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Christophe Muller

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Christophe Muller

Aix-Marseille Univ, CNRS, EHESS, Ecole Centrale, IRD, AMSE, Marseille, France.
christophe.muller@univ-amu.fr

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Abstract:
The main two methods of endogeneity correction for linear quantile regressions with their advantages and drawbacks are reviewed and compared. Then, we discuss opportunities of alleviating the constant effect restriction of the fitted-value approach by relaxing identification conditions.

Key words: Two-Stage Estimation, Quantile Regression, Fitted-Value Approach, Endogeneity.

1 Introduction

Endogeneity issues in regression models have been well studied in econometrics, though they may have been less fully investigated in the biometric and biostatistic literature. Endogeneity in regression model estimation may arise from reverse or feed-back causality, correlated measurement errors in dependent and independent variables, and unobserved heterogeneity correlated with dependent and independent variables.

It is not difficult to find examples in which endogeneity is a problem in biological sciences. For example, in observations of natural phenomena in biological sciences, the presence of an unobserved variable correlated with both the dependent and independent variables is often likely to generate endogeneity biases in regression estimates. Consider for instance the study of infant weights in some tropical country. In this context, heavy rains can simultaneously cause higher travelling costs because of flooded roads, on the one hand, and, worse health status, because of malaria spurred by a larger number anopheles mosquitoes, on the other hand. Then, lower observed weights may result from higher malaria incidence. However, it may also come from inefficient health care delivery caused by transportation delays. In that case, in a regression of observed infant weights on malaria spells, one expects some endogeneity of this latter variable associated with unobserved transport costs. Moreover, information on rains can be used an instrument for malaria.

In this example, investigating low quantiles of baby weights, and not only the mean weight, is crucial as these weights are excellent measures of child nutrition status, and what matters is that the weight does not fall under a minimal threshold.

More generally, analysing distributions of outcomes in biological or medicine studies is fundamental as the global average may hide many interesting and vital phenomena. Quantile regressions have been found a convenient statistical tool for such explorations and for better understanding the heterogeneity of the observed individual units in general. The practical statistical use of quantile regressions was popularised by Bassett and Koenker (1978) and Koenker and Bassett (1978) who

brought to the fore tractable computational techniques and derived asymptotic properties for these methods.

Quantile regressions allow for any given regressor having different effects for different individual units. Therefore, using quantile regressions increases the flexibility of the models. It also enables researchers to explore specific locations of the conditional distribution of the outcome variable, in particular the lower and upper tails. In that case, more substantial explanations of the variability of the studied phenomenon can be obtained, particularly in the case of nonconstant effect; that is: with regression coefficients varying across quantiles.

Different approaches have been pursued for dealing with endogeneity issues in quantile regressions. On the one hand, an analogue of the instrumental regression approach, based on exclusion restrictions, has been developed by Chernozhukov and Hansen (2005, 2006, 2008a,b). It is associated with specifications of the conditional quantile function of main equation of interest.[1] It allows for nonconstant quantile effects.

On the other hand, the fitted-value approach corresponds to another analogue of the typical two-stage least-square estimator for quantile regression. It was pioneered by Amemiya (1982) and Powell (1983) who laid their theoretical properties for two-stage least-absolute-deviations estimators in a simple setting. First, fitted-values of the endogenous regressors are estimated using a set of exogenous independent variables. Then, the estimation of the quantile regression of interest is performed by substituting the endogenous regressors with their fitted-values. This approach can be seen as imposing restrictions on the quantile of the reduced-form error. It makes sense to pay particular attention to reduced-form equations in experimental settings or policy design. Blundell and Powell (2006) pointed out that the reduced-form is of interest when control variables for the policy maker include instrumental variables. In social statistics, the pro-poor targeting of social programs can be improved by relying on predictions of living conditions based on well focused quantile regressions of reduced forms (Muller, 2005, Muller and Bibi, 2010). Similarly, in public health interventions, predictive quantile equations of health outcomes are useful; and so on for other biological sciences in which targeting interventions may be important.

