, and stored energy subject to conversion losses–to maximize consumer utility over a single production cycle. This cycle represents a repetitive pattern of renewable generation with a peak around which output fluctuates, reflecting the intermittency of resources such as wind or sunlight (see Helm and Mier 2021). Formulated as a standard optimal control problem, the model determines the optimal electricity consumption path under three regimes: no storage, non-binding storage and binding storage capacities.

The no storage case occurs because the potential welfare gain from transferring energy from peak production is outweighed by conversion losses. Consumer demand is met by combining intermittent renewable generation with dispatchable energy. At each instant, consumption follows renewable output when it is high and is supplemented by dispatchable energy when renewable production falls short. This regime first illustrates how dispatchable generation, together with demand-side adjustment, can accommodate renewable variability. Second, it provides a baseline for assessing the welfare effects of adding storage, particularly since a higher share of renewable energy substituting dispatchable generation reduces consumer welfare.

In the second regime, storage is abundant. Unlike the previous case, where all consumption is directly supplied by renewable and dispatchable energy, here the planner can shift energy from periods of high renewable production to periods of scarcity without hitting the storage limit. This results in a consumption path with two distinct levels: a lower level when renewable output is insufficient and is supplemented by stored and dispatchable energy, and a higher level when renewable production is abundant and energy is being stored. This regime demonstrates that storage can partially offset the variability of renewable energy and reduce welfare losses as the share of renewables replacing dispatchable generation increases.

Finally, we consider the case when storage is used but reaches its capacity limit. In this case, the social planner would like to shift energy from periods of high renewable output to periods of low output, but the storage constraint prevents a full transfer. As a result, the levels of consumption during low and high demand periods are determined not only by renewable and dispatchable supply but also by the storage limit. The lower consumption level falls below the threshold observed when storage is unconstrained, while the higher level reflects the maximum energy that can be stored. This regime shows how binding storage capacity restricts the planner's ability to smooth consumption over time, with the optimal consumption path and storage strategy jointly shaped by dispatchable energy, renewable capacity, and storage size. Furthermore, it shows that a moderate expansion of storage capacity can fully offset the welfare losses associated with substituting dispatchable generation with renewables.

Taken together, the three regimes clarify that while consumers inevitably bear part of the adjustment burden from renewable variability, this burden can be alleviated through storage. In addition, each regime is associated with a distinct level of consumer welfare, allowing us to directly assess how variations in energy mixes, including storage capacities, shape consumption patterns and overall consumer welfare.

The rest of the paper is organized as follows. In Section 2, we present the main features of the model followed by the definition of the optimal electricity consumption path in Section 3. We then characterize this path under three regimes: no storage (Section 4), non-binding storage (Section 5), and binding storage (Section 6). A welfare analysis across different energy mixes is provided in Section 7 and Section 8 discuss some broader implications of our results. The paper concludes in Section 9. All proofs are relegated to the appendices.

2. The model

We consider a social planner who is concerned with the optimal allocation of electricity for consumption in an economy endowed with three technologies: dispatchable power plants, renewable generators and energy storage systems.

The consumption side is represented by a consumer[2] who derives utility from electricity consumed over a period of time which we normalize to the interval $[0, 1]$. This normalization aligns with the temporal cycle of the renewable energy production, as explained later in this section. Our focus is therefore on analyzing the consumer's *electricity consumption path* over $[0, 1]$. We assume that the utility, U, obtained from an entire consumption path, $x(\cdot)$, is the integral over the time interval $t \in [0, 1]$ of the instantaneous utility, $u(x(t))$, where $x(t)$ is the instantaneous electricity consumption. Given the short duration of the cycle, intertemporal discounting is considered negligible, and the utility function is specified as:

$$U(x(\cdot)) = \int_0^1 u(x(t))\, dt \tag{1}$$

We model $u(x)$ as an increasing and concave function that satisfies the Inada conditions: $u'(x) > 0$, $\lim_{x\to 0} u'(x) = \infty$, $\lim_{x\to\infty} u'(x) = 0$ and $u''(x) < 0$.

[2]This representative agent assumption is formulated for explanatory simplicity. Section 8.3 shows that, within our specific problem, this framework is equivalent to a setting involving several heterogeneous consumers.

As we do not impose any functional form on consumption dynamics, the consumer is, in principle, free to vary her consumption along its path. However, it can be observed that the consumer prefers a constant path. Since u is concave, Jensen's inequality implies that a constant consumption path equal to the mean, x^{cst}, of any fluctuating consumption path $x(\cdot)$ yields at least as much utility as the fluctuating path itself.

$$U\left(x^{cst}\right) \geq U\left(x\left(\cdot\right)\right), \text{ where } x^{cst} = \int_0^1 x\left(t\right) dt \tag{2}$$

Nevertheless, due to fluctuations in renewable production, it may not always be possible for the consumer to achieve a perfectly constant stream of consumption. In such cases, it is useful to consider the intertemporal marginal rate of substitution (MRS) between any two periods t and t'. In our model, the intertemporal MRS does not depend on the entire consumption path but only on the consumption levels at times t and t':

$$MRS_{x(t)/x(t')} = \frac{u'\left(x\left(t'\right)\right)}{u'\left(x\left(t\right)\right)}, \quad \forall t, t' \in [0, 1] \tag{3}$$

We hence use the MRS in our analysis to characterize how the consumer is willing to substitute consumption intertemporally in response to fluctuations.

To ensure the provision of electricity required for consumption, the social planner considers three technologies, each with capacities fixed at an exogenous level.

The first is a *renewable technology* with installed capacity R. The volume of electricity it produces varies over time due to uncontrollable natural factors such as wind speed or solar radiation. To model this variability, we define a time-varying unit productivity function, $\epsilon\left(t\right)$, which represents the proportion of a unit of installed capacity utilized for electricity production at each instant. We assume that this renewable productivity exhibits a peak, ϵ_M, around which it varies in a repetitive cycle over time. To simplify the analysis, we normalize the minimum of $\epsilon\left(t\right)$ to zero so that $\epsilon\left(t\right) \in [0, \epsilon_M]$. We also normalize the duration of the cycle to 1. This also explains the earlier alignment of the temporal cycle of the consumer's electricity consumption to $[0, 1]$. More formally, if $T_M \in [0, 1]$ denotes the time at which renewable production reaches its peak, we have $\epsilon\left(0\right) = \epsilon\left(1\right) = 0$ and $\forall t \in (0, T_M)$, $\epsilon'\left(t\right) > 0$ while for $\forall t \in (T_M, 1)$, $\epsilon'\left(t\right) < 0$. The instantaneous volume of electricity production from this technology at any time t is therefore $\epsilon\left(t\right) R$ so that the total volume of renewable production over the time period $[0, 1]$ is given by $R\int_0^1 \epsilon\left(t\right)\, dt$.

The second technology is a *dispatchable technology*, where dispatchability refers to the ability to control the energy resource, allowing electricity production to be adjusted freely at any instant.[3] Given that the interval $[0, 1]$ is a normalized representation of a

[3]This is characteristic of conventional power plants such as nuclear, coal, and gas, where fuel input can be controlled to adjust electricity output.

short-term cycle in our analysis and that the installed capacity of the technology is fixed, we assume it generates a total volume D of electricity over the period. So, while this total volume can be allocated flexibly over time, the total quantity dispatched across the interval must be equal to D. Letting $d(t)$ denote the instantaneous dispatch at time t, this constraint is formalized as:

$$\int_0^1 d(t)\ dt = D \tag{4}$$

We also have an *energy storage technology* of an installed capacity S that does not produce electricity but rather stores it for later use. We denote by $s^+(t)$ the volume of energy supplied to the storage system at time t and $s^-(t)$ the volume withdrawn from it. The storage process is subject to conversion inefficiencies: it takes $\sigma^+ > 1$ units of electricity to store one unit of energy and only a fraction $\sigma^- \in (0,1)$ of each unit withdrawn from storage is effectively usable for consumption. Accordingly, the overall round-trip efficiency is given by $\frac{\sigma^-}{\sigma^+} < 1$, capturing the total energy losses incurred in the storage and withdrawal process. To ensure temporal feasibility over the cycle, we assume that the storage system is empty at both the beginning and end of the period. This implies that the net accumulation of stored energy over the interval is zero:

$$\int_0^1 \left(s^+(t) - s^-(t)\right) dt = 0 \tag{5}$$

Moreover, the planner is constrained by the system's capacity, which limits the total volume of electricity that can be allocated to storage over the period:

$$\int_0^1 s^+(t)\, dt \leq S \tag{6}$$

Also, storage decisions are not tied to strict chronological ordering. It may be optimal, for example, to withdraw energy early in the cycle—before any has technically been stored—if this helps achieve a smoother consumption path. This apparent paradox reflects the stationarity of the model where time represents position within a cycle rather than absolute sequence, and all decisions are made consistently within this closed temporal structure.

Finally, we denote by $e = (D, R, S) \in \mathcal{E}$ the energy mix which represents the short-term exogenous capacity composition of the supply side. Since our analysis focuses on the short-term consumption adjustments induced by fluctuations in renewable production, we introduce two additional assumptions on this set of parameters. These assumptions ensure that neither the storage capacity nor the available dispatchable energy is sufficient to fully cover the fluctuations in electricity production.

The first condition simply says that the maximum storage capacity, including conversion losses, is bounded above by the total renewable energy produced during the cycle and available for storage. This leads to the following constraint:

$$\sigma^+ S \leq R \int_0^1 \varepsilon(t)\, dt \tag{7}$$

The second condition rules out cases in which dispatchable energy is sufficiently abundant to offset fluctuations caused by renewable production. Since the consumer is averse to fluctuations in electricity consumption (see Eq.(2)), the planner must, in such cases, be able to deliver a constant flow of consumption c throughout the entire period. Moreover, this constant flow must be greater than the maximum renewable production, i.e., $c > \varepsilon_M R$. However, feasibility over the whole cycle requires that the total quantity of dispatchable energy covers this constant consumption path net of renewable production so that: $D = c - R\int_0^1 \varepsilon(t)\, dt$[4]. Since $c > \varepsilon_M R$, this case is excluded if we assume:

$$D \leq R \int_0^1 (\varepsilon_M - \varepsilon(t))\, dt \tag{8}$$

3. The optimal electricity consumption path

The social planner faces a constrained intertemporal allocation problem: how to distribute limited energy resources—dispatchable production, intermittent renewable output, and imperfectly efficient storage—so as to maximize the representative consumer's utility from electricity consumption over the cycle $[0, 1]$. This optimal consumption path solves:

$$W(e) = \max_{(d(t), s^+(t), s^-(t))} \int_0^1 u(\underbrace{d(t) - \sigma^+ s^+(t) + \sigma^- s^-(t) + \varepsilon(t)R}_{x(t)})dt \tag{9}$$

subject to the 3 iso-perimetric constraints given by Eqs.(4),(5) and (6). Moreover, each of these constraints can be replaced by a differential equation on $[0, 1]$ with terminal state conditions. Hence, for Eqs.(4),(5) and (6), we respectively obtain:

$$\begin{cases} \int_0^1 d(t)dt = D \\ \int_0^1 (s^+(t) - s^-(t))\, dt = 0 \\ \int_0^1 s^+(t)dt \leq S \end{cases} \Leftrightarrow \begin{cases} \dot{D}(t) = -d(t),\ D(0) = D,\ D(1) = 0 \\ \dot{\Sigma}(t) = s^+(t) - s^-(t),\ \Sigma(0) = \Sigma(1) = 0 \\ \dot{S}(t) = -s^+(t),\ S(0) = S,\ S(1) \geq 0 \end{cases} \tag{10}$$

This leads to a standard optimal control problem with 3 state variables. If we denote by $\lambda_D(t)$, $\lambda_\Sigma(t)$ and $\lambda_S(t)$ the associated co-states, the Hamiltonian writes:

$$\mathcal{H} = u(x(t)) - \lambda_D(t)d(t) + \lambda_\Sigma(t)\left(s^+(t) - s^-(t)\right) - \lambda_S s^+(t) \tag{11}$$

Moreover, if we denote respectively by $\mu_d(t)$, $\mu_{s^+}(t)$ and $\mu_{s^-}(t)$ the multipliers associated to the non-negativity conditions on $d(t), s^+(t)$ and $s^-(t)$, the Lagrangian associated to this Hamiltonian becomes:[5]

[4]Recall that we work on $[0, 1]$, so that $\int_0^1 c\, dt = c$.

[5]Since we have assumed that $\lim_{x\to 0} u'(x) = +\infty$, we do not consider the constraint $x \geq 0$. It can easily be shown that this one is always satisfied.

$$\mathcal{L} = \mathcal{H} + \mu_d(t)d(t) + \mu_{s^+}(t)s^+(t) + \mu_{s^-}(t)s^-(t) \tag{12}$$

Let us now turn to the optimality conditions starting with the dynamics of the co-states. As is usual with isoperimetrical constraints, the co-states are constant and using the transversality conditions, we have:

$$\dot{\lambda}_D(t) = -\frac{\partial \mathcal{L}}{\partial D} = 0,\ \lambda_D(1) \text{ free} \tag{13}$$
$$\dot{\lambda}_\Sigma(t) = -\frac{\partial \mathcal{L}}{\partial \Sigma} = 0,\ \lambda_\Sigma(1) \text{ free} \tag{14}$$
$$\dot{\lambda}_S(t) = -\frac{\partial \mathcal{L}}{\partial S} = 0,\ \lambda_S(1)S(1) = 0,\ \lambda_S(1), S(1) \geq 0 \tag{15}$$

Hereafter, we consider the co-states as constant. It follows that the control variables are optimal if we have:

$$\frac{\partial \mathcal{L}}{\partial d} = u'(x(t)) - \lambda_D + \mu_d(t) = 0 \tag{16}$$
$$\frac{\partial \mathcal{L}}{\partial s^+} = -\sigma^+ u'(x(t)) + \lambda_\Sigma - \lambda_S + \mu_{s^+}(t) = 0 \tag{17}$$
$$\frac{\partial \mathcal{L}}{\partial s^-} = \sigma^- u'(x(t)) - \lambda_\Sigma + \mu_{s^-}(t) = 0 \tag{18}$$

and if we satisfy the associated slackness conditions:

$$\mu_d(t)d(t) = 0,\ \mu_d(t) \geq 0,\ d(t) \geq 0 \tag{19}$$
$$\mu_{s^+}(t)s^+(t) = 0,\ \mu_{s^+}(t) \geq 0,\ s^+(t) \geq 0 \tag{20}$$
$$\mu_{s^-}(t)s^-(t) = 0,\ \mu_{s^-}(t) \geq 0,\ s^-(t) \geq 0 \tag{21}$$

Moreover, with regard to this optimization problem, we can immediately observe that all the constraints are linear. So, if we can show that the Hamiltonian is concave with respect to the control and the stock variables, we obtain the following Mangasarian-type of sufficient conditions:

Proposition 1. *If, for a continuous and piecewise differentiable control path given by $(d(t), s^+(t), s^-(t))$, there exists constant co-states $(\lambda_D, \lambda_\Sigma, \lambda_S)$, and continuous and piecewise differentiable multipliers $(\mu_d(t), \mu_{s^+}(t), \mu_{s^-}(t))$ that satisfy Eqs.(10), (15) and (16) to (21), then $(d(t), s^+(t), s^-(t))$ is a solution to the previous problem.*

Looking at the result of the optimization problem, we can already observe interesting specifications, namely that consumption can be constant on some time intervals and that some composition of controls are impossible.

First, suppose that on some interval (t, t') dispatchable energy is used, i.e. $d(t) > 0$. Then, by the slackness condition (19), we have $\mu_d(t) = 0$, which implies that consumption $x(t)$ is constant on this interval, since $x(t) = u^{-1}(\lambda_D)$ from Eq.(16). Similarly, if on another interval (t, t') storage is used, then the slackness condition implies $\mu_s^+(t) = 0$ or $\mu_s^-(t) = 0$, so that consumption is again constant, as the co-state in Eqs.(17) and (18) is constant.

We can also obtain further restrictions: if storage and discharge occur simultaneously, one can always improve the allocation by setting the smallest of them to zero and adjusting the other so that the net storage is preserved. This reduces the conversion losses and therefore increases consumption along the cycle. Finally, if production comes from dispatchable generation, then storing part of it ($s^+ > 0$) would transform dispatchable energy $d(t)$ into stored electricity recovered at efficiency $\sigma^-/\sigma^+ < 1$, which entails a net loss and is therefore dominated.

The next Lemma summarizes these observations

Lemma 1. *On any interval (t, t'), the following properties hold:*
(i) If dispatchable production is used, then $\mu_d(t) = 0$ and consumption is constant, with : $x(t) = \bar{c}$.
(ii) If storage is used, then $\mu_s^+(t) = 0$ or $\mu_s^-(t) = 0$, and consumption is again constant.
(iii) Storage and discharge cannot occur simultaneously: $s^+(t)\, s^-(t) = 0$.
(iv) Dispatchable production and storage cannot occur simultaneously: $s^+(t)\, d(t) = 0$.

4. Conversion losses vs. storage

This section focuses on scenarios without energy storage due to conversion losses and low fluctuations in renewable energy production. The main objective is to define the conditions under which this case occurs, and conversely, under which storage is part of the problem of optimal electricity allocation.

We have seen in the model section that there are some energy mix e that we discard. Notably the one where storage is not needed because dispatchable is relatively larger than the renewable allowing to have the consumption stream c^a above the peak of renewable generation $\varepsilon_M R$ as well as no fluctuation produced by the renewable on the consumption stream. But this raises a second question: if a feasible constant consumption path, $c^a < \varepsilon_M R$, does not reach the peak of renewable generation, does storage necessarily take place? This is where the conversion losses associated with storage come into play. In fact, a unit of electricity stored, for example, during a production peak, only returns a potential electricity consumption of $\frac{\sigma^-}{\sigma^+} < 1$ when it is withdrawn. This prompts the question whether it is in the interests of consumers to make a transfer of electricity from the peak of renewable generation (without dispatchable), $\varepsilon_M R$, to a lower constant consumption path, $c^a < \varepsilon_M R$, supported by the use of dispatchable energy. Recall now that the marginal rate of substitution $MRS_{c^a/\epsilon_M R} = \frac{u'(\varepsilon_M R)}{u'(c^a)}$ measures the increase in consumption at c^a that keeps the utility constant after a one-unit decrease in consumption at the peak production, $\varepsilon_M R$. So, if the quantity of electricity $\left(\frac{\sigma^-}{\sigma^+}\right)$ that can actually be transferred due to conversion losses is less than this $MRS_{c^a/\epsilon_M R}$, there is no storage. Let us now introduce the level of electricity consumption $\bar{x} < \varepsilon_M R$ given by:

$$MRS_{\bar{x}/\epsilon_M R} = \frac{u'(\varepsilon_M R)}{u'(\bar{x})} = \frac{\sigma^-}{\sigma^+} \Leftrightarrow \bar{x} = (u')^{-1}\left(\frac{\sigma^+}{\sigma^-} u'(\varepsilon_M R)\right) \tag{22}$$

Since $MRS_{c/\epsilon_M R}$ increases in c, $\bar{x}$ can be viewed as the highest consumption plateau at which storage occurs. Conversely, if the optimal consumption path $x(t)$ solving program (16) satisfies $x(t) \geq \bar{x}$, no storage occurs.

