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Knowledge-Based Structural Change

Kevin Genna
Christian Ghiglino
Kazuo Nishimura
Alain Venditti

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Kevin GENNA

Aix-Marseille Univ., CNRS, AMSE, Marseille, France

Christian GHIGLINO

Department of Economics, University of Essex, UK

Kazuo NISHIMURA

RIEB, Kobe University, Japan

and

Alain VENDITTI

Aix-Marseille Univ., CNRS, AMSE, Marseille
& EDHEC Business School, France

Abstract: *How will structural change unfold beyond the rise of services? Motivated by the observed dynamics within the service sector we propose a model of structural change in which productivity is endogenous and output is produced with two intermediate substitutable capital goods. In the progressive sector the accumulation of knowledge leads to an unbounded increase in TFP, as sector becoming asymptotically dominant. We are then able to recover the increasing shares of workers, the increasing real and nominal shares of the output observed in progressive service and IT sectors in the US. Interestingly, the economy follows a growth path converging to a particular level of wealth that depends on the initial price of capital and knowledge. As a consequence, countries with the same fundamentals but lower initial wealth will be characterized by lower asymptotic wealth.*

Keywords: *Two-sector model, technological knowledge, constant elasticity of substitution, non-balanced endogenous growth, structural change, Kaldor and Kuznets facts*

Journal of Economic Literature Classification Numbers: C62, E32, O41.

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1 Introduction

The historical coexistence of structural transformation and aggregate balanced growth is a remarkable feature of economic development, the so-called Kaldor [28] and Kuznets [30] facts. There is now a large literature providing mechanisms that successfully explain the big picture of structural transformation over the past two centuries. However, recent literature questions whether structural change will continue with the same pattern despite the dominance of services and IT industries.[1] To substantiate this argument is the data on the dynamics of the different sub-sectors in service and IT industries which fails to show that stagnant sectors absorb growing quantities of factor inputs. A plausible explanation is that services are to some extend substitutable in their intermediate use or final consumption.

In this paper we want to shed light on the patterns of structural change when intermediate sectors produce goods that are substitutable inputs in a final output. The structural transformation literature points to two broad mechanisms able to generate large economy wide transformations. On the demand side, when preferences are not of the Gorman type and relative prices are constant, the growth of income affects differently the consumption of the various goods.[2] On the supply side, differences in sectoral productivity growth rates and in the degree of capital-labor substitutability induce variations in the relative prices and relocation of factors.[3] It is likely that both mechanisms operate in the real world, a route pursued in some recent literature (eg. Comin *et al.* [19]).

In this paper we focus on the role of supply. The literature has shown that differences in sectoral productivity growth rates, in the degree of capital-labor substitutability and in factor intensities determines the characteristics of the equilibrium path. The main take is that to capture the historical trend, differences in productivity growth and factor intensity should be accompanied by complementarity in intermediate goods in the production of final goods or in consumption. Indeed, non-substitution is the main force leading to pouring factors into stagnant sectors.

Our model allows for substitution, in which case productivity growth differences take centre stage.[4] Focusing on productivity growth requires further attention to the underlying economic process. We take note that the empirical evidence on the effect of years of schooling on productivity is not conclusive. For this reason, rather than characterizing and aggregating sectors according to their skilled intensity we aggregate sectors according to their knowledge

[1] See e.g. León-Ledesma and Moro [31] and Sen [41].

[2] Examples of this literature are Alonso-Carrera and Raurich [7], Boppart [12], Buera and Kaboski [13, 14], Echevarria [22], Foellmi and Zweimueller [23] and Kongsamut *et al.* [29]

[3] See for instance Acemoglu and Guerrieri [4], Alvarez-Cuadrado *et al.* [8], Caselli and Coleman [18], Ngai and Pissarides [38]) or as in Buera *et al.* [15] and the survey of Herrendorf *et al.* [27].

[4] As we will explore the role of heterogenous sectoral productivity growth, progress can be seen as a type of sectoral human capital accumulation but not captured by years of schooling.

intensity, revealed by their productivity growth.[5]

In Section 2 we provide empirical evidence that can be summarized as follows. GDP growth in developed economies is accompanied by: 1) a decrease in the price of the progressive sectors, 2) a rise in the share of value added of these sectors, 3) a rise in the number of workers working in the progressive sectors, and 4) path dependence leading to conditional convergence to non-balanced growth paths characterized by different levels.

We now use these stylized facts of economic development to motivate the structure of our model. We adopt a two intermediate sector model *à la* Acemoglu and Guerrieri [2]. The mechanism relies on relative price effects resulting from differential productivity growth across sectors. Each sector uses capital and labour. Differential growth is endogenous and results from individual choices, similar to specific knowledge accumulation in Lucas [23]. Agents working in the "progressive" sector need to accumulate technological knowledge and thus split their individual unit of time between accumulation of technological knowledge (research) and work. We assume, as in Lucas [23] and Romer [28], that the equation governing knowledge accumulation is linear. By contrast, agents working in the second sector do not need to accumulate knowledge and therefore devote all their individual unit of time to work. While the total population is growing at an exogenously given rate, the number of agents working in each sector is endogenously determined over time. The final good, which is consumed, is produced through a CES technology with a constant elasticity of substitution.

The analysis of our model proceeds in several steps. First, we define a workable equilibrium concept. As in Lucas, knowledge accumulation is an externality to the firm, and we consider the planner's solution in which this externality is internalized. We then provide a detailed characterization of the asymptotic NBGP and of the transitional path converging toward this NBGP. A numerical calibrated example provides some further insights into the transition dynamics and its empirical plausibility.

The first finding is that the pattern of labour and capital reallocations along the equilibrium path dramatically depends on whether the two intermediate sectors are complements or substitutes. We show that when inputs are substitutes, technological knowledge ensures that output growth is larger within the "knowledge-intensive" sector which becomes dominant in the long run. The long run growth rate of the final good sector is determined by the growth rate of the "knowledge-intensive" sector and there is capital deepening in this sector. As a result, in the long run the real and nominal shares of the "knowledge-intensive" sector are constant while they are decreasing for the second sector. Along the transition, the real and nominal shares of the "knowledge-intensive" sector are therefore increasing while the real and nominal shares of the second sector are decreasing. Similarly, the relative prices of the knowledge intensive and lagging sectors are respectively decreasing and increasing.

[5] Herrendorf *et al.* [27] adopts a similar partition of sectors.

These conclusions therefore fit with the pattern of labour and capital shares and value added presented in Section 2 below.

When inputs are complements, the productivity lag between the two intermediate sectors has to be compensated by larger capital and labor allocations in the second sector implying that in the long run the growth rate of the final good sector is now determined by the growth rate of the second sector. However, because knowledge accumulation acts as a TFP driver, the "knowledge-intensive" sector again becomes dominant in the long run and is characterized by the largest growth rate, but here there is no capital deepening. These properties imply that in the long run, the real and nominal shares of the "knowledge-intensive" sector are respectively increasing and decreasing while they are both constant for the second sector. These patterns are not compatible with the empirical facts of structural change we want to model here. A similar conclusion holds for the case with unitary elasticity of substitution.

The analysis also reveals the path dependence of economic development. First, we detrend all the variables by their respective endogenous non-balanced growth rates, and prove that there exists a manifold of steady states parameterized by the initial value of the price of knowledge. Each steady states is saddle-point stable and is associated with a set of unique non-balanced growth rates, which depend on the initial value of capital and knowledge. Thus for each initial condition the economy will follow a particular growth path converging to a particular level of wealth. As a consequence, countries with the same fundamentals but lower initial wealth will be characterized by lower asymptotic wealth. In the case with inputs substitutability, the long run values of knowledge and human capital depend on the initial price of knowledge and long run inequality concerns both stocks. Contrary to Acemoglu and Guerrieri [4] but like Lucas [32], we obtain the existence of non-convergence across countries in a framework with structural change. Again, only the substitutable case appears to be in line with data that shows heterogeneity of physical capital and knowledge across countries.

We finally provide a numerical illustration in the substitutable case to characterize the transitional dynamics of the main variables. Our aim is to test whether our model is able to replicate qualitatively and possibly quantitatively the data and stylized facts described in Section 2. Our calibration generates increases of the capital and labor shares and of the real and nominal shares in GDP of the knowledge intensive sector that are qualitatively consistent with the empirical evidence provided in Section 2. Similarly, we obtain decreasing and increasing relative prices of the knowledge intensive and lagging sectors respectively as shown in Section 2.

The paper is organized as follows. In Section 2 we present empirical evidences that support our main theoretical results. In Section 3 we discuss the comparisons between our formulation and the recent literature. Section 4 presents the model. Section 5 characterizes the intertemporal equilibrium. Section 6 shows that our model generates non-balanced growth and structural change consistent with Kaldor and Kuznets facts. Section 7 establishes the existence of a manifold of steady states of the stationarized dynamical system and provides a local stability analysis. Section 8 contains our numerical illustration. Section 9 presents conclusions and the Appendix contains all the proofs.

2 Empirical evidence

2.1 Labour, capital and value added

Before to set up our model, we explore the patterns revealed by the data inside the service economy, when sectors are ranked according to their knowledge intensity. We follow the intuition of Duernecker *et al.* [21] by aggregating all services into two broad categories, progressive and stagnant services. However, our classification is slightly different as we refer to knowledge accumulation instead of productivity growth. Using the EUKLEMS 2019 database,[6] we look at the evolution of the labour compensation per worker, corrected by capital accumulation, as a way to discriminate between knowledge intensive and non-knowledge intensive sectors. The idea is simple: we consider that if workers exhibit a higher growth of compensation per capita than average, which is not explained by a capital deepening mechanism, they are accumulating knowledge and becoming more productive. Sub-sectors that perform better than the average service sector are ranked as progressive, the other as stagnant (see Appendix 10.1 for the precise explanation of sectors' ranking).

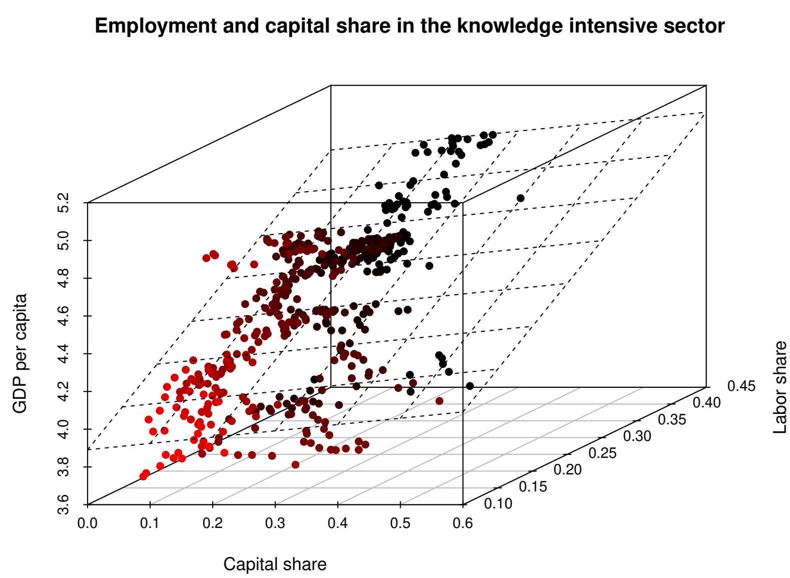


Figure 1: Labour and capital shares of progressive sectors

[6]This database covers the period 1995-2017 for numerous European countries, Japan and the US, on a yearly basis

This classification led us to observe interesting stylized facts regarding the structural change mechanism. The first one, described in figure 1, shows that the share of labor and capital in the progressive sector is increasing along the development process. We use a 3D representation to highlight the correlation between both capital share, labour share and GDP per capita. We provide the same evidence for the share of value added in the progressive sector on figure 2.

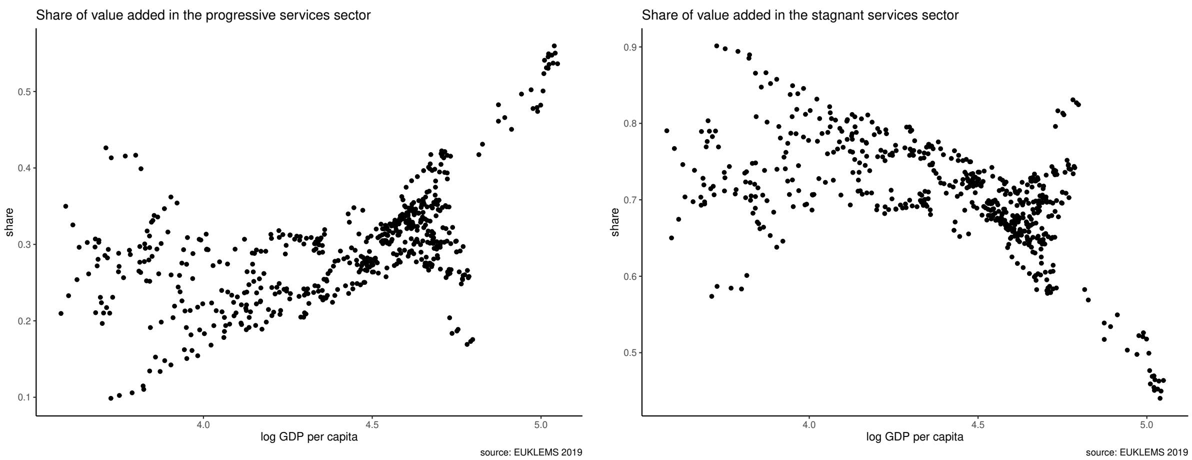


Figure 2: Share of value added in the progressive and stagnant sectors

Within the services economy it seems that the progressive, or knowledge intensive, sector is attracting labour and capital as long as a country is developing. These findings are in line with Duernecker *et al.* [21] but it is yet too soon to conclude on the presence of substitutability between progressive and stagnant services. For this purpose we look at figure 2 that captures the relationship between log GDP per capita and the share of value added in progressive sector (left panel) and in the stagnant sector (right panel). Finally we observe that both capital, labour and value added are accumulating more rapidly in the progressive services sector, along the development process. These findings let us consider that the stagnant and the progressive sectors are gross substitutes, as it was already shown by Duernecker *et al.* [21] even if our classification slightly differs. We also believe it provides evidence that our ranking captures increases in specific human capital of workers, as modeled by Lucas [32]. As our ranking corrects for the presence of capital deepening, the increase of compensation per workers is only due to workers' productivity and to their human capital.

Lastly, looking at figure 2, we are unable to reject the presence of a U-shape representation of our data. Therefore investigation of historical data for one country might be useful to depict the presence of an eventual U-shape in the progressive services sector. Our analysis being based on data from the last 25 years we do not capture previous trends and we remain agnostic on the behavior of the data when log GDP per capita is below 4, considering that the trend is always increasing.

2.2 Prices

A second interesting fact concerns relative prices. Using the same categories than for labor and capital shares, we construct the price index for the progressive sector by weighting each category, according to their share in total value added, and aggregating them into one index for the sector. Price index of one category affects the global price of the sector at the level of its importance for value added. Using the same procedure for the lagging sector and dividing by the global price index, we obtain the relative price of both progressive and stagnant sectors. However, prices in the EUKLEMS 2019 are given in 2010 price indices, meaning that we do not have the same basis to compare prices between them, a basic plot of prices according to GDP per capita give figure 3 where the trends are parallel, due to the lack of harmonization between countries.

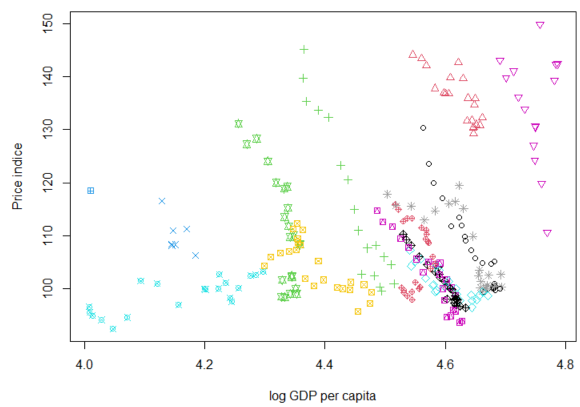


Figure 3: Relative prices of knowledge intensive sectors for many countries

To get rid of this harmonization problem we decided to make a projection of the linear regression of prices according to log GDP per capita, with a country fixed effect to capture variation between countries. Results are plotted in figure 4. We observe that the relative price of the progressive sector is decreasing while the relative price of the stagnant sector is increasing.

These projections have been obtained using the "predictoreffect" command in R which derives also the confidence interval of the relationship. While the decreasing trend for prices is always decreasing with quite a wide range, the increase of the relative price of the stagnant sector exhibits a narrower interval. Accumulation of knowledge in the progressive sector has lead to a decrease of its relative price, comparing to the stagnant sector.