Using this approach, Chen (1988) and Chen and Portnoy (1996) studied two-stage quantile regression in which trimmed least squares (TLS) and least absolute deviations (LAD) estimators are employed as the first-stage estimators. To reduce the variance of two-stage quantile regression estimators, Kim and Muller (2018) constructed a weighted average of the dependent variable with its fitted value from a preliminary estimation, which is employed as the dependent variable in a final two-stage quantile regression. Kim and Muller (2004) used a similar approach with instead quantile regression in the first stage. We now turn to a more precise discussion of the conditions in which these estimation methods yield consistent estimation and other useful properties.

2 Results and Discussion

2.1 Models and assumptions

Let us consider the estimation of the parameter (α) in the following linear equation by using quantile regression:

$$\begin{aligned} y_t &= x_{1t}'\beta + Y_t'\gamma + u_t \\ &= z_t'\alpha + u_t, \end{aligned} \tag{1}$$

[1] See Abadie et al. (2002), Hong and Tamer (2003), Honore and Hu (2004), Ma and Koenker (2006), Chernozhukov, Imbens and Newey (2007), Lee (2007), Sakata (2007).

where $t = 1, ..., T$, $[y_t, Y_t']$ is a $(G+1)$ row vector of endogenous variables, x_{1t}' is a K_1 row vector of exogenous variables, and u_t is an error term. We denote by x_{2t}' the row vector of the K_2 exogenous variables excluded from (1).To shorten notations, let $z_t = [x_{1t}', Y_t']'$, $\alpha = [\beta', \gamma']'$. Assume the first element of x_{1t} is 1.

By assumption, the following linear equation, which is assumed to be correctly specified, can be used to generate an exogenous fitted-value for Y_t:

$$Y_t' = x_t'\Pi + V_t', \tag{2}$$

where $x_t' = [x_{1t}', x_{2t}']$ is a K row vector with $K = K_1 + K_2$. Matrix Π is a $K \times G$ matrix of unknown parameters, while V_t' is a G row vector of unknown error terms. Assumptions 2 and 4 below will complete the DGP. However, let us first discuss the reduced form.

Using (1) and (2) yields:

$$y_t = x_t'\pi + v_t, \tag{3}$$

for $t = 1, ..., T$,

$$\begin{aligned} \pi &= H(\Pi)\alpha \text{ with } H(\Pi) = \left[\begin{pmatrix} I_{K_1} \\ 0 \end{pmatrix}, \Pi\right] \\ \text{and } v_t &= u_t + V_t'\gamma. \end{aligned} \tag{4}$$

Let $\rho_\theta(z) = z\psi_\theta(z)$, where $\psi_\theta(z) = \theta - 1_{[z \leq 0]}$, for any quantile index $\theta \in (0, 1)$ and $1_{[.]}$ is the indicator function. If the orthogonality conditions, $E(z_t\psi_\theta(u_t)) = 0$, were satisfied, then the one-stage quantile regression estimator would be consistent. However, when u_t and Y_t are correlated under endogeneity of Y_t, these conditions are generally not satisfied, and the quantile regression estimator of α is not consistent.

The Two-Stage Quantile Regression estimator $\hat{\alpha}$ of α is defined, for any quantile θ, as a solution to:

$$\min_{\alpha} \sum_{t=1}^{T} \rho_\theta(y_t - x_t'H(\hat{\Pi})\alpha), \tag{5}$$

where $\hat{\Pi}$ is a first-stage estimator. Let us state a few hypotheses and regularity assumptions.

Assumption 1. The sequence $\{(x_t', u_t, v_t)\}$ is $\alpha-$mixing with mixing numbers $\{\alpha(s)\}$ of size $-2(4K+1)(K+1)$.

