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On Information Aggregation in International Alliances

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Abstract

I develop a model of strategic communication to study information aggregation in an alliance between multiple players. An alliance exhibits four features: i) imperfect private information among players; ii) substitutability in actions; iii) constraints on the action set; and iv) preference heterogeneity (biases). The main result of the paper derives conditions for full information aggregation within the alliance under a public communication protocol. Full information aggregation ensues as long as players' biases are sufficiently *cohesive* with respect to the constraints on the action set. When players can (costlessly) choose an action set ex ante, I derive the precise conditions on the minimal action set such that there is full information aggregation. Comparative statics uncovers two sources for the differences in the size of the minimal action set between players: bias over outcomes (preference effect) and degree of interdependency (interdependency effect). The results are discussed in the context of burden sharing incentives during military interventions within NATO.

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1 Introduction

Information sharing is an integral part of international alliances. Countries in an alliance typically have common objectives (e.g. collective defense, joint military intervention, peacekeeping), and share private information in order to coordinate their actions towards achieving these shared objectives. In order to share and aggregate information efficiently, contributing to the alliance (e.g. fiscal or military resources) plays an important role.[1] Take the example of NATO's joint military intervention in Libya to overthrow the Qaddafi regime. The NATO alliance coordinated the actions of 18 countries under an unified command, including partners in the region and by sharing the military burden among its members (Daalder and Stavridis, 2012).

However, some pertinent problems remain. For example, a majority of the military resources for the Libya war—and NATO in general—were contributed by the United States while European allies were heavily reliant on them. In fact, this *contributions gap* has been glaringly evident in NATO since the early 2000's. In 2007, the U.S. contributed roughly 68% of the total defense spending of NATO and this increased to 72% by 2012.[2] In this context, some pertinent questions arise with respect to information aggregation in alliances. Specifically, how is information aggregation within an alliance affected by the availability of resources among the allies? Given the need for efficient information aggregation and coordination, what is the relationship between the individual preferences of members and their contributions to the alliance? What factors might determine the divergence in ex ante contributions to the alliance?

Surprisingly, theoretical literature on informational incentives in international alliances is limited, if not non-existent. In this paper, I develop a stylized model to study information aggregation incentives in international alliances *á* la NATO. The main findings of the paper are twofold. First,

[1] NATO's *2010 Strategic Concept* document captures this idea succinctly: "The alliance will engage actively to enhance international security, through partnership with relevant countries and other international organizations; by contributing actively to arms control, non-proliferation and disarmament."; further, it adds "Any security issue of interest to any ally can be brought to the NATO table, to share information, exchange views and, where appropriate, forge common approaches."

[2] As of 2012, U.S. spent over four per cent of its GDP on defense compared to European counterparts that averaged around 1.6%, and some of whom spent less than one per cent towards defense spending. This growing *transatlantic gap* in defense spending was duly pointed out by the Secretary General of NATO in the 2012 Annual report.

the model provides an intuitive closed form characterization of the conditions for full information aggregation with constraints. Second, the paper identifies two possible sources for the *contributions gap* to emerge: *preference heterogeneity* and *degree of interdependency* in the alliance.

An alliance between a set of N players incorporates four key features: information asymmetry about an unobservable state of nature, strategic interdependency in actions, preference heterogeneity (*biases*) over final outcomes, and constraints on actions. The setup induces a multi-player version of a coordination game (see e.g. Venkatesh, 2019) in which each player has a joint action function that depends on the actions of all other players in the alliance. Further, each player faces a quadratic loss whenever the joint action function deviates from the players' expectation of the (biased) state of nature.

Information about the underlying state of the world (e.g. intelligence on the location of military targets) is *soft* in nature and *disaggregated* among the players. That is, conditional on the draw of the (unobserved) state of the world, each player receives a private signal—*low* or *high*—that imperfectly informs them about the true underlying state. The goal of the alliance is to fully aggregate the private information of all the players. In the communication stage, each player, publicly and simultaneously, sends a cheap talk message about their private information to the alliance. After the communication stage, conditional on the private information and the messages exchanged, each player takes an action, where actions of players are substitutable.

The first theorem characterizes the players' actions for any generic communication equilibrium of the game when the actions are unconstrained. For any generic equilibrium, the communication round splits the players into two sets: *truthful* and *babbling*. Since the communication protocol is public, in equilibrium, all players possess the same set of truthful signals. This implies that the posterior expectation of all the truthful players is the same while the babbling players possess an additional (unrevealed) signal. Exploiting this information structure induced by communication, I formulate a guessing strategy such that each players' action is simply a linear function of their own bias, the weighted sum of all biases, and the posterior expectation of the underlying state conditional on the information (truthful signals) available.

The second theorem provides the *main result* of the paper: the set of equilibrium conditions for full information aggregation (henceforth FIA) in the presence of action constraints. The characterization of equilibrium actions without constraints is an useful starting point to derive the conditions for FIA. Specifically, I impose (homogeneous) constraints on the actions of players and extend the set of truthful players to N. The main finding is that information is fully aggregated as long as the distance between the bias of an individual player and the weighted average of biases of the group falls within a certain bound. FIA implies that each player reveals both the signals truthfully and takes an action such that the joint action function is perfectly coordinated with their preferred state, conditional on every other player doing the same.

The intuition for the result is as follows. In order to reveal any information truthfully, it must be that the subsequent action of the players must be within their set of available actions. In other words, the action set acts as an incentive compatibility constraint for truth-telling.[3] Accordingly, players can be grouped into two types: those who always reveal the low signal but may lie about the high signal, and vice-versa. For the former, the only relevant constraint of is when the action goes above the lower bound for the lowest posterior expectation of the state conditional on the player holding a high signal. Similarly, for the latter set of players, the only constraint of interest is for the action to be below the upper bound for the highest possible posterior expectation of the state, conditional on them holding a low signal. In the case of homogeneous constraints, the set of conditions for FIA boils down to just *two* constraints, one each for the least biased (with a high signal) and the most biased player (with a low signal).

The equilibrium results show that an important determinant for efficient information aggregation is the available set of actions (or constraints). That is, a greater upper bound on the actions of players eases the incentive constraints for truthtelling. In other words, apportioning resources and contributing to the alliance via ex ante *commitments* to defense spending, for example, improves credibility of communication and aids efficient information aggregation. The paper therefore provides a novel informational rationale for alliances.[4] The aggregation result also depends crucially

[3] Refer to Theorem 2 and Theorem 3 of Venkatesh (2019).

[4] Traditionally, in the international relations literature (Walt, 1985; Walt, 1990; Waltz, 2010), alliance formation

on the distribution of biases in the alliance. The presence of extreme players (either *too* hawkish or *too* dovish) breaks the FIA condition.[5]

The next result relaxes the assumption of homogeneous action set to allow for heterogeneity in the upper bound of the action space of players (the lower bound is still zero). Doing so does not change any of the earlier logic of the main result and a similar set of incentive constraints apply for the characterization of FIA. Heterogeneity in the upper bound simply implies that instead of two there are now a total of $(N+1)$ constraints that have to be satisfied. This includes the least biased agent's IC constraint for the high signal as before, and a set of N constraints on the upper bound for each players' low signal.

The heterogeneous constraints result provides the *minimal set of actions*—the smallest upper bound—for each player such that FIA result goes through. The minimal set has an intuitive ordering. The upper bounds are increasing in the biases such that the most hawkish player in the group has the largest set of actions while the most dovish player's action set is the smallest.[6] That is, the preferences of players determines their contributions to the alliance (*preference effect*). Crucially, these differences are *also* exacerbated by the degree of interdependence in actions (*interdependence effect*). That is, when the actions of players become more interdependent, the minimal set of the hawks in the alliance *expands* while that of the doves *shrinks* giving rise to greater divergence in the size (specifically, upper bounds) of the action sets.

The paper identifies two critical strategic sources for the gap in resource contributions to arise in international alliances. The first is driven by pure preferences of countries in an alliance. The second is due to the nature of interventions that countries undertake. To put it precisely, any joint action that increases interdependencies (for example, military strikes or peacekeeping) allows for a form of *piggybacking* in which countries that care less about the issue tend to piggyback on the contributions of those countries for whom the intervention is more salient. An important

has primarily been studied within the purview of state capacity - either align in order to balance against a powerful state or bandwagon with a threatening state (or coalition).

[5] This is fairly intuitive. In alliances like NATO, the United States is seen as the most hawkish on issues like military intervention while countries like Germany, Denmark, the Netherlands are all viewed as more dovish.

[6] A *hawk* in this paper is any player whose bias is greater than the average of the group; a *dove* is one with a bias lower than the average bias.

implication of the result is that a natural divergence in contributions is inevitable in alliances. Burden sharing rules like the "*two per cent of GDP*" adopted by NATO that proscribes member countries to target defense contributions to the alliance according to their GDP's might not be feasible, given the varying preferences.[7] Further, these rules might become harder to implement given the high degree of interdependencies in military and external security related interventions that are a main feature of NATO type alliances.

Related Literature

The paper is related to the cheap talk literature with multiple senders and receivers. The two closest papers are the ones by Hagenbach and Koessler (2010) and Galeotti et al. (2013). Though the information and communication structure I adopt in the model are identical to Galeotti et al. (2013), a fundamental difference is that in their work the actions of players were independent of each other. Instead I allow actions to be interdependent, in which case a player's message affects her own action by shifting beliefs of other players. Hagenbach and Koessler (2010) study a model of strategic with multiple players and interdependent actions. However, they study strategic complementarities in actions, while my setup considers actions that are substitutable. Further, in their information framework, private signals of players are independent and communication is private. On the other hand, I model the case where the private signals of players are conditionally independent but correlated, and the communication protocol is public.

The paper is also related to information sharing (Konrad, 2012; Konrad, 2014) and role of capacity constraints (Konrad and Kovenock, 2009) in alliances. Konrad (2012) study alliance formation in which there is information exchange (about their budget) between members and actions of members are independently taken. They characterize stable alliance formations in the presence of truthful information sharing within the alliance. On the other hand, Konrad and Kovenock (2009) analyze the trade off between increased capacity of an alliance versus higher post alliance

[7]As of 2014, for example, only four countries in NATO contributed over 2% of GDP to defense (U.S., Greece, United Kingdom, and Estonia).

conflict with members. My paper instead focuses on comparative statics with respect to biases and interdependencies of players within the alliance. Further, the paper studies information aggregation incentives and abstracts away from the problem of alliance formation.

Finally, many papers have looked into the problem of information aggregation in the context of elections (Bhattacharya, 2013; Feddersen and Pesendorfer, 1997), polling (Morgan and Stocken, 2008), financial markets (Angeletos and Werning, 2006; Dasgupta and Prat, 2008), and organizations (Dewan et al., 2015; Jehiel, 1999). None of the papers look at interdependent action environments, and more specifically, the case of imperfectly substitutable actions. Further, the relation between information aggregation and action constraints, and their application to international organizations like NATO is novel and not been explored previously.

The rest of the paper is organized as follows. Section 2 presents the basic framework of the model. Section 3 characterizes the equilibrium actions in the absence of constraints. Section 4 provides the main result pertaining to information aggregation and related comparative statics results. Concluding remarks are contained in Section 5.

2 Model

An alliance consists of a group of players, $N = \{1, 2, ..., n\}$. The players belonging to the alliance have to work together on a joint project. The payoff of every player is dependent on an unknown common state of the world θ distributed uniformly on $[0,1]$. Each player chooses an action $x_i \in V$. The players' actions are interdependent in a way that there is a joint action function for each player $\phi_i : V^N \to \mathbb{R}$. Specifically, this function is dependent on the action vector of all the players. Finally, each player has a heterogeneous preference b_i over the final state that captures the extent to which a player cares about the outcome (without loss of generality, $0 \leqslant b_1 \leqslant b_2 \leqslant .... \leqslant b_n$). For sake of exposition let $\underline{b} = b_1$ and $\bar{b} = b_n$. The joint action function of each player, $\phi_i(\mathbf{x})$, assumes the following functional form:

$$\phi_i(\mathbf{x}) = \frac{x_i + \eta \sum\limits_{j \neq i} x_j}{1 + (n-1)\eta}$$

The utility function itself is given by,

$$u_i(\mathbf{x};\theta,b_i) = -\left[\phi_i(\mathbf{x}) - \theta - b_i\right]^2$$

This utility form captures the essential features of an alliance between countries. The action x_i, for example, captures the extent of military resources supplied by a country during an intervention. The novelty in the setup is that there is a positive spillover in the actions of players captured by η in the joint action function. This measures the degree of interdependencies in the alliance. For example, in the Libya war conducted by NATO allies, a total of eighteen countries participated in the military operation to overthrow the Qaddafi regime. The payoff structure intuitively captures this interdependence in actions. Further, each player i's action x_i has a higher marginal effect on their joint function $\phi_i(.)$, compared to other players' actions.

