Managing a common-pool resource with no stock externality: The case of artesian aquifers

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Abstract

This paper studies a specific class of common-pool resources whereby rivalry is not characterized by competition for the resource stock. Artesian aquifers have been identified as a typical example, since the stock is never depleted, even when part of the resource is extracted. We first propose a dynamic model to account for relevant features of such aquifers - like water pressure, or well yield - and to characterize the corresponding dynamics. We then compare the social optimum and the private exploitation of an open-access aquifer. The comparison of these two equilibria allows us to highlight the existence of a new source of inefficiency. We refer to this as *pressure externality*. This externality results in the long run in an additional number of wells for the same water consumption, and hence additional costs. Finally, we characterize a specific stock-depend tax to neutralize the pressure externality.

JEL Classification: H21, H23, Q15, Q25, C61

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1. Introduction

Common-pool resources (CPRs) have been studied extensively since the seminal papers by Gordon [10] and Hardin [11] in the 1950s and 1960s. Most of these studies have addressed issues related to the characteristics of non-excludability and rivalry in consumption. Indeed, it is difficult to assign adequate property rights to control access to resource stock, and any amount of resources that is extracted is no longer available to

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others. However, the degree of rivalry can differ, as well as the resulting externalities. Private appropriation in particular reduces the available stock, generating a series of externalities associated with stock variation (Gordon [10], Smith [24], or recently, Huang and Smith [12]). Such *stock externalities* occur, for example, when harvesting most CPRs. This is, of course, not the case for resources where the stock is infinite (e.g., solar energy resources). However, it may also not be the case for some resources that are finite. This is particularly true for *confined aquifers*, that is, aquifers confined between an upper and a lower impermeable layer that recharge from a distant and more elevated aquifer that is often unconfined. For these aquifers, water naturally flows out from a well, without any pumping (the *artesian property*), and withdrawal from the confined aquifer is immediately compensated so that there is no dewatering at all. The absence of stock externality therefore suggests that this resource does not suffer overexploitation under open access and needs no regulation. The main objective of this paper is to show that this intuition is wrong. The basic point is that the artesian property of a single well is mainly related to the global water pressure in a confined aquifer. This *pressure externality* leads, under an open-access regime, to an excessive number of wells and to additional costs. This result is obtained by contrasting the open-access outcome with the socially optimal equilibrium, which internalizes this externality. In other words, even if there is no decrease in groundwater stock, we show that the behaviors of economic agents are not aligned with the socially optimal outcome.

Most of the literature on groundwater management never considers this case. Since Smith's 1969 study [24], stock externalities have reflected the effects that a *reduction* of a resource stock may have on economic decisions. A lower stock actually creates two distinct effects (Burt and Provencher [6]): a reduction in extraction opportunities to all others, and an increase in future extraction costs for all users. Stock externality represents the former effect and arises because exploitation of a resource is constrained by a finite resource stock. The latter effect represents the pumping cost externality and arises because the cost of extraction depends on the resource stock. However, most of the studies refer irrespectively to one or the other, but include only pumping cost externality in their models (e.g., Gisser and Sanchez [9] or, more recently, Brown [5]).

Groundwater has been studied extensively since Gisser and Sanchez's 1980 paper [9], in which they concluded that welfare gains from public management are negligible. A large part of the follow-up literature continues to analyze the potential role of water management under different assumptions, but no clear-cut answer has yet been provided. A number of studies compare perfect competition with socially optimal management outcomes. For instance, Provencher [19] and Provencher and Burt [20] respectively show that property rights allow the recovery of a large part of welfare gains, but these gains remain relatively low. Allen and Gisser [1] obtain a small difference between competition and social planning, while Brill and Burness [3] find an increased divergence between both scenarios under different hydrological and economic assumptions, including demand growth, declining well yields, and low social discount rates. Another part of the literature contrasts the social planner solution with strategic behaviors (Negri [17], Rubio and Casino [22]). Rubio and Casino [22], for instance, confirms Gisser and Sanchez's [9] result. Some

papers expand on this in a number of directions, including uncertainty (Knapp and Olson [15], Tsur and Graham-Tomasi [27]), conjunctive surface water use (Azaiez [2], Pongki-jvorasin and Roumasset [18], Stahn and Tomini [25],[26]), and water quality management (Roseta-Palma [21]). Nevertheless, all of these works assume that an aquifer behaves as a "bathtub" with perfect hydraulic conductivity. This assumption is equivalent to assuming a bottomless aquifer that is capable of capturing only one effect resulting from stock variation, that is, the pumping cost externality. Recent papers have introduced a spatial representation of the aquifer taking into account that transmissivity is not infinite, such that the stock available to users and the impact on users' decisions may differ according to the location of extraction (Brozovic et al. [4], Chakravorty and Roumasset [7], Chakravorty and Umetsu [8], Pfeiffer and Lin [16]). Brozovic et al. [4] found that welfare gains may be misestimated under the bathtub assumption. As a consequence, these studies show that policy recommendations based on such basic hydrological assumptions may be irrelevant. Nevertheless, all of these studies analyze the magnitude of externalities and inefficiency within the specific context of unconfined aquifers.

To the best of our knowledge, only Worthington et al. [29] have considered confined aquifers. Adopting a dynamic programming system applied to the Crow Creek Valley aquifer, they characterize the optimal seasonal water management and contrast it with a competitive outcome. They show that there may be significant welfare gains from public management. However, they fail to capture specific features of confined aquifers, since they merely expand the standard model, based on a simple equation of balancing water stock, by considering the effect of pumping activities on groundwater stock. Artesian aquifers are groundwater reservoirs confined by impermeable layers where water is under pressure, such that water may naturally flow over the top of a drilling well, without the need for pumping. However, well water yields depend on the pressure of the confined aquifer. When a new well is drilled, the pressure of the whole aquifer decreases, which consequently exerts a negative externality on the yields of existing wells. This is what we refer to as the pressure externality, which requires overall management of the number of wells.

There exist many artesian aquifers across the globe. The Great Artesian Basin in Australia is considered the largest and deepest in the world, underlying 22% of the continent. Other important systems are the Edwards Aquifer in Texas, USA, and the Northern Sahara Aquifer System in the north of Africa. There are also artesian wells in the south of the city of Vancouver, Canada, and in several areas in India, which provide water for millions of people. Nevertheless, this resource is subject to the "rule of capture" and thus threatened by human pressure. Moreover, in many of the wells, the flow is still uncontrolled. As such, optimizing the exploitation of this resource and the production of groundwater reservoirs is a difficult challenge. Some regulations have already been implemented in some areas to limit adverse impacts. For instance, the St. Johns River Water Management District encourages well owners to control flow, and even abandon problem wells by plugging them.

In this paper, we develop a hydro-economic dynamic model based on fluid mechanics

to effectively consider the time evolution of water pressure in an aquifer. Specifically, we use fluid dynamics to describe the relationship between water stock, pressure, and water discharge. On this basis, we first assume that this resource is under an open-access regime. This means that additional wells are drilled until the marginal cost of an additional well is equal to the private marginal returns, this last quantity being, under a purely competitive assumption, related to the water price. Second, we introduce an optimal management problem for the number of wells that takes into account this pressure externality. For both management regimes, we characterize the long-run steady values and their properties with respect to changes in the economic and hydrological parameters of the model. Finally, we compare these two management regimes by introducing two rates that capture the main differences. The first rate measures the yield losses per well compared with the optimal management case, while the second rate measures the rate of increase in the long-run water cost. These rates act, respectively, as proxies for the loss of pressure and the social cost of overextraction. We analyze the sensitivity of these rates to the economic and hydrological parameters of the model and propose a state-dependent tax that aligns the open-access steady state with optimal management.

The remainder of this paper is organized as follows. The specific features of a confined aquifer, in particular concerning the consequences of the water pressure model, are presented in Section 2. Section 3 presents the main economic characteristics of our system and analyzes the short- and long-run properties of this hydro-economic model under free access. Section 4 analyzes the centralized water management problem, and Section 5 describes the long-run properties of this optimal management. Section 6 contrasts these two regimes in terms of pressure losses and social cost, and Section 7 concludes. Proofs and computations can be found in the appendix.