Assumption 2. $E(\psi_\theta(v_t)|x_t) = 0$, for an arbitrary θ.

This is the main identifying condition of the fitted-value approach with quantile regression.

Assumption 3. (i) $H(\Pi + B_\Pi)$ is of full column rank.
(ii) Let $F_t(.|x)$ be the conditional cumulative distribution function (CDF) and $f_t(.|x)$ be the conditional probability density function (PDF) of v_t. The conditional PDF $f_t(\cdot|x)$ is assumed to be Lipschitz continuous for all x, strictly positive and bounded by a constant f_0 (i.e., $f_t(\cdot|x) < f_0$, for all x).
(iii) The matrices $Q = \lim_{T\to\infty} E\left[\frac{1}{T}\sum_{t=1}^{T} x_t x_t'\right]$ and $Q_0 = \lim_{T\to\infty} E\left[\frac{1}{T}\sum_{t=1}^{T} f_t(0|x_t)x_t x_t'\right]$ are finite and positive definite.

(iv) There exists $C > 0$, such that $E(\|x_t\|^3) < C < \infty$, for any t.

In Kim and Muller (2018), a general asymptotic expansion is derived that can be used to compute the particular case in the following theorem by plugging the asymptotic expansion of a first stage OLS estimator $\hat{\Pi}$ in it, to obtain:

Theorem 1. Under Assumptions 1-3, the asymptotic representation for the two-stage quantile regression estimator is:

$$T^{1/2}(\hat{\alpha} - \alpha) = RT^{-1/2}\sum_{t=1}^{T} x_t\psi_\theta(v_t) - RQ_0Q^{-1}T^{-1/2}\sum_{t=1}^{T} x_t(v_t - u_t) + o_p(1),$$

where $R = Q_{zz}^{-1}H(\Pi)'$ and $Q_{zz} = H(\Pi)'Q_0H(\Pi)$.

Assumption 4. $E(V_t|x_t) = E(V_t)$.

Assumption 4 imposes the independence of the reduced-form errors with all non-constant exogenous variables and it corresponds to the use of unbiased OLS in the first stage.

Assumption 5. (i) There are finite constants Δ, such that $E|x_{ti}V_{jt}|^3 < \Delta$, for all i, j and t.
(ii) The covariance matrix $V_T = var\left(T^{-1/2}\sum_{t=1}^{T} S_t\right)$ is positive definite for T large enough, where $S_t = (\psi_\theta(v_t), v_t - u_t)' \otimes x_t, u_t = v_t - V_t^{'}\gamma$ and $\otimes$ is the Kronecker product.

Theorem 3. (Kim and Muller, 2018) Under Assumptions 1-5,

$$D_T^{-1/2}T^{1/2}(\hat{\alpha} - \alpha) \xrightarrow{d} N(0, I),$$

where $D_T = MV_TM'$ *and* $M = R[I, -Q_0Q^{-1}]$.

Therefore, calculating the estimator and performing asymptotic inference is straightforward with this approach. In contrast, the instrumental variable approach in that case assume, instead of Assumption 2, that the conditional quantile of u_t with respect to z_t is constant: $Q_{U|Z}(\theta|z) =$ constant, where u_t can be seen as an error in (0,1), - or another interval - and for any $\theta \in (0, 1)$. This model can also be extended to nonseparable (in error) models. Practically, the IV approach is performed by approximating the first-order conditions by iterating some ancillary quantile regressions.

Specifically, the IV-QR estimator of the coefficient vector for the endogenous regressors is

$\tilde{\gamma}_\theta = \arg\min_{\gamma} \delta(\gamma, \theta)'A\delta(\gamma, \theta)$, where A is a positive definite matrix,

and $\left(\tilde{\beta}(\gamma_\theta), \tilde{\delta}(\gamma_\theta)\right) = \arg\min_{\beta_\theta, \delta_\theta} \sum_{t=1}^{T} \rho_\theta\left(y_t - Y_t\gamma_\theta - x_{1t}\beta_\theta - x_{2t}\delta_\theta\right)$.