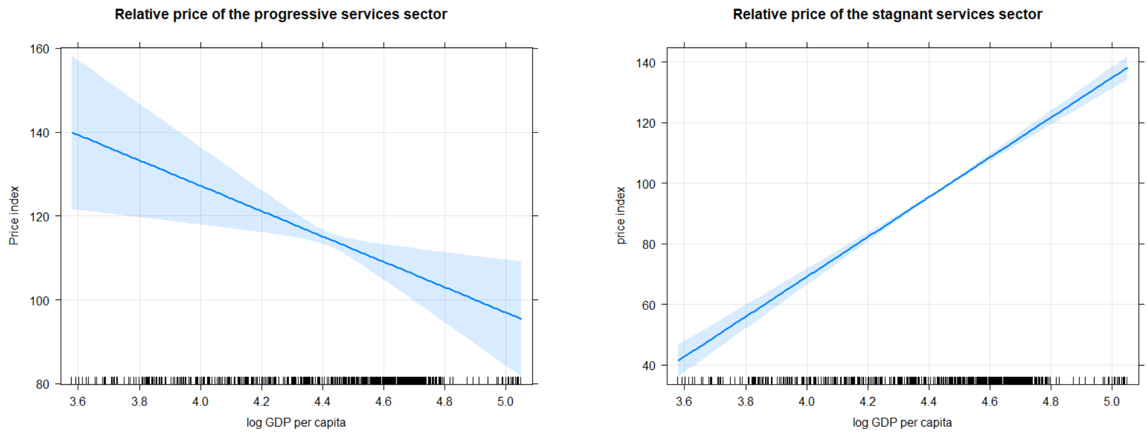


Figure 4: Relative prices of the knowledge intensive sector

2.3 Multiplicity of trends

The last stylized facts we obtain using this classification and EUKLEMS 2019 data are appealing. We decided to reverse the x and y axis of the previous graphs and groups countries according to their income group: low, middle, high.[7] This transformation is useful to see if we obtain the same level of development for the same share of labour, capital or value added in the progressive sector. Results are obtained in figure 5.

We observe that for the same share of workers, capital or value added in the knowledge intensive sector, a country might not exhibit the same level of development. We also observe that there is no sharp evidence of any convergence process, especially between high and low income groups. We believe this observation is linked to Lucas [32], for which endogenous accumulation of knowledge lead countries to the same growth rate but not to the same level of wealth. With these 3 graphs we have some evidences of a multiplicity of equilibrium according to the income level of the country. It seems that accumulating workers and capital in the progressive services sector has a positive impact on GDP per capita but it does not help to correct past inequalities, especially between low and high income groups.

Such results advocate for the use of an endogenous growth model in order to reproduce this multiplicity of equilibrium property present in the seminal paper of Lucas [32]. These observations has led us to consider the interest of an endogenous growth model to study structural change.

[7] The low income group contains Montenegro, Bulgaria, Romania, Latvia, Slovakia, Lithuania, Hungary and Slovenia. The middle income group is composed by Portugal, Poland, Czech Republic, Greece and Estonia. The other countries available in the EUKLEMS data (Except Cyprus) are composing the high income group.

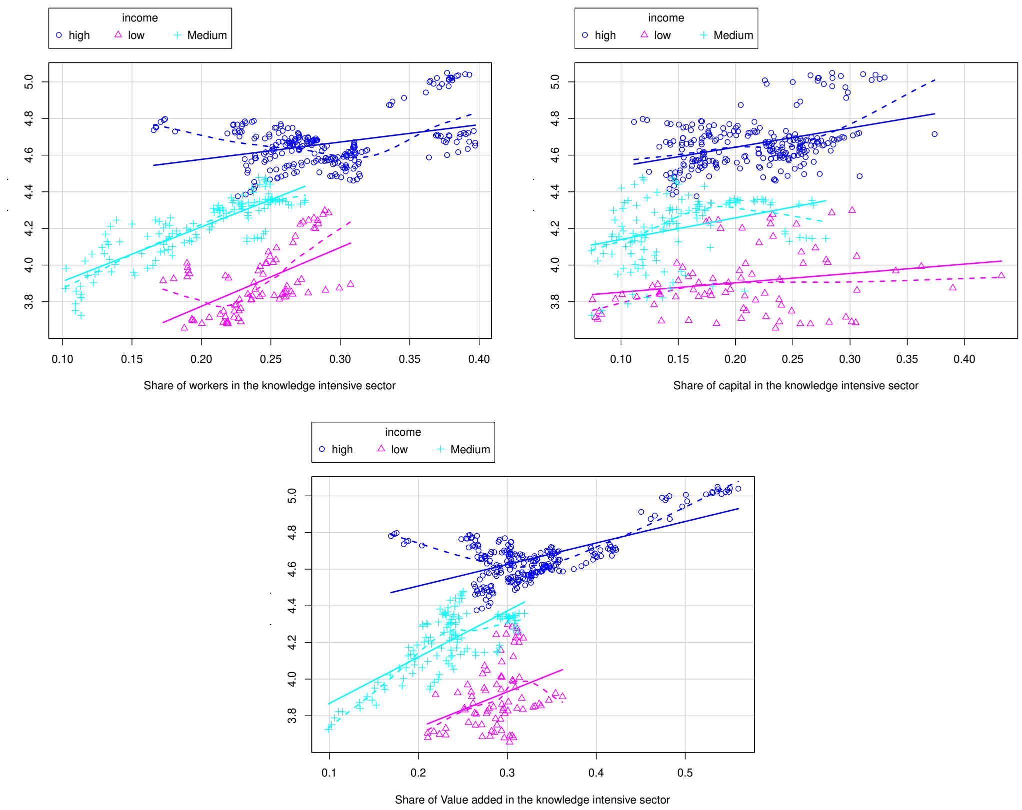


Figure 5: Development level according to the share of workers, capital and value added in the progressive sector

3 Comparisons with the recent literature

We may now review the connection of our model with the most recent literature. Acemoglu and Guerrieri [4] focus on the role of differences in factor intensity in an environment with modest differences in technological progress so that capital deepening is the main cause for changes in relative price. Their dynamics depend on the "labour augmented rate of technological progress". When intermediate goods are complements, the change in relative prices due to capital deepening is more than proportional with the change in quantities and this encourages more of the factors to be allocated towards the sector with small adjusted technological progress. Their calibrated exercise requires that capital intensive sector has a slightly higher productivity growth rate, still allowing the other sector to become dominant and determine the growth rate.

Acemoglu and Guerrieri [4] also consider the case of substitutable goods, formalised by Assumption 2, ii). However, they assume that the sector with highest labour intensity has a much larger productivity growth, so that this sector has the largest augmented rate of technological progress. Therefore, the dominant sector has also in this case high labour intensity.

Still with substitutable goods, we rather focus on the case in which the sector intensive in capital has also the largest productivity growth, a case excluded from their Assumption 2. Indeed, this allows us to capture well progressive services.

Acemoglu and Guerrieri [4] do address the question whether this outcome is driven by the exogenously fixed difference in technological progress. In the endogenous growth version of their model ([3]) they find that technological change tends to offset the non-balanced nature of economic growth. In particular, technological change is stronger in the sector that is growing less which is the labour intensive sector. However, even on the long run this sector grows more slowly than the more capital-intensive and endogenous growth is non-balanced. In fact, endogenous growth does not restore balance between the two sectors as long as capital intensity differences between the two sectors remain. Their analysis is weakened by the failure to prove dynamic stability of their equilibrium.

Another related contribution is Ngai and Pissarides [38] who study a more standard multisector model of growth with differences in TFP growth rates across sectors. They give sufficient conditions on utility and production for the coexistence of structural change, characterized by sectoral labor reallocation and balanced aggregate growth. The sectoral employment changes are consistent with the historical trend of structural change,[8] if substitutability between the final goods produced by each sector is low. Note that, balanced aggregate growth requires, in addition, a logarithmic intertemporal utility function. Along the balanced growth path, labor employed in the production of consumption goods gradually moves to the sector with the lowest TFP growth rate, until in the limit it is the only sector with nontrivial employment of this kind. Underlying the balanced aggregate growth there is a shift of employment away from sectors with a high rate of technological progress toward sectors with low growth, and eventually, in the limit, all employment converges to only two sectors, the sector producing capital goods and the sector with the lowest rate of productivity growth.

Ngai and Pissaridis [38] state that the key requirement for these results is low substitutability between final goods. In this case, their model also predicts that labor would move from the low stagnant sector to the progressive sector. The sectors that produce these goods also retain some employment in the limit, which is used to produce the intermediate goods. These results are consistent with the observation of simultaneous growth in the relative prices and employment shares of stagnant sectors such as community services, with the near constancy of real consumption shares when compared with nominal shares. Ngai and Pissarides [38] accept that for these sub-sectors, there is structural change toward the high TFP goods, but between the aggregates the flow is from high to low TFP sectors.

As in our model, Duernecker *et al.* [21] adopt a decomposition of sectors based on pro-

[8] The decline of agriculture's employment share, the rise and then fall of the manufacturing share, and the rise in the service share (Kuznets [30] and Maddison [36])

ductivity. In particular, they consider two service sectors based on their productivity growth: a "progressive" service sector composed of services industries with above average productivity growth whereas "stagnant" services comprise the services industries with below average productivity growth.

Their model consists of three sectors producing goods, progressive services, and stagnant services such that the value added is produced linearly with labor services only via a sector specific "total" factor productivity. The linear specification implies that sectoral TFP equals labor productivity but as this is provided by the data, the role of capital accumulation is brought back into the model. They find that progressive services are necessities, stagnant services are luxuries, and the two services are substitutes.

The approach has many shortcomings. Importantly, they do not distinguish between TFP growth and investment as separate determinants of sectoral labor productivity growth. Their view is that for the U.S. and along a balanced growth path, aggregate TFP and aggregate capital both grow at the same constant rate. In addition, most models assume that the capital-labor ratios are equalized across sectors so that relative sectoral TFPs and labor productivities are equal. They acknowledge that distinguishing between Sectoral TFP growth and capital growth is likely to matter in middle-income and developing countries where capital is scarce and transition dynamics.

Their main result is that provided productivity growth is below some threshold value, substitution away from the stagnant services is dominant and drives the sector down to zero. They conclude that the future effect of structural change on productivity growth remains limited.

Our model contributes to this discussion as in our model growth is endogenous and capital accumulation plays its role. We show that even in this case, the economy moves away from the stagnant sector, and we do not need the upper bound on productivity growth mentioned by Duernecker *et al.* [21].

Note that these authors argue that "there is an advantage of working with our disaggregation into services with low and high productivity growth instead of working with existing disaggregation of services." Indeed, they establish that "none of the traditional alternatives is as informative about productivity growth as our two-sector split."[9]

As a model of endogenous growth, our model is also related to a literature on heterogenous skills. Indeed, a possible interpretation of our progressive sector would be to consider that it employs a larger proportion of high skilled workers. In this case, our sector decomposition would be comparable to the one used in the recent contributions of Buera and Kabovski [13] and Buera *et al.* [15] where heterogeneous skill levels are considered.

[9] These alternatives include traditional versus non-traditional services as suggested by Duarte and Restuccia [20]; market versus non–market services as used by the guidelines of the System of National Accounts; high-skill-intensive versus low-skill-intensive services as suggested by Buera and Kabovski [13] and Buera *et al.* [15].

However, our classification differs from Buera *et al.* [15]. Indeed, these authors focus on the stock of knowledge, based on the number of schooling years of workers obtained away from this sectors. For example, they include real estate in their high skilled sector because it exhibits a share of labor compensation for more educated workers higher than average, but this industry is out of our classification due to the lack of TFP growth: techniques to sell houses are barely the same than 40 years ago. We are looking at the development of new techniques, and new skills, within the industry instead of the actual schooling level of workers.

On the other hand, our definition of skills is based on a measure of technical development within the sector instead of within workers. This theoretical difference is at the origin of our data classification, and explain partially why relative prices in our paper are decreasing while they are increasing in Buera *et al.* [15]. The second explanation lies in the absence of demand-side mechanisms in our paper, which eliminates the price effect of non-homothetic utility function present in Buera *et al.* [15].

4 The model

We consider an economy in which at each time t there is a continuum $[0, N(t)]$ of infinitely-lived agents characterized by homogeneous preferences. We assume a standard formulation for the utility function which is compatible with endogenous growth such that

$$u(c_i(t)) = \int_0^{+\infty} \frac{c_i(t)^{1-\theta}}{1-\theta} e^{-\rho t} dt \tag{1}$$

where $c_i(t)$ is consumption of agents of type $i \in [0, N(t)]$ at time t, $\theta > 0$ is the inverse of the elasticity of intertemporal substitution in consumption and $\rho > 0$ is the discount factor. We assume that the total population grows at the constant exponential rate $n \in [0, \rho)$, so that $N(t) = e^{nt}N(0)$. Agents are heterogeneous only with respect to the sector in which they work. As explained below, this difference arises from different decisions on the allocation of labor time.

Our economy consists of two sectors, with three factors of production, capital, $K(t)$, labor, $L(t)$, and individual technological knowledge, $a(t)$. In line with Lucas [32], we interpret $a(t)$ as the outcome of an individual's choice. Final output, $Y(t)$, is an aggregate of the output of two intermediate sectors, $Y_H(t)$ a "knowledge-intensive sector" (or "progressive") (e.g. sector H), and $Y_L(t)$ a "stagnant sector" (e.g. sector L). The numbers of workers $N_H(t)$ and $N_L(t)$ in these two sectors, together with their respective growth rates $\dot{N}_H(t)/N_H(t) = n_H(t)$ and $\dot{N}_L(t)/N_L(t) = n_L(t)$, are endogenously determined at the equilibrium.

Agents working in the "knowledge-intensive sector" devote a fraction $u(t) \in (0, 1)$ of their unit of time to production. Total labor in this sector is $L_H(t) = u(t)N_H(t)$. The rest of time $1 - u(t)$ is devoted to the accumulation of individual technological knowledge $a(t)$. Here, $L_H(t)$ denotes the total number of hours worked in the "knowledge-intensive sector" and differs from the number $N_H(t)$ of workers. Similarly to how Lucas [32] treats newborns, in our model each agent entering the knowledge-intensive sector at any time t_0 acquires the available knowledge $a(t_0)$. We assume that

$$\dot{a}(t) = z[1 - u(t)]a(t) - \eta a(t) \tag{2}$$

with $z > 0$ and $\eta > 0$ the depreciation rate of knowledge. By contrast, agents working in the second sector will spend all their unit of time working so that total labor in this sector is $L_L(t) = N_L(t)$. Contrary to the "knowledge-intensive sector", in this sector the number of hours worked is identical to the number of workers, since the total individual available working time is normalized to one.

The final good is produced through a CES technology such that

$$Y(t) = \left(\gamma Y_H(t)^{\frac{\epsilon-1}{\epsilon}} + (1-\gamma)Y_L(t)^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}} \tag{3}$$

with $\gamma \in (0,1)$ and ϵ the constant elasticity of substitution. The value of ϵ is important as it determines whether the inputs are substitutable ($\epsilon > 1$) or complementary ($\epsilon < 1$). Sector H produces the knowledge-intensive intermediate good using capital, labor and knowledge through the following Cobb-Douglas technology:

$$Y_H(t) = [L_H(t)a(t)]^\alpha K_H(t)^{1-\alpha} \tag{4}$$

with $\alpha \in (0,1)$. The product $L_H(t)a(t) = u(t)N_H(t)a(t)$ then represents total efficient labor.[10] Note that, considering capital and labor (hours worked) as inputs, the total factor productivity (TFP) is given by $a(t)^\alpha$. Sector L produces the second intermediate good using only capital and labor through the following Cobb-Douglas technology:

$$Y_L(t) = L_L(t)^\beta K_L(t)^{1-\beta} \tag{5}$$

with $L_L(t) = N_L(t)$ and $\beta \in (0,1)$. Contrary to the "knowledge-intensive" sector, the second sector has a constant TFP. While we do not impose any restriction on the capital intensity difference $(\alpha - \beta)$ across sectors, we assume $\alpha, \beta > 1/2$ to match standard empirical estimates of labor shares.

