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Ethical Voting in Heterogenous Groups

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Abstract

Voting in large elections appears to be both ethically motivated and influenced by strategic considerations. One way to capture this interplay postulates a rule-utilitarian calculus, which abstracts away from voters' heterogeneity in the intensity of support (Feddersen and Sandroni 2006, Coate and Conlin 2004). I argue that this approach is limited when such heterogeneity is considered, because it implies that the intensity of preferences is irrelevant for participation, in contrast to the empirical evidence. I compare the rule-utilitarian framework with a different model of ethical voting, in which agents maximize their individual utility under a moral constraint given by a universalization principle. Such a model predicts instead higher turnout rates among voters with higher intensity of support, thus linking ethical motivation to the spatial theory of voting.

Keywords: Voting, Turnout, Ethical Voter, Rule-utilitarian, Kantian Optimization
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1 Introduction

Explaining voting in large elections has proven difficult. The instrumental model, with citizens moved only by the desire to affect the outcome, clashes against the probability of being pivotal becoming negligible as the number of voters grows. Hence, even small voting costs bind the predicted turnout rate close to zero. To resolve the impasse, previous research has suggested a prominent role for citizens' ethical motivation to vote, as an act of civic duty (Blais 2000, François and Gergaud 2019, Blais and Daoust 2020). Ethics, in turn, explains little alone, since voter turnout does vary in ways that suggest the presence of strategic considerations. For example, participation is typically increasing in the expected closeness of the election, in line with a higher perceived likelihood of affecting the outcome (Shachar and Nalebuff 1999, Fauvelle-Aymar and François 2006, Arnold 2018, Bursztyn et al. 2017). The challenge posed by the empirical evidence is thus to model the interaction between voters' ethical and strategic reasoning.

In two seminal contributions, Feddersen and Sandroni (2006) and Coate and Conlin (2004) proposed to do so through a *rule-utilitarian calculus* of voting. Rule-utilitarian supporters follow a turnout rule which dictates participation if the voting cost is below a threshold, and choose the threshold in order to maximize aggregate utility. Voting is thus both ethically motivated and, since the threshold cost is endogenous, responsive to the characteristics of the election. While such a model captures the empirical properties of aggregate turnout well, the framework of both papers features two assumptions that it is interesting to question. The first is that the groups of supporters are given exogenously, which ignores voters' choice of the candidate they support. The second is that groups are homogenous, in that all supporters of the same candidate obtain the same benefit from the outcome of the election.

Typically, instead, differences in the underlying policy preferences of voters, and thus in how much they feel represented by candidates, affect both their vote choice and their likelihood of participating. The spatial voting literature, for example, has long since highlighted how citizens are less likely to vote the more they feel equally close to the competing candidates or too far from all of them (Riker and Ordeshook 1973, Zipp 1985, Plane and Gershtenson 2004, Adams, Dow and Merrill 2006). In line with this perspective, this paper studies voter turnout in a *spatial* framework, in which the two previous assumptions are relaxed. Citizens have idiosyncratic preferences and form supporters' groups endogenously, depending on the distance between their preferred policy and those proposed by the candidates. As a consequence, the intensity of support differs among supporters of the same candidate. How is the ethical motivation to vote balanced with policy considerations that push voters towards abstention? How does turnout behavior reflect the heterogeneity in the intensity of support for a candidate?

I argue that the rule-utilitarian approach is limited in explaining participation within groups of heterogenous agents. Specifically, I show that if voting costs differ and are ex-ante unknown, as typically assumed, the utilitarian turnout rule is independent of voters' preferences and thus of the intensity of support. The maximization of aggregate utility, indeed, requires that aggregate costs be minimized by disregarding any heterogeneity in preferences. This result, however, clashes with the previous evidence of differential participation along a spatial dimension. Moreover, the implications are also inconvenient under the simpler assumption that voting costs are fixed and identical. In this case, the optimal turnout rule pins down only the number of votes, and an infinity of participation patterns can emerge at the individual level. Such a loose prediction, however, due again to the irrelevance of idiosyncratic preferences for a utilitarian calculus, practically fails to answer the research questions of the paper.

In light of the previous results, I examine then an alternative framework of ethical voting, in which the underlying principle consists in maximizing individual utility under a constraint of universalization of behavior. Specifically, I follow the model of Kantian optimization by Roemer (2010, 2015, 2019) in assuming that citizens, when evaluating deviations from a turnout rule, look at the consequences resulting if all supporters in their group deviated similarly. I obtain unique predictions with both fixed and variable costs of voting. Crucially, the supporters' probability of voting depends on their policy preferences and is increasing in the intensity of support, given by the difference in utility from the candidates' policies. Unlike in the rule-utilitarian approach, the result is then consistent with the patterns of participation established by the previous literature at the individual level. Moreover, if the voting cost is fixed, the aggregate number of votes coincides with the one derived from the rule-utilitarian calculus. In this case, thus, Kantian optimization can be interpreted as a selection device that specifies a unique rule on which even rule-utilitarian supporters can coordinate. Overall, the model offers a very tractable way to study ethical participation in a spatial setting and matches the empirical properties of turnout as a function of voters' policy preferences.

The rule-utilitarian approach builds on the work of Harsanyi (1977, 1980). I follow Coate and Conlin (2004) in considering *group rule-utilitarian* agents who maximize group utility.[1] Extensions of the basic model to richer frameworks are in Ali and Lin (2013), Jorgenson and Saavedra (2018), and Bierbrauer, Tsyvinsky, and Werquin (2021). The theoretical foundations of the rule-utilitarian calculus rely on the thought experiment of choosing under a *veil of ignorance*, which concerns the realizations of the costs of voting. In the standard model, the cost of voting is the only dimension of (ex-post) heterogeneity; hence, all agents within a group are ex-ante identical and benefit equally from maximizing aggregate utility. As such, the rule-utilitarian model works best when the analysis focuses on aggregate turnout, whose empirical patterns are indeed well captured by the comparative statics results. In a spatial framework, however, the policy space represents the conflict of interests inherent to politics, and thus a heterogeneity in preferences for which the application of the veil of ignorance becomes less plausible. In this case, Kantian optimization outperforms the rule-utilitarian model because agents maximize their individual utility. As such, idiosyncratic preferences, which determine the intensity of support, are taken into account in the ethical calculus and translate into heterogenous turnout rates.