The assumption made by Chernozhukov and Hansen (2006) for a quantile model $y = Q(Y, x_1, U_Y)$ are as follows:

Assumption B1: Given $X_1 = x_1$ for each value $\bar{Y}$ of $Y, y = Q(\bar{Y}, x_1, U_Y)$, where $U_{\bar{Y}} \sim U(0,1)$, and $Q(\bar{Y}, x_1, \theta)$ is strictly increasing in θ.

Assumption B2: Given $X_1 = x_1$, $U_{\bar{Y}}$ is independent of X_2.

Assumption B3: Given $X_1 = x_1$ and $X_2 = x_2$, for an unknown function G and a random vector v, $Y = G(X_2, X_1, \nu)$.

Assumption B4: For any values $\bar{Y}$ and $\bar{Y}'$, given (ν, X_1, X_2), $U_{\bar{Y}} \sim U_{\bar{Y}'}$.

Assumption B5: $(y_t, Y_t, X_{1t}, X_{2t})$ are iid on a compact set.

Assumption B6: For any θ, $(\beta_\theta, \gamma_\theta)$ is in the interior of a compact convex set.

Assumption B7: Assume that y has almost surely a bounded conditional density $fy_{|X_1,Y,X_2}$, and let $\pi \equiv (\beta, \gamma, \delta), \alpha \equiv (\beta', \gamma')$, $\Psi_t(\theta) \equiv V_t(\theta)\left[\Phi_t(\theta)', X_{1t}'\right]'$, where $\Phi_t(\theta) \equiv \Phi_t(\theta, X_{1t}, X_{2t})$ is a transformation of instrument information, $V_t(\theta) \equiv V_t(\theta, X_{1t}, X_{2t})$ is a positive weight function, and $\Sigma(\pi, \theta) \equiv E\left[\theta - 1_{\left\{y - Y'\gamma - X_1'\beta + \Phi_t(\theta)'\delta < 0\right\}}\Psi(\theta)\right]$ and $\Sigma(\eta, \theta) \equiv E\left[\theta - 1_{\left\{y - Y'\gamma - X_1'\beta\right\}}\Psi(\theta)\right]$.

Assume that $\frac{\partial}{\partial(\beta', \gamma')}\Sigma(\alpha, \theta)$ and $\frac{\partial}{\partial(\gamma', \alpha')}\Sigma(\pi, \theta)$ are continuous and have uniformy full rank, and that the image of $(\beta, \gamma) \longmapsto \Pi(\alpha, \theta)$ is simply connected.

Assumption B8: Almost surely, the following estimated function, denoted $f(\theta, X_1, X_2)$, converge in probability uniformy in (θ, X_1, X_2) over compact sets: $\hat{\Phi}(\theta, X_1, X_2)$ and $\hat{V}(\theta, X_1, X_2)$.

Assume that these functions $f(\theta, X_1, X_2)$ are uniformy smooth functions in (X_1, X_2) with the uniform smoothness order greater than $\dim(Y, X_1, X_2)/2$, and $\left\|f(\theta', X_1, X_2) - f(\theta, X_1, X_2)\right\| < C\left\|\theta - \theta'\right\|^a$, where $C > 0$ and $a > 0$, for all $\theta, \theta', X_1, X_2$.