Denoting total capital by $K(t)$ and total labor by $L(t)$, capital and labor market clearing require at each date $K(t) \geq K_H(t) + K_L(t)$ and $L(t) \geq L_H(t) + L_L(t) = N_H(t)u(t) + N_L(t)$. The capital accumulation equation is standard

$$\dot{K}(t) = Y(t) - \delta K(t) - N_H(t)c_H(t) - N_L(t)c_L(t) \tag{6}$$

[10] As in Lucas [32], we define each worker's output according to $(u(t)a(t))^\alpha k(t)^{1-\alpha}$ where $k(t) = K_H(t)/N_H(t)$.

where $\delta > 0$ is the depreciation rate of capital and c_i the consumption of any agent working in sector $i = H, L$.

5 Planner solution and intertemporal equilibrium

In the model, as in Lucas [32], the welfare theorems hold and the intertemporal competitive equilibrium can be found via the planner's problem.[11] Assume that the planner has Benthamite objective function and consider the following intertemporal optimization problem

$$\begin{array}{ll} \max\limits_{\{c_i(t),K_i(t),L_i(t)\}_{i=H,L},u(t),a(t)} & \int_0^{+\infty} \left(N_H(t)\dfrac{c_H(t)^{1-\theta}}{1-\theta} + N_L(t)\dfrac{c_L(t)^{1-\theta}}{1-\theta} \right) e^{-\rho t} dt \\ s.t. & (2), (3), (4), (5), (6) \text{ and} \\ & K(t) \geq K_H(t) + K_L(t) \\ & L(t) \geq L_H(t) + L_L(t) = N_H(t)u(t) + N_L(t) \\ & K(0), a(0), N(0) \text{ given} \end{array} \tag{7}$$

From now on, let us denote the value of increments in aggregate capital by the "price" P, the price of the knowledge-intensive good by P_H, the price of the second good by P_L and the price of knowledge by Q. Solving the first order conditions of this optimization problem allows to state the following Proposition:

Proposition 1. *All agents, no matter which sector they work in, have the same labor and capital income at the equilibrium and have the same intertemporal profile of consumption as given by* $c_H(t) = c_L(t) = P^{-1/\theta}$ *for any* $t \geq 0$. *Aggregate consumption is thus* $C(t) = N(t)P(t)^{-\frac{1}{\theta}}$. *Moreover, for any given initial conditions* $(K(0), a(0))$, *and considering the rental rate of capital*

$$R(t) \quad = \quad (1-\alpha)\gamma \left(\frac{Y}{Y_H}\right)^{\frac{1}{\epsilon}} \frac{Y_H}{K_H} = (1-\beta)(1-\gamma) \left(\frac{Y}{Y_L}\right)^{\frac{1}{\epsilon}} \frac{Y_L}{K_L}, \tag{8}$$

any path $\{K(t), a(t), P(t), Q(t)\}_{t\geq 0}$ *that satisfies the following system of differential equations*

$$\frac{\dot{P}}{P} = -\left[R - \delta - \rho\right], \quad \frac{\dot{Q}}{Q} = -(z - \eta - \rho) \tag{9}$$

$$\frac{\dot{K}}{K} = \frac{Y}{K} - \delta - \frac{NP^{-\frac{1}{\theta}}}{K}, \quad \frac{\dot{a}}{a} = z(1-u) - \eta \tag{10}$$

together with the transversality conditions

$$\lim_{t\to+\infty} P(t)K(t)e^{-\rho t} = 0 \text{ and } \lim_{t\to+\infty} Q(t)a(t)e^{-\rho t} = 0 \tag{11}$$

is an optimal solution of problem (7) and therefore an intertemporal equilibrium.

So far we have interpreted knowledge accumulation along the lines of Lucas [32] (see

[11] The economic interpretation of competitive equilibrium is as in Lucas [32].

footnote 2). Another interpretation of the model is as follows. Knowledge now becomes an aggregated variable, a public good used by the firms in the high-tech sector. Since there is no private incentive to invest in $a(t)$, the regulator has to step in. The government manages the production of $a(t)$ by imposing lump-sum taxes on consumers and use the revenue to hire workers from the high-tech sector. As the two other sectors (low- and high-tech) are perfectly competitive, the wage per hour needs to be identical across sectors. Note that the tax revenue used to buy efforts from consumers working in the high-tech industry might involve a time dependent tax system.

6 Non-balanced growth paths

We now provide the existence result and characterize the non-balanced growth paths along which the variables P, Q, K, a grow at constant rates. Note that this implies that C, Y, C_i, K_i and Y_i also grow at constant rates. We introduce the following notations valid along the non-balanced growth path, denoted NBGP, which highlights the fact that the growth rates of the various variables can differ:

$$g_C = \frac{\dot{C}}{C}\ , g_Y = \frac{\dot{Y}}{Y}, g_K = \frac{\dot{K}}{K},\ g_{Y_i} = \frac{\dot{Y}_i}{Y_i},\ g_{K_i} = \frac{\dot{K}_i}{K_i},\ g_{c_i} = \frac{\dot{c}_i}{c_i},\ g_a = \frac{\dot{a}}{a}$$

and

$$g_P = \frac{\dot{P}}{P},\ g_Q = \frac{\dot{Q}}{Q}$$

for $i = H, L$. Along an NBGP we have $K(t) = k(t)e^{g_K t}$, $a(t) = x(t)e^{g_a t}$, $Q(t) = q(t)e^{g_Q t}$ and $P(t) = p(t)e^{g_P t}$. Note that NBGP can differ in their levels but still have the same 8-tuple in $\mathbb{R}^8$ of growth rates.

As the final good is produced via a CES function, the dynamics strongly depends on the value of the elasticity of substitution ϵ between the two intermediary sectors. When inputs are substitute, i.e., $\epsilon > 1$, as technological progress only occurs in the knowledge intensive sector, it is optimal to reallocate capital and labor inputs to sector H. Thus, eventually sector H becomes dominant. On the contrary, when sectors are complement, i.e. $\epsilon < 1$, both inputs are necessary to produce efficiently the final good. In this case, in addition to the advantage of sector H due to technological progress, complementarity between inputs provides incentive to increase the production and use of sector L. The total effect is therefore a priori not determined.

6.1 The case of substitutable inputs

We consider first the case of substitutable inputs, $\epsilon > 1$, which will turn out to be the one delivering outcomes in line with the recent data on services we described in the introduction. We introduce the following restrictions which guarantee positiveness of growth rates and interiority of the share u of time devoted to work by agents in the knowledge-intensive sector:

Assumption 1. *$\epsilon > 1$, $z > \eta + \rho - n$ and*

$$\theta > \max\left\{1 - \frac{\rho - n}{z - \eta}, \frac{\beta(\epsilon - 1)(z - \eta - \rho + n)}{n}\right\}$$

From the first-order conditions (14)-(21) and the differential equations (9)-(10) we derive:

Theorem 1. *Under Assumption 1, the set of non-balanced growth paths (NBGP) is non-empty and is characterised by the same 8-tuple of growth rates. The growth rates are such that $g_Y = g_{Y_H} = g_K = g_{K_H} = g_C = n + g_{c_H} = n + g_{c_L} = n - \frac{g_P}{\theta} = n + g_a, g_Y = g_{Y_H} > g_{Y_L}, g_K = g_{K_H} > g_{K_L}$ and $n_H > n_L$. Moreover, the time devoted to production in the knowledge-intensive sector and the rental rate of capital are both constant.*

Proof. See Appendix 10.3.

We can derive a number of important implications from Theorem 1. First, we obtain that $g_Y = g_{Y_H} > g_{Y_L}$, $g_K = g_{K_H} > g_{K_L}$ and $n_H > n_L$. Sector H exhibits a higher growth rate than sector L and capital and labor are allocated more intensively to the knowledge-intensive sector, which becomes dominant in the economy. Consequently, in the long run there is capital deepening, as the capital-labor ratio of both sectors is increasing and the share of skilled labor is increasing while the share of unskilled-labor is decreasing. This pattern of structural change is observed in the data, as shown by Figure 1 in Section 2.

Clearly, as $g_K = g_Y = g_{K_H} = g_{Y_H} = n + g_a$, technical knowledge, a, is the engine of growth. It is worth noting, however, that although the knowledge-intensive sector is asymptotically dominant in output, the amount of inputs used is not vanishing and workforce does not shrink. Finally, the price of the final good decreases, $g_P < 0$. In fact, if we decompose this price effect using equations (10) and (11), we see that this drop is mainly due to the decreasing price of the intermediate good H.

We obtain the following characterisation of the NBGP which includes the Kaldor facts:

Corollary 1. *Under Assumption 1, the non-balanced growth path has the following properties:*

1. *There is capital deepening, i.e., the ratio K/L is increasing.*
2. *The growth rate of real GDP, or Y, is constant.*
3. *The capital-output ratio K/Y is constant.*
4. *The nominal share of capital income in GDP is constant and equal to $s_K = 1 - \alpha$.*
5. *The real interest rate R is constant.*

6. *The relative prices P_H/P and P_L/P are respectively constant and increasing. As a result, P_L/P_H is increasing.*

7. *The real and nominal shares in GDP of the knowledge-intensive sector and of the stagnant sector, respectively Y_H/Y, $P_H Y_H/PY$, and Y_L/Y, $P_L Y_L/PY$, are respectively constant and decreasing.*

Proof. See Appendix 10.4.

The mechanism at work in the economy can be described as follows. The accumulation of individual knowledge in the knowledge-intensive sector H is used as labor-augmenting technological progress. Knowledge accumulation thus leads to an unbounded increase in TFP in the knowledge-intensive sector and to capital deepening. The relative price of the knowledge-intensive good decreases because of knowledge accumulation and TFP growth, while the relative price of the less knowledge-intensive good increases. The change in relative prices is associated with changes in the demand for both intermediate goods by the final sector. This mechanism endogenously determines the growth rates of the two sectors such that the growth rate of real GDP remains constant and determined by the growth rate of sector H. The consequences of this are that the relative real and nominal shares of the knowledge-intensive sector with respect to sector L, Y_H/Y_L and $P_H Y_H/P_L Y_L$, are both increasing, and the real and nominal shares of the dominated sector L are both decreasing. Finally, endogenous technological progress makes production of the final good more and more efficient, so that its price decreases.

These results are in line with documented facts of recent structural change in the services which shows that both real and nominal shares of the goods producing sector (here sector L) have fallen (see Figure 2 in Section 2). Moreover, the relative number of hours (and number of workers) in stagnant service sectors have fallen (see Figure 1). Our model is also able to explain the decreasing relative price of the knowledge intensive sector and the increasing relative price of the stagnant sector (see Figure 4). Finally, our results are compatible with the fact that the relative share of modern market services (here sector H) has increased significantly with respect to the other sectors (here sector L) (see again Figure 2). Although the mechanism at work is quite different, we get similar conclusions than Buera *et al.* [15] who derive along the exogenous growth process a systematic shift in the composition of value added to sectors that are intensive in knowledge-intensive labor.

Note that Acemoglu *et al.* [1] also supports input substitution. However, their results are obtained in a framework that focuses on the role of the environment and directed technical change and assume two intermediary sectors characterized by dirty and clean technologies. In our framework, we conjecture that substitution between inputs will be favored by the develoment digital goods and artificial intelligence.

6.2 The case of complementary inputs

We now consider the case of complementary inputs, i.e., $\epsilon < 1$. We introduce the following restrictions which guarantee positiveness of growth rates and interiority of the share u of time devoted to work by agents in the knowledge-intensive sector:

Assumption 2. $\epsilon < 1$ *and* $z \in \left\{\eta + \rho - n, \eta + \rho + \frac{n[\epsilon + \alpha(1-\epsilon)}{1+\alpha(1-\epsilon)}\right\}$

We obtain the following theorem:

Theorem 2. *Under Assumption 2, the set of NBGR is non-empty and is characterised by the same* 8*-tuple of growth rates. The growth rates are constant across this set and given by:* $g_K = g_Y = g_{K_L} = g_{Y_L} = n_L = n$ *and* $g_{K_H} = n_H < g_{K_L}$ *and* $n_H < n_L . g_P = g_C = 0$*. Moreover, the time devoted to production in the knowledge-intensive sector and the rental rate of capital are both constant.*

Proof. See Appendix 10.5.

The mechanism at work in this case can be described as follows. When $\epsilon < 1$ there exist two opposite forces: complementarity of inputs and endogenous technical change. The accumulation of individual knowledge in the knowledge-intensive sector is used as labor-augmenting technological progress. Knowledge accumulation thus leads to an unbounded increase in TFP in the knowledge-intensive sector. But as the secon intermediate sector output is also needed in the production of the final good because of the complementarity assumption, the productivity gap has to be compensated by a stronger reallocation of capital and labor in that sector, $g_{K_H} < g_{K_L}$ and $n_H < n_L$. Importantly, there is no capital deepening in any sector, i.e., the ratio K/L is constant in the long run. In addition, the share of skilled labor is decreasing while the share of unskilled-labor is increasing. These results here are clearly not in line with the US data.

The relative price of the knowledge-intensive good still decreases because of TFP growth, while the relative price of the less knowledge-intensive good increases. The striking result here is that these two effects offset each other so that the price of the final good P and thus consumption are constant. The change in relative prices is however still associated with changes in the demand for both intermediate goods by the final sector. This mixed mechanism endogenously determines the growth rates of the two sectors such that the growth rate of real GDP remains constant and now determined by the growth rate of sector L, but the knowledge-intensive sector remains dominant in the long run. It is also worth noting that the productivity lag effect is more than compensated by the technical change effect as the knowledge intensive sector grows faster, i.e. $g_{Y_H} > g_{Y_L}$, which implies that the dominant sector is again sector H.

The consequences of this are that the relative real and nominal shares of sector L with respect to the knowledge-intensive sector, Y_L/Y_H and $P_L Y_L / P_H Y_H$, are both decreasing, and the real and nominal shares of the knowledge-intensive sector are respectively increasing and decreasing. Again, these properties are not compatible with the well documented facts of the structural change literature mentioned in Section 2.

6.3 The case of a unitary elasticity

For reference, we also consider the limit case $\epsilon = 1$ where the production function of the final good sector is Cobb-Douglas such that

$$Y(t) = Y_H(t)^{\gamma} Y_L(t)^{1-\gamma} \tag{12}$$

with $\gamma \in (0, 1)$, and the two intermediary sectors have an elasticity of substitution equal to one. Perfect substitution between the two intermediate inputs implies that the only driving force determining the long run growth properties is the knowledge accumulation in the knowledge intensive sector. As a consequence, the asymptotic dominant sector is still the knowledge intensive one but compared to Theorems 1 and 2, there is a discontinuity since the growth rate of the final sector is now determined by a convex combinaison of the growth rates of the two intermediate sectors. It can be shown indeed that there exists a unique set of NBGR such that

$$g_Y = g_K = g_{K_L} = g_{K_H} = g_C = n - \frac{g_p}{\theta} = \gamma g_{Y_H} + (1-\gamma) g_{Y_L},\ n_H = n_L = n,$$
$$g_{Y_H} = \frac{\alpha[\gamma + (1-\gamma)\beta](z-\eta+n-\rho)}{\theta\gamma\alpha + (1-\gamma)\beta} + n,\ g_{Y_L} = \frac{\gamma\alpha(1-\beta)(z-\eta+n-\rho)}{\theta\gamma\alpha + (1-\gamma)\beta} + n,$$
$$g_a = \frac{[\gamma\alpha + (1-\gamma)\beta](z-\eta+n-\rho)}{\theta\gamma\alpha + (1-\gamma)\beta}$$

with $g_{Y_H} > g_Y > g_{Y_L}$. In this configuration, the ratios K_H/K_L, L_H/L_L and N_H/N_L are constant along the NBGP and along the transition implying that the labour share and the share of capital in the knowledge-intensive sector are constant. Moreover, the nominal value added of the knowledge-intensive sector is also constant. We disregard this case as the characteristics of the NBGP are not in line with the empirical evidence.

7 Local dynamics

The asymptotic properties of the NBGP described in the previous section only provide a long-run necessary condition. Yet interesting and empirically relevant properties also occur along the transition path. Note that in the rest of the paper we will focus on the case with substitution, i.e., $\epsilon > 1$. Indeed, given the analysis in the previous section only this case generates a growth pattern compatible with the empirical evidence. However, we will mention some results for the case $\epsilon \leq 1$ for completeness.

We now aim to generate structural change along the transition path. We can reformulate the dynamical system given by equations (9)-(10) using the normalization of variables introduced by Caballe and Santos [16]. The stationarized NBGP is obtained by "removing" the NBGR trend from the variables, namely: $k(t) = K(t)e^{-g_K t}$, $x(t) = a(t)e^{-g_a t}$, $q(t) = Q(t)e^{-g_Q t}$ and $p(t) = P(t)e^{-g_P t}$, for all $t \geq 0$, with $k(t)$, $x(t)$, $q(t)$ and $p(t)$ the stationarized values for $K(t)$, $a(t)$, $Q(t)$ and $P(t)$. As the price of knowledge Q is characterized by a constant growth rate g_Q, the solution of the corresponding equation in (9) is given by $Q(t) = Q(0)e^{-g_Q t}$ and its stationarized value is constant with $\dot{q}(t) = 0$. We then get $q(t) = q(0) = q_0$ for all $t \geq 0$. Recall that as the population is growing at the exponential rate n, we have $N(t) = e^{nt}N(0)$ with $N(0) = N_0$ given.

