

**Pollution in a globalized world:
Are debt transfers among countries a solution?**

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Abstract

This article analyzes the impacts of debt relief on production and pollution. We develop a two-country overlapping generations model with environmental externalities, public debts and perfect mobility of assets. Pollutant emissions arise from production, but agents may invest in pollution mitigation. Could debt relief be an efficient tool to encourage less developed countries to engage in the fight against climate change? We consider a decrease of the debt of the poor country balanced by an increase of the richer country's debt. We show that debt relief makes it possible to engage poor countries in the process of pollution abatement. Capital, environmental quality and welfare can increase in both countries. This result relies on the environmental sensitivity and the discount factor in the poor country relative to the rich one: the greater they are the more beneficial the debt relief is.

JEL classification: F43, H23, Q56.

Keywords: Pollution; Abatement; Overlapping generations; Public debt; Capital market integration.

1 Introduction

The trend in CO₂ emissions predicts a global temperature increase of more than 3°C by the end of the century with severe consequences. Loss of agricultural yield, loss of biodiversity, sea level rise, climate migration, extreme weather events... are all phenomena that will increase and the brunt of the economic damages will be probably borne by poorest countries (Schelling, 1992;

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Mendelsohn, et al., 2006). To achieved a 1.5°C consistent pathway, the international community calls for immediate global response. Meeting this challenge would require to accelerate the deployment of mitigation actions and to have an active participation of all countries (see Coninck et al., 2018). Regarding engagement of countries, important disparities prevail, as illustrated by Table 1 that presents the Environmental Performance Index (EPI) for climate and energy for several countries. This index provides a good proxy of country’s environmental engagement.

Countries	Index	Countries	Index
Switzerland	90.55	Republic Czech	54.65
Sweden	86.8	Ireland	54.16
France	70.46	Netherlands	52.55
China	68.62	South Korea	46.75
Spain	67.77	United States of America	45.81
Denmark	67.56	Japan	45.08
Italy	65.14	India	44.77
Poland	64.33	Greece	42.17
Norway	63.58	Turkey	39.99
United Kingdom	63.06	Canada	39.88
Austria	62.79	Brazil	33.24
Portugal	62.33	Burkina Faso	26.24
Germany	55.47	Madagascar	16.23

Table 1 – *The Climate & Energy Environmental Performance Index (EPI), 2018.*
<https://epi.envirocenter.yale.edu/>

Environmental objectives are a particular concern for poor countries in which the environmental challenges are compounded by high levels of external economic vulnerability and public debt. They recurrently use public debt to absorb the impact of external macroeconomic shocks and, now, of more frequent natural disasters. In turn, higher levels of public debt associated with weak macroeconomic situation increase fiscal pressure and constrain the capacity of countries to address vulnerabilities and to mitigate pollution. According to IMF classification, a large part of poor countries presents a high risk of experiencing a debt crisis, what underlines the importance to compose with countries debt profiles to address environmental challenge.¹

When we examine the diversity of instruments used to provide climate finance, debt transfers, in the form of debt reliefs granted by developed countries to developing one, currently represent only an insignificant proportion of financing strategies (see the Table 2 built by Fenton et al., 2014).

Question then arises, does a debt transfer in favor of less developed countries is a relevant macroeconomic policy lever to accompany environmental actions? This article contributes to this question by using a theoretical approach combining debt transfers and global pollution issues. We aim at analyzing the consequences of a debt transfer from rich to poor countries on mitigation

¹See the debt sustainability framework of IMF and World Bank for public and external debt sustainability analysis in low-income countries.

Financial instruments	Amount (million, US\$)	Share
Grants and related instruments	14379.1	45.2%
Unknown	614.4	1.9%
Multiple	419.3	1.3%
Loans, guarantees and insurance	14840.2	46.7%
Debt relief	82.5	0.3%
Capital contribution	1457.4	4.6%
Total	31792.9	100%

Table 2 – *Instrument used to fulfill climate finance commitments.*

From “Debt relief and financing climate change action”, A. Fenton, H. Wright, S. Afionis, J. Paavola and S. Huq, *Nature Climate Change* (4), 2014. Based on data collected by the World Resource Institute

<http://go.nature.com.inshs.bib.cnrs.fr/rMhVxK>

decision of agents, as well as on pollution, GDP and well-being.

We develop a two-country overlapping generations (OLG) model with global pollution externalities and private mitigation. Production deteriorates the environmental quality, harming the welfare of future generations. There is a perfect mobility of assets between countries. In line with arguments previously presented, countries exhibit different macroeconomic profiles. They have different saving rates, heterogeneous green awareness as well as productivities. In addition, they are characterized by a different level of debt, which is financed by a tax on agents’ income and by new debt emissions. To keep the analysis simple, we do not formalize public spending for environmental protection, such that fiscal pressure is mainly determined by the debt level. The relative productivity between countries determines their profile: we define the poor (resp. the rich) country as those with the lower (resp. the higher) level of productivity.

To limit the effects of macroeconomic policy on fully integrated financial markets, we assume that overall debt remains constant. This implies a constant global debt supply and no direct effect of the debt policies on the international interest rate, even if it does not exclude composition effects of debt among countries. This assumption can also be seen as a way to rule out unsustainable global debt.

Accordingly, we study the environmental and economic consequences of a debt relief in favor of the poorer country, funded by the richer country. Our research question leads us to focus on situations in which, without any transfer, the poor country does not engage in environmental action while the rich one does it. We show that debt relief for this less involved country is never sufficient to lead him to engage in the process of pollution abatement when its sensitivity to environmental quality is low. When green awareness in the poor country is high enough, a debt transfer can induce him to engage in actions to fight against climate change. This result is confirmed even when the aid donor country faces high level of debt, as are most North-American and Western European countries. In any case, such a policy can be good for both capital accumulation and the environment depending on the preferences of the different countries. The debt transfer translates into income tax variation and hence implies a redistribution of net income

between countries. Given that bonds are used as debt instrument, the debt level has an effect on the capital market equilibrium. It affects debt services, and therefore, the level of taxes. If the poor country is the more patient, it increases net income and saving for the recipient while reduces it for the donor. As poor and rich countries share a common international asset market, global saving, capital stock and hence global pollution abatement increases only if the transfer is in favor of the population that saves more. In such a case, welfare benefits are clear for the recipient. However, when the poor country is relatively impatient, it is not appropriate to implement a debt transfer without any other economic and environmental actions, because even if it can help him to engage in mitigation activities, the transfer reduces the global stock of capital and the global level of effort to mitigate pollution. As the efficiency of abatement activities in reducing emission is sufficiently important, the policy is damaging for environmental quality. Our analysis sustains the interest of debt reliefs to address climate change issues, by providing the conditions under which a debt transfer is an efficient macroeconomic instrument to promote environmental actions.

As we have seen, our paper adopts a theoretical approach to contribute to two strands of the economic literature, namely the interplay between public debt and environmental quality, and the links between economic development and the environment in an international context. Connecting these two strands, we provide new intuitions about the effects of international transfers on welfare and the environment.

Debt financing has been introduced in dynamic models with environmental concerns (Bovenberg and Heijdra, 1998; Heijdra et al., 2006; Fernandez et al., 2010). Debt policy only makes possible to redistribute welfare gains from future to existing generations: there is no debt financing of the pollution mitigation sectors. Fodha and Seegmuller (2012, 2014) and Fodha et al. (2018) analyze debt financing schemes for public and private mitigation. They show that efficient environmental tax reforms may be designed. This existing literature, however, considers closed economies, which is a limitation as capital markets are interconnected and globalized, as climate.

Our work complements prior contributions that deal with global environmental issues in international frameworks. This paper most directly extends three existing papers. John and Pecchenino (1997) focus on the effects of cooperation between two countries in mitigating global pollution. Their approach is simpler because there is no capital or savings arbitrage and they only assume lump-sum transfers between countries. They conclude that the developing countries must be compensated for their environmental expenditures. In our article, we focus on financial markets and debt levels. Debt reduction is interesting because it has two effects. A direct one plays the same role as a lump-sum transfer of income. An indirect one goes through the debt service reduction. The second is the work of Bednar-Friedl et al. (2010). Countries differ in their levels of public debt per capita such that one country is a net creditor and the other one is a net debtor to the world economy. They find that if the country which unilaterally reduces its emissions permits is a net creditor to the world economy, the domestic welfare costs are smaller and the foreign welfare costs are larger than if it was a net debtor. While in this work mitigation efforts

are exogenous, we differ by taking into account endogenous mitigation expenditure. Especially, we study in details the effect of debt transfers when one country makes no effort to reduce its net emissions. Third, Muller-Furstenberger and Schumacher (2017) propose a dynamic model where all agents contribute to a global externality, but only those in a specific region suffer from it. They develop an overlapping generations model with two types of agents. In the non integrated economy, even though agents have the same technologies, if agents affected by the externality are sufficiently poor in terms of initial capital endowments, they may be stuck in an environmental poverty trap. Capital market integration helps to eliminate it. This article, although close to our work, considers heterogeneity in the consequences of the climate change while we are interested in heterogeneity in preferences. Moreover, in addition to the integrated capital market assumption, we add public debt that influences savings and capital accumulation.

The consequences of international transfers have been particularly examined in the development economic literature, to see if it provides development support for indebted countries. While direct aid to poor countries are often criticized (see e.g Boone, 1996; Burnside and Dollar, 2000; Rajan and Subramanian, 2006; Dreher and Langlotz, 2017), Cassimon et al. (2015) and Channing et al. (2016) conclude that the gains from debt cancellation promote health spending, public investment, and tax mobilization within the recipient countries. We extend the discussion by considering climate change issues and we give conditions such that untied debt cancellation promotes also environmental quality and GDP per capita worldwide.

The literature has also questioned the characteristics of international transfers to protect the environment. Baraano and San Martin (2015) develop a dynamic equilibrium model in which public investments in both infrastructure and pollution abatement can be financed using international aid. They find that transfers linked to both infrastructure and pollution abatement may be the best welfare-enhancing alternative. Sakamoto et al. (2017) show that financial aid to enhance adaptation capacity of vulnerable countries is efficient. They find that Norths assistance to South can facilitate pollution mitigation in both regions. Another branch of the literature consider the interplay between environmental finance and development aid transfers. Bretschger and Suphaphiphat (2014) develop a two country growth model with two policy outcomes: climate mitigation by the North to reduce global pollution and traditional development aid under the form of income transfer. Because of low learning capacities in the South, they obtain that income transfer is without effect on growth while active climate policy favors capital accumulation in both countries and hence growth. In the same vein, Eyckmans et al. (2016) develop a theoretical model in which North-South development transfers are complemented by mitigation and adaptation transfers. Concerning the effects of a transfer for mitigation support, they underline that climate change finance works as a pure income transfer, without effectiveness in terms of emission cuts unless associated with matching grant. In our article, we depart from this literature as we limit aid transfers to untied debt cancellation while technology against pollutant emission corresponds to mitigation. Our work differs from these contributions by highlighting the role of private agents in mitigation decisions, which depend on individuals preferences and net income.

Finally, in our paper, debt relief is undertaken without any counterpart. Hence, our model does not take into account debt for nature swaps (Deacon and Murphy, 1997; Cassimon et al., 2011, 2014). The literature on economic development underlines the inefficiencies of this type of aid as soon as donor country monitoring is not put in place. The risk of inefficiency of this policy is all the higher as the country faces problems of quality governance, or extreme poverty. We show the possibility of economic and environmental improvements even if debt relief is considered without promise of any environmental action.

The rest of this paper is organized as follows. Section 2 presents the two-country OLG model with environmental externalities. Section 3 defines the equilibrium. Section 4 studies the situations where agents in the poor country have a weak environmental sensitivity. Section 5 presents the opposite configuration. Technical details are relegated to an Appendix.

2 The model

The world consists of two competitive economies indexed by $i \in \{D, F\}$. Within each country, a new generation of two-period lived agents is born at each period of time. Therefore, two generations alive in each period t : the workers and the retired people. In each country, the population size of the generation is constant and normalized to one. There is no mobility of labor between countries, whereas there is a perfect mobility of the assets and the unique final good, as for instance in Persson (1985). The assumption of perfect capital mobility can be seen as a good approximation of what happens in countries with highly integrated financial markets and few capital controls.²

2.1 Environmental quality

We consider the environment as a global public good, such as climate. We measure the evolution of this aggregate by an index, which deteriorates with pollution from global production and improves with private abatement. We assume that both the rates of pollutant emissions and the rates of abatement are equal among countries. In addition, we neglect natural degradation, so the environment is a stock whose accumulation varies with net pollution. Mitigation consists in reducing greenhouse gas emissions into the atmosphere, by increasing energy efficiency, replacing fossil fuels with new energies and enhancing the sinks that capture and store these gases. Mitigation could be reforestation, insulation of buildings or the development of carbon capture and sequestration technologies. Global environmental quality E_t evolves according to:

$$E_{t+1} = E_t - \theta(y_{Dt} + y_{Ft}) + \phi(a_{Dt} + a_{Ft}) \quad (1)$$

where y_{it} and a_{it} represent production and abatement of country i respectively, and $E_0 > 0$. The pollution flow resulting from production is given by the emission factor $\theta > 0$ and efficiency of

²At the end of Section 5.2, we will provide intuitions about the mechanisms that operate when capital mobility between countries is not perfect to compare with the situation with perfect mobility.

abatement is given by factor $\phi > 0$. We keep these parameters equal in order to focus on the role played by debt, savings and mitigation decisions. Taking into account different values for these parameters would be possible, but would only accentuate the effects of specialization. The country with a high ϕ would have to mitigate, while the country with a low θ would have to produce more.