The b_i's capture the extent of alignment of interest or the heterogeneity in preferences in the alliance. Specifically, each player would like their $\phi_i(.)$ to match a biased final outcome $\theta + b_i$. Through out the rest of the analysis, a hawks (doves) refers to players whose biases are greater (lesser) than the average bias of the group, $Avg(\mathbf{b})$. A higher b_i indicates greater relative hawkishness. If there were no biases (or were all equal to b) then there exists no strategic problem since every country would want to perfectly aggregate their information and take an action equal to θ (or $\theta + b$).

Another important feature of the setup is that there is a *joint cost* for each player that depends on the action vector $\mathbf{x}$. This captures the joint/collective nature of decision-making in alliances. For each player this amounts to a cost that is proportional to the value of the joint action function $\phi_i(\mathbf{x})$. This can be interpreted as a reputational cost of being involved in a joint operation. Alternatively, this could be purely the operational costs of coordinating multiple entities during a military intervention. An interesting feature of the preferences is that these joint costs are directly proportional to each players' joint action function. Since the players are coordinating their actions such that the joint function is equal to $\theta + b_i$, it is clear that the costs they incur are also proportional to this outcome and is indirectly determined by the actions of all other players in the group.

Finally, the action space V can be thought of as the ex ante capacity (or sunk investment) of a country within the alliance or the initial contribution made by the partnering countries.[8] The model, however, does not include an investment stage nor an individual cost of contributing to this budget. Instead, the focus is on the information aggregation question and how it depends on the overall stock of actions available to each player. The initial stock of actions therefore can be interpreted as the ex ante contribution to defense spending made by the players to the alliance. I assume this to be a fixed cost which would neither affect the incentives for information transmission nor the subsequent actions of the players.

This setup lends itself naturally to situations in international alliances that involve countries coordinating with each other to resolve a common foreign policy objective. In such scenarios, each country in the alliance has potentially varying degrees of information and alignment of interests. Sharing private information enables countries in an alliance to coordinate their actions effectively.[9] The problem of information aggregation is critical since information about the underlying state is imperfect and distributed among the players in the alliance. The precise nature of the game — the information structure, communication protocol, and the strategy of players — is described below.

2.1 Information Structure, Communication, and Actions

Information structure The information asymmetry among players is modeled along the lines of Galeotti et al. (2013). Specifically, the underlying state $\theta \in \mathbb{U}[0,1]$ is not directly observable. Each player i receives an imperfect private signal $s_i \in S_i \equiv \{0,1\}$ about the state of the world such that: $s_i = 1$ with probability θ, and $s_i = 0$ with probability $1-\theta$. Clearly, when there is no interdependence in actions (i.e. $\eta = 0$) there is no information aggregation problem in that the incentives of players to reveal their respective private information are perfectly aligned. The

[8] In NATO, for example, countries pledged in the Riga Summit of 2006 to commit two per cent of their GDP towards the NATO collective defense budget. Refer to the Annual Report 2012 released by the NATO Secretary General.

[9] For example, consider NATO's Partnership Action Plan against Terrorism, drafted post the September 2011 attacks. It clearly delineates the vital element of information sharing as one of the key requirements for effectively fighting terrorism and other security related challenges. For more, see http://www.nato.int/cps/en/natohq/official_texts_19549.htm.

players' signals are conditionally independent but correlated in the following way: $\Pr(s_j = 1 \mid s_i = 1) = \frac{2}{3}, \Pr(s_j = 0 \mid s_i = 1) = \frac{1}{3}$ and $\Pr(s_j = 0 \mid s_i = 0) = \frac{2}{3}, \Pr(s_j = 1 \mid s_i = 0) = \frac{1}{3}$. Given this structure, every player can be classified into one of two types - high ($s_i = 1$) and low ($s_i = 0$). This information structure provides a parsimonious way to capture disaggregated information within the group about a common underlying state. In alliances, for example, intelligence information about a possible terror target is usually disaggregated among different allies and the military operation requires aggregating this information between the members.[10]

Communication round After each player receives their signal s_i, they *publicly* and *simultaneously* communicate their information through a cheap talk message to the group. In this paper, I focus on pure messaging strategies in which each player simultaneously sends a public message $m_i(s_i)$ to every other player in the group. Player i's messaging strategy is given by $m_i : \{0,1\} \longrightarrow \{0,1\}$. A truthful message by i to the group implies $m_i(s_i) = s_i$ for $s_i = 0$ and 1, and a babbling message is one where $m_i(s_i) = m_i(1 - s_i)$.[11] Let $\mathbf{m} = (m_1, m_2, ..., m_n)$ be the joint communication strategy of the n players.

Action round Once messages have been exchanged publicly, each player decides on their individual action x_i simultaneously. I will once again focus on pure strategies. Since the utility function is strictly concave in x_i, the best response correspondences exist and are unique. The strategy for a player can be defined as $\tau_i : S_i \times \{0,1\}^N \rightarrow V$. That is, $\tau_i(s_i, (m_i, m_{-i}))$ is the action of player i with private signal s_i, having sent message m_i and received $(N-1)$ messages $m_{-i} = (m_j)_{j \neq i}$ from the group. Let $\tau(\mathbf{s}, \mathbf{m}) = (\tau_i(s_i, (m_i, m_{-i})))_{i \in N}$ be the strategy profile of the players.

The bias vector $\mathbf{b}$, the interdependency parameter η, and the action set of players V are all

[10] The Joint Intelligence, Surveillance and Reconnaissance (JISR) initiative within NATO is tasked with the responsibility of sharing information during military and other security related joint operations. NATO Secretary General's *Annual Report 2012* makes the case for an enhanced JISR; he writes, "a new JISR initiative was launched to help coordinate the gathering, analysis and dissemination of information. It will use information gathered by the Alliance Ground Surveillance system and other ISR assets in support of Alliance operations, integrating operations and intelligence. This will permit the coordinated collection, processing, dissemination and sharing within NATO of ISR material, in direct support of current and future operations."

[11] Public communication protocols are very common in the real-world. For example, international forums like UN, NATO or PESCO often get together and share private information about a common issue of interest. Public diplomacy remains a main feature of such organizations. See Walt (1990) and Wittmann (2009) for more on the role of public diplomacy in alliances.

assumed to be common knowledge. The sequence of the game can be summarized as follows:

1. The state of nature θ is drawn from an uniform distribution on $[0,1]$. Conditional on θ each player observes a private signal $s_i \in \{0,1\}$.

2. The players simultaneously send a public message $m_i(s_i)$ to the group. The vector $\mathbf{m} = (m_1, m_2, ..., m_n)$ is the information publicly available to all players at the end of the communication stage.

3. Upon observing the private signal s_i and set of messages $\mathbf{m}$ each player decides on the action $x_i(s_i, \mathbf{m}) \in V$ simultaneously. Payoffs are realized.

2.2 Equilibrium

Given the above structure of messaging, the players can be grouped post the communication round into two sets - truthful set and babbling set. I define them in the following way:

Definition 1. *Truthful set,* $T = \{i : m_i(0) = 0, m_i(1) = 1\}$

Definition 2. *Babbling set,* $B = \{j : m_j(0) = m_j(1)\}$

The first is just the set of players whose messages are believed in equilibrium as informative, and messages from the second are ignored as uninformative. Given this, the vector of messages after communication consists of $|T|$ truthful messages $m_T = \{m_i : i \in T\}$ and $|B|$ babbling messages $m_B = \{m_j : j \in B\}$. The equilibrium concept is perfect Bayesian equilibrium in pure strategies (henceforth equilibrium). An equilibrium is defined as a strategy profile $(m, \tau) = ((m_i)_{i \in N}, (\tau_i)_{i \in N})$ such that,

1. Actions are sequentially rational, given messages and beliefs:

   $\forall i \in N,\ m_{-i} \in M_{-i}:$

$$\tau_i(s_i, (m_i, m_{-i})) \in \arg\max_{x_i \in V} \int_0^1 \sum_{s_{-i} \in \{0,1\}^{n-1}} u_i(x_i, (\tau_j(s_j, (m_j, m_{-j})))_{j \neq i}; \theta, b_i)$$
$$\Pr(s_{-i} \mid \theta) f(\theta | m_{-i}, s_i) d\theta$$

2. Messages are truthful iff they satisfy the *IC* for truth-telling:

$\forall i \in N, s_i \in \{0,1\}$:

$$-\int_0^1 \sum_{s_{T-1}\in\{0,1\}^{t-1}} \sum_{s_B\in\{0,1\}^{n-t}} u_i(\tau_i(s_i,(s_i,m_{-i})),(\tau_j(s_j,(s_i,m_{-i})))_{j\in T-1},$$
$$(\tau_k(s_B(k),(s_i,m_{-i})))_{k\in B};\theta,b_i)f(\theta,s_{T-1},s_B|s)d\theta$$
$$\geq$$
$$-\int_0^1 \sum_{s_{T-1}\in\{0,1\}^{t-1}} \sum_{s_B\in\{0,1\}^{n-t}} u_i(\tau_i(s_i,(1-s_i,m_{-i})),(\tau_j(s_j,(1-s_i,m_{-i})))_{j\in T-1},$$
$$(\tau_k(s_B(k),(1-s_i,m_{-i})))_{k\in B};\theta,b_i)f(\theta,s_{T-1},s_B|s)d\theta$$

where s_{T-1} is the set of $(T-1)$ truthful signals, apart from player i and s_B is the set of babbling signals.

3 Equilibrium Characterization without constraints

I proceed by first characterizing the equilibrium actions of the players in the final stage of the game, given a set of truthful messages. In order to do so, I assume that players are unconstrained in their action space, i.e., $V \equiv \mathbb{R}$ and $x_i \in \mathbb{R}$. This is done in order to fully characterize in closed form the solution to the post communication action stage, for a generic communication equilibria. Let $t = |T|$ and $(n-t) = |B|$ be the number of truthful players and babbling players respectively, post the communication stage.

An intuitive way to think about the action stage is to abstract away from communication, and assume the following. Suppose all agents were exogenously given the information (a set of t signals) m_T, and a sub-group of $(n-t)$ agents were additionally provided with a private signal 0 or 1. Subject to this exogenous information structure what are the optimal actions of each player? The solution to the problem is then a Bayesian Nash equilibrium (BNE) that is equivalent to solving the case where there are t truthful players and $(n-t)$ babbling players.

Let $x_i^*(m_T)$ and $x_j^*(s_j, m_T)$ be the equilibrium actions of truthful players $i \in T$ and babbling players $j \in B$ respectively. The equilibrium actions of the truthful players are solutions to the following:

$$\forall i \in T : x_i^*(m_T) \equiv \underset{x_i \in \mathbb{R}}{\operatorname{argmax}} \mathbb{E}_{\theta, s_B} \left[u_i \left(\phi_i \left(x_i, x^*_{T\setminus\{i\}}(m_T), x^*_{j\in B}(s_j, m_T) \right), \theta, b_i \right) \mid m_T \right] \tag{1}$$

$x^*_{T\setminus\{i\}}(m_T) = \{x^*_{i'}(m_T) : i' \in T\setminus\{i\}\}$ is the vector of actions of the remaining $(t-1)$ truthful players in equilibrium, and $x^*_{j\in B}(s_j, m_T)$ is the vector of actions for the babbling players. Analogously, the maximization problem for a babbling player j with private signal s_j is given by,

$$\begin{aligned} &\forall j \in B, s_j \in \{0,1\} : \\ &x_j^*(s_j, m_T) \equiv \underset{x_j \in \mathbb{R}}{\operatorname{argmax}} \mathbb{E}_{\theta, s_B} \left[u_j \left(\phi_j \left(x_j, x_T^*(m_T), x^*_{j'\in B\setminus\{j\}}(s_{j'}, m_T) \right), \theta, b_j \right) \mid m_T, s_j \right] \end{aligned} \tag{2}$$

As before, $x_T^*(m_T)$ is the vector of equilibrium actions of the T truthful players and $x_{j'\in B\setminus\{j\}}(s_{j'}, m_T)$ is the vector of actions of $(n-t-1)$ babbling players besides j. For exposition sake, the actions can be represented without the truthful message set m_T that is publicly observable. Clearly, the equilibrium profile of actions is then given by,

$$\left(\{x_i^*\}_{i\in T}, \left\{x_j^*(0), x_j^*(1)\right\}_{j\in B} \right)$$

Hence, players choose an action that solves a system of equations $(t+2b)$ given by their respective maximization problems stated above. The usefulness of public communication can be gleaned from the above equations. Every player in the group knows precisely who the set of truthful players are (in terms of equilibrium beliefs), and therefore also the set of babbling players. Moreover, given the Beta-binomial distribution, each truthful player has the same information given by the set of messages m_T, and the babbling player has one extra piece of information which is their private signal s_j. Critically, the players can form expectations over this private information of babbling players. The babbling player is then one of two types—0 or 1—such that every player in the group has the same posterior expectation over the two babbling types.