Substituting these stationarized variables into (9)-(10), we obtain an equivalent stationarized system of differential equations that characterizes the equilibrium path. Of course, the expression of this dynamical system slightly differs depending on the value of the elasticity of substitution ϵ. As assumed above we focus on the case with $\epsilon > 1$.

Lemma 1. *Let N_0 be given and suppose Assumption 1 holds. Along a stationarized equilibrium path and for any given q_0, knowledge x, capital k and its price p are solutions of the following dynamical system*

$$\begin{array}{rcl} \frac{\dot{p}}{p} & = & -\left[(1-\alpha)\gamma\psi^{\frac{1}{\epsilon}}x^{\alpha}\left(\frac{L_H(k,x,p,q_0,N_0)}{K_H(k,x,p,q_0,N_0)}\right)^{\alpha} + g_P - \rho - \delta\right] \\ \frac{\dot{k}}{k} & = & \psi x^{\alpha}\left(\frac{L_H(k,x,p,q_0,N_0)}{K_H(k,x,p,q_0,N_0)}\right)^{\alpha}\frac{K_H(k,x,p,q_0,N_0)}{k} - \delta - g_K - \frac{N_0 p^{-\frac{1}{\theta}}}{k} \\ \frac{\dot{x}}{x} & = & z\Big(1 - u(k,x,p,q_0,N_0)\Big) - g_a - \eta \end{array} \qquad (13)$$

with $\psi = \gamma^{\frac{\epsilon}{\epsilon-1}}\left(1 + (\frac{1-\alpha}{1-\beta})\frac{k - K_H(k,x,p,q_0,N_0)}{K_H(k,x,p,q_0,N_0)}\right)^{\frac{\epsilon}{\epsilon-1}}$.

Proof. See Appendix 10.6.[12]

Note also that the transversality conditions (11) become

$$\lim_{t\to+\infty} p(t)k(t)e^{-[\rho-g_K-g_P]t} = 0 \text{ and } \lim_{t\to+\infty} x(t)e^{-(\rho-g_a-g_Q)t} = 0$$

Using the expressions of the growth rates g_K, g_a, g_Q and g_P given in Theorem 1, we easily derive under Assumption 1 that $zu^* = z - g_a - \eta = \rho - g_K - g_P = \rho - g_a - g_Q > 0$.

Considering the stationarized dynamical systems given in Lemma 1, we can now focus on proving the existence of a steady state solution, i.e., $\dot{p}/p = \dot{k}/k = \dot{x}/x = 0$, and $\dot{q}(t) = 0$ which obviously corresponds to the NBGR exhibited in Theorems 1.

Theorem 3. *Let N_0 be given and suppose Assumption 1 holds. The projection of the set of NBGP on the subspace (k, x, p, q) is a one-dimensional manifold, noted $\mathcal{M} \subset \mathcal{R}^4$, parameterized by q_0. Fur-*

[12] Analogous results for the case with complementarity ($\epsilon < 1$) or the case with a unitary elasticity of substitution ($\epsilon = 1$) can be obtained.

thermore, for any given $q_0 > 0$, there exists a unique steady state $(k^*(q_0), x^*(q_0), p^*(q_0))$, solution of the dynamical system (13). Moreover, $k^{*'}(q_0) < 0$, $x^{*'}(q_0) < 0$ and $p^{*'}(q_0) > 0$.

Proof. See Appendix 10.7.

Theorem 3 proves that when $\epsilon > 1$, there exists a one-dimensional manifold of steady states for the capital stock k, technological knowledge x and the price of capital p parameterized by the constant price of knowledge q_0. Importantly, the asymptotic amount of time devoted to production in the knowledge-intensive sector u^* and the asymptotic rental rate of capital R^*, as given in Theorem 1, do not depend on q_0. Note that an analogous result to Theorem 3 for the case $\epsilon < 1$ can be obtained and shows that there still exists a manifold of steady state but this is degenerate as only technological knowledge x depends on q_0 while the capital stock k and the price p do not.

The existence of a manifold of steady states in levels is fairly standard in endogenous growth models (see for instance Lucas [32]) where the asymptotic equilibrium of the economy depends on some initial conditions. We will show that there exists a set $\mathcal{K}$ containing the set $\mathcal{M}$ such that for initial values of physical capital $k(0)$ and technological knowledge $x(0)$ in $\mathcal{K}$, a value of q_0 is "automatically" selected in order for the economy to leap onto the optimal path (i.e., the stable manifold) and then converge to the particular steady state $(k^*(q_0), x^*(q_0), p^*(q_0))$ situated on the manifold $\mathcal{M}$. Note that for any given pair $(k(0), x(0)) \in \mathcal{K}$ there exists a unique value of the price of knowledge q_0 compatible with the equilibrium conditions.

To prove such a result, we need to study the local stability properties of the steady state. Linearizing the dynamical systems (13) around the steady state $(k^*(q_0), x^*(q_0), p^*(q_0))$ for a given $q_0 > 0$, the local stability property of $(k^*(q_0), x^*(q_0), p^*(q_0))$ is appraised through the characteristic roots of the associated Jacobian matrix. As shown by Martinez-Garcia [34] (see also Bond, Wang and Yip [11] and Xie [45]), since we have two state variables, k and x, and two forward variables, p and q, with q being constant, the standard saddle-point stability occurs if there exists a one-dimensional stable manifold, i.e. if only one characteristic root is negative.

Lemma 2. *Let Assumption 1 hold. Then for any given $q_0 > 0$, the steady state $(k^*(q_0), x^*(q_0), p^*(q_0))$ is saddle-point stable.*

Proof. See Appendix 10.8.[13]

In the dynamical system (13), the predetermined variables are the capital stock and the level of individual knowledge. For given initial conditions $K(0) = k(0) = k_0$ and $a(0) = x(0) = x_0$, we generically cannot find a value of q_0 such that $(k_0, x_0) = (k^*(q_0), x^*(q_0))$ and

[13] The same conclusion holds for the complementary case ($\epsilon < 1$), i.e., Assumption 2, or the case with a unitary elasticity of substitution ($\epsilon = 1$).

the economy is not in the set of NBGP from the initial date. In other words, non-trivial transitional dynamics generically occurs starting from $(k_0, x_0) \neq (k^*, x^*)$. The initial values of the forward variables $p(0) = p_0$ and $q(0) = q_0$ are chosen such that the one-dimensional stable optimal path converging toward an NBGP is selected. As the stable manifold is one-dimensional, for any given $K(0) = k(0) = k_0$ and $x(0) = x_0$ there exists a unique pair (q_0, p_0) compatible with an equilibrium path. This property therefore defines a conditional convergence depending on the value of the initial price of knowledge q_0.

The arguments supporting the truth of the previous statement are as follows. The dynamical system has two state and two forward looking variables. The steady state is then a 4-uple (k, x, p, q). There is a one-dimensional manifold of steady states. Each of these is saddle-path stable and q is constant on any equilibrium path. For each of these q there is a one-dimensional stable manifold leading to the NBGP. When q spans the feasible set, this describes a stable planar manifold. Generically there is a two-dimensional plane in space of dimension four with a given $(x(0), k(0))$. The intersection of the two planes in dimension four is a set of dimension zero, a point. So for a given $(x(0), k(0))$ there is a unique $(p(0), q(0))$.

A striking property is that, although the steady state values $(k^*(q_0), x^*(q_0), p^*(q_0))$ depend on the selected q_0, the eigenvalues do not. It follows that the rate of convergence along any transitional path is the same, regardless of the initial conditions and thus of the asymptotic value of the steady state.

Building on Theorem 3, it can be shown (see Appendix 10.7) that $k^*(x^*)$ is a linear function of x^*. The conditional convergence property can be illustrated by the following Figure.

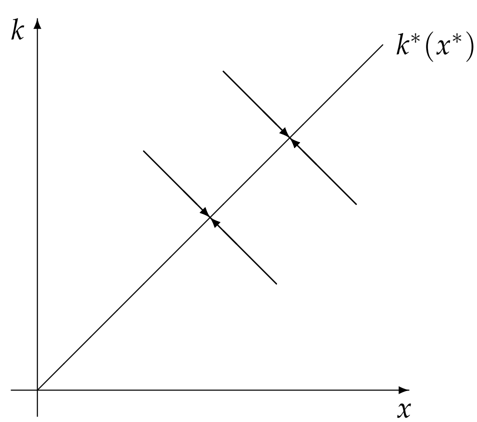


Figure 6: Manifold of steady states when $\epsilon > 1$.

The economy is characterized by two initial conditions, $k(0) = k_0$ and $x(0) = x_0$, while $p(0)$ and $q(0)$ are pinned down by the equilibrium conditions. All pairs (k^*, x^*) satisfying $k^* = k^*(x^*)$ correspond to a common asymptotic NBGP but different optimal paths along the transition according to the initial condition (k_0, x_0). For a given (k_0, x_0), the optimal path will converge toward an asymptotic position located on the curve $k^*(x^*)$ which depends on the initial position (k_0, x_0) that defines the admissible initial values p_0 and q_0. Arrows in Figure 6 illustrate some possible trajectories. It can be shown indeed that $k(t)$ and $x(t)$ evolve in opposite directions. Such a property is easy to explain. As k and x are stationnarized values of K and a, if we consider initial values k_0 and x_0 that are above the curve $k^*(x^*)$, the transitory growth rate of $a(t)$ will be lower than the long run growth rate g_a and will increase progressively to g_a while $x(t)$ converges to the long run value x^*. On the contrary, the transitory growth rate of $K(t)$ will be initially higher than its long run value and decrease progressively converging toward g_K while $k(t)$ converges to the long run value k^*. Opposite results are of course obtained if we consider initial values k_0 and x_0 that are below the curve $k^*(x^*)$.

Remark: In the case of complementarity with $\epsilon < 1$, k^* is independent from x^* and the manifold is horizontal in the (x, k) space. The intuition on this degenerate manifold can easily be explained as follows: as sector L is dominant in the long run, it determines the level of capital independently from the level of technical knowledge. There is therefore no synergy in the long run between knowledge and capital accumulation. In the case of a unitary elasticity of substitution $\epsilon = 1$, we get a similar manifold as in Figure 6 but which is a non-linear increasing concave function.

The following Theorem summarizes the results.

Theorem 4. *Let Assumption 1 hold. There exists a set $\mathcal{K}$ containing the set $\mathcal{M} \subset \mathbb{R}^4$ such that for initial values of physical capital $k(0)$ and technological knowledge $x(0)$ in $\mathcal{K}$, the economy converges to the steady state $(k^*(q_0), x^*(q_0), p^*(q_0))$ situated on the manifold $\mathcal{M}$, where the price of knowledge q_0 and p_0 are uniquely determined and the optimal path converging to the NBGP is unique.*

Proof. See Appendix 10.9.

The existence of a manifold of steady states, while standard in the endogenous growth literature, is a fundamental difference with respect to exogenous growth models where the technological progress is exogenous, the steady state is unique and countries with the same fundamentals but different initial endowments of physical capital will all converge toward the same asymptotic level of wealth and the same asymptotic growth rate. By contrast, in the present model, while all countries with the same fundamentals are characterized by the same growth rate, they will follow different optimal paths along the transition and be asymptotically characterized by different wealth levels. Importantly, the long run heterogeneity of wealth will concern both physical capital and knowledge.[14]

[14]Note that when $\epsilon < 1$, countries will converge to the same capital stock but will have heterogeneous levels of knowledge, which is less plausible from an empirical perspective.

8 Transitional structural change

Having shown the existence of paths converging to the NBGP, we focus here on the properties of these paths. When inputs are substitutable, for any given pair (k_0, x_0) the optimal path will converge toward an asymptotic position located on the manifold $\mathcal{M}$. From the property that the manifold is parameterized by q_0 and that $k^{*'}(q_0) < 0$, $x^{*'}(q_0) < 0$ in Theorem 3, we know that the set of NBGP is such that greater value of capital along the NBGP is associated with greater value of knowledge (also seen in Fig. 6). It is also likely that an economy with initially low levels of physical capital and technological knowledge will remain permanently behind an initially better-endowed economy, as suggested by Fig. 6. In other words, q_0 is decreasing when $k(0)$ and $a(0)$ increase.

Theorems 1, 3 and 4, and Corollary 1 allows us to give a characterisation of the transition path toward the non-balanced growth path.

Corollary 2. *Under Assumption 1, the transition toward the non-balanced growth path is characterized by the following properties:*

1. *The shares of capital and labor in the knowledge-intensive sector, $\kappa = K_H/K$ and $\lambda = L_H/L$, are increasing.*
2. *The real and nominal shares in GDP of the knowledge-intensive sector, respectively Y_H/Y and $P_H Y_H/PY$, are increasing.*
3. *The nominal share of capital income in GDP, $s_K = RK/Y$, is increasing (decreasing) if and only if $\beta > (<)\alpha$.*
4. *The relative price of the knowledge-intensive sector P_H/P is decreasing while the relative price of the stagnant sector P_L/P is increasing.*

Proof. See Appendix 10.10.

The results 1, 2 and 3 in Corollary 2 are in line with the empirical evidence collected in Section 2. First, along the transition path, the share of workers in the knowledge-intensive sector increases while converging toward its stationary value, and the labor force in sector L decreases. As illustrated in Section 2, the share of workers in service sectors in most countries over the period 1997-2017 (see Figure 1) follows this pattern. Other patterns reinforce this fact. The number of hours and workers in the stagnant sectors relative to those in progressive service sectors have fallen and as both real and nominal shares of service sectors (here sector L) have also fallen. Moreover, the relative share of modern market services plus durable goods (here sector H) has increased dramatically with respect to the other sectors (here sector L). Our results are thus also fully in line with these empirical facts (see Section 2).

Results 4 is also well in line with the dynamics of relative prices highlighted in Section 2 (see Figure 4). Over the period $1997 - 2017$, the relative prices of services of the progressive sector and of the stagnant sector are indeed respectively increasing and decreasing as derived from our model. Our results are clearly explained by the productivity growth differential across the two intermediate sectors. Indeed, as the stagnant sector has a relatively lower growth rate than the knowledge-intensive sector, its relative price obviously has to increase over time.

8.1 Transitional dynamics: an illustrative calibration

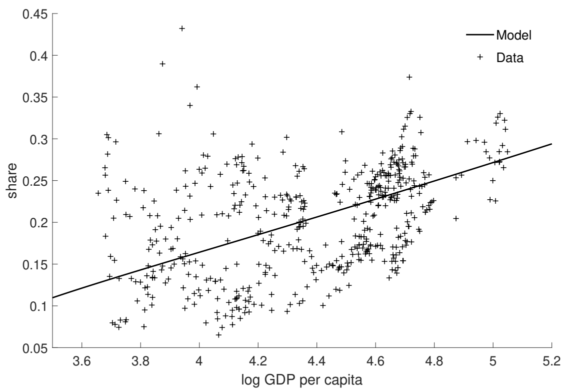
We now provide an illustrative calibration to investigate whether the equilibrium dynamics generated by our model is consistent with the patterns shown by the data described in Section 2. The first step is to obtain a plausible NBGR. Our model is characterized by 10 parameters, namely $\epsilon, \delta, \rho, \theta, \gamma, \alpha, \beta, n, z$ and η. Following Barro and Sala-i-Martin [9], we adopt the standard values for the annual depreciation rate $\delta = 0.05$, the annual discount rate $\rho = 0.02$ and the long-run annual interest rate $R^* = 0.08$. We use the EUKLEMS database to evaluate the annual population growth rate over the period 1997-2017 and we find $n = 0.01$. Concerning the sectoral labor shares, we focus on the US data and we use[15] the National Income and Product Accounts (NIPA) between 1948 and 2005 where industries are classified according to the North American Industrial Classification System at the 22-industry level. We classify industries according to the requirement of technological knowledge by the workers. That is, we consider an industry to be knowledge-intensive if workers exhibit a higher growth of compensation per capita than average. The Table provided in Appendix 10.11 shows the average capital share of each industry together with the sector classification. This classification allows us to compute average shares of capital for two "aggregate sectors" in which $\alpha = 0.62$ and $\beta = 0.64$.[16] Recent contributions by Mulligan [36], Vissing-Jorgensen and Attanasio [44] and Gruber [25] provide robust estimates of the elasticity of intertemporal substitution in consumption between 1 and 2.3. We consider here an intermediate value of 1.5 leading therefore to $\theta = 10/15$. As we do not have any empirical evidence to calibrate the values of the parameters z and η characterizing the accumulation of individual knowledge, we adjust these values to match the endogenous annual output growth rate g_Y. The total output growth between 1997 and 2017 in the EUKLEMS database is 2.5%, leading to $g_Y = 0.025$. The corresponding values of z and η are then $z = 0.11$ and $\eta = 0.09$.