Roemer (2015, 2019) develops the theory of Kantian optimization in broader terms and presents it as a cooperative protocol that yields Pareto-efficient outcomes. Previous references to the underlying principle of universalization include Laffont (1975), Sugden (1984, 1991), and Bilodeau and Gravel (2004). Alger and Weibull (2013, 2016) and Alger, Weibull, and Lehmann (2020) investigate the evolutionary foundations of morality and characterize stable preferences as partially *Kantian*. Two applications of Kantian optimization to political competition and voting are in Roemer (2006, 2010). De Donder and Roemer (2016) apply it to lobbying in order to study the evolution of the income distribution. Grafton et al. (2017) use a similar model to study the dynamics of climate change mitigation, while Eichner and Pethig (2020) examine the implications on tax competition. Sher (2020) provides a valuable discussion of the normative aspects of Roemer's theory.

The remainder of the paper is organized as follows. Section 2 lays down the spatial framework and shows the limitations of the rule-utilitarian approach in such a setting. Section 3 develops the Kantian optimization model and presents the main results. Section 4 discusses the endogenization of candidates' policies, the possibility of amending the rule-utilitarian calculus, and the Kantian label that economists typically assign to the universalization principle. Section 5 concludes.

[1] The results and insights of the paper are analogous in the framework of Feddersen and Sandroni (2006), in which supporters maximize a more aggregate welfare measure that includes all social costs of voting.

2 Voting in a spatial setting

2.1 Framework

The policy space is the interval $[0,1]$. Two candidates, A and B, compete by proposing policies a and b. The electorate is composed of a continuum of citizens, distributed according to a density function $f(z)$, where z denotes their preferred policy (or bliss point). Each citizen z evaluates any policy x with a utility function $u(z-x)$, denoted shortly $u_z(x)$. That is, the policy benefits $u_z(a)$ and $u_z(b)$ depend on the distance between the bliss point z and the proposed policies a and b.

Assumption 1. *The utility function $u_z(x)$ is continuous and single-peaked around the bliss point z. The function u is the same for all citizens z.*

Denote the groups of supporters $\mathcal{Z}_A = \{z : u_z(a) > u_z(b)\}$ and $\mathcal{Z}_B = \{z : u_z(b) > u_z(a)\}$. Assumption 1 guarantees that the two groups are separated on the policy space by an indifferent citizen z^*. If without loss of generality $a \leq b$, then $\mathcal{Z}_A = [0, z^*)$ and $\mathcal{Z}_B = (z^*, 1]$.[2]

The consequences of the election are probabilistic and depend on the number of votes v_a and v_b targeted by the two groups. Specifically, policy a is implemented with some probability $P(v_a, v_b)$, while policy b is implemented with probability $1-P(v_a, v_b)$. Because voting only affects the probability of either policy a or b to be implemented, the vote choice is sincere in favor of the preferred candidate. The participation behavior is described by two functions $p_a(z) : \mathcal{Z}_A \to [0,1]$ and $p_b(z) : \mathcal{Z}_B \to [0,1]$ giving the probability of voting for each supporter in each group, which we are ultimately interested in deriving endogenously. The number of votes are then obtained by aggregating the probability of voting within each group, i.e. $v_a = \int_{\mathcal{Z}_A} p_a(z) f(z) dz$ and $v_b = \int_{\mathcal{Z}_B} p_b(z) f(z) dz$.

Assumption 2. *(i) $P(v_a, v_b)$ is increasing in v_a, decreasing in v_b, and equal to $\frac{1}{2}$ if $v_a = v_b$. (ii) $P(v_a, v_b)$ is continuous, concave in v_a, and convex in v_b.*

Assumption 2(i) is standard. Assumption 2(ii) is more restrictive but guarantees the tractability of the model, as the first order conditions imply optimality. A natural candidate for $P(v_a, v_b)$ is a *contest success function* $\frac{{v_a}^\gamma}{{v_a}^\gamma + {v_b}^\gamma}$ with noise parameter $\gamma > 0$. Note that, in this case, Assumption 2(ii) is satisfied only for $\gamma \leq 1$, although in general

[2]If $u_z(x)$ is also symmetric around the bliss point z, i.e. if $u_z(x) = u(|z-x|)$, then $z^* = \frac{a+b}{2}$.

an optimal solution exists also if $\gamma > 1$.[3]

In terms of interpretation, the uncertainty can arise at the legislative stage, because the probability of passing either policy depends on the number of votes obtained by each candidate. Or it can arise at the voting stage, because *targeted* votes do not map perfectly into *effective* votes. As an example of a stochastic voting stage, assume that for targeted votes v_a and v_b in the two groups, the effective number of votes are

$$\frac{v_a + v_b}{\theta_a v_a + \theta_b v_b}\theta_a v_a \quad , \quad \frac{v_a + v_b}{\theta_a v_a + \theta_b v_b}\theta_b v_b \tag{1}$$

for group A and B respectively, with θ_a, θ_b iid $\sim$ exp(λ). That is, the total number of votes is deterministic but the shares of votes for the two groups are affected by two aggregate shocks. Then, the probability that the effective number of votes for A is greater than the effective number of votes for B is given by a *Tullock contest success function* (i.e. $\gamma = 1$). Indeed

$$P(v_a, v_b) = P(\theta_a v_a > \theta_b v_b) = \frac{v_a}{v_a + v_b}$$

since the shocks are independent and exponentially distributed (see Konrad 2007 for a proof).[4] The main results in the next sections require only Assumptions 1 and 2, while I will use this example to derive closed-form solutions and comparative statics results.

Finally, each citizen has a positive cost of voting. The standard assumption about voting costs in the rule-utilitarian model is that they are drawn independently from a uniform distribution $c \sim \mathcal{U}[0, \bar{c}]$. Citizens decide their participation behavior before learning the realizations of their costs. The turnout rule in each group is thus given by a threshold cost: supporters vote if their realization is below the threshold and abstain if it is above. In our spatial framework, voting costs are also independent of policy preferences. But the threshold costs in both groups shall be allowed to vary as a function of the supporters' preferences. Let us consider thus threshold costs that are functions $c_a(z)$ and $c_b(z)$. Moreover, since results change in interesting ways, I will compare the analysis under heterogenous voting costs with the simpler case of a fixed voting cost equal to c for all citizens.[5]

[3]Herrera, Morelli and Palfrey (2014), Herrera, Morelli and Nunnari (2016), and Bouton, Castanheira, and Drazen (2020) also model uncertainty in elections via a contest success function.