2.2 Firms

In each country, producers use capital and labor for the production of a unique final good, which is the numéraire. The technology used is Cobb-Douglas. Taking into account that labor is unit, the production function writes $y_{it} = A_i k_{it}^\alpha$, where k_{it} denotes capital, $A_i > 0$ the global productivity and $\alpha \in (0, 1)$ the capital share in income. Profit maximization gives:

$$w_{it} = (1 - \alpha)A_i k_{it}^{\alpha-1} \quad (2)$$

$$R_{it} = \alpha A_i k_{it}^{\alpha-1} \quad (3)$$

with w_{it} the wage and R_{it} the return of capital in country i .³

2.3 Households

A generation born at period t derives utility from consumption when young c_{it} and old d_{it+1} and from environmental quality at both periods. Accordingly, the lifetime utility is given by:

$$\ln c_{it} + \beta_i \ln d_{it+1} + \delta_i \ln E_t + \gamma_i \ln E_{t+1} \quad (4)$$

where $\beta_i \in (0, 1)$ denotes the discount factor in country i , and $\delta_i \geq 0$ and $\gamma_i \geq 0$ the sensitivity to environmental quality when young and old respectively. We assume that these preference parameters are country specific, which means that $\beta_D \neq \beta_F$, $\delta_D \neq \delta_F$ and $\gamma_D \neq \gamma_F$. On the one hand, heterogeneity between β_i is empirically supported by the work of Wang et al. (2016), which present results from a large-scale international survey on time preference, conducted in 53 countries, and find that the waiting tendency is correlated with country specific characteristics, like innovation, environmental protection, and crediting rating. On the other hand, δ_i and γ_i aggregate both sensitivity and vulnerability to climate change which are also different among rich and poor countries (Schelling, 1992; Mendelsohn et al., 2006).

When young, each agent inelastically supplies one unit of labor and receives real wage w_{it} . A lump-sum tax τ_{it} is levied on this income, which is shared between consumption c_{it} , savings s_{it} and private abatement a_{it} , as in John and Pecchenino (1994). Indeed, young people take care about the environmental quality they will face when old, but suffer from the past accumulation of pollution. Consumption when old d_{it+1} is entirely financed by the remunerated savings.

³For simplification and taking into account the duration of a period, we assume full depreciation of capital after one period of use.

Therefore, the two budget constraints faced by an agent born at period t write:

$$c_{it} + s_{it} + a_{it} = w_{it} - \tau_{it} \quad (5)$$

$$d_{it+1} = R_{it+1}s_{it} \quad (6)$$

A young agent maximizes her utility (4) taking into account the two budget constraints (5) and (6), the environmental quality (1) and the non-negativity of abatement $a_{it} \geq 0$. We obtain:

$$d_{it+1} = \beta_i R_{it+1} c_{it} \quad (7)$$

$$E_{t+1} \geq \gamma_i \phi c_{it}, \text{ with an equality when } a_{it} > 0 \quad (8)$$

which also ensures $E_{t+1} > 0$ for all $t \geq 0$.

We deduce that we have two main situations. Either $a_{it} > 0$ and:

$$s_{it} = \frac{\beta_i}{\gamma_i \phi} E_{t+1}, \quad c_{it} = \frac{E_{t+1}}{\gamma_i \phi}, \quad d_{it+1} = \frac{\beta_i}{\gamma_i \phi} R_{it+1} E_{t+1} \quad (9)$$

$$a_{it} = w_{it} - \tau_{it} - \frac{1 + \beta_i}{\gamma_i \phi} E_{t+1} \quad (10)$$

or $a_{it} = 0$ and:

$$s_{it} = \frac{\beta_i}{1 + \beta_i} (w_{it} - \tau_{it}), \quad c_{it} = \frac{1}{1 + \beta_i} (w_{it} - \tau_{it}) \quad (11)$$

$$d_{it+1} = \frac{\beta_i}{1 + \beta_i} R_{it+1} (w_{it} - \tau_{it}), \quad E_{t+1} > \frac{\gamma_i \phi}{1 + \beta_i} (w_{it} - \tau_{it}) \quad (12)$$

When environmental quality is high enough with respect to the net income, an agent has no incentive to mitigate pollution. In contrast, if the net income is high enough and environmental quality too low, she engages in abatement activities. The latter case is all more likely as γ_i is high.

2.4 Government

The evolution of public debt in domestic and foreign countries and its consequences, especially on economic growth, is a major economic concern. Public debt issues are also included in international debates on the environment, in which the connection between public debt and the environment is recursively discussed. To keep analysis simple, we do not formalize public spending for environmental protection. Rather, we focus on interactions between public debt and the environment in an international context by assuming that final action to reduce global pollution is private. In this way, the government cannot directly use debt to finance future environmental quality and the direct link between public debt and environmental policy is ignored.

In each country, the government faces the following budget constraint:

$$b_{it+1} = R_{it} b_{it} - \tau_{it} \quad (13)$$

with the initial public debt level $b_{i0} \geq 0$ given. The government collects lump sum tax τ on workers and uses bonds as debt instrument. Its expenditures include repayment of debt and interest payments.

3 Equilibrium

As international capital mobility is assumed to be perfect, foreign and domestic assets yield the same rate of return. Market clearing requires world savings equal to world investment:

$$R_{Dt} = R_{Ft} \quad (14)$$

$$s_{Dt} + s_{Ft} = k_{Dt+1} + b_{Dt+1} + k_{Ft+1} + b_{Ft+1} \quad (15)$$

Given the production technologies in each country, the law of motion for environmental quality is given by:

$$E_{t+1} = E_t - \theta(A_D k_{Dt}^\alpha + A_F k_{Ft}^\alpha) + \phi(a_{Dt} + a_{Ft}) \quad (16)$$

and $E_t > 0$. Using (3), equation (14) means that:

$$k_{Ft} = k_{Dt} \left(\frac{A_F}{A_D} \right)^{\frac{1}{1-\alpha}} \quad (17)$$

We will investigate policies where debt in both countries are exogenous policy parameters, i.e. $b_{Dt} = b_D$ and $b_{Ft} = b_F$ for all t , while τ_{Dt} and τ_{Ft} are endogenous and satisfy the government budgets. In this way, public debt is a policy variable and the economic sustainability of public debt is satisfied at each period. Equation (13) becomes:

$$\tau_{it} = (\alpha A_i k_{it}^{\alpha-1} - 1) b_i \quad (18)$$

We consider a domestic economy richer than the foreign one. Numerous studies emphasize that the major part of the difference in incomes between rich and poor countries is due to differences in total factor productivity (see the survey by Caselli, 2005, and the study by Hsieh and Klenow, 2010).

Assumption 1 $A_D > A_F$.

Despite this assumption on the productivities between countries, we do not impose any ranking on the respective debt levels, i.e. b_D higher or smaller than b_F . We will in particular analyze the effect of an increase of b_D which allows to decrease b_F . Such a measure implies a redistribution of income net of tax from the rich to the poor country, as the tax paid in each country linearly depends on b_i . However, we will see that the effects of a debt transfer quantitatively differ from a standard redistribution of income because it goes through different channels compare to a simple transfer of income. In addition, qualitative differences emerge if we consider imperfect capital mobility, as discussed at the end of Section 5.2.

We focus on equilibria with $a_{Dt} > 0$ and either $a_{Ft} > 0$ or $a_{Ft} = 0$. Equilibria without any abatement activities are excluded from our analysis, while situations in which only one of the country do not invest in environmental protection are examined. Indeed, when both countries do not engage in mitigation, the net pollution flow is positive at each date, leading to a perpetual decrease in the quality of the environment that rules out any stationary equilibrium.

3.1 Equilibrium with $a_{Dt} > 0$ and $a_{Ft} = 0$

We focus here on an asymmetric equilibrium where only the domestic households are engaged in pollution mitigation. This means that:

$$\frac{\phi\gamma_D}{1+\beta_D} [(1-\alpha)A_D k_{Dt}^\alpha - \tau_{Dt}] > E_{t+1} > \frac{\phi\gamma_F}{1+\beta_F} \left[(1-\alpha)A_F k_{Dt}^\alpha \left(\frac{A_F}{A_D}\right)^{\frac{1-\alpha}{1-\alpha}} - \tau_{Ft} \right] \quad (19)$$

Using (2), (11), (10) and (17), equilibrium on the asset market (15) satisfies:

$$\begin{aligned} k_{Dt+1} \left[1 + \left(\frac{A_F}{A_D}\right)^{\frac{1}{1-\alpha}} \right] - \frac{\beta_D}{\gamma_D\phi} E_{t+1} + b_{Dt+1} + b_{Ft+1} \\ = \frac{\beta_F}{1+\beta_F} \left[(1-\alpha)A_F k_{Dt}^\alpha \left(\frac{A_F}{A_D}\right)^{\frac{1-\alpha}{1-\alpha}} - \tau_{Ft} \right] \end{aligned} \quad (20)$$

Substituting (2), (10), (17) and $a_{Ft} = 0$, equation (16) becomes:

$$\left(1 + \frac{1+\beta_D}{\gamma_D}\right) E_{t+1} = E_t - \phi\tau_{Dt} + A_D k_{Dt}^\alpha \left[\phi(1-\alpha) - \theta \left(1 + \left(\frac{A_F}{A_D}\right)^{\frac{1}{1-\alpha}}\right) \right] \quad (21)$$

An equilibrium with $a_{Dt} > 0$ and $a_{Ft} = 0$ satisfies equations (20) and (21), taking into account that inequalities (19) hold and the government budget constraint is given by (18).

3.2 Equilibrium with $a_{Dt} > 0$ and $a_{Ft} > 0$

Using inequality (12), we deduce that $a_{Dt} > 0$ and $a_{Ft} > 0$ require:

$$E_{t+1} < \phi \min \left\{ \frac{\gamma_D}{1+\beta_D} [(1-\alpha)A_D k_{Dt}^\alpha - \tau_{Dt}]; \frac{\gamma_F}{1+\beta_F} \left[(1-\alpha)A_F k_{Dt}^\alpha \left(\frac{A_F}{A_D}\right)^{\frac{1-\alpha}{1-\alpha}} - \tau_{Ft} \right] \right\} \quad (22)$$

Using (9) and (17), the equilibrium condition (15) rewrites:

$$\frac{1}{\phi} E_{t+1} \left(\frac{\beta_D}{\gamma_D} + \frac{\beta_F}{\gamma_F} \right) = k_{Dt+1} \left[1 + \left(\frac{A_F}{A_D}\right)^{\frac{1}{1-\alpha}} \right] + b_{Dt+1} + b_{Ft+1} \quad (23)$$

Now, using (2), (10) and (17), the dynamics of environmental quality (16) becomes:

$$\begin{aligned} \left(1 + \frac{1 + \beta_D}{\gamma_D} + \frac{1 + \beta_F}{\gamma_F}\right) E_{t+1} &= E_t - \phi(\tau_{Dt} + \tau_{Ft}) \\ &+ A_D k_{Dt}^\alpha [\phi(1 - \alpha) - \theta] \left[1 + \left(\frac{A_F}{A_D}\right)^{\frac{1}{1-\alpha}}\right] \end{aligned} \quad (24)$$

An equilibrium with $a_{Dt} > 0$ and $a_{Ft} > 0$ satisfies equations (23) and (24), taking into account the government budget constraint (18) and inequalities (22).

4 A foreign economy with low environmental sensitivity (γ_F arbitrarily small)

We consider first a foreign economy with green awareness lower than in the domestic economy, and even arbitrarily small and close to zero.

Assumption 2 γ_F arbitrarily small, satisfying $\gamma_F < \gamma_D$, and close to 0.

Substituting (18) in (19), the economy is in the asymmetric case $a_{Dt} > 0$ and $a_{Ft} = 0$ if the following inequalities are satisfied:

$$E_{t+1} < \frac{\phi\gamma_D}{1 + \beta_D} \left[(1 - \alpha)A_D k_{Dt}^\alpha - b_D \left(\alpha A_D k_{Dt}^{\alpha-1} - 1 \right) \right] \quad (25)$$

$$E_{t+1} > \frac{\phi\gamma_F}{1 + \beta_F} \left[(1 - \alpha)A_D k_{Dt}^\alpha \left(\frac{A_F}{A_D} \right)^{\frac{1}{1-\alpha}} - b_F \left(\alpha A_D k_{Dt}^{\alpha-1} - 1 \right) \right] \quad (26)$$

Under Assumption 2, we note that any steady state (k_D, E) will satisfy equation (26), meaning that there will be no steady state with $a_F > 0$ when γ_F is small and close to zero. This leads us to restrict our attention to equilibria such that $a_{Dt} > 0$ and $a_{Ft} = 0$ when Assumption 2 is satisfied.

