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# A Dynamic Theory of The Balassa-Samuelson Effect

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## A Dynamic Theory of The Balassa-Samuelson Effect\*

By

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#### Abstract

The Balassa-Samuelson effect is still an important phenomenon in the theory of economic development, as Balassa states, "As economic development is accompanied by greater inter-country differences in the productivity of tradable goods, differences in wages and service prices increase, and correspondingly so do differences in purchasing power parity and exchange rates." To the best of our knowledge, the Balassa-Samuelson effect has not been formally examined in the framework of optimal growth theory. By embedding the Balassa-Samuelson's original model in an optimal growth model setting, we investigate the validity of the Balassa-Samuelson effect in such a case and show that the Balassa-Samuelson effect follows from one of the properties of the optimal steady state.

JEL. C61, F31, O41

**Keywords.** Two sector optimal growth, optimal steady state, saddle-point stability, phase diagram, Hamiltonian, capital intensity

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## 1. Introduction

The Balassa-Samuelson effects (BS effects, hereafter) is still an important phenomenon in the theory of economic development, as Balassa (1964) states, "As economic development is accompanied by greater inter-country differences in the productivity of tradable goods, differences in wages and service prices increase, and correspondingly so do differences in purchasing power parity and exchange rates." Formally it can be expressed as the following formula (I).

(I) 
$$\frac{\dot{p}}{p} = \left(\frac{1-\beta}{1-\alpha}\right)\frac{A_T}{A_T} - \frac{A_N}{A_N}$$

where  $\dot{x}$  indicates a time derivative; dx/dt, p is the relative price of non-tradable goods, where the price of tradable goods is numéraire,  $\alpha$  is the capital share of the tradable goods output,  $\beta$  is the capital share of the non-tradable output, and  $A_i$  (i=T, N) is the TFP of the sector  $i^3$ .

This problem is addressed in the framework of optimal growth theory. To our knowledge, the BS effect has never been formally examined in the framework of twosector optimal growth theory in the case of a small country that cannot determine interest rates but can give access to world capital markets. We prove that the BS effect holds as a property of the long-run optimal steady state. This shows that the BS property follows from one of the properties of the optimal steady state.

## 2. Optimal growth problem

Here we will study the small country case, originally studied separately by Balassa (1964) and Samuelson (1964). The following are standard assumptions often made in this study.

Assumption 1.  $\alpha > \beta$  and  $r^*$  is given in the world market.

The assumption means that the tradable goods sector is more capital intensive than the non-tradable goods sector and the interest rate is determined in the world market.

We may set up and solve the following two-sector optimal growth problem with a non-homothetic objective function. Note that all outputs are measured in efficiency units.

<sup>&</sup>lt;sup>3</sup> For the standard derivation of the equation (I) and (II), please refer to Asea et al. (1990) and Coreharde et al. (2020) for comprehensive explanations.

In addition, the standard discussion of the BS effect solely depends on the supply side. In contrast, the model here incorporates the demand side by introducing the Stone-Geary utility function as the objective function, which is non-homothetic. Therefore, the model includes the possibility that the BS effect also depends on demand-side parameters associated with economic development.

$$(P) \begin{cases} \int_{t=0}^{\infty} \rho^{t} \Big[ \xi \ln \Big( c_{T} - \gamma_{T} \Big) + (1 - \xi) \ln \Big( \widetilde{c}_{N} - \gamma_{N} \Big) \Big] dt, \\ \text{where } 0 < \xi < 1, c_{T} - \gamma_{T} > 0 \text{ and } c_{N} - \gamma_{N} > 0. \\ \text{st} \\ (1) \widetilde{y}_{T} = k_{T}^{\alpha} \ell_{T}^{1-\alpha}, \ \widetilde{y}_{T} = \frac{y_{T}}{A_{T}}, \ (2) \ \widetilde{c}_{N} = k_{N}^{\beta} \ell_{N}^{1-\beta}, \ \widetilde{c}_{N} = \frac{c_{N}}{A_{N}} \\ (3) \ \ell_{T} + \ell_{N} = 1, \ (4) \ k_{T} + k_{N} = k, \\ (5) \ \widetilde{k} = \widetilde{y}_{T} - c_{T} - \delta k, \text{ the initial stock } k(0) \text{ is given.} \end{cases}$$