[4]Note that the term $\frac{v_a+v_b}{\theta_a v_a+\theta_b v_b}$ in (1) serves only a normalization purpose. Note also that the expected values of the effective number of votes are v_a and v_b for any value of the exponential distribution's parameter λ.

[5]Considering different distributions from the uniform does not yield, instead, additional insights.

2.2 The limitations of the rule-utilitarian approach

I now derive the implications of the rule-utilitarian calculus in our spatial framework. I consider *group* rule-utilitarians supporters, who set the turnout rule in the form of a threshold cost function $c_a(z)$ in order to maximize the group aggregate utility. Under a uniform distribution of voting costs $c \sim \mathcal{U}[0, \bar{c}]$, the probability of voting for a supporter z in group A is given by the cumulative distribution function evaluated at the threshold, i.e. $p_a(z) = \frac{1}{\bar{c}} c_a(z)$. The expected voting cost is $\int_0^{c_a(z)} \frac{1}{\bar{c}} c \, dc = \frac{1}{2\bar{c}} c_a(z)^2$ and the supporter's expected utility is then

$$P(v_a, v_b) u_z(a) + (1 - P(v_a, v_b)) u_z(b) - \frac{1}{2\bar{c}} c_a(z)^2$$

where $v_a = \int_{\mathcal{Z}_A} \frac{1}{\bar{c}} c_a(z) f(z) dz$ and $v_b = \int_{\mathcal{Z}_B} \frac{1}{\bar{c}} c_b(z) f(z) dz$. By aggregation, group A utility is given by

$$P(v_a, v_b) \int_{\mathcal{Z}_A} u_z(a) f(z) dz + (1 - P(v_a, v_b)) \int_{\mathcal{Z}_A} u_z(b) f(z) dz - \frac{1}{2\bar{c}} \int_{\mathcal{Z}_A} c_a(z)^2 f(z) dz \quad (2)$$

The first result is that, despite the presence of heterogenous policy preferences, the turnout rule maximizing aggregate utility must necessarily be constant among supporters, i.e. independent of their bliss points z.

Lemma 1. *Consider the group rule-utilitarian model with heterogenous voting costs. If there exists $c_a(z)$ such that group A utility in (2) is maximized, then $c_a(z) = k$. Hence, the probability of voting is the same for all supporters in a group.*

A proof is provided in the appendix. To grasp the intuition, consider the case of a non-constant $c_a(z)$. Clearly, equalizing the threshold costs across different supporters z while keeping the same aggregate number of votes v_a lowers the expected costs, since group members voting with high costs are substituted by members with low costs. Hence, the model's prediction is stark: supporters should ignore their idiosyncratic preferences in order to make sure that aggregate voting costs are minimized. This conclusion follows from the nature of the rule-utilitarian approach, which maximizes aggregate utility: what matters for group utility is only the number of votes v_a and the aggregate voting costs.

Let us now compare the previous framework to the simpler case of a fixed cost of voting c for all citizens. Consider again group A and assume that in this case supporters choose a turnout rule directly in the form of a probability of voting $p_a(z)$ in $[0, 1]$. The individual expected utility is then

$$P(v_a, v_b)u_z(a) + (1 - P(v_a, v_b))u_z(b) - c\, p_a(z)$$

and group A aggregate utility is

$$P(v_a, v_b) \int_{\mathcal{Z}_A} u_z(a) f(z) dz + (1 - P(v_a, v_b)) \int_{\mathcal{Z}_A} u_z(b) f(z) dz - c\, v_a \tag{3}$$

where $v_a = \int_{\mathcal{Z}_A} p_a(z) f(z) dz$ by aggregation. As we see from equation (3), the aggregate utility depends now on the function $p_a(z)$ only through the number of votes v_a. Hence, the maximization of group utility determines only an optimal v_a. Clearly, insofar as the optimal v_a is an interior solution, there exists an infinity of different functions $p_a(z)$ that are consistent with it. That is, in the case of a fixed voting cost, the rule-utilitarian calculus pins down only the aggregate number of votes, but not the individual probability of voting given by $p_a(z)$.

Lemma 2. *Consider the group rule-utilitarian model with a fixed voting cost. If group A utility in (3) is maximized at an interior solution for the number of votes v_a, there exists an infinity of solutions for the probability of voting $p_a(z)$at the individual level, i.e. all those for which the aggregate relation $v_a = \int_{\mathcal{Z}_A} p_a(z) f(z) dz$ holds.*

Hence, with a fixed voting cost the problem is one of equilibrium selection. The model does not yield any specific prediction on the relationship between policy preferences and participation at the individual level. Again, this is due to its utilitarian nature: what matters for the aggregate group utility is now only the number of votes. How supporters share the voting costs via the probability of voting is irrelevant, since costs are identical.

Let me summarize the two previous results. If voting costs are heterogenous and ex-ante unknown, group utility maximization requires the threshold cost (and thus the probability of voting) to be the same among supporters of the same candidate, independently of their idiosyncratic preferences. Solving for the optimal costant threshold c_a pins down the optimal constant probability of voting p_a and the optimal number of votes v_a. Instead, if the voting cost is fixed and identical among supporters, the model pins down the optimal number of votes v_a but is silent on the individual probability of voting: an infinite number of functions $p_a(z)$ is consistent with the optimal v_a, with the model giving no further prediction. Both results are unsatisfactory. The first depicts turnout behavior as independent of voters' distance from the candidates, in contrast with the paradigm of spatial voting and the evidence of differential participation. The second, while not being necessarily inconsistent with such evidence, is too loose and offers no specific answer to how supporters balance ethical and policy motives in their turnout decision.

3 An alternative model

Let us then investigate the extent to which a model of ethical voting based on the theory of Kantian optimization by Roemer (2010, 2015, 2019) delivers better predictions than the rule-utilitarian approach on the relationship between supporters' idiosyncratic preferences and their probability of voting.

3.1 The Kantian Optimization Protocol

Kantian optimization builds on the idea that ethical agents envision a universalization of their behavior. In a framework of heterogenous agents, this universalization concerns potential deviations from a prescribed rule, which are evaluated by the consequences that would result if other agents deviated similarly. In this paper, the universalization applies only within groups of supporters, while the voting behavior in the opposing group is taken as given, as in Nash optimization. In line with Roemer's terminology, I call the solution concept a Nash-Kantian equilibrium.