2. Aquifers with flowing artesian wells

Confined aquifers are a relevant example of a CPR for which rivalry may be open to question. Indeed, although units of water withdrawn from such aquifers are no longer available, the same amount of total stock in the ground is still available for others. In other words, although there may be private appropriation of water, the availability of the resource is not reduced. Nevertheless, what is reduced is water pressure. More precisely, confined (artesian) aquifers are overlain by a relatively impermeable layer, such that water is under a pressure greater than atmospheric. Consequently, water may rise above the top of the aquifer, even above the land surface, when a drill hole penetrates an artesian aquifer. In particular, we can observe wells that naturally flow to (or above) the land surface, without the need for pumping, when the *potentiometric line*[1] is above the surface. This level represents the pressure exerted by water, given the force of gravity. Consequently, the confined layer remains saturated, even with exploitation. However,

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[1]The potentiometric line is an imaginary line where water pressure is equal to atmospheric pressure. Water will rise to this specific level. In an unconfined aquifer, the potentiometric line is equivalent to the water table level.

abstraction of groundwater resources implies variations in pressure, which in turn trigger falls in water flow at ground surface.

These confined aquifers never dewater, since they are fed by water that moves from a distant *outcrop* area, also called the recharge zone. This area is usually located at a high elevation (e.g., near mountains), exposed at ground surface, and a considerable distance away from the confined aquifer, which is at a lower elevation beneath an impermeable layer. This difference in elevation generates a water column, h, the weight of which exerts a pressure on the confined aquifer. More precisely, we define the *water column* as the vertical distance between the top of the water table in the outcrop area and the upper layer of the confined aquifer. The elevation of this upper layer is for simplicity normalized to 0 so that $h \in (0, h_{\max})$, with $h_{\max}$ the upper level of the outcrop area. If this area behaves as a bathtub unconfined aquifer with a flat bottom of area A_r perpendicular sides and storativity rate s, the weight of the water column that exerts this pressure is sA_rh. The water column may rise and fall according to the recharge and the discharge toward the confined aquifer. We denote R as the *potential* recharge, that is, all water available at the surface, which may or may not reach the ground. The part of the potential recharge that effectively soaks into the soil represents the *actual* recharge, while the remaining water runs off over land.[2] In other words, only the proportion ρR recharges the outcrop area, where ρ is the infiltration rate.

This net recharge moves slowly down toward the confining layer, replacing the water flow out of the artesian wells because of the pressure exerted by the weight of the water column. Artesian groundwater exploitation results in changes in volume storage in the outcrop area only, and consequently, in declining pressure. Since there is less pressure to cause a well to flow naturally to the land surface, less water will rise to the surface. From fluid dynamics, we can characterize the relationship between water pressure and the water column, and then deduce the *maximum water yield* flow out of a well using Torricelli's theorem.[3] Let us assume that k is the diameter of the drill hole, that is, a technological characteristic, and g is the acceleration due to gravity, then the maximum water yield, r, is approximately given by

$$r(h) = k\sqrt{2gh} \tag{1}$$

We can easily observe that the lower the water column, the lower is the maximum water yield ($r'(h) > 0$). Water yield approximatively captures the pressure (given k), and change in water yield is equivalent to change in artesian pressure. Moreover, if there are n active wells operating at capacity, the total water discharge is $w(h) = r(h)n$. Hereafter, we always consider n to be a real number. This means, for instance, that the last well is only partially active. Figure 1 illustrates this specific structure.

Hence, we can characterize the hydrodynamics of confined aquifers by two equivalent formulations: a law of motion based on changes in the water column, $h(t)$, or in the outcrop

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[2]This distinction is commonly used in the hydrogeological literature (e.g., Rushton [23]).

[3]This theorem is an application of Bernoulli's principle relating pressure, velocity, and elevation for a steady-flow system. Torricelli's theorem describes fluid flow out of an orifice.
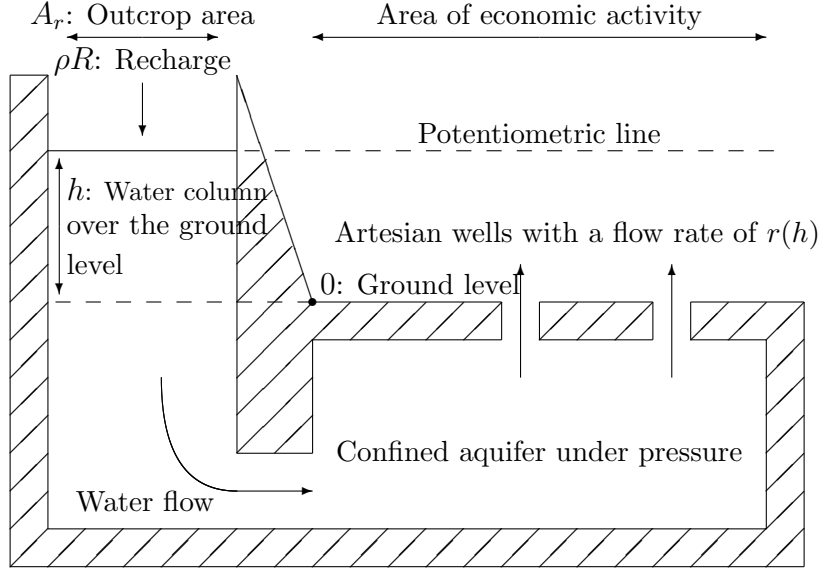
Figure 1: Schematic representation of a confined aquifer

area; or a law of motion based on changes in the maximum water yield, $r(t)$, of a well over the confining aquifer. The dynamics of the water column result from a physical water balance between inflows and outflows: The actual recharge ρR is the source of incoming water, while outflows are the sum of water discharge flowing at the surface in each well, $w(h(t)) = r(h(t))n(t)$. Within this context, the time evolution of the water column (as long as $h \geq 0$) in the outcrop area is associated with changes in the potentiometric water head of the confined aquifer and is described as

$$\dot{h}(t) = \frac{\rho R - w\left(h(t)\right)}{sA_r} \tag{2}$$

We can see that these dynamics are state dependent, in contrast to the law of motion of the water table in unconfined aquifers.

Using Torricelli's theorem (1), we can characterize the time evolution of the maximum water yield:

$$\dot{r}(t) = \frac{kg\dot{h}(t)}{\sqrt{2gh(t)}} = \frac{k^2 g\dot{h}(t)}{r(t)} \tag{3}$$

Using the dynamics of Eq. (2), it follows that the dynamics of the yield of an artesian well are given by

$$\dot{r}(t) = \frac{k^2 g}{r(t)sA_r}\left[\rho R - r(t)n(t)\right] \tag{4}$$

These second dynamics sound more economically oriented since outflows result from the economic decision to build new wells, but they are unusual because they are based on

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the accumulation of resources flowing out of a reservoir. However, more water out of the aquifer means that the pressure correspondingly changes. Interestingly, this formulation captures the idea that the pressure of the water behaves as a CPR. Consequently, we may expect the existence of a new externality, different from the pumping cost commonly observed in analyses of unconfined aquifers. This *pressure externality* results from the effect of an additional well on future yields.

It remains to set the initial condition of this system. For simplicity, we assume for Eq. (2) that the initial level of the water column is $h(0) = h_{\max}$ or, equivalently, for Eq. (4) that the initial yield is $r(0) = k\sqrt{2gh_{\max}}$.

3. Confined aquifer as an open-access resource

As recharge occurs at elevated outcrop areas, we assume that water extraction occurs only above the confined aquifer, for example, in distant lower areas such as plains or valleys where the economic activity takes place. The water demand is described by a decreasing inverse water demand with the overall production of water, $P(w)$, with $P'(w) < 0$, and providing a social benefit $\int_0^w P(\omega)d\omega$. Moreover, we ignore possible conveyance losses, and we thus assume that all water flow at the surface is the actual volume of water used by consumers.

Individual wells are drilled to fulfill this demand, such that the number of wells corresponds to the number of well owners. The well owner faces an instantaneous profit function:

$$\Pi = p(t)r(t) - c \tag{5}$$

with $p(t)$ the market price the owner receives for the yield $r(t) = k\sqrt{2gh(t)}$ of their well, and $c > 0$ the annual exploitation cost of their well. Following the literature on irrigation networks (e.g., Chakravorty and Roumasset [7] or Jandoc, Juarez, and Roumasset [13]), this cost includes, among others, the per-period equivalent of construction cost, operating and maintenance cost, and water provision because of irrigation networks, conveyance structures linking all wells, and distribution to consumers. Now consider that a confined aquifer is exploited by myopic well owners, who decide to exploit the aquifer when there is a positive profit. As the resource is under condition of free access, this also means that new wells are drilled as long as the observed profit remains positive. This free-entry condition therefore induces the following dynamics of the number of wells:[4]

$$\dot{n}(t) = \alpha \left(P\left(n(t)r(t)\right) r(t) - c \right) \tag{6}$$

with $\alpha > 0$ an adjustment parameter, given the market-clearing price $p(t) = P\left(n(t)r(t)\right)$.