4.1 Dynamics and steady states

Substituting (18) in equations (20) and (21), we obtain the dynamic behavior of the model:

$$\begin{aligned} k_{Dt+1} &\left[1 + \left(\frac{A_F}{A_D}\right)^{\frac{1}{1-\alpha}}\right] - \frac{\beta_D}{\gamma_D \phi} E_{t+1} + b_D + \frac{b_F}{1 + \beta_F} \\ &= \frac{\beta_F}{1 + \beta_F} \left[(1 - \alpha)A_F k_{Dt}^\alpha \left(\frac{A_F}{A_D} \right)^{\frac{\alpha}{1-\alpha}} - b_F \alpha A_D k_{Dt}^{\alpha-1} \right] \end{aligned} \quad (27)$$

$$\begin{aligned} \left(1 + \frac{1 + \beta_D}{\gamma_D}\right) E_{t+1} &= E_t - \phi b_D \left[\alpha A_D k_{Dt}^{\alpha-1} - 1 \right] \\ &+ A_D k_{Dt}^\alpha \left[\phi(1 - \alpha) - \theta \left(1 + \left(\frac{A_F}{A_D} \right)^{\frac{1}{1-\alpha}} \right) \right] \end{aligned} \quad (28)$$

Substituting (28) into (27), we get:

$$\begin{aligned}
k_{Dt+1} \left[1 + \left(\frac{A_F}{A_D} \right)^{\frac{1}{1-\alpha}} \right] &= -\frac{b_D(1+\gamma_D)}{1+\beta_D+\gamma_D} - \frac{b_F}{1+\beta_F} + \frac{\beta_D}{1+\beta_D+\gamma_D} \frac{E_t}{\phi} \\
+ k_{Dt}^\alpha \left[(1-\alpha)A_F \left(\frac{A_F}{A_D} \right)^{\frac{\alpha}{1-\alpha}} \frac{\beta_F}{1+\beta_F} + A_D \left(1 - \alpha - \frac{\theta}{\phi} \left(1 + \left(\frac{A_F}{A_D} \right)^{\frac{1}{1-\alpha}} \right) \right) \frac{\beta_D}{1+\beta_D+\gamma_D} \right] \\
- \alpha A_F k_{Dt}^{\alpha-1} \left[\frac{b_F \beta_F}{1+\beta_F} + \frac{b_D \beta_D}{1+\beta_D+\gamma_D} \right] & \quad (29)
\end{aligned}$$

The dynamic system (28) and (29) explicitly gives (E_{t+1}, k_{Dt+1}) as functions of (E_t, k_{Dt}) . Now, we use it to analyse the existence and the number of steady states, and their stability properties.

Using (28), we deduce that $E_{t+1} \geq E_t$ if and only if:

$$\begin{aligned}
E_t \leq \varphi(k_{Dt}) \equiv \\
\left\{ A_D k_{Dt}^\alpha \left[\phi(1-\alpha) - \theta \left(1 + \left(\frac{A_F}{A_D} \right)^{\frac{1}{1-\alpha}} \right) \right] - \phi b_D \left[\alpha A_D k_{Dt}^{\alpha-1} - 1 \right] \right\} \frac{\gamma_D}{1+\beta_D} & \quad (30)
\end{aligned}$$

To be able to ensure $E_t > 0$ even if there is under-accumulation of capital ($\alpha A_D k_{Dt}^{\alpha-1} > 1$), we assume:

Assumption 3 $\phi(1-\alpha) > \theta \left(1 + \left(\frac{A_F}{A_D} \right)^{\frac{1}{1-\alpha}} \right)$.

This assumption can be easily interpreted. Indeed, it can be rewritten as $\phi w_D > \theta(y_D + y_F)$ meaning that when the rich country spends all its income on mitigation, it has to absorb more than the total flow of pollution. This is empirically relevant because when the domestic advanced economy does not save, capital is only financed by the poorest economy, meaning that the capital stock and hence the pollution flow are low.

Using now (29), the inequality $k_{Dt+1} \geq k_{Dt}$ is equivalent to $E_t \geq \psi(k_{Dt})$, with:

$$\begin{aligned}
\frac{\beta_D}{1+\beta_D+\gamma_D} \frac{1}{\phi} \psi(k_{Dt}) &= k_{Dt} \left[1 + \left(\frac{A_F}{A_D} \right)^{\frac{1}{1-\alpha}} \right] + \frac{b_D(1+\gamma_D)}{1+\beta_D+\gamma_D} + \frac{b_F}{1+\beta_F} \\
- k_{Dt}^\alpha \left[(1-\alpha)A_F \left(\frac{A_F}{A_D} \right)^{\frac{\alpha}{1-\alpha}} \frac{\beta_F}{1+\beta_F} + A_D \left(1 - \alpha - \frac{\theta}{\phi} \left(1 + \left(\frac{A_F}{A_D} \right)^{\frac{1}{1-\alpha}} \right) \right) \frac{\beta_D}{1+\beta_D+\gamma_D} \right] \\
+ \alpha A_F k_{Dt}^{\alpha-1} \left[\frac{b_F \beta_F}{1+\beta_F} + \frac{b_D \beta_D}{1+\beta_D+\gamma_D} \right] & \quad (31)
\end{aligned}$$

We note that $\varphi(k_{Dt})$ is increasing and concave, while $\psi(k_{Dt})$ is U-shaped. Now, using (28),

inequalities (25) and (26) are equivalent to:

$$\begin{aligned}
E_t &< A_D k_{Dt}^\alpha \left[\frac{\phi \gamma_D}{1 + \beta_D} (1 - \alpha) + \theta \left(1 + \left(\frac{A_F}{A_D} \right)^{\frac{1}{1-\alpha}} \right) \right] - (\alpha A_D k_{Dt}^{\alpha-1} - 1) \frac{\phi \gamma_D b_D}{1 + \beta_D} \equiv C_D(k_{Dt}) \\
E_t &> -A_D k_{Dt}^\alpha \left[\phi (1 - \alpha) \left(1 - \left(\frac{A_F}{A_D} \right)^{\frac{1}{1-\alpha}} \frac{(1 + \beta_D + \gamma_D) \gamma_F}{(1 + \beta_F) \gamma_D} \right) - \theta \left(1 + \left(\frac{A_F}{A_D} \right)^{\frac{1}{1-\alpha}} \right) \right] \\
&+ (\alpha A_D k_{Dt}^{\alpha-1} - 1) \phi \left[b_D - b_F \frac{(1 + \beta_D + \gamma_D) \gamma_F}{(1 + \beta_F) \gamma_D} \right] \equiv C_F(k_{Dt})
\end{aligned} \tag{33}$$

where $C_D(k_{Dt})$ is increasing and concave and $C_F(k_{Dt})$ is decreasing and convex. Moreover, $C_D(k_{Dt}) > \varphi(k_{Dt})$ for all $k_{Dt} > 0$, which means that a steady state satisfies $a_D > 0$ whatever the value of k_D . As shown in Appendix 7.1, there is $\underline{k} > 0$ such that for all $k_{Dt} > \underline{k}$, we have $C_D(k_{Dt}) > \varphi(k_{Dt}) > C_F(k_{Dt})$. We also show whether $\psi(k_{Dt}) \geq C_F(k_{Dt})$. See Figure 1 for a representation.

To complete the picture, a steady state is of course a solution $(E_t, k_{Dt}) = (E_{t+1}, k_{Dt+1}) = (E, k_D)$ satisfying equations (27) and (28), which corresponds to the intersections between $\varphi(k_{Dt})$ and $\psi(k_{Dt})$. This allows us to deduce the number of steady states depending on the importance of the policy parameters b_D and b_F (see also Figure 1 for the case with two steady states):

Proposition 1 *Under Assumptions 1-3, there exist two steady states with $a_D > 0$ and $a_F = 0$, namely (k_j, E_j) with $j = \{l, h\}$ and $0 < k_l < k_h$, if public debt (at least in one country) is positive, with b_D and b_F low enough. If public debt (in one or the two countries) is too large, there is no steady state with $a_D > 0$ and $a_F = 0$.*

If we further assume that:

$$\frac{\alpha}{1 - \alpha} > \frac{\frac{\beta_D}{1 + \beta_D} + \frac{\beta_F}{1 + \beta_F} \left(\frac{A_F}{A_D} \right)^{\frac{1}{1-\alpha}}}{1 + \left(\frac{A_F}{A_D} \right)^{\frac{1}{1-\alpha}}} \tag{34}$$

the two existing steady states are characterized by under-accumulation.

Proof. See Appendix 7.2 ■

Proposition 1 shows that there are two steady states with $a_D > 0$ and $a_F = 0$ when the public debt in both countries is positive but not too high. This is a common feature of dynamic models with a constant level of public debt, including those with pollution (see Fodha and Seegmuller 2012, 2014). Using the ingredients of this section and Appendix 7.1, we can also deduce the stability properties of the steady states, drawing the phase diagram as depicted in Figure 1. The steady state with the highest level of capital is stable, whereas the one with the lowest level of capital is a saddle. Given that there are two predetermined variables, this last one is generically unstable.

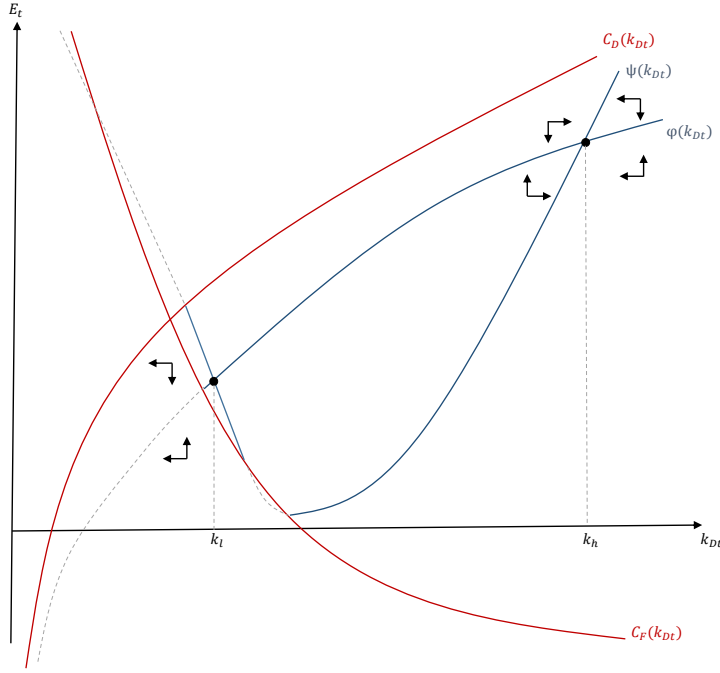


Figure 1 – Global dynamics under γ_F arbitrarily small.

As generations overlap, agents may over- or under-accumulate capital relative to the usual golden rule. The sufficient condition to have under-accumulation seems to be quite standard since it requires that capital income over labor income has to be higher than a weighted sum of the saving rates $\beta_i/(1 + \beta_i)$. Under-accumulation ensures in particular that the tax levels $\tau_i = b_i(\alpha A_i k_i^{\alpha-1} - 1)$, for $i = D, F$, are positive as governments are indebted.

4.2 Policy implications

This section explores how capital, environmental quality and the welfare of consumers could be affected by changes of the level of public debt in both countries. More precisely, we examine the effect of a bilateral debt relief in favor of the less advanced country *i.e.* the foreign one $i = F$. We can justify this policy scheme using two arguments. First, the debt burden in poorest countries can act as a brake on development, and richest countries must express support to relax this constraint. Second, for fairness motives: the biggest contributor to climate change are rich-more technological advanced- countries. This is consistent with the concept of climate justice promoted by United Nations (2009): “[...] the world’s richest countries have contributed most to the problem, they have a greater obligation to take action and to do so more quickly.”

Considering that Proposition 1 holds, we investigate the effect of a debt transfer, *i.e.* an increase of b_D associated to a decrease of b_F of the same amount, on the stationary stock of

capital and on the environment.⁴ Examining the government budget constraint given by (18), a debt transfer directly translates into a variation of disposable income, positive for the country that receives the transfer and negative for the contributor.

These comparative statics are evaluated at the steady state with the highest level of capital, since it is the only one stable.

Proposition 2 *Under Assumptions 1-3, b_D and b_F not too high and inequality (34):*

- *When the domestic economy is more patient than the foreign one, $\beta_D > \beta_F$, an increase of b_D associated to a decrease of b_F of the same amount reduces E_h and k_h .*
- *When the domestic economy is less patient than the foreign one, $\beta_D < \beta_F$, an increase of b_D associated to a decrease of b_F of the same amount increases k_h , and E_h if⁵:*

$$\frac{\beta_F}{1 + \beta_F} - \frac{\beta_D}{1 + \beta_D} > \frac{1}{b_D \mathcal{X}} \quad (35)$$

Proof. See Appendix 7.3 ■

We first discuss the effect of the debt transfer on the level of capital. To understand the economic mechanism, we recall the expressions of the savings in the two countries. Using (9) and (10) on the one hand, and (11) on the other hand, we have:

$$s_D = \frac{\beta_D}{1 + \beta_D} (w_D - \tau_D - a_D), \quad (36)$$

$$s_F = \frac{\beta_F}{1 + \beta_F} (w_F - \tau_F) \quad (37)$$

In addition, the equilibrium conditions (15) and (16) evaluated at a steady state can be written:

$$s_D + s_F = k_D \left[1 + \left(\frac{A_F}{A_D} \right)^{\frac{1}{1-\alpha}} \right] + b_D + b_F, \quad (38)$$

$$a_D = \frac{\theta}{\phi} A_D k_D^\alpha \left[1 + \left(\frac{A_F}{A_D} \right)^{\frac{1}{1-\alpha}} \right] \quad (39)$$

The increase of the public debt in the domestic economy means a larger taxation. We observe the reverse mechanism in the foreign economy, where the level of public debt goes down. Debt transfer entails a fall of savings in the domestic economy and an increase in the foreign one, while the aggregate level of debt $b_D + b_F$ is unchanged. The effect on global savings depends on differences in time preferences, *i.e.* β_D compare to β_F , which determine the saving rates in the two countries. When $\beta_D > \beta_F$, the domestic economy is more patient, global savings go down, and, given the perfect capital mobility, so does the stock of capital. We get the reverse result when

⁴Note that we would get the reverse result following an increase of b_F associated to a decrease of b_D .