An important aspect is what a similar way of deviating means: in the model, deviations from a turnout rule are represented by a multiplicative factor and a similar deviation is one by the same factor.[6] The equilibrium is thus characterized by the absence of profitable collective deviations, i.e. by the condition that no group member would want to deviate from the group turnout rule by any scalar factor, given that all other members would deviate by the same factor. Hence, while rule-utilitarian agents are ethical in that they maximize aggregate utility, Kantian agents are ethical in that they maximize their hypothetical utility, which results when their behavior is universalized within their group. It is by keeping the focus on each individual utility that such a model overcomes the limitations of the rule-utilitarian approach described in the previous section.

Consider now first the case of a fixed voting cost, in which supporters choose directly a probability of voting, e.g. a rule $p_a(z)$ in group A. An additional technicality concerns

[6] Roemer (2015, 2019) studies also deviations in the form of additive factors and a simpler notion of Kantian Equilibrium that applies in symmetric frameworks of identical agents, for which the universalization concerns actions and not deviations.

the fact that supporters have compact strategy sets. To ensure that probabilities remain lower than 1, assume that, for a deviation factor σ, all supporters z deviate from their voting probability $p(z)$ by the $\min\{\sigma, \frac{1}{p(z)}\}$. Moreover, since different voters (might) vote with different probabilities, assume that each supporter z, when evaluating potential deviations, considers only deviation factors that are bounded above by $\frac{1}{p(z)}$.[7] It is then useful to state a precise definition of the equilibrium concept. An equivalent definition holds, after minimal adjustments, for the case of heterogenous voting costs.

Definition. *A Nash-Kantian voting equilibrium is a pair of probability functions $p_a(z), p_b(z)$ such that for each group $X \in \{A, B\}$ and policy $x \in \{a, b\}$, no supporter $z \in \mathcal{Z}_X$ would prefer all supporters $z' \in \mathcal{Z}_X$ to vote with probability $\min\{\sigma p_x(z'), 1\}$ for any deviation factor $\sigma \in [0, \frac{1}{p_x(z)}]$, $\sigma \neq 1$, given the voting behavior in the opposing group.*
This can be expressed concisely as

$$\forall z \in \mathcal{Z}_A \quad \arg \max_{\sigma \in [0, \frac{1}{p_a(z)}]} U_z \left(\min\{\sigma p_a(\cdot), 1\}, p_b(\cdot) \right) = 1$$

$$\forall z \in \mathcal{Z}_B \quad \arg \max_{\sigma \in [0, \frac{1}{p_b(z)}]} U_z \left(p_a(\cdot), \min\{\sigma p_b(\cdot), 1\} \right) = 1$$

where U_z is supporter z's expected utility.

Note that the analytical conditions operationalize the absence of deviations by imposing that the argument σ of the maximization problem be equal to one. As a final remark, note also that the pair of probability functions $p_a(z) = 0$, $p_b(z) = 0$ for all supporters in both groups is always a Nash-Kantian equilibrium, as multiplicative deviations are in this case ineffective. In the following sections, uniqueness of the equilibrium will be claimed by restricting the analysis to strictly positive probability functions.

3.2 Fixed Cost of Voting

Equipped with the previous definition of Nash-Kantian equilibrium, let us reconsider the model with a fixed cost of voting equal to c for all citizens. Recall that, in this case, the expected utility of a supporter z in group A is

$$P(v_a, v_b)u_z(a) + (1 - P(v_a, v_b))u_z(b) - c\, p_a(z)$$

[7] This assumption prevents a voter who is voting with a high probability from proposing a high deviation factor, which is less costly for himself than for those who vote with lower probability, because of the bound at 1. This approach follows the generalized definition of the equilibrium concept for compact strategy sets (Roemer 2010)

By definition, in equilibrium z would not want all supporters $z' \in \mathcal{Z}_A$ to deviate from their voting probability $p_a(z')$ by any factor $\sigma \in [0, \frac{1}{p_a(z)}]$. For technical convenience, however, we can neglect the fact that other voters z' would deviate by $\min\{\sigma, \frac{1}{p_a(z')}\}$ and work out the analysis assuming that all voters would deviate by σ, i.e. as if voters could vote with probability higher than 1. Intuitively, indeed, the benefit of a deviation by a factor σ is greater for a voter z when the constraint given by $\min\{\sigma, \frac{1}{p_a(z')}\}$ is not considered; hence if such a deviation is not profitable, it won't be when the probability of voting is bounded by 1. If a deviation by σ is followed by all voters in $\mathcal{Z}_A$, then, as $\int_{\mathcal{Z}_A} \sigma p_a(z) f(z) dz = \sigma v_a$, the expected utility of supporter z as a function of the deviation factor σ is equal to

$$P(\sigma v_a, v_b) u_z(a) + (1 - P(\sigma v_a, v_b)) u_z(b) - c\, \sigma p_a(z) \tag{4}$$

The solution concept requires the expression in (4) to be maximized at $\sigma = 1$. Note that, since $P(\cdot)$ is concave in σv_a, the previous expression is concave in σ, hence the optimality condition is given by the first order condition evaluated at $\sigma = 1$, that is

$$\frac{\partial}{\partial v_a} P(v_a, v_b) \cdot v_a [u_z(a) - u_z(b)] - p_a(z) c = 0 \tag{5}$$

Equation (5) does not pin down $p_a(z)$ directly, as v_a also depends on the function $p_a(\cdot)$. However, since v_a is a definite integral over $\mathcal{Z}_A$, the equation implies that $p_a(z)$ has to be proportional to the utility differential $u_z(a) - u_z(b)$, with the coefficient of proportionality to be determined in equilibrium. A similar analysis for any voter $z \in \mathcal{Z}_\mathcal{B}$ yields analogous results and implications due to the complete symmetry of the framework. That is, in equilibrium we must have

$$\begin{aligned} p_a(z) &= \pi_a [u_z(a) - u_z(b)] \\ p_b(z) &= \pi_b [u_z(b) - u_z(a)] \end{aligned} \tag{6}$$

By aggregation, the number of votes must then solve

$$\begin{aligned} v_a &= \pi_a u_{\mathcal{Z}_A} \\ v_a &= \pi_b u_{\mathcal{Z}_B} \end{aligned} \tag{7}$$

where the terms $u_{\mathcal{Z}_A}$ and $u_{\mathcal{Z}_B}$ denote the aggregate utility differentials in group A and B, respectively, i.e.

$$\begin{aligned} u_{\mathcal{Z}_A} &:= \int_{\mathcal{Z}_A} [u_z(a) - u_z(b)] f(z) dz \\ u_{\mathcal{Z}_B} &:= \int_{\mathcal{Z}_B} [u_z(b) - u_z(a)] f(z) dz \end{aligned} \tag{8}$$

Finally, the equilibrium values of the proportionality coefficients π_a and π_b are obtained by substituting (6) and (7) into the first order conditions, which can be rewritten together as

$$\begin{aligned} \frac{\partial}{\partial v_a} P(v_a, v_b) \cdot u_{\mathcal{Z}_A} &= c \\ -\frac{\partial}{\partial v_b} P(v_a, v_b) \cdot u_{\mathcal{Z}_B} &= c \end{aligned} \tag{9}$$

We have thus the following result.