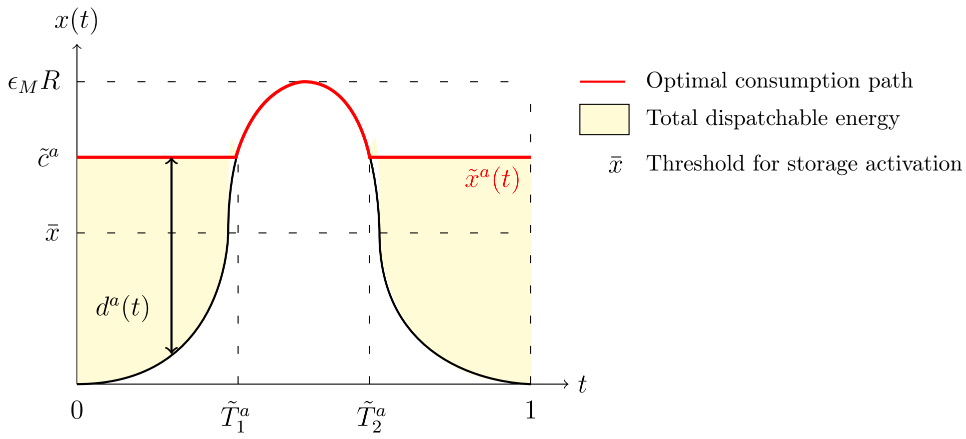

Figure 1: Consumption without storage

Two open questions remain: (1) what is the optimal consumption path for this case, and (2) under which condition it satisfies $x^a(t) \geq \bar{x}$? As long as the dispatchable technology is solicited, we know from Lemma 1(i) that the consumption path is constant at c^a and we can conjecture that without the dispatchable, the renewable peak is absorbed (see Fig. 1). This one is given by:

$$\forall t \in [0,1], \quad x(c^a, t) = \max\{c^a, \varepsilon(t)R\} \tag{23}$$

In other words, the use of dispatchable energy adjusts the renewable production to this constant path as long as this amount is higher than the energy produced by the renewable, i.e.,

$$\forall t \in [0,1], \quad d(c^a, t) = \max\{c^a - \varepsilon(t)R, 0\} \tag{24}$$

This consumption plateau c^a in nevertheless an endogenous variable since the total dispatchable energy used at each $t \in [0,1]$ must be equal to the amount, D, of dispatchable energy dedicated to the cycle (see the yellow area in Fig. 1). In other words, c^a satisfies:

$$\int_0^1 d(c^a, t)\, dt = \int_0^1 \max\{c^a - \varepsilon(t)R, 0\}\, dt = D \tag{25}$$

Now, recall that no storage occurs if $\forall t \in [0,1]$, $x(t) \geq \bar{x}$. This therefore simply means, by Eq.(23) that the solution $\tilde{c}^a$ to Eq.(25) is larger than $\bar{x}$. As $\int_0^1 \max\{c^a - \varepsilon(t)R, 0\}\, dt$ increases in c^a, a necessary (and sufficient) condition which ensures that no storage occurs, is given by:

$$\int_0^1 \max\{\bar{x} - \varepsilon(t)R, 0\}\, dt \leq D \tag{26}$$

To sum up this discussion, we can say:

Proposition 2. *If* $D \geq \int_0^1 \max\{\bar{x} - \varepsilon(t)R, 0\}\, dt$,
(i) There exists a unique solution, $\tilde{c}^a$*, to Eq.(25) with the property that* $\tilde{c}^a \geq \bar{x}$*.*
(ii) The optimal electricity consumption path which solves program (57) is given by $\forall t \in [0,1]$*,* $\tilde{x}^a(t) = \max\{\tilde{c}^a, \varepsilon(t)R\}$*.*
(iii) The path $\tilde{x}(t)$ *requires no storage,* $\forall t \in [0,1]$*,* $(\tilde{s}^-)^a(t) = (\tilde{s}^+)^a(t) = 0$*, but a use of dispatchable energy given by* $\forall t \in [0,1]$*,* $\tilde{d}^a(t) = \max\{\tilde{c}^a - \varepsilon(t)R, 0\}$*.*

To prove this proposition (see Appendix B), we first show that there exists a unique consumption plateau $\tilde{c}^a \geq \bar{x}$ solution to Eq.(25). We then verify that the induced paths, $\tilde{x}^a(t)$ and $\tilde{d}^a(t)$, verify the sufficient optimality conditions provided by Proposition 1.

According to (i) of Proposition 3, the plateau $\tilde{c}^a(D,R)$ that solves Eq.(25) is also a function of the amount of dispatchable, D, and renewable, R, energy available during the period $[0,1]$. Therefore, it is interesting to know the effect of these parameters on this plateau. The direction of these effects is fairly obvious: an increase in dispatchable, D or renewable R increases the consumption plateau, $\tilde{c}^a$. A detailed analysis however prepares the section 7, in which we study how the substitution of dispatchable energy by renewable energy affects the consumer's welfare.

To conduct this analysis, let us define the times at which consumption switches from one regime to another as illustrated in Fig. 1. Using the properties of $\varepsilon(t)$, we can define $\tilde{T}_i^a(D,R) = \varepsilon^{-1}\left(\frac{\tilde{c}^a(D,R)}{R}\right)$ as the two solutions of $\varepsilon(t)R = \tilde{c}^a(D,R)$ in t, respectively, for $i = 1,2$, $\varepsilon'(\tilde{T}_1^a(D,R)) > 0$ and $\varepsilon'(\tilde{T}_2^a(D,R)) < 0$. With these additional notations, Eq.(25) is now written:

$$\int_0^{\tilde{T}_1^a(D,R)} (\tilde{c}^a(D,R) - \varepsilon(t)R)\, dt + \int_{\tilde{T}_2^a(D,R)}^1 (\tilde{c}^a(D,R) - \varepsilon(t)R)\, dt - D = 0 \qquad (27)$$

and becomes a differentiable identity in (D,R). A simple calculation (see Appendix F) yields the results presented in Table 1. As expected, both derivatives are positive. However, it should be noted that the magnitudes of these two effects are not the same. The increase in dispatchable energy is spread over the intervals $[0, T_1^a]$ and $[T_2^a, 1]$, which explains their presence in the derivatives. Secondly, an increase in renewable capacity increases the global energy production and redistributes dispatchable energy on the segment where it is used.

	∂D	∂R
$\partial \tilde{c}^a$	$\frac{1}{1+\tilde{T}_1^a(D,R)-\tilde{T}_2^a(D,R)} > 0$	$\frac{\int_0^{\tilde{T}_1^a(D,R)} \varepsilon(t)dt + \int_{\tilde{T}_2^a(D,R)}^1 \varepsilon(t)dt}{1+\tilde{T}_1^a(D,R)-\tilde{T}_2^a(D,R)} > 0$

Table 1: Comparative statics without storage

5. Optimal consumption with abundant storage

Our previous discussion delineates the case where no storage occurs which is given by condition (26). If the contrary holds, i.e.,

$$\int_0^1 \max\{\bar{x} - \varepsilon(t)R, 0\}\, dt > D \text{ with } \bar{x} \text{ given by } \bar{x} = (u')^{-1}\left(\frac{\sigma^+}{\sigma^-} u'(\varepsilon_M R)\right) \tag{28}$$

we can expect the storage occurs. But what would the energy consumption path be in this case, especially when storage is abundant?

From Lemma 1, we know that the use of dispatchable energy and/or stored energy cannot be simultaneous with storage. In addition, after the available stock is down-scaled by the conversion losses, the leftover energy can be viewed as an additional stock of dispatchable energy. This suggests that both stored and dispatchable energies are simultaneously used when renewable energy production is low, while storage takes place during the peak production. Lemma 1 also suggests a time invariant consumption when dispatchable energy is used or when energy is stored. This means that there should be now two endogenous consumption plateaus $\tilde{c}_1^b$ and $\tilde{c}_2^b$. The first occurs when renewable energy production is low and mainly requires stored and dispatchable energy, while the second occurs when renewable energy production is high and is achieved through storage (see Fig.2).

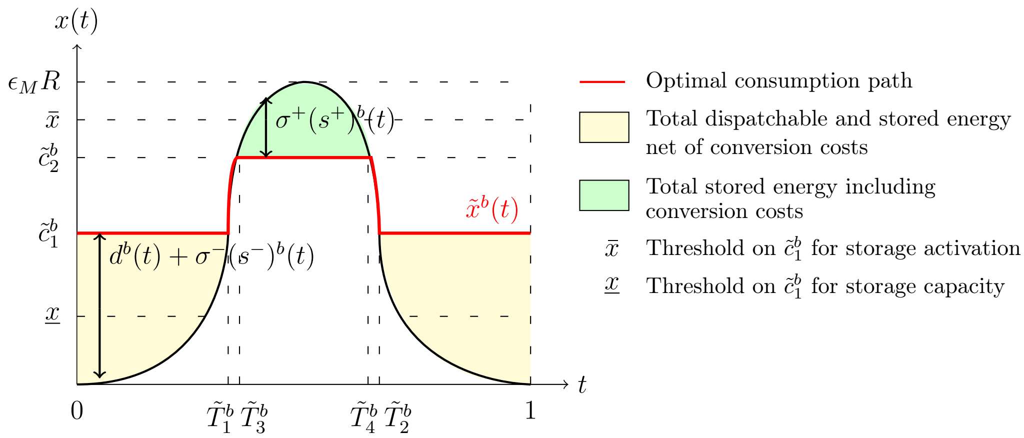

Figure 2: Consumption with abundant storage

More formally, this largely suggests (see Fig.2) that the optimal consumption path looks like:

$$\forall t \in [0,1]\,,\; x\left(c_1^b, c_2^b, t\right) = \min\left\{\max\{c_1^b, \varepsilon(t)R\}, c_2^b\right\} \text{ with } c_1^b < c_2^b \tag{29}$$

with of course

- a supplement to renewable generation when this one is low, i.e. $\varepsilon(t)R < c_1^b$, by stored and dispatchable energy given by:

$$\forall t \in [0,1]\,,\; \left(d(c_1^b, t) + \sigma^- s^-(c_1^b, t)\right) = \max\left\{c_1^b - \varepsilon(t)R, 0\right\} \tag{30}$$

- storage of renewable energy when this production is high, i.e. $\varepsilon(t)R > c_2^b$, so that the storage path is given by :

$$\forall t \in [0,1]\,,\;\; s^+(c_2^b, t) = \tfrac{1}{\sigma^+} \max\left\{\varepsilon(t)R - c_2^b, 0\right\} \tag{31}$$

- a continuous adjustment between the two levels, where only renewable energy is consumed.

The question that remains open is the identification of the two endogenous consumption plateaus c_1^b and c_2^b.

Let us first argue that there exists a relationship between c_1^b and c_2^b when the storage capacity is not binding. Since energy is transferred from the consumption plateau of c_2^b to that of c_1^b without any capacity constraint, the social planner now take care to the fact that the marginal rate of substitution between the two consumption levels is equal to the conversion losses. This means that:

$$MRS_{c_1^b/c_2^b} = \frac{u'\left(c_2^b\right)}{u'\left(c_1^b\right)} = \tfrac{\sigma^-}{\sigma^+} \tag{32}$$

Otherwise, the planner has an incentive to adjust these two consumptions levels and to improve the situation of the consumer by either increasing or decreasing storage. We can therefore say that the following relation between c_1^b and c_2^b must hold:

$$c_2^b = f(c_1^b) = (u')^{-1}\left(\tfrac{\sigma_-}{\sigma_+} u'(c_1^b)\right) \tag{33}$$

We can even note that this function is increasing since:

$$f'(c_1^b) = \tfrac{\sigma_-}{\sigma_+}\left(u''\left((u')^{-1}\left(\tfrac{\sigma_-}{\sigma_+} u'(c_1^b)\right)\right)\right)^{-1} u''(c_1^b) > 0 \tag{34}$$

The existence of this relation has also several consequences. Without surprise, we can claim that $c_2^b > c_1^b$ since $MRS_{c_1^b/c_2^b}$ is decreasing in c_2^b and $MRS_{c_1^b/c_1^b} = 1 > \tfrac{\sigma^-}{\sigma^+}$. This confirms the idea that when storage occurs, the consumer increases her consumption to partially limit the conversion losses. Moreover, consistency requires that the consumption level, c_2^b, at which storage occurs is lower than the maximum production of renewable energy, i.e., $c_2^b < \varepsilon_M R$. Since $f(c_1^b)$ is increasing, this requires, from Eq.(22), that $c_1^b < \bar{x}$. This now means that the consumption plateau at which dispatchable energy is used is lower than smallest consumption plateau, $\bar{x}$, introduced in the previous section and for which no storage occurs. This validate the idea the condition given by Eq.(28 is a minimal condition ensuring the existence of storage.

It now remains to show how the endogenous plateau, c_2^b, is obtained. At this plateau, the renewable energy production is insufficient and the gap is offset by the use of dispatchable and stored energies (net of the conversion losses) in an instantaneous amount of $\forall t \in [0,1]$, $\max\left\{c_1^b - \varepsilon(t)R, 0\right\}$ (see Eq.(30)). These energy sources are nevertheless available in limited quantities in the renewable energy cycle. Dispatchable energy over $[0, 1]$ is clearly limited to D, while, under our stationarity assumption, stored energy is limited by the provision made during peak production. Using Eqs.(30) and (31), this requires that:

$$\int_0^1 \max\left\{c_1^b - \varepsilon(t)R, 0\right\} dt = D + \sigma^- \int_0^1 \tfrac{1}{\sigma^+} \max\left\{\varepsilon(t)R - f(c_1^b), 0\right\} dt = 0 \tag{35}$$

However, let us not forget that the main argument of this section is based on the assumption that the storage capacity is not binding. By using Eqs.(31) and (33), that $\int_0^1 s^+(f(c_1^b), t)dt < S$. So let us now introduce a second threshold $\underline{x}$ for which the storage capacity is saturated, i.e.,

$$\int_0^1 s^+(f(\underline{x}), t)dt = \int_0^1 \max\left\{\varepsilon(t)R - f(\underline{x}), 0\right\} = S \tag{36}$$

It can be shown that this unique threshold satisfies $\underline{x} < \bar{x}$ and that for $c_1^b \geq \underline{x}$ the storage capacity is not strictly binding. This observation provides a condition on the parameters of our model, ensuring that the capacity constraint is never binding. This condition states that if the sum of dispatchable energy, D, and storage capacity net of conversion costs, $\sigma^- S$, is greater than the amount of energy required to sustain a consumption plateau at $\underline{x}$, then the storage capacity is not binding. This condition can be written as:

$$\int_0^1 \max\left\{\underline{x} - \varepsilon(t)R, 0\right\} dt \leq D + \sigma^- S \tag{37}$$

If the conditions given by Eqs.(28) and (37) are satisfied, we can state that:

Proposition 3. *If the conditions given by Eqs.(28) and (37) are verified, then*
(i) There exists a unique solution, $\tilde{c}_1^b$, to Eq.(35) with the property that $\underline{x} \leq \tilde{c}_1^b < \bar{x}$. The second consumption plateau is given by $\tilde{c}_2^b = (u')^{-1}\left(\frac{\sigma_-}{\sigma_+} u'(\tilde{c}_1^b)\right)$.
(ii) The optimal electricity consumption path which solves program (57) is given by $\forall t \in [0, 1]$, $\tilde{x}^b(t) = \min\left\{\max\left\{\tilde{c}_1^b, \varepsilon(t)R\right\}, \tilde{c}_2^b\right\}$.
(iii) The optimal storage strategy occurs at the peak production and is given by $\forall t \in [0, 1]$, $(\tilde{s}^+)^b(t) = \frac{1}{\sigma^+} \max\left\{\varepsilon(t)R - \tilde{c}_2^b, 0\right\}$ and the storage capacity is non binding.
(iv) There are several ways to allocate dispatchable and stored energy as long as $\forall t \in [0, 1]$, $\tilde{d}^b(t) + \sigma^-(\tilde{s}^-)^b(t) = \max\left\{\tilde{c}_1^b - \varepsilon(t)R, 0\right\}$, $\int_0^1 \tilde{d}^b(t)dt = D$ and $\int_0^1 (\tilde{s}^-)^b(t)dt = \int_0^1 \tilde{s}^+(t)dt$.

Point (iv) suggests that the solution to the optimization problem is not unique. This is not really surprising: once stored, electricity operates as an alternative source of dispatchable energy. The two types of energy can therefore be used indifferently to reach the consumption plateau $\tilde{c}_1^b$ corresponding to a low level of renewable energy production. In Appendix D, we show that any continuous and positive selection satisfying the conditions given by (iv) of Proposition 3 can be part of the optimal solution.

As in the previous scenario, let us now characterize the effect of a change in dispatchable or renewable energy sources on the two consumption plateaus $c_1^b(D,R)$ and $c_2^b(D,R)$. This requires, in a first step, the introduction of the four switching times given respectively by $\tilde{T}_i^b(D,R) = \varepsilon^{-1}\left(\frac{\tilde{c}_1^b(D,R)}{R}\right)$, $i = 1,2$ and $\tilde{T}_i^b(D,R) = \varepsilon^{-1}\left(\frac{\tilde{c}_2^b(D,R)}{R}\right)$, $i = 3,4$ (see Fig.2). With this additional notations, Eq.(35), now writes:

$$\int_0^{\tilde{T}_1^b(D,R)} \left(\tilde{c}_1^b(D,R) - \varepsilon(t)R\right) dt + \int_{\tilde{T}_4^b(D,R)}^1 \left(\tilde{c}_1^b(D,R) - \varepsilon(t)R\right) dt + \tfrac{\sigma^-}{\sigma^+} \int_{\tilde{T}_2^b(D,R)}^{\tilde{T}_3^b(D,R)} \left(\tilde{c}_2^b(D,R) - \varepsilon(t)R\right) dt - D = 0 \tag{38}$$

and, by Eq.(33), we know that:

$$\tilde{c}_2^b(D,R) = f(\tilde{c}_1^b(D,R)) = (u')^{-1}\left(\tfrac{\sigma_-}{\sigma_+} u'(\tilde{c}_1^b(D,R))\right) \tag{39}$$

The results described in Table 2 follow from the differentiation of these two equations with respect to D and R (see Appendix F). Unsurprisingly, we again obtain that $\tilde{c}_1^b(D,R)$ increases in both D and R simply because, in each case, more electricity is available. The results for $\tilde{c}_2^b(D,R)$ follow from the positive relationship between $\tilde{c}_2^b(D,R)$ and $\tilde{c}_1^b(D,R)$, i.e. $f'(\tilde{c}_1^b) > 0$ (see Eq.(34)). We use these results in the discussion of the consumer's welfare in section 7.