Definition 3. *Let* $\tilde{b}(\eta) = \frac{\eta}{1+(n-1)\eta} \sum_{j \in N} b_j$ *be the weighted average of the alignment of interests in the group. Let* $A_i(\eta) = \left[b_i - \tilde{b}(\eta)\right]$ *be a measure of the misalignment of interests for player i with respect to the group.*

$A_i(\eta)$ provides an intuitive way to think about the misalignment of interests within the alliance. Specifically it captures the distance of each player's individual bias from that of the weighted average of the group. This measure is critical for understanding the incentives within the alliance. When alliances are more cohesive then the individual biases are much closer to the weighted average while in less cohesive alliances the dispersion in biases is quite high. Theorem 1 below completely characterizes the actions of players for a generic communication equilibrium with no constraints.

Theorem 1. *Under unconstrained domain of actions ($x_i \in \mathbb{R}$) the players' sequentially rational action after receiving t truthful messages and $(n-t)$ babbling messages is given by:*

Truthful player:

$$x_i^* = \frac{(1+(n-1)\eta)}{1-\eta} A_i(\eta) + \mathbb{E}[\theta \mid m_T]$$

Babbling player with low signal:

$$x_j^*(0) = \frac{(1+(n-1)\eta)}{1-\eta} A_i(\eta) + \frac{h(t)}{1+h(t)} \mathbb{E}[\theta \mid m_T]$$

Babbling player with high signal:

$$x_j^*(1) = \frac{(1+(n-1)\eta)}{1-\eta} A_i(\eta) + \frac{1}{1+h(t)} \mathbb{E}[\theta \mid m_T]$$

where $h(t) = \frac{\frac{(2+t(1-\eta))}{(1+(n-1)\eta)}}{\left(1+\frac{(2+t(1-\eta))}{(1+(n-1)\eta)}\right)}$

Proof. See Appendix A.1. □

When $x_i \in \mathbb{R}$, players are unconstrained in what actions they can take in the final stage. This implies that they can truthfully reveal information in equilibrium and then choose the optimal action dictated by the first equation of Theorem 1. If every player in group reveals their private information and acts according to Theorem 1, then $\phi_i(\mathbf{x})$ exactly matches the expected ideal state for any set of truthful messages. Therefore, with unconstrained domain there always exists a fully revealing equilibrium in which every player reveals their private information to the group. In a fully revealing equilibrium with unrestricted domain, $T = N$ and every player plays the following action post the communication round:

$$x^*_{i \in N} = \frac{(1+(n-1)\eta)}{1-\eta} A_i(\eta) + \mathbb{E}[\theta \mid m_N] \tag{3}$$

4 Full Information Aggregation with Constraints

Most countries in international alliances face some form of constraints on how much they can contribute to any joint task. Take the leading example I consider in this paper, the Libya war intervention by NATO allies. It is immediately obvious that if any one country could unilaterally intervene then it would do so without bothering about an alliance. One main purpose of engaging in alliances is that there is a *joint contributions* (coordination component) and a *shared costs* feature. However, military budgets of most countries are constrained by economic and local political economy concerns. This implies that countries rely on each other to also contribute to the joint defense budgets.

To capture the role of constraints, let the actions available to players be $V = [0,1]$ and $\forall i \in N:$ $x_i \in [0,1]$. The action set is chosen deliberately in order to understand the relationship between constraints and information aggregation. Since the underlying state of the world is in $[0,1]$, the expectation of the state of the world given any set of truthful messages, $\mathbb{E}[\theta | m_N] \in (0,1)$. As a result the equilibrium actions of players in any fully truthful equilibrium (given by Equation 3) is dependent on the constraint imposed on the actions. When $x_i \in [0,1]$, it is intuitively clear that whether the actions dictated by Equation 3 are indeed within the bounds is driven by the sign of $A_i(\eta)$. Specifically, if $A_i(.) < 0$ then the actions can never exceed the upper bound ($\sup V = 1$) while if $A_i(.) > 0$ then the action is always above the lower bound ($\inf V = 0$).

Given this observation, what is necessary (and indeed sufficient) for FIA is for the actions to be within the bounds for any set of signal realizations and truthful revelation of the same. That is, FIA implies each players' (expected) joint action function $\mathbb{E}[\phi_i(\mathbf{x})]$ be exactly equal to $\mathbb{E}[\theta|m_N] + b_i$ for every possible set of truthful messages m_T. This implies no player can do better by misreporting their private signal. However, the moment any player's action goes above (below) the upper (lower) bound, in equilibrium, the other players readjust their actions. This increases (in expectation) the variance of players for whom the bound on actions were binding since $\mathbb{E}[\phi_i(\mathbf{x})]$ is different from their ideal $\mathbb{E}[\theta|m_N] + b_i$.

Let the players be separated into two types based on their incentives for revealing the low and high signal. I define $0-type$ and $1-type$ set of players in the following way:

Definition 4. $0-type = \{i \in N : A_i(\eta) < 0\}$

Definition 5. $1-type = \{i \in N : A_i(\eta) > 0\}$

The players in $0-type$ reveal their low signal but may face incentives to misrepresent their high signal $s_i = 1$. This is driven by the earlier observation that for some message realizations m_N of the group it might be that the optimal action may fall below the lower bound. If this happens, they face an additional variance of deviating from $\mathbb{E}[\theta \mid m_N] + b_i$. Instead if the player misrepresents information and instead sends a low signal $m_i = 1 - s_i = 0$, they would induce other players to choose a smaller action. This way, they can readjust their own actions thereby reducing or completely eliminating the additional variance arising from miscoordination. On the other hand players in the set $1-type$ always reveal their high signal, but may misrepresent their low signal for analogous reasons. FIA requires the balancing of these opposing incentives for truth-telling for two sets of players in the group. The main result below provides an intuitive closed form characterization for the necessary and sufficient conditions for FIA.

Theorem 2. *Under public communication protocol with a given alignment of interests* $\mathbf{b}$ *and constraints* $[0,1]$*, there is FIA if and only if:*

$$\forall i \in N: \qquad |A_i(\eta)| \leqslant \frac{(1-\eta)}{(1+(n-1)\eta)} \cdot \frac{2}{n+2}$$

Proof. See Appendix A.2. □

I provide a brief intuition in words for Theorem 2. The key logic in deriving this result involves looking at the broad incentives for truth-telling given the action constraints. This involves identifying the *pivotal IC* constraint for each of the players. I begin by explaining the result in the case of two players, as presented in Lemma 1. With two players and biases $(b_1, b_2) = (0, b)$, it is straightforward to see that player 1 is a $0-type$ and player 2 is a $1-type$. To check player 1's IC constraint for truth-telling it is enough to focus on $s_1 = 1$ and look for conditions under which there are no profitable deviations. This involves two steps. First, I show that player 1 has an incentive to under-report the high signal whenever for some realization of player 2's signal, player 1's action under truth-telling is below the lower bound. In this case, $x_1(s) = 0$ and player 2 readjusts her actions accordingly. Crucially, this readjustment results in $\phi_1(\mathbf{x}) > \mathbb{E}[\theta \mid s]$, increasing the overall ex ante variance. This is sufficient to induce player 1 to deviate and under-report. Second, I identify precisely the *pivotal* IC constraint. Since the utility functions satisfy single crossing, if player 1 is truthful and her action is below the bound for $s_2 = 1$, then it is also below the bound for $s_2 = 0$. The vice versa need not be true. As a result, the pivotal IC constraint for player 1, is the case when $s_2 = 0$ and $x_1(1,0) < 0$. A similar argument for player 2 implies that her pivotal IC is when $s_2 = 0$ and $s_1 = 1$ such that $x_2(1,0) > 1$. Together these conditions gives the characterization for the two player case.

The general case extends this intuition to a group of N players. FIA simply implies that every player in the group must reveal their information $s_i = \{0,1\}$ truthfully for every possible signal realization of the other $(n-1)$ players, conditional on them revealing their signals truthfully. In other words, each player's action must be within the bounds for all possible truthful message realizations of $(n-1)$ other players, conditional on their own truthful message. Since players are ordered according to their bias parameter, it is clear that there exists a threshold player with bias b_z, where $z \in 0-type$ such that every player with bias above b_z is in $1-type$ set.

Let a player, say i, from the set $0-type$ hold a signal $s_i = 1$. In any equilibrium where all $(n-1)$ other players reveal truthfully, it must hold that the equilibrium action of player i is greater than zero, for every possible signal realization[12] of the remaining $(n-1)$ truthful players. Otherwise, there is an incentive to lie for player i in the communication stage. Moreover, what matters is the sufficient statistic (given by $\sum_{j \in N \setminus \{i\}} s_j$) of the $(n-1)$ signals. Given this formulation, truth-telling for a $0-type$ player has to satisfy the tightest IC constraint, meaning her action under the tightest constraint has to be within the action set $[0,1]$. If this is not so, there are positive deviations for the $0-type$ player. The tightest constraint for a $0-type$ player occurs when $k=1$, meaning all other $(n-1)$ signals are 0 ($\sum_{j \in N \setminus \{i\}} s_j = 0$) and $s_i = 1$, implying that $\mathbb{E}[\theta \mid m_N] = \frac{2}{n+2}$. Once this is satisfied, all other IC constraints are satisfied automatically, by single crossing property of the utility function. The IC is then simply written as,

$$x_i(m_N) \geq 0 \implies -A_i(\eta) \leq \frac{(1-\eta)}{(1+(n-1)\eta)}.\frac{2}{n+2}$$

An analogous logic ensues for any truth-telling player belonging to the set $1-type$. For a player $i \in 1-type$ to separate her messages in equilibrium, her equilibrium actions have to be weakly below one for every possible realization of the remaining $(n-1)$ signals. This further implies that the tightest IC in this case is one where the sufficient statistic of the rest of the truthful messages is the highest possible, ie, $\sum_{j \in N \setminus \{i\}} s_j = (n-1)$, and $s_i = 0$. Again, as before, if this IC is satisfied, every other constraint would also be satisfied because of the single crossing property. The corresponding expectation of the state under the pivotal IC is $\mathbb{E}[\theta \mid m_N] = \frac{n}{n+2}$. The constraint can be written as follows:

$$x_i(m_N) \leq 1 \implies A_i(\eta) \leq \frac{(1-\eta)}{(1+(n-1)\eta)}.\frac{2}{n+2}$$

[12] Given the nature of Beta-binomial distribution, signals are ex-ante correlated, and that given a signal, all possible contingencies of remaining $(n-1)$ signals occur with a positive probability.

Letting $D(\eta) = \frac{(1-\eta)}{(1+(n-1)\eta)}$, the FIA condition can be reformulated as,

$$\left|b_i - \tilde{b}(\eta)\right| \leqslant D(\eta)\frac{2}{n+2} \tag{4}$$

Theorem 2 clearly shows the importance of alignment of interests for information aggregation within an alliance. Despite varying degrees of interest about the ideal state of the world, it is possible for alliances to aggregate information as long as the dispersion in the biases is within a bound. To this effect, players in the group must be 'closely' aligned. A_i provides a measure of this alignment of interests required for efficient aggregation of information. When the action set is $[0,1]$ for all players, the above equation has to be satisfied only for the extreme players — lowest in $0-type$ and highest in $1-type$ — in the group. The following two conditions are sufficient to capture the incentives for FIA.[13]

$$\tilde{b}(\eta) - \underline{b} \leqslant D(\eta)\frac{2}{n+2}$$

$$\bar{b} - \tilde{b}(\eta) \leqslant D(\eta)\frac{2}{n+2}$$

Heterogeneous Constraints

When the action set is such that $V \in [0,\bar{R}]$, then the pivotal IC for the players in the $1-type$ set can be rewritten as follows.

$$x_i(m_N) \leq \bar{R} \implies A_i(\eta) \leq D(\eta).\left(\bar{R} - \frac{n}{n+2}\right)$$

Again, taking only the most extreme type and rewriting the equation, I get,

$$\bar{b} - \tilde{b}(\eta) \leqslant D(\eta).\left(\bar{R} - \frac{n}{n+2}\right) \implies \bar{R} \geq \frac{n}{n+2} + \frac{\bar{b} - \tilde{b}(\eta)}{D(\eta)}$$

[13]Given an alignment of interests for a group of players, it is trivial to compute the conditions for FIA. All that is required is to compute $\tilde{b}(\eta)$ and $D(\eta)$, and ensure that these equations are satisfied for the extreme players in the group.