Even if there is no dewatering of the confined aquifer, however, water flows from any additional well reduce the water column ($h(t)$) in the outcrop area. A lower water column

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[4]We adopt the standard free-entry condition based on the average rent earned from the exploitation resource. See, among others, Weitzamn [28] for a static characterization of free access and Gordon [10], who characterized the bioeconomic equilibrium of a fishery.

in this area consequently reduces pressure in the confined aquifer and therefore lowers the water yields of each existing well. This means that a complete description of the dynamics of the number of wells must include both the free-entry dynamics (Eq. (6)) and the dynamics of the water column in the outcrop area (Eq. (2)). With the help of Torricelli's theorem, which links the yields to the water column, we get

$$
\begin{cases}
\dot{n}(t) = \alpha \left(P \left(n(t)k\sqrt{2gh(t)} \right) k\sqrt{2gh(t)} - c \right) \\
sA_r \dot{h}(t) = \rho R - n(t)k\sqrt{2gh(t)}
\end{cases}
\tag{7}
$$

The open-access hydro-economic equilibrium is thus obtained by setting the time variation of the number of wells and that of the water column in the outcrop area equal to 0: $\dot{n} = \dot{h} = 0$. We can easily deduce that the number of wells under open access is

$$
n^m(c, \rho, R) = \frac{P(\rho R)}{c} \rho R
\tag{8}
$$

This number depends on the actual recharge, weighted somehow by a profitability rate. We then derive the steady-state value for the water column:

$$
h^m(c, k, \rho, P) = \frac{1}{2g} \left(\frac{c}{kP(\rho R)} \right)^2
\tag{9}
$$

At this point, it should be noted that we need a consistency assumption for the parameters of the model, which simply states that $h_{\max} > h^m$. Finally, using Torricelli's theorem (Eq. (1)), we can derive the long-run maximum water yield

$$
r^m(c, \rho, R) = \frac{c}{P(\rho R)}
\tag{10}
$$

and the global water consumption

$$
w^m(\rho, R) = \rho R
\tag{11}
$$

We can even prove that the open-access hydro-economic equilibrium is stable, as summarized in the following proposition.

Proposition 1. *The dynamical system given by Eq. (7) has a unique steady state given by Eqs. (8) and (9) that is locally asymptotically stable. The effects of any change in the economic parameters, that is, the unit cost c and the diameter k of a well, and the hydraulic parameters, that is, the recharge R and the infiltration rate ρ, are given in Table 1. The notation $+/-$ means that an effect can be either positive or negative depending on whether the elasticity $\varepsilon_P(w)$ of the inverse demand evaluated at the steady-state consumption ρR is higher or smaller than -1.*

The impact of an increase in the unit cost per well is clear. It provides less incentives to drill additional wells, which typically constraints water in the outcrop area, the

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Table 1: Effects of economic and hydrological parameters on the open-access equilibrium

Parameters	water column	number of wells	water consumption	water yield
Cost (c)	$+$	$-$	0	$+$
Diameter of the well (k)	$-$	0	0	0
Recharge (R)	$+$	$+/-$	$+$	$+$
Infiltration rate (ρ)	$+$	$+/-$	$+$	$+$

(The header "Effects on" spans the last four columns.)

consequence of which is an increase in the pressure in the confined aquifer and therefore of the yield. To understand the effect of an increase in the diameter of the well on the steady state, it is important to note that the steady-state yield of a well is, under free entry, independent of k (see Eq. (10)). This means that any increase in k induces transitory dynamics toward a new steady state with unchanged yield. However, during this transitory adjustment, yield first increases, meaning that the new steady state can be reached only at a lower potentiometric level, h. Moreover, bearing in mind that the water consumption at the steady state is equal to the recharge, the number of wells must be unchanged. An increase in the infiltration rate or in the recharge increases the amount of water in the aquifer. At the steady state, this contributes to an increase in the water column, yield, and water consumption. The effect on the number of wells depends on the elasticity of the inverse demand evaluated at the net recharge, that is, $\varepsilon_P(\rho R) = \frac{P'(\rho R)\rho R}{P(\rho R)}$. If $\varepsilon_P(\rho R) > -1$, the number of wells increases, or decreases if $\varepsilon_P(\rho R) < -1$.

4. Optimal management of an artesian aquifer

The objective of the social planner is to choose the number of active wells within the confined aquifer at each time period t, accounting for the water flow dynamics (Eq. (4)). If $\delta > 0$ denotes the social discount rate, he/she maximizes the discounted sum of social benefits net of exploitation costs:

$$\max_{n(t)} \int_0^{+\infty} \left(\int_0^{r(t)n(t)} P(\omega)d\omega - cn(t) \right) e^{-\delta t}dt$$

$$\text{s.t.} \quad \dot{r}(t) = \frac{k^2 g}{r(t)sA_r}\left[\rho R - r(t)n(t)\right] \tag{12}$$

In this formulation, the objective of the social planner is to control the amount of water flowing out of the wells, given the time variation of artesian well yields. Considering now the total water discharge, we can write

$$\int_0^{r(t)n(t)} P(\omega)d\omega = \int_0^{w(t)} P(\omega)d\omega$$

and using the hydrological relationship between the confining aquifer and the outcrop area, given by Torricelli's theorem (Eq. (1)), the social planner can take a more standard

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approach:

$$\max_{w(t)} \int_0^{+\infty} \left(\int_0^{w(t)} P(\omega)d\omega - \frac{cw(t)}{k\sqrt{2gh(t)}} \right) e^{-\delta t} dt$$

$$\text{s.t.} \quad sA_r\dot{h}(t) = \rho R - w(t) \tag{13}$$

This formulation is convenient to analyze the optimal exploitation of a confined aquifer since the optimization program is defined only over water quantity, rather than water and the number of wells. Moreover, the dynamics of the resource is now independent of the state variable. This formulation enables us to derive (i) the optimal number of wells from the social planner's choice of water supply and (ii) water pressure in the confined aquifer resulting from the optimal exploitation of the aquifer. More precisely, we know that the potentiometric line of the aquifer corresponds to the level of the water column in the recharge area, h, and is the point to which water naturally flows from a single well. Given this level, we can first deduce well yield and the number of wells to obtain the total water discharge, w. Second, the potentiometric line directly allows us to capture water pressure. More relevantly, this formulation associates the potentiometric line with the constant marginal cost of drilling an additional well, c, which enables us to capture, not pumping cost externality, but pressure externality. For instance, a decrease in the potentiometric line reduces the water yield. This requires, for a given water production level, $w(t)$, additional wells to be drilled, therefore increasing globally the well exploitation cost.

However, this new setting has one problem. The co-state variable, $\lambda(t)$, associated with the water column dynamics in the program (Eq. (13)) now stands for the shadow price of the decrease of one unit in the water column in the recharge area. This variable only partially captures the shadow price of a unit of water naturally rising at the surface from an artesian well. This value is actually given by the co-state variable, $\Gamma(t)$, associated with the dynamics used in the optimization problem (Eq. (12)). This variable measures the monetary consequences of a decrease in the yield of an artesian well. However, let us denote the future values at t of the two programs (Eqs. (12) and (13)) along the optimal path by, respectively, $V_1^*(t) = V_1\left((n^*(t)), r^*(t), \Gamma^*(t)\right)$ and $V_2^*(t) = V_2\left((w^*(t)), h^*(t), \lambda^*(t)\right)$. We know from the equivalence of both programs that $\forall t, V_1^*(t) = V_2^*(t)$. It follows from Torricelli's theorem (Eq. (1)) and the usual property of the co-states that

$$\Gamma^*(t) = \frac{\partial V_1^*}{\partial r} = \frac{\partial V_2^*}{\partial h}\frac{\partial h}{\partial r} = \lambda^*(t)\left(\frac{r^*(t)}{k^2 g}\right) = \lambda^*(t)\left(\frac{\sqrt{2gh^*(t)}}{kg}\right) \tag{14}$$

In other words, we can obtain the shadow price of a unit of water naturally rising at the surface from an artesian well by using the solution to the program given by Eq. (13).