Proposition 1. *Consider the Kantian optimization model with a fixed voting cost. There exists a (strictly positive) Nash-Kantian equilibrium and it is such that the probability of voting of each supporter in both groups is proportional to the utility differential from the candidates' policies, as given by* (6).

Existence of the equilibrium is guaranteed, for c within an appropriate range of values, by the Poincaré-Miranda theorem, as shown in the Appendix. Uniqueness of the (strictly positive) equilibrium typically holds, as in the example below, although a proof would require more technical assumptions. Unlike the rule-utilitarian model, which was silent on turnout behavior at the indivudal level, Kantian optimization yields thus an intuitive prediction. Citizens' probability of voting is proportional to the utility differential from the two candidates' policies: the higher the intensity of support for a candidate, measured by the utility differential, the higher the contribution to the group in terms of probability of voting.

Note that the result recovers a dependence of voters' behavior on the utility differential from candidates' policies, which is at the core of the spatial theory of voting. In the standard spatial theory of voting, however, this relationship is analyzed in a framework of instrumental voting, in which the utility differential is discounted by the probability of being pivotal. Kantian optimization endogenizes it in a model of ethical participation that overcomes the issue of pivotality. The dependence on the utility differential is what makes the predictions consistent with the patterns of differential participation observed empirically. The shape of the function $u_z(x)$ determines how the intensity of support relates to the distance between supporters and candidates, and thus accounts for a higher likelihood of abstention motivated by being equally close to the candidates or too far from them.[8]

Moreover, an important link between Kantian optimization and the rule-utilitarian model emerges under the assumption of a fixed cost of voting. Indeed, the first equation in (9) coincides with the first order condition from the maximization of group utility in equation (3) with respect to v_a. The same holds in group B. This implies that the solutions for the aggregate number of votes v_a and v_b are the same in the two models.

Proposition 2. *Consider the Kantian optimization model with a fixed voting cost. At the Nash-Kantian equilibrium, the aggregate number of votes v_a and v_b in the two groups correspond to the solutions of a group rule-utilitarian calculus.*

Hence, even if Kantian agents do not commit themselves ex-ante to maximizing aggregate utility, they do maximize it in equilibrium. In a sense, under a fixed voting cost, Kantian optimization can be interpreted as complementary to the group rule-utilitarian model, in that it specifies how heterogeneous supporters share the burden of voting in order to maximize group utility.

Let us then calculate the Nash-Kantian equilibrium for the example introduced in section 2, in which the uncertainty takes the form of a Tullock contest success function $P(v_a, v_b) = \frac{v_a}{v_a+v_b}$. We readily obtain the following unique pair of positive solutions for the probability of voting in the two groups

$$p_a(z) = \frac{u_{\mathcal{Z}_A} u_{\mathcal{Z}_B}}{c(u_{\mathcal{Z}_A} + u_{\mathcal{Z}_B})^2}[u_z(a) - u_z(b)]$$
$$p_b(z) = \frac{u_{\mathcal{Z}_A} u_{\mathcal{Z}_B}}{c(u_{\mathcal{Z}_A} + u_{\mathcal{Z}_B})^2}[u_z(b) - u_z(a)]$$

By aggregation, the number of votes v_a and v_b are equal to

$$v_a = \frac{{u_{\mathcal{Z}_A}}^2 u_{\mathcal{Z}_B}}{c(u_{\mathcal{Z}_A} + u_{\mathcal{Z}_B})^2} \quad , \quad v_b = \frac{u_{\mathcal{Z}_A} {u_{\mathcal{Z}_B}}^2}{c(u_{\mathcal{Z}_A} + u_{\mathcal{Z}_B})^2} \tag{10}$$

Hence, for a Tullock contest success function, the coefficient of proportionality is the same for all supporters in both groups.[9] Both the coefficient of proportionality and the

[8]See Grillo (2021) for a more detailed discussion on the notions of abstention due to indifference and alienation, and on how the convexity of the utility function $u_z(x)$ captures voters' propensity to alienation.

[9]Note that, being probability functions, $p_a(z)$ and $p_b(z)$ should not be greater than 1, but to this end it suffices to assume that the voting cost c is big enough.

aggregate number of votes v_a, v_b are decreasing in the cost of voting and increasing in both groups' aggregate utility differentials defined in (8). These comparative statics results are intuitive but offer nonetheless additional insights with respect to the standard ethical voter model. The spatial framework, indeed, allows for a richer analysis of participation behavior, as a function of candidates' proposed policies a and b, voters' policy preferences $u_z(x)$, and their distribution on the policy space $f(z)$. These elements are all captured by the aggregate utility differentials $u_{\mathcal{Z}_A}$ and $u_{\mathcal{Z}_B}$.