The notations are as follows:

 $\gamma_i (i = N, T)$ : a basic consumption of good *i* and  $c_i > \gamma_i$ .  $\rho$ : a discount rate (0< $\rho$ <1),  $c_T$ : *per* – *capita consumption of the tradable goods*,  $c_N$ : *per* – *capita consumption of the non* – *tradable goods*,  $\ell_i$ : *labor input in the i th sector* (*i*=N,T), *and*  $k_i$ : *capital input in the i th sector* (*i*=N,T),

Lemma 1 below states that under our production constraints, the efficiency allocation problem of production (P') shown below can be summarized as a social production function:  $\tilde{y}_T = T(\tilde{c}_N, k)$ , as demonstrated by Baierl, Nishimura and Yano (1998).

$$(P') \begin{cases} (1) \tilde{y}_{T} = k_{T}^{\alpha} \ell_{T}^{1-\alpha}, \tilde{y}_{T} = \frac{y_{T}}{A_{T}}, \\ (2) \tilde{c}_{N} = k_{N}^{\beta} \ell_{N}^{1-\beta}, \tilde{c}_{N} = \frac{c_{N}}{A_{N}} \\ (3) \ell_{T} + \ell_{N} = 1, \\ (4) k_{T} + k_{N} = k, \end{cases}$$

Lemma 1. The problem (P') is summarized as the following social production function.

$$\widetilde{y}_{T} = \left[\frac{\beta(1-\alpha)}{\Delta(\widetilde{c}_{N},k)}\right]^{1-\alpha}(k-k_{N}) \equiv T(\widetilde{c}_{N},k)$$

where

 $\Delta(\tilde{c}_N,k)\equiv\beta(1-\alpha)\,k+(\alpha-\beta)\,k_N$ 

and  $k_N$  is obtained by solving the following equation:

$$(*) \left[ \alpha(1-\beta) \right]^{1-\beta} k_N = \widetilde{c}_N \left[ \Delta(\widetilde{c}_N, k) \right]^{1-\beta}.$$

Note that solving the implicit function (\*) with respect to  $k_N \Rightarrow k_N = f(\tilde{c}_N, k)$ . Also hold the following properties:

$$\frac{\partial T}{\partial k} = T_k = r = \alpha \left[ \frac{\beta(1-\alpha)}{\Delta(\tilde{c}_N, k)} \right]^{1-\alpha} \left( \because \alpha \tilde{y}_T = \tilde{r}k_T \Rightarrow \tilde{r} = \frac{\alpha \tilde{y}_T}{k_T} \right),$$
  
and

$$\frac{\partial T}{\partial c_{N}} = T_{c_{N}} = -\tilde{p} = -\frac{T_{k}}{\beta} \left[ \frac{\Delta(\tilde{c}_{N}, k)}{\alpha(1-\beta)} \right]^{1-\beta} \left( \begin{array}{c} \because \frac{\partial L}{\partial k_{N}} = p\beta k_{N}^{\beta-1} \mathcal{C}_{N}^{1-\beta} - \tilde{r} = 0 \\ \Rightarrow \tilde{p} = \frac{\tilde{r}k_{N}}{\beta \tilde{c}_{N}} \text{and } k_{N} = \tilde{c}_{N} \left[ \frac{\Delta}{\alpha(1-\beta)} \right]^{1-\beta} \text{from } (*) \end{array} \right)$$

From Lemma 1 and Assumption 1,

$$T_{k} = r^{*} = \alpha \left[ \frac{\beta(1-\alpha)}{\Delta(\tilde{c}_{N},k)} \right]^{1-\alpha} \Rightarrow \Delta = \left( \frac{\alpha}{r^{*}} \right)^{\frac{1}{1-\alpha}} \left[ \beta(1-\alpha) \right]$$
(1)