The current-value Hamiltonian becomes

$$\mathcal{H}\left(w(t), h(t), \lambda(t)\right) = \int_0^{w(t)} P(\omega)d\omega - \frac{cw(t)}{k\sqrt{2gh(t)}} + \frac{\lambda(t)}{sA_r}\left[\rho R - w(t)\right] \tag{15}$$

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where $\lambda(t)$ is the current-value shadow price associated with the water column of the recharge area. We derive the following first-order conditions:

$$P\left(w(t)\right) = \frac{c}{k\sqrt{2gh(t)}} + \frac{\lambda(t)}{sA_r} \tag{16}$$

$$\dot{\lambda}(t) = \delta\lambda(t) - \frac{cgw(t)}{k\sqrt{(2gh(t))^3}} \tag{17}$$

Eq. (16) represents the usual optimality condition, which yields a marginal benefit in each period equal to the total marginal costs, the sum of the marginal exploitation cost and the water shadow price, $\frac{\lambda(t)}{sA_r}$.[5]

Eq. (17) describes the behavior of the shadow value. This equation shows that the time evolution of the shadow price depends on the discount factor, and as usual in the literature on unconfined aquifers, it depends on the marginal exploitation cost. In fact, a stock-dependent recharge impacts the value of the resource, since current exploitation of the confined aquifer affects the water column in the outcrop area, which in turn affects pressure and, consequently, future flow.

5. Sustainable artesian water exploitation

We can now investigate the sustainable management of artesian aquifer by characterizing the optimal steady state, that is, by setting $\dot{h} = \dot{\lambda} = 0$. Indeed, in the steady state, we characterize the highest rate of exploitation without depleting the aquifer, or rather, without reducing to zero the water pressure in the long run. From Eq. (2), we can directly derive the steady-state level of global water consumption:

$$w^o = \rho R \tag{18}$$

Regarding the standard model of an unconfined aquifer, we can see that global water consumption corresponds to the actual recharge, but now of the distant outcrop area. Using Eq. (17) and the stationary state for water discharge (Eq. (18)), we also obtain the steady state for the shadow value of water:

$$\lambda(h^o; c, k, \delta, \rho, R) = \frac{cg\rho R}{\delta k\sqrt{(2gh^o)^3}} \tag{19}$$

Then, using Torricelli's theorem (Eq. (1)), we can derive the long-run maximum water yield of a well

$$r(h^o; k) = k\sqrt{2gh^o} \tag{20}$$

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[5]Recall that λ measures the shadow price of one additional drop of water in the ground, and $\frac{\lambda}{sA_r}$ is consequently the shadow value of a marginal water column elevation in the outcrop area.

the shadow value of the pressure

$$\Gamma(h^o; c, k, \delta, \rho, R) = \frac{c\rho R}{2\delta g k^2 h^o} \tag{21}$$

and the number of wells

$$n(h^o; k, \rho, R) = \frac{\rho R}{k\sqrt{2gh^o}} \tag{22}$$

All of these steady-state values depend on the long-run level of the water column in the distant outcrop area, h^o. On the basis of previous observations and using Eq. (16), we obtain the condition required for the level of the water column at the steady state:

$$P(\rho R) = \frac{c}{k\sqrt{2gh^o}}\left(1 + \frac{\rho R}{2\delta h^o s A_r}\right) \tag{23}$$

Hence, we can investigate the existence of a steady state based on this single condition, and examine the stability properties. Of course, we again need some consistency assumption for the parameters to ensure that $h^o < h_{\max}$.[6] All of the results are introduced in the proposition below.

Proposition 2. *Any optimal water consumption path $w^o(t)$ and water column path $h^o(t)$ that statisfy the optimal first conditions (16) and (17), with the dynamics (2), admit a unique steady state, which is a local saddle point.*

We can now analyze the long-run impact of variations in the parameters of the model on the water column, water consumption, water yield of a well, and the number of wells. More precisely, we want to study the effect of the hydrological parameters $\{A_r, R, \rho\}$ and the economic parameters $\{c, k, \delta\}$.[7]

Regarding **the long-run water column**, Eq. (23) shows that all of the parameters, except the natural recharge R and ρ, affect the long-run costs only. This specifically modifies the incentives to exploit artesian water by affecting the full marginal cost of a unit of water, that is, the sum of the marginal exploitation cost and the marginal user cost. Typically, the two parameters associated with exploitation, that is, $\{c, k\}$, affect both parts of the full marginal cost. First, an increase in the cost of drilling a well increases the full marginal cost and leads to an increase in the water column, for the optimality condition (23) to hold in the long run. Conversely, a larger drill hole diameter, k, decreases the full marginal cost of a unit of water and leads to a decline in the long-run water column.

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[6]In the appendix, we explicitly compute the solution to Eq. (23) and provide this consistency assumption.

[7]Detailed computations can be found in Appendix C.

Second, the hydrogeological parameter A_r and the discount rate δ both affect the time variation of the shadow value of the resource (Eq. (17)), and thus they impact its long-run value. Basically, a larger recharge area, A_r, gives a greater incentive to exploit water because the resource has a lower future value. Likewise, the higher the discount rate, the lower the present value of the future benefit. Both parameters therefore lead to a lower water column in the long run.

Finally, an increased potential recharge R or a larger infiltration rate ρ simultaneously impacts both water demand and the long-run social cost. An increase in R or ρ allows more water to infiltrate the outcrop area, which consequently decreases water price. Intuitively, at the steady state, the higher the inflows, the higher the outflows of the system. This drives up marginal costs. The overall effect on demand and costs decreases the incentive to exploit artesian groundwater, thus leading to a higher water column in the long run. Table 2 summarizes these first results of the comparative statics.

Table 2: Effects of economic and hydrological parameters on the optimal long-run water column

Parameters	Effect
Cost of a well (c)	$+$
Diameter of the well (k)	$-$
Discount rate (δ)	$-$
Recharge area (A_r)	$-$
Recharge (R)	$+$
Infiltration rate (ρ)	$+$

Let us now analyze the effect of variations in each parameter on **the long-run water consumption**, w^o. Eq. (18) shows that only the recharge and the infiltration rate matter. In fact, a higher potential recharge, R, or a larger infiltration rate, ρ, increases the net recharge and thus the water consumption.

Concerning **the long-run water yield of a well**, Eq. (20) shows that all of the parameters except k indirectly modify the yield consecutively to a change in the water column, which is positively related to the yield. It follows that for the parameters $\{c, \delta, A_r, R, \rho\}$, the signs of the effects are exactly those found in Table 2. Finally, a single parameter has an *a priori* ambiguous impact: A larger drill hole results in a positive direct impact on the yields, and a negative indirect impact, because of an abatement of the water column level. However, both effects alter the cost structure, and the impact of these opposite variations offset such that we observe a positive overall effect. Consequently, an increased drill hole diameter drives a higher long-run water yield.

The analysis of impacts on **the long-run number of wells** is a little more tricky. Recall that $n(h^o) = \frac{w^o}{r(h^o)}$. Variations in the number of wells will thus depend on simultaneous changes in water consumption and water yield. For the parameters $\theta = \{c, \delta, A_r\}$, this is fairly straightforward, since it depends on effects only on the water column. More precisely, we observe a decline in the number of wells in the long run. Now recall from our

previous result that changes in the diameter of the drill hole, k, positively affect water yield and, consequently, decrease the number of wells. The effect of an increase in the recharge R or in the infiltration rate ρ is more ambiguous. Both increase the long-run water consumption and the yield of a well. It is therefore difficult to estimate which effect dominates the other. To illustrate this ambiguity, we provide in Appendix C a class of examples in which this effect can be either positive or negative.

Table 3 summarizes the results.

Table 3: Variations in water consumption, water yield, and the number of wells

Parameters	Impact on water consumption (w)	Impact on water yield (r)	Impact on the number of wells (n)
Marginal cost (c)	0	+	-
Diameter of the drill hole (k)	0	+	$-$
Recharge area (A_r)	0	-	+
Discount rate (δ)	0	-	+
Recharge (R)	+	+	?
Infiltration rate (ρ)	+	+	?