3.3 Heterogenous Costs of Voting

Let us consider now the case of heterogenous costs of voting, iid drawn from a uniform distribution, $c \sim \mathcal{U}[0, \bar{c}]$. I show that the model's predictions are consistent with those in the case of a fixed cost of voting, although the equivalence result for the aggregate number of votes given by Proposition 2 fails to hold. Recall that with heterogenous costs, a turnout rule in group A is given by a threshold cost function $c_a(z)$ and that, given the uniform distribution, the probability of voting is $p_a(z) = \frac{1}{\bar{c}} c_a(z)$. Thus, for a multiplicative deviation factor σ applied to the threshold cost, it still holds that the number of votes v_a scales up by σ. The expected utility of a supporter $z \in \mathcal{Z}_A$, when a deviation by σ is followed by all voters in group A, is then equal to

$$P(\sigma v_a, v_b) u_z(a) + (1 - P(\sigma v_a, v_b)) u_z(b) - \frac{1}{2\bar{c}} (\sigma c_a(z))^2$$

where the last term is the expected cost of voting for a member. Taking the first order condition with respect to σ and imposing $\sigma = 1$ yields

$$\frac{\partial}{\partial v_a} P(v_a, v_b) \cdot v_a [u_z(a) - u_z(b)] - \frac{1}{\bar{c}} c_a(z)^2 = 0$$

By comparing the previous expression with the one in (5), we see that under uniformly distributed costs, the threshold cost must now be proportional to the square root of the utility differential. Given $p_a(z) = \frac{1}{\bar{c}} c_a(z)$, we thus have

$$p_a(z) = \tilde{\pi}_a \sqrt{[u_z(a) - u_z(b)]} \tag{11}$$

and by aggregation

$$v_a = \tilde{\pi}_a \tilde{u}_{\mathcal{Z}_A}$$

where $\tilde{u}_{\mathcal{Z}_A} = \int_{\mathcal{Z}_A} \sqrt{[u_z(a) - u_z(b)]} f(z)dz$ denotes an aggregate utility differential which is 'adjusted' with respect to (8) by taking the square root within the integral. Clearly, the probability of voting in (11) is still an increasing function of the utility differential. An equivalent calculation concerns group B, and the proportionality coefficients $\tilde{\pi}_a, \tilde{\pi}_b$ are determined by solving jointly the following first order conditions

$$\begin{aligned} \frac{\partial}{\partial v_a} P(v_a, v_b) \cdot \frac{\tilde{u}_{\mathcal{Z}_A}}{\tilde{\pi}_a} &= \bar{c} \\ -\frac{\partial}{\partial v_b} P(v_a, v_b) \cdot \frac{\tilde{u}_{\mathcal{Z}_B}}{\tilde{\pi}_b} &= \bar{c} \end{aligned} \tag{12}$$

given $v_a = \tilde{\pi}_a \tilde{u}_{\mathcal{Z}_A}$ and $v_b = \tilde{\pi}_b \tilde{u}_{\mathcal{Z}_B}$, with $\tilde{u}_{\mathcal{Z}_B} = \int_{\mathcal{Z}_B} \sqrt{[u_z(b) - u_z(a)]} f(z)dz$. As before, existence of an equilibrium is guaranteed by the Poincaré-Miranda theorem. The conditions in (12), however, are now different than what would result from the maximization of group utility. To compare the two, let me first define the size of the two groups as $|\mathcal{Z}_A| = \int_{\mathcal{Z}_A} f(z)dz$ and $|\mathcal{Z}_B| = \int_{\mathcal{Z}_B} f(z)dz$, i.e. the shares of population belonging to each group. In light of Lemma 1, rule-utilitarian members of group A would target v_a votes by setting a constant threshold equal to $c_a = \frac{\bar{c}}{|\mathcal{Z}_A|} v_a$. We can then substitute this voting rule into (2) and proceed similarly for group B to obtain the following pair of first order conditions for the rule-utilitarian calculus

$$\begin{aligned} \frac{\partial}{\partial v_a} P(v_a, v_b) \cdot \frac{u_{\mathcal{Z}_A} |\mathcal{Z}_A|}{v_a} &= \bar{c} \\ -\frac{\partial}{\partial v_b} P(v_a, v_b) \cdot \frac{u_{\mathcal{Z}_B} |\mathcal{Z}_B|}{v_b} &= \bar{c} \end{aligned} \tag{13}$$

Technically, the difference between (12) and (13) arises because in the Kantian optimization model the voting probabilities are aggregated after taking the square root of the utility differential, while the aggregation occurs before in the rule-utilitarian model. The solution is not invariant to the order of the two operations. I summarize the results as follows.

Proposition 3. *Consider the Kantian optimization model with heterogenous voting costs $c \sim \mathcal{U}[0, \bar{c}]$. There exists a (strictly positive) Nash-Kantian equilibrium and it is such that the probability of voting is an increasing function (proportional to the square root) of the utility differential from the candidates' policies. The aggregate number of votes in the two groups do not correspond, in this case, to the solutions from a rule-utilitarian calculus.*

The result confirms the advantage of Kantian optimization for studying electoral participation in a spatial framework. Indeed, while the rule-utilitarian model with heterogenous costs predicts the same turnout rate among all supporters in each group, Kantian optimization yields heterogenous participation as an increasing function of the intensity of support. Such a positive correlation between the intensity of support and the probability of voting is more in line with the participation patterns theorized and observed by the previous literature.

Let us turn again to the example of a Tullock contest success function $P(v_a, v_b) = \frac{v_a}{v_a+v_b}$. In this case, the unique positive Nash-Kantian equilibrium is given by

$$p_a(z) = \frac{\sqrt{\tilde{u}_{\mathcal{Z}_A}\tilde{u}_{\mathcal{Z}_B}}}{\sqrt{\bar{c}}(\tilde{u}_{\mathcal{Z}_A}+\tilde{u}_{\mathcal{Z}_B})}\sqrt{[u_z(a)-u_z(b)]}$$
$$p_b(z) = \frac{\sqrt{\tilde{u}_{\mathcal{Z}_A}\tilde{u}_{\mathcal{Z}_B}}}{\sqrt{\bar{c}}(\tilde{u}_{\mathcal{Z}_A}+\tilde{u}_{\mathcal{Z}_B})}\sqrt{[u_z(b)-u_z(a)]}$$

As with a fixed cost of voting, the coefficient of proportionality is the same for all supporters in both groups, it is positively related to the aggregate utility differential in both groups and negatively to the cost of voting. The aggregate number of votes are in this case

$$v_a = \frac{\tilde{u}_{\mathcal{Z}_A}\sqrt{\tilde{u}_{\mathcal{Z}_A}\tilde{u}_{\mathcal{Z}_B}}}{\sqrt{\bar{c}}(\tilde{u}_{\mathcal{Z}_A}+\tilde{u}_{\mathcal{Z}_B})} \quad , \quad v_b = \frac{\tilde{u}_{\mathcal{Z}_B}\sqrt{\tilde{u}_{\mathcal{Z}_A}\tilde{u}_{\mathcal{Z}_B}}}{\sqrt{\bar{c}}(\tilde{u}_{\mathcal{Z}_A}+\tilde{u}_{\mathcal{Z}_B})}$$

which, despite being analytically different from the rule-utilitarian solutions[10], offer however the same comparative statics properties: the number of votes in both groups is increasing in the aggregate utility differentials and decreasing in the cost of voting.