⁵The expression for \mathcal{X} is given in Appendix 7.3.

$\beta_D < \beta_F$. This is due to a redistribution of net income from country D to country F. At a steady state, environmental preferences do not modify this conclusion, even if savings in the domestic economy depend on the environmental quality. As it is clear from equation (39), abatement along the steady state exactly compensates pollution flows coming from aggregate production. Because of perfect mobility of assets, this implies that abatement is determined by the level of capital and not the distribution of income.

Then, we emphasize the effects on the environmental quality. A policy that leads economies to a state with a lower stock of capital reduces labor income and the environmental contribution of the domestic economy. Given the efficiency of abatement compare to the polluting intensity of production (Assumption 3), a fall in capital stock damages the environment. This is why a debt transfer from the domestic to the foreign economy damages environmental quality when $\beta_D > \beta_F$. When the foreign economy is more patient ($\beta_D < \beta_F$), an additional condition is required to ensure that a debt transfer from the domestic to the foreign economy is efficient to improve the environment. Indeed, the domestic economy is the only contributor to environmental protection and a policy aiming at increase its debt level would entail direct negative income effect, with adverse consequences on abatement activities. The condition (35) presented in Proposition 2 implies that the elasticity of the capital stock to debt variation is sufficiently high to overtake the direct negative effect of debt variation on abatement spendings through the variation of taxes.⁶ It especially requires a significant difference between β_F and β_D to ensure, following the debt transfer, a high enough increase of global savings and, therefore, capital.

We investigate now the welfare effects of the debt transfer. Debt policy affects welfare through its impact on the environment and on consumption spendings. The consumption part of the welfare is affected by debt transfer through three effects: First, a direct debt effect, because any variation in the level of debt changes the tax on income; Second, an income effect because the variation of capital stock entailed by the policy modifies the return on labor and on capital; Finally, a cost of debt effect because of the variation in the interest rate and hence of the country's debt burden. These effects can be competing. The welfare analysis is presented in the following proposition:

Proposition 3 *Under Assumptions 1-3, b_D and b_F not too high, and $\alpha/(1-\alpha) > \max\left\{\frac{\beta_D}{1+\beta_D}; \frac{\beta_F}{1+\beta_F}\right\}$, we have the following results:*

- *For $\beta_D > \beta_F$, an increase of b_D associated to a decrease of b_F of the same amount reduces the welfare in the domestic country while the impact is ambiguous for the foreign one.*
- *For $\beta_D < \beta_F$ and under condition (35), an increase of b_D associated to a decrease of b_F of the same amount increases the welfare in the foreign country while the impact is ambiguous for the domestic one.*

⁶For more details, see Appendix 7.3.

Proof. See Appendix 7.4 ■

When the foreign economy is impatient relative to the domestic one, a debt transfer in its favor leads economies to converge to a lower stock of capital and to a worsen environmental quality. Nevertheless, if the debt reduction is sufficiently high, these negative effects could, at least partially, be compensated in the foreign country by the direct increase in consumption spendings associated to a higher after-tax income.

The result is different if the foreign economy is relatively more patient and the variation of the capital stock with respect to debt is sufficiency high to ensure that it drives an improvement of the environmental quality. In such a situation, welfare improves in the foreign economy, while the impact remains unclear for the welfare of the domestic economy because it suffers from an increase in its debt. If it is highly sensitive to climate change issue (γ_D high enough), the better environmental quality could a priori compensate the increase of tax burden.

We emphasize in this section that when the poorest economy is characterized by low environmental concern (γ_F is arbitrarily small), the two steady states are always characterized by the same configuration, *i.e.* domestic country contributes to environmental protection while the foreign economy does not. As long as green awareness of foreign agents is too low, a debt transfer from the domestic to the foreign economy is not an efficient policy tool to induce them to contribute to the environment. Moreover, it reduces capital stock and environmental quality if the richest economy is more patient. On the contrary, if the advanced economy is less patient, such a transfer allows to converge to a higher stock of capital and to a better environmental quality. In such a configuration, the debt relief is an efficient policy even if the poor country remains not engaged in mitigation.

5 A foreign economy with a significant environmental sensitivity (γ_F significant)

Now, households living in the foreign country are significantly concerned by environmental quality, *i.e.* the preference parameter γ_F is no more arbitrarily small. We will highlight the policy implications it will imply in comparison with the previous case. We especially emphasize some configurations where debt transfers between the two countries may induce significant changes in the private engagement concerning environmental mitigation. To be more specific, we assume:

Assumption 4 γ_F is high enough, satisfying the following inequality:

$$\frac{\gamma_F}{1 + \beta_F} > \max \left\{ \left(\frac{A_D}{A_F} \right)^{\frac{1}{1-\alpha}} ; \frac{b_D}{b_F} \right\} \frac{\gamma_D}{1 + \beta_D}$$

By inspection of equation (32), we observe that the frontier defining whether a_{Dt} is positive or not, namely $E_t = C_D(k_{Dt})$, is not affected by γ_F . Therefore, it has the same properties than

in Section 4. However, under Assumption 4, $E_t = C_F(k_{Dt})$, defined in equation (33) and such that $a_{Ft} = 0$, becomes an increasing and concave function.⁷ We will see that this will have strong implications on the existence of steady states and on the effectiveness of policies making some transfers of debt between the two countries.

We start by investigating the region where $a_{Dt} > 0$ and $a_{Ft} > 0$, which becomes relevant because γ_F is high enough. We focus on the existence of steady states, their stability properties and the effectiveness of debt transfers keeping $b_D + b_F$ constant. Then, we will investigate whether debt transfers allow to promote abatements in country F at the stable steady state with the highest level of capital. We will analyze the consequences of such policies on capital, environmental quality and welfare.

5.1 The regime with $a_{Dt} > 0$ and $a_{Ft} > 0$

Under Assumption 4, we study equilibria where both countries are engaging in abatements. Using (18), equations (23) and (24) rewrite:

$$\frac{1}{\phi} E_{t+1} \left(\frac{\beta_D}{\gamma_D} + \frac{\beta_F}{\gamma_F} \right) = k_{Dt+1} \left[1 + \left(\frac{A_F}{A_D} \right)^{\frac{1}{1-\alpha}} \right] + b_D + b_F \quad (40)$$

$$\begin{aligned} \left(1 + \frac{1 + \beta_D}{\gamma_D} + \frac{1 + \beta_F}{\gamma_F} \right) E_{t+1} &= E_t - \phi(b_D + b_F) \left[\alpha A_D k_{Dt}^{\alpha-1} - 1 \right] \\ &+ A_D k_{Dt}^\alpha [\phi(1 - \alpha) - \theta] \left[1 + \left(\frac{A_F}{A_D} \right)^{\frac{1}{1-\alpha}} \right] \end{aligned} \quad (41)$$

Equation (41) gives the dynamics of environmental quality, whereas equation (40) gives a static link between E_t and k_{Dt} . Using (41), $E_{t+1} \geq E_t$ if and only if $E_t \leq \Omega(k_{Dt})$, with:

$$\begin{aligned} \left(\frac{1 + \beta_D}{\gamma_D} + \frac{1 + \beta_F}{\gamma_F} \right) \Omega(k_{Dt}) &= -\phi(b_D + b_F) \left[\alpha A_D k_{Dt}^{\alpha-1} - 1 \right] \\ &+ A_D k_{Dt}^\alpha [\phi(1 - \alpha) - \theta] \left[1 + \left(\frac{A_F}{A_D} \right)^{\frac{1}{1-\alpha}} \right] \end{aligned} \quad (42)$$

where $\Omega(k_{Dt})$ is strictly increasing and concave under Assumption 3. Substituting (41) into (40), $k_{Dt+1} \geq k_{Dt}$ is equivalent to $E_t \geq \Pi(k_{Dt})$, with:

$$\begin{aligned} \Pi(k_{Dt}) &\equiv \phi \frac{\gamma_D \gamma_F + \gamma_F(1 + \beta_D) + \gamma_D(1 + \beta_F)}{\gamma_F \beta_D + \gamma_D \beta_F} \left[k_{Dt} \left(1 + \left(\frac{A_F}{A_D} \right)^{\frac{1}{1-\alpha}} \right) + b_D + b_F \right] \\ &+ \phi(b_D + b_F) (\alpha A_D k_{Dt}^{\alpha-1} - 1) - A_D k_{Dt}^\alpha [\phi(1 - \alpha) - \theta] \left[1 + \left(\frac{A_F}{A_D} \right)^{\frac{1}{1-\alpha}} \right] \end{aligned} \quad (43)$$

where $\Pi(k_{Dt})$ is a U-shaped function. Of course, all dynamic trajectory (k_{Dt}, E_t) with $a_{Dt} > 0$ and $a_{Ft} > 0$ should satisfy $E_t < \min \{C_F(k_{Dt}), C_D(k_{Dt})\}$.

⁷We will also have $\lim_{k_{Dt} \rightarrow 0} C_F(k_{Dt}) = -\infty$ and $\lim_{k_{Dt} \rightarrow +\infty} C_F(k_{Dt}) = +\infty$.

We examine now if some long term steady states in which both countries contribute to environmental protection may exist. To this aim, we analyze if positive abatement activities in the foreign and the domestic good are compatible with a stationary stock of capital and a constant value of environmental index.

Proposition 4 *Under Assumptions 1, 3 and 4, there exist two steady states with $a_D > 0$ and $a_F > 0$, namely (k_j^s, E_j^s) with $j = \{l, h\}$ and $0 < k_l^s < k_h^s$, if public debt (at least in one country) is positive, with b_D and b_F low enough. If public debt (in one or the two countries) is too large, there is no steady states with $a_D > 0$ and $a_F > 0$.*

These steady states are admissible if $E_j^s < \min\{C_F(k_j^s), C_D(k_j^s)\}$ is satisfied. If we further assume that:

$$\phi(1 - \alpha) - \theta < \alpha \frac{(1 + \beta_D)\gamma_F + (1 + \beta_F)\gamma_D}{\gamma_F\beta_D + \gamma_D\beta_F} \quad (44)$$

they are characterized by under-accumulation.

Proof. See Appendix 7.6 ■

Note that the conditions for the existence of two steady states when $a_D > 0$ and $a_F > 0$ is closely related to what happens when $a_D > 0$ and $a_F = 0$. In both cases, the debt levels should be positive but not too high. Therefore, the question will be whether the steady states in each regime are admissible or not, i.e. really satisfy $a_F > 0$ or $a_F = 0$ respectively. We also remark that the sufficient condition to have under-accumulation is compatible with Assumption 3, for an appropriate level of ϕ , if $\alpha \frac{(1 + \beta_D)\gamma_F + (1 + \beta_F)\gamma_D}{\gamma_F\beta_D + \gamma_D\beta_F} > \theta(A_F/A_D)^{\frac{1}{1-\alpha}}$, which is for instance satisfied for θ low enough.

All the ingredients of this section also allow to deduce the dynamics when $a_{Dt} > 0$ and $a_{Ft} > 0$ showing that the steady state with the highest level of capital is stable, whereas the other one is a saddle.

We can investigate the effects of public debt transfer when both countries contribute to the environmental protection.

Proposition 5 *Under Assumptions 1, 3 and 4, assume that b_D and b_F are low enough. Any increase of b_D associated to a decrease of b_F of the same amount has no effect on an equilibrium with $a_{Dt} > 0$ and $a_{Ft} > 0$.*

When both countries invest to improve environmental quality, a transfer of debt from the domestic to the foreign country that let the total amount of debt $b_D + b_F$ unchanged is useless. By inspection of the equilibrium conditions (40) and (41), it does not modify any dynamic trajectory characterized by $a_{Dt} > 0$ and $a_{Ft} > 0$.

Using (9), (10), (15) and (16), the equilibrium conditions can be written:

$$s_{Dt} + s_{Ft} = k_{Dt+1} \left(1 + \left(\frac{A_F}{A_D} \right)^{\frac{1}{1-\alpha}} \right) + b_D + b_F \quad (45)$$

$$E_{t+1} = E_t - \theta A_D k_{Dt}^\alpha \left(1 + \left(\frac{A_F}{A_D} \right)^{\frac{1}{1-\alpha}} \right) + \phi(a_{Dt} + a_{Ft}) \quad (46)$$

where s_{it} and a_{it} are given by (9) and (10). Since s_{it} can be expressed as functions of environmental quality, aggregate savings do not depend on the distribution of income across countries. Therefore, this heterogeneity of incomes between countries D and F has no effect on the asset equilibrium, i.e. on capital. By inspection of (10), abatement linearly depends on income. Since the efficiency of abatement on environmental quality ϕ is the same in both countries, the distribution of income has also no effect on environmental quality.