It is not unreasonable to consider the minimal constraint required to ensure FIA. For example, countries contributing to an alliance have opportunity costs of over-committing costly resources that may not be used up. By allowing the above equation to bind, it is possible to compute the *smallest* upper bound necessary to sustain FIA in an alliance. Intuitively, this corresponds with the most extreme player in the group. For any set of alignments, it is possible to compute the minimal contribution to the alliance that is required to ensure FIA. This is just given by the expression above, i.e., $\bar{R} = \frac{n}{n+2} + \frac{\bar{b}-\tilde{b}(\eta)}{D(\eta)}$. In the above setup, if each player were allowed to choose their $V_i = [0, R_i]$, then it is straightforward to observe that the R_i's would differ according to the alignment parameter b_i. Clearly, this captures an intuitive *informational* rationale for differences in contributions R_i to emerge between members in an alliance. The next proposition characterizes the conditions for FIA with heterogeneous constraints.

Proposition 1. *With heterogeneous constraints, there is FIA if the following conditions hold:*

$$\tilde{b}(\eta) - \underline{b} \leqslant D(\eta)\frac{2}{n+2}$$

$$\forall i \in 0-type: \qquad \tilde{b}(\eta) - b_i \geq D(\eta)\left(\frac{n}{n+2} - R_i\right) \tag{5}$$

$$\forall i \in 1-type: \qquad b_i - \tilde{b}(\eta) \leq D(\eta)\left(R_i - \frac{n}{n+2}\right) \tag{6}$$

and for at least one agent Equation 5 or Equation 6 holds strictly.

Proof. See Appendix A.3. □

The conditions follow from employing a similar logic as in the proof of Theorem 2. Specifically, the IC constraint for the upper bound is modified according to the R_i and given by $x_i(m_N) \leq R_i$. The IC constraint for the lower bound is the same as before and must be non-binding for player 1 with bias $\underline{b}$, i.e. $x_1(m_N) \geq 0$. As a result there are $(n+1)$ conditions that have to be satisfied in total: one constraint ($\underline{b}$) for the lower bound, and n constraints for the upper bound (one each for every player). The minimal $\bar{R}_i$'s required for FIA can be rewritten as,

$$\forall i \in 0-type: \qquad \bar{R}_i = \frac{n}{n+2} - \frac{\tilde{b}(\eta) - b_i}{D(\eta)} \tag{7}$$

$$\forall i \in 1-type: \qquad \bar{R}_i = \frac{n}{n+2} + \frac{b_i - \tilde{b}(\eta)}{D(\eta)} \tag{8}$$

The *minimal set* for each player is simply $V_i \equiv [0, \bar{R}_i]$. The $\bar{R}_i$'s capture in essence the ex ante (costless) contributions that are required to ensure FIA in the group. They are clearly ordered according to the bias parameter b_i. The more hawkish a player is the greater are the ex ante contributions required to achieve FIA. The more dovish a player becomes (smaller b_i), the greater are the incentives to misrepresent the high signal. At the same time, a smaller b_i also implies lesser contributions required to satisfy the IC constraint associated with exaggeration. By taking the differences in $\bar{R}_i$'s,

$$\forall i, j \in N, j > i: \qquad \bar{R}_j - \bar{R}_i = \frac{b_j - b_i}{D(\eta)} \tag{9}$$

The differences exhibit an intuitive ordering. The greater is the difference in biases, the greater are the differences in the *minimal set* of actions. This clearly provides a strategic rationale for why alliances like NATO struggle with enforcing the two per cent guideline among the member nations.[14] The contributions are indexed to the GDP of the countries while in reality what matters is how much each country cares about military interventions or other defense related policies. Countries like Germany, Norway, or Denmark for example have GDPs comparable to the UK and France but choose not to contribute according to the two per cent rule. A simple explanation that the above analysis uncovers is that what matters is the biases of these countries towards defense related causes. If countries care much more about climate change related cooperation compared

[14] In a speech to NATO allies in June 2014, the then US President Obama said, and I quote, "Obviously, we all have different capacities. The United States is going to have different capacities than Poland; Poland is going to have a different capacity than Latvia. But everyone has the capacity to do their fair share, to do a proportional amount to make sure that we have the resources, the planning, the integration, the training in order to be effective....". He further continued, "The United States is proud to bear its share of the defense of the Transatlantic Alliance. It is the cornerstone of our security. But we can't do it alone. And we're going to need to make sure that everybody who is a member of NATO has full membership. They expect full membership when it comes to their defense; then that means that they've also got to make a contribution that is commensurate with full membership."

to defense related policies (Petersson (2015)), then their willingness to contribute to joint defense initiatives would diminish. As a result more dovish countries (low b_i in the model) would commit to lesser compared to more hawkish members (i.e. US, UK, or France).

The differences in contributions are also affected by how interdependent the actions are, captured by the η parameter. A higher interdependence exacerbates the differences in contributions between players. However, it does not say much about how an individual player's $\bar{R}_i$ changes with the interdependency parameter. The next proposition characterizes this comparative statics.

Proposition 2. *Let $Avg(\mathbf{b}) > \tilde{b}(\eta)$ be the average alignment of interests of the players.*

(a) If $b_i > Avg(\mathbf{b})$ (player i is a hawk), then $\frac{d\bar{R}_i}{d\eta} > 0$

(b) If $b_i \leq Avg(\mathbf{b})$ (player i is a dove), then $\frac{d\bar{R}_i}{d\eta} \leq 0$

Proof. Take equations 7 and 8. Simple differentiation yields,

$$\frac{d\bar{R}_i}{d\eta} > 0 \quad \text{iff} \quad nA_i(\eta) > \frac{(1-\eta).n}{(1+(n-1)\eta)}.Avg(\mathbf{b})$$

Rearranging gives the required result.

$$\frac{d\bar{R}_i}{d\eta} > 0 \qquad \text{iff} \qquad b_i > Avg(\mathbf{b})$$

□

The result throws light on how greater interdependencies affect individual contributions to the alliance. Intuitively, the *hawks* in the group increase their contribution while the *doves* decrease theirs. While Equation 9 shows that the differences in contributions within hawks or doves increases as η increases, Proposition 2 captures this increasing gap between hawks and doves. Interestingly, greater interdependency brings about opposite responses from the two subgroups. The results can be seen through the pattern of defense expenditures within NATO over the years. For example, at the end of the cold war in 1989/90, the United States contributed 6% of its GDP and European members of NATO around 3% of their GDP towards joint-defense expenditures. However, from 1990 onwards there was a steady decline in these expenditures - in 2000 the US contributed 3.2% of GDP and NATO Europe around 2% of GDP. The September 2001 attack reversed the trend for the United States, which began committing more resources to fighting the war on terror. As a result, by 2009, the United States was spending about 5.3% of GDP while the expenditure by European members gradually decreased to approximately 1.8% of GDP. The US increased its contributions quite drastically while contributions from NATO Europe as a block continued to decline in the aggregate.[15]

These trends are in line with the theoretical predictions of this paper. In particular, the near universal drop in contributions between 1990-2000 could be almost entirely attributed to the decreased Soviet threat (post the cold war breakup of USSR). The ending of the Cold war is equivalent to a decline in the b_i's of the countries in NATO. However, when the terror attacks of September 2001 happened, the salience of defense related spending increased again. Correspondingly, it disproportionately affected some NATO members more than others. This explains why there was a continued decline in defense contributions by European partner countries compared to US and the UK. The nature of defense policy also changed with the '*war on terror*' in that it required coordination of allies and troops on the ground (in Afghanisthan and Iraq), which was not the case during Cold war. This is akin to an increase in interdependency which diminished the incentives of dovish members and exacerbated those of the hawks to contribute towards the war and subsequent military interventions.

5 Conclusion

In recent times, the issue of differences in contributions made by different countries to alliances like NATO have received attention from world leaders. In this paper, I study information sharing incentives between multiple players, each of whom has imperfect information about a common

[15]Within NATO Europe, UK which was an important ally of the US on the war of terror increased its share of defense commitments to NATO.

(unknown) state of the world. Further, the players have heterogeneous preferences over the final state. The key departure from the models of Galeotti et al. (2013) and Hagenbach and Koessler (2010) is that the actions of players are imperfectly substitutible and players face constraints on the actions they can take.

In the baseline setup, the actions are unconstrained. For any generic communication equilibrium, the players' actions are purely a linear function of their bias, the sum of biases of all the players, and the posterior expectation of the state. When constraints are imposed on the actions, I characterize the precise conditions under which there exists full information aggregation. The full information aggregation result is useful since it highlights the importance of within alliance *cohesion* for successful information sharing.

When players can costlessly choose their (minimal) action set subject to full information aggregation, they do so according to the extent of their biases. Specifically, players that are more biased end up choosing a greater set of actions compared to less biased ones. As a result, players with more biased preferences over the final outcomes tend to, ex ante, be willing to commit to an action set with greater upper bounds (preference effect). Further, a greater degree of interdependence also affects the size of the action set players choose. With greater interdependency, the hawkish player's action set expands while those of dovish players become smaller (interdependency effect). The paper therefore identifies two crucial mechanisms through which greater divergence in contributions emerge in international alliances.

This paper's analysis can be expanded upon in many possible directions. I suggest two possibilities here. The first would be to relax the assumption of costless action set and consider a case where the agents invest in the upper bound $\bar{R}_i$ at some marginal cost. The other possible extension is one where the players invest a cost in acquiring information and choose strategically how much to acquire (see, e.g. Argenziano et al., 2016), while the actions themselves are not constrained. In this case, the interesting trade off is between the incentives to acquire more precise information and the subsequent noisy communication of costly signals. These questions remain pertinent and unanswered, and await future research.

A Proofs

A.1 Proof of Theorem 1

Before proceeding to prove Theorem 1, I provide some basic insights into the nature of maximization problem that each type of players face, and in general, lay out some important properties of the Beta-Binomial distribution that I employ in the paper. I start by reformulating the maximization problem faced by a truthful player, given in equation 1, as follows:

$$\max_{x_i} \int_0^1 \sum_{s_B \in \{0,1\}^{n-t}} u_i((x_i, x_{T\setminus\{i\}}, x_B(s_B)); \theta, b_i) \Pr(s_B|\theta) f(\theta|m_T) d\theta$$

The conditional density $f(\theta|m_T)$ belongs to a standard Beta-binomial distribution. Letting $k = \sum_{i\in T} s_i$, the number of signals s_i with $i \in T$ that are equal to one, the posterior distribution of θ with uniform prior on $[0,1]$, given k successes in t trials, is a Beta distribution with parameters $k+1$ and $t-k+1$. As a consequence, $f(\theta|m_T) = \frac{(t+1)!}{k!(t-k)!}\theta^k (1-\theta)^{t-k}$ and $E[\theta|m_T] = [k+1]/[t+2]$. Further, for any s_B, letting $\ell(s_B) = \sum_{q\in B} s_q$, it is the case that $\Pr(s_B|\theta) = \theta^{\ell(s_B)} (1-\theta)^{n-t-\ell(s_B)}$. In a similar way, the problem of every babbling player $j \in B$ with a private signal s_j, stated in equation 2, can be expanded as the following:

$$\max_{x_j(s_j)} \int_0^1 \sum_{s_{B\setminus\{j\}} \in \{0,1\}^{n-t-1}} u_j((x_j(s_j), x_T, x_{B\setminus\{j\}}(s_{B\setminus\{j\}})); \theta, b_j) \Pr(s_{B\setminus\{j\}}|\theta)$$
$$f(\theta|m_T, s_j) d\theta$$

Again, the posterior density $f(\theta|m_T, s_j)$ belongs to the beta family, with $k+s_j$ successes in $t+1$ signals, and is a Beta distribution with parameters $k+s_j+1$ and $(t-k-s_j+2)$. Consequently, $f(\theta|m_T, s_j) = \frac{(t+2)!}{(k+s_j)!\left(t+1-k-s_j\right)!}\theta^{k+s_j}(1-\theta)^{t+1-k-s_j}$ and $E\left[\theta|m_T, s_j\right] = [k+s_j+1]/[t+3]$. As before, for any $s_{B\setminus\{j\}}$, $\Pr(s_{B\setminus\{j\}}|\theta) = \theta^{\ell(s_{B\setminus\{j\}})}(1-\theta)^{n-t-\ell(s_{B\setminus\{j\}})}$.

The characterization involves solving the best responses of each of the three types of players from Equation 1 and Equation 2.