4 Discussion

I address here a few relevant issues, before concluding. I first discuss the endogenization of candidates' proposed policies a and b. I then examine whether one could amend the rule-utilitarian approach, instead of abandoning it, in order to improve its predictions at the individual level. Finally, I offer a critical perspective on the *Kantian* label that economists usually assign to agents' reasoning based on a universalization principle.

[10]The rule-utilitarian solutions under heterogenous costs of voting are

$$v_a = \frac{(u_{\mathcal{Z}_A}|\mathcal{Z}_A|)^{\frac{3}{4}}(u_{\mathcal{Z}_B}|\mathcal{Z}_B|)^{\frac{1}{4}}}{\bar{c}(\sqrt{u_{\mathcal{Z}_A}|\mathcal{Z}_A|}+\sqrt{u_{\mathcal{Z}_B}|\mathcal{Z}_B|})} \quad , \quad v_b = \frac{(u_{\mathcal{Z}_A}|\mathcal{Z}_A|)^{\frac{1}{4}}(u_{\mathcal{Z}_B}|\mathcal{Z}_B|)^{\frac{3}{4}}}{\bar{c}(\sqrt{u_{\mathcal{Z}_A}|\mathcal{Z}_A|}+\sqrt{u_{\mathcal{Z}_B}|\mathcal{Z}_B|})}$$

4.1 Candidates' Choice of Policies

In order to focus on citizens' participation behavior, I have taken as given candidates' policies a and b. The advantage of a spatial framework, however, is also to study how candidates choose their platforms on the policy space. While determining policies endogenously is beyond the scope of this paper, I sketch here some preliminary considerations. An equilibrium analysis of policy choices requires assumptions on candidates' objective. In the classical Downsian model, candidates are purely office-motivated and maximize therefore their probability of winning. In the framework of this paper, office-motivation corresponds to maximizing $P(v_a, v_b)$ for candidate A and to minimizing it for candidate B. Note that this implies that candidates care only about the aggregate number of votes v_a and v_b, and not about the specific distribution of individual voting probabilities given by $p_a(z)$ and $p_b(z)$.

In our analysis, independently of citizens' ethical calculus and in both cases of fixed and heterogenous voting costs, the aggregate number of votes v_a and v_b depend crucially on the groups' aggregate utility differentials. Consider the example of a Tullock contest function $P(v_a, v_b) = \frac{v_a}{v_a+v_b}$ in the case of a fixed cost of voting, whose solutions for the number of votes are given in (10). It is easy to check that candidates' maximization problems correspond to

$$\max_a \frac{u_{\mathcal{Z}_A}}{u_{\mathcal{Z}_A} + u_{\mathcal{Z}_B}} \quad , \quad \max_b \frac{u_{\mathcal{Z}_B}}{u_{\mathcal{Z}_A} + u_{\mathcal{Z}_B}}$$

The aggregate utility differentials $u_{\mathcal{Z}_A}$ and $u_{\mathcal{Z}_B}$, in turn, depend on the primitive elements of the model, such as citizens' distribution on the policy space $f(z)$ or the shape of their utility function $u_z(x)$. In general, thus, also the existence and properties of the political equilibrium for candidates' policies a and b depend on such primitive elements. In a related paper (Grillo 2021), I examine in more detail candidates' strategies and provide conditions on $f(z)$ and $u_z(x)$ for a result of turnout-driven polarization, occurring when candidates pursue an electoral strategy of mobilization.

4.2 Why not *subgroup* rule-utilitarianism?

Could we amend somehow the rule-utilitarian approach in order to obtain heterogeneity of supporters' turnout behavior as a function of their preferences, without resorting to Kantian optimization? The answer is in some sense affirmative, although ultimately it implies moving to a lesser degree in the same direction, i.e. reducing the level at which supporters' utility is aggregated from group utility to a finer dimension. Consider for example a simpler model in which each group is further divided in two subgroups: a subgroup supports the candidate strongly, while the other only weakly. In this case, *subgroup* rule-utilitarians agents would maximize the aggregate subgroup utility taking as given the voting behavior both in the other subgroup of supporters of the same candidate and in the two subgroups of supporters of the opposing candidate. Under the same assumptions as in the previous sections, one can easily conjecture that the logic behind Lemma 1 makes the probability of voting constant among members of a subgroup but does not prevent different threshold costs between subgroups. In this case, strong supporters could vote with a higher probability than weak supporters, showing a positive relationship between participation and the intensity of support for a candidate.

There are, however, two inconveniences of a *subgroup* rule-utilitarian model. The first is the analytical complexity, as already in our simple example the strategic interaction involves four different maximization problems. The second is that when the reduction of the level of aggregation is taken to the limit, one stumbles back onto the issue of pivotality. If the distribution of citizens over the policy space is continuous, as in the model of this paper, every set of citizens sharing the same bliss point has mass zero. Hence, one cannot take the level of aggregation down to voters' bliss point, because the aggregate utility would then coincide with the individual utility, and supporters within a subgroup would even collectively be unable to affect the outcome of the election. Kantian optimization, instead, provides a very tractable model to account for voters' idiosyncratic preferences, even in the presence of a continuous distribution.

4.3 On the ***Kantian*** Label

A reference to Kant is customary in the economic literature to denote the type of counterfactual reasoning that agents display in the model when they envision a universalization of their behavior. With respect to the ethical voting model, it is interesting to note that an analogous mention of Kant is made by Feddersen (2004) in justifying the rule-utilitarian approach, which follows Harsanyi's (1977) tribute to Kant's intellectual tradition of claiming a requirement of universality for moral rules. Indeed, while the universalization principle is explicit in Roemer's Kantian optimization, it is nonetheless implicit in the rule-utilitarian calculus, as the prescribed rule is optimal only if followed by everyone.

An explicit Kantian label, however, also risks generating some misunderstanding, as Kantian moral philosophy is generally understood as non-consequentialist. In my opinion, instead, both approaches build on a consequentialist interpretation of the universalization principle, more in line with rule-consequentialism. As such, I would tentatively describe Kantian optimization as a model of egoistic (non-utilitarian) rule-consequentialist agents.[11] A valuable discussion on Kantian rationality is in Sugden (1991), while a critique of the Kantian label has been expressed by Wolfelsperger (1999) and Ballet and Jolivet (2003). As a comparison, White (2004, 2019) discusses how the paradigm of homo economicus can relate to Kantian moral philosophy in its orthodox deontological interpretation.