Theorem 4. (Chernozhukov and Hansen):

Under Assumptions B1-B8, let $\varepsilon_t(\theta) \equiv y_t - X_{1t}'\beta(\theta) - Y_t'\gamma(\theta)$ and $l_t(\theta, \eta(\theta)) \equiv \theta - 1_{\{\varepsilon_t(\theta) < 0\}}$, then:

$\sqrt{T}\left(\hat{\alpha}(.) - \alpha(.)\right) = -J(.)^{-1}\frac{1}{\sqrt{T}}\sum_{t=1}^{T} l_t(., \alpha(.))\Psi_t(.) + o_p(1)$ converges in distribution to $b(.)$, which is a centered Gaussian process with covariance function $E\left[b(\theta)b(\theta')'\right] = J(\theta)^{-1}S(\theta, \theta')\left[J(\theta)^{-1}\right]'$, where $J(\theta) \equiv E\left[f_{\varepsilon_t(\theta)}(0 \mid X_1, Y, X_2)\Psi(\theta)\left[Y', X_1'\right]\right]$
and $S(\theta, \theta') = \left(\min(\theta, \theta') - \theta\theta'\right)E\left[\Psi(\theta)\Psi(\theta')'\right]$.

Let us now compare the two approaches in the next subsection.

2.2 Advantages and drawbacks of the two approaches

On the one hand, the fitted-value approach is often convenient. It corresponds to an elementary OLS and quantile regressions, which is practically analogous to the two-stage least-square procedure. For this reason, it has been used by empirical researchers keen to avoid computation problems.[2] In particular, no non-parametric estimation, no simulations, no numerous iterations of computation

[2] See: Arias, Hallock and Sosa-Escudero (2001), Garcia, Hernandez and Lopez (2001), Chortareas, Magonis and Panagiotidis (2012), Chepatrakul, Kim and Mizon, (2012).

steps nor optimisation grid are needed. The fitted-value approach also allows the use of a new method of variance reduction for quantile regressions proposed by Kim and Muller (2018) under very general conditions.

In contrast, an issue with the IV approach is that it may be costly in terms of numerical computations to perform. For example, the following table, extracted from Kim and Muller (2012) compares computation times using the fitted-value approach in Kim and Muller (2004, 2018) and the Chernozhukov and Hansen (2005, 2006) procedure and programs, for a simple simulation setting.

Table 1: Comparison of computation times

	Nb of observations				
Number of endogenous regressors:		1	2	3	4
Kim and Muller Test: Time *in seconds*	100	0.1	0.06	0.06	0.1
	500	0.18	0.38	0.54	0.82
Chernozhukov and Hansen Test: Time *in hours!*	100	0.0007	0.0215	1	82
	500	0.0013	0.1299	9	703

The computation time for the Chernozhukov and Hansen test is much higher because of the iterations for the numerical approximation of the first-order conditions, especially with more than one endogenous regressor. Although this drawback may be alleviated by using more efficient algorithms, it remains an issue when there are many endogenous regressors.

On the other hand, the fitted-value approach is often plagued by the occurrence of constant effects, as claimed by Lee (2007). That is: all coefficients, except for the intercept, should be the same for all considered quantiles, which makes the model less flexible and therefore less realistic. However, even if this drawback is real, this is not completely so. Muller (2017) showed that it is possible to estimate two-stage quantile regressions using the fitted-value approach that are consistent with a particular form of nonconstant effects. In that case, heterogeneous coefficients can be allowed for some of the model regressors only. This can be obtained by assuming weaker instrumental variable restrictions than usual. Under these weakened conditions, the endogeneity can be treated by using the fitted-value approach, although the nonconstant effects have to correspond to parameters that are in the model but cannot be identified precisely. However, nonconstant effects, varying with the quantile index, can still be present in the true model. Another shortcoming of the fitted-value approach is that the first-stage equation must be well specified, whereas this is not required for the instrumental variable approach.

However, there is also some common ground between the two approaches. Indeed, even under constant effects, quantile regressions can be useful when only one given quantile is of interest, for example when the considered intervention or experiment is targeted to this quantile. In that case, both methods are appropriate. Moreover, when one is only interested in the individual mean, then the two approaches can be seen as equivalent under exact identification (Galvao and Montes-Rosas, 2012).