6. Pressure externality

We now show that the behaviors of economic agents under open access are not aligned with the socially optimal outcome. Well owners are not rivals in water exploitation in the confined aquifer, since this aquifer never dewaters. However, this activity reduces the water column in the outcrop area, which decreases the pressure in the confined aquifer and reduces water yields for other well owners. The extra decline of pressure, which arise under an open-access regime is what we call the pressure externality. This externality has a fairly intuitive interpretation. As the owners do not take into account this pressure externality, they have an incentive to drill more wells than the number required by the social optimum. However, at the steady state, water consumption is the same in both cases. With more active wells for the same consumption, yields must be lower in the open-access case. Moreover, according to Torricelli's theorem (Eq. (1)), which links yields to the water column, this also means that this water column is lower. Therefore, under open access, there is less pressure in the confined aquifer.

The question is now how to measure the impact of this externality. We can either measure the hydrological consequences, or the economic consequences. First, it may be interesting to know the yield reduction rate of a well at the steady state with respect to optimal management. Using Eq. (10), which describes the steady-state yield under open access, and Eq. (20) for the optimal management regime (h_0 being the solution to Eq. (23)), this rate is given by

$$\Delta_{Yield}^{-} (c, k, \delta, A_r, R, \rho) = 1 - \frac{r^m (c, R, \rho)}{r^o (c, k, \delta, A_r, R, \rho)} \tag{24}$$

Second, we know that under open access, there are more wells in the long run for the same water consumption. It thus may be interesting to know the rate by which the cost of one unit of water is augmented with respect to optimal management. Without a pumping cost due to artesianism, this rate is

$$\Delta_{Cost}^{+} = \left(\frac{cn^m}{w^m} \right) \Big/ \left(\frac{cn^o}{w^o} \right) - 1 \qquad (25)$$

with n^m, w^m, n^o, $and w^o$ given by, respectively, Eqs. (8), (11), (22), and (23). If we now recall that water consumption is the same at both steady states, this rate of increase in the water cost coincides with the proportion of additional wells, that is

$$\Delta_{Cost}^{+} (c, k, \delta, A_r, R, \rho) = \frac{n^m (c, R, \rho)}{n^o (c, k, \delta, A_r, R, \rho)} - 1 \qquad (26)$$

We can even go a step further by looking at the impact of the different parameters on these two rates. Concerning the diameter of the well, k, the discount rate, δ, and the size of the outcrop area, A_r, the results are fairly straightforward since these parameters modify only the steady-state yield and the number of wells under optimal management. Since Δ_{Yield}^{-} and Δ_{Cost}^{+} respectively increase and decrease in r^0, we can immediately assert, by using Table 3, that the rate of yield losses, Δ_{Yield}^{-}, and the rate of additional water cost, Δ_{Cost}^{+}, both increase with the diameter of the well, k, and decrease with the discount rate, δ, and the size of the outcrop area, A_r. However, the effect of the cost per well, c, or that of the net recharge, ρR, is less obvious, since these quantities modify the yield and the number of wells in both steady states. Nevertheless, we are able to show that an increase in c reduces both rates, while the sign of the effect of an increase in the net recharge depends again on the elasticity of the inverse demand assessed at the net recharge, $\varepsilon(\rho R) = \frac{P'(\rho R)\rho R}{P(\rho R)}$.

The following proposition summarizes our discussion.

Proposition 3. *By comparing the open-access steady-state values to those of the socially optimal one, there are more wells, $n^m > n^0$, a lower yield, $r^m < r^o$, and a lower water column, $h^m < h^0$, in the outcrop area. The effects of the parameters on the rate of yield losses and on the rate of increase in the water cost are described in Table 4. The notation $+/-$ means that the effect can be either positive or negative, depending on whether the elasticity, $\varepsilon_P(w)$, of the inverse demand evaluated at the steady-state consumption, ρR, is higher or smaller than $-1/2$.*

A quick look at Table 4 may also suggest that the so-called Gisser–Sanchez effect is again at work since the rate of yield losses and the rate of additional social costs both decrease with the area, A_r. However, contrary to the Gisser–Sanchez effect, this quantity is not the size of the confined aquifer, but the size of only the recharge area, which, in the case of artesian systems, is often relatively narrow with respect to the area covered

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Table 4: Impact of the parameters on the rate of yield losses and additional social cost rate

Parameters	Yield losses, Δ_{Yield}^-	Additional water cost, Δ_{Cost}^+
Marginal cost (c)	$-$	$-$
Diameter of the drill hole (k)	$+$	$+$
Recharge area (A_r)	$-$	$-$
Discount rate (δ)	$-$	$-$
Recharge (R)	$+/-$	$+/-$
Runoff (ρ)	$+/-$	$+/-$

by the confined aquifer. This last observation suggests that the two equilibria would be very different, thus requiring public regulation.

If the objective of a policy is to limit this additional water cost, it should, on the basis of Eq. (26), refrain from providing incentives to drill new wells and instead ensure that their number does not exceed that recommended by the steady state of an optimal management policy. This means that under open access, either the well extension is limited by a standard command-and-control policy or the profitability of an addition well is reduced (see Eq. (5)) by imposing a tax per well. However, this last incentive-based policy must provide some signals related to pressure losses. Since pressure is inversely proportional to the height of the water column in the outcrop area, $h(t)$, this tax rate must decrease with $h(t)$. More precisely, we can claim the following.

Proposition 4. *A state-dependent tax per well given by* $\tau\left(h(t)\right) = \frac{c\rho R}{2\delta h(t)sA_r}$ *ensures that the steady state of open-access management of an aquifer coincides with the steady state under optimal management.*

7. Concluding remarks

This paper contributes to the literature on CPRs and externalities arising from exploitation of these resources, as well as to the literature on groundwater management. In this paper, we have analyzed the role of public policy when CPRs exhibit a low degree of rivalry in terms of consumption. We have thus introduced a specific type of CPR, artesian aquifers, whose stock remains fully saturated even when the resource is exploited. Moreover, if such an aquifer is tapped by a well, the water will naturally rise above the land surface since the water is stored under pressure. Losses of pressure will consequently alter the outflow at the surface. Economic agents therefore do not compete to appropriate part of the resource stock, but suffer from a reduction in artesian pressure as the number of wells increases. The tragedy of the commons results here from the existence of a pressure externality. We have evaluated the impact of this externality as the difference between the open-access outcome and the socially optimal solution. In particular, we have used fluid mechanics, particularly Torricelli's theorem, to introduce water pressure and water yields flowing out at the surface in a dynamic optimization model. We have shown that the number of wells depends on the level to which water will rise, which corresponds to the water

column in the recharge area. In the long run, the height of this water column is therefore lower under open access than under socially optimal management. This externality results in the long run in additional wells for the same water consumption, and hence in additional costs, thus requiring public regulation. We even provide a state-dependent tax scheme that corrects this externality at the steady state.

We have considered a very simple setting in which to isolate the pressure externality. First, it would be interesting to extend this analysis to the case of other management regimes to include rational agents and strategic externality to analyze, for instance, how water users compete for a resource that is never depleted. Second, we could also consider more complex dynamics of such aquifers by introducing either an endogenous infiltration rate or leakages. Finally, we have characterized situations only where artesian properties are held in the long run. It may be interesting to analyze situations in which these properties disappear, that is, when a confined aquifer becomes an unconfined aquifer in the long run, implying the need for pumping.

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Appendix A. Proof of Proposition 1

(i) <u>Local stability</u>

From Eq.(7), observe that the linear approximation of these dynamics arround the steady state is given by:

$$\begin{pmatrix} \dot{n} \\ \dot{h} \end{pmatrix} = \underbrace{\begin{bmatrix} \alpha P'(w^m) k^2 2gh^m & \alpha \left(P'(w^m) n^m k^2 g + P(w^m) k\sqrt{g} (2h^m)^{-1/2} \right) \\ -\frac{k\sqrt{2gh^m}}{sA_r} & -\frac{1}{sA_r} \left(n^m k\sqrt{g} (2h^m)^{-1/2} \right) \end{bmatrix}}_{=D_m} \begin{pmatrix} \dot{n} - n^m \\ \dot{h} - h^m \end{pmatrix} \quad \text{(A.1)}$$

Under our assumption, it is immediate that:

$$trace(D_m) = \alpha P'(w^m) k^2 2gh^m - \frac{1}{sA_r} \left(n^m k\sqrt{g} (2h^m)^{-1/2} \right) < 0 \quad \text{(A.2)}$$

Moreover, after some computations:

$$\det(D_m) = \frac{\alpha P(w^m) k^2 g}{sA_r} > 0 \quad \text{(A.3)}$$

We can therefore conclude that our dynamical system is locally asymptotically stable.