5.2 The interplay between debt and the existence of steady states with environmental mitigation

We investigate now if a policy consisting in debt transfers can promote the environmental maintenance of the foreign economy, moving from an equilibrium with $a_{Ft} = 0$ to an equilibrium with positive abatement $a_{Ft} > 0$. The underlying question is of course to know whether such a policy is relevant to improve capital, environmental quality and welfare in a globalized world.

To be more specific, we will start with a configuration where there exist two steady states with $a_D > 0$ and $a_F = 0$. Then, we will show that a debt transfer among countries will lead to a configuration where the stable steady state with the highest level of capital moves from a situation where $a_D > 0$ and $a_F = 0$ to a situation where $a_D > 0$ and $a_F > 0$. We will investigate what are the implications of such a change of regime on capital, environmental quality and welfare.

Our explanations are based on phase diagrams, represented in Figures 2 and 3, which allows to have a global picture of the existence of steady states and dynamics. We start by focusing on equilibria with $a_{Dt} > 0$ and $a_{Ft} = 0$ when γ_F satisfies Assumption 4 and highlight the differences with the previous case where γ_F is arbitrarily low. By inspection of equations (30)-(32), we observe that $\varphi(k_{Dt})$, $\psi(k_{Dt})$ and $C_D(k_{Dt})$ are not affected by γ_F , meaning that they have the same properties than in Section 4. To locate these curves in the (k_{Dt}, E_t) plane, especially with respect to $C_D(k_{Dt})$ and $C_F(k_{Dt})$, we further assume:

Assumption 5

$$\frac{b_D}{b_F} < \frac{\phi(1-\alpha) - \theta \left(1 + \left(\frac{A_F}{A_D} \right)^{\frac{1}{1-\alpha}} \right)}{\phi(1-\alpha) \left(\frac{A_F}{A_D} \right)^{\frac{1}{1-\alpha}}} \equiv \left(\frac{b_D}{b_F} \right)^+$$

This assumption implies that $b_D/b_F < (A_D/A_F)^{\frac{1}{1-\alpha}}$, meaning that the debt per GDP in

advanced country, $b_D/A_D^{\frac{1}{1-\alpha}}$, is lower than in foreign country, $b_F/A_F^{\frac{1}{1-\alpha}}$.⁸

As shown in Appendix 7.5, where some technical details to construct the phase diagram when γ_F is significant are relegated, we note that, on the one hand, $\varphi(k_{Dt})$ and $\Omega(k_{Dt})$, that correspond to $E_{t+1} = E_t$, are both increasing and concave functions and cross the curve $C_F(k_{Dt})$ at the same point given by $k_{Dt} = k_1$. Moreover, $\varphi(k_{Dt}) > 0$ for $k_{Dt} > k_0$, $\varphi(k_{Dt})$ is above $C_F(k_{Dt})$ for $k_{Dt} \leq k_1$ and $\Omega(k_{Dt})$ is below $C_F(k_{Dt})$ for $k_{Dt} \geq k_1$, where $k_1 > k_0$ under Assumption 5. On the other hand, $\psi(k_{Dt})$ and $\Pi(k_{Dt})$, that correspond to $k_{Dt+1} = k_{Dt}$, are both convex U-shaped functions that cross the curve $C_F(k_{Dt})$ at the same points. We denote $k_{Dt} = k_a$ and $k_{Dt} = k_b$, with $k_a > k_b$ these two points.

Using the constraint (26), a steady state with $a_D > 0$ and $a_F = 0$ requires⁹:

$$E > \frac{\phi\gamma_F}{1 + \beta_F} \left[(1 - \alpha)A_D k_D^\alpha \left(\frac{A_F}{A_D}\right)^{\frac{1}{1-\alpha}} - b_F (\alpha A_D k_D^{\alpha-1} - 1) \right] \quad (47)$$

Using (28), such a steady state should satisfy $\Theta(k_D) < 0$, with:

$$\begin{aligned} \Theta(k_D) \equiv & A_D k_D^\alpha \left[(1 - \alpha) \left(\frac{\gamma_F}{1 + \beta_F} \left(\frac{A_F}{A_D}\right)^{\frac{1}{1-\alpha}} - \frac{\gamma_D}{1 + \beta_D} \right) + \frac{\gamma_D}{1 + \beta_D} \frac{\theta}{\phi} \left(1 + \left(\frac{A_F}{A_D}\right)^{\frac{1}{1-\alpha}} \right) \right] \\ & + (\alpha A_D k_D^{\alpha-1} - 1) \left(\frac{\gamma_D b_D}{1 + \beta_D} - \frac{\gamma_F b_F}{1 + \beta_F} \right) \end{aligned} \quad (48)$$

where $\Theta(k_D)$ is an increasing function under Assumption 4. Of course, a steady state with $a_D > 0$ and $a_F > 0$ requires the opposite inequality, i.e. $\Theta(k_D) > 0$.

Using Appendices 7.2 and 7.6, there exist \hat{k} and \hat{k}^s such that the possible steady states k_l and k_h with $a_F = 0$ and $a_D > 0$ are such that $k_l < k_h < \hat{k}$ and the possible steady states k_l^s and k_h^s with $a_F > 0$ and $a_D > 0$ are such that $k_l^s < k_h^s < \hat{k}^s$.

Therefore, only the steady states k_l and k_h exist if $\Theta(\hat{k}^s) < 0$ and $\Theta(\hat{k}) < 0$. Let:

$$\Sigma \equiv \frac{\gamma_F b_F}{1 + \beta_F} - \frac{\gamma_D b_D}{1 + \beta_D} \quad (49)$$

These two inequalities are equivalent to $\Sigma > \Sigma_0$ and $\Sigma > \Sigma_1$, with:

$$\Sigma_0 \equiv \frac{A_D \left(\frac{J_1}{J_2}\right)^{\frac{\alpha}{1-\alpha}} \left[(1 - \alpha) \left(\frac{\gamma_F}{1 + \beta_F} \left(\frac{A_F}{A_D}\right)^{\frac{1}{1-\alpha}} - \frac{\gamma_D}{1 + \beta_D} \right) + \frac{\gamma_D}{1 + \beta_D} \frac{\theta}{\phi} \left(1 + \left(\frac{A_F}{A_D}\right)^{\frac{1}{1-\alpha}} \right) \right]}{\alpha A_D \frac{J_2}{J_1} - 1} > 0 \quad (50)$$

$$\Sigma_1 \equiv \frac{A_D \left(\frac{G_1}{F_1}\right)^{\frac{\alpha}{1-\alpha}} \left[(1 - \alpha) \left(\frac{\gamma_F}{1 + \beta_F} \left(\frac{A_F}{A_D}\right)^{\frac{1}{1-\alpha}} - \frac{\gamma_D}{1 + \beta_D} \right) + \frac{\gamma_D}{1 + \beta_D} \frac{\theta}{\phi} \left(1 + \left(\frac{A_F}{A_D}\right)^{\frac{1}{1-\alpha}} \right) \right]}{\alpha A_D \frac{F_1}{G_1} - 1} > 0 \quad (51)$$

⁸Note that Assumption 5 implies $\max \left\{ \left(\frac{A_D}{A_F}\right)^{\frac{1}{1-\alpha}} ; \frac{b_D}{b_F} \right\} = \left(\frac{A_D}{A_F}\right)^{\frac{1}{1-\alpha}}$. Thus, when Assumption 5 holds, Assumption 4 simplifies to $\frac{\gamma_F}{1 + \beta_F} > \left(\frac{A_D}{A_F}\right)^{\frac{1}{1-\alpha}} \frac{\gamma_D}{1 + \beta_D}$.

⁹In such a case, Propositions 2 and 3 will still apply, even if Assumption 2 does no more hold.

Hence, there are only two steady states with $a_D > 0$ and $a_F = 0$ if $\Sigma > \max\{\Sigma_0, \Sigma_1\}$. In such a case, k_1 is higher than k_a (see Figure 2). Starting from this situation, consider some debt transfers, which is to reduce b_F and increase b_D keeping $b_D + b_F$ identical. It implies a reduction of Σ . As shown in Appendix 7.5, this lowers k_1 . When b_D/b_F increases and tends to its upper bound $(b_D/b_F)^+$, k_1 decreases towards k_0 . This means that there exists a value Σ^* such that k_1 is lower than k_a for $\Sigma < \Sigma^*$. It is clear that for $\Sigma < \Sigma^*$ and Σ sufficiently close to Σ^* , one steady state k_l ($< k_1$) with $a_D > 0$ and $a_F = 0$ coexists with one steady state k_h^s ($> k_1$) with $a_D > 0$ and $a_F > 0$ (see Figure 3). Of course, we have $\Sigma^* < \min\{\Sigma_0, \Sigma_1\}$. Indeed, if $\Sigma > \Sigma_0$, k_h is an admissible steady state, whereas if $\Sigma > \Sigma_1$, k_h^s cannot be an admissible steady state.

We deduce the following proposition¹⁰:

Proposition 6 *Under Assumptions 1 and 3-5, b_D and b_F not too high, inequalities (34) and (44), there exists $\Sigma^* \in (0, \min\{\Sigma_0, \Sigma_1\})$, such that:*

1. *If $\Sigma > \max\{\Sigma_0, \Sigma_1\}$, there exist two steady states with $a_D > 0$ and $a_F = 0$, k_l and k_h ;*
2. *If $\Sigma < \Sigma^*$ and Σ sufficiently close to Σ^* , there exists one steady state k_l with $a_D > 0$ and $a_F = 0$ and one steady state k_h^s with $a_D > 0$ and $a_F > 0$.*

Therefore, an increase of b_D associated to a decrease of b_F of the same amount may allow to move from a configuration where the two steady states are characterized by $a_D > 0$ and $a_F = 0$ to a configuration where the steady state with the lowest level of capital is characterized by $a_D > 0$ and $a_F = 0$ and the steady state with the highest level of capital by $a_D > 0$ and $a_F > 0$.

This proposition shows that following a decrease of debt in country F exactly compensated by an increase of debt in country D, the long-run stable steady state moves from a configuration where environmental abatement is constrained in country F to a configuration where it is no more. The stability properties of the steady states presented in Proposition 6 are deduced from the analysis of Sections 5.1 and 5.2 and are illustrated in Figures 2 and 3.

¹⁰We are not interested in situations where environmental abatement of country F at the lowest steady state becomes positive, because we think that it is more relevant to analyse the policy implications on the stable steady state, which is characterized by the highest level of capital.

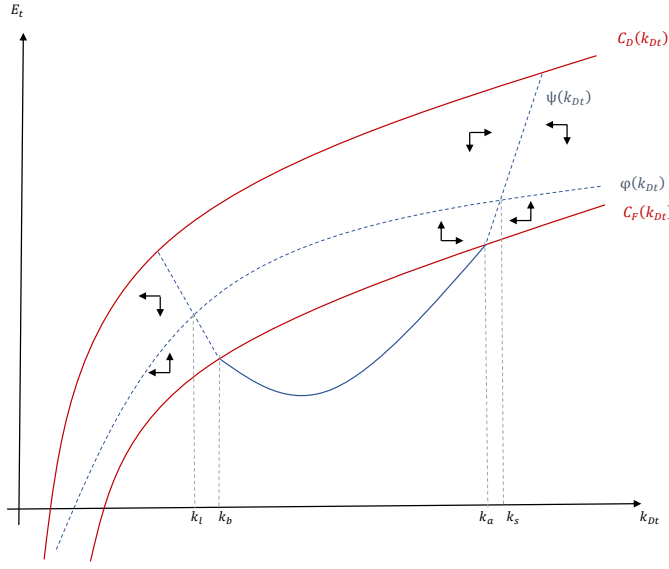


Figure 2 – Global dynamics for $\Sigma > \max\{\Sigma_0, \Sigma_1\}$

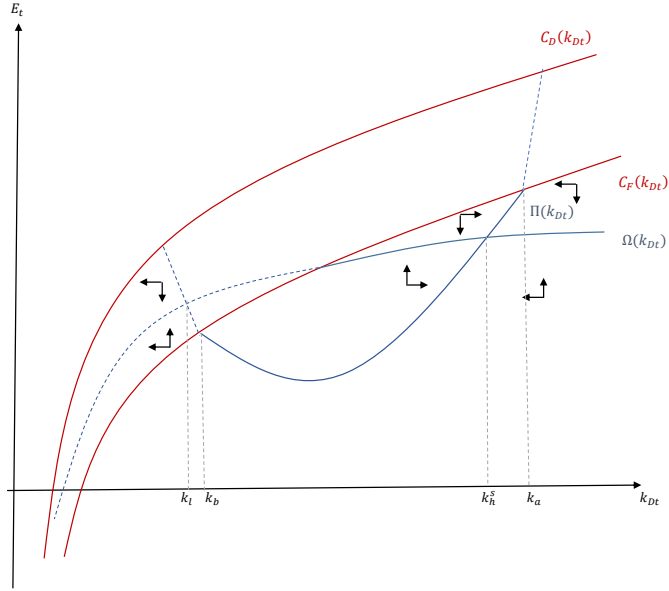


Figure 3 – Global dynamics for $\Sigma < \Sigma^*$

Proposition 6 requires a set of assumptions. They are compatible with each other. Indeed, Assumptions 1 and 5 imply that $A_D/A_F > \max\{(b_D/b_F)^{1-\alpha}; 1\}$, which is satisfied for an appropriate set of the productivity parameters. Assumption 5 also requires the inequality $b_D/b_F < (b_D/b_F)^+$ which involves appropriate debt levels. Taking into account these restrictions, Assumption 4 which simplifies to $\frac{\gamma_F}{1+\beta_F} > \left(\frac{A_D}{A_F}\right)^{\frac{1}{1-\alpha}} \frac{\gamma_D}{1+\beta_D}$ is fulfilled for an appropriate set of the preference pa-

rameters γ_F and γ_D . Assumption 3 means a sufficiently low value of θ . In this case, inequalities (34) and (44) are satisfied for β_i low and α high enough, which are standard conditions to have under-accumulation.