As in Acemoglu and Guerrieri [4] we compute the capital share of the knowledge-intensive sector, $\kappa(t)$, considering equation (45). This allows us to derive the labour share of the knowledge-intensive sector, $\lambda(t)$, as given by (41), the nominal (value added) share

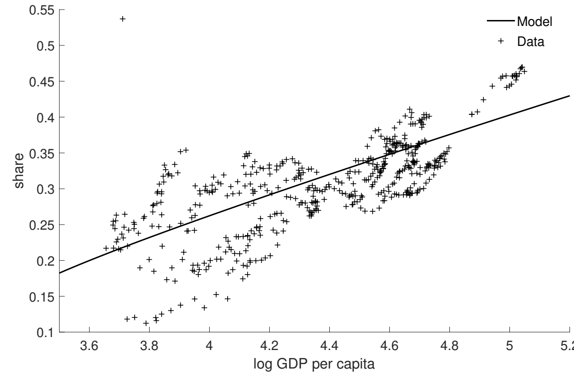
[15] As Acemoglu and Guerrieri [4]

[16] These two numbers are obtained as the weighted average of the shares mentioned in Table 1 according to the relative size of each sector.

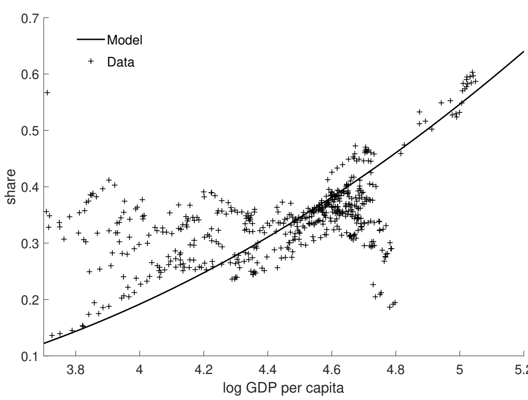
in GDP, $P_H(t)Y_H(t)/P(t)Y(t)$, and the real (value added) share in GDP of the knowledge-intensive sector, $Y_H(t)/Y(t)$ as given by (62) (in Appendix 10.10). The model allows for a numerical caracterisation of the dynamics of these quantities along the transition for $\epsilon = 1.8$ (reported in the following Figures). Importantly the predictions are not very sensitive to the choice of ϵ within the range $(1.5, 2)$.



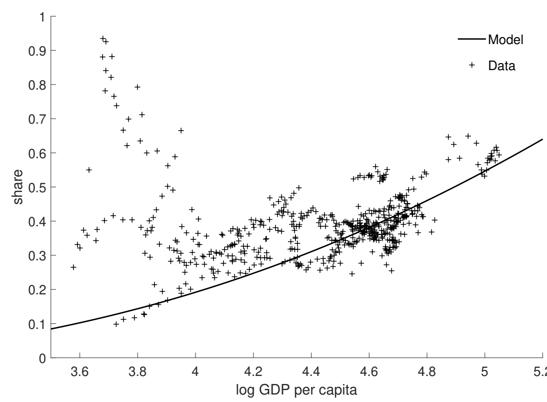
(a) Capital share



(b) Labour share



(c) Real value added share



(d) Nominal value added share

Figure 7: Transitional dynamics of capital, labour, real and nominal value added share in the progressive sector

A number of features are worth noting. The dynamics of capital and labor shares and of the real and nominal shares of value added in GDP of the knowledge-intensive sector match well the data. Our simulations in particular show an increasing real share in GDP of the knowledge-intensive sector, which is in line with the fact that the relative share of modern market services plus durable goods (here sector H) has increased with respect to the other sectors (here sector L). Even if the data are spread over the model trend we capture quite well the increasing path along the development process implied by the data. The use of an elasticity of substitution larger than one is validated by such figures.

Finally, the following figure contain the dynamics of the relative prices of progressive and stagnant sectors obtained with the simulations.

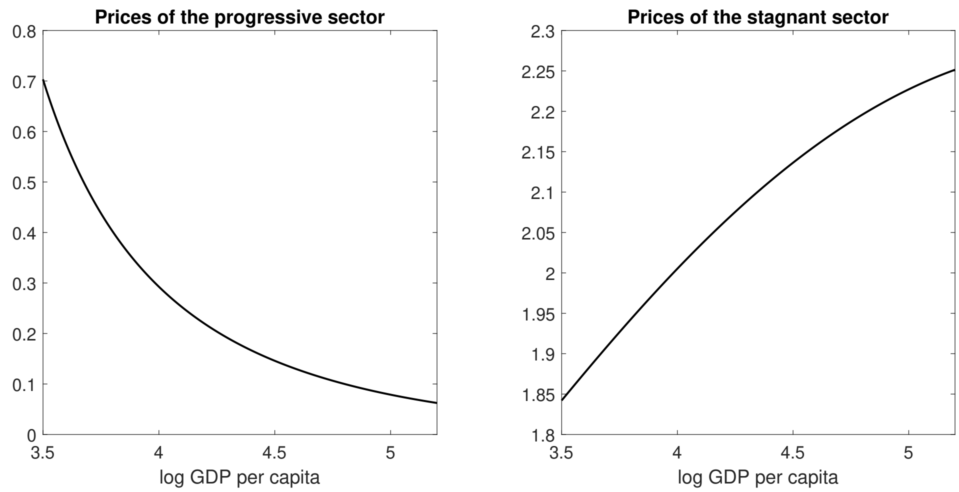


Figure 8: Transitional dynamics of the relative prices

As proved in Corollary 2 and in agreement with the data, we find that the relative price of the knowledge-intensive sector P_H/P is decreasing while the relative price of the stagnant sector P_L/P is increasing. When compared to the data given in Section 2, we clearly overestimate the decreasing rate of the knowledge-intensive sector relative price while we underestimate the increasing rate of stagnant sector relative price. The causes of the discrepancies concerning the relative prices can be twofold, first it can be due to our choice of ignoring demand side mechanisms. Differences in technical progress across sectors are the dominant forces behind structural transformations (see, e.g., Herrendorf et al. [26]). However, as shown by Swiecki [42], income effects caused by non-homothetic preferences may be the leading factor behind changes in relative prices. Second, EUKLEMS data cover a short period of 20 years and the price index available in figure 4 is only an aggregation of all the countries with a fixed effect for each of them. Therefore, we do not have prices data for a country that start with a log GDP per capita of 3.5 and end at 5.2, while our model is able to simulate such situation. The nature of prices data make it impossible to compare our model and the reality, as it have been done for κ, λ, nominal and real value added share.

9 Conclusion

Motivated by the recent dynamics of progressive sectors, we propose a two-sector model of non-balanced endogenous growth. The final good is produced through a CES technology using two intermediate sectors. The progressive sector is knowledge-intensive as in Romer

[40] and the accumulation of knowledge leads to an unbounded increase in TFP. Along the NBGP capital and labor reallocate across sectors. When intermediary capital goods are substitutable, the real and nominal shares of the "knowledge-intensive" sector are increasing while the real and nominal shares of the second sector are decreasing. Interestingly, the shares of capital and labor in the knowledge intensive sector depend on the initial value of capital and knowledge. As a consequence, countries with the same fundamentals but lower initial wealth will be characterized by lower asymptotic wealth. We finally provided a numerical illustration to characterize the transitional dynamics of the main variables.

In this paper we explore the dynamics of an economy in which productivity is endogenous and intermediate sectors are all substitutes in the production of a final output. The next step is to extend the economy to include also complementary intermediates. We expect that the general dynamics will depend on the details of the model as there are obvious conflicting tendencies. A realistic model would also include demand side effect generated by non-homotheticities in preferences, in the spirit of Comin *et al.* [19].

A more satisfactory theory of structural change would require deeper microeconomic foundations. A promising approach is to follow recent developments able to endogenize the linkages in an Input output economy.[17] Indeed, the way innovation pushes to new or better substitutes or complements depends on these details. Similarly, the role of automation on structural change through its impact on labor productivity could be precisely analyzed.[18]

10 Appendix

10.1 Sectors ranking

For the ranking of our sectors we use the EUKLEMS 2019 data, that cover the period 1995-2017 for Austria, Belgium, Bulgaria, Cyprus, Czech Republic, Germany, Denmark, Estonia, Greece, Spain, Finland, France, Croatia, Hungary, Ireland, Italy, Lithuania, Luxembourg, Latvia, Montenegro, Netherlands, Poland, Portugal, Romania, Sweden, Slovenia, Slovakia, United Kingdom, Japan and United States. Instead of relying on a temporal index and on a panel data analysis we decided to follow the literature of structural change by working with a log GDP per capita index to study structural change along the development process, see Herrendorf et al. [27] and Buera et al. [15] among others.

In order to determine if a sector should be considered as "progressive" or "stagnant" we decided to look at the evolution of compensation per worker. Our idea was simple, if we consider that workers are paid according to their marginal productivity, if the compensation per worker increases more rapidly in one sector it means that workers are becoming more

[17] See for instance Acemoglu and Azar [2], Carvalho and Tahbaz-Salehi [17], Ghiglino [24], Miranda-Pinto [35], Oberfield [39].

[18] See Acemoglu and Restrepo [5].

productive. However, it might be due to a capital deepening mechanism, if a sector provide more capital to its workers, they will be more productive. Therefore we decide to "deflate" compensation growth by capital deepening. In fine, we were able to compute compensation increase that was due to workers productivity itself. Then we compute the average growth of compensation per worker in each sector, and industries that perform better than average are then belonging to the "progressive" sector. Industries composing the "progressive" services sector can be found in table 1.

Code	Industry name
G46	Wholesale trade, except of motor vehicle and motorcycles
H51	Air transport
H53	Postal and courier activities
J58-J60	Publishing, audio-visual and broadcasting activities
J61	Telecommunications
J62-J63	IT and other information services
K	Financial and insurances activities
M_N	Professional, scientific, technical, administrative and support service activities

Table 1: Industries in the progressive sector

These industries are composing the progressie services sector of our economy, and every time we refer to the "progressive" or "knowledge intensive" sector, we will refer to this list of industries.

10.2 Proof of Proposition 1

The Hamiltonian in current value associated to the optimization problem (7) is (we omit subscript for t to simplify notations):

$$\begin{aligned}
\mathbb{H} = & \; N_H \frac{c_H^{1-\theta}}{1-\theta} + N_L \frac{c_L^{1-\theta}}{1-\theta} + P_H\Big[(L_H a)^\alpha K_H^{1-\alpha} - Y_H\Big] + P_L\Big[L_L^\beta K_L^{1-\beta} - Y_L\Big] \\
& + P\Big[\Big(\gamma Y_H^{\frac{\epsilon-1}{\epsilon}} + (1-\gamma)Y_L^{\frac{\epsilon-1}{\epsilon}}\Big)^{\frac{\epsilon}{\epsilon-1}} - \delta K - N_H c_H - N_L c_L\Big] + Q\Big[z(1-u) - \eta\Big]a \\
& + \mu_K\left[K - K_H - K_L\right] + \mu_L[L - L_H - L_L]
\end{aligned}$$

with $L_H = uN_H$ and μ_K and μ_L the Lagrange multipliers associated with the capital and labor market clearing conditions. The first-order conditions with respect to the control variables $c_H, c_L, u, L_H, L_L, K_H, K_L, Y_H$ and Y_L give:

$$
\begin{aligned}
c_i^{-\theta} &= P \text{ for any } i = H, L \quad (14)\\
P_H \alpha \frac{Y_H}{L_H a} N_H &= Qz \quad (15)\\
P_H &= P\gamma \left(\frac{Y}{Y_H}\right)^{\frac{1}{\epsilon}} \quad (16)\\
P_L &= P(1-\gamma) \left(\frac{Y}{Y_L}\right)^{\frac{1}{\epsilon}} \quad (17)\\
\mu_K = P_H(1-\alpha)\frac{Y_H}{K_H} &= P_L(1-\beta)\frac{Y_L}{K_L} \quad (18)\\
\mu_L = P_H \alpha \frac{Y_H}{L_H} &= P_L \beta \frac{Y_L}{L_L} \quad (19)
\end{aligned}
$$

Substituting (16) and (17) into (18) and (19) gives

$$
\begin{aligned}
(1-\alpha)\gamma \left(\frac{Y}{Y_H}\right)^{\frac{1}{\epsilon}} \frac{Y_H}{K_H} &= (1-\beta)(1-\gamma) \left(\frac{Y}{Y_L}\right)^{\frac{1}{\epsilon}} \frac{Y_L}{K_L} \quad (20)\\
\alpha\gamma \left(\frac{Y}{Y_H}\right)^{\frac{1}{\epsilon}} \frac{Y_H}{L_H} &= \beta(1-\gamma) \left(\frac{Y}{Y_L}\right)^{\frac{1}{\epsilon}} \frac{Y_L}{L_L} \quad (21)
\end{aligned}
$$

In the "knowledge-intensive sector", the rental rate of capital r_H and the individual wage rate w_H are given by

$$
r_H = (1-\alpha)\tfrac{Y_H}{K_H}, \quad w_H = \alpha \tfrac{Y_H}{N_H u a} \quad (22)
$$

while in the second sector, r_L and w_L are given by

$$
r_L = (1-\beta)\tfrac{Y_L}{K_L}, \quad w_L = \beta \tfrac{Y_L}{N_L} \quad (23)
$$

Note first that substituting (16) and (17) into (22) and (23) allows (21) to be written as follows

$$
w_H(t) a(t) P_H(t) = w_L(t) P_L(t)
$$

which gives the equality between nominal wages per hour devoted to production in the relevant sector. Similarly (20) is equivalent to

$$
r_H(t) P_H(t) = r_L(t) P_L(t) \quad (24)
$$

which gives the equality between the capital return in the two sectors. These two properties imply that despite the fact that agents work in different sectors, they all consume the same amount since $c_H(t) = c_L(t) = P^{-1/\theta}$ for any $t \geq 0$.

The equilibrium rental rate of capital can then be defined as

$$
\begin{aligned}
R(t) &= (1-\alpha)\gamma \left(\tfrac{Y}{Y_H}\right)^{\frac{1}{\epsilon}} \tfrac{Y_H}{K_H} = (1-\beta)(1-\gamma) \left(\tfrac{Y}{Y_L}\right)^{\frac{1}{\epsilon}} \tfrac{Y_L}{K_L}\\
&= \tfrac{P_K r_K}{P} = \tfrac{P_L r_L}{P}
\end{aligned} \quad (25)
$$

From the Hamiltonian, we also derive the optimality conditions that provide differential equations for the prices P and Q of aggregate capital and knowledge:

$$\begin{aligned} \dot{P} &= \rho P - \frac{\partial \mathbb{H}}{\partial K} = \rho P - \frac{\partial \mathbb{H}}{\partial K} = P(\rho+\delta) - P_H(1-\alpha)\frac{Y_H}{K_H} && (26) \\ \dot{Q} &= \rho Q - \frac{\partial \mathbb{H}}{\partial a} = -Q(z-\eta-\rho) + Qzu - P_H\alpha\frac{Y_H}{a} && (27) \end{aligned}$$

Substituting equation (16) into (26) and using (25) then gives

$$\dot{P} = -P\left[R-\delta-\rho\right] \tag{28}$$

Note now that using $L_H = uN_H$, equation (15) becomes $P_H\alpha Y_H/(ua) = Qz$. Substituting this expression into (27) gives

$$\dot{Q} = -Q(z-\eta-\rho) \tag{29}$$

Moreover, since $c_H(t) = c_L(t) = P^{-\frac{1}{\theta}}$, we get aggregate consumption as

$$C = C_H + C_L = N_H c_H + N_L c_L = NP^{-\frac{1}{\theta}}$$

We finally obtain the following differential equations for prices and stocks as given by (9)-(10). The result follows.