(ii) <u>Comparative statics at the steady state</u>

From Eqs. (8), (9), (10) and (11), we immediately obtain:

	∂c	∂k	∂R	$\partial \rho$
∂h^m	$\frac{c}{g} \left(\frac{1}{kP(w^m)} \right)^2 > 0$	$-\frac{1}{gk^3} \left(\frac{c}{P(w^m)} \right)^2 < 0$	$-\frac{\left(\frac{c}{k} \right)^2 P'(w^m)\rho}{g(P((w^m))^3} > 0$	$-\frac{\left(\frac{c}{k} \right)^2 RP'(w^m)}{g(P(w^m))^3} > 0$
∂w^m	0	0	$\rho > 0$	$R > 0$
∂r^m	$\frac{1}{P(w^m)} > 0$	0	$-\frac{c\rho P'(w^m)}{(P(\rho R))^2} > 0$	$-\frac{cRP'(\rho R)}{(P(w^m))^2} > 0$
∂n^m	$-\frac{w^m P(w^m)}{c^2} < 0$	0	$\frac{\rho}{c} (P(w^m) + w^m P'(w^m))$	$\frac{R}{c} (P(w^m) + w^m P'(w^m))$

Finally remark that $\frac{\partial n^m}{\partial R} \lesseqgtr 0 \Leftrightarrow \varepsilon_P(w^m) := \frac{w^m P'(w^m)}{P(w^m)} \lesseqgtr -1$ and $\frac{\partial n^m}{\partial \rho} \lesseqgtr 0 \Leftrightarrow \varepsilon_P(w^m) \lesseqgtr -1$.

Appendix B. Proof of Proposition 2

(i) Existence and uniqueness of the steady state

From our discussion, there exists a unique state if and only if Eq.(23), given by:

$$P(\rho R) - \frac{c}{k\sqrt{2gh}}\left(1 + \frac{\rho R}{2h\delta s A_r}\right) = 0 \tag{B.1}$$

admits a unique solution in h. By setting $x = \frac{1}{\sqrt{2h}}$, this equation becomes:

$$P(\rho R) - \left(\frac{c}{k\sqrt{g}}\right)x - \left(\frac{c\rho R}{k\delta s A_r \sqrt{g}}\right)x^3 = 0 \Leftrightarrow x^3 + \underbrace{\frac{\delta s A_r}{\rho R}}_{=p>0}x + \underbrace{\left(-\frac{k\delta s A_r \sqrt{g}P(\rho R)}{c\rho R}\right)}_{=q<0} = 0 \tag{B.2}$$

Since $4p^3 + 27q^2 > 0$, we knows from Cardano formula that this equation admits a unique solution in the reals and since $p > 0$ and $q < 0$, this solution, x^o, is strictly positive and is given by:

$$x^o = \sqrt[3]{-\frac{q^2}{2} + \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}} + \sqrt[3]{-\frac{q^2}{2} - \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}} \tag{B.3}$$

As a consequence $h^o = \frac{1}{2(x^o)^2}$ and the consistency assumption on our parameters now becomes $\frac{1}{2(x^o)^2} < h^{\max}$.

(ii) Local saddle point stability

Let us first remember that the dynamics is given by:

$$\begin{cases} \dot{\lambda} = \delta\lambda - \partial_h \mathcal{H}(w,h,\lambda)|_{w=w(h,\lambda)} \\ \dot{h} = \partial_\lambda \mathcal{H}(w,h,\lambda)|_{w=w(h,\lambda)} \end{cases} \quad \text{with } w(h,\lambda) \text{ solution to } \partial_w \mathcal{H}(w,h,\lambda) = 0 \tag{B.4}$$

It follows that the dynamics around the steady state can be approximated by:

$$\begin{pmatrix} \dot{\lambda} \\ \dot{h} \end{pmatrix} = \underbrace{\left[\begin{array}{cc} \delta - \left(\partial_{h,\lambda}^2 \mathcal{H} - \partial_{h,w}^2 \mathcal{H}\frac{\partial_{w,\lambda}^2 \mathcal{H}}{\partial_{w,w}^2 \mathcal{H}}\right) & -\left(\partial_{h,h}^2 \mathcal{H} - \partial_{h,w}^2 \mathcal{H}\frac{\partial_{w,h}^2 \mathcal{H}}{\partial_{w,w}^2 \mathcal{H}}\right) \\ -\partial_{\lambda,w}^2 \mathcal{H}\frac{\partial_{w,\lambda}^2 \mathcal{H}}{\partial_{w,w}^2 \mathcal{H}} & \partial_{\lambda,h}^2 \mathcal{H} - \partial_{\lambda,w}^2 \mathcal{H}\frac{\partial_{w,h}^2 \mathcal{H}}{\partial_{w,w}^2 \mathcal{H}} \end{array}\right]\Bigg|_{(w^o,h^o,\lambda^o)}}_{=D_o} \begin{pmatrix} \dot{\lambda} - \lambda^o \\ \dot{h} - h^o \end{pmatrix} \tag{B.5}$$

with:

$$\begin{cases} \partial_{h,\lambda}^2 \mathcal{H} = 0 & \partial_{h,w}^2 \mathcal{H} = \frac{c}{k\sqrt{g}}(2h)^{-\frac{3}{2}} & \partial_{h,h}^2 \mathcal{H} = -\frac{3cw}{k\sqrt{g}}(2h)^{-\frac{5}{2}} \\ \partial_{\lambda,w}^2 \mathcal{H} = -\frac{1}{sA_r} & \partial_{w,w}^2 \mathcal{H} = P'(w) & \partial_{\lambda,\lambda}^2 \mathcal{H} = 0 \end{cases} \tag{B.6}$$

By substitution:

$$D_o = \left[\begin{array}{cc} \delta - \frac{c(2h)^{-\frac{3}{2}}}{sA_r k\sqrt{g}P'(w)} & \frac{3cw(2h)^{-\frac{5}{2}}}{k\sqrt{g}} + \left(\frac{c(2h)^{-\frac{3}{2}}}{k\sqrt{g}}\right)^2\frac{1}{P'(w)} \\ -\left(\frac{1}{sA_r}\right)^2\frac{1}{P'(w)} & \frac{c(2h)^{-\frac{3}{2}}}{sA_r k\sqrt{g}P'(w)} \end{array}\right] \tag{B.7}$$

It follows that $trace(D_o) = \delta > 0$ and, after some simplifications, that:

$$\det(D_o) = \left(\frac{c(2h)^{-\frac{3}{2}}}{sA_r k\sqrt{g}}\right)\left(\delta + \frac{3w}{2sA_r h}\right)\frac{1}{P'(w)} < 0 \tag{B.8}$$

Finally, since $trace(D_o) > 0$ and $\det(D_o) < 0$, we know that our steady state is a locally stable saddle point.

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Appendix C. Proof of the sensitivity analysis of section 5

Let us introduce

$$\phi\left(h, c, k, \delta, A_r, R, \rho\right) = P\left(\rho R\right) - \frac{c}{k\sqrt{2gh}}\left(1 + \frac{\rho R}{2h\delta s A_r}\right) \tag{C.1}$$

Impact on the water column level

Let us first observe that:

$$\frac{\partial\phi}{\partial h} = \frac{c}{k\sqrt{g}}\left(2h\right)^{-3/2}\left(1 + \frac{3\rho R}{2h\delta s A_r}\right) > 0 \tag{C.2}$$