	∂D	∂R
$\partial \tilde{c}_1^b$	$\frac{u"(\tilde{c}_2^b)}{\left(1+\tilde{T}_1^b-\tilde{T}_2^b\right)u"(\tilde{c}_2^b)+\left(\frac{\sigma^-}{\sigma^+}\right)^2(\tilde{T}_4^b-\tilde{T}_3^b)u"(\tilde{c}_1^b)} > 0$	$\frac{\left(\int_0^{\tilde{T}_1^b}\varepsilon(t)dt+\int_{\tilde{T}_2^b}^1\varepsilon(t)d+\frac{\sigma^-}{\sigma^+}\int_{\tilde{T}_3^b}^{\tilde{T}_4^b}\varepsilon(t)dt\right)u"(\tilde{c}_2^b)}{\left(1+\tilde{T}_1^b-\tilde{T}_2^b\right)u"(\tilde{c}_2^b)+\left(\frac{\sigma^-}{\sigma^+}\right)^2(\tilde{T}_4^b-\tilde{T}_3^b)u"(\tilde{c}_1^b)} > 0$
$\partial \tilde{c}_2^b$	$\frac{\frac{\sigma^-}{\sigma^+}u"(\tilde{c}_1^b)}{\left(1+\tilde{T}_1^b-\tilde{T}_2^b\right)u"(\tilde{c}_2^b)+\left(\frac{\sigma^-}{\sigma^+}\right)^2(\tilde{T}_4^b-\tilde{T}_3^b)u"(\tilde{c}_1^b)} > 0$	$\frac{\left(\int_0^{\tilde{T}_1^b}\varepsilon(t)dt+\int_{\tilde{T}_2^b}^1\varepsilon(t)d+\frac{\sigma^-}{\sigma^+}\int_{\tilde{T}_3^b}^{\tilde{T}_4^b}\varepsilon(t)dt\right)\frac{\sigma^-}{\sigma^+}u"(\tilde{c}_1^b)}{\left(1+\tilde{T}_1^b-\tilde{T}_2^b\right)u"(\tilde{c}_2^b)+\left(\frac{\sigma^-}{\sigma^+}\right)^2(\tilde{T}_4^b-\tilde{T}_3^b)u"(\tilde{c}_1^b)} > 0$

Table 2: Comparative statics with non binding storage

6. Binding storage capacity

This last case is in some respects close to the previous one, since storage occurs in both cases but now with binding capacity. This suggests that the discussion at the beginning of the previous section resulting from Lemma 1 and concerning the properties of the solution is still appropriate. In particular, we can state that the general form of the solutions is the same as that described by Eqs.(29),(30) and (31). The main difference is that the construction and the levels of the two consumption plateaus c_1^c and c_2^c are totally different.

For instance in Fig. 3, the green area corresponds to the storage capacity augmented by the conversion cost, the smallest consumption plateau is now below $\underline{x}$, and the relation between c_1^c and c_2^c is not as simple as in the previous section. But let us come back on these points, by showing how c_1^c and c_2^c are constructed.

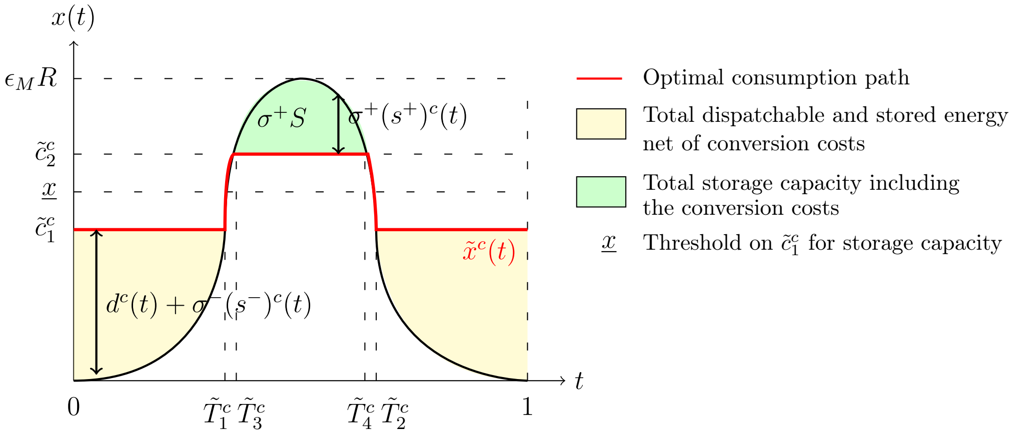

Figure 3: Consumption with binding storage

First note that a binding storage capacity restricts the transfer of electricity consumption over time, notably between the two consumption plateaus c_2^c and c_1^c. This means that we are in a situation where the consumer is still willing to transfer electricity from the high consumption plateau c_2^c to the low consumption plateau c_1^c, but is unable to do this because of the capacity constraint. In other words, we end up with the marginal rate of substitution between c_1^c and c_2^c being below the conversion losses, and based on equations (32) and (33), we have :

$$MRS_{c_1^c/c_2^c} = \frac{u'\left(c_2^c\right)}{u'\left(c_1^c\right)} < \tfrac{\sigma^-}{\sigma^+} \text{ or } c_2^c > (u')^{-1}\left(\tfrac{\sigma_-}{\sigma_+}u'(c_1^c)\right) > c_1^c \tag{40}$$

It also implies that the total energy stored at the high consumption plateau c_2^c must be equal to the storage capacity (see Fig. 3), i.e.:

$$\int_0^1 s^+(c_2^c, t)\, dt = \tfrac{1}{\sigma^+}\int_0^1 \max\left\{\varepsilon(t)R - c_2^c, 0\right\}\, dt = S \tag{41}$$

This equation corresponds to Eq.(36), which, in the previous section, introduced the lowest consumption plateau, c_1^b, for which storage is not constraining. This first means that c_2^c can be deduced from our previous results by simply setting $c_2^c = f(\underline{x})$ with $f(x)$ given by Eq.(33). Furthermore, since $\underline{x}$ is a threshold for c_1^c below which the storage constraint is binding, we now need to ensure that the lowest consumption plateau satisfies $c_1^c < \underline{x}$.

Finally, to characterize the optimal consumption trajectory, it still remains to define c_1^c. This is done by stating that the total dispatchable and stored energy that is needed to sustain the low-consumption plateau (the yellow area in Fig. 3) equates the amount of dispatchable energy and the storage capacity since the latter is now binding, i.e.:

$$\int_0^1 \tilde{d}^c(t) + \sigma^-(\tilde{s}^-)^c(t)dt = \int_0^1 \max\left\{\tilde{c}_1^c - \varepsilon(t)R, 0\right\} dt = D + \sigma^- S \tag{42}$$

Moreover, to ensure that $\tilde{c}_1^c < \underline{x}$, we need the contrary of condition (37), that is:

$$\int_0^1 \max\left\{\underline{x} - \varepsilon(t)R, 0\right\} dt > D + \sigma^- S \tag{43}$$

Let us however recall that $\underline{x} = f^{-1}(c_2^c)$ and is independent of D (see Eq.(41)). This means that $\int_0^1 \max\left\{\underline{x} - \varepsilon(t)R, 0\right\} dt - \sigma^- S$ may be negative in particular if S is large and induces a contradiction with $D \geq 0$. In other words, if the storage capacity is too large, it cannot be binding and this last case is vacuous. So, to consistently treat the three cases, we add, in the rest of the paper, an additional restriction on the set $\mathcal{E}$ of energy mix given by:

$$\int_0^1 \max\left\{f^{-1}(c_2^c(R,S)) - \varepsilon(t)R, 0\right\} dt > \sigma^- S \text{ with } c_2^c(R,S) \text{ solution to Eq.(41)} \tag{44}$$

The following proposition summarizes our results.

Proposition 4. *If the condition given by Eqs.(43) and (44) are verified, then:*
(i) There exists a unique solution, $\tilde{c}_1^c$, to Eq.(42) with the property that $\tilde{c}_1^c < \underline{x}$. The second consumption plateau is given by $\tilde{c}_2^c = f(\underline{x})$ with $f(x)$ given by Eq.(33)
(ii) The optimal electricity consumption path which solves program (57) is given by $\forall t \in [0,1]$, $\tilde{x}^c(t) = \min\left\{\max\left\{\tilde{c}_1^c, \varepsilon(t)R\right\}, \tilde{c}_2^c\right\}$.
(iii) The optimal storage strategy is limited by the storage capacity and is given by $\forall t \in [0,1]$, $(\tilde{s}^+)^c(t) = \frac{1}{\sigma^+} \max\left\{\varepsilon(t)R - \tilde{c}_2^c, 0\right\}$.
(iv) There are again several ways to affect dispatchable and stored energy as long as $\forall t \in [0,1]$, $\tilde{d}^c(t) + \sigma^-)(\tilde{s}^-)^c(t) = \max\left\{\tilde{c}_1^c - \varepsilon(t)R, 0\right\}$, $\int_0^1 \tilde{d}^c(t)dt = D$ and $\int_0^1 (\tilde{s}^-)^c(t)dt = S$.

Since storage is limited in the latter case, let's examine how dispatchable energy, renewable generation capacity, and storage capacity affect the consumption plateaus $\tilde{c}_1^c(D,R,S)$ and $\tilde{c}_2^c(R,S)$. This again requires the construction of the four switching times, given by $\tilde{T}_i^c(D,R,S) = \varepsilon^{-1}\left(\frac{\tilde{c}_1^c(D,R,S)}{R}\right)$, $i = 1,2$ and $\tilde{T}_i^c(R,S) = \varepsilon^{-1}\left(\frac{\tilde{c}_2^c(R,S)}{R}\right)$, $i = 3,4$ (see Fig.3) and the introduction of the two identities induced by the definitions of $\tilde{c}_1^c$ and $\tilde{c}_2$, respectively. These identities are given by:

$$\int_0^{\tilde{T}_1^c(D,R,S)} \left(\tilde{c}_1^c(D,R,S) - \varepsilon(t)R\right) dt + \int_{\tilde{T}_2^c(D,R,S)}^1 \left(\tilde{c}_1^c(D,R,S) - \varepsilon(t)R\right) dt - D - \sigma^- S = 0 \tag{45}$$

$$\int_{\tilde{T}_3^c(D,R,S)}^{\tilde{T}_4^c(D,R,S)} \max\left\{\varepsilon(t)R - \tilde{c}_2^c(R,S), 0\right\} dt - \sigma^+ S = 0 \tag{46}$$

By calculating the derivatives of the two identities with respect to D, R, and S (see Appendix F), we find the results shown in Table 3. As usual, $\tilde{c}_1^c$ remains increasing in dispatchable energy, D, and renewable capacity, R, but this plateau also increases with storage capacity, S. The intuition is obvious: an increase in storage capacity induces additional storage, as stored electricity is fully dispatchable, consumption increases when this one is required, i.e, when $\tilde{c}_1^c$ is reached. The main changes concern $\tilde{c}_2^c$. When storage is limited, the marginal rate of substitution between $\tilde{c}_1^c$ and $\tilde{c}_2^c$ is no longer related to conversion losses. Instead, it is now determined by the storage constraint. Thus, $\tilde{c}_2^c$ becomes independent of the level of dispatchable energy and decreases with storage capacity to expand stored electricity. Finally, if renewable capacity increases, so does peak production. With limited storage capacity, the only option is to increase $\tilde{c}_2^c$.

	∂D	∂R	∂S
$\partial \tilde{c}_1^c$	$\frac{1}{\tilde{T}_1^c+1-\tilde{T}_2^c} > 0$	$\frac{\int_0^{\tilde{T}_1^c} \varepsilon(t)dt+\int_{\tilde{T}_2^c}^1 \varepsilon(t)dt}{\tilde{T}_1^c+1-\tilde{T}_2^c} > 0$	$\frac{\sigma^-}{\tilde{T}_1^c+1-\tilde{T}_2^c} > 0$
$\partial \tilde{c}_2^c$	0	$\frac{\int_{\tilde{T}_3^c}^{\tilde{T}_4^c} \varepsilon(t)dt}{\tilde{T}_4^c-\tilde{T}_3^c} > 0$	$-\frac{\sigma^+}{\tilde{T}_4^c-\tilde{T}_3^c} < 0$

Table 3: Comparative statics with binding storage

7. Energy mix and consumers' welfare

This section proceeds in two steps. We first summarize our previous results so as to construct the consumer welfare function for various energy mixes $e = (D, R, S) \in \mathcal{E}$ and show that this function is continuously differentiable. This enables us, in a second step, to assess the effect on consumer welfare of substituting dispatchable energy with renewable energy.

7.1. The consumer welfare function

To construct this function, let us briefly review our previous results. We start with an energy mix, $e = (D, R, S) \in \mathcal{E}$, where fluctuations in renewable energy affect consumptions. This is done by limiting the storage capacity (see Eq.(7)) and the availability of dispatchable energy (see Eq.(8)). We even add an additional restriction (see Eq.(44) that ensures that storage can be binding [6]. Sections 4 to 6 identify the subsets $\mathcal{E}^a$, $\mathcal{E}^b$ and $\mathcal{E}^c$, respectively, corresponding to situations without storage or in which storage capacity is sufficiently large or binding. If these three subsets form a partition of the set $\mathcal{E}$ of the

[6] If this restriction is not satisfied, it simply means that the binding storage case is empty and that all results associated with this case can be ignored. However, we believe it is appropriate to allow this situation, as we are only at the beginning of the development of storage capacities.

energy mix under consideration (see preliminary remarks in Appendix G), we can build the welfare function $W(e)$ resulting from the program outlined in section 3

Moreover, to unify the notation, we introduce, in the no-storage case, a virtual plateau, $\tilde{c}_2^a(e)$, defined by the highest renewable production, i.e., $\forall e \in \mathcal{E}^a$, $\tilde{c}_2(e) = \varepsilon_M R$. This harmless trick enables us to define, for all $e \in \mathcal{E}$, piecewise functions $\tilde{c}_1(e)$ and $\tilde{c}_2(e)$ describing the two consumption plateaus occurring in each of the three sub-cases. With this new notation, the optimal consumption path becomes for each $e \in \mathcal{E}$:

$$\forall t \in [0,1], \; \tilde{x}(t;e) = \min\{\max\{\tilde{c}_1(e), \varepsilon(t)R\}, \tilde{c}_2(e)\} \tag{47}$$

We can even calculate the consumer's welfare for each energy mix along a renewable cycle. This function is given by:

$$W(e) = \int_0^1 u(\tilde{x}(t;e))\,dt \tag{48}$$

More interestingly, if we can show that $\tilde{c}_1(e)$ and $\tilde{c}_2(e)$ are continuous over the entire domain $\mathcal{E}$, then the same is true for $\tilde{x}(t;e)$ and $W(e)$. We will even go further. Using the results in Tab. 1 to 3, we calculate $\partial W(e)$ piecewise and show that this gradient remains globally continuous. More precisely:

Lemma 2. *For all $e \in \mathcal{E}$, we can say that:*
(i) The two plateaus $\tilde{c}_1(e)$, $\tilde{c}_2(e)$ the consumption process $\tilde{x}(t;e)$ and the consumer's welfare $W(e)$ are continuous functions
(ii) The welfare function $W(e)$ is at least of class C^1 and its gradient is given by:

$$\partial W(e) = \begin{pmatrix} u'(\tilde{c}_1(e)) \\ \int_0^1 u'(\min\{\max\{\tilde{c}_1(e), \varepsilon(t)R\}, \tilde{c}_2(e)\})\,\varepsilon(t)dt \\ \sigma^+ u'(\tilde{c}_1(e)) \max\left\{\frac{\sigma^-}{\sigma^+} - \frac{u'(\tilde{c}_2(e))}{u'(\tilde{c}_1(e))}, 0\right\} \end{pmatrix} \tag{49}$$

This technical lemma induces an interesting observation: even if different energy sources are substitutes for consumers (see Eq.57), changes in their availabilities over the renewable production cycle produce different welfare effects. Indeed, each shift in one of these resources has a different impact on the optimal consumption path defined over $[0,1)$. Higher dispatchable capacity provides additional consumption outside peak hours, so its marginal contribution to welfare is estimated by the marginal utility of the lowest-consumption plateau. Changes in renewable capacity increase production at each t and therefore the consumption path. The effect on welfare is then the "sum" of the different marginal utilities along the optimal consumption path. Lastly the storage capacity only matters in cases of a binding constraint because, in this case, $\frac{\sigma^-}{\sigma^+} > \frac{u'(\tilde{c}_2(e))}{u'(\tilde{c}_1(e))}$.

7.2. The welfare effect of energy transition on consumers

This Lemma also helps us understand the impact of the energy transition on consumer welfare, particularly in the presence of a storage system. Although we cannot examine this transition process in our short-term approach, we can nevertheless study the effect on welfare of a substitution of dispatchable energy by renewable production capacities that keep the total supply of energy constant over the renewable production cycle. In this setting, we essentially try to address three questions. Does this substitution reduce consumers' welfare? Is this effect attenuated when there is energy storage? Is it possible, when the storage capacity is binding, to reverse this negative effect through a reasonable extension of this capacity?

To answer the first question, we consider any energy mix $e \in \mathcal{E}$ and simultaneously increase renewable production capacity by ΔR and decrease dispatchable energy availability by $\Delta D = -\left(\int_0^1 \varepsilon(t)\right) \Delta R$. In this way, the total available energy remains constant over $[0, 1]$ and the change in consumer welfare is $\forall e \in \mathcal{E}$ given by:

$$
\begin{aligned}
\tfrac{\Delta W}{\Delta R} &= -\left(\int_0^1 \varepsilon(t)\right) u'\left(\tilde{c}_1(e)\right) + \int_0^1 u'\left(\min\left\{\max\left\{\tilde{c}_1(e), \varepsilon(t)R\right\}, \tilde{c}_2(e)\right\}\right) \varepsilon(t)dt \\
&= \int_0^1 \left(u'\left(\min\left\{\max\left\{\tilde{c}_1(e), \varepsilon(t)R\right\}, \tilde{c}_2(e)\right\}\right) - u'\left(\tilde{c}_1(e)\right)\right) \varepsilon(t)dt \qquad (50)
\end{aligned}
$$

Now recall that $\tilde{c}_1(e) \leq \tilde{c}_2(e)$ so that $\min\left\{\max\left\{\tilde{c}_1(e), \varepsilon(t)R\right\}, \tilde{c}_2(e)\right\} \geq \tilde{c}_1(e)$. Since the marginal utility, u', decreases, we can say that:

$$
\tfrac{\Delta W}{\Delta R} = \int_0^1 \underbrace{\left(u'\left(\min\left\{\max\left\{\tilde{c}_1(e), \varepsilon(t)R\right\}, \tilde{c}_2(e)\right\}\right) - u'\left(\tilde{c}_1(e)\right)\right)}_{\leq 0} \varepsilon(t)dt \leq 0 \qquad (51)
$$

We can therefore conclude that for any energy mix $e \in \mathcal{E}$, replacing dispatchable energy with additional renewable capacity at constant energy generation reduces consumers' welfare. This is mainly because this substitution increases the variability of the optimal consumption path. Let us now recall that $\partial_R W(e) \geq 0$. Thus, if the goal is to maintain consumer welfare, this result also suggests that the decrease in the availability of dispatchable energy should be less than the total increase in renewable production. In that case, the total increase in energy production over the cycle compensates, from the consumers' perspective, the extra fluctuations.

We can also ask whether the presence of active storage mitigates these welfare losses. In other words, it might be interesting to compare the welfare losses of an energy mix, $e \in \mathcal{E}^b \cap \mathcal{E}^c$, with those of a hypothetical case in which storage is forbidden. In the latter case, the optimal electricity allocation is similar to that described section 4, except it now applies to an energy mix $e \in \mathcal{E}^b \cap \mathcal{E}^c$. Moreover, as it can be seen in Fig.1 and either Fig.2 or 3, that the single plateau without storage $c^{ns}(e)$ is, in both cases, lower than $\tilde{c}_1(e)$. This follows from the fact that the yellow area in the case without storage (see Fig.1) corresponds to the amount of dispatchable energy. In the case with storage (Fig.2 or 3), this area corresponds to the amount of dispatchable and stored energy (see Appendix H for a proof). Let us now denote the welfare losses when storage is forbidden by $\frac{\Delta W^{ns}}{\Delta R}$. This quantity is typically given by Eq.(50), where $\tilde{c}_1(e) = c^{ns}(e)$ and $\tilde{c}_2(e) = \varepsilon_M R$. We can therefore construct the difference in welfare losses for each e in $\mathcal{E}^b \cap \mathcal{E}^c$. After some manipulations (see Appendix H), we obtain that:

$$\begin{aligned} \frac{\Delta W^{ns}}{\Delta R} - \frac{\Delta W}{\Delta R} = & \int_0^1 \left(u'\left(\max\left\{\min\left\{\tilde{c}_1(e), \varepsilon(t)R\right\}, c^{ns}(e)\right\}\right) - u'\left(c^{ns}(e)\right)\right)\varepsilon(t)\, dt \\ & + \int_0^1 \left(u'\left(\max\left\{\tilde{c}_2(e), \varepsilon(t)R\right\}\right) - u'\left(\tilde{c}_2(e)\right)\right)\varepsilon(t)\, dt \end{aligned} \tag{52}$$

Now recall from our previous discussion that $\tilde{c}_1(e) \geq c^{ns}(e)$ so that:

$$\max\left\{\min\left\{\tilde{c}_1(e), \varepsilon(t)R\right\}, c^{ns}(e)\right\} \geq c^{ns}(e) \tag{53}$$

and observe that $\max\left\{\tilde{c}_2(e), \varepsilon(t)R\right\} \geq \tilde{c}_2(e)$. Since $u' < 0$, both integrals are negative, leading to $\frac{\Delta W^{ns}}{\Delta R} - \frac{\Delta W}{\Delta R} \leq 0$. Given that both terms are negative, we can conclude that the welfare losses when storage is forbidden, $\left|\frac{\Delta W^{ns}}{\Delta R}\right|$, are greater than the welfare losses with storage, $\left|\frac{\Delta W}{\Delta R}\right|$. Thus, we can say that active storage facilitates the energy transition from the consumer's point of view.