□

10.3 Proof of Theorem 1

From (10) we immediately get $g_a = z(1-u) - \eta$. Differentiating equation (14) gives using (9)

$$\begin{aligned} g_{c_H} = g_{c_L} = -\tfrac{1}{\theta} g_P &= \tfrac{1}{\theta}\left[(1-\alpha)\gamma\left(\tfrac{Y}{Y_H}\right)^{\frac{1}{\epsilon}}\tfrac{Y_H}{K_H} - \delta - \rho\right] \\ &= \tfrac{1}{\theta}\left[(1-\beta)(1-\gamma)\left(\tfrac{Y}{Y_L}\right)^{\frac{1}{\epsilon}}\tfrac{Y_L}{K_L} - \delta - \rho\right] \end{aligned} \tag{30}$$

It follows that $g_P = -\theta g_{c_H} = -\theta g_{c_L}$. Since g_P is constant along a NBGP, we get

$$g_{K_H} = \tfrac{\epsilon-1}{\epsilon} g_{Y_H} + \tfrac{1}{\epsilon} g_Y \tag{31}$$

and

$$g_{K_L} = \tfrac{\epsilon-1}{\epsilon} g_{Y_L} + \tfrac{1}{\epsilon} g_Y \tag{32}$$

The capital accumulation equation (6) can be written as

$$g_K = \tfrac{Y}{K} - \delta - \tfrac{NP^{-1/\theta}}{K}$$

Differentiating this expression using the fact that along a NBGP $\dot{g}_K = 0$ yields

$$g_Y = n - g_P/\theta \tag{33}$$

Since aggregate consumption is given by $C = NP^{-1/\theta}$ we conclude that $g_C = g_Y$. Differentiating (4) gives:

$$g_{Y_H} = \alpha g_a + \alpha N_H + (1-\alpha) g_{K_H} \tag{34}$$

Combining (15) and (16) and differentiating gives:

$$g_P + \tfrac{\epsilon-1}{\epsilon} g_{Y_H} + \tfrac{1}{\epsilon} g_Y - g_a = g_Q = -(z-\eta-\rho) \tag{35}$$

Now differentiating equations (20) and (21) yields:

$$\begin{aligned} \tfrac{\epsilon-1}{\epsilon} g_{Y_H} - g_{K_H} &= \tfrac{\epsilon-1}{\epsilon} g_{Y_L} - g_{K_L} \\ \tfrac{\epsilon-1}{\epsilon} g_{Y_H} - N_H &= \tfrac{\epsilon-1}{\epsilon} g_{Y_L} - N_L \end{aligned} \tag{36}$$

The differentiation of (5) gives

$$g_{Y_L} = \beta N_L + (1-\beta) g_{K_L} \tag{37}$$

and using (36) we get

$$g_{Y_L} = (1-\beta)\epsilon g_{K_H} + \beta\epsilon n + (1-\epsilon) g_{Y_H} \tag{38}$$

Equations (31)-(38) are not enough to determine explicitly the values of the growth rates. We need also to determine the relationship between the growth rate of K and K_H, L and L_H, Y and Y_H. We thus use the same methodology than in Acemoglu and Guerrieri.

Considering the first order conditions (15)-(19) allows to define the following maximized value of current output given the physical capital stock $K(t)$ and the human capital stock $a(t)$ at time t as

$$\Phi(K(t), a(t), t) \quad = \quad \max_{K_H(t), K_L(t), L_H(t), L_L(t), u(t)} \left(\gamma Y_H^{\frac{\epsilon-1}{\epsilon}} + (1-\gamma) Y_L^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}}$$

Let us define the shares of capital and labor allocated to the knowledge-intensive sector (sector H) as

$$\kappa(t) \equiv \tfrac{K_H(t)}{K(t)}, \qquad \lambda(t) \equiv \tfrac{L_H(t)}{L(t)} \tag{39}$$

We also have $1-\kappa(t) \equiv \frac{K_L(t)}{K(t)}$ and $1-\lambda(t) \equiv \frac{L_L(t)}{L(t)}$. And combining this statement with the equations (20) and (21) we obtain:

$$\kappa(t) = \left[1 + \left(\frac{1-\beta}{1-\alpha} \right) \left(\frac{1-\gamma}{\gamma} \right) \left(\frac{Y_L(t)}{Y_H(t)} \right)^{\frac{\epsilon-1}{\epsilon}} \right]^{-1} \tag{40}$$

and

$$\lambda(t) = \left[1 + \left(\frac{1-\alpha}{1-\beta} \right) \left(\frac{\beta}{\alpha} \right) \left(\frac{1-\kappa(t)}{\kappa(t)} \right) \right]^{-1} \tag{41}$$

Using equation (3), we can write the maximized value of current ouput $\Phi(K(t), a(t), t)$

as follows:

$$Y(t) = \Phi(K(t), a(t), t) = \psi(t)a(t)^{\alpha}\lambda(t)^{\alpha}\kappa(t)^{1-\alpha}L(t)^{\alpha}K(t)^{1-\alpha} \tag{42}$$

with

$$\psi(t) \quad = \quad \gamma^{\frac{\epsilon}{\epsilon-1}}\left(1+\frac{1-\alpha}{1-\beta}\frac{1-\kappa(t)}{\kappa(t)}\right)^{\frac{\epsilon}{\epsilon-1}} \tag{43}$$

Considering that $c_H = c_L = c$, we can rewrite the Hamiltonian in maximized value:

$$\mathbb{H}(c, K, a, P) = N\tfrac{c^{1-\theta}}{1-\theta} + P\Big[\Phi(K(t), a(t), t) - \delta K - Nc\Big]$$

and the first order conditions with respect to consumption gives:

$$\frac{\dot{c}}{c} = \frac{1}{\theta}\Big[(1-\alpha)\gamma\psi(t)^{\frac{1}{\epsilon}}a^{\alpha}\lambda^{\alpha}\kappa^{-\alpha}L^{\alpha}K^{-\alpha} - \delta - \rho\Big] = -\frac{1}{\theta}\frac{\dot{P}}{P} \tag{44}$$

Since along the NBGP $\dot{g_c} = 0$, we get:

$$\tfrac{1}{\epsilon}\tfrac{\dot{\psi}(t)}{\psi(t)} + \alpha g_a + \alpha\tfrac{\dot{\lambda}(t)}{\lambda(t)} - \alpha\tfrac{\dot{\kappa}(t)}{\kappa(t)} + \alpha n - \alpha g_K = 0$$

Differentiating equations (41) and (43) gives

$$\begin{aligned}
\tfrac{\dot{\psi}(t)}{\psi(t)} &= \tfrac{\dot{\kappa}(t)}{\kappa(t)}\left(\tfrac{\epsilon(1-\alpha)}{[\Delta\kappa+(1-\alpha)](1-\epsilon)}\right)\\
\tfrac{\dot{\lambda}(t)}{\lambda(t)} &= \tfrac{\dot{\kappa}(t)}{\kappa(t)}\left(\tfrac{(1-\beta)\alpha}{(1-\alpha)\beta} + \tfrac{1-\kappa}{\kappa}\right)^{-1}\\
\tfrac{\dot{K}(t)}{K(t)} &= \tfrac{\dot{\kappa}(t)}{\kappa(t)}\left[\tfrac{(1-\alpha)\epsilon}{\alpha(1-\epsilon)[\Delta\kappa+1-\alpha]} + \left(\tfrac{(1-\beta)\alpha}{(1-\alpha)\beta} + \tfrac{1-\kappa}{\kappa}\right)^{-1}\right] + \Big(z(1-u) - \eta + n\Big)
\end{aligned}$$

where $\Delta = \alpha - \beta$. Now differentiating (40) and substituting with what we have just found allow to write the law of motion of κ as follows:

$$\tfrac{\dot{\kappa}(t)}{\kappa(t)} \quad = \quad G(\kappa(t))\beta g_a \tag{45}$$

with

$$G(\kappa(t)) \quad = \quad \tfrac{(1-\kappa)(\epsilon-1)}{\epsilon+(\epsilon-1)\left(\Delta KS(1-\kappa)-\delta\kappa-(1-\alpha)-\lambda S(\alpha-\lambda\Delta)\left(\frac{(1-\alpha)\beta+\Delta\kappa}{(1-\alpha)\beta}\right)\right)} \tag{46}$$

where

$$KS = \tfrac{(1-\alpha)\epsilon}{\alpha(1-\epsilon)[\Delta\kappa+1-\alpha]} + \left(\tfrac{(1-\beta)\alpha}{(1-\alpha)\beta} + \tfrac{1-\kappa}{\kappa}\right)^{-1}, \quad \lambda S = \left(\tfrac{(1-\beta)\alpha}{(1-\alpha)\beta} + \tfrac{1-\kappa}{\kappa}\right)^{-1} \tag{47}$$

It is then easy to check under $\alpha, \beta > 1/2$ that $G(0) > 0$, $G(1) = 0$ and $G'(\kappa) < 0$ for any κ. This implies that equation (45) has a unique solution such that $\lim_{t\to\infty}\kappa(t) = \kappa^* = 1$. Using this asymptotic value into λ and ψ give the following results: $\lim_{t\to\infty}\lambda(t) = \lambda^* = 1$ and $\lim_{t\to\infty}\psi(t) = \psi^* = \gamma^{\frac{\epsilon}{\epsilon-1}}$.

We conclude therefore that $g_K = g_{K_H}$ and $n = n_H$, and using the maximized value of the output $\Phi(K(t), a(t), t)$ as in given by (42), we obtain $Y(t) = \psi^* Y_H(t)$, which gives $g_{Y_H} = g_Y$. Now we can replace all these equalities into the equations (31)-(38) to obtain the explicit values of the growth rates. From (31), (33) and (34), we have $g_K = g_Y$, $g_P = \theta(n - g_K)$ and

$g_K = g_a + n$. Using (35) we then get $g_P = g_Q - n = -(z - \eta - \rho + n)$ and thus

$$g_a = \frac{z-\eta-\rho+n}{\theta} \text{ and } g_K = n + \frac{z-\eta-\rho+n}{\theta}$$

From (36) and (38) we derive

$$g_{Y_L} = n + (1-\beta\epsilon)g_a,\, g_{K_L} = n + (1+\beta-\beta\epsilon)g_a \text{ and } n_L = n + (1-\epsilon)\beta g_a$$

Using $g_a = z(1-u) - \eta$ we finally obtain:

$$u^* = \frac{1}{z}\left[z - \eta - \frac{1}{\theta}(n - \rho + z - \eta)\right]$$

and from (9)

$$R^* = \delta + \rho - g_P = \delta + \rho - \theta(n - g_K)$$

□

10.4 Proof of Corollary 1

Let us consider the first-order conditions (15)-(21) and Theorem 1.

i) We derive from Theorem 1 that the growth rate of K/L is equal to

$$g_{K/L} = g_K - n = g_a > 0$$

ii) This result is obvious from Theorem 1.

iii) Along the NBGP the capital-output ratio is given by

$$g_{K/Y} = g_K - g_Y = 0$$

iv) The share of capital income in GDP is given by

$$s_K = \frac{RK}{Y} = (1-\alpha)\gamma\left(\frac{Y}{Y_H}\right)^{\frac{1-\epsilon}{\epsilon}}\frac{1}{\kappa}$$

Along the NBGP with $\kappa = 1$ and $Y = \psi^* Y_H$ we then get

$$s_K = (1-\alpha)$$

v) This result is obvious from Theorem 1.

vi) From (16) and (17) we derive that

$$\frac{P_H}{P} = \gamma\left(\frac{Y}{Y_H}\right)^{\frac{1}{\epsilon}},\text{ and } \frac{P_L}{P} = (1-\gamma)\left(\frac{Y}{Y_L}\right)^{\frac{1}{\epsilon}} \tag{48}$$

Since $g_{Y_H} = g_Y > g_{Y_L}$, we conclude that $g_{P_H/P} = 0$, $g_{P_L/P} > 0$ and thus $g_{P_L/P_H} > 0$.

vii) From the results of point vi), we immediately derive that the real shares of the knowledge-intensive and the second sectors in GDP, Y_H/Y and Y_L/Y, are respectively constant and decreasing. It is also obvious that the nominal shares of sector H, $P_H Y_H / PY$, is constant. However, we derive from point vi) that

$$g_{\frac{P_L Y_L}{PY}} = \frac{1-\epsilon}{\epsilon}(g_Y - g_{Y_L}) < 0$$

which implies that the nominal share of sector L, $P_L Y_L / PY$, is decreasing.

□

10.5 Proof of Theorem 2

Basically, we use the same type of methodology to find the NBGP when inputs are complementary. We define however in a different way the output of the final good as follows

$$Y(t) = \phi(t)(1-\lambda)^{\beta}(1-\kappa)^{1-\beta}K^{1-\beta}L^{\beta} \tag{49}$$

with

$$\phi = (1-\gamma)^{\frac{\epsilon}{\epsilon-1}}\left(1 + \left(\frac{1-\beta}{1-\alpha}\right)\left(\frac{\kappa}{1-\kappa}\right)\right)^{\frac{\epsilon}{\epsilon-1}}$$

To simplify notations let us define $v = 1-\kappa$ and $w = 1-\lambda$.

Using again the hamiltonian in maximized value, and the fact that along the NBGP $g_{\dot{C}} = 0$, we obtain:

$$\frac{\dot{\phi}(t)}{\phi(t)} + \beta\frac{\dot{w}(t)}{w(t)} - \beta\frac{\dot{v}(t)}{v(t)} + \beta\frac{\dot{L}(t)}{L(t)} - \beta\frac{\dot{K}(t)}{K(t)} = 0$$

We need to transform equations (20) and (21) to express them in terms of v and w, then we differentiate these transformed equations and also the definition of ϕ. We modify the formulations and we express them in terms of $\frac{\dot{v}}{v}$. Once we have these 3 differential equations we use the previous equality to express $\frac{\dot{K}}{K}$ in terms of $\frac{\dot{v}}{v}$. We have:

$$\begin{aligned}
\frac{\dot{v}(t)}{v(t)} &= -\frac{\epsilon-1}{\epsilon}(1-v)(g_{Y_H} - g_{Y_L}) \\
\frac{\dot{\phi}(t)}{\phi(t)} &= \frac{\dot{v}(t)}{v(t)}\left(\frac{\epsilon(1-\beta)}{[\Delta v + (1-\alpha)](1-\epsilon)}\right) \equiv D_1\frac{\dot{v}(t)}{v(t)} \\
\frac{\dot{w}(t)}{w(t)} &= \frac{\dot{v}(t)}{v(t)}\left(\frac{(1-\alpha)\beta}{(1-\beta)\alpha} + \frac{1-v}{v}\right)^{-1} \equiv D_2\frac{\dot{v}(t)}{v(t)} \\
\frac{\dot{K}(t)}{K(t)} &= \frac{1}{\beta}\frac{\dot{\phi}(t)}{\phi(t)} + \frac{\dot{w}(t)}{w(t)} - \frac{\dot{v}(t)}{v(t)} + n
\end{aligned}$$

The last step is to prove that $\frac{\dot{v}(t)}{v} = 0$ when $v = 1$ implying that $\kappa = 0$ and $\lambda = 0$. To this end, we compute $(g_{Y_H} - g_{Y_L})$ and we obtain the following equation:

$$\frac{\dot{v}(t)}{v(t)} = \frac{(1-\epsilon)\alpha g_a (1-v)}{\epsilon + (1-\epsilon)\left(D_2\alpha\frac{1-v}{1-w} + 1 - \alpha + \frac{\Delta}{\beta}(1-v)D_1\right)} \equiv G(v)$$

Under $\alpha, \beta > 1/2$, $G(.)$ is a positive and either decreasing or hump-shaped function in v, with $G(0) > 0$, $G'(1) < 0$ and $G(1) = 0$. As v is defined over a compact subset $[0,1]$, we conclude that $v = 1$ is the steady state value of the model, which implies that $\kappa^* = 0$, $\lambda^* = 0$ and $\phi^* = (1-\gamma)^{\frac{\epsilon}{\epsilon-1}}$. We conclude therefore that $g_K = g_{K_L}$ and $n = n_L$, and using equation (49) we obtain $Y(t) = \phi^* Y_L(t)$, which gives $g_{Y_L} = g_Y$. Now we can replace all these equalities into the equations (31)-(38) to obtain the explicit values of the growth rates. From (37) and

(32), (33), we have $g_K = g_Y$, $g_K = n$ and $g_P = 0$. From (36) and (34) we get $g_{K_H} = n_H$, $g_{Y_H} = \epsilon\alpha g_a + n$ and $n_H = n + (\epsilon - 1)g_a$. Using (35) we get $n - g_Q = g_a[1 + \alpha(1 - \epsilon)]$ and thus

$$g_a = \frac{z-\eta-\rho+n}{1+\alpha(1-\epsilon)} \text{ and } n_H = \frac{n[\epsilon+\alpha(1-\epsilon)]-(1-\epsilon)(z-\eta-\rho)}{1+\alpha(1-\epsilon)}$$