Case 1. *Truthful player's problem:*

$$\mathbb{E}_{\theta,s_B}\left[u_i(\mathbf{x},\mathbf{m})\right] =$$
$$-\int_0^1 \sum_{s_B\in\{0,1\}^{n-t}} \left(\frac{x_i+\eta\sum_{j\in T\setminus\{i\}}x_j+\eta\sum_{j\in B}x_j(s_j)}{1+(n-1)\eta}-\theta-b_i\right)^2 \Pr(s_B|\theta)f(\theta|m_T)d\theta$$

where $f(\theta|m_T)=\frac{(t+1)!}{k!(t-k)!}\theta^k(1-\theta)^{t-k}$, iff $0\le\theta\le 1$.

Differentiating the above with respect to x_i, we get the following FOC:

$$\int_0^1 \sum_{s_B\in\{0,1\}^{n-t}} \left(\frac{x_i+\eta\sum_{j\in T\setminus\{i\}}x_j+\eta\sum_{j\in B}x_j(s_j)}{1+(n-1)\eta}-\theta-b_i\right)$$
$$\Pr(s_B|\theta)f(\theta|m_T)d\theta=0$$

Simplifying, we obtain:

$$x_i+\eta\left[\sum_{j\in T\setminus\{i\}}x_j+\int_0^1\sum_{s_B\in\{0,1\}^{n-t}}\sum_{j\in B}x_j(s_j)\Pr(s_B|\theta)f(\theta|m_T)d\theta\right]= \tag{10}$$
$$(1+(n-1)\eta)\left(b_i+\mathbb{E}[\theta|m_T]\right)$$

Case 2. *Babbling player's problem:*

With analogous procedures, the expected utility of a babbling player i with signal s_i is:

$$\mathbb{E}_{\theta,s_B}\left[u_i(\mathbf{x},\mathbf{m})\right]=-\mathbb{E}_{\theta,s_{B\setminus\{i\}}}\left[\left(\frac{x_i(s_i)+\eta\sum_{j\in T}x_j+\eta\sum_{j\in B\setminus\{i\}}x_j(s_j)}{1+(n-1)\eta}-\theta-b_i\right)^2 \mid m_T,s_i\right]$$
$$=-\int_0^1\sum_{s_{B\setminus\{i\}}\in\{0,1\}^{n-t-1}}\left(\frac{x_i(s_i)+\eta\sum_{j\in T}x_j+\eta\sum_{j\in B\setminus\{i\}}x_j(s_j)}{1+(n-1)\eta}-\theta-b_i\right)^2\Pr(s_{B\setminus\{i\}}|\theta)f(\theta|m_T,s_i)d\theta \tag{11}$$

Again, the density $f(\theta|m_T,s_i)$ belongs to the Beta-binomial family such that,

$$f(\theta|m_T,s_i) = \frac{(t+2)!}{(k+s_i)!\,(t+1-k-s_i)!}\theta^{k+s_i}(1-\theta)^{t+1-k-s_i}$$

Differentiating Equation 11 with respect to $x_i(s_i)$,

$$\int_0^1 \sum_{s_{B\setminus\{i\}}\in\{0,1\}^{n-t-1}} \left(\frac{x_i(s_i)+\eta\sum_{j\in T}x_j+\eta\sum_{j\in B\setminus\{i\}}x_j(s_j)}{1+(n-1)\eta} - \theta - b_i\right)\Pr(s_{B\setminus\{i\}}|\theta) f(\theta|m_T,s_i)d\theta = 0$$

Simplifying yields,

$$x_i(s_i)+\eta\left[\sum_{j\in T}x_j+\int_0^1 \sum_{s_{B\setminus\{i\}}\in\{0,1\}^{n-t-1}}\sum_{j\in B\setminus\{i\}} x_j(s_j)\Pr(s_{B\setminus\{i\}}|\theta)f(\theta|m_T,s_i)d\theta\right] = (b_i+E[\theta|m_T,s_i])\,[1+(n-1)\eta] \quad (12)$$

I focus on linear equilibrium strategies of the form where x_i and $x_i(s_i)$ are both only functions of the individual bias b_i, the vector of group biases $\mathbf{b}$, and the expectation of the state given the information – m_T for truthful players and (m_T,s_i) for the babbling players.[16] Since the signals are conditionally independent, the information contained in m_T captures everything that the players know about each babbling players' privately held signal. As a result I can rewrite Equation 10 as the following:

$$x_i+\eta\sum_{j\in T\setminus\{i\}}x_j+\eta\int_0^1\sum_{j\in B}\sum_{s_j\in\{0,1\}}x_j(s_j)\Pr(s_j|\theta)f(\theta|m_T)d\theta = (1+(n-1)\eta)\,(b_i+\mathbb{E}[\theta|m_T])$$

[16] For example, a linear functional form where $x_i = [P(b_i+E[\theta|m_T])+Q]$ and $x_i(s_i) = [P_{s_i}(b_i+\mathbb{E}[\theta|m_T,s_i])+Q_{s_i}]$ could be applied to the best response equations. I would like to thank Francesco Squintani for pointing this out.

Substituting in the expressions $\Pr(s_j|\theta) = \theta$ and $f(\theta|m_T) = \frac{(t+1)!}{k!(t-k)!}\theta^k(1-\theta)^{t-k}$,

$$(1+(n-1)\eta)(b_i + \mathbb{E}[\theta|m_T]) = x_i + \eta \sum_{j\in T\setminus\{i\}} x_j + \quad \eta \int_0^1 \sum_{j\in B} x_j(0)(1-\theta)\frac{(t+1)!}{k!(t-k)!}\theta^k(1-\theta)^{t-k}d\theta$$
$$+\eta \int_0^1 \sum_{j\in B} x_j(1)\theta\frac{(t+1)!}{k!(t-k)!}\theta^k(1-\theta)^{t-k}d\theta$$

Since $\int_0^1 (1-\theta)\frac{(t+1)!}{k!(t-k)!}\theta^k(1-\theta)^{t-k}d\theta = 1 - \mathbb{E}[\theta|m_T]$ and $\int_0^1 \theta\frac{(t+1)!}{k!(t-k)!}\theta^k(1-\theta)^{t-k}d\theta = \mathbb{E}[\theta|m_T]$, the above equation can be further simplified as:

$$(1+(n-1)\eta)(b_i + \mathbb{E}[\theta|m_T]) = x_i + \eta \sum_{j\in T\setminus\{i\}} x_j + \eta(1-\mathbb{E}[\theta|m_T])\sum_{j\in B} x_j(0) + \eta\mathbb{E}[\theta|m_T]\sum_{j\in B} x_j(1)$$

Similarly, applying the same principles to Equation 12 and making the substitution $f(\theta|m_T, s_i) = \frac{(t+2)!}{(k+s_i)!(t+1-k-s_i)!}\theta^{k+s_i}(1-\theta)^{t+1-k-s_i}$,

$$\begin{aligned}(1+(n-1)\eta)(b_i + \mathbb{E}[\theta|m_T, s_i]) =& x_i(s_i) + \eta \sum_{j\in T} x_j \\ &+ \eta \int_0^1 \sum_{j\in B\setminus\{i\}} \sum_{s_j\in\{0,1\}} x_j(s_j)\Pr(s_j|\theta) f(\theta|m_T, s_i)d\theta\end{aligned}$$

The manipulations on this equation are analogous to the truthful player's characterization. Performing similar substitutions:

$$(1+(n-1)\eta)\,(b_i+\mathbb{E}[\theta|m_T,s_i]) = x_i(s_i)+\eta\sum_{j\in T}x_j+\eta\,(1-\mathbb{E}\,[\theta|m_T,s_i])\sum_{j\in B\setminus\{i\}}x_j(0)$$
$$+\,\eta\mathbb{E}[\theta|m_T,s_i]\sum_{j\in B\setminus\{i\}}x_j(1)$$

Together, we can sum up the best responses for the three types of players as the following:

1. Truthful player $i\in T$:

$$x_i = (b_i+\mathbb{E}[\theta|m_T])\,[1+(n-1)\eta]-\eta\sum_{j\in T\setminus\{i\}}x_j-\eta\,(1-\mathbb{E}\,[\theta|m_T])\sum_{j\in B}x_j(0)$$
$$-\,\eta\mathbb{E}[\theta|m_T]\sum_{j\in B}x_j(1) \tag{13}$$

2. Babbling player with low signal $i\in B$, $s_i=0$:

$$x_i(0) = (b_i+E[\theta|m_T,0])\,[1+(n-1)\eta]-\eta\sum_{j\in T}x_j-\eta\,(1-E\,[\theta|m_T,0])\sum_{j\in B\setminus\{i\}}x_j(0)$$
$$-\,\eta E[\theta|m_T,0]\sum_{j\in B\setminus\{i\}}x_j(1) \tag{14}$$

3. Babbling player with high signal $i\in B$, $s_i=1$:

$$x_i(1) = (b_i+\mathbb{E}[\theta|m_T,1])\,[1+(n-1)\eta]-\eta\sum_{j\in T}x_j-\eta\,(1-\mathbb{E}\,[\theta|m_T,1])\sum_{j\in B\setminus\{i\}}x_j(0)$$
$$-\,\eta\mathbb{E}[\theta|m_T,1]\sum_{j\in B\setminus\{i\}}x_j(1) \tag{15}$$

There are essentially three types post the communication round – the truthful type, the babbling type with low private signal, and one with high private signal. Let $\mathbb{E}\,[\theta|m_T]=\theta_T$, $\mathbb{E}\,[\theta|m_T,0]=\theta_T^0$ and $\mathbb{E}\,[\theta|m_T,1]=\theta_T^1$. I apply the following linear guessing strategies for the players respectively:

$$x_i = Pb_i+Q\sum_{j\neq i}b_j+K$$

$$x_i(0) = Pb_i + Q\sum_{j\neq i} b_j + K_0$$

$$x_i(1) = Pb_i + Q\sum_{j\neq i} b_j + K_1$$

Plugging the above functional forms into equations 13, 14 and 15, I get the following:

Case. *Truthful player best response:*

$$x_i = (1+(n-1)\eta)(\theta_T + b_i) - \eta\left[\sum_{j\in T\setminus\{i\}} x_j + \theta_T\sum_{j\in B} x_j(1) + (1-\theta_T)\sum_{j\in B} x_j(0)\right]$$

$$\sum_{j\in T\setminus\{i\}} x_j = (t-1)Q.b_i + P.\sum_{j\in T\setminus\{i\}} b_j + (t-2)Q.\sum_{j\in T\setminus\{i\}} b_j + (t-1)Q.\sum_{j\in B} b_j + (t-1)K$$

$$\begin{aligned}\theta_T\sum_{j\in B} x_j(1) + (1-\theta_T)\sum_{j\in B} x_j(0) = P.\sum_{j\in B} b_j + (n-t-1)Q.\sum_{j\in B} b_j + (n-t)Q.b_i \\ + (n-t)Q.\sum_{j\in T\setminus\{i\}} b_j + (n-t)(\theta_T K_1 + (1-\theta_T)K_0)\end{aligned}$$

Adding the above two expressions above and simplifying yields,

$$\begin{aligned}\left[\sum_{j\in T\setminus\{i\}} x_j + \theta_T\sum_{j\in B} x_j(1) + (1-\theta_T)\sum_{j\in B} x_j(0)\right] &= (n-1)Q.b_i + (P+(n-2)Q).\sum_{j\in T\setminus\{i\}} b_j \\ &\quad + (P+(n-2)Q).\sum_{j\in B} b_j + (t-1)K \\ &\quad + (n-t).(\theta_T K_1 + (1-\theta_T)K_0)\end{aligned}$$

$$\begin{aligned}x_i = (1+(n-1)(1-Q)\eta)b_i - \eta(P+(n-2)Q).\sum_{j\neq i} b_j - \eta(t-1)K \\ + (1+(n-1)\eta)\theta_T - \eta b.(\theta_T K_1 + (1-\theta_T)K_0)\end{aligned} \tag{16}$$

Case. *Babbling player with low signal*

$$x_i(0) = (1+(n-1)\eta)(\theta_T^0 + b_i) - \eta \left[\sum_{j\in T} x_j + \theta_T^0 \sum_{j\in B\setminus\{i\}} x_j(1) + (1-\theta_T^0) \sum_{j\in B\setminus\{i\}} x_j(0) \right]$$

$$\sum_{j\in T} x_j = P. \sum_{j\in T} b_j + (t-1)Q. \sum_{j\in T} b_j + tQ.b_i + tQ. \sum_{j\in B\setminus\{i\}} b_j + tK$$

$$\left[\theta_T^0 \sum_{j\in B\setminus\{i\}} x_j(1) + (1-\theta_T^0) \sum_{j\in B\setminus\{i\}} x_j(0) \right] = P. \sum_{j\in B\setminus\{i\}} b_j + (n-t-2)Q. \sum_{j\in B\setminus\{i\}} b_j + (n-t-1)Q.b_i$$
$$+(n-t-1)Q. \sum_{j\in T} b_j + (n-t-1).(\theta_T^0 K_1 + (1-\theta_T^0)K_0)$$