Finally, let us recall from Eq.(50) that any increase in storage capacity has a positive effect on welfare as long as this capacity remains binding (i.e., when $\frac{\sigma^-}{\sigma^+} > MRS_{\tilde{c}_1/\tilde{c}_2}$). This means that we can offset the negative impact of substituting dispatchable energy with renewable energy thanks to a paired increase in storage capacity. To illustrate this point, we again take a situation in which renewable capacity rises by ΔR, while dispatchable capacity falls by $\left(\int_0^1 \varepsilon(t)\, dt\right)\Delta R$. But now, we couple this change with an expansion of storage capacity proportional to the change in renewable energy production over the cycle $[0,1]$ and given by $k\left(\int_0^1 \varepsilon(t)\, dt\right)\Delta R$. In this case, the variation in consumer welfare becomes:

$$\begin{aligned} \frac{\Delta W}{\Delta R} = & -u'\left(\tilde{c}_1(e)\right)\left(\int_0^1 \varepsilon(t)\, dt\right) + \int_0^1 u'\left(\min\left\{\max\left\{\tilde{c}_1(e), \varepsilon(t)R\right\}, \tilde{c}_2(e)\right\}\right)\varepsilon(t)\, dt \\ & + k\sigma^+ u'\left(\tilde{c}_1(e)\right)\left(\frac{\sigma^-}{\sigma^+} - MRS_{\tilde{c}_1/\tilde{c}_2}\right)\left(\int_0^1 \varepsilon(t)\, dt\right) \end{aligned} \tag{54}$$

Since the first two terms are negative in sum and the last is positive due to the binding storage capacity, we can easily identify the proportional expansion of storage capacity that offsets the initial negative welfare effect. This factor k is given by:

$$k = \frac{u'(\tilde{c}_1(e))\left(\int_0^1 \varepsilon(t)\, dt\right) - \int_0^1 u'(\min\{\max\{\tilde{c}_1(e), \varepsilon(t)R\}, \tilde{c}_2(e)\})\varepsilon(t)\, dt}{\sigma^+ u'(\tilde{c}_1(e))\left(\frac{\sigma^-}{\sigma^+} - MRS_{\tilde{c}_1/\tilde{c}_2}\right)\left(\int_0^1 \varepsilon(t)\, dt\right)} > 0 \tag{55}$$

But this clearly raises the question of whether the size of this factor is reasonable. To bring an answer, let us observe that $\min\left\{\max\left\{\tilde{c}_1(e), \varepsilon(t)R\right\}, \tilde{c}_2(e)\right\} \leq \tilde{c}_2(e)$. It follows that:

$$k \leq \frac{(u'(\tilde{c}_1(e))-u'(\tilde{c}_2(e)))\left(\int_0^1 \varepsilon(t)\,dt\right)}{\sigma^+ u'(\tilde{c}_1(e))\left(\frac{\sigma^-}{\sigma^+}-MRS_{\tilde{c}_1/\tilde{c}_2}\right)\left(\int_0^1 \varepsilon(t)\,dt\right)}$$

$$= \frac{1}{\sigma^+}\frac{1-MRS_{\tilde{c}_1/\tilde{c}_2}}{\frac{\sigma^-}{\sigma^+}-MRS_{\tilde{c}_1/\tilde{c}_2}} < 1 \quad \text{since } \frac{\sigma^-}{\sigma^+} > 1 \text{ and } \sigma^+ > 1 \tag{56}$$

In conclusion, we can state that the negative effect on welfare of substituting dispatchable energy sources with renewable energy sources can be offset by increasing storage capacity, with this expansion being less than the additional renewable production over the cycle.

The next proposition summarizes this discussion:

Proposition 5. *If renewable energy replaces dispatchable energy over the cycle while keeping total energy production constant, then:*
(i) Consumer welfare decreases in the different scenarios that we have identified.
(ii) These welfare losses are nevertheless smaller when storage is activated during the renewable production cycle.
(iii) If storage capacity is limited, these welfare losses can be offset by adjusting the storage capacity through an increment that is lower than the rise in renewable energy production.

8. Further discussions

In this section, we discuss two extensions and a technical aspect of the model. We first examine the optimal choice of energy capacities, and then characterize the pricing structure associated with the optimal consumption path. We finally explain why, in a framework that focuses on consumption, it is possible to restrict, without loss of generality, to a representative consumer.

8.1. The energy portfolio choice

In the analysis up to now, we have assumed away the costs associated with capacity installation and the production of dispatchable energy. In order to take these into consideration, we need to introduce explicit cost functions C_S, C_R, and C_D corresponding respectively to storage, renewable, and dispatchable capacities. As we assume stationarity we can represent these capacity cost as primarily reflect operating expenses and the amortized capital repayment over the relevant time horizon. For dispatchable technologies, the cost function must additionally incorporate fuel expenditures required to supply D units of energy over the cycle. This function, as in the seminal paper of Ambec and Crampes (2012), can be taken as linear. In this simple case, we can write the following optimization program:

$$\max_{(D,R,S)\,\in\,\mathcal{E}} \; W(D,R,S) - C_D(D) - C_R(R) - C_S(S) \tag{57}$$

where $W(D,R,S)$ is the welfare function obtained in Lemma 2 and C_i, $i = D, R, S$, the cost function associated with the capacity levels over the cycle. If these cost functions satisfy standard properties, especially differentiability, then Lemma 2 tells us that for interior solutions, the marginal welfare of each energy source must be equal to its marginal cost. This creates a system of equations which enables us to find the optimal energy mix.

More sophisticated specifications such as capacity accumulation could also be introduced (see, for instance, Helm and Mier 2021 or Pommeret and Schubert 2022). Under capacity accumulation, the cost functions C_S, C_R, and C_D would represent the investment undertaken in each period, while total installed capacity becomes the cumulative sum of past investments depreciated at different constant rates. This extension introduces an intertemporal trade off in which the social planner balances the upfront investment costs against the consumer gains generated by smoother consumption enabled by the different production technologies.

In addition, we can allow for heterogeneous dispatchable generation technologies, such as coal, gas, or nuclear, each characterized, by its own linear marginal cost. This naturally yields the classical merit-order structure : technologies with the lowest marginal cost are dispatched first up to their capacity constraint, followed sequentially by higher cost units (see, for instance Léautier 2019). One could further extend the framework by introducing a merit order within renewable technologies, see Abrell et al. (2019), whereby the most productive units produce first. A similar merit order could also be applied to heterogeneous storage technologies, see Newbery (2018).

8.2. Electricity tariffs along the cycle

Throughout this work we study the problem from the perspective of a social planner. As long, as we introduce price taking behaviors, we can even associate to this economy a competitive equilibrium. Moreover without externality, like environmental damages, we know, by the Welfare Theorem, that the electricity price obtained at the competitive equilibrium is obtained by the marginal utility of the consumer along the optimal consumption path. This yields a tariff structure consisting of two constant levels, a high and a low price, with real-time pricing between them.

In the case when storage operates the optimal consumption trajectory itself traces out the competitive tariff (see Figure 2 or 3). Between $[0, T_1]$, given that the marginal utility is decreasing in consumption, the price is constant at its upper level. Over the interval $[T_1, T_3]$, the consumer's flexibility allows consumption to adjust smoothly to the rising profile of renewable production, implying a decreasing price path up to a second plateau. On $[T_3, T_4]$, the price remains at its lower constant level, corresponding to the period of maximum renewable availability. Over $[T_4, T_2]$, the price increase as renewable output falls, eventually reconnecting with the higher plateau. Finally, from $[T_2, 1]$, the price again remains constant at the high level. For the first regime the tariff is straightforward with a unique off-peak price and real time price adjustment elsewhere (see figure 1).

A key modeling assumption in our analysis is that the consumer is perfectly flexible in its level of consumption. The tariff we derive therefore directly implements the planner's optimal consumption trajectory. This stands in contrast to the traditional Real-Time Pricing literature, such as Borenstein and Holland (2005), which focuses on how consumers adjust their load in response to supply-driven price fluctuations. In their framework, peak demand is taken as exogenous and drives the shape of the optimal price path where flat pricing is not efficient contrary to RTP. Our results show that, even under full flexibility of consumers, the efficient allocation features consumption plateaus generated by renewable, dispatchable and storage which translate into fixed prices such as in Time of Use (see for instance Nicolson et al. 2018). As a consequence, the full RTP schedule can be closely approximated by a ToU augmented with a limited amount of real-time adjustment. The presence of, storage capacities able to store excess renewable energy is what creates this tariff structure.

8.3. Back to the representative consumer assumption

Now let us show how we can, without loss of generality, restrict our attention to a representative consumer. Suppose there are n heterogeneous consumers. Their utility function is given by $U_i\left(x_i\left(\cdot\right)\right) = \int_0^1 u_i\left(x_i\left(t\right)\right) dt$, where each $u_i(x)$ is increasing, strictly concave, and satisfies the usual Inada conditions. The social planner's problem can now be written as follows:

$$\max_{\left((x_i(t))_{i=1}^n, d(t), s^+(t), s^-(t)\right)} \sum_{i=1}^n \int_0^1 u_i\left(x_i\left(t\right)\right) dt \tag{58}$$

This problem is subject to the same set of dynamic constraints given by Eq.(10), but with an additional constraint. This constraint stipulates that, for each t, distributed electricity corresponds to the amount of available energy due to a particular use of dispatchable energy and storage and is given by:

$$\sum_{i=1}^n x_i\left(t\right) = \underbrace{d(t) - \sigma^+ s^+(t) + \sigma^- s^-(t) + \varepsilon(t)R}_{=X(t)} \tag{59}$$

Note that the allocation of dispatchable energy and the storage strategy are independent of how energy is distributed among consumers. This allows us to separate the questions of distribution and production, and consider the following problem:

$$V\left(X(\cdot)\right) = \max_{(x_i(\cdot))_{i=1}^n} \sum_{i=1}^n \int_0^1 u_i\left(x_i\left(t\right)\right) dt \text{ s.t } \forall t,\ \sum_{i=1}^n x_i\left(t\right) = X(t) \tag{60}$$

The function $V\left(X(\cdot)\right)$ stands for the aggregate indirect utility be used in the production choice problem. In other words, it is the utility of our representative agent. We also observe that the previous problem is separable in time. This means we can restrict our attention to the subproblem given by:

$$v(X) = \max_{(x_i)_{i=1}^n} \sum_{i=1}^n u_i\left(x_i\right) \text{ s.t } \sum_{i=1}^n x_i = X \tag{61}$$

and define the aggregate indirect utility as $V\left(X(\cdot)\right) = \int_0^1 v\left(X\left(t\right)\right) dt$. So, if $v\left(X\left(t\right)\right)$ shares the same restrictions as those we have imposed on our representative agent, both approaches lead to the same results. In other words, it remains to verify that $v(X(t))$ is increasing, strictly concave and satisfies the usual Inada conditions. This technical point is verified in Appendix I.

9. Concluding remarks

In this paper, our primary focus has been the investigation into the implications on consumer welfare of the substitution of dispatchable energy sources, mainly obtained from fossil fuels, by renewable energy sources that are frequently intermittent in nature. We started with two observations. On the one hand, the additional variability in supply requires consumers to be more flexible, which leads to welfare losses. Conversely, the development of storage capacity serves to mitigate these short-term fluctuations, thereby curtailing such welfare losses. To address this issue, we examined how intermittent renewable energy generation, dispatchable power supply and energy storage work together to determine the optimal electricity consumption path over a renewable production cycle. We then characterized the resulting level of consumer welfare. By solving this optimal control problem with fixed production and storage capacities, we identified three different regimes (no storage, abundant storage, and binding storage capacity) and their corresponding energy mixes. The effects on consumer welfare of replacing dispatchable energy with intermittent renewable energy sources were then discussed using these scenarios. As expected, consumer welfare declines with this substitution due to the need for additional flexibility. These welfare losses are smaller when storage is active, but they nevertheless persist. Finally, when storage capacity is limited, a moderate expansion of this capacity can fully offset these losses.

For a more comprehensive analysis, as suggested in Section 8.1, our model would require a deeper study of the composition of the electricity mix. In this paper, we considered only the optimal short-term adjustment of dispatchable, renewable, and storage capacities to supply variability. A more general framework would incorporate capacity accumulation, whereby new capacities result from investment decisions made in preceding cycles. This would introduce an intertemporal trade-off across the costs of different energy sources and yield an optimal time path for the energy mix, rather than the parametric analysis developed here. The resulting dynamics could then be compared with the existing literature (see, for instance, Helm and Mier 2021 or Pommeret and Schubert 2022).

It should also be noted that our discussion of electricity prices in Section 8.2 applies only when the conditions of the second welfare theorem are satisfied. This excludes several types of market failures, such as price making decisions or environmental externalities which would both call for policy interventions. Regarding the latter, it is straightforward to introduce a damage function related to the production of dispatchable energy which is fossil-based. Such a damage function can be readily integrated into our optimization problem, as in Pommeret and Schubert (2022). However, addressing the policy issues induced by this externality would require explicitly modeling the price-taking behaviors of the different agents. To address market power, we even need to go a step further by identifying competitive and non-competitive behaviors (e.g., Ito and Reguant 2016, Acemoglu et al. 2017, Andrés-Cerezo and Fabra 2025). Both market failures could also be studied concomitantly as in García-Alaminos and Rubio (2021).

Finally, it is important to emphasize that the welfare losses highlighted in our model stem from an aversion to consumption flexibility. Consumers prefer a constant stream of consumption since the instantaneous utility does not depend explicitly on time but is defined only through the consumption path. If this condition does not hold, it may be possible to identify alternative consumption profiles, correlated or not, with the short-term renewable production cycle. For such analyzes, one may refer to Trotta (2020) or Reguant et al. (2025).

References

Abrell, J., Rausch, S., Streitberger, C., 2019. The economics of renewable energy support. Journal of Public Economics 176, 94–117.

Acemoglu, D., Kakhbod, A., Ozdaglar, A., 2017. Competition in electricity markets with renewable energy sources. The Energy Journal 38, 137–155.

Ambec, S., Crampes, C., 2012. Electricity provision with intermittent sources of energy. Resource and Energy Economics 34, 319–336.

Ambec, S., Crampes, C., 2019. Decarbonizing electricity generation with intermittent sources of energy. Journal of the Association of Environmental and Resource Economists 6, 1105–1134.

Ambec, S., Crampes, C., 2021. Real-time electricity pricing to balance green energy intermittency. Energy Economics 94, 105074.

Andrés-Cerezo, D., Fabra, N., 2025. Storage and renewable energies: Friends or foes. Technical Report. Working Paper.

Borenstein, S., 2005. The long-run efficiency of real-time electricity pricing. The Energy Journal 26, 93–116.

Borenstein, S., Holland, S., 2005. On the efficiency of competitive electricity markets with time-invariant retail prices. RAND Journal of Economics 36, 69–493.

Cabot, C., Villavicencio, M., 2024. The demand-side flexibility in liberalised power market: A review of current market design and objectives. Renewable and Sustainable Energy Reviews 201, 114643.

Carson, R.T., Novan, K., 2013. The private and social economics of bulk electricity storage. Journal of Environmental Economics and Management 66, 404–423.

Crampes, C., Moreaux, M., 2010. Pumped storage and cost saving. Energy Economics 32, 325–333.

Denholm, P., 2015. Greening the grid: The role of storage and demand response.

García-Alaminos, Á., Rubio, S.J., 2021. Emission taxes and feed-in subsidies in the regulation of a polluting monopoly. SERIEs 12, 255–279.

Helm, C., Mier, M., 2021. Steering the energy transition in a world of intermittent electricity supply: Optimal subsidies and taxes for renewables and storage. Journal of Environmental Economics and Management 109, 102497.

IEA, 2020. Introduction to system integration of renewables. International Energy Agency.

IEA, 2024. The clean energy economy demands massive integration investments now. International Energy Agency.

IRENA, 2019. Innovation landscape brief: Time-of-use tariffs.

Ito, K., Reguant, M., 2016. Sequential markets, market power, and arbitrage. American Economic Review 106, 1921–57.

Jiang, T., Cao, Y., Yu, L., Wang, Z., 2014. Load shaping strategy based on energy storage and dynamic pricing in smart grid. IEEE Transactions on smart grid 5, 2868–2876.

Joskow, P.L., Wolfram, C.D., 2012. Dynamic pricing of electricity. American Economic Review 102, 381–385.

Léautier, T.O., 2019. Imperfect markets and imperfect regulation: An introduction to the microeconomics and political economy of power markets. MIT Press.

Lund, P.D., Lindgren, J., Mikkola, J., Salpakari, J., 2015. Review of energy system flexibility measures to enable high levels of variable renewable electricity. Renewable and Sustainable Energy Reviews 45, 785–807.

Maghami, M.R., Pasupuleti, J., Ekanayake, J., 2024. Energy storage and demand response as hybrid mitigation technique for photovoltaic grid connection: Challenges and future trends. Journal of Energy Storage 88, 111680.

Newbery, D., 2018. Shifting demand and supply over time and space to manage intermittent generation: The economics of electrical storage. Energy Policy 113, 711–720.

Nicolson, M.L., Fell, M.J., Huebner, G.M., 2018. Consumer demand for time of use electricity tariffs: A systematized review of the empirical evidence. Renewable and Sustainable Energy Reviews 97, 276–289.

Pommeret, A., Schubert, K., 2022. Optimal energy transition with variable and intermittent renewable electricity generation. Journal of Economic Dynamics and Control 134, 104273.

Puller, S.L., West, J., 2013. Efficient retail pricing in electricity and natural gas markets. American Economic Review 103, 350–355.

Reguant, M., Fabra, N., Wang, J., 2025. The distributional impacts of time-varying electricity pricing: A novel approach to estimating household income .

Schittekatte, T., Mallapragada, D., Joskow, P.L., Schmalensee, R., 2024. Electricity retail rate design in a decarbonizing economy: an analysis of time-of-use and critical peak pricing. The Energy Journal 45, 25–56.

Sioshansi, R., 2014. When energy storage reduces social welfare. Energy Economics 41, 106–116.

Steffen, B., Weber, C., 2013. Efficient storage capacity in power systems with thermal and renewable generation. Energy Economics 36, 556–567.

Trotta, G., 2020. An empirical analysis of domestic electricity load profiles: Who consumes how much and when? Applied Energy 275, 115399.