Using $g_a = z(1-u) - \eta$ we finally obtain:

$$u^* = \frac{1}{z}\left[z - \eta - \frac{z-\eta-\rho+n}{1+\alpha(1-\epsilon)}\right]$$

and from (9)

$$R^* = \delta + \rho - g_P = \delta + \rho$$

□

10.6 Proof of Lemma 1

Let us consider the stationarized values for $K(t)$, $a(t)$ and $P(t)$ as defined by $k(t) = K(t)e^{-g_K t}$, $x(t) = a(t)e^{-g_a t}$ and $p(t) = P(t)e^{-g_P t}$, for all $t \geq 0$. Recall also that as population is growing at the exponential rate n, we have $N(t) = e^{nt}N(0)$ with $N(0) = N_0$ given. Let Assumption 1 hold and let us substitute the maximized output $Y(t) = \Phi(K(t), a(t), t)$ as given by (42) into equations (23) and (26). We obtain

$$\begin{aligned}
\frac{\dot{P}}{P} &= -\left[(1-\alpha)\gamma\psi^{\frac{1}{\epsilon}}x^{\alpha}\left(\frac{L_H}{K_H}\right)^{\alpha} - \rho - \delta\right] \\
\frac{\dot{K}}{K} &= \psi a^{\alpha}\left(\frac{L_H}{K_H}\right)^{\alpha}\frac{K_H}{K} - \delta - \frac{NP^{-\frac{1}{\theta}}}{K}
\end{aligned}$$

We need to prove that $L_H = L_H(k, a, p, q_0, N_0)$, $K_H = K_H(k, a, p, q_0, N_0)$ and $u = u(k, a, p, q_0, N_0)$, and to compute the derivatives of these functions. From (14) and (15) we derive:

$$\begin{aligned}
(14) \quad &\Leftrightarrow \quad \frac{\gamma(1-\alpha)}{(1-\gamma)(1-\beta)}\frac{[(L_H a)^{\alpha}K_H^{1-\alpha}]^{\frac{\epsilon-1}{\epsilon}}}{K_H} = \frac{[(L-L_H)^{\beta}(K-K_H)^{1-\beta}]^{\frac{\epsilon-1}{\epsilon}}}{K-K_H} \\
(15) \quad &\Leftrightarrow \quad \frac{\gamma\alpha}{(1-\gamma)\beta}\frac{[(L_H a)^{\alpha}K_H^{1-\alpha}]^{\frac{\epsilon-1}{\epsilon}}}{L_H} = \frac{[(L-L_H)^{\beta}(K-K_H)^{1-\beta}]^{\frac{\epsilon-1}{\epsilon}}}{L-L_H} \\
\frac{(14)}{(15)} \quad &\Leftrightarrow \quad \frac{B_H}{B_L}L_H(K-K_H) = K_H(L-L_H)
\end{aligned}$$

with $B_H = \frac{\gamma(1-\alpha)}{(1-\gamma)(1-\beta)}$ and $B_L = \frac{\gamma\alpha}{(1-\gamma)\beta}$. Let us then denote

$$\begin{aligned}
\omega_H &= \frac{B_H}{B_L}L_H(K-K_H) - K_H(L-L_H) = 0 \\
\omega_L &= B_H\frac{[(L_H a)^{\alpha}K_H^{1-\alpha}]^{\frac{\epsilon-1}{\epsilon}}}{K_H} - \frac{[(L-L_H)^{\beta}(K-K_H)^{1-\beta}]^{\frac{\epsilon-1}{\epsilon}}}{K-K_H} = 0
\end{aligned}$$

If the matrix

$$J_1 = \begin{pmatrix} \frac{\partial\omega_H}{\partial K_H} & \frac{\partial\omega_H}{\partial L_H} \\ \frac{\partial\omega_L}{\partial K_H} & \frac{\partial\omega_L}{\partial L_H} \end{pmatrix}$$

is non-singular, there exists locally unique functions $K_H = \tilde{K}_H(K, a, L)$ and $L_H = \tilde{L}_H(K, a, L)$, with:

$$\begin{pmatrix} \frac{\partial \tilde{K}_H}{\partial K} & \frac{\partial \tilde{K}_H}{\partial a} & \frac{\partial \tilde{K}_H}{\partial L} \\ \frac{\partial \tilde{L}_H}{\partial K} & \frac{\partial \tilde{L}_H}{\partial a} & \frac{\partial \tilde{L}_H}{\partial L} \end{pmatrix} = J_1^{-1} \begin{pmatrix} \frac{\partial \omega_H}{\partial K} & \frac{\partial \omega_H}{\partial a} & \frac{\partial \omega_H}{\partial L} \\ \frac{\partial \omega_L}{\partial K} & \frac{\partial \omega_L}{\partial a} & \frac{\partial \omega_L}{\partial L} \end{pmatrix}$$

Tedious but straightforward computations then give

$$\begin{aligned}
\frac{\partial \tilde{K}_H}{\partial K} &= \frac{\tilde{K}_H}{Y}\Big[(\alpha-\beta)(\epsilon-1)(L-L_H)-L\Big] \\
\frac{\partial \tilde{K}_H}{\partial a} &= -\frac{1}{Y}\alpha(\epsilon-1)(K-\tilde{K}_H)\tilde{K}_H L\frac{1}{a} \\
\frac{\partial \tilde{K}_H}{\partial L} &= -\frac{\tilde{K}_H}{Y}(\epsilon-1)(\alpha-\beta)(K-\tilde{K}_H) \\
\frac{\partial \tilde{L}_H}{\partial K} &= \frac{1}{Y}\frac{B_H\tilde{L}_H}{B_L\tilde{K}_H}\tilde{L}_H(\alpha-\beta)(\epsilon-1)(K-\tilde{K}_H) \\
\frac{\partial \tilde{L}_H}{\partial a} &= -\frac{1}{Y}\frac{B_H\tilde{L}_H}{B_L\tilde{K}_H}\alpha(\epsilon-1)K\tilde{L}_H(K-\tilde{K}_H)\frac{1}{a} \\
\frac{\partial \tilde{L}_H}{\partial L} &= \frac{\tilde{L}_H}{Y}\Big[(\alpha-\beta)(\epsilon-1)(K-\tilde{K}_H)-K\Big]
\end{aligned}$$

with $Y = Y(L, K, L_H, K_H) = \Big[(\epsilon-1)(\alpha-\beta)(\frac{K_H}{K}-\frac{L_H}{L})-1\Big]LK$.

We need now to prove that $K_H = K_H(K, a, P, Q, N)$ and $L_H = L_H(K, a, P, Q, N)$. We have just shown that $K_H = \tilde{K}_H(K, a, L)$ and $L_H = \tilde{L}_H(K, a, L)$. We use market clearing conditions to assess $L = N - (1-u)N_H$ in order to obtain $K_H = \tilde{L}_H(K, a, N-(1-u)N_H)$ and $L_H = \tilde{L}_H(K, a, N-(1-u)N_H)$. The first step is to prove that $N_H = N_H(K, a, u, N)$, and thus $u = u(K, a, P, Q, N)$. Let us denote $H \equiv uN_H - \tilde{L}_H(K, a, N-(1-u)N_H) = 0$. Assuming $\partial H/\partial N_H \neq 0$ implies that here exists a locally unique function $N_H = \tilde{N}_1(K, a, u, N)$ such that

$$\frac{\partial \tilde{N}_1}{\partial K} = \frac{\frac{\partial \tilde{L}_H}{\partial K}}{u+(1-u)\frac{\partial \tilde{L}_H}{\partial L}}, \quad \frac{\partial \tilde{N}_1}{\partial a} = \frac{\frac{\partial \tilde{L}_H}{\partial a}}{u+(1-u)\frac{\partial \tilde{L}_H}{\partial L}}$$

$$\frac{\partial \tilde{N}_1}{\partial N} = \frac{\frac{\partial \tilde{L}_H}{\partial L}}{u+(1-u)\frac{\partial \tilde{L}_H}{\partial L}}, \quad \frac{\partial \tilde{N}_1}{\partial u} = -\frac{\tilde{N}_1(1-\frac{\partial \tilde{L}_H}{\partial L})}{u+(1-u)\frac{\partial \tilde{L}_H}{\partial L}}$$

Let us consider now equation (15) such that:

$$G(K, a, u, P, Q, N) = \frac{\alpha\gamma}{zau}PY(K, a, u, N)^{\frac{1}{\epsilon}}Y_H(K, a, u, N)^{\frac{\epsilon-1}{\epsilon}} - Q = 0$$

Assuming $\partial G/\partial u \neq 0$, there exists a locally unique function $u = u(K, a, P, Q, N)$ such that:

$$\frac{\partial u}{\partial K} = -\frac{\frac{\partial G}{\partial K}}{\frac{\partial G}{\partial u}}; \quad \frac{\partial u}{\partial a} = -\frac{\frac{\partial G}{\partial a}}{\frac{\partial G}{\partial u}}; \quad \frac{\partial u}{\partial P} = -\frac{\frac{\partial G}{\partial P}}{\frac{\partial G}{\partial u}}; \quad \frac{\partial u}{\partial Q} = -\frac{\frac{\partial G}{\partial Q}}{\frac{\partial G}{\partial u}}; \quad \frac{\partial u}{\partial N} = -\frac{\frac{\partial G}{\partial N}}{\frac{\partial G}{\partial u}}$$

with:

$$\begin{aligned}
\frac{\partial G}{\partial K} &= Q\Big[\frac{1}{\epsilon}\frac{1}{Y}\frac{\partial Y}{\partial K} + \frac{1-\epsilon}{\epsilon}\frac{1}{Y_H}\frac{\partial Y_H}{\partial K}\Big],\ \frac{\partial G}{\partial a} = Q\Big[-\frac{1}{a} + \frac{1}{\epsilon}\frac{1}{Y}\frac{\partial Y}{\partial a} + \frac{1-\epsilon}{\epsilon}\frac{1}{Y_H}\frac{\partial Y_H}{\partial a}\Big],\ \frac{\partial G}{\partial P} = \frac{Q}{P}, \\
\frac{\partial G}{\partial u} &= Q\Big[\frac{1}{\epsilon}\frac{1}{Y}\frac{\partial Y}{\partial u} + \frac{1-\epsilon}{\epsilon}\frac{1}{Y_H}\frac{\partial Y_H}{\partial u} - \frac{1}{u}\Big],\ \frac{\partial G}{\partial Q} = -1,\ \frac{\partial G}{\partial N} = Q\Big[\frac{1}{\epsilon}\frac{1}{Y}\frac{\partial Y}{\partial N} + \frac{1-\epsilon}{\epsilon}\frac{1}{Y_H}\frac{\partial Y_H}{\partial N}\Big]
\end{aligned}$$

Let us then denote

$$\begin{aligned} & K_H = \tilde{K}_H(K, a, N - (1 - u(K, a, P, Q, N)\tilde{N}_1(K, a, u(K, a, P, Q, N))) \\ \Leftrightarrow \quad & K_H = K_H(K, a, P, Q, N) \\ & L_H = \tilde{L}_H(K, a, N - (1 - u(K, a, P, Q, N)\tilde{N}_1(K, a, u(K, a, P, Q, N))) \\ \Leftrightarrow \quad & L_H = L_H(K, a, P, Q, N) \end{aligned} \tag{50}$$

Tedious but straightforward computations allow therefore to express all the derivatives we need. For example, considering the derivatives with respect to K we get:

$$\begin{aligned} \frac{\partial Y}{\partial K} &= \left(\gamma \frac{\partial Y_H}{\partial K} Y_H^{-\frac{1}{\epsilon}} + (1-\gamma) \frac{\partial Y_L}{\partial K} Y_L^{-\frac{1}{\epsilon}}\right) Y^{\frac{1}{\epsilon}} \\ \frac{\partial Y_H}{\partial K} &= Y_H \left(\alpha \frac{1}{L_H} \frac{\partial L_H}{\partial K} + (1-\alpha) \frac{1}{K_H} \frac{\partial K_H}{\partial K}\right) \\ \frac{\partial Y_L}{\partial K} &= -Y_L \left(\beta \frac{1}{L - L_H} \frac{\partial L_H}{\partial K} + (1-\beta) \frac{1}{K - K_H} \frac{\partial K_H}{\partial K}\right) \\ \frac{\partial L_H}{\partial K} &= \frac{\partial \tilde{L}_H}{\partial K} \left(\frac{u}{u + (1-u) \frac{\partial \tilde{L} L_H}{\partial L}}\right) \\ \frac{K_H}{\partial K} &= \frac{\partial \tilde{K}_H}{\partial K} - (1-u) \frac{\partial \tilde{K}_H}{\partial L} \frac{\frac{\partial \tilde{L}_H}{\partial K}}{u + (1-u) \frac{\partial \tilde{L}_H}{\partial L}} \end{aligned}$$

We do similar computations to obtain all the derivatives K_H and L_H with respect to K, a, P, Q and N. Recalling that N_H as follow: $N_H = \tilde{N}_1(K, a, u(K, a, P, Q, N), N) = N_H(K, a, P, Q, N)$ we finally derive:

$$\begin{aligned} \frac{\partial N_H}{\partial K} &= \frac{\partial \tilde{N}_1}{\partial K} + \frac{\partial \tilde{N}_1}{\partial u} \frac{\partial u}{\partial K}, \ \frac{\partial N_H}{\partial A} = \frac{\partial \tilde{N}_1}{\partial A} + \frac{\partial \tilde{N}_1}{\partial u} \frac{\partial u}{\partial A}, \ \frac{\partial N_H}{\partial P} = \frac{\partial \tilde{N}_1}{\partial u} \frac{\partial u}{\partial P}, \\ \frac{\partial N_H}{\partial Q} &= \frac{\partial \tilde{N}_1}{\partial u} \frac{\partial u}{\partial Q}, \ \frac{\partial N_H}{\partial N} = \frac{\partial \tilde{N}_1}{\partial N} + \frac{\partial \tilde{N}_1}{\partial u} \frac{\partial u}{\partial N} \end{aligned}$$

Using these results into (50) we obtain:

$$\begin{aligned} \frac{\partial K_H}{\partial K} &= \frac{\partial \tilde{K}_H}{\partial K} + \frac{\partial \tilde{K}_H}{\partial L} \left[\frac{\partial u}{\partial K} N_H - (1-u) \frac{\partial N_H}{\partial K}\right] \\ \frac{\partial K_H}{\partial A} &= \frac{\partial \tilde{K}_H}{\partial A} + \frac{\partial \tilde{K}_H}{\partial L} \left[\frac{\partial u}{\partial A} N_H - (1-u) \frac{\partial N_H}{\partial A}\right] \\ \frac{\partial K_H}{\partial P} &= \frac{\partial \tilde{K}_H}{\partial L} \left[\frac{\partial u}{\partial P} N_H - (1-u) \frac{\partial N_H}{\partial P}\right] \\ \frac{\partial K_H}{\partial Q} &= \frac{\partial \tilde{K}_H}{\partial L} \left[\frac{\partial u}{\partial Q} \tilde{N}_H - (1-u) \frac{\partial N_H}{\partial Q}\right] \\ \frac{\partial K_H}{\partial N} &= \frac{\partial \tilde{K}_H}{\partial N} + \frac{\partial \tilde{K}_H}{\partial L} \left[\frac{\partial u}{\partial N} N_H - (1-u) \frac{\partial N_H}{\partial N}\right] \end{aligned}$$

and

$$
\begin{aligned}
\frac{\partial L_H}{\partial K} &= \frac{\partial \tilde{L}_H}{\partial K} + \frac{\partial \tilde{L}_H}{\partial L}\left[\frac{\partial u}{\partial K}N_H - (1-u)\frac{\partial N_H}{\partial K}\right] \\
\frac{\partial L_H}{\partial A} &= \frac{\partial \tilde{L}_H}{\partial A} + \frac{\partial \tilde{L}_H}{\partial L}\left[\frac{\partial u}{\partial A}N_H - (1-u)\frac{\partial N_H}{\partial A}\right] \\
\frac{\partial L_H}{\partial P} &= \frac{\partial \tilde{L}_H}{\partial L}\left[\frac{\partial u}{\partial P}N_H - (1-u)\frac{\partial N_H}{\partial P}\right] \\
\frac{\partial L_H}{\partial Q} &= \frac{\partial \tilde{L}_H}{\partial L}\left[\frac{\partial u}{\partial Q}N_H - (1-u)\frac{\partial N_H}{\partial Q}\right] \\
\frac{\partial L_H}{\partial N} &= \frac{\partial \tilde{L}_H}{\partial N} + \frac{\partial \tilde{L}_H}{\partial L}\left[\frac{\partial u}{\partial N}N_H - (1-u)\frac{\partial N_H}{\partial N}\right]
\end{aligned}
$$