Adding the above two expressions and simplifying gives,

$$\left[\sum_{j\in T} x_j + \theta_T^0 \sum_{j\in B\setminus\{i\}} x_j(1) + (1-\theta_T^0) \sum_{j\in B\setminus\{i\}} x_j(0) \right] =$$
$$(n-1)Q.b_i + (P+(n-2)Q). \sum_{j\neq i} b_j + tK + (n-t-1). \left(\theta_T^0 K_1 + (1-\theta_T^0)K_0 \right)$$

$$x_i(0) = (1+(n-1)(1-Q)\eta).b_i - \eta(P+(n-2)Q). \sum_{j\neq i} b_j - \eta t K$$
$$-\eta(n-t-1).(\theta_T^0 K_1 + (1-\theta_T^0)K_0) + (1+(n-1)\eta).\theta_T^0 \quad (17)$$

Case. *Babbling player with high signal*

This is very similar to the low signal case, except for one expression. Following the same steps as in the case with the low signal,

$$x_i(1) = (1+(n-1)(1-Q)\eta).b_i - \eta(P+(n-2)Q). \sum_{j\neq i} b_j - \eta t K$$
$$-\eta(n-t-1).(\theta_T^1 K_1 + (1-\theta_T^1)K_0) + (1+(n-1)\eta).\theta_T^1 \quad (18)$$

COMPARING COEFFICIENTS:

Using the equations 16, 17, and 18 to compare coefficients:

$$P = (1+(n-1)(1-Q)\eta) \qquad Q = -\eta(P+(n-2)Q)$$

$$\implies Q = -\eta(1+(n-1)(1-Q)\eta+(n-2)Q)$$

Solving the above equations:

$$P = \frac{(1+(n-2)\eta)}{1-\eta} \qquad Q = -\frac{\eta}{1-\eta}$$

Henceforth, for brevity, I will denote $c = \theta_T \quad c_0 = \theta_T^0 \quad c_1 = \theta_T^1$. Proceeding similarly, I solve for three equations in three unknowns (K, K_0, K_1).

From Equation 16,

$$K = -\eta(t-1)K - \eta(n-t).(cK_1+(1-c)K_0)+(1+(n-1)\eta)c$$

$$K = \frac{(1+(n-1)\eta)c}{(1+(t-1)\eta)} - \frac{\eta(n-t)(1-c)}{(1+(t-1)\eta)}K_0 - \frac{\eta(n-t)c}{(1+(t-1)\eta)}K_1 \tag{19}$$

From Equation 17,

$$K_0 = -\eta tK - \eta(n-t-1).(c_0K_1+(1-c_0)K_0)+(1+(n-1)\eta).c_0$$

$$K_0 = \frac{(1+(n-1)\eta)c_0}{(1+(n-t-1)(1-c_0)\eta)} - \frac{\eta t}{(1+(n-t-1)(1-c_0)\eta)}K - \frac{\eta(n-t-1)c_0}{(1+(n-t-1)(1-c_0)\eta)}K_1 \tag{20}$$

From Equation 18,

$$K_1 = -\eta t K - \eta(n-t-1).(c_1 K_1 + (1-c_1)K_0) + (1+(n-1)\eta).c_1$$

$$K_1 = \frac{(1+(n-1)\eta)c_1}{(1+(n-t-1)c_1\eta)} - \frac{\eta t}{(1+(n-t-1)c_1\eta)}K - \frac{\eta(n-t-1)(1-c_1)}{(1+(n-t-1)c_1\eta)}K_0 \tag{21}$$

Solving Equation 19, Equation 20, and Equation 21 for the three unknowns,

$$\begin{pmatrix} 1 & \frac{\eta(n-t)(1-c)}{(1+(t-1)\eta)} & \frac{\eta(n-t)c}{(1+(t-1)\eta)} \\ \frac{\eta t}{(1+(n-t-1)(1-c_0)\eta)} & 1 & \frac{\eta(n-t-1)c_0}{(1+(n-t-1)(1-c_0)\eta)} \\ \frac{\eta t}{(1+(n-t-1)c_1\eta)} & \frac{\eta(n-t-1)(1-c_1)}{(1+(n-t-1)c_1\eta)} & 1 \end{pmatrix} \begin{pmatrix} K \\ K_0 \\ K_1 \end{pmatrix} = \begin{pmatrix} \frac{(1+(n-1)\eta)c}{(1+(t-1)\eta)} \\ \frac{(1+(n-1)\eta)c_0}{(1+(n-t-1)(1-c_0)\eta)} \\ \frac{(1+(n-1)\eta)c_1}{(1+(n-t-1)c_1\eta)} \end{pmatrix}$$

$$\begin{pmatrix} K \\ K_0 \\ K_1 \end{pmatrix} = \begin{pmatrix} 1 & \frac{\eta(n-t)(1-c)}{(1+(t-1)\eta)} & \frac{\eta(n-t)c}{(1+(t-1)\eta)} \\ \frac{\eta t}{(1+(n-t-1)(1-c_0)\eta)} & 1 & \frac{\eta(n-t-1)c_0}{(1+(n-t-1)(1-c_0)\eta)} \\ \frac{\eta t}{(1+(n-t-1)c_1\eta)} & \frac{\eta(n-t-1)(1-c_1)}{(1+(n-t-1)c_1\eta)} & 1 \end{pmatrix}^{-1} \begin{pmatrix} \frac{(1+(n-1)\eta)c}{(1+(t-1)\eta)} \\ \frac{(1+(n-1)\eta)c_0}{(1+(n-t-1)(1-c_0)\eta)} \\ \frac{(1+(n-1)\eta)c_1}{(1+(n-t-1)c_1\eta)} \end{pmatrix}$$

Therefore, the solutions to the 3 unknowns are,

$$K = \frac{(1+\eta(n-t-1))(1-\eta(c_1-c_0))c - (n-t)\eta c_0}{(1-\eta)\,(1+\eta(n-t-1)(c_1-c_0))}$$

$$K_0 = \frac{(1+\eta(t-1))c_0 - \eta t(1-\eta(c_1-c_0))c}{(1-\eta)((1+\eta(n-t-1)(c_1-c_0))}$$

$$K_1 = \frac{((n-1)\eta-(n-t-1))\eta c_0 + (1+(n-1)\eta)(1-\eta)c_1 - \eta t(1-\eta(c_1-c_0))c}{(1-\eta)\,(1+\eta(n-t-1)(c_1-c_0))}$$

Suppose, out of the T truthful messages, k signals are 1, then $E[\theta \mid k,t] = \frac{k+1}{t+2}$. Similarly, the babbling player with low signal then has an expectation given by $E[\theta \mid k,t,0] = \frac{k+1}{t+3}$, and the babbling player with a high signal has $E[\theta \mid k, t, 1] = \frac{k+2}{t+3}$.

$$c = \frac{k+1}{t+2} \qquad c_0 = \frac{k+1}{t+3} \qquad c_1 = \frac{k+2}{t+3} \qquad c_1 - c_0 = \frac{1}{t+3}$$

Substituting these values in the expressions for K, K_0 and K_1,

$$K = \frac{k+1}{t+2}$$

$$K_0 = \frac{k+1}{t+2} \frac{1}{(1 + \frac{1+(n-1)\eta}{2+t(1-\eta)})}$$

$$K_1 = \frac{\left(1 + \frac{(k+1)}{(t+2)} \frac{(2+t(1-\eta))}{(1+(n-1)\eta)}\right)}{\left(1 + \frac{2+t(1-\eta)}{1+(n-1)\eta}\right)}$$

Truthful players' equilibrium action:

$$x_i = \frac{(1+(n-2)\eta)}{1-\eta} b_i - \frac{\eta}{1-\eta} \sum_{j \neq i} b_j + \mathbb{E}[\theta \mid m_T] \tag{22}$$

Low signal babbling players' equilibrium action:

$$x_i(0) = \frac{(1+(n-2)\eta)}{1-\eta} b_i - \frac{\eta}{1-\eta} \sum_{j \neq i} b_j + \frac{\frac{(2+t(1-\eta))}{(1+(n-1)\eta)}}{\left(1 + \frac{(2+t(1-\eta))}{(1+(n-1)\eta)}\right)} . \mathbb{E}[\theta \mid m_T] \tag{23}$$

High signal babbling players' equilibrium action:

$$x_i(1) = \frac{(1+(n-2)\eta)}{1-\eta} b_i - \frac{\eta}{1-\eta} \sum_{j \neq i} b_j + \frac{\frac{(2+t(1-\eta))}{(1+(n-1)\eta)}}{\left(1 + \frac{(2+t(1-\eta))}{(1+(n-1)\eta)}\right)} . \mathbb{E}[\theta \mid m_T] + \frac{1}{\left(1 + \frac{(2+t(1-\eta))}{(1+(n-1)\eta)}\right)} \tag{24}$$

Substituting for $h(t) = \frac{(2+t(1-\eta))}{(1+(n-1)\eta)}$ in the equilibrium actions above gives the required expressions. This completes the proof. QED

Lemma 1. *Suppose $N = 2$ such that $b_1 = 0$ and $b_2 = b > 0$. Then the following condition is necessary and sufficient for FIA with two players:*

$$b \leq \frac{(1-\eta)}{2}$$

Proof. Since there are only two players and second player is more hawkish than the first, it directly follows that the latter is a $0-type$ and the former is a $1-type$. This implies the first player always has incentives to reveal the low signal while the second, the high one. Further, the signals s_i are conditionally independent but correlated in that $Pr(s_2 = 1|s_1) = \frac{2}{3}$ and $Pr(s_2 = 0|s_1) = \frac{1}{3}$, and vice versa for s_2. Suppose signals $s = (s_1, s_2)$ are *publicly* observed such that,

$$\mathbb{E}[\theta|s] = \frac{s_1 + s_2 + 1}{4}$$

The actions of players in this case is given by,

$$x_1(s) = \mathbb{E}[\theta|s] - \frac{\eta}{1-\eta}b < 1 \tag{25}$$

$$x_2(s) = \mathbb{E}[\theta|s] + \frac{1}{1-\eta}b > 0 \tag{26}$$

The actions depend crucially on whether the bounds on actions $x_i \in [0,1]$ are binding or not. It is very clear from the above equations that $x_1(s)$ is always less than one while $x_2(s)$ is always greater than zero. As long as actions remain within the bound, it is straightforward to observe that $\phi_1(x_1(s), x_2(s)) = \mathbb{E}[\theta|s]$ and $\phi_2(x_1(s), x_2(s)) = \mathbb{E}[\theta|s] + b$.

However, suppose the constraints are binding for certain signal realizations. In particular $x_1(s) < 0$ and/or $x_2(s) > 1$ for some realization of signals s. Suppose $x_1(s) < 0$, then $x_1 = 0$ and $x_2(s) = (1+\eta)(\mathbb{E}[\theta|s] + b)$. Further, $x_1(s) < 0$ implies $\mathbb{E}[\theta|s] < \eta.(\mathbb{E}[\theta|s] + b)$. It can be verified trivially that $\phi_i(0, x_2(s)) = \eta.(\mathbb{E}[\theta|s] + b) > \mathbb{E}[\theta|s]$. Similarly, if $x_2(s) > 1$, then $x_2 = 1$ and $x_1(s) = (1+\eta)\mathbb{E}[\theta|s] - \eta$. As before, $\phi_2(x_1(s), 1) = \eta.\mathbb{E}[\theta|s] + (1-\eta) < \mathbb{E}[\theta|s] + b$ since $x_2(s) > 1$ implies

$\mathbb{E}[\theta|s]+b>\eta.\mathbb{E}[\theta|s]+(1-\eta)$.