Zerrahn, A., Schill, W.P., Kemfert, C., 2018. On the economics of electrical storage for variable renewable energy sources. European Economic Review 108, 259–279.

Appendix A. Proof of Lemma 1

Points (i) and (ii) follow directly from our discussion.
(iii) Assume that $\exists\,(t,t')$ for which $\forall t \in (t,t')$, $s^+(t)s^-(t) \neq 0$. In this case $s^+(t) > 0$ and $s^-(t) > 0$, and, according to the slackness conditions (see Eqs.(20) and (21)), we have $\mu_{s^+}(t) = \mu_{s^-}(t) = 0$. But if we sum Eq.17 and Eq.18, we get the following:

$$\lambda_S = \underbrace{\left(\sigma^- - \sigma^+\right)}_{<0}\underbrace{u'(x(t))}_{>0} < 0 \tag{A.1}$$

which contradicts the fact that λ_s is a non-negative constant (see Eq.15).
(iv) Now assume that $\exists\,(t_0,t_1)$ for which $\forall t \in (t_0,t_1)$, $s^+(t)d(t) \neq 0$. It follows from the slackness conditions given by Eqs.(19) and (20) that $\mu_d(t) = \mu_{s^+}(t) = 0$. Using the optimality conditions given by Eq.16 and Eq.17, we can therefore claim that:

$$-\sigma^+\lambda_D + \lambda_\Sigma - \lambda_S = 0 \tag{A.2}$$

Now, recall from Eq.(10) that all the stored energy is discharged during the cycle. As storage and discharge cannot occur simultaneously (see point (iii) of this proof), $\exists\,(t_2,t_3)$ for which $\forall t \in (t_2,t_3)$, $s^+(t) > 0$. and therefore $\mu_{s^-}(t) = 0$. From the optimality conditions given by Eq.18 and Eq.16, and the slackness condition (19), we obtain, respectively, $u'(x(t)) = \frac{\lambda_\Sigma}{\sigma^-}$ and $\frac{\lambda_\Sigma}{\sigma^-} - \lambda_D \leq 0$. As the co-states are constant on $[0,1]$ (see Eqs. (13) to (15)), we can say from Eq.(A.2) that:

$$\lambda_S = -\sigma^+\lambda_D + \lambda_\Sigma \leq -\sigma^+\lambda_D + \sigma^-\lambda_D < 0 \tag{A.3}$$

and contradicts the fact that λ_s is a non-negative constant (see Eq.15).

Appendix B. Proof of Proposition 1

Since the constraints of the optimization problem are linear, it remains to check that the Hamiltonian is concave with respect to the control and stock variables. This Hamiltonian

$$\mathcal{H}\left(d, s^+, s^-, \lambda_D, \lambda_\Sigma, \lambda_S, t\right) = u(d - \sigma^+ s^+ + \sigma^- s^- + \varepsilon(t)R) - \lambda_D d + \lambda_\Sigma\left(s^+ - s^-\right) - \lambda_S s^+ \tag{B.1}$$

is also independent of the stock variables $(D(t), \Sigma(t), S(t))$. We only have to verify that this function is concave with respect to the controls (d, s^+, s^-). A simple computation shows that its Hessian is given by:

$$\partial^2_{d,s^+,s^-}\mathcal{H} = u''(x(t)) \begin{bmatrix} 1 & -\sigma^+ & \sigma^- \\ -\sigma^+ & (\sigma^+)^2 & -\sigma^+\sigma^- \\ \sigma^- & -\sigma^+\sigma^- & (\sigma^-)^2 \end{bmatrix} = u''(x(t)) \begin{bmatrix} 1 \\ -\sigma^+ \\ \sigma^- \end{bmatrix} \begin{bmatrix} 1 & -\sigma^+ & \sigma^- \end{bmatrix} \quad \text{(B.2)}$$

It follows that:

$$\forall h \in \mathbb{R}^3,\ h^t \cdot \partial^2_{d,s^+,s^-}\mathcal{H} \cdot h = u''(x(t)) \left(h_1 - \sigma^+ h_2 + \sigma^- h_3\right)^2 \leq 0 \quad \text{(B.3)}$$

Appendix C. Proof of Proposition 2

Point (i). Let us show that:

$$\phi_1(c) = \int_0^1 \max\{c - \varepsilon(t)R, 0\}\, dt - D \quad \text{(C.1)}$$

admits a unique solution $\tilde{c}^a$ with the property that $\tilde{c}^a \geq \bar{x} = (u')^{-1}\left(\frac{\sigma^+}{\sigma^-} u'(\varepsilon_M R)\right)$. Obviously $\lim_{c\to\infty} \phi_1(c) = +\infty$ and, under the no storage condition (Eq.(26)), by construction $\lim_{c\to\bar{x}} \phi_1(c) \leq 0$. So, if $\phi_1(c)$ is increasing, there exists a unique $\tilde{c}^a \geq \bar{x}$ solution to $\phi_1(c) = 0$. To verify this point, divide the domain of $\phi_1(c)$ into two intervals: $[0, \varepsilon_M R)$ and $[\varepsilon_M R, +\infty)$. On the second, $c \geq \varepsilon_M R \geq \varepsilon(t)R$. It follows that:

$$\forall c \in [\varepsilon_M R, +\infty),\ \phi_1(c) = \int_0^1 (c - \varepsilon(t)R)\, dt - D = c - R\int_0^1 \varepsilon(t)dt - D \quad \text{(C.2)}$$

so that $\phi_1'(c) = 1$. Let us now move to the interval $[0, \varepsilon_M R)$. Since $c < \varepsilon_M R$, we can, by construction of the cycle $\varepsilon(t)$, assert that there exists, for each $c \in (0, \varepsilon_M R)$, a unique couple $(T_1(c), T_2(c))$ such that for $i = 1, 2$, $\varepsilon(T_i)R = c$. If T_M denotes the time of the peak of renewable production, we also have $0 < T_1(c) < T_M < T_2(c) < 1$. Moreover, since $\varepsilon(t)$ increases and decreases on, respectively, $(0, T_M)$ and $(T_M, 1)$, we can also say that $T_1'(c) > 0$ and $T_2'(c) < 0$. It follows that:

$$\forall c \in (0, \varepsilon_M R),\ \phi_1(c) = \int_0^{T_1(c)} (c - \varepsilon(t)R)\, dt + \int_{T_2(c)}^1 (c - \varepsilon(t)R)\, dt - D \quad \text{(C.3)}$$

and therefore that:

$$\begin{aligned} \phi_1'(c) &= T_1(c) + T_1'(c)\,(c - \varepsilon(t)R)_{t=T_1(c)} + (1 - T_2(c)) - T_2'(c)\,(c - \varepsilon(t)R)_{t=T_2(c)} \\ &= 1 - T_2(c) + T_1(c) > 0 \text{ since } 0 < T_1(c) < T_2(c) < 1 \end{aligned} \quad \text{(C.4)}$$

Point (ii) and (iii). To save notation, we will omit the superscript a characterizing the case. So, let us now construct the consumption path $\forall t \in [0,1]$, $\tilde{x}(t) = \max\{\tilde{c}, \varepsilon(t)R\}$, the allocation of dispatchable $\forall t \in [0,1]$, $\tilde{d}(t) = \max\{\tilde{c} - \varepsilon(t)R, 0\}$ and the storage strategy $\forall t \in [0,1]$, $\tilde{s}^-(t) = \tilde{s}^+(t) = 0$. By Proposition 1, if constant co-states $(\tilde{\lambda}_D, \tilde{\lambda}_\Sigma, \tilde{\lambda}_S)$, and continuous and piecewise differentiable multipliers $(\tilde{\mu}_d(t), \tilde{\mu}_{s^+}(t), \tilde{\mu}_{s^-}(t))$ exist and satisfy Eqs.(10), (15) and (16) to (21), then $\tilde{x}(t)$ is the solution to our electricity allocation problem.

Let us start with the isoperimetrical conditions given Eq.(10). By construction of $\tilde{c}$, we have $\int_0^1 \tilde{d}(t)dt = D$ and, since $\tilde{s}^+(t) = \tilde{s}^-(t) = 0$, we satisfy $\int_0^1 (s^+(t) - s^-(t))\, dt = 0$ and $\int_0^1 s^+(t)dt \leq S$. The lack of storage also implies that $\int_0^1 \tilde{s}^+(t)dt = 0 < S$ and therefore, by Eq.(15), that $\tilde{\lambda}_S = 0$. Since $\tilde{\lambda}_D$ is free (see Eq.(13)), we set $\tilde{\lambda}_D = u'(\tilde{c})$.

Let us now concentrate on Eqs.(16) and (19). Since the electricity consumption is given by $\tilde{x}(t) = \max\{\tilde{c}, \varepsilon(t)R\}$ and $\tilde{\lambda}_D = u'(\tilde{c})$, Eq.(16) provides the path of $\tilde{\mu}_d(t)$, given by:

$$\forall t \in [0,1], \, \tilde{\mu}_d(t) = u'(\tilde{c}) - u'(\max\{\tilde{c}, \varepsilon(t)R\}) \tag{C.5}$$

Obviously, $\tilde{\mu}_d(t) \geq 0$ since $\tilde{c} \leq \max\{\tilde{c}, \varepsilon(t)R\}$ and $u"(x) < 0$. It remains to verify that $\forall t \in [0,1]$, $\tilde{d}(t)\tilde{\mu}_d(t) = 0$. Since $u"(x) < 0$, we can say that $\forall t \in [0,1]$

$$\tilde{d}(t) = \max\{\tilde{c} - \varepsilon(t)R, 0\} > 0 \Leftrightarrow \tilde{c} > \varepsilon(t)R \Leftrightarrow \tilde{\mu}_d(t) = u'(\tilde{c}) - u'(\max\{\tilde{c}, \varepsilon(t)R\}) = 0 \tag{C.6}$$

and, since $\tilde{d}(t) \geq 0$ and $\tilde{\mu}_d(t) \geq 0$, we also have $\tilde{\mu}_d(t) > 0 \Leftrightarrow \tilde{d}(t) = 0$. It follows that $\forall t \in [0,1]$, $\tilde{d}(t)\tilde{\mu}_d(t) = 0$.

Finally, let us concentrate on Eqs.(17),(18) and (20), (21). Since $\forall t \in [0,1]$, $\tilde{s}^+(t) = \tilde{s}^-(t) = 0$, the slackness conditions given by Eqs.(20) and (21) only require that $\forall t \in [0,1]$, $\tilde{\mu}_{s^+}(t), \tilde{\mu}_{s^-}(t) \geq 0$. If we now move to Eqs.(17) and (18), we get:

$$\forall t \in [0,1], \, \begin{cases} \tilde{\mu}_{s^+}(t) = \sigma^+ u'(\tilde{x}(t)) - \lambda_\Sigma \\ \tilde{\mu}_{s^-}(t) = \lambda_\Sigma - \sigma^- u'(\tilde{x}(t)) \end{cases} \tag{C.7}$$

Since λ_Σ is free (see Eq.(14)), the non-negativity of $\tilde{\mu}_{s^+}(t)$ and $\tilde{\mu}_{s^-}(t)$ therefore requires that:

$$\forall t \in [0,1], \, \lambda_\Sigma \in \left[\sigma^- u'(\tilde{x}(t)), \, \sigma^+ u'(\tilde{x}(t))\right] \tag{C.8}$$

In order to show that such a λ_Σ exists, first recall that no storage occurs if $\bar{x} \leq \tilde{x}(t) \Leftrightarrow u'(\bar{x}) \geq u'(\tilde{x}(t))$ since $u"(x) < 0$. Moreover, by Eq.(22), $\bar{x} = (u')^{-1}\left(\frac{\sigma^+}{\sigma^-} u'(\varepsilon_M R)\right)$. It follows that:

$$\max_{t \in [0,1]} \sigma^- u'(\tilde{x}(t)) \leq \sigma^- u'(\bar{x}) = \sigma^+ u'(\varepsilon_M R) \tag{C.9}$$

Since $\tilde{x}(t) = \max\{\tilde{c}, \varepsilon(t)R\} \leq \max\{\tilde{c}, \varepsilon_M R\}$ and $u"(x) < 0$, we can also say that:

$$\min_{t \in [0,1]} \sigma^+ u'(\tilde{x}(t)) \geq \sigma^+ u'(\max\{\tilde{c}, \varepsilon_M R\}) \tag{C.10}$$

Finally, since $u'(\varepsilon_M R) \leq u'(\max\{\tilde{c}, \varepsilon_M R\})$, we can define a λ_Σ which satisfies Eq.(C.8) given, for instance, by:

$$\lambda_\Sigma = \frac{\sigma^+}{2}\left(u'(\varepsilon_M R) + u'(\max\{\tilde{c}, \varepsilon_M R\})\right) \tag{C.11}$$

Appendix D. Proof of Proposition 3

Point (i). We proceed in two steps. First, we construct the threshold $\underline{x}$ and exhibit its property. In a second step, we show that there exists a unique $c_1 \in [\underline{x}, \bar{x})$. satisfying Eq.(35)

Let us first study $\underline{x}$ which solves $\int_0^1 s^+(f(\underline{x}), t) - S = 0$. By Eqs.(33) and (34), we know that $f(x)$ is a bijection from $(0, +\infty)$ into $(0, +\infty)$, so let us concentrate, by the definition of $s^+(y,t)$ (see Eq.(31)), on:

$$\varphi(y) = \frac{1}{\sigma^+} \int_0^1 \max\{\varepsilon(t)R - y, 0\}\, dt - S \tag{D.1}$$

If there exists a unique $\underline{y} > 0$ solution to $\varphi(y) = 0$ then $\underline{x} = f^{-1}(\underline{y})$ exists and is unique. Let us observe that (i) $\forall y \geq \varepsilon_M R$, $\varphi(y) = -S$ and (ii) by our assumption (see Eq.(7), $\lim_{x \to 0} \varphi(y) = \frac{R}{\sigma^+}\int_0^1 \varepsilon(t)dt - S > 0$. It therefore remains to verify that $\varphi(y)$ is strictly decreasing on $(0, \varepsilon_M R)$. From the property of our cycle $\varepsilon(t)$ on $[0,1]$, we can construct, for each $y \in (0, \varepsilon_M R)$, a unique couple of times $T_3(y) < T_M < T_4(y)$ with the property that for $i = 3, 4$, $\varepsilon(T_i)R = y$. It follows that $\varphi(y)$ becomes $\varphi(y) = \frac{1}{\sigma^+}\int_{T_3(y)}^{T_4(y)} (\varepsilon(t)R - y)\, dt - S$ and :

$$\varphi'(y) = \tfrac{1}{\sigma^+}(-\underbrace{(\varepsilon(t)R - y)|_{t=T_3(x)}}_{=0}T_3'(x) + \underbrace{(\varepsilon(t)R - y)|_{t=T_4(x)}}_{=0}T_4'(x) - (T_4(x) - T_3(x))\, y) < 0 \tag{D.2}$$

Let us now show that there exists a unique $\tilde{c}_1^b \in [\ \underline{x}\ , \bar{x})$ satisfying Eq.(35). So, define:

$$\phi_2(c) = \int_0^1 \max\{c - \varepsilon(t)R, 0\}\, dt - \sigma^- \int_0^1 \tfrac{1}{\sigma^+} \max\{\varepsilon(t)R - f(c), 0\}\, dt - D \tag{D.3}$$

From Eqs.(C.1) and (D.1), we observe that $\phi_2(c) = \phi_1(c) - \sigma^-(\varphi(f(c)) + S)$. Since, by Eq.(22), $f(\bar{x}) = \varepsilon_M R$ and $f'(c) > 0$, we can say, by Eq.(D.1), that $\forall c \geq \bar{x}$, $\varphi(f(c)) = -S$. It follows, by the condition given by Eq.(28), that:

$$\forall c \geq \bar{x},\ \phi_2(c) = \int_0^1 \max\{c - \varepsilon(t)R, 0\}\, dt - D \geq \int_0^1 \max\{\bar{x} - \varepsilon(t)R, 0\}\, dt - D > 0 \tag{D.4}$$

Now let us consider $c < \underline{x}$. Since $f'(c) > 0$, we can state, by construction of φ (see Eq.(D.1)), that $\forall c < \underline{x}$, $\varphi(f(c)) \geq \varphi(f(\underline{x})) = 0$. Moreover the condition given by Eq.((37) implies that $\forall c < \underline{x}$, $\phi_1(c) < \sigma^- S$ and it follows that $\forall c < \underline{x}$, $\phi_2(c) < \sigma^- S - \sigma^- S = 0$. We can therefore claim that there exists $\tilde{c}_1^b$ solution to $\phi_2(c) = 0$ with the property that $\tilde{c}_1^b \in [\ \underline{x}, \bar{x})$. Moreover from Eqs.(34),(C.4) and (D.2, we can say $\forall c \in (\underline{x}, \bar{x})$, $\phi_2'(c) > 0$. This ensures uniqueness.

Point (ii) to (iv). We again neglect the superscript b of the case and show that our solution candidates satisfy the sufficient optimality conditions of proposition 1. So, let us set $\tilde{x}(t) = \min\{\max\{\tilde{c}_1, \varepsilon(t)R\}, \tilde{c}_2)\}$ and $\tilde{s}^+(t) = \frac{1}{\sigma^+}\max\{\varepsilon(t)R - \tilde{c}_2), 0\}$ with $\tilde{c}_2 = f(\tilde{c}_2)$ given by Eq.(33). The choice of $\tilde{d}(t)$ and $\tilde{s}^-(t)$ (see (iv) of proposition 3) is less obvious due to multiple solutions. To solve this problem, we consider any continuous and positive selections $\tilde{d}(t)$ and $\tilde{s}^-(t)$ that verify the conditions given in (iv) of Proposition 3. [7]

The isoperimetrical constraints given by Eq.(10) are obviously satisfied. $\tilde{d}(t)$ and $\tilde{s}^-(t)$ verify their respective condition by construction of these selections. Moreover since $\tilde{c}_1 \geq \underline{x}$, the storage capacity is not binding so that $\int_0^1 \tilde{s}^+(t) \leq S$.

Now let us set $\tilde{\lambda}_D = u'(\tilde{c}_1)$, $\tilde{\lambda}_\Sigma = \sigma^+ u'(\tilde{c}_2))$ and $\tilde{\lambda}_S = 0$ (so that condition (15) is satisfied). If we now concentrate on the Lagrangian multiplier, we immediately obtain from Eqs (16) to (18) that:

$$\forall t \in [0,1],\ \tilde{\mu}_d(t) = u'(\tilde{c}_1) - u'(\tilde{x}(t)) \tag{D.5}$$

$$\forall t \in [0,1],\ \tilde{\mu}_{s^+}(t) = -\sigma^+ u'(\tilde{c}_2) + \sigma^+ u'(\tilde{x}(t)) \tag{D.6}$$

$$\forall t \in [0,1],\ \tilde{\mu}_{s^-}(t) = \sigma^+ u'(\tilde{c}_2) - \sigma^- u'(\tilde{x}(t)) \tag{D.7}$$

It remains to show that the slackness conditions given by Eqs.(19) to (21) hold.