Considering that all these assumptions are satisfied, Proposition 6 shows that a high enough debt transfer allows country F to engage into environmental mitigation. This occurs if either β_D is higher or lower than β_F . Indeed, as we have already seen, the debt transfer means a lower fiscal pressure in country F. Therefore, if the debt transfer is significant, the after-tax income in country F is sufficiently high to finance also abatement. The marginal cost of engaging in abatement becomes equal to its marginal benefit (see the first order conditions (9)-(10) and (11)-(12)).

What is interesting is that we can further deduce some implications of the effect of such a policy on the levels of capital, environmental quality and welfare at the stable steady state. Using also Proposition 2,¹¹ we see that when $\beta_D > \beta_F$, such a policy reduces the levels of capital and environmental quality at the steady state k_h with $a_D > 0$ and $a_F = 0$. When the transfer of debt becomes high enough, the stable steady state is characterized by positive abatement in both countries and is no more affected by the debt transfer between countries that we consider (Proposition 5). In contrast, when $\beta_D < \beta_F$, the levels of capital and environmental quality at the steady state k_h with $a_D > 0$ and $a_F = 0$ increase. Again, when the debt transfer is sufficiently high, the stable steady state enters the regime where both countries are engaged in environmental mitigation and becomes independent of the debt transfer considered. Using Proposition 3 and the property that the welfare can be expressed as a function of capital and environmental quality, we can deduce the effect of the debt transfer on capital, environmental quality and the welfare of each country.

Corollary 1 *Under Assumptions 1 and 3-5, b_D and b_F not too high, $\alpha/(1-\alpha) > \max\left\{\frac{\beta_D}{1+\beta_D}; \frac{\beta_F}{1+\beta_F}\right\}$, inequalities (34) and (44), a transfer of debt such that the economy moves from a stable steady state with no foreign contribution to environmental protection ($a_F = 0$) to a stable steady state with foreign contribution ($a_F > 0$), has the following implications:*

- *For $\beta_D > \beta_F$, capital, environmental quality and the welfare in the domestic country decreases while the impact is ambiguous for the welfare in the foreign one as long as $a_F = 0$. When a_F becomes strictly positive, all these variables become invariant to the policy.*
- *For $\beta_D < \beta_F$ and under condition (35), capital, environmental quality and the welfare in the foreign country increases while the impact is ambiguous for the welfare in the domestic country as long as $a_F = 0$. When a_F becomes strictly positive, all these variables become invariant to the policy.*

Such a policy of transferring debt from country F to country D is relevant in terms of the level of capital, and also environmental quality, when country D is the most impatient one (β_D lower

¹¹As we have already explained, Propositions 2 and 3 also hold when Assumption 2 is not satisfied.

than β_F). The debt transfer from one country to the other one implies a reallocation of income net of taxes from country D to country F, since taxes increase in the first country, whereas it decreases in the other one. Taking into account the equilibrium on the international asset market (14) and (15), we know that capitals in both countries move in the same direction because the returns are equal. This means that capital will increase if global saving raises. This is possible if the redistribution of net income between countries is in favor of the population that saves more. Such a debt transfer also allows to raise welfare in country F, whereas the effect on the welfare of country D is not clear-cut. In addition, debt relief for poor countries can be used as a policy lever by rich countries to help them to engage in environmental protection. Nonetheless, we reveal that this instrument is useless once the unconstrained regime has been achieved. Therefore, the effect of public transfer is maximum when the economy switches from the regime with $a_F = 0$ to the regime with $a_F > 0$.

When country D is the most patient one (β_D higher than β_F), we have some opposite conclusions and economic mechanisms on capital and environmental quality. In addition, the negative effect of debt transfer is the highest on capital, environmental quality and the welfare of country D when one moves from the regime with $a_F = 0$ to the regime with $a_F > 0$. The impact on the welfare of country F is however unclear.

We could observe that while the effect of the debt transfer on the welfare of foreign country may be positive, it is never the case for the welfare of domestic country, without introducing some adding restrictions on the parameters. This asks the question of the implementability of the debt transfer for country D.

Short remarks on perfect mobility of assets and redistribution of income

Our setting with perfect capital mobility is a good approximation of what happens in countries with highly developed financial markets and few capital controls. To have a more general view, assuming imperfect capital mobility is a relevant option. When a debt transfer is implemented in such a case, capital accumulation is in part crowded out in the donor country. Assuming that the overall debt level is unchanged, this negative effect is offset if capital mobility is free, i.e. if the donor shares a common asset market with the beneficiary. As long as capital is not perfectly mobile between countries, the donor would suffer from a crowding-out effect, in addition to a negative net income effect due to a higher level of taxes. Reversely, the program would favor the capital stock in the recipient country. The implementation of a debt transfer in a context of imperfect capital mobility would affect severely the capital stock and hence mitigation efforts in the rich country, while reverse results are observed in the poor country. Assuming that the poor country is initially less engaged in environmental action, this policy would not favor the global environmental quality. This suggests that perfect mobility of assets seems to be the most favorable configuration to implement a debt relief policy.

In this case of perfect mobility, a debt transfer implies a redistribution of income net of tax from the rich to the poor country. Indeed, recall that taxes paid by each country are given by

$\tau_i = (\alpha A_i k_i^{\alpha-1} - 1)b_i$, i.e. linearly depend on b_i . Therefore, any increase of b_D exactly compensated by a decrease of b_F means a redistribution of net income from country D to country F. There is however an adding effect that goes through the interest factor $\alpha A_i k_i^{\alpha-1}$, which (in)decreases if and only if capital (de)increases. As we have just seen above, when the mobility of assets becomes imperfect, the debt relief has also an effect through some crowding-out effect in some countries. The debt transfer does no more consist only in a redistribution of income as in the case of perfect mobility of asset. It is a way to direct financial help in favor of capital accumulation in the recipient country. Therefore, debt transfer can be seen as more general than just a redistribution of income through taxes.

6 Concluding remarks

Starting from the difficulties to implement effective environmental policies to fight pollutant emissions, the use of macroeconomic policy instruments. In an international context, we question the effect of debt transfer in favor of poor countries, the overall debt level remaining unchanged. Despite the fact that debt reliefs from rich to poor countries is actually an insignificant instrument used to promote climate change actions, we ask theoretically whether it should be more intensively considered.

We show that debt relief for poor countries, with low productivity, makes it possible to engage these countries in the process of pollution abatement. The efficiency of this policy strongly depends on the preferences of households in each country. Not only the sensitivity to environmental quality plays a key role, but also the time preference, that is a determinant of the saving rate. We highlight that, if the poor country is sufficiently patient, richer countries should help poorer countries to better fight pollutant emissions, even if indebted. This configuration gives interest to a debt relief in favor of poor countries. This conclusion does not applied if agents in the poor country are impatient. In that case, we reveal that there are some limits to implement debt transfer to achieve global environmental objectives. Debt transfer should be accompanied by other policy programs, aiming at modifying behaviors for instance. Otherwise, it is damaging for the environment and reduces the stock of capital, even if it can favor the participation of the poor-impatient country to mitigate climate change. Thus, our results provide intuitions about the difficulties of aid transfer programs to improve the donor and the recipient well-being, and the conditions to make them effective.

7 Appendix

7.1 Phase diagram when γ_F is arbitrarily small and close to 0

We easily see from (30), that $\varphi(k_{Dt})$ is strictly increasing ($\varphi(k_{Dt})' > 0$) and concave ($\varphi(k_{Dt})'' < 0$), with $\lim_{k_{Dt} \rightarrow 0} \varphi(k_{Dt}) = -\infty$ and $\lim_{k_{Dt} \rightarrow +\infty} \varphi(k_{Dt}) = +\infty$.

Using equation (31), we have $\lim_{k_{Dt} \rightarrow 0} \psi(k_{Dt}) = +\infty$ and $\lim_{k_{Dt} \rightarrow +\infty} \psi(k_{Dt}) = +\infty$, with

$\lim_{k_{Dt} \rightarrow +\infty} \psi(k_{Dt})/\varphi(k_{Dt}) = +\infty$. We can also show that $\psi''(k_{Dt}) > 0$. Since $\lim_{k_{Dt} \rightarrow 0} \psi'(k_{Dt}) = -\infty$ and $\lim_{k_{Dt} \rightarrow +\infty} \psi'(k_{Dt}) > 0$, $\psi(k_{Dt})$ is a convex function, decreasing for low values of k_{Dt} and increasing for high values of k_{Dt} .

By direct inspection of (30) and (32), we easily see that $C_D(k_{Dt})$ is increasing and concave, with $\lim_{k_{Dt} \rightarrow 0} C_D(k_{Dt}) = -\infty$, $\lim_{k_{Dt} \rightarrow +\infty} C_D(k_{Dt}) = +\infty$ and $C_D(k_{Dt}) > \varphi(k_{Dt})$ for all $k_{Dt} > 0$.

By direct inspection of equation (33), under Assumptions 1-3, the two terms into brackets are positive. It implies that $C_F(k_{Dt})$ is decreasing and convex, with $\lim_{k_{Dt} \rightarrow 0} C_F(k_{Dt}) = +\infty$ and $\lim_{k_{Dt} \rightarrow +\infty} C_F(k_{Dt}) = -\infty$

Now, we examine the conditions such that $\varphi(k_{Dt}) > C_F(k_{Dt})$:

Lemma 1 *Under Assumptions 1-3, there exists $\bar{k} > 0$ such that $\varphi(k_{Dt}) > \max\{0; C_F(k_{Dt})\}$ for all $k_{Dt} > \bar{k}$.*

Proof. Using (30) and (33), $\varphi(k_{Dt}) \geq C_F(k_{Dt})$ is equivalent to $F(k_{Dt}) \leq \tilde{F}$, with:

$$F(k_{Dt}) \equiv \alpha k_{Dt}^{-1} - \frac{1}{A_D} k_{Dt}^{-\alpha} \quad (52)$$

$$\tilde{F} \equiv \frac{\left[\phi(1-\alpha) - \theta \left(1 + \left(\frac{A_F}{A_D} \right)^{\frac{1}{1-\alpha}} \right) \right] \frac{\gamma_D}{1+\beta_D} - \phi(1-\alpha) \left(\frac{A_F}{A_D} \right)^{\frac{1}{1-\alpha}} \frac{\gamma_F}{1+\beta_F}}{\phi \left(\frac{b_D \gamma_D}{1+\beta_D} - \frac{b_F \gamma_F}{1+\beta_F} \right)} > 0 \quad (53)$$

We can show that $F(k_{Dt})$ decreases from $+\infty$ to $\underline{F} \equiv (\alpha - 1) A_D^{-\frac{1}{1-\alpha}} (< 0)$ when k_{Dt} increases from 0 to $\hat{k} \equiv A_D^{\frac{1}{1-\alpha}}$ and increases to 0 when k_{Dt} increases from \hat{k} to $+\infty$. Using these properties, there is a unique $k_1 > 0$ such that $F(k_{Dt}) \leq \tilde{F}$ for all $k_t \geq k_1$.