This means that for all $\theta \in \{c, k, \delta, A_r, R, \rho\}$, $sign\left(\frac{\partial h}{\partial\theta}\right) = sign\left(-\frac{\partial\phi}{\partial\theta}\Big/\frac{\partial\phi}{\partial h}\right) = -sign\{\frac{\partial\phi}{\partial\theta}\}$. Moreover by computation:

$$\begin{cases} \frac{\partial\phi}{\partial c} = -\frac{1}{k\sqrt{2gh}}\left(1 + \frac{\rho R}{2h\delta s A_r}\right) < 0 & \frac{\partial\phi}{\partial A_r} = \frac{c\rho R}{(2h)^{3/2}k\sqrt{g}\delta s(A_r)^2} > 0 \\ \frac{\partial\phi}{\partial k} = \frac{c}{k^2\sqrt{2gh}}\left(1 + \frac{\rho R}{2h\delta s A_r}\right) > 0 & \frac{\partial\phi}{\partial R} = \rho P'\left(w\right) - \frac{c}{k\sqrt{2gh}}\left(\frac{\rho}{2h\delta s A_r}\right) < 0 \\ \frac{\partial\phi}{\partial\delta} = \frac{c}{k\sqrt{2gh}}\left(\frac{\rho R}{2h\delta^2 s A_r}\right) > 0 & \frac{\partial\phi}{\partial\rho} = RP'\left(w\right) - \frac{c}{k\sqrt{2gh}}\left(\frac{R}{2h\delta s A_r}\right) < 0 \end{cases} \tag{C.3}$$

hence

$$\begin{cases} \frac{\partial h^o}{\partial c} > 0 & \frac{\partial h^o}{\partial k} < 0 & \frac{\partial h^o}{\partial\delta} < 0 & \frac{\partial h^o}{\partial A_r} < 0 & \frac{\partial h^o}{\partial R} > 0 & \frac{\partial h^o}{\partial\rho} > 0 \end{cases} \tag{C.4}$$

Impact on the water consumption

At the steady state $w^o = \rho R$, thus all $\theta \neq R, \rho \ \frac{\partial w^o}{\partial\theta} = 0$. Moreover, $\frac{\partial w^o}{\partial R} = \rho > 0$ and $\frac{\partial w^o}{\partial\rho} = R > 0$

Impact on the yield of an artesian well

At the steady state $r^o = k\sqrt{2gh^o}$. It follows that for all $\theta \neq k$, $\frac{\partial r^o}{\partial\theta} = \frac{kg}{\sqrt{2gh^o}} \cdot \frac{\partial h^o}{\partial\theta}$. From Eq. (C.4), we immediately obtain:

$$\begin{cases} \frac{\partial h^o}{\partial c} > 0 & \frac{\partial h^o}{\partial\delta} < 0 & \frac{\partial h^o}{\partial A_r} < 0 & \frac{\partial h^o}{\partial R} > 0 & \frac{\partial h^o}{\partial\rho} > 0 \end{cases} \tag{C.5}$$

It remains to study $\frac{\partial r^o}{\partial k} = \frac{kg}{\sqrt{2gh^o}} \cdot \frac{\partial h^o}{\partial k} + \sqrt{2gh^o}$ with $\frac{\partial h^o}{\partial k} = -\frac{\partial\phi}{\partial k}\Big/\frac{\partial\phi}{\partial h}$ and $\frac{\partial\phi}{\partial h} > 0$ (see Eq.(C.2)). We can therefore say that:

$$sign\left\{\frac{\partial r^o}{\partial k}\right\} = signe\left\{\underbrace{\sqrt{\frac{g}{2h^o}}}_{>0}\right\} \times sign\left\{k\frac{\partial h^o}{\partial k} + 2h^o\right\} = sign\left\{\underbrace{-k\frac{\partial\phi}{\partial k} + 2h^o\frac{\partial\phi}{\partial h}}_{=D}\right\} \tag{C.6}$$

By using Eqs.(C.2) and (C.3), we obtain:

$$D = -\frac{c}{k\sqrt{2gh^o}}\left(1 + \frac{\rho R}{2h^o\delta s A_r}\right) + \frac{c}{k\sqrt{2gh^o}}\left(1 + \frac{3\rho R}{2h^o\delta s A_r}\right) = \frac{c}{k\sqrt{2gh^o}}\frac{\rho R}{h^o\delta s A_r} > 0 \tag{C.7}$$

and conclude that $\frac{\partial r^o}{\partial k} > 0$.

Impact on the number of artesian wells

At the steady state $n^o = \frac{\rho R}{k\sqrt{2gh^o}}$. It follows that $\forall\theta \neq R, \rho, k$, $\frac{\partial n^o}{\partial\theta} = -\frac{\rho R}{k\sqrt{g}}\left(2h^o\right)^{-3/2}\frac{\partial h^o}{\partial\theta}$. From Eq. (C.4), we immediately obtain:

$$\begin{cases} \frac{\partial n^o}{\partial c} < 0 & \frac{\partial n^o}{\partial\delta} > 0 & \frac{\partial n^o}{\partial A_r} > 0 \end{cases} \tag{C.8}$$

Let us now concentrate on $\frac{\partial n^o}{\partial k}$. By computation:

$$\frac{\partial n^o}{\partial k} = -\frac{\rho R}{k\sqrt{g}}\left(2h^o\right)^{-3/2}\frac{\partial h^o}{\partial k} - \frac{\rho R}{k^2\sqrt{2gh^o}} \tag{C.9}$$

It follows that:

$$sign\left(\frac{\partial n^o}{\partial k}\right) = -sign\left(\frac{\partial h^o}{\partial k} + \frac{2h^o}{k}\right) = -sign\left(-\frac{\partial \phi}{\partial k} + \frac{2h^o}{k}\frac{\partial \phi}{\partial h}\right) \tag{C.10}$$

Moreover by Eqs.(C.2) and (C.3)

$$-\frac{\partial \phi}{\partial k} + \frac{2h^o}{k}\frac{\partial \phi}{\partial h} = -\frac{c}{k^2\sqrt{2gh^o}}\left(1 + \frac{\rho R}{2h^o\delta sA_r}\right) + \frac{c}{k^2\sqrt{2gh^o}}\left(1 + \frac{3\rho R}{2h^o\delta sA_r}\right) = \frac{c}{k^2\sqrt{2gh^o}}\left(\frac{\rho R}{h^o\delta sA_r}\right) > 0 \tag{C.11}$$

We can therefore say that $\frac{\partial n^0}{\partial k} < 0$.

In order to show that $\frac{\partial n^0}{\partial R}$ and $\frac{\partial n^0}{\partial \rho}$ are unsigned, let us introduce the following example. We set $P(w) = w^{-a}$ with $\alpha > 1$, choose the parameters $\{c, k, \delta, A_r\}$ such that $\frac{c}{k\sqrt{g}} = \frac{1}{2}$ and $\delta sA_r = 1$. Under these assumptions, we know from Eq.(C.1), that h^o solves

$$\phi_1(h; \rho, R) = (\rho R)^{-\alpha} - \frac{1}{2}(2h)^{-\frac{1}{2}} - \frac{\rho R}{2}(2h)^{-\frac{3}{2}} = 0 \tag{C.12}$$

Moreover, if we take a particular configuration of ρ and R such that $\rho R = 1$, it is easy to check that $h^o = \frac{1}{2}$ solves Eq.(C.12). If we now change R, the change of h^0, in the neighborhood of this solution, is given by:

$$\frac{\partial h^o}{\partial R} = -\frac{\partial \phi_1}{\partial R}\Big/\frac{\partial \phi_1}{\partial h}\Big|_{h=\frac{1}{2},\rho R=1} = -\frac{-\alpha\rho - \frac{\rho}{2}}{\frac{1}{2}+\frac{3}{2}} = \rho\frac{2\alpha+1}{4} \tag{C.13}$$

If we now look at the change in the number of wells (see Eq.(22)), we obtain:

$$\frac{\partial n^o}{\partial R} = \left(\frac{\rho}{k\sqrt{2gh}} - \frac{\rho R}{k\sqrt{g}}(2h)^{-3/2}\frac{\partial h^o}{\partial R}\right)\Big|_{h=\frac{1}{2},\rho R=1} = \left(\frac{\rho}{k\sqrt{g}} - \frac{1}{k\sqrt{g}}\rho\frac{2\alpha+1}{4}\right) = \frac{\rho}{4k\sqrt{g}}(3-2\alpha) \tag{C.14}$$

This clearly shows that $\frac{\partial n^o}{\partial R}$ can be either positive or negative, depending, in this example, whenever the elasticity of the demand $\alpha \lesseqgtr \frac{3}{2}$. Finally, the reader also observes that the result is identical for $\frac{\partial n^0}{\partial R}$ as long as we replace ρ by R in Eq.C.14.