Concerning the non-negativity of these multipliers, recall first that $\tilde{x}(t) = \min\{\max\{\tilde{c}_1, \varepsilon(t)R\}, \tilde{c}_2\}$ and that $\tilde{c}_1 < \tilde{c}_2$. This implies that (i) $\tilde{x}(t) = \tilde{c}_1$ if $\varepsilon(t)R \leq \tilde{c}_1$ and (ii) $\tilde{x}(t) = \min\{\varepsilon(t)R, \tilde{c}_2\}$ else. In any case $\tilde{x}(t) \geq \tilde{c}_1$, and, since $u''(x) < 0$, $u'(\tilde{x}(t)) \leq u'(\tilde{c}_1)$. It follows from Eq.(D.5) that:

[7] Readers may wonder whether such selections exist. The example given by:

$$\tilde{d}(t) = \tfrac{D}{D + \sigma^- \int_0^1 \tilde{s}^+(t)dt} \max\{\tilde{c}_1 - \varepsilon(t)R, 0\} \text{ and } \tilde{s}^-(t) = \tfrac{\int_0^1 \tilde{s}^+(t)dt}{D + \sigma^- \int_0^1 \tilde{s}^+(t)dt} \max\{\tilde{c}_1 - \varepsilon(t)R, 0\}$$

satisfy the conditions given in (iv) of Proposition 3.

$$\forall t \in [0,1]\,,\ \tilde{\mu}_d(t) = \begin{cases} 0 \text{ if } \varepsilon(t)R \leq \tilde{c}_1 \\ u'(\tilde{c}_1) - u'\left(\tilde{x}\left(t\right)\right) \text{ else} \end{cases} \geq 0 \tag{D.8}$$

Since $\tilde{c}_1 < \tilde{c}_2$, we can also deduce, from the definition $\tilde{x}(t)$, that (i) $\tilde{x}(t) = \tilde{c}_2$ if $\varepsilon(t)R \geq \tilde{c}_2$ and (ii) $\tilde{x}(t) < \tilde{c}_2$ else. As $u"(x) < 0$, we get $u'\left(\tilde{x}(t)\right) \geq u'(\tilde{c}_2))$. Hence, Eq.(D.6) implies that:

$$\forall t \in [0,1]\,,\ \tilde{\mu}_{s^+}(t) = \begin{cases} 0 \text{ if } \varepsilon(t)R \geq \tilde{c}_2 \\ \sigma^+\left(u'\left(\tilde{x}\left(t\right)\right) - u'(\tilde{c}_2) \text{ else} \right. \end{cases} \geq 0 \tag{D.9}$$

Finally, since $\tilde{c}_2 = (u')^{-1}\left(\frac{\sigma_-}{\sigma_+}u'(\tilde{c}_1)\right)$, Eq.(D.7) becomes:

$$\forall t \in [0,1]\,,\ \tilde{\mu}_{s^-}(t) = \sigma^- u'(\tilde{c}_1) - \sigma^- u'\left(\tilde{x}(t)\right) = \sigma^- \tilde{\mu}_d(t) \geq 0 \tag{D.10}$$

It remains to verify the three exclusion conditions. First recall that the selections, $\tilde{d}(t)$ and $\tilde{s}^-(t)$, verify:

$$\forall t \in [0,1]\,,\ d(t) + \sigma^- s^-(t) = \max\left\{\tilde{c}_1 - \varepsilon(t)R, 0\right\} \tag{D.11}$$

Since $\tilde{d}(t), \tilde{s}^-(t) \geq 0$, we can therefore claim that $\tilde{d}(t) = \tilde{s}^-(t) = 0$ if $\varepsilon(t)R \geq \tilde{c}_1$. It follows respectively by Eq.(D.8) and Eq.(D.10) that $\forall t \in [0,1]\,,\ \tilde{\mu}_d(t)\tilde{d}(t) = 0$ and $\tilde{\mu}_{s^-}(t)\tilde{s}^-(t) = 0$. Finally, since $\tilde{s}^+(t) = \frac{1}{\sigma^+}\max\left\{\varepsilon(t)R - \tilde{c}_2, 0\right\}$, we can say that $\tilde{s}^+(t) = 0$ if $\tilde{c}_2 \geq \varepsilon(t)R$. Hence by Eq.(D.9), $\forall t \in [0,1]$, $\tilde{\mu}_{s^+}(t)\tilde{s}^+(t) = 0$.

Appendix E. Proof of Proposition 4

Point (i). Let us start the existence and uniqueness of $\tilde{c}_2^c$. From Eq.(41), $\tilde{c}_2^c$ solves $\varphi(\tilde{c}_2^c) = 0$ as defined in Eq.(D.1) of Appendix D. By using the first step of point (i) of this Appendix, we immediately conclude that $\tilde{c}_2^c = f(\underline{x})$ with,$f(x)$ given by Eqs.(33).

Now observe that $\tilde{c}_1^c$ solves $\phi_3(c) = 0$ with

$$\phi_3(c) = \int_0^1 \max\left\{c - \varepsilon(t)R, 0\right\} dt - \sigma^- S - D = \phi_1(c) - \left(D + \sigma^- S\right) \tag{E.1}$$

From (i) of Appendix C, we know that $\phi_1'(c) > 0$, the same therefore hold for $\phi_3(c)$. Moreover $\phi_3(0) = -\left(D + \sigma^- S\right) < 0$ and, with the condition given by Eq.(43), $\phi_3(\underline{x}) > 0$. It follows that there exists a unique $\tilde{c}_1^c <\underline{x}$ solving this equation.

Point (ii) to (iv). To spare notation, we omit the superscript c of the case under consideration and, as in Appendix D, we set $\tilde{x}(t) = \min\left\{\max\left\{\tilde{c}_1, \varepsilon(t)R\right\}, \tilde{c}_2\right\}$ and $\tilde{s}^+(t) = \frac{1}{\sigma^+}\max\left\{\varepsilon(t)R - \tilde{c}_2, 0\right\}$ and select two continuous function $\tilde{d}(t)$ and $\tilde{s}^-(t)$ that satisfy $\forall t \in [0,1]$, $\tilde{d}(t) + \sigma^- \tilde{s}^-(t) = \max\left\{\tilde{c}_1 - \varepsilon(t)R, 0\right\}$, $\int_0^1 \tilde{d}(t)dt = D$ but with now $\int_0^1 \tilde{s}^-(t)dt = S$. It follows that the isoperimetrical constraints given by Eq.(10) are again satisfied at the difference that the storage constraint is now binding.

Since $\tilde{\lambda}_D$ and $\tilde{\lambda}_\Sigma$ are free (see Eqs.(13 and 14), we set $\tilde{\lambda}_D = u'(\tilde{c}_1)$ and , $\tilde{\lambda}_\Sigma = \sigma^- u'(\tilde{c}_1)$. We now fix $\tilde{\lambda}_S = \sigma^- u'(\tilde{c}_1) - \sigma^- u'(\tilde{c}_2)$. By Eq.(40), we know that $\tilde{\lambda}_S \geq 0$, and, because the storage constraint is binding, $S(1) = 0$. It follows that Eq.(15) is satisfied.

It follows, by Eqs.(16) to 18) that the Lagrangian multipliers are given by:

$$\tilde{\mu}_d(t) = u'(\tilde{c}_1) - u'(\tilde{x}(t)) \tag{E.2}$$

$$\tilde{\mu}_{s^+}(t) = \sigma^+\left(u'(\tilde{x}(t)) - u'(\tilde{c}_2)\right) \tag{E.3}$$

$$\tilde{\mu}_{s^-}(t) = \sigma^-\left(u'(\tilde{c}_1) - u'(\tilde{x}(t))\right) = \sigma^- \tilde{\mu}_d(t) \tag{E.4}$$

Concerning $\tilde{\mu}_d(t)$, we can, since $\tilde{c}_1 < \tilde{c}_2$, apply the same argument as in Appendix D and obtain that:

$$\forall t \in [0,1]\,,\ \tilde{\mu}_d(t) = \begin{cases} 0 \text{ if } \varepsilon(t)R \leq \tilde{c}_1 \\ u'(\tilde{c}_1) - u'(\tilde{x}(t)) \text{ else} \end{cases} \geq 0 \tag{E.5}$$

From Eq.(E.4), we immediately observe that $\tilde{\mu}_{s^-}(t) \geq 0$. Finally, concerning $\tilde{\mu}_{s^+}(t)$, let us recall that $\tilde{x}(t) = \min\{\max\{\tilde{c}_1, \varepsilon(t)R\}, \tilde{c}_2\}$ and that $\tilde{c}_1 < \tilde{c}_2$. This implies that (i) $\tilde{x}(t) = \max\{\tilde{c}_1, \varepsilon(t)R\}$ if $\varepsilon(t)R \leq \tilde{c}_2$ and (ii) $\tilde{x}(t) = \tilde{c}_2$ if $\varepsilon(t)R > \tilde{c}_2$. In any case, $\tilde{x}(t) \leq \tilde{c}_2$, or $u'(\tilde{x}(t)) \geq u'(\tilde{c}_2)$ since $u''(x) < 0$. It follows, from Eq.(E.3), that:

$$\forall t \in [0,1]\,,\ \tilde{\mu}_{s^+}(t) = \begin{cases} \sigma^+ \left(u'(\tilde{x}(t)) - u'(\tilde{c}_2)\right) \text{ if } \varepsilon(t)R < \tilde{c}_2 \\ 0 \text{ else} \end{cases} \geq 0 \tag{E.6}$$

It remains to verify the three exclusions conditions contained in Eqs.(19) to (21) hold. With the same argument as in Appendix D, we conclude that $\forall t \in [0,1]\,,\ \tilde{\mu}_d(t)\tilde{d}(t) = 0$ and $\tilde{\mu}_{s^-}(t)\tilde{s}^-(t) = 0$. Finally, since $\tilde{s}^+(t) = \frac{1}{\sigma^+}\max\{\varepsilon(t)R - \tilde{c}_2, 0\}$, we can say that $\tilde{s}^+(t) = 0$ if $\tilde{c}_2 \geq \varepsilon(t)R$. Hence by Eq.(E.6), $\forall t \in [0,1]\,,\ \tilde{\mu}_{s^+}(t)\tilde{s}^+(t) = 0$.

Appendix F. Consumption plateaus and comparative statics

Case (a): Without storage.

The derivative of Eq.(27) with respect to D is given by: [8]

$$\int_0^{\tilde{T}_1^a} \partial_D \tilde{c}^a dt + \underbrace{\left(\tilde{c}^a - \varepsilon(t)R\right)\big|_{t=\tilde{T}_1^a}}_{=0} \partial_D \tilde{T}_1^a + \int_{\tilde{T}_2^a}^1 \partial_D \tilde{c}^a dt - \underbrace{\left(\tilde{c}^a - \varepsilon(t)R\right)\big|_{t=\tilde{T}_2^a}}_{=0} \partial_D \tilde{T}_1^a - 1 = 0 \tag{F.1}$$

and from the definition of $\tilde{T}_i^a$, we know that $\left(\tilde{c}^a - \varepsilon(t)R\right)\big|_{t=\tilde{T}_1^a} = \left(\tilde{c}^a - \varepsilon(t)R\right)\big|_{t=\tilde{T}_2^a} = 0$. Moreover, $\partial_D \tilde{c}^a$ is independent of t and $\tilde{T}_2^a < 1$. It follows that:

$$\partial_D \tilde{c}^a \left(1 + \tilde{T}_1^a - \tilde{T}_2^a\right) - 1 = 0 \Leftrightarrow \partial_D \tilde{c}^a = \left(1 + \tilde{T}_1^a - \tilde{T}_2^a\right)^{-1} > 0 \tag{F.2}$$

Using a similar argument, the derivative of Eq.(27) with respect to R becomes:

$$\int_0^{\tilde{T}_1^a} \left(\partial_R \tilde{c}^a - \varepsilon(t)\right) dt + \int_{\tilde{T}_2^a}^1 \left(\partial_R \tilde{c}^a - \varepsilon(t)\right) dt = 0 \tag{F.3}$$

We deduce that:

$$\partial_R \tilde{c}^a = \left(1 + \tilde{T}_1^a - \tilde{T}_2^a\right)^{-1} \left(\int_0^{\tilde{T}_1^a} \varepsilon(t) dt + \int_{\tilde{T}_2^a}^1 \varepsilon(t) dt\right) > 0 \tag{F.4}$$

Case (b): With non binding storage.

First, let us substitute $\tilde{c}_2^b$ in Eq.(38) by its value given by Eq.(39). Using the definition of switching times $\tilde{T}_i^b$ $i = 1, \ldots, 4$, the derivative of this new identity with respect to D is:

$$\int_0^{\tilde{T}_1^b} \partial_D \tilde{c}_1^b dt + \underbrace{\left(\tilde{c}_1^b - \varepsilon(t)R\right)\big|_{t=\tilde{T}_1^b}}_{=0} \partial_D \tilde{T}_1^b + \int_{\tilde{T}_2^b}^1 \partial_D \tilde{c}_1^b dt - \underbrace{\left(\tilde{c}_1^b - \varepsilon(t)R\right)\big|_{t=\tilde{T}_2^b}}_{=0} \partial_D \tilde{T}_2^b$$

$$+ \tfrac{\sigma^-}{\sigma^+} \int_{\tilde{T}_3^b}^{\tilde{T}_4^b} f'(\tilde{c}_1^b) \partial_D \tilde{c}_1^b dt - \underbrace{\left(f(\tilde{c}_1^b) - \varepsilon(t)R\right)\big|_{t=\tilde{T}_3^b}}_{=0} \partial_D \tilde{T}_3^b + \underbrace{\left(f(\tilde{c}_1^b) - \varepsilon(t)R\right)\big|_{t=\tilde{T}_4^b}}_{=0} \partial_D \tilde{T}_4^b - 1 = 0 \tag{F.5}$$

[8] To simplify the notations, we omit in what follow the argument (D, R) of the different functions unless it is necessary for a better understanding.

Since $\tilde{c}_1^b$ and $\partial_D \tilde{c}_1^b$ are time-independent, we obtain:

$$\left(\tilde{T}_1^b + 1 - \tilde{T}_2^b + \tfrac{\sigma^-}{\sigma^+}(\tilde{T}_3^b - \tilde{T}_4^b) f'(\tilde{c}_1^b)\right) \partial_D \tilde{c}_1^b - 1 = 0 \tag{F.6}$$

Now observe that $f'(\tilde{c}_1^b)$ is given byEq.(34, and recall that $u" < 0$ and $\tilde{T}_2^b < 1$, $\tilde{T}_3^b < \tilde{T}_4^b$. It follows that:

$$\partial_D \tilde{c}_1^b = \frac{u"(\tilde{c}_2^b)}{(1+\tilde{T}_1^b - \tilde{T}_2^b)u"(\tilde{c}_2^b) + \left(\frac{\sigma^-}{\sigma^+}\right)^2 (\tilde{T}_4^b - \tilde{T}_3^b)u"(\tilde{c}_1^b)} > 0 \tag{F.7}$$

We now compute the derivative of Eq.(38) with respect to R. Using the definition of the switching times $\tilde{T}_i^b$ $i = 1, \dots, 4$ and the fact that $\tilde{c}_1^b$ and $\partial_D \tilde{c}_1^b$ are time-independent, we get that:

$$\left(\tilde{T}_1^b + 1 - \tilde{T}_2^b + \tfrac{\sigma^-}{\sigma^+}(\tilde{T}_4^b - \tilde{T}_3^b) f'(\tilde{c}_1^b)\right) \partial_R \tilde{c}_1^b - \int_0^{\tilde{T}_1^b} \varepsilon(t)dt - \int_{\tilde{T}_2^b}^1 \varepsilon(t)dt - \tfrac{\sigma^-}{\sigma^+}\int_{\tilde{T}_3^b}^{\tilde{T}_4^b} \varepsilon(t)dt = 0 \tag{F.8}$$

since $u" < 0$ and $\tilde{T}_2^b < 1$, $\tilde{T}_3^b < \tilde{T}_4^b$, we can conclude that:

$$\partial_R \tilde{c}_1^b = \frac{\left(\int_0^{\tilde{T}_1^b} \varepsilon(t)dt + \int_{\tilde{T}_2^b}^1 \varepsilon(t)d + \frac{\sigma^-}{\sigma^+}\int_{\tilde{T}_3^b}^{\tilde{T}_4^b} \varepsilon(t)dt\right) u"(\tilde{c}_2^b)}{(1+\tilde{T}_1^b - \tilde{T}_2^b)u"(\tilde{c}_2^b) + \left(\frac{\sigma^-}{\sigma^+}\right)^2 (\tilde{T}_4^b - \tilde{T}_3^b)u"(\tilde{c}_1^b)} > 0 \tag{F.9}$$

To obtain the derivatives of $\tilde{c}_2^b$ with respect to D and R, we simply use Eq.(39). By the chain rule, we get $\partial_D \tilde{c}_2^b = f'(\tilde{c}_1^b)\partial_D \tilde{c}_1^b$ and $\partial_R \tilde{c}_2^b = f'(\tilde{c}_1^b)\partial_R \tilde{c}_1^b$. Since $u" < 0$ and $\tilde{T}_2^b < 1$, $\tilde{T}_3^b < \tilde{T}_4^b$, it follows that:

$$\partial_D \tilde{c}_2^b = \frac{\frac{\sigma^-}{\sigma^+} u"(\tilde{c}_1^b)}{(1+\tilde{T}_1^b - \tilde{T}_2^b)u"(\tilde{c}_2^b) + \left(\frac{\sigma^-}{\sigma^+}\right)^2 (\tilde{T}_4^b - \tilde{T}_3^b)u"(\tilde{c}_1^b)} > 0 \tag{F.10}$$

and

$$\partial_R \tilde{c}_1^b = \frac{\left(\int_0^{\tilde{T}_1^b} \varepsilon(t)dt + \int_{\tilde{T}_2^b}^1 \varepsilon(t)d + \frac{\sigma^-}{\sigma^+}\int_{\tilde{T}_3^b}^{\tilde{T}_4^b} \varepsilon(t)dt\right) \frac{\sigma^-}{\sigma^+} u"(\tilde{c}_1^b)}{(1+\tilde{T}_1^b - \tilde{T}_2^b)u"(\tilde{c}_2^b) + \left(\frac{\sigma^-}{\sigma^+}\right)^2 (\tilde{T}_4^b - \tilde{T}_3^b)u"(\tilde{c}_1^b)} > 0 \tag{F.11}$$

Case (c): With binding storage.