Whenever $x_1(s)<0$ there is *over-provision* concern for player 1 and whenever $x_2(s)>1$ there is *under-provision* concern for player 2. To see how this can lead to babbling in equilibrium, let us consider the case where the signals are privately observed and the players communication with each other via simultaneous cheap talk messages. Say truthful messages are such that m_i^T: $m(s_i)=s_i$ for $s_i=0$ or 1. Consider the case where $s_1=1$.[17] Let $\mathbb{E}U_1(1,m_1^T)$ be the ex ante EU of player 1 with information $s_1=1$ under truthful messaging, conditional on the other player reporting truthfully.

$$\mathbb{E}U_1(1,m_1^T)=-\sum_{s_2\in\{0,1\}}Pr(s_2|s_1=1)\int_0^1\left(\frac{x_1(1,s_2)+\eta x_2(1,s_2)}{1+\eta}-\theta\right)^2 f(\theta\mid 1,s_2)d\theta$$

Given the interim expectation of s_2 conditional on s_1, I can expand the above equation as follows:

$$\begin{aligned}\mathbb{E}U_1(1,m_1^T)=&-\frac{1}{3}\int_0^1\left(\frac{x_1(1,0)+\eta x_2(1,0)}{1+\eta}-\theta\right)^2 f(\theta\mid 1,0)d\theta\\&-\frac{2}{3}\int_0^1\left(\frac{x_1(1,1)+\eta x_2(1,1)}{1+\eta}-\theta\right)^2 f(\theta\mid 1,1)d\theta\end{aligned}$$

Suppose $x_1(1,1)>0$ but $x_1(1,0)<0$ according to Equation 25 and Equation 26. Then it follows from previous arguments that,

$$\frac{x_1(1,1)+\eta x_2(1,1)}{1+\eta}=\mathbb{E}[\theta|(1,1)]$$

$$\frac{x_1(1,0)+\eta x_2(1,0)}{1+\eta}=\mathbb{E}[\theta|(1,0)]+(\eta b-(1-\eta)\mathbb{E}[\theta|(1,0)])$$

[17] An analogous argument follows for the player 2.

Where,

$$\Delta_1(1,0) = (\eta b - (1-\eta)\mathbb{E}[\theta|(1,0)]) > 0$$

$$\mathbb{E}U_1(1,m_1^T) = -\frac{1}{3}\int_0^1 (\mathbb{E}[\theta|1,0] - \theta + \Delta_1(1,0))^2 f(\theta \mid 1,0)d\theta - \frac{2}{3}\int_0^1 (\mathbb{E}[\theta|1,1] - \theta)^2 f(\theta \mid 1,1)d\theta$$

The extra term $\Delta_1(1,0)$ increases the expected losses over and above the standard variance term $(\mathbb{E}[\theta|1,0]-\theta)^2$. This provides incentives for under-reporting the high signal. To see this clearly, suppose player 1 misrepresents her signal and sends a deviation message m_1^D: $m(1) = m(0) = 0$. Player 2 treats the deviation message as if it were *on equilibrium path*. This implies that her action is simply given by $x_2(0,0)$ and $x_2(0,1)$ when the signal $s_2 = 0$ or 1 respectively. Of course, if $x_2(1,0)$ and $x_2(1,1)$ were both above the upper bound implying $x_2(1,0) = x_2(1,1) = 1$, then it is also possible $x_2(0,0) = x_2(0,1) = 1$. If so, then the deviation strategy does not alter the expected utility and player 1 is indifferent. However, if $x_2(0,0) < x_2(0,1) < 1$, then player 1's deviation actions $x_1^D(1,0)$ and $x_1^D(1,1)$ anticipate player 2's actions and correspondingly is readjusted. That is,

$$x_2(0,0) = \frac{1}{4} + \frac{1}{1-\eta}b \implies x_1^D(1,0) = (1+\eta)\frac{1}{2} - \eta .x_2(0,0)$$

$$x_2(0,1) = \frac{1}{2} + \frac{1}{1-\eta}b \implies x_1^D(1,1) = (1+\eta)\frac{3}{4} - \eta .x_2(0,1)$$

This implies that $\phi_1\left(x_1^D(1,0), x_2(0,0)\right) = \mathbb{E}[\theta|1,0]$ or $\phi_1\left(x_1(1,0), x_2(1,0)\right) \geq \phi_1\left(x_1^D(1,0), x_2(0,0)\right) \geq \mathbb{E}[\theta|1,0]$. In the former case, it is obvious $\Delta_1^D(1,0) = 0$ while in the latter $\Delta_1^D(1,0) \leq \Delta_1(1,0)$. That is,

$$\mathbb{E}U_1^D(1,m_1^D) = -\frac{1}{3}\int_0^1 \left(\mathbb{E}[\theta|1,0] - \theta + \Delta_1^D(1,0)\right)^2 f(\theta \mid 1,0)d\theta - \frac{2}{3}\int_0^1 (\mathbb{E}[\theta|1,1] - \theta)^2 f(\theta \mid 1,1)d\theta$$

$$\geq \mathbb{E}U_1(1,m_1^T)$$

As a result player 1 will always prefer the deviation message and truthful revelation breaks down. In the case where $x_1(1,0) < x_1(1,1) < 0$ the deviation message weakly benefits player 1 when $s_2 = 0$ and $s_2 = 1$. That is with truthful messaging the expected utility is,

$$\mathbb{E}U_1(1,m_1^T) = -\frac{1}{3}\int_0^1 (\mathbb{E}[\theta|1,0] - \theta + \Delta_1(1,0))^2 f(\theta \mid 1,0)d\theta$$

$$-\frac{2}{3}\int_0^1 (\mathbb{E}[\theta|1,1] - \theta + \Delta_1(1,1))^2 f(\theta \mid 1,1)d\theta$$

Under the deviation message, the expected utility is instead,

$$\mathbb{E}U_1^D(1,m_1^D) = -\frac{1}{3}\int_0^1 \left(\mathbb{E}[\theta|1,0] - \theta + \Delta_1^D(1,0)\right)^2 f(\theta \mid 1,0)d\theta$$

$$-\frac{2}{3}\int_0^1 \left(\mathbb{E}[\theta|1,1] - \theta + \Delta_1^D(1,1)\right)^2 f(\theta \mid 1,1)d\theta$$

For analogous arguments made above, $\Delta_1(1,0) \geq \Delta_1^D(1,0) \geq 0$ and $\Delta_1(1,1) \geq \Delta_1^D(1,1) \geq 0$. Therefore it follows that $\mathbb{E}U_1^D(1,m_1^D) \geq \mathbb{E}U_1(1,m_1^T)$. Finally,it can be inferred from the arguments that the relevant IC constraints for truth-telling are $s_1 = 1$ for player 1 and $s_2 = 0$ for player 2.

That is to check if full revelation is possible by both players, it is necessary and sufficient to check for IC constraint of player 1 with the high signal and player 2 with low signal. Given the incentives for player 1 to deviate from revealing the high signal, as described above, it is also clear that the *pivotal* IC is the one where $s = (1,0)$ and $x_1(s) < 0$, i.e the one in which the other player holds a low signal and the resulting action of player 1 after truthfully revealing the high signal is below the lower bound of actions. The vice-versa holds for player 2. The pivotal constraint to check is one in which $s = (1,0)$ and $x_2(1,0) > 1$. (Remember that for player 2 the following inequalities hold when the actions are above the upper bound: $1 \leq x_2(0,0) \leq x_2(1,0)$.) The reason is intuitive. Though $x_2(0,0) < 1$ (or $x_1(1,1) > 0$) is a necessary condition for truthful revelation, it is not sufficient. Simply put, as in the case analyzed earlier, it could be that $x_1(1,1) > 0$ but $x_1(1,0) < 0$ ($x_1(0,0) < 1$ and $x_2(1,0) > 1$ in the case of player 2). In this case, sufficiency breaks down since player's have an incentive to deviate since $x_1(1,0) < 0$ ($x_2(1,0) > 1$). Therefore the pivotal case is one in which the signal $s = (1,0)$ and the actions corresponding to the truthful revelation of s is such that,

$$x_1(1,0) = \frac{1}{2} - \frac{\eta}{1-\eta} b \geq 0 \qquad x_2(1,0) = \frac{1}{2} + \frac{1}{1-\eta} b \leq 1$$

Rearranging gives,

$$b \leq \frac{(1-\eta)}{\eta} \frac{1}{2} \qquad \text{and} \qquad b \leq (1-\eta) \frac{1}{2}$$

Since $\eta < 1$ the condition for x_2 is the one that is binding. This completes the proof.

□

A.2 Proof of Theorem 2

Sufficiency: As argued in Section 3, a $0-type$ player always reveals the low signal and the $1-type$ player never misreports a high signal. The only cases of relevance then is one where $0-type$ ($1-type$) gets a high (low) signal.

Take the case of a $0-type$ player. For i to reveal a high signal $s_i = 1$, it must be that, for any possible realization of the other $(n-1)$ players' signals, sending a truthful message $m_i = s_i = 1$ must be optimal. This means that the equilibrium action of i, $x_i(1,1,m_{-i}) \geqslant 0$ for any set of (truthful) messages from the other players, m_{-i}. Since the posterior on the state θ is a beta-binomial distribution, what matters is the sufficient statistic k, the number of 1's in the set of messages (m_i, m_{-i}).

Therefore, for i to reveal $s_i = 1$, a set of n constraints (corresponding to $k = 1$ to n). However, the tightest constraint that would ensure this is when every other player reveals 0, meaning that $\sum m_{-i} = 0$. In this case, if $m_i = 1$, then $k = \sum_{j \in N} m_j = 1$ and therefore the expected value of θ, $E[\theta \mid m] = \frac{2}{n+2}$. Once this constraint is satisfied, every other *IC* for player i must be satisfied.

From Equation 3, it must be that,

$$\frac{(1+(n-1)\eta)}{1-\eta}.A_i(\eta)+\frac{2}{(n+2)}\geqslant 0$$

$$-A_{i\in 0-type}(\eta)\leqslant\frac{2}{(n+2)}.\frac{(1-\eta)}{(1+(n-1)\eta)} \tag{27}$$

A similar argument ensues for a player $j\in 1-type$. For i to reveal a low signal truthfully, it must be that for any other order of $(n-1)$ truthful signals from the other players, player i's optimal action upon sending the message $m_j=s_j=0$ must be within the upper bound of the action set. As before, we only need to concentrate on the tightest *IC* that satisfies this condition. In the case of j, this is the constraint when $\sum m_{-j}=(n-1)$, that is, every other player reveals a high signal. In this case, if $m_j=0$, then $k=\sum_N m=(n-1)$ and therefore the expected value of θ, $E[\theta\mid m]=\frac{n}{n+2}$. Once this constraint is satisfied, every other *IC* for player j must be satisfied. Again applying the upper bound condition for Equation 3,

$$\frac{(1+(n-1)\eta)}{1-\eta}.A_i(\eta)+\frac{n}{(n+2)}\leqslant 1$$

$$A_{i\in 1-type}(\eta)\leqslant(1-\frac{n}{(n+2)}).\frac{(1-\eta)}{(1+(n-1)\eta)}=\frac{2}{(n+2)}.\frac{(1-\eta)}{(1+(n-1)\eta)} \tag{28}$$

Equation 27 and Equation 28 together imply that for every possible signal realization, every player's action is within the action set $[0,1]$. This means that $\phi_i(\mathbf{x})=\mathbb{E}[\theta\mid m_N]$ for all $i\in N$ and $m_N\in\{0,1\}^N$. From Lemma 1, there is no additional residual variance (in expectation) and the players cannot do better by misrepresenting their signals. Since $A_{i\in 0-type}(\eta)<0$ and $A_{j\in 1-type}(\eta)>0$ by definition, by combining equations 27 and 28, we conclude that there is full information aggregation if:

$$\forall i\in N:\qquad |A_i(\eta)|\leqslant\frac{(1-\eta)}{(1+(n-1)\eta)}\frac{2}{(n+2)} \tag{29}$$

Necessity:

I prove by contradiction. Suppose there is FIA equilibrium in which for $(n-1)$ players Equation 29 is satisfied and for some player $i \in N$, this condition is violated. It is then enough to show a profitable deviation for this player i conditional on truthful messaging strategy of the other players in the group. Without loss of generality, let the condition be violated for player n, with conflict of interest b_n.[18] Then, given that each of remaining $(n-1)$ players are being truthful and the sufficient condition holding for them, it requires to be checked if n has an incentive to misreport her signal. Since $b_n = \sup\{b_i : i \in N\}$, n is a $1-type$ player. Further, as before, $s_n = 0$ and n reports truthfully. Then, if each of the other signals are such that $\sum m_{-n} = (n-1)$, then the equilibrium action of n is $x_n(n-1) = \min\{1, \frac{(1+(n-1)\eta)}{1-\eta}.A_j + \frac{n}{(n+2)}\} = 1$, since Equation 29 is violated by construction. This implies that the other $(n-1)$ players readjust their action by compensating for player n's lower action as opposed to one dictated by Equation 3.

I proceed in two steps. First, I will characterize exactly how the rest of the players readjust their actions when $x_n(n-1) = 1$ and the under-provision for player n as a result of this readjustment process. In other words, $\phi_n(.) < \phi_n(\mathbf{x}^*)$, where $\mathbf{x}^*$ is the vector of actions in the case of unrestricted domain given by Equation 3. Let $x_j(k)$ be the readjusted actions of players $j \in \{1, 2, ...(n-1)\}$ when the sufficient statistic — number of 1's in the truthful message set — is k, and let $x_{-n}(k)$ be the joint vector of (readjusted) actions of the $(n-1)$ players.