2.3 Relaxing identification conditions

The above-mentioned interest in relaxing identification requirements for quantile regression under endogeneity invites to pursue the discussion in this direction. Assumption 2 imposes that zero is

the given θ^{th}-quantile of the conditional distribution of $v_{t\theta}$, where the quantile index θ has been added to v_t to show well that Assumption 2 characterizes a given quantile index θ.

Assumption 6: *For a given quantile index θ, the cdf of v_{t_θ} conditional on x_t, denoted $F_{v_{t\theta}|x_t}$, the cdf of $v_{t\theta}$ conditional on x_{1t}, denoted $F_{v_{t\theta}|x_{1t}}$, and the marginal cdf of x_{2t}, denoted $F_{x_{2t}}$, are continuous and strictly increasing.*

Now, *instead of Assumption 2*, the weaker Assumption 2' can be used.

Assumption 2': *For a given quantile θ and under Assumption 1:*

$$v_{t\theta} \text{ is independent of } x_{2t}, \text{ conditionally on } x_{1t}. \tag{6}$$

In Muller (2007), it is shown that

Theorem 5: *Under Assumptions 6 and 2', for a quantile regression process of the reduced form (3):*

(a) There is a constant effect for the variables in x_{2t}.

(b) A nonconstant effect is possible for the variables in x_{1t}.

(c) For all θ, $F^{-1}_{v_{t\theta}|x_{1t}}(\theta)$ is linear in x_{1t}.

The popular 'linear location-scale hypothesis' in the quantile regression literature on the non-constant effect (e.g., Koenker, 2005, pp. 104–105) is consistent with Result (c) in Proposition 1. Moreover, Result *(c)* may be easily relaxed by including polynomial terms in x_{1t} in the model. Alternatively, the reduced form in (3) could be specified as being partially linear in x_{2t}, and nonlinear in x_{1t}, with an unknown nonlinear functional form. This still yields a constant effect for x_{2t} and an unrestricted nonlinear effect for x_{1t}. In that case, Result *(c)* could be discarded. Finally, instead of imposing Assumption 2', one may first test for which coefficients the hypothesis of constant effect is rejected or not in typical quantile regression estimation, so as to guide the precise specification of this assumption. Finally, the results in Muller (2017) are:

Theorem 6: *Under Assumptions 1 and 2:*

(a) *The components of π_θ in the reduced form (3) for any quantile index θ can be identified, for the constant coefficients $\pi_{2\theta} = \pi_2$ of x_{2t}, but not for the non-constant coefficients $\pi_{1\theta}$ of x_{1t}.*

(b) *For the quantile model (1), the coefficient vector γ_θ of the endogenous regressors Y_t in the quantile model is identified, while constant with respect to the quantile index θ: $\gamma_\theta = \gamma$, for all $\theta \in (0, 1)$.*

(c) *The coefficient vector β_θ of the exogenous regressors x_{1t} in the quantile model can be non-constant with respect to the quantile index θ, while it is not identified in general.*

In biological sciences, obtaining a condition like in Assumption 2', or a similar one by reversing the roles of x_{1t} and x_{2t}, should be much easier than in social sciences. Indeed, experimental settings or specification of controls could be designed accordingly.

3 Conclusion

The design of identification conditions for solving endogeneity issues in quantile regressions is still an open research area. The diverse methods used in the literature to deal with the problems

discussed here correspond to non-encompassing restrictions. It seems therefore fruitful to further investigate and develop each kind of approach. We have sketched the state of the question from which such extension could be built on.

In particular, a few reservations would deserve further investigation. First, studying some practical applications based on actual policy data or experiment data that are characterised by constant effect of treatment variables would assist in clarifying the potential of the respective methods. Second, the rigid distinction between constant effect and nonconstant effect for endogenous and exogenous regressors could be relaxed to generate more flexible specifications. Third, as always, finding instruments, even if weakened ones, is still hard in general. However, owing to controlled experiments, this may be easier in biological sciences than in other study areas.

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