Since $\tilde{c}_2^c$ is not in Eq.(45) and vice-versa for $\tilde{c}_1^c$ and Eq.(46), we can work identity per identity. So if we compute the derivative of Eq.(45) with respect to D we get:

$$\int_0^{\tilde{T}_1^c} \partial_D \tilde{c}_1^c dt + \underbrace{(\tilde{c}_1^c - \varepsilon(t)R)|_{t=\tilde{T}_1^b}}_{=0} \partial_D \tilde{T}_1^c + \int_{\tilde{T}_2^c}^1 \partial_D \tilde{c}_1^c dt - \underbrace{(\tilde{c}_1^c - \varepsilon(t)R)|_{t=\tilde{T}_2^c}}_{=0} \partial_D \tilde{T}_2^c - 1 = 0 \tag{F.12}$$

Since $\partial_D \tilde{c}_1^c$ is independent of t and $\tilde{T}_2^c < 1$, we can say that:

$$\partial_D \tilde{c}_1^c = \frac{1}{(\tilde{T}_1^c + 1 - \tilde{T}_2^c)} > 0 \tag{F.13}$$

Now by differentiating with respect to R, we have:

$$\int_0^{\tilde{T}_1^c} (\partial_R \tilde{c}_1^c - \varepsilon(t))\, dt + \int_{\tilde{T}_2^c}^1 (\partial_R \tilde{c}_1^c - \varepsilon(t))\, dt = 0 \Rightarrow \partial_R \tilde{c}_1^c = \frac{\int_0^{\tilde{T}_1^c} \varepsilon(t)dt + \int_{\tilde{T}_2^c}^1 \varepsilon(t)dt}{(\tilde{T}_1^c + 1 - \tilde{T}_2^c)} > 0 \tag{F.14}$$

Finally, working with S gives the following:

$$\int_0^{\tilde{T}_1^c} \partial_S \tilde{c}_1^c dt + \int_{\tilde{T}_2^c}^1 \partial_S \tilde{c}_1^c dt - \sigma^- = 0 \Rightarrow \partial_S \tilde{c}_1^c = \frac{\sigma^-}{(\tilde{T}_1^c + 1 - \tilde{T}_2^c)} > 0 \tag{F.15}$$

If we now move to Eq.(46), we immediately observe that $\partial_D \tilde{c}_2^c = 0$. Moreover, the derivative of Eq.(46) with respect to R is given by:

$$\int_{\tilde{T}_3^c}^{\tilde{T}_4^c} \left(\varepsilon(t) - \partial_R \tilde{c}_2^c\right) dt = 0 \Rightarrow \partial_R \tilde{c}_2^c = \frac{\int_{\tilde{T}_3^c}^{\tilde{T}_4^c} \varepsilon(t)dt}{\tilde{T}_4^c - \tilde{T}_3^c} > 0 \tag{F.16}$$

By doing the same computation with respect to S, we get:

$$\int_{\tilde{T}_3^c}^{\tilde{T}_4^c} \left(-\partial_S \tilde{c}_2^c\right) dt - \sigma^+ = 0 \Rightarrow \partial_S \tilde{c}_2^c = -\frac{\sigma^+}{\tilde{T}_4^c - \tilde{T}_3^c} < 0 \tag{F.17}$$

Appendix G. Proof of Lemma 2

(i) Preliminary remarks

From the conditions given by Eqs.(26), (28),(37) and 43), and the upper bounded on dispatchable energy given by Eq.(8), we can say that the different energy mix domains corresponding to no-storage, non-binding storage and binding storage are respectively given by:

$$\mathcal{E}^a = \left\{ e \in \mathcal{E} : \int_0^1 \max\left\{\bar{x}(e) - \varepsilon(t)R, 0\right\} dt \leq D \leq R \int_0^1 \left(\varepsilon_M - \varepsilon(t)\right) dt \right\} \tag{G.1}$$

$$\mathcal{E}^b = \left\{ e \in \mathcal{E} : \int_0^1 \max\left\{\underline{x}(e) - \varepsilon(t)R, 0\right\} dt - \sigma^- S \leq D < \int_0^1 \max\left\{\bar{x}(e) - \varepsilon(t)R, 0\right\} dt \right\} \tag{G.2}$$

$$\mathcal{E}^c = \left\{ e \in \mathcal{E} : 0 \leq D < \int_0^1 \max\left\{\underline{x}(e) - \varepsilon(t)R, 0\right\} dt - \sigma^- S \right\} \tag{G.3}$$

with $\bar{x}(e) = f^{-1}(\varepsilon_M R) = (u')^{-1}\left(\frac{\sigma^+}{\sigma^-} u'\left(\varepsilon_M R\right)\right)$ (see Eq.(22)) and $\underline{x}$ solving $\int_0^1 s^+(f(\underline{x}(e)), t) - S = 0$ (see (i) of Appendix D).

In addition, note that, by construction, $\bar{x}(e)$ and $\underline{x}(e)$ are continuous functions, but neither of them takes D as an argument. This means that the left and right terms of each inequality that define these domains are independent of D. Recall now that $\varepsilon_M R \geq \bar{x}(e) > \underline{x}(e) > 0$, it follows that :

$$R \int_0^1 \left(\varepsilon_M - \varepsilon(t)\right) dt \ \geq \int_0^1 \max\left\{\bar{x}(e) - \varepsilon(t)R, 0\right\} dt \tag{G.4}$$

and

$$\int_0^1 \max\left\{\bar{x} - \varepsilon(t)R, 0\right\} dt > \int_0^1 \max\left\{\underline{x}(e) - \varepsilon(t)R, 0\right\} dt - \sigma^- S \tag{G.5}$$

Finally, by condition (43), we know that:

$$\int_0^1 \max\left\{\underline{x}(e) - \varepsilon(t)R, 0\right\} dt - \sigma^- S > 0 \tag{G.6}$$

These observations first ensures that $\mathcal{E}^a$, $\mathcal{E}^b$ and $\mathcal{E}^c$ form a partition of $\mathcal{E}$. But they also say that (i) $\mathcal{E}^a$ has only boundary with $\mathcal{E}^b$ given by $\int_0^1 \max\left\{\bar{x}(e) - \varepsilon(t)R, 0\right\} dt = D$, (ii) the boundary between $\mathcal{E}^b$ and $\mathcal{E}^c$ is given by $\int_0^1 \max\left\{\underline{x}(e) - \varepsilon(t)R, 0\right\} dt = D + \sigma^- S$, and (iii) $\mathcal{E}^a$ and $\mathcal{E}^c$ has no common boundary. This simplifies the proof of the continuity of $\tilde{c}_1(e)$ and $\tilde{c}_2^b(e)$ since we only have to look at "what happens" on each side of the common boundary.

(ii) Continuity of $\tilde{c}_1(e)$ and $\tilde{c}_2(e)$

The boundary $\int_0^1 \max\left\{\bar{x}(e) - \varepsilon(t)R, 0\right\} dt = D$ separates $\mathcal{E}^a$ from $\mathcal{E}^b$. So let us consider two sequences $e_n^a \to e$ and $e_n^b \to e$ that verify $e_n^a \in \mathcal{E}^a$, $e_n^b \in \mathcal{E}^b$ and e is a point of this boundary and let us show that

$(\tilde{c}_1(e_n^a), \tilde{c}_2(e_n^a))_{n\in\mathbb{N}}$ and $\left(\tilde{c}_1\left(e_n^b\right), \tilde{c}_2\left(e_n^b\right)\right)_{n\in\mathbb{N}}$ converge to the same vector $(\tilde{c}_1(e)\,\tilde{c}_2(e))$. Let us start with the sequence $(\tilde{c}_1(e_n^a), \tilde{c}_2(e_n^a))_{n\in\mathbb{N}}$. By construction, $\forall e \in \mathcal{E}^a$, $\tilde{c}_2(e^a) = \varepsilon_M R$, hence $\tilde{c}_2(e_n^a) \to \tilde{c}_2(e) = \varepsilon_M R$. Concerning $\tilde{c}_1(e_n^a)$, let us first observe by Eq.(C.1) that $\phi_1(\bar{x}(e)) = 0$ so that $\tilde{c}_1(e) = \bar{x}(e)$. Moreover $\forall n$, $\tilde{c}_1(e_n^a)$ solves $\phi_1(\tilde{c}_1(e_n^a)) = 0$. Since ϕ_1 is continuous and admits a unique zero (see (i) of Appendix C), we can say that $\tilde{c}_1(e_n^a) \to \tilde{c}_1(e) = \bar{x}(e)$. Let us now move to the sequence $\left(\tilde{c}_1\left(e_n^b\right), \tilde{c}_2\left(e_n^b\right)\right)_{n\in\mathbb{N}}$. From Eq.D.3, we know that $\forall n$, $\phi_2(\tilde{c}_1\left(e_n^b\right)) = 0$, hence by continuity of ϕ_2, we have $\phi_2(\tilde{c}_1(e)) = 0$. Since e belongs to the boundary, we can say that:

$$\phi_2(\tilde{c}_1(e)) = -\sigma^- \int_0^1 \tfrac{1}{\sigma^+} \max\{\varepsilon(t)R - f(\tilde{c}_1(e)), 0\}\, dt = 0 \tag{G.7}$$

It follows that $\tilde{c}_1^b(e) \geq f^{-1}(\varepsilon_M R) = \bar{x}(e)$. But we also know (see (i) of proposition 3) that $\forall n$, $\tilde{c}_1\left(e_n^b\right) \leq \bar{x}(e_n^b) = f^{-1}(\varepsilon_M R_n)$, hence, by pushing at the limit, continuity says that $\tilde{c}_1(e) \leq \bar{x}(e) = f^{-1}(\varepsilon_M R)$. In other words we can conclude that $\tilde{c}_1\left(e_n^b\right) \to \tilde{c}_1(e) = \bar{x}(e)$. Concerning $\tilde{c}_2\left(e_n^b\right)$, let us recall, by (i) of proposition 3, that $\forall n$, $\tilde{c}_2\left(e_n^b\right) = f\left(\tilde{c}_1\left(e_n^b\right)\right)$. As $\tilde{c}_1\left(e_n^b\right) \to \bar{x}(e) = f^{-1}(\varepsilon_M R)$, it follows that $\tilde{c}_2\left(e_n^b\right) \to \varepsilon_M R$.

Let now move to the boundary $\int_0^1 \max\{\underline{x}(e) - \varepsilon(t)R, 0\}\, dt = \sigma^- S + D$ that separates $\mathcal{E}^b$ from $\mathcal{E}^c$ and let us consider two sequences $e_n^b \to e$ and $e_n^c \to e$ that verify $e_n^b \in \mathcal{E}^b$, $e_n^c \in \mathcal{E}^c$ and e is a point of this boundary. We first start with the study $\left(\tilde{c}_1\left(e_n^b\right), \tilde{c}_2\left(e_n^b\right)\right)_{n\in\mathbb{N}}$. So let us observe, by Eq.(D.1), that $\varphi\left(f(\underline{x}(e)\right) = 0$. By Eq.D.3 and the definition of the boundary under consideration, we can claim that:

$$\phi_2(\underline{x}(e)) = \int_0^1 \max\{\underline{x}(e) - \varepsilon(t)R, 0\}\, dt - \sigma^- \left(\varphi(f(\ \underline{x}(e))) + S\right) = 0 \tag{G.8}$$

It follows that $\tilde{c}_2(e) = \underline{x}(e)$. Uniqueness of the solution (see (i) of Appendix D) and continuity of ϕ_2 ensure that $\tilde{c}_1\left(e_n^b\right) \to \tilde{c}_1(e) = \underline{x}(e)$. Moreover, by continuity of $f(c_1)$, we get that $\tilde{c}_2\left(e_n^b\right) = f\left(\tilde{c}_1\left(e_n^b\right)\right) \to f\left(\underline{x}(e)\right)$. Now, let us concentrate on $(\tilde{c}_1(e_n^c), \tilde{c}_2(e_n^c))_{n\in\mathbb{N}}$. From Eq.**??** , we know that $\forall n$, $\phi_3(\tilde{c}_1(e_n^c)) = 0$. By continuity of ϕ_3, this implies, at the limit, that $\tilde{c}_1(e)$ solves:

$$\phi_3(\tilde{c}_1(e)) = \int_0^1 \max\{\tilde{c}_1^c(e) - \varepsilon(t)R, 0\}\, dt - \sigma^- S - D = 0 \tag{G.9}$$

As the solution to this equation is unique (see (i) of Appendix E), we deduce, from the definition of the boundary under consideration that $\tilde{c}_1(e) = \underline{x}(e)$. Moreover by continuity of ϕ_3 and f, $\tilde{c}_1(e_n^c) \to \underline{x}(e)$. Finally, recall from (i) of proposition 4 that $\tilde{c}_2^c(e_n^c) = f\left(\underline{x}(e_n^c)\right)$, hence, by continuity, $\tilde{c}_2^c(e_n^c) \to \tilde{c}_2^c(e) = f\left(\underline{x}(e)\right)$

At that point we can conclude that $\tilde{c}_1(x)$ and $\tilde{c}_2(x)$ are continuous functions. As the consumption path writes $\tilde{x}(t,e) = \min\{\max\{\tilde{c}_1(e), \varepsilon(t)R\}, \tilde{c}_2(e)\}$ this one is also a continuous function in both t and e and the same holds for the consumer welfare $W(e) = \int_0^1 u\left(\tilde{x}(t,e)\right) dt$.

(iii) The welfare function $W(e)$

Now we calculate the gradient of $W(e)$. We begin by studying this gradient in each of the three cases and verify, in a final step, that this one is globally continuous. The notation, particularly concerning switching times, is the same as in Appendix F. We omit the arguments of the various functions in order to lighten the notation.

Case (a) : Without storage

In this case $W^a = \left(\tilde{T}_1^a + 1 - \tilde{T}_2^a\right) u\left(\tilde{c}^a\right) + \int_{\tilde{T}_1^a}^{\tilde{T}_2^a} u\left(\varepsilon(t)R\right) dt$. Taking the derivative with respect to D, we get the following.

$$\begin{aligned}\partial_D W^a &= \left(\partial_D \tilde{T}_1^a - \partial_D \tilde{T}_2^a\right) u(\tilde{c}_1^a) + \left(\tilde{T}_1^a + 1 - \tilde{T}_2^a\right) u'(\tilde{c}_1^a)\partial_D \tilde{c}_1^a - \partial_D \tilde{T}_1^a u\left(\varepsilon(\tilde{T}_1^a)R\right) + \partial_D \tilde{T}_2^a u\left(\varepsilon(\tilde{T}_2)R\right)\\ &= \left(\tilde{T}_1^a + 1 - \tilde{T}_2^a\right) u'(\tilde{c}_1^a)\partial_D \tilde{c}_1^a \text{ (since } \varepsilon(\tilde{T}_i^a)R = \tilde{c}_1^a \text{ for } i = 1,2)\\ &= u'(\tilde{c}_1^a) \text{ (see } \partial_D \tilde{c}_1^a \text{ in Tab. 1)}\end{aligned} \tag{G.10}$$

Working now with R, we obtain:

$$\begin{aligned}
\partial_R W^a &= \left(\partial_R \tilde{T}_1^a - \partial_R \tilde{T}_2^a\right) u(\tilde{c}_1^a) + \left(\tilde{T}_1^a + 1 - \tilde{T}_2^a\right) u'(\tilde{c}_1^a)\partial_R \tilde{c}^a + \int_{\tilde{T}_1^a}^{\tilde{T}_2^a} u'(\varepsilon(t)R)\varepsilon(t)dt \\
&\quad -\partial_R \tilde{T}_1^a u(\varepsilon(\tilde{T}_1^a)R) + \partial_R \tilde{T}_2^a u'(\varepsilon(\tilde{T}_2^a)R) \\
&= \left(1 + \tilde{T}_1^a - \tilde{T}_2^a\right) u'(\tilde{c}_1^a)\partial_R \tilde{c}_1^a + \int_{\tilde{T}_1^a}^{\tilde{T}_2^a} u'(\varepsilon(t)R)\varepsilon(t)dt \text{ (since } \varepsilon(\tilde{T}_1^a)R = \varepsilon(\tilde{T}_2^a)R = \tilde{c}_1^a) \\
&= u'(\tilde{c}_1^a)\left(\int_0^{\tilde{T}_1^a} \varepsilon(t)dt + \int_{\tilde{T}_2^a}^1 \varepsilon(t)dt\right) + \int_{\tilde{T}_1^a}^{\tilde{T}_2^a} u'(\varepsilon(t)R)\varepsilon(t)dt \text{ (see } \partial_D \tilde{c}_1^a \text{ in Tab. 1)} \\
&= \int_0^1 u'\left(\max\{\tilde{c}_1^a, \varepsilon(t)R\}\right)\varepsilon(t)dt \qquad \text{(G.11)}
\end{aligned}$$

Finally, by construction, $\partial_S W^a = 0$.
Case (b) : With non binding storage

In this case $W^b = \left(\tilde{T}_1^b + 1 - \tilde{T}_2^b\right) u\left(\tilde{c}_1^b\right) + \left(T_4^b - T_3^b\right) u\left(\tilde{c}_2^b\right) + \int_{\tilde{T}_1^b}^{\tilde{T}_3^b} u\left(\varepsilon(t)R\right) dt + \int_{\tilde{T}_4^b}^{\tilde{T}_2^b} u\left(\varepsilon(t)R\right) dt$. Taking the derivative with respect to D, we get the following.

$$\begin{aligned}
\partial_D W^b &= \left(\partial_D \tilde{T}_1^b - \partial_D \tilde{T}_2^b\right) u\left(\tilde{c}_1^b\right) + \left(\tilde{T}_1^b + 1 - \tilde{T}_2^b\right) u'\left(\tilde{c}_1^b\right) \partial_D \tilde{c}_1^b + \left(\partial_D T_4^b - \partial_D T_3^B\right) u\left(\tilde{c}_2^b\right) \\
&\quad + \left(T_4^b - T_3^b\right) u'\left(\tilde{c}_2^b\right) \partial_D \tilde{c}_2^b - \partial_D \tilde{T}_1^b u\left(\varepsilon(\tilde{T}_1^b)R\right) + \partial_D \tilde{T}_3^b u\left(\varepsilon(\tilde{T}_3^b)R\right) \\
&\quad -\partial_D \tilde{T}_4^b u\left(\varepsilon(\tilde{T}_4)R\right) + \partial_D \tilde{T}_2^b u\left(\varepsilon(\tilde{T}_2^b)R\right) \qquad \text{(G.12)}
\end{aligned}$$

Now remark, by construction, that $\varepsilon(\tilde{T}_i^b)R = \tilde{c}_1^b$ for $i = 1, 2$ and $\varepsilon(\tilde{T}_i^b)R = \tilde{c}_2^b$ for $i = 3, 4$. It follows that:

$$\begin{aligned}
\partial_D W^b &= \left(\tilde{T}_1^b + 1 - \tilde{T}_2^b\right) u'\left(\tilde{c}_1^b\right) \partial_D \tilde{c}_1^b + \left(T_4^b - T_3^b\right) u'\left(\tilde{c}_2^b\right) \partial_D \tilde{c}_2^b \\
&= u'\left(\tilde{c}_1^b\right) \frac{\left(\tilde{T}_1^b + 1 - \tilde{T}_2^b\right) u"\left(\tilde{c}_2^b\right) + \left(T_4^b - T_3^b\right) \frac{\sigma^-}{\sigma^+} u"\left(\tilde{c}_1^b\right) \frac{u'\left(\tilde{c}_2^b\right)}{u'\left(\tilde{c}_s^b\right)}}{\left(1 + \tilde{T}_1^b - \tilde{T}_2^b\right) u"\left(\tilde{c}_2^b\right) + \left(\frac{\sigma^-}{\sigma^+}\right)^2 (\tilde{T}_4^b - \tilde{T}_3^b) u"\left(\tilde{c}_1^b\right)} \text{ (see } \partial_D \tilde{c}_1^b,\ \partial_D \tilde{c}_1^b \text{ in Tab. 2)} \\
&= u'\left(\tilde{c}_1^b\right) \text{ (since } \frac{u'\left(\tilde{c}_2^b\right)}{u'\left(\tilde{c}_1^b\right)} = \frac{\sigma^-}{\sigma^+}) \qquad \text{(G.13)}
\end{aligned}$$

Taking the derivative with respect to D, we get the following.

$$\begin{aligned}
\partial_R W^b &= \left(\partial_R \tilde{T}_1^b - \partial_R \tilde{T}_2^b\right) u\left(\tilde{c}_1^b\right) + \left(\tilde{T}_1^b + 1 - \tilde{T}_2^b\right) u'\left(\tilde{c}_1^b\right) \partial_R \tilde{c}_1^b + \left(\partial_R T_4^b - \partial_R T_3^b\right) u\left(\tilde{c}_2^b\right) \\
&\quad + \left(T_4^b - T_3^b\right) u'\left(\tilde{c}_2^b\right) \partial_R \tilde{c}_2^b + \int_{\tilde{T}_1^b}^{\tilde{T}_3^b} u'\left(\varepsilon(t)R\right)\varepsilon(t)dt - \partial_R \tilde{T}_1^b u\left(\varepsilon(\tilde{T}_1^b)R\right) \\
&\quad + \partial_R \tilde{T}_3^b u\left(\varepsilon(\tilde{T}_3^b)R\right) + \int_{\tilde{T}_4^b}^{\tilde{T}_2^b} u'\left(\varepsilon(t)R\right)\varepsilon(t)dt - \partial_R \tilde{T}_4^b u\left(\varepsilon(\tilde{T}_4^b)R\right) + \partial_R \tilde{T}_2^b u\left(\varepsilon(\tilde{T}_2^b)R\right) \qquad \text{(G.14)}
\end{aligned}$$