STEP 1:

Clearly, the readjusted action $x_j(n-1) \geq x_j^*(n-1)$ (it binds for player $n-1$ if $x_{n-1}^*(n-1) = 1$ according to Equation 3). I drop the sufficient statistic k where possible in order to simplify notation. Let $\varepsilon_{n-1}^* = x_n^* - 1$ be the residual action that player n could not take. Then the extra action that remaining $(n-1)$ players have to compensate is given by $\frac{\eta}{1+(n-1)\eta}\varepsilon_{n-1}^*$. Since the marginal spillover of each player's action on every other players' $\phi(.)$ function is homogeneous, the $(n-1)$ players must share this extra equally among themselves. Let X_{n-1}^* be the extra action

[18] For example, the same set of arguments are valid for players in the set $0-type$.

of each of the remaining players. Then, the following condition solves for X^*_{n-1}:

$$\frac{1}{1+(n-1)\eta}X^*_{n-1}+\frac{(n-2)\eta}{1+(n-1)\eta}X^*_{n-1}=\frac{\eta}{1+(n-1)\eta}\varepsilon^*_{n-1}$$

The above expression simply implies that the total sum of the actions must equal to the residual that is required to be compensated. Solving gives,

$$X^*_{n-1}=\frac{\eta}{1+(n-2)\eta}\varepsilon^*_{n-1}$$

There are two cases to be considered. If $x_{n-1}=x^*_{n-1}+X^*_{n-1}\leq 1$ then each of the other $(n-1)$ players take an action that is given by $x_i=x^*_i+X^*_{n-1}$ and $\phi_n(x)$ is,

$$\phi_n(x)=\left[\frac{(x^*_n-\varepsilon^*_{n-1})+\eta\left(\sum_{j\in N\backslash n}x^*_j+(n-1)X^*_{n-1}\right)}{(1+(n-1)\eta)}\right]$$

$$\phi_n(x)=\phi_n(x^*)-\frac{1-\eta}{1+(n-1)\eta}\varepsilon^*_{n-1}-\frac{\eta}{1+(n-1)\eta}\left[\varepsilon^*_{n-1}-(n-1)X^*_{n-1}\right]$$

$$\phi_n(x)=\phi_n(x^*)-\frac{1-\eta}{1+(n-1)\eta}\varepsilon^*_{n-1}-\frac{\eta}{1+(n-1)\eta}\left[\frac{1-\eta}{1+(n-2)\eta}\varepsilon^*_{n-1}\right]$$

In this case, as described in Lemma 1 I can write the above as a residual variance $\Delta^q_n(n-1)$, where the superscript q denotes the number of players (including n) for whom the constraint binds. $\Delta^1_n(n-1)$ is the residual variance when the constraint binds only for one player, n. Since by construction $\phi_n(x^*)=\mathbb{E}[\theta\mid k=(n-1)]$, $\Delta^1_n(n-1)$ is simply given by,

$$\Delta^1_n(n-1)=\frac{1-\eta}{1+(n-1)\eta}\varepsilon^*_{n-1}+\frac{\eta}{1+(n-1)\eta}\left[\frac{1-\eta}{1+(n-2)\eta}\varepsilon^*_{n-1}\right]$$

$$\phi_n(x)=\phi_n(x^*)-\Delta_n(n-1)$$

In the case where $x_{n-1}=x^*_{n-1}+X^*_{n-1}>1$, but $x^*_j+X^*_{n-1}\leq 1$ for all $j\in\{1,2,...(n-2)\}$, I can define ε_{n-2}^* in a similar manner, i.e.,

$$\varepsilon_{n-2}^* = x_{n-1}^* + X_{n-1}^* - 1$$

As before X_{n-2}^* solves:

$$\frac{1}{1+(n-1)\eta} X_{n-2}^* + \frac{(n-3)\eta}{1+(n-1)\eta} X_{n-2}^* = \frac{\eta}{1+(n-1)\eta} \varepsilon_{n-2}^*$$

$$X_{n-2}^* = \frac{\eta}{1+(n-3)\eta} \varepsilon_{n-2}^*$$

The actions of players $j \in \{1,2,....,(n-2)\} : x_j = x_j^* + X_{n-1}^* + X_{n-2}^*$, that of $(n-1)$ is $x_{n-1} = (x_{n-1}^* + X_{n-1}^*) - \varepsilon_{n-2}^*$, and as before $x_n = x_n^* - \varepsilon_{n-1}^*$. Again if $x_j = x_j^* + X_{n-1}^* + X_{n-2}^* \leq 1$,

$$\phi_n(x) = \phi_n(x^*) - \frac{1-\eta}{1+(n-1)\eta} \varepsilon_{n-1}^* - \frac{\eta}{1+(n-1)\eta} \left[(\varepsilon_{n-1}^* - (n-1)X_{n-1}^*) + (\varepsilon_{n-2}^* - (n-2)X_{n-2}^*) \right]$$

This can be simplified as before,

$$\phi_n(x) = \phi_n(x^*) - \frac{1-\eta}{1+(n-1)\eta} \varepsilon_{n-1}^* - \frac{\eta}{1+(n-1)\eta} \left[\frac{1-\eta}{1+(n-2)\eta} \varepsilon_{n-1}^* + \frac{1-\eta}{1+(n-3)\eta} \varepsilon_{n-2}^* \right]$$

Suppose the constraint binds on every player except player 1. Recursively writing the above equation I get,

$$\phi_n(x) = \phi_n(x^*) - \frac{1-\eta}{1+(n-1)\eta} \varepsilon_{n-1}^* - \frac{\eta(1-\eta)}{1+(n-1)\eta} \sum_{j=1}^{n-1} \frac{\varepsilon_{n-j}^*}{1+(n-j-1)\eta}$$

In this case,

$$\Delta_n^{n-1}(n-1) = \frac{1-\eta}{1+(n-1)\eta} \varepsilon_{n-1}^* + \frac{\eta(1-\eta)}{1+(n-1)\eta} \sum_{j=1}^{n-1} \frac{\varepsilon_{n-j}^*}{1+(n-j-1)\eta}$$

$$\Delta_n^{n-1}(n-1) = \frac{1-\eta}{1+(n-1)\eta}\left[\varepsilon_{n-1}^* + \eta\sum_{j=1}^{n-1}\frac{\varepsilon_{n-j}^*}{1+(n-j-1)\eta}\right]$$

The generic expression for $\Delta_n^q(n-1)$, the residual variance when q players' action constraint binds, is given by,

$$\Delta_n^q(n-1) = \frac{1-\eta}{1+(n-1)\eta}\left[\varepsilon_{n-1}^* + \eta\sum_{j=1}^{q}\frac{\varepsilon_{n-j}^*}{1+(n-j-1)\eta}\right]$$

The action of the players in this case is,

$$\forall j \in \{1,2,...,(n-q)\} : x_j = x_j^* + \sum_{h=1}^{q} X_{n-q}^*$$

$$\forall j \in \{(n-q+1),(n-q+2)....,n\} : x_j = 1$$

Clearly, irrespective of how the readjustment takes place there is a residual variance. Further, this residual variance $\Delta_n^q(n-1)$ is increasing in q, the number of players facing a binding constraint as a result of the readjustment process. This implies there is always under-provision from n's point of view for arguments similar to one made in Lemma 1.

STEP 2:

Now instead if n misreports her signal and sends a message $m_n = 1 - s_n = 1$, and as before the rest of the players all have a signal $s_i = 1$, then the actions of every other player apart from n is increased in equilibrium. The additional increase is just $\mathbb{E}[\theta \mid k = n] - \mathbb{E}[\theta \mid k = (n-1)] = \frac{1}{(n+2)}$. As in *STEP 1*, the players $\forall j \in \{(n-q+1),(n-q+2)....,n\} : x_j = 1$ and the remaining players have to compensate for an additional $\frac{1}{n+2}$ increase in the expectation of θ. For all players $j \in \{1,2,...,(n-q)\}$, the action is $x_j(n) = x_j(n-1) + \gamma(n)$ where $\gamma(n)$ solves,

$$\left[\frac{1}{1+(n-1)\eta} + \frac{(n-q-1)\eta}{1+(n-1)\eta}\right]\gamma(n) = \frac{1}{n+2}$$

$$\gamma(n) = \frac{1+(n-1)\eta}{(n+2)(1+(n-q-1)\eta)} < 1$$

It is straightforward to see that when each player takes $\gamma(n)$ extra, the marginal increase in $\phi_n(.)$ is just $\chi_n^d(q) = \frac{(n-q)\eta}{(1+(n-1)\eta)}\gamma(n)$. The overall value of the joint function for player n from playing the deviation strategy is given by,

$$\phi_n(x_{-n}(n-1), x_n(n-1)) = \phi_n(x_{-n}^*(n-1), x_n^*(n-1)) - \Delta_n^q(n-1) + \chi_n^d(q)$$

If $\Delta_n^q(n-1) > \chi_n^d(q)$, residual variance from deviating and sending the higher message, given by $\Delta_n^d = \Delta_n^q(n-1) - \chi_n^d(q)$, is lower and therefore there is a gain from deviation. In the other case, if $\chi_n^d(q) > \Delta_n^q(n-1)$ then since $\chi_n^d(q) < \frac{1}{1+(n-1)\eta}$, it immediately implies that $\chi_n^d(q) - \Delta_n^q(n-1) < \frac{1}{1+(n-1)\eta}$. Player n can readjust her actions and choose a deviation action $x_n^d \in (0,1)$ such that

$$\phi_n(x_{-n}, x_n^d) = \phi_n\left(x_{-n}^*(n-1), x_n^*(n-1)\right)$$

This concludes steps 1 and 2. A similar argument holds true for every other signal realization of the remaining $(n-1)$ players. This can be observed by noting that q — the number of players for whom the constraint is binding — is weakly increasing in k, the sufficient statistic of the Beta-binomial distribution. It is straightforward to note that as before, either $\Delta_n^q(k) > \chi_n^d(q)$ in which case the residual variance is smaller from the deviation strategy, or $\Delta_n^q(k) < \chi_n^d(q)$ in which case since $\chi_n^d(q) < \frac{1}{1+(n-1)\eta}$ irrespective of the value of q. That is player n can always readjust her action according to the realization of k and reduce the residual variance, or eliminate it, for all possible realizations of the other signals from the remaining players. It concludes to observe that irrespective of whether $x_n(s_n, m_{-n}, 1) \leqslant 1$ or not, n is better off by deviating to the higher message when $s_n = 0$, since the actions of other players have unequivocally risen and decreases the residual variance as a consequence. Thus, n benefits from deviating to $m_n = 1$ when $s_n = 0$. But if this is true, then a $n-$player equilibrium ceases to exist, contradicting the starting presumption.

An analogous argument holds for players $j \in 0-type$. This concludes the proof. QED

A.3 Proof of Proposition 1

With heterogeneous constraints, the arguments hold forth for sufficiency in exactly the same manner. As long as $0 \leqslant x_i(m_N) \leqslant R_i$ for all players and signal/message realizations m_N, the joint action function $\phi_i(\mathbf{x})$ is exactly equal to $\mathbb{E}[\theta \mid m_N] + b_i$ and players cannot do better by misreporting their private signal. To show that the condition is sufficient, consider the case where $R_i \geq \bar{R}_i$ for all $i \in \{1, 2, ...(n-1)\}$ and $R_i > \bar{R}_i$ for at least one i, but $R_n < \bar{R}_n$. This implies that except for player n, every other player's constraint is not binding and they reveal their information truthfully. Suppose n reveals her information $s_n = 0$ truthfully. Clearly, when the other messages are all such that $s_{-n} = \{1\}^{(n-1)}$ then $k = (n-1)$ and $\mathbb{E}[\theta \mid s_n, s_{-n}] = \frac{n}{n+2}$. The corresponding actions as before is just $x_i(s_n, s_{-n}) = \bar{R}_i$ and $x_n(s_n, s_{-n}) = R_n < \bar{R}_n$. This implies,

$$\phi_n(\mathbf{x_{-n}}, \mathbf{R_n}) < \phi_n(\mathbf{x_{-n}}, \mathbf{\bar{R}_n}) = \mathbb{E}[\theta \mid s_n, s_{-n}] + b_n$$

This induces an additional variance term since $\Delta_1(s_{-n}, s_n) > 0$. In the case where $k = (n-2)$, that is, out of the $(n-1)$ other players there are $(n-2)$ high signals. And suppose $x_n(n-2) \leq R_n$, where $x_n(n-2)$ is the action of n when $k = (n-2)$. Then for all other signal realizations such that $k < (n-2)$ the action of n is below R_n. There is therefore no gain from misreporting. However, since the player n gains from misreporting for the realization $k = (n-1)$ (the intuition follows from arguments made in the proof of Theorem 2), there is an incentive to misreport and exaggerate. QED

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