5 Conclusion

I have argued in favor of a Kantian optimization model of electoral participation within heterogenous groups, in which supporters share a preference for a candidate but have different intensities of support. In a spatial framework, the difference in intensities comes from voters' underlying idiosyncratic preferences on the policy space. A rule-utilitarian model of voter turnout, typically praised for its ability to generate good comparative statics properties on aggregate participation, fails to account for the heterogeneity of behavior at the individual level. Kantian optimization, instead, predicts participation as an increasing function of voters' utility differential from candidates' policies. The results are consistent with the theoretical and empirical literature on spatial voting showing how voters' distance from candidates affect their likelihood of participating. Furthermore, if voting costs are identical among supporters, an equivalence result emerges for the solutions of the aggregate number of votes in both Kantian optimization and the rule utilitarian model. In this case, Kantian optimization can complement the aggregate predictions from the rule-utilitarian model by specifying a unique rule at the individual level on which supporters can coordinate.

What makes Kantian optimization deliver better predictions at the individual level is its focus on individual utility. Indeed, while rule-utilitarians follow the rule that maximizes aggregate utility, Kantian optimizing agents maximize their individual utility. Their

[11] Roemer (2019, Ch.1) acknowledges using the term Kantian for its suggestive meaning and not to imply a deeper Kantian justification.

ethical principle consists in constraining such optimization by universalizing (within their group) their potential deviations from a participation rule. With respect to Roemer's broader approach, I have only considered deviations in a multiplicative form. From a theoretical standpoint, considering the case in which the universalization does not imply the same scalar deviation for all concerned agents but allows for heterogeneity in deviations represents a promising direction for future research. Moreover, in line with the positive rather than normative perspective of the analysis, the model calls for empirical evidence that could more precisely associate voters' actual reasoning to either a utilitarian or a Kantian ethical principle.

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A Appendix

Proof of Lemma 1:

Consider any non-constant continuous $c_a(z)$ and the resulting aggregate number of votes $v_a = \int_{\mathcal{Z}_A} \frac{1}{c} c_a(z) f(z) dz$. There always exists a constant turnout rule k that yields the same number of votes v_a but lower aggregate voting costs, i.e. such that $\int_{\mathcal{Z}_A} \int_0^{c_a(z)} \frac{1}{c} c \, dc f(z) dz >$

$\int_{\mathcal{Z}_A}\int_0^k \frac{1}{\bar{c}} c\, dc f(z)dz$. Hence the aggregate group utility is higher under the voting rule k than under the voting rule $c_a(z)$. One can alternatively solve the corresponding calculus of variations problem

$$\min_{c_a(z)\in\mathcal{C}} \int_{\mathcal{Z}_A} F(z, c_a(z))dz \quad \text{subject to} \quad \int_{\mathcal{Z}_A} G(z, c_a(z))dz = v_a$$

where $F(z, c_a(z)) = \int_0^{c_a(z)} \frac{1}{\bar{c}} c\, dc$ and $G(z, c_a(z)) = \int_0^{c_a(z)} \frac{1}{\bar{c}} dc$. The augmented Lagrangian is $\int_{\mathcal{Z}_A} F(z, c_a(z))dz + \lambda\, G(z, c_a(z))dz$ and the Euler-Lagrange equation gives

$$\frac{\partial F}{\partial c_a(z)} + \lambda \frac{\partial G}{\partial c_a(z)} = \frac{(c_a(z) + \lambda)}{\bar{c}} = 0$$

from which we obtain that $c_a(z)$ is constant. Note that the second variation equals $\int_{\mathcal{Z}_A} \frac{1}{\bar{c}} v^2 dz$ which is positive definite for all variations $v(z)$, and hence the solution is indeed a minimizer.

Proof of Proposition 1:

Rewrite system (9) as

$$\frac{\partial}{\partial v_a} P(v_a, v_b) \cdot u_{\mathcal{Z}_A} - c = 0$$
$$-\frac{\partial}{\partial v_b} P(v_a, v_b) \cdot u_{\mathcal{Z}_B} - c = 0$$

We have $v_a \in [0, |\mathcal{Z}_A|]$ and $v_b \in [0, |\mathcal{Z}_B|]$ where $|\mathcal{Z}_A| = \int_{\mathcal{Z}_A} f(z)dz$ and $|\mathcal{Z}_B| = \int_{\mathcal{Z}_B} f(z)dz$ are the population shares of the two groups, with $|\mathcal{Z}_A| + |\mathcal{Z}_B| = 1$. By assumption 2(ii), $\frac{\partial}{\partial v_a} P(v_a, v_b)$ is decreasing in v_a and $-\frac{\partial}{\partial v_b} P(v_a, v_b)$ is decreasing in v_b. Then if c takes a value in the range satisfying

$$\max_{v_b} \frac{\partial}{\partial v_a} P(|\mathcal{Z}_A|, v_b) \cdot u_{\mathcal{Z}_A} < c < \min_{v_b} \frac{\partial}{\partial v_a} P(0, v_b) \cdot u_{\mathcal{Z}_A}$$

and

$$\max_{v_a} -\frac{\partial}{\partial v_b} P(v_a, |\mathcal{Z}_B|) \cdot u_{\mathcal{Z}_B} < c < \min_{v_a} -\frac{\partial}{\partial v_b} P(v_a, 0) \cdot u_{\mathcal{Z}_B}$$

we have $\frac{\partial}{\partial v_a} P(0, v_b) \cdot u_{\mathcal{Z}_A} - c > 0$ and $\frac{\partial}{\partial v_a} P(|\mathcal{Z}_A|, v_b) \cdot u_{\mathcal{Z}_A} - c < 0\ \forall v_b$, and as well $-\frac{\partial}{\partial v_b} P(v_a, 0) \cdot u_{\mathcal{Z}_B} - c > 0$ and $\frac{\partial}{\partial v_b} P(v_b, |\mathcal{Z}_B|) \cdot u_{\mathcal{Z}_B} - c < 0\ \forall v_a$. Hence, by the Poincaré-Miranda theorem, system (9) has a solution $(v_a, v_b) \in [0, |\mathcal{Z}_A|] \times [0, |\mathcal{Z}_B|]$ and thus, given (7), a solution for the proportionality coefficients π_a, π_b.

The proof of Proposition 3 is analogous.