Moreover, $\varphi(k_{Dt}) > 0$ is equivalent to $F(k_{Dt}) < F_0$, with:

$$F_0 \equiv \frac{\phi(1-\alpha) - \theta \left(1 + \left(\frac{A_F}{A_D} \right)^{\frac{1}{1-\alpha}} \right)}{\phi b_D} \quad (54)$$

there is a unique $k_0 > 0$ such that $F(k_{Dt}) < F_0$ for all $k_t > k_0$. We deduce the lemma by noting $\underline{k} = \max\{k_1; k_0\}$. ■

For all $k_{Dt} > k_1$, we have $C_D(k_{Dt}) > \varphi(k_{Dt}) > C_F(k_{Dt})$. To construct a phase digram and be able to have a picture of global dynamics, we also need to analyse whether $\psi(k_{Dt}) \geq C_F(k_{Dt})$. Using (31) and (33), this inequality is equivalent to $\Gamma(k_{Dt}) \geq 0$, with:

$$\begin{aligned} \Gamma(k_{Dt}) \equiv & k_{Dt} \left[1 + \left(\frac{A_F}{A_D} \right)^{\frac{1}{1-\alpha}} \right] - k_{Dt}^\alpha (1-\alpha) A_D \left(\frac{A_F}{A_D} \right)^{\frac{1}{1-\alpha}} \left(\frac{\beta_F}{1+\beta_F} + \frac{\beta_D}{1+\beta_F} \frac{\gamma_F}{\gamma_D} \right) \\ & - \alpha k_{Dt}^{\alpha-1} \left[A_D \left(\frac{\beta_D b_D}{1+\beta_D + \gamma_D} - \frac{\beta_D \gamma_F b_F}{(1+\beta_F) \gamma_D} \right) - A_F \left(\frac{\beta_F b_F}{1+\beta_F} + \frac{\beta_D b_D}{1+\beta_D + \gamma_D} \right) \right] \\ & + b_D + \frac{b_F}{1+\beta_F} \left[1 - \frac{\gamma_F \beta_D}{\gamma_D} \right] \end{aligned} \quad (55)$$

We deduce that $\lim_{k_{Dt} \rightarrow +\infty} \Gamma(k_{Dt}) = +\infty$, which means that there is a value of k above which $\psi(k_{Dt}) \geq C_F(k_{Dt})$.

7.2 Proof of Proposition 1

A steady state is a solution $(E_t, k_{Dt}) = (E_{t+1}, k_{Dt+1}) = (E, k_D)$ satisfying equations (27) and (28):

$$\begin{aligned} \frac{\beta_D}{\gamma_D \phi} E &= k_D \left[1 + \left(\frac{A_F}{A_D} \right)^{\frac{1}{1-\alpha}} \right] \\ &+ b_D + \frac{b_F}{1 + \beta_F} - \frac{\beta_F}{1 + \beta_F} \left[(1 - \alpha) A_F k_D^\alpha \left(\frac{A_F}{A_D} \right)^{\frac{\alpha}{1-\alpha}} - b_F \alpha A_D k_D^{\alpha-1} \right] \end{aligned} \quad (56)$$

$$\frac{1 + \beta_D}{\gamma_D} E = -\phi b_D \left[\alpha A_D k_D^{\alpha-1} - 1 \right] + A_D k_D^\alpha \left[\phi(1 - \alpha) - \theta \left(1 + \left(\frac{A_F}{A_D} \right)^{\frac{1}{1-\alpha}} \right) \right] \quad (57)$$

Using these two equations and substituting E , a steady state solves $H_1(k_D) = H_2(k_D)$, where:

$$H_1(k_D) = G_1 k_D^\alpha - F_1 k_D \quad (58)$$

$$H_2(k_D) = G_2 k_D^{\alpha-1} + F_2 \quad (59)$$

with:

$$F_1 = 1 + \left(\frac{A_F}{A_D} \right)^{\frac{1}{1-\alpha}} > 0 \quad (60)$$

$$G_1 = A_F \frac{\beta_F}{1 + \beta_F} (1 - \alpha) \left(\frac{A_F}{A_D} \right)^{\frac{\alpha}{1-\alpha}} + A_D \frac{\beta_D}{1 + \beta_D} \left[1 - \alpha - \frac{\theta}{\phi} \left(1 + \left(\frac{A_F}{A_D} \right)^{\frac{1}{1-\alpha}} \right) \right] > 0 \quad (61)$$

$$F_2 = \frac{b_D}{1 + \beta_D} + \frac{b_F}{1 + \beta_F} > 0 \quad (62)$$

$$G_2 = \alpha A_D \left(\frac{b_D \beta_D}{1 + \beta_D} + \frac{b_F \beta_F}{1 + \beta_F} \right) > 0 \quad (63)$$

See also Figure 4 which illustrates this steady state analysis with $X_1 = H_1$ and $X_2 = H_2$.

When $b_D = b_F = 0$, we have $F_2 = G_2 = 0$. Steady states are solutions solving $H_1(k_D) = 0$. There are two solutions, $k_D = 0$ and $k_D = (G_1/F_1)^{\frac{1}{1-\alpha}} \equiv \hat{k}$.¹² We note that $H_1(k_D)$ is an inverse U-shaped strictly concave function which attains its maximum value for $k_D = (\alpha G_1/F_1)^{\frac{1}{1-\alpha}} \equiv \bar{k}$. This maximum value is equal to:

$$H_1(\bar{k}) = \alpha^{\frac{\alpha}{1-\alpha}} G_1^{\frac{1}{1-\alpha}} F_1^{-\frac{\alpha}{1-\alpha}} (1 - \alpha) \equiv \bar{H}_1 \quad (64)$$

By direct inspection of (59), we also see that, for a strictly positive b_D and/or b_F , $H_2(k_D)$ is strictly decreasing and convex, with $\lim_{k_D \rightarrow 0} H_2(k_D) = +\infty$ and $\lim_{k_D \rightarrow +\infty} H_2(k_D) = F_2$. In

¹²We recover the result presented in Muller-Furstenberger and Schumacher (2017), i.e without any debt there is one unique non-trivial steady state.

addition, we have $H_2(\bar{k}) = F_2 + G_2F_1/(\alpha G_1)$. Then, $H_2(\bar{k}) \leq H_1(\bar{k})$ if:

$$\alpha F_2 G_1 + G_2 F_1 \leq (1 - \alpha) \alpha^{\frac{1}{1-\alpha}} G_1^{\frac{2-\alpha}{1-\alpha}} F_1^{-\frac{\alpha}{1-\alpha}} \quad (65)$$

Using (62) and (63), we easily deduce that this last inequality is satisfied if b_D and b_F are low enough. In this case, there are two stationary solutions to the equation $H_1(k_D) = H_2(k_D)$. Since a steady state satisfies (57), inequality (25) is fulfilled. We also note that since (56) and (57) are independent of the parameter γ_F , a steady state (k_D, E) does not depend on γ_F . Therefore, inequality (26) evaluated at a steady state is satisfied for γ_F small enough. More precisely, for a small γ_F , each steady state satisfies the condition to be in an asymmetric regime, *i.e.* $\varphi(k_D) > C_F(k_D)$.

From (65), if the following sufficient condition holds:

$$F_2 > (1 - \alpha) \alpha^{\frac{\alpha}{1-\alpha}} G_1^{\frac{1}{1-\alpha}} F_1^{-\frac{\alpha}{1-\alpha}}, \quad (66)$$

we have $H_2(k_D) > H_1(k_D)$ for all $k_D > 0$ and there is no steady state. This last inequality is satisfied if b_D and/or b_F are sufficiently high.

We further note that since both steady states are lower than $\hat{k} = (G_1/F_1)^{\frac{1}{1-\alpha}}$, they are characterized by under-accumulation if $\hat{k} < (\alpha A_D)^{\frac{1}{1-\alpha}}$. Using (60) and (61), it is equivalent to:

$$\frac{\alpha}{1 - \alpha} > \frac{\frac{\beta_D}{1+\beta_D} + \frac{\beta_F}{1+\beta_F} \left(\frac{A_F}{A_D}\right)^{\frac{1}{1-\alpha}}}{1 + \left(\frac{A_F}{A_D}\right)^{\frac{1}{1-\alpha}}}$$

7.3 Proof of Proposition 2

We examine the impact of debt variation on the stationary capital stocks and environmental quality index. Using (58)-(63), we obtain:

$$\frac{dk_D}{db_i} = \frac{\frac{\beta_i}{1+\beta_i} \alpha A_D k_D^{\alpha-1} + \frac{1}{1+\beta_i}}{H_1'(k_D) - H_2'(k_D)}, \text{ for } i = \{D, F\} \quad (67)$$

We consider the following variation: $db_D = -db_F = db$. Using (67), we get:

$$\frac{dk_D}{db} = \frac{(\alpha A_D k_D^{\alpha-1} - 1) \left(\frac{\beta_D}{1+\beta_D} - \frac{\beta_F}{1+\beta_F} \right)}{H_1'(k_D) - H_2'(k_D)} \quad (68)$$

We deduce the effect of debt transfer on capital taking into account that $H_1'(k_D) < H_2'(k_D)$ when $k_D = k_h$.

Then, we analyze how the environmental quality that prevails at the high steady state, *i.e.*

E_h , evolves with a debt transfer. Using (57) we have:

$$\begin{aligned} \frac{1 + \beta_D}{\gamma_D} \frac{dE_h}{db} = & -\phi(\alpha A_D k_h^{\alpha-1} - 1) + \phi b_D A_D \alpha (1 - \alpha) k_h^{\alpha-2} \frac{dk_h}{db} \\ & + \alpha A_D k_h^{\alpha-1} \frac{dk_h}{db} \left[\phi(1 - \alpha) - \theta \left(1 + \left(\frac{A_F}{A_D} \right)^{\frac{1}{1-\alpha}} \right) \right] \end{aligned}$$

When $\beta_D > \beta_F$, we deduce that $dE_h/db < 0$. We rewrite the previous expression to examine the case $\beta_F > \beta_D$:

$$\frac{1 + \beta_D}{\gamma_D} \frac{dE_h}{db} = \phi + \phi A_D \alpha k_h^{\alpha-1} \left[(1 - \alpha) \epsilon_{k_h/b} - 1 \right] + \alpha A_D k_h^{\alpha-1} \frac{dk_h}{db} \left[\phi(1 - \alpha) - \theta \left(1 + \left(\frac{A_F}{A_D} \right)^{\frac{1}{1-\alpha}} \right) \right]$$

with $\epsilon_{k_h/b} \equiv \frac{dk_h}{db} \frac{b_D}{k_h}$. Under Assumptions 1-3, $\epsilon_{k_h/b}$ is decreasing with k_h . By inspection of equation (68), this is easily shown by observing that $H_2''(k_D) > 0$, $H_1''(k_D) < 0$ and the steady state level of capital is characterized by under-accumulation.

We also have $\frac{dk_h}{db} > 0$ if inequality (34) is satisfied and $\beta_F > \beta_D$. As a result, the condition $\epsilon_{k_h/b} > 1/(1 - \alpha)$ guarantees $\frac{dE_h}{db} > 0$. As the elasticity $\epsilon_{k_h/b}$ is a decreasing function of k_h and that $k_h < \hat{k}$, a sufficient condition is to have

$$\epsilon_{\hat{k}/b} \equiv \frac{d\hat{k}}{db} \frac{b_D}{\hat{k}} > 1/(1 - \alpha)$$

Using (58)-(63) and (67), this condition corresponds to

$$\frac{(1 + \beta_D)(1 + \beta_F)}{b_D(\beta_F - \beta_D)} < \frac{\alpha A_D F_1 - G_1}{G_1^{\frac{2-\alpha}{1-\alpha}} F_1^{\frac{-\alpha}{1-\alpha}} - G_2 F_1} \equiv \mathcal{X}$$

and Proposition 2 follows.

7.4 Proof of Proposition 3

We then examine the effect of a debt transfer on the agent's welfare in both economies. Using agent's consumption choices, its indirect utility function along the high steady state is given by:

$$V_i = (1 + \beta_i) \ln(c_i) + \beta_i \ln(R_i) + (\delta_i + \gamma_i) \ln(E_h) + \beta_i \ln \beta_i$$

We decompose the welfare into a consumption (V_{C_i}) and an environmental (V_{E_i}) components.

$$V_i = \underbrace{(1 + \beta_i) \ln(c_i) + \beta_i \ln(R_i)}_{V_{C_i}} + \underbrace{(\delta_i + \gamma_i) \ln(E_h) + \beta_i \ln \beta_i}_{V_{E_i}} \quad (69)$$

The environmental part of the welfare evolves as the environment. We have $Sign \frac{dV_{E_i}}{db} = Sign \frac{dE_h}{db}$.