As $\varepsilon(\tilde{T}_i^b)R = \tilde{c}_1^b$ for $i = 1, 2$ and $\varepsilon(\tilde{T}_i^b)R = \tilde{c}_2^b$ for $i = 3, 4$, we obtain the following:

$$\begin{aligned}
\partial_R W^b = \underbrace{\left(\tilde{T}_1^b + 1 - \tilde{T}_2^b\right) u'\left(\tilde{c}_1^b\right) \partial_R \tilde{c}_1^b + \left(T_4^b - T_3^b\right) u'\left(\tilde{c}_2^b\right) \partial_R \tilde{c}_2^b}_{=A} \\
+ \int_{\tilde{T}_1^b}^{\tilde{T}_3^b} u'\left(\varepsilon(t)R\right)\varepsilon(t)dt + \int_{\tilde{T}_4^b}^{\tilde{T}_2^b} u'\left(\varepsilon(t)R\right)\varepsilon(t)dt \qquad \text{(G.15)}
\end{aligned}$$

If we now use the results of Tab.2 and the fact that $\frac{u'\left(\tilde{c}_2^b\right)}{u'\left(\tilde{c}_1^b\right)} = \frac{\sigma^-}{\sigma^+}$, A becomes:

$$\begin{aligned}
A &= \frac{\left(\int_0^{\tilde{T}_1^b} \varepsilon(t)dt + \int_{\tilde{T}_2^b}^1 \varepsilon(t)d + \frac{\sigma^-}{\sigma^+} \int_{\tilde{T}_3^b}^{\tilde{T}_4^b} \varepsilon(t)dt\right) u'\left(\tilde{c}_1^b\right)\left(u"\left(\tilde{c}_2^b\right)\left(\tilde{T}_1^b + 1 - \tilde{T}_2^b\right) + \frac{\sigma^-}{\sigma^+} u"\left(\tilde{c}_1^b\right)\left(T_4^b - T_3^b\right)\frac{u'\left(\tilde{c}_2^b\right)}{u'\left(\tilde{c}_1^b\right)}\right)}{\left(1 + \tilde{T}_1^b - \tilde{T}_2^b\right)u"\left(\tilde{c}_2^b\right) + \left(\frac{\sigma^-}{\sigma^+}\right)^2 \left(\tilde{T}_4^b - \tilde{T}_3^b\right)u"\left(\tilde{c}_1^b\right)} \\
&= \left(\int_0^{\tilde{T}_1^b} \varepsilon(t)dt + \int_{\tilde{T}_2^b}^1 \varepsilon(t)dt + \frac{\sigma^-}{\sigma^+} \int_{\tilde{T}_3^b}^{\tilde{T}_4^b} \varepsilon(t)dt\right) u'\left(\tilde{c}_1^b\right) \\
&= \int_0^{\tilde{T}_1^b} u'\left(\tilde{c}_1^b\right)\varepsilon(t)dt + \int_{\tilde{T}_2^b}^1 u'\left(\tilde{c}_1^b\right)\varepsilon(t)d + \int_{\tilde{T}_3^b}^{\tilde{T}_4^b} u'\left(\tilde{c}_2^b\right)\varepsilon(t)dt
\end{aligned} \tag{G.16}$$

We can therefore say that:

$$\begin{aligned}
\partial_R W^b &= \int_0^{\tilde{T}_1^b} u'\left(\tilde{c}_1^b\right)\varepsilon(t)dt + \int_{\tilde{T}_2^b}^1 u'\left(\tilde{c}_1^b\right)\varepsilon(t)d + \int_{\tilde{T}_3^b}^{\tilde{T}_4^b} u'\left(\tilde{c}_2^b\right)\varepsilon(t)dt \\
&\quad + \int_{\tilde{T}_1^b}^{\tilde{T}_3^b} u'\left(\varepsilon(t)R\right)\varepsilon(t)dt + \int_{\tilde{T}_4^b}^{\tilde{T}_2^b} u'\left(\varepsilon(t)R\right)\varepsilon(t)dt \\
&= \int_0^1 u'\left(\min\left\{\max\left\{\tilde{c}_1^b, \varepsilon(t)R\right\}, \tilde{c}_2^b\right\}\right)\varepsilon(t)dt \quad \text{(since } \tilde{c}_1^b \leq \tilde{c}_2^b\text{)}
\end{aligned} \tag{G.17}$$

Finally, by construction, $\partial_S W^b = 0$.

Case (c) : With binding storage

Like in the previous case $W^c = \left(\tilde{T}_1^c + 1 - \tilde{T}_2^c\right) u\left(\tilde{c}_1^c\right) + \left(T_4^c - T_3^c\right) u\left(\tilde{c}_2^c\right) + \int_{\tilde{T}_1^c}^{\tilde{T}_3^c} u\left(\varepsilon(t)R\right)dt + \int_{\tilde{T}_4^c}^{\tilde{T}_2^c} u\left(\varepsilon(t)R\right)dt$. So if we take the derivative with respect to D, we obtain the same result as in Eq.(G.13). The same simplifications apply since $\varepsilon(\tilde{T}_i^c)R = \tilde{c}_1^c$ for $i = 1, 2$ and $\varepsilon(\tilde{T}_i^c)R = \tilde{c}_2^c$ for $i = 3, 4$. But $\partial_D \tilde{c}_1^c$ and $\partial_D \tilde{c}_2^c$ are now taken from Tab. 3. It follows that:

$$\partial_D W^c = \left(\tilde{T}_1^c + 1 - \tilde{T}_2^c\right) u'\left(\tilde{c}_1^c\right)\partial_D \tilde{c}_1^c + \left(T_4^c - T_3^c\right) u'\left(\tilde{c}_2^c\right)\partial_D \tilde{c}_2^c = u'\left(\tilde{c}_1^c\right) \tag{G.18}$$

If we now move to the derivative with respect to R, the same simplifications apply as in Eq.(G.15) and we get:

$$\begin{aligned}
\partial_R W^c &= \left(\tilde{T}_1^c + 1 - \tilde{T}_2^c\right) u'\left(\tilde{c}_1^c\right)\partial_R \tilde{c}_1^c + \left(T_4^c - T_3^c\right) u'\left(\tilde{c}_2^c\right)\partial_R \tilde{c}_2^c \\
&\quad + \int_{\tilde{T}_1^c}^{\tilde{T}_3^c} u'\left(\varepsilon(t)R\right)\varepsilon(t)dt + \int_{\tilde{T}_4^c}^{\tilde{T}_2^c} u'\left(\varepsilon(t)R\right)\varepsilon(t)dt
\end{aligned} \tag{G.19}$$

By using now the results of Tab. 3, we observe that:

$$\begin{aligned}
\partial_R W^c &= \int_0^{\tilde{T}_1^c} u'\left(\tilde{c}_1^c\right)\varepsilon(t)dt + \int_{\tilde{T}_2^c}^1 u'\left(\tilde{c}_1^c\right)\varepsilon(t)dt + \int_{\tilde{T}_3^c}^{\tilde{T}_4^c} u'\left(\tilde{c}_2^c\right)\varepsilon(t)dt \\
&\quad + \int_{\tilde{T}_1^c}^{\tilde{T}_3^c} u'\left(\varepsilon(t)R\right)\varepsilon(t)dt + \int_{\tilde{T}_4^c}^{\tilde{T}_2^c} u'\left(\varepsilon(t)R\right)\varepsilon(t)dt \\
&= \int_0^1 u'\left(\min\left\{\max\left\{\tilde{c}_1^c, \varepsilon(t)R\right\}, \tilde{c}_2^c\right\}\right)\varepsilon(t)dt \quad \text{(since } \tilde{c}_1^b \leq \tilde{c}_2^b\text{)}
\end{aligned} \tag{G.20}$$

Finally, if we move to the derivative with respect to S, we obtain by apply the same simplifications that:

$$\partial_S W^c = \left(\tilde{T}_1^c + 1 - \tilde{T}_2^c\right) u'\left(\tilde{c}_1^c\right)\partial_S \tilde{c}_1^c + \left(T_4^c - T_3^c\right) u'\left(\tilde{c}_2^c\right)\partial_S \tilde{c}_2^c \tag{G.21}$$

Using again Tab. 3, we can say that:

$$\partial_S W^c = \sigma^- u'\left(\tilde{c}_1^c\right) - \sigma^+ u'\left(\tilde{c}_2^c\right) = u'\left(\tilde{c}_1^c\right)\sigma^+\left(\frac{\sigma^-}{\sigma^+} - \frac{u'\left(\tilde{c}_2^c\right)}{u'\left(\tilde{c}_1^c\right)}\right) \tag{G.22}$$

Pooling together these results

By Eqs.(G.10), (G.13) and (G.18), $\partial_D V = u'\left(\tilde{c}_1\right)$. Moreover by our extension of $\tilde{c}_2$ to case (a), we can say by Eqs.(G.11), (G.17) and (G.20) that $\partial_R W = \int_0^1 u'\left(\min\left\{\max\left\{\tilde{c}_1, \varepsilon(t)R\right\}, \tilde{c}_2\right\}\right)\varepsilon(t)dt$. By Eqs. (22), (32) and (40), we know that in case (a) $\frac{\sigma^-}{\sigma^+} < \frac{u'(\varepsilon_M R)}{u'\left(\tilde{c}_1^a\right)}$, in case (b) $\frac{\sigma^-}{\sigma^+} = \frac{u'\left(\tilde{b}_2^c\right)}{u'\left(\tilde{b}_1^c\right)}$ and in case (c) $\frac{\sigma^-}{\sigma^+} > \frac{u'\left(\tilde{c}_2^c\right)}{u'\left(\tilde{c}_1^c\right)}$. We can therefore say that $\partial_S W = \max\left\{u'\left(\tilde{c}_1\right)\sigma^+\left(\frac{\sigma^-}{\sigma^+} - \frac{u'\left(\tilde{c}_2\right)}{u'\left(\tilde{c}_1\right)}\right), 0\right\}$. Finally, we know that $\tilde{c}_1$ and $\tilde{c}_2$ are continuous functions, meaning that the gradient of W is also continuous.

Appendix H. Proof of Proposition 5

The proofs of point (i) and (iii) directly follow from our discussion while, for point (ii), it remains to show that $c^{ns}(e) \leq \tilde{c}_1(e)$ and that Eq.(52) holds.

(i) $c^{ns}(e) \leq \tilde{c}_1(e)$

Let us first recall the definition of $c^{ns}(e)$. We consider an energy mix $e \in \mathcal{E}^b \cup \mathcal{E}^c$ and assume that storage is forbidden. This consumption plateau therefore solves $\phi_1(c) = \int_0^1 \max\left\{c - \varepsilon(t)R, 0\right\}dt - D = 0$. From our previous results in (i) of Appendix C and the fact that $\lim_{c\to 0}\phi_1(c) = -D$, we can conclude that $c^{ns}(e)$ is well-defined for each $e \in \mathcal{E}^b \cup \mathcal{E}^c$.

Now let us recall, respectively, from (i) of Appendix D and Appendix E that $\tilde{c}_1(e)$ either solves $\phi_2(c) = \phi_1(c) - \sigma^-\left(\varphi(f(c)) + S\right) = 0$ or $\phi_3(c) = \phi_1(c) - \sigma^- S = 0$. Now observe that $\phi_2(c^{ns}(e)) = -\sigma^-\left(\varphi(f(c^{ns}(e))) + S\right) \leq 0$ and $\phi_3(c^{ns}(e)) = -\sigma^- S \leq 0$. As both $\phi_2(c)$ and $\phi_3(c)$ are increasing in c, we can conclude that $\forall e \in \mathcal{E}^b \cup \mathcal{E}^c$, $c^{ns}(e) \leq \tilde{c}_1(e)$.

(ii) The construction of Eq.(52)

As usually, let us first introduce the following switching times $T_i^{ns}(e) = \varepsilon^{-1}\left(\frac{c^{ns}(e)}{R}\right)$, $i = 1, 2$, $\tilde{T}_i(e) = \varepsilon^{-1}\left(\frac{\tilde{c}_1(e)}{R}\right)$, $i = 1, 2$ and $\tilde{T}_i^b(e) = \varepsilon^{-1}\left(\frac{\tilde{c}_2(e)}{R}\right)$, $i = 3, 4$. From (i) and since we know that $\tilde{c}_1(e) < \tilde{c}_2(e)$, we have that, $T_1^{ns}(e) < \tilde{T}_1(e) < \tilde{T}_3(e) < \tilde{T}_4(e) < \tilde{T}_2(e) < T_2^{ns}(e)$.

Let us now move to the computation of Eq.(52). By neglecting e in the later notation, we know, by Eq.(50), that:

$$\begin{aligned}
\frac{\Delta W^{ns}}{\Delta R} &= -u'\left(c^{ns}\right)\int_0^1 \varepsilon(t)dt + \int_0^{T_1^{ns}} u'\left(c^{ns}\right)\varepsilon(t)dt + \int_{T_1^{ns}}^{T_2^{ns}} u'\left(\varepsilon(t)R\right)\varepsilon(t)dt + \int_{T_2^{ns}}^1 u'\left(c^{ns}\right)\varepsilon(t)dt \\
&= -u'\left(c^{ns}\right)\int_{T_1^{ns}}^{T_2^{ns}} \varepsilon(t)dt + \int_{T_1^{ns}}^{T_2^{ns}} u'\left(\varepsilon(t)R\right)\varepsilon(t)dt
\end{aligned} \tag{H.1}$$

and

$$\begin{aligned}
\frac{\Delta W}{\Delta R} &= -u'\left(\tilde{c}_1\right)\int_0^1 \varepsilon(t)dt + \int_0^{\tilde{T}_1} u'\left(\tilde{c}_1\right)\varepsilon(t)dt + \int_{\tilde{T}_1}^{\tilde{T}_3} u'\left(\varepsilon(t)R\right)\varepsilon(t)dt + \int_{\tilde{T}_3}^{\tilde{T}_4} u'\left(\tilde{c}_2\right)\varepsilon(t)dt \\
&\quad + \int_{\tilde{T}_4}^{\tilde{T}_2} u'\left(\varepsilon(t)R\right)\varepsilon(t)dt + \int_{\tilde{T}_2}^1 u'\left(\tilde{c}_1\right)\varepsilon(t)dt \\
&= -u'\left(\tilde{c}_1\right)\int_{\tilde{T}_1}^{\tilde{T}_2} \varepsilon(t)dt + \int_{\tilde{T}_1}^{\tilde{T}_3} u'\left(\varepsilon(t)R\right)\varepsilon(t)dt + \int_{\tilde{T}_3}^{\tilde{T}_4} u'\left(\tilde{c}_2\right)\varepsilon(t)dt + \int_{\tilde{T}_4}^{\tilde{T}_2} u'\left(\varepsilon(t)R\right)\varepsilon(t)dt
\end{aligned} \tag{H.2}$$

By computing the difference, we get after reorganization and simplification that:

$$\frac{\Delta W^{ns}}{\Delta R}-\frac{\Delta W}{\Delta R}=\int_{T_1^{ns}}^{\tilde{T}_1}\left(u'\left(\varepsilon(t)R\right)-u'\left(c^{ns}\right)\right)\varepsilon(t)dt+\left(u'\left(\tilde{c}_1\right)-u'\left(c^{ns}\right)\right)\int_{\tilde{T}_1}^{\tilde{T}_2}\varepsilon(t)dt$$
$$+\int_{\tilde{T}_2}^{T_2^{ns}}\left(u'\left(\varepsilon(t)R\right)-u'\left(c^{ns}\right)\right)\varepsilon(t)dt+\int_{\tilde{T}_3}^{\tilde{T}_4}\left(u'\left(\varepsilon(t)R\right)-u'\left(\tilde{c}_2\right)\right)\varepsilon(t)dt \tag{H.3}$$

By grouping together the three first terms (recall that $c^{ns}\leq\tilde{c}_1$ and using the definition of $\tilde{c}_2$, we finally get that:

$$\frac{\Delta W^{ns}}{\Delta R}-\frac{\Delta W}{\Delta R}=\int_0^1\left(u'\left(\max\left\{\min\left\{\tilde{c}_1,\varepsilon(t)R\right\},c^{ns}\right\}\right)-u'\left(c^{ns}\right)\right)\varepsilon(t)dt$$
$$+\int_0^1\left(u'\left(\max\left\{\tilde{c}_2,\varepsilon(t)R\right\}\right)-u'\left(\tilde{c}_2\right)\right)\varepsilon(t)dt \tag{H.4}$$

Appendix I. The representative agent assumption

Consider the following optimization problem:

$$U(X)=\max_{(x_i)_{i=1}^n\geq 0,}\sum_{i=1}^n u_i(x)\text{ s.t. }\sum_{i=1}^n x_i=X \tag{I.1}$$

Assume that $\forall i$, $u_i'(x)>0$, $u_i"(x)>0$, $\lim_{x\to 0}u_i'(x)=+\infty$ and $\lim_{x\to+\infty}u_i'(x)=0$ and let us show that $U(X)$ has the same properties. If λ denotes the Lagrangian multiplier, the first order necessary and sufficient optimality conditions are given by $\forall i=1,\ldots,n$, $u_i'(x_i)=\lambda$ and $\sum_{i=1}^n x_i=X$. By solving this set of equations, we construct $(x_i(X))_{i=1}^n$ and $\lambda(X)$. It follows that:

$$U'(X)=\sum_{i=1}^n u_i'\left(x_i(X)\right)x_i'(X)=\lambda(X)\sum_{i=1}^n x_i'(X)=\lambda(X)>0 \tag{I.2}$$

since $\sum_{i=1}^n x_i(X)=X$ and so $\sum_{i=1}^n x_i'(X)=1$. Moreover, if we differentiate the first order conditions, we get:

$$\begin{cases}\forall i=1,\ldots,n,\quad u_i^{"}(x_i)dx_i=d\lambda\\ \sum_{i=1}^n dx_i=dX\end{cases} \tag{I.3}$$

$$\Leftrightarrow\begin{cases}\forall i=1,\ldots,n,\quad \frac{dx_i}{dX}=\frac{1}{u_i^{"}(x_i)}u_i^{"}(x_i)\left(\sum_{i=1}^n\frac{1}{u_i^{"}(x_i)}\right)^{-1}>0\\ \frac{d\lambda}{dX}=\left(\sum_{i=1}^n\frac{1}{u_i^{"}(x_i)}\right)^{-1}<0\end{cases} \tag{I.4}$$

It follows that $U"(X)=\lambda'(X)<0$. It remains to verify the Inada conditions. First suppose that $X\to 0$, then, by the constraint, $\forall i$, $x_i\to 0$. As $\lim_{x\to 0}u_i'(x)=+\infty$, the first order conditions say that $\lambda\to+\infty$ and Eq.(I.2) leads to $\lim_{X\to 0}U'(X)=+\infty$. Now, suppose that $X\to+\infty$ then at least of one i, $x_i\to+\infty$. As $\lim_{x\to+\infty}u_i'(x)=0$, then, with the same argument, $\lambda\to 0$ and $\lim_{X\to+\infty}U'(X)=0$.