We have proved therefore that K_H and L_H are functions of K, a, P, Q and N, and we have all the derivatives of our main variables. Using the property that a homogeneous of degree 1 CES function generates input demand functions which are homogeneous of degree 0, we state finally:

$$
\begin{aligned}
K_H(K, a, P, Q, N) &= K_H(k, x, p, q_0, N_0) \\
L_H(K, a, P, Q, N) &= L_H(k, x, p, q_0, N_0) \\
u(K, a, P, Q, N) &= u(k, x, p, q_0, N_0)
\end{aligned}
$$

The stationarized dynamical system (13) is then easily derived. □

10.7 Proof of Theorem 3

Consider the stationarized dynamical system as given by (13). Using (42) we can rewrite it as follows

$$
\begin{aligned}
\frac{\dot{p}}{p} &= -\left[(1-\alpha)\gamma\psi^{\frac{1}{\epsilon}}x^{\alpha}\lambda^{\alpha}\kappa^{1-\alpha}l^{\alpha}k^{-\alpha} + g_P - \rho - \delta\right] \\
\frac{\dot{k}}{k} &= \psi x^{\alpha}\lambda^{\alpha}\kappa^{1-\alpha}l^{\alpha}k^{-\alpha} - \delta - g_K - \frac{N_0 p^{-\frac{1}{\theta}}}{k} \\
\frac{\dot{x}}{x} &= z(1-u) - g_a - \eta
\end{aligned}
\tag{51}
$$

A steady-state is therefore a solution of the following system

$$
(1-\alpha)\gamma\psi^{\frac{1}{\epsilon}}x^{\alpha}\lambda^{\alpha}\kappa^{-\alpha}l^{\alpha}k^{-\alpha} = \rho + \delta - g_P \tag{52}
$$

$$
\psi x^{\alpha}\lambda^{\alpha}\kappa^{1-\alpha}l^{\alpha}k^{-\alpha} = \delta + g_K + \frac{N_0 p^{-\frac{1}{\theta}}}{k} \tag{53}
$$

$$
z(1-u) = g_a + \eta \tag{54}
$$

From (54) we get $u = (z - g_a - \eta)/z \equiv u^*$ as given by (**??**). Since at steady state we have $\lambda^* = 1$ we derive $L_H = L$ and $N_H = N$. Recalling that $L = N - (1-u)N_H$, we get $l^* = u^* N_0$ with N_0 the initial value of $N(t)$. Taking the ratio of (53) on (52) gives after simplification

$$g_K + \delta + \frac{N_0 p^{-\frac{1}{\theta}}}{k} = \frac{\delta+\rho-g_P}{(1-\alpha)\gamma}\kappa\psi^{\frac{\epsilon-1}{\epsilon}} \tag{55}$$

Using the fact that $\kappa^* = 1$ and $\psi^* = \gamma^{\frac{\epsilon}{\epsilon-1}}$, substituting (55) into (53) and solving for k gives

$$k = \left(\frac{(1-\alpha)\gamma^{\frac{\epsilon}{\epsilon-1}}}{\rho+\delta-g_P}\right)^{\frac{1}{\alpha}} l^* x \equiv \mathcal{Z}_1 x \tag{56}$$

Solving (55) with respect to p using (56) gives

$$p = \left[\frac{(1-\alpha)N_0}{[\delta+\rho-g_P-(1-\alpha)(\delta+g_K)]\mathcal{Z}_1}\right]^{\theta} x^{-\theta} \equiv \mathcal{Z}_2 x^{-\theta} \tag{57}$$

Consider finally equation (15) which can be written at the steady state as

$$p\gamma\psi^{\frac{1}{\epsilon}}\alpha l^{\alpha} x^{\alpha} l^{\alpha} k^{1-\alpha} = q_0 z u x \tag{58}$$

Substituting (56) and (57) into (58) and solving for x finally gives

$$x^* = \left(\frac{\mathcal{Z}_2 \mathcal{Z}_1^{1-\alpha}\gamma^{\frac{\epsilon}{\epsilon-1}}\alpha l^{*\alpha}}{q_0 z u^*}\right)^{\frac{1}{\theta}} \equiv x^*(q_0) \tag{59}$$

Therefore, substituting (59) into (57) and (56), we find

$$\begin{aligned} k^* &= \mathcal{Z}_1\left(\frac{\mathcal{Z}_2 \mathcal{Z}_1^{1-\alpha}\gamma^{\frac{\epsilon}{\epsilon-1}}\alpha l^{*\alpha}}{q_0 z u^*}\right)^{\frac{1}{\theta}} \equiv k^*(q_0) \\ p^* &= \frac{q_0 z u^*}{Z_1^{1-\alpha}\gamma^{\frac{\epsilon}{\epsilon-1}}\alpha l^{*\alpha}} \equiv p^*(q_0) \end{aligned} \tag{60}$$

We conclude that for any given $q_0 > 0$, there exists a unique steady state $(k^*(q_0), x^*(q_0), p^*(q_0))$ with $k^{*'}(q_0) < 0$, $x^{*'}(q_0) < 0$ and $p^{*'}(q_0) > 0$.

□

10.8 Proof of Lemma 2

Under Assumption 1, let us consider the dynamical system

$$\begin{aligned} \dot{p} &= -p\left\{(1-\alpha)\gamma\psi^{\frac{1}{\epsilon}}x^{\alpha}\left(\frac{L_H(k,x,p,q_0,N_0)}{K_H(k,x,p,q_0,N_0)}\right)^{\alpha} + g_P - \rho - \delta\right\} \equiv \mathcal{F}(p,k,x) \\ \dot{k} &= k\left\{\psi x^{\alpha}\left(\frac{L_H(k,x,p,q_0,N_0)}{K_H(k,x,p,q_0,N_0}\right)^{\alpha}\frac{K_H(k,x,p,q_0,N_0)}{k} - \delta - g_K - \frac{N_0 p^{-\frac{1}{\theta}}}{k}\right\} \equiv \mathcal{G}(p,k,x) \\ \dot{x} &= x\left\{z\Big(1-u(k,x,p,q_0,N_0)\Big) - g_a - \eta\right\} \equiv \mathcal{H}(p,k,x) \end{aligned}$$

The linearization around the steady state yields the following Jacobian matrix:

$$\mathcal{J} = \begin{pmatrix} \mathcal{F}_1(p^*,k^*,x^*) & \mathcal{F}_2(p^*,k^*,x^*) & \mathcal{F}_3(p^*,k^*,x^*) \\ \mathcal{G}_1(p^*,k^*,x^*) & \mathcal{G}_2(p^*,k^*,x^*) & \mathcal{G}_3(p^*,k^*,x^*) \\ \mathcal{H}_1(p^*,k^*,x^*) & \mathcal{H}_2(p^*,k^*,x^*) & \mathcal{H}_3(p^*,k^*,x^*) \end{pmatrix}$$

with

$$\begin{aligned}
\mathcal{F}_1(p^*,k^*,x^*) &= -(\rho+\delta-g_P)\tfrac{\alpha}{1-\alpha} < 0,\ \mathcal{F}_2(p^*,k^*,x^*) = 0 \\
\mathcal{F}_3(p^*,k^*,x^*) &= (\rho+\delta-g_P)\tfrac{\alpha}{1-\alpha}\tfrac{p^*}{x^*} > 0 \\
\mathcal{G}_1(p^*,k^*,x^*) &= \tfrac{k^*}{p^*}\left[(\rho+\delta-g_P)\tfrac{\alpha}{(1-\alpha)^2} + \tfrac{1}{\theta}N_0\tfrac{p^{*\frac{-1}{\theta}}}{k^*}\right] > 0 \\
\mathcal{G}_2(p^*,k^*,x^*) &= (\rho+\delta-g_P)\tfrac{1}{1-\alpha} - \delta - g_K > 0 \\
\mathcal{G}_3(p^*,k^*,x^*) &= (\rho+\delta-g_P)\tfrac{1}{(1-\alpha)^2}\tfrac{k^*}{x^*} > 0 \\
\mathcal{H}_1(p^*,k^*,x^*) &= -\tfrac{x^*}{p^*}\tfrac{zu^*}{1-\alpha} < 0,\ \mathcal{H}_2(p^*,k^*,x^*) = -\tfrac{x^*}{k^*}zu^* < 0 \\
\mathcal{H}_3(p^*,k^*,x^*) &= z(1-u^*) - g_a - \eta + \tfrac{zu^*}{1-\alpha} = \tfrac{zu^*}{1-\alpha} > 0
\end{aligned}$$

We then derive the characteristic polynomial

$$\mathcal{Q}(\lambda) = \lambda^3 - \lambda^2\mathcal{T} + \lambda\mathcal{S} - \mathcal{D} \tag{61}$$

with

$$\begin{aligned}
\mathcal{T} &= \mathcal{F}_1(p^*,k^*,x^*) + \mathcal{G}_2(p^*,k^*,x^*) + \mathcal{H}_3(p^*,k^*,x^*) \\
\mathcal{S} &= \mathcal{F}_1(p^*,k^*,x^*)\mathcal{G}_2(p^*,k^*,x^*) \\
&+ \mathcal{G}_2(p^*,k^*,x^*)\mathcal{H}_3(p^*,k^*,x^*) - \mathcal{H}_2(p^*,k^*,x^*)\mathcal{G}_3(p^*,k^*,x^*) \\
&+ \mathcal{F}_1(p^*,k^*,x^*)\mathcal{H}_3(p^*,k^*,x^*) - \mathcal{H}_1(p^*,k^*,x^*)\mathcal{F}_3(p^*,k^*,x^*) \\
\mathcal{D} &= \mathcal{F}_1(p^*,k^*,x^*)\left[\mathcal{G}_2(p^*,k^*,x^*)\mathcal{H}_3(p^*,k^*,x^*) - \mathcal{G}_3(p^*,k^*,x^*)\mathcal{H}_2(p^*,k^*,x^*)\right] \\
&+ \mathcal{F}_3(p^*,k^*,x^*)\left[\mathcal{G}_1(p^*,k^*,x^*)\mathcal{H}_2(p^*,k^*,x^*) - \mathcal{G}_2(p^*,k^*,x^*)\mathcal{H}_1(p^*,k^*,x^*)\right]
\end{aligned}$$

Consider first the expression of $\mathcal{T}$. We get

$$\mathcal{T} = \rho - g_p - g_K + \tfrac{u^*z}{1-\alpha} = u^*z + \tfrac{u^*z}{1-\alpha} > 0$$

which does not depend on q_0. Consider now the expression of $\mathcal{D}$. We get

$$\begin{aligned}
\mathcal{G}_2\mathcal{H}_3 - \mathcal{G}_3\mathcal{H}_2 &= \tfrac{u^*z}{(1-\alpha)^2}\left[(\rho+\delta-g_P)(1+\alpha) + (1-\alpha)zu^*\right] > 0 \\
\mathcal{G}_1\mathcal{H}_2 - \mathcal{G}_2\mathcal{H}_1 &= \tfrac{x^*}{p^*}u^*z\left[\tfrac{\rho-g_P-g_K}{1-\alpha} - \tfrac{1}{\theta}N_0\tfrac{p^{*\frac{-1}{\theta}}}{k^*}\right]
\end{aligned}$$

and thus

$$\mathcal{D} = -(\rho+\delta-g_P)\tfrac{\alpha zu^*}{1-\alpha}\left[\tfrac{1}{\theta}N_0\tfrac{p^{*\frac{-1}{\theta}}}{k^*} + (\rho+\delta-g_P)\tfrac{1+\alpha}{(1-\alpha)^2}\right] < 0$$

which does not depend on q_0 either. Finally, straightforward computations also show that $\mathcal{S}$ does not depend on q_0 either. We conclude that the eigenvalues do not depend on the value of q_0 and nor on the value of the steady state $(p^*(q_0), k^*(q_0), x^*(q_0))$. Therefore, since $\mathcal{T} > 0$ and $\mathcal{D} < 0$, we conclude that there exists a unique negative eigenvalue and thus that any given steady state $(p^*(q_0), k^*(q_0), x^*(q_0))$ on the manifold is a saddle-point.

□

10.9 Proof of Theorem 4

In the case $\epsilon > 1$, we have shown in the proof of Lemma 2 that the eigenvalues of the linearized dynamical system do not depend on the value of q_0 and nor, therefore, on the value of the steady state $(p^*(q_0), k^*(q_0), x^*(q_0))$. Therefore, since $\mathcal{T} > 0$ and $\mathcal{D} < 0$, we conclude that for any given steady state $(p^*(q_0), k^*(q_0), x^*(q_0))$ on the manifold, the local stability properties are the same. For any given $q_0 > 0$, there exists a unique characteristic root with negative real part and the steady state $(p^*(q_0), k^*(q_0), x^*(q_0))$ is saddle-point stable. Therefore, for any given $q_0 > 0$, there exists a unique $p_0 > 0$ such that the unique converging path is on the stable manifold of dimension one. Along this converging path all the variables are bounded and the transversality conditions are satisfied. Therefore this converging path is the unique optimal solution. □

10.10 Proof of Corollary 2

Let Assumption 1 holds.

i) From Theorems 3 and 4, and Corollary 1, we know that along the NBGP, $\kappa = \lambda = 1$, and that the unique steady state is saddle-point stable. As $\kappa(0)$ and $\lambda(0)$ are necessarily less than 1, the result follows.

ii) From (3), (16) and (40) we get

$$\frac{Y_H}{Y} = \gamma^{\frac{\epsilon}{1-\epsilon}} \left(\frac{(1-\beta)\kappa}{(1-\beta)\kappa + (1-\alpha)(1-\kappa)} \right)^{\frac{\epsilon}{\epsilon-1}}, \ \frac{P_H Y_H}{PY} = \frac{(1-\beta)\kappa}{(1-\beta)\kappa + (1-\alpha)(1-\kappa)} \tag{62}$$

and thus

$$\frac{\partial Y_H/Y}{\partial \kappa} = \frac{\epsilon}{\epsilon-1} \frac{Y_H}{Y} \frac{(1-\alpha)}{\kappa[(1-\beta)\kappa + (1-\alpha)(1-\kappa)]} > 0, \ \frac{\partial P_H Y_H/PY}{\partial \kappa} = \frac{(1-\beta)(1-\alpha)}{[(1-\beta)\kappa + (1-\alpha)(1-\kappa)]^2} > 0$$

The result follows from i).

iii) From (8) and (62) we get

$$s_K = \frac{RK}{Y} = \frac{(1-\beta)(1-\alpha)}{(1-\beta)\kappa + (1-\alpha)(1-\kappa)} \text{ and thus } \frac{\partial s_K}{\partial \kappa} = \frac{(1-\beta)(1-\alpha)(\beta-\alpha)}{[(1-\beta)\kappa + (1-\alpha)(1-\kappa)]^2} \tag{63}$$

The result follows.

iv) From (48) and (62) we derive

$$\frac{P_H}{P} = \gamma^{\frac{\epsilon}{\epsilon-1}} \left[1 + \frac{(1-\kappa)(1-\alpha)}{\kappa(1-\beta)}\right]^{\frac{1}{\epsilon-1}} \text{ and } \frac{P_L}{P} = (1-\gamma)^{\frac{\epsilon}{\epsilon-1}} \left[1 + \frac{\kappa(1-\beta)}{(1-\kappa)(1-\alpha)}\right]^{\frac{1}{\epsilon-1}} \tag{64}$$

It follows therefore

$$\frac{\partial P_H/P}{\partial \kappa} < 0 \text{ and } \frac{\partial P_L/P}{\partial \kappa} > 0 \tag{65}$$

The result follows.

□

10.11 Data used for the computation of the capital shares

Considering as Acemoglu and Guerrieri [4] the National Income and Product Accounts (NIPA) between 1948 and 2005 where industries are classified according to the North American Industrial Classification System at the 22-industry level, we classify industries according to the requirement of technological knowledge by the workers. That is, we consider an industry to be knowledge-intensive if workers exhibit a higher growth of compensation per capita than average. The following Table shows the average capital share of each industry together with the sector classification.

Industry	Sector	Capital share
Management of companies and enterprises	H	0.20
Professional, scientific and technical services	H	0.34
Transportation and warehousing	H	0.35
Finance and insurance	H	0.45
Wholesale trade	H	0.46
Information	H	0.53
Educational services	L	0.10
Health care and social assistance	L	0.22
Administrative and waste management services	L	0.28
Other services, except government	L	0.33
Accommodation and food services	L	0.36
Retail trade	L	0.42
Arts, entertainment and recreation	L	0.42
Utilities	L	0.77

Table 2: Industry capital shares

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