Then, we examine how the consumption part of the welfare changes with a debt transfer.

$$\text{Sign} \frac{dV_{Ci}}{db} = \text{Sign} \left\{ (1 + \beta_i) \frac{dc_i}{db} R_i + \beta_i \frac{dR_i}{db} c_i \right\}$$

Given (14), we have

$$\frac{dR_F}{db} = \frac{dR_D}{db} = A_D \alpha (\alpha - 1) k_h^{\alpha-2} \frac{dk_h}{db}$$

Using (9), (11) and (57) we have:

$$c_D = \frac{E}{\gamma_D \phi} = \frac{1}{1 + \beta_D} \left[A_D k_D^\alpha \left(1 - \alpha - \frac{\theta}{\phi} \left(1 + \left(\frac{A_F}{A_D} \right)^{\frac{1}{1-\alpha}} \right) \right) - b_D (\alpha A_D k_D^{\alpha-1} - 1) \right]$$

$$c_F = \frac{1}{1 + \beta_F} (w_F - \tau_F) = \frac{1}{1 + \beta_F} \left[(1 - \alpha) A_D k_D^\alpha \left(\frac{A_F}{A_D} \right)^{\frac{1}{1-\alpha}} - b_F (\alpha A_D k_D^{\alpha-1} - 1) \right]$$

Differentiating these equations, we get:

$$(1 + \beta_D) \frac{dc_D}{db} = \alpha A_D k_h^{\alpha-1} \left[(1 - \alpha) - \frac{\theta}{\phi} \left(1 + \left(\frac{A_F}{A_D} \right)^{\frac{1}{1-\alpha}} \right) \right] \frac{dk_h}{db} - (R_D - 1) + b_D \alpha (1 - \alpha) A_D k_D^{\alpha-2} \frac{dk_h}{db}$$

$$(1 + \beta_F) \frac{dc_F}{db} = \alpha A_D k_h^{\alpha-1} (1 - \alpha) \frac{dk_h}{db} \left(\frac{A_F}{A_D} \right)^{\frac{1}{1-\alpha}} + (R_F - 1) + b_F \alpha (1 - \alpha) A_D k_h^{\alpha-2} \frac{dk_h}{db}$$

Finally, the sign of dV_{CD}/db is given by the sign of:

$$\underbrace{\left(1 - \frac{(1 - \alpha)\beta_D}{\alpha(1 + \beta_D)} \right) \left[(1 - \alpha) - \frac{\theta}{\phi} \left(1 + \left(\frac{A_F}{A_D} \right)^{\frac{1}{1-\alpha}} \right) \right] \frac{dk_h}{db} R_D^2}_{\text{income effect}} \quad (70)$$

$$+ \underbrace{\frac{b_D R_D (1 - \alpha)}{k_h} \left(R_D + \frac{(R_D - 1)\beta_D}{1 + \beta_D} \right) \frac{dk_h}{db}}_{\text{debt burden effect}} - \underbrace{\frac{(R_D - 1)R_D}{db}}_{\text{direct debt effect}}$$

and the sign of dV_{CF}/db by the sign of:

$$\left(1 - \frac{(1 - \alpha)\beta_F}{\alpha(1 + \beta_F)} \right) \frac{dk_h}{db} \left(\frac{A_F}{A_D} \right)^{\frac{1}{1-\alpha}} (1 - \alpha) R_F^2 \quad (71)$$

$$+ \frac{b_F R_F (1 - \alpha)}{k_h} \left(R_F + \frac{(R_F - 1)\beta_F}{1 + \beta_F} \right) \frac{dk_h}{db} + (R_F - 1) R_F$$

By inspection of (70) and (71), the income effect in both countries is positive when $\alpha/(1 - \alpha) > \max \left\{ \frac{\beta_D}{1 + \beta_D}; \frac{\beta_F}{1 + \beta_F} \right\}$. This condition ensures that the two countries under-accumulate capital. In this case, the condition (34) that guarantees that the capital at the international level is necessarily lower than the golden rule is always satisfied.

Proposition 3 follows from these last results.

7.5 Phase diagram when γ_F is significant

Lemma 2 *Under Assumptions 1, 3 and 4, there exists $k_1 > 0$ such that $C_F(k_{Dt}) < \varphi(k_{Dt})$ for all $k_{Dt} < k_1$. This upper bound k_1 increases with $\frac{b_F \gamma_F}{1 + \beta_F} - \frac{b_D \gamma_D}{1 + \beta_D}$. In addition, there exists $k_0 (< k_1)$ such that $\varphi(k_{Dt}) > 0$ for all $k_{Dt} > k_0$ if:*

$$\frac{b_D}{b_F} < \frac{\phi(1 - \alpha) - \theta \left(1 + \left(\frac{A_F}{A_D} \right)^{\frac{1}{1 - \alpha}} \right)}{\phi(1 - \alpha) \left(\frac{A_F}{A_D} \right)^{\frac{1}{1 - \alpha}}} \quad (72)$$

Proof. Since $\frac{b_F \gamma_F}{1 + \beta_F} > \frac{b_D \gamma_D}{1 + \beta_D}$, $\varphi(k_{Dt}) > C_F(k_{Dt})$ is equivalent to $F(k_{Dt}) > \tilde{F}$, where $F(k_{Dt})$ is given by (52) and \tilde{F} by (53). Under Assumption 4, $\tilde{F} > 0$ which means that there exists a unique $k_1 > 0$ such that $F(k_{Dt}) > \tilde{F}$ for all $k_{Dt} < k_1$. Using (52) and (53), we also deduce that k_1 increases with $\frac{b_F \gamma_F}{1 + \beta_F} - \frac{b_D \gamma_D}{1 + \beta_D}$.

Moreover, $\varphi(k_{Dt}) > 0$ is equivalent to $F(k_{Dt}) < F_0$, with $F_0 > 0$ given by (54). $F_0 > \tilde{F}$ is ensured by inequality (72). In this case, there exists k_0 lower than k_1 such that $F(k_{Dt}) < F_0$ for all $k_{Dt} > k_0$. ■

We still have that $\psi(k_{Dt}) \geq C_F(k_{Dt})$ is equivalent to $\Gamma(k_{Dt}) \geq 0$, where $\Gamma(k_{Dt})$ is given by (55). However, under Assumption 4, we deduce that $\lim_{k_{Dt} \rightarrow 0} \Gamma(k_{Dt}) = +\infty$, $\lim_{k_{Dt} \rightarrow +\infty} \Gamma(k_{Dt}) = +\infty$ and $\Gamma(k_{Dt})$ is convex:

$$\begin{aligned} \Gamma''(k_{Dt}) &= k_{Dt}^{\alpha-2} \alpha(1 - \alpha)^2 A_D \left(\frac{A_F}{A_D} \right)^{\frac{1}{1 - \alpha}} \left(\frac{\beta_F}{1 + \beta_F} + \frac{\beta_D}{1 + \beta_F} \frac{\gamma_F}{\gamma_D} \right) \\ &\quad - \alpha(\alpha - 1)(\alpha - 2) k_{Dt}^{\alpha-3} \left[A_D \left(\frac{\beta_D b_D}{1 + \beta_D + \gamma_D} - \frac{\beta_D \gamma_F b_F}{(1 + \beta_F) \gamma_D} \right) - A_F \left(\frac{\beta_F b_F}{1 + \beta_F} + \frac{\beta_D b_D}{1 + \beta_D + \gamma_D} \right) \right] \geq 0 \end{aligned}$$

This means that $\psi(k_{Dt})$ is higher than $C_F(k_{Dt})$ at least for k_{Dt} low or high enough.

Under Assumption 3, we observe, using (42), that $\Omega(k_{Dt})$ is strictly increasing ($\Omega(k_{Dt})' > 0$) and concave ($\Omega(k_{Dt})'' < 0$), with $\lim_{k_{Dt} \rightarrow 0} \Omega(k_{Dt}) = -\infty$ and $\lim_{k_{Dt} \rightarrow +\infty} \Omega(k_{Dt}) = +\infty$.

By inspection of equation (43), we have $\lim_{k_{Dt} \rightarrow 0} \Pi(k_{Dt}) = +\infty$ and $\lim_{k_{Dt} \rightarrow +\infty} \Pi(k_{Dt}) = +\infty$, with $\lim_{k_{Dt} \rightarrow +\infty} \Pi(k_{Dt})/\Omega(k_{Dt}) = +\infty$. We can also show that $\Pi''(k_{Dt}) > 0$. Since $\lim_{k_{Dt} \rightarrow 0} \Pi'(k_{Dt}) = -\infty$ and $\lim_{k_{Dt} \rightarrow +\infty} \Pi'(k_{Dt}) > 0$, $\Pi(k_{Dt})$ is a convex function, decreasing for low values of k_{Dt} and increasing for high values of k_{Dt} .

We can further study whether $C_F(k_{Dt}) \geq \Omega(k_{Dt})$. After some computations, we can show that when $\frac{b_F \gamma_F}{1 + \beta_F} > \frac{b_D \gamma_D}{1 + \beta_D}$, this inequality is equivalent to $F(k_{Dt}) \leq \tilde{F}$. Using Lemma 2, we immediately deduce that this is satisfied for $k_{Dt} \geq k_1$. This means of course that $\varphi(k_1) = \Omega(k_1)$.

7.6 Proof of Proposition 4

A steady state is defined by $E = \Omega(k_D)$ and

$$\frac{1}{\phi}E \left(\frac{\beta_D}{\gamma_D} + \frac{\beta_F}{\gamma_F} \right) = k_D \left[1 + \left(\frac{A_F}{A_D} \right)^{\frac{1}{1-\alpha}} \right] + b_D + b_F \quad (73)$$

with $E < \min \{C_F(k_D), C_D(k_D)\}$. Using these equations, a steady state is a solution k_D to the equation $I(k_D) = J(k_D)$, with:

$$I(k_D) = I_1 + I_2 k_D^{\alpha-1} \quad (74)$$

$$J(k_D) = J_1 k_D^\alpha - J_2 k_D \quad (75)$$

and

$$I_1 = (b_D + b_F) \frac{\gamma_F + \gamma_D}{(1 + \beta_D)\gamma_F + (1 + \beta_F)\gamma_D} > 0 \quad (76)$$

$$I_2 = (b_D + b_F) \frac{\gamma_F \beta_D + \gamma_D \beta_F}{(1 + \beta_D)\gamma_F + (1 + \beta_F)\gamma_D} \alpha A_D > 0 \quad (77)$$

$$J_1 = \frac{(\gamma_F \beta_D + \gamma_D \beta_F) A_D [\phi(1 - \alpha) - \theta] \left[1 + \left(\frac{A_F}{A_D} \right)^{\frac{1}{1-\alpha}} \right]}{(1 + \beta_D)\gamma_F + (1 + \beta_F)\gamma_D} > 0 \quad (78)$$

$$J_2 = 1 + \left(\frac{A_F}{A_D} \right)^{\frac{1}{1-\alpha}} \equiv F_1 > 0 \quad (79)$$

The analysis for the existence and multiplicity of steady states is quite similar to the case with $a_{Dt} > 0$ and $a_{Ft} = 0$. Figure 4 illustrates the steady state analysis for the symmetric case, with $X_1 = I$ and $X_2 = J$. Indeed, $J(k_D)$ is an inverse U-shaped strictly concave function which attains its maximum value for $k_D = (\alpha J_1 / J_2)^{\frac{1}{1-\alpha}} \equiv k_J$, with:

$$J(k_J) = \alpha^{\frac{\alpha}{1-\alpha}} J_1^{\frac{1}{1-\alpha}} J_2^{-\frac{\alpha}{1-\alpha}} (1 - \alpha) \quad (80)$$

If $b_D + b_F = 0$, we have $I_1 = I_2 = 0$. Steady states are solutions solving $J(k_D) = 0$. There are two solutions, $k_D = 0$ and $k_D = (J_1 / J_2)^{\frac{1}{1-\alpha}} \equiv \hat{k}^s$.

If $b_D + b_F > 0$, we see that $I(k_D)$ is strictly decreasing and convex, with $\lim_{k_D \rightarrow 0} I(k_D) = +\infty$ and $\lim_{k_D \rightarrow +\infty} I(k_D) = I_1$. We have $I(k_J) \leq J(k_J)$ iff:

$$\alpha J_1 I_1 + J_2 I_2 \leq (1 - \alpha) \alpha^{\frac{1}{1-\alpha}} J_1^{\frac{2-\alpha}{1-\alpha}} J_2^{-\frac{\alpha}{1-\alpha}} \quad (81)$$

Using (76)-(79), we note that J_1 and J_2 do not depend on b_D and b_F , while I_1 and I_2 both linearly increase with $b_D + b_F$. Therefore, inequality (81) is satisfied if $b_D + b_F$ is low enough. In this case, there are two stationary solutions to the equation $H_1(k_D) = H_2(k_D)$. The steady states are admissible if $E < \min \{C_F(k_D), C_D(k_D)\}$ is satisfied for each steady state.

On the contrary, if:

$$I_1 > \alpha^{\frac{\alpha}{1-\alpha}} J_1^{\frac{1}{1-\alpha}} J_2^{-\frac{\alpha}{1-\alpha}} (1 - \alpha) \quad (82)$$

we have $I(k_D) > J(k_D)$ for all $k_D > 0$. This last inequality is satisfied if $b_D + b_F$ is sufficiently high. In this case, there is no steady state.

In the case in which there exist two steady states, we know that their associated levels of capital are lower than $\hat{k}^s = (J_1/J_2)^{\frac{1}{1-\alpha}}$. Thus, they are characterized by under-accumulation if $\hat{k}^s < (\alpha A_D)^{\frac{1}{1-\alpha}}$. Using (78) and (79), this condition can be written as inequality (44) and Proposition 4 follows.

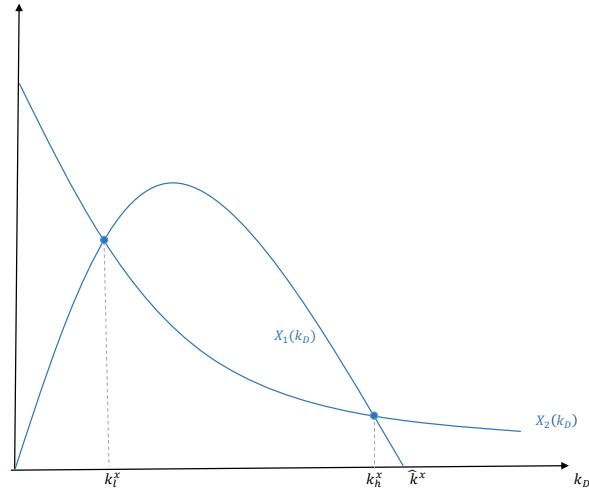


Figure 4 – Steady state analysis

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