Appendix D. Proof of Proposition 3

The ranking of the solution

In order to verify that $h^0 > h^m$, let us remember (i) that h^0 solves $\phi(h^0) = 0$ (see Eq.(C.1)) with $\phi'(h) > 0$ (see Eq.(C.2)) and that (ii) $h^m = \frac{1}{2g}\left(\frac{c}{kP(\rho R)}\right)^2$ (see Eq.(9)). Now observe that:

$$\phi(h^m) = P(\rho R) - \frac{c}{k\sqrt{2gh^m}}\left(1 + \frac{\rho R}{2h^m\delta sA_r}\right) = -\frac{gk^2\rho R(P(\rho R))^3}{c^2\delta sA_r} < 0 \tag{D.1}$$

Since $\phi'(h) > 0$, we deduce that $h^0 > h^m$. From Torricelli formula (see Eq.(??)), we also know that $r = k\sqrt{2gh}$, so if $h^0 > h^m$ we must also have $r^0 > r^m$. Finally remember that at the steady state, the number of wells is given by $n = \frac{\rho R}{r}$. So if $r^0 > r^m$, we must have $n^0 < n^m$.

The rate of yield losses

Let us now concentrate on $\frac{\partial \Delta_{Yield}^-}{\partial \theta}$ with $\theta \in \{c, k, A_r, \delta, R, \rho\}$. This computation requires some preliminary observations. First, by Torricelli formula (see Eq.(??)), Eq.(B.1) can be written as:

$$P(\rho R) - \frac{c}{r^0} - \frac{c}{(r^0)^2}\frac{k^2 g\rho R}{\delta sA_r} = 0 \tag{D.2}$$

Moreover, we know from Eq.(10) that $r^m = \frac{c}{P(\rho R)}$. By inserting this quantity in Eq.(D.2), we get:

$$P(\rho R) - c\left(\frac{P(\rho R)}{c}\right)\left(\frac{r^m}{r^0}\right) - \frac{ck^2 g\rho R}{\delta sA_r}\left(\frac{P(\rho R)}{c}\right)^3\left(\frac{r^m}{r^0}\right)^3 = 0 \tag{D.3}$$

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or after simplifications:

$$\psi\left(\left(\tfrac{r^m}{r^0}\right), X\right) = 1 - \left(\tfrac{r^m}{r^0}\right) - \underbrace{\tfrac{k^2 g \rho R (P(\rho R))^2}{c^2 \delta s A_r}}_{=X>0} \left(\tfrac{r^m}{r^0}\right)^3 \tag{D.4}$$

Moreover we observe that $\frac{\partial \psi}{\partial (r^m/r^0)} = -1 - 2X \left(\tfrac{r^m}{r^0}\right)^2 < 0$ and $\frac{\partial \psi}{\partial X} = -\left(\tfrac{r^m}{r^0}\right)^3 < 0$.

Now let us come back to $\frac{\partial \Delta_{Yield}^-}{\partial \theta}$. From our previous results and the definition of $\Delta_{Yield}^- = 1 - \left(\tfrac{r^m}{r^0}\right)$, we deduce that:

$$sign\left(\frac{\partial \Delta_{Yield}^-}{\partial \theta}\right) = -sign\left(\frac{\partial\left(r^m/r^0\right)}{\partial X}\right) sign\left(\frac{\partial X}{\partial \theta}\right) \tag{D.5}$$

$$= -sign\left(-\tfrac{\partial \psi}{\partial (r^m/r^0)}\Big/\tfrac{\partial \psi}{\partial X}\right) sign\left(\frac{\partial X}{\partial \theta}\right) = sign\left(\frac{\partial X}{\partial \theta}\right) \tag{D.6}$$

and by computation, we easily check that:

$$\begin{cases} \frac{\partial X}{\partial c} = -\frac{2k^2 g \rho R (P(\rho R))^2}{c^3 \delta s A_r} < 0 & \frac{\partial X}{\partial k} = \frac{2k g \rho R (P(\rho R))^2}{c^2 \delta s A_r} > 0 \\ \frac{\partial X}{\partial A_r} = -\frac{k^2 g \rho R (P(\rho R))^2}{c^2 \delta s A_r^2} < 0 & \frac{\partial X}{\partial \delta} = -\frac{k^2 g \rho R (P(\rho R))^2}{c^2 \delta^2 s A_r} < 0 \end{cases} \tag{D.7}$$

Up to now, it follows that :

$$\begin{cases} \frac{\partial \Delta_{Yield}^-}{\partial c} < 0 & \frac{\partial \Delta_{Yield}^-}{\partial k} > 0 & \frac{\partial \Delta_{Yield}^-}{\partial A_r} < 0 & \frac{\partial \Delta_{Yield}^-}{\partial \delta} < 0 \end{cases} \tag{D.8}$$

Concerning $\frac{\partial X}{\partial R}$ and $\frac{\partial X}{\partial \rho}$, let us set $w = \rho R$. It follows that:

$$\frac{\partial X}{\partial w} = \frac{k^2 g}{c^2 \delta s A_r}\left(P(w)^2 + 2w P(w) P'(w)\right) = \frac{k^2 g P(w)^2}{c^2 \delta s A_r}\left(1 + 2\varepsilon_P(w)\right) \tag{D.9}$$

We can therefore conclude that $\frac{\partial X}{\partial R} = \rho \left.\frac{\partial X}{\partial w}\right|_{w=\rho R} \gtreqless 0$ and $\frac{\partial X}{\partial \rho} = R \left.\frac{\partial X}{\partial w}\right|_{w=\rho R} \gtreqless 0$ if and only if $\varepsilon_P(w) = \frac{w P'(w)}{P(w)} \gtreqless -\frac{1}{2}$ evaluated at $w = \rho R$ or equivalently that

$$\frac{\partial \Delta_{Yield}^-}{\partial R} \gtreqless 0 \text{ and } \frac{\partial \Delta_{Yield}^-}{\partial \rho} \gtreqless 0 \text{ if and only if } \varepsilon_P(w) \gtreqless -\frac{1}{2} \tag{D.10}$$

The rate of increase of the water disposal cost

Since the number of wells is in both cases given by the net recharge ρR divided by the yield and since $\Delta_{Yield}^- = 1 - \left(\tfrac{r^m}{r^0}\right)$, we can say that:

$$\Delta_{Cost}^+ = \frac{n^m}{n^o} - 1 = \frac{\frac{\rho R}{r^m}}{\frac{\rho R}{r^0}} - 1 = \frac{r^o}{r^m} - 1 = \frac{1}{1 - \Delta_{Yield}^-} - 1 = \frac{\Delta_{Yield}^-}{1 - \Delta_{Yield}^-} \tag{D.11}$$

If we now observe that $\frac{\partial \Delta_{Cost}^+}{\partial \Delta_{Yield}^-} > 0$, we can immediately say that $sign\left(\frac{\partial \Delta_{Cost}^+}{\partial \theta}\right) = sign\left(\frac{\partial \Delta_{Yield}^-}{\partial \theta}\right)$ for all $\theta \in \{c, k, A_r, \delta, R, \rho\}$

Appendix E. Proof of proposition 4

By applying a tax of $\tau(h(t)) = \frac{c \rho R}{2 \delta h(t) s A_r}$ per well, the instantaneous profit of a well owner under free access (see Eq.(5)) becomes:

$$\Pi = p(t) r(t) - c - \tau(h(t)) = p(t) r(t) - c\left(1 + \frac{\rho R}{2 \delta h(t) s A_r}\right) \tag{E.1}$$

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It follows that the dynamics of the free access system (see Eq.(7)) writes:

$$\begin{cases} \dot{n}(t) = \alpha \left(P\left(n(t)k\sqrt{2gh(t)} \right) k\sqrt{2gh(t)} - c\left(1 + \frac{\rho R}{2\delta h(t)sA_r} \right) \right) \\ sA_r \dot{h}(t) = \rho R - n(t)k\sqrt{2gh(t)} \end{cases} \tag{E.2}$$

If we now concentrate on the new open access steady state, we obtain by the second equation that n^m given by:

$$n^m = \frac{\rho R}{k\sqrt{2gh^m}} \tag{E.3}$$

with h^m which solves

$$P\left(n^m k\sqrt{2gh^m} \right) k\sqrt{2gh^m} - c\left(1 + \frac{\rho R}{2\delta h^m sA_r} \right) = 0$$

$$\Longleftrightarrow P\left(\rho R \right) - \frac{c}{k\sqrt{2gh^m}} \left(1 + \frac{\rho R}{2\delta h^m sA_r} \right) = 0 \tag{E.4}$$

But this last equation in the same as the one which define the optimal management water column (see Eq.(23)). It follows that under this tax rate the open access steady state coincides with the optimal management steady state.