Also, it holds that

$$\left[\alpha(1-\beta)\right]^{1-\beta}k_{N} = \tilde{c}_{N}\Delta^{1-\beta} = \tilde{c}_{N}\left(\frac{\alpha}{r^{*}}\right)^{\frac{1-\beta}{1-\alpha}} \left[\beta(1-\alpha)\right]^{1-\beta}.$$

Thus, we have

$$k_{N} = \left\{ \left( \frac{\alpha}{r^{*}} \right)^{\frac{1-\beta}{1-\alpha}} \left[ \frac{\alpha(1-\beta)}{\beta(1-\alpha)} \right]^{1-\beta} \right\} \widetilde{c}_{N}$$

$$\tag{2}$$

From the definition of  $\Delta$ ,

$$\beta(1-\alpha) k + (\alpha - \beta) k_N \equiv \Delta = \left(\frac{\alpha}{r^*}\right)^{\frac{1}{1-\alpha}} \left[\beta(1-\alpha)\right].$$

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Solving this with respect to  $k_N$ ,

$$k_{N} = -\left[\frac{\beta(1-\alpha)}{\alpha-\beta}\right]k + \left(\frac{\alpha}{r^{*}}\right)^{\frac{1}{1-\alpha}}\left[\frac{\beta(1-\alpha)}{\alpha-\beta}\right]$$
(3)

From Eq. (2) and Eq. (3), it follows that

$$\tilde{c}_{N} = -\frac{\left[\frac{\beta(1-\alpha)}{\alpha-\beta}\right]}{\Omega}k + \frac{\left(\frac{\alpha}{r^{*}}\right)^{\frac{1}{1-\alpha}}\left[\frac{\beta(1-\alpha)}{\alpha-\beta}\right]}{\Omega},$$

where  $\Omega \equiv \left(\frac{\alpha}{r^*}\right)^{\frac{1-\beta}{1-\alpha}} \left[\frac{\alpha(1-\beta)}{\beta(1-\alpha)}\right]^{1-\beta}$ .

Finally, the following expression yields.

$$\widetilde{c}_{N} = \underbrace{-\frac{\left[\beta(1-\alpha)\right]^{2-\beta}}{(\alpha-\beta)\left[\alpha(1-\beta)\right]^{1-\beta}} \left(\frac{\alpha}{r^{*}}\right)^{\frac{1-\beta}{1-\alpha}} k}_{\Gamma} + \underbrace{\left(\frac{\alpha}{r^{*}}\right)^{\frac{\beta-\alpha}{1-\alpha}} \frac{\left[\beta(1-\alpha)\right]^{2-\beta}}{(\alpha-\beta)\left[\alpha(1-\beta)\right]^{1-\beta}}}_{\Pi}$$

$$= \Gamma k + \Pi.$$
(4)

Eq. (4) indicates that  $\tilde{c}_N$  is a function of k, denoted as  $\tilde{c}_N(k)$ . And the following properties can be also shown.

$$\frac{\partial \widetilde{c}_N}{\partial k} = -\frac{\left[\beta(1-\alpha)\right]^{2-\beta}}{(\alpha-\beta)\left[\alpha(1-\beta)\right]^{1-\beta}} \left(\frac{\alpha}{r^*}\right)^{\frac{1-\beta}{1-\alpha}} < 0.$$

and

$$T_{\tilde{c}_{N}} = -\frac{T_{k}}{\beta} \left[ \frac{\Delta(\tilde{c}_{N},k)}{\alpha(1-\beta)} \right]^{1-\beta} = -\left(r^{*}\right)^{\frac{(\beta-\alpha)}{1-\alpha}} \frac{\alpha^{\left(\frac{1-\beta}{1-\alpha}\right)}}{\beta} \left[ \frac{\beta(1-\alpha)}{\alpha(1-\beta)} \right]^{1-\beta} < 0$$

#### 2.1 Small country optimal problem

Because of  $\tilde{c}_N(k)$ , based on the original problem (P), we present it as a small country

optimal problem (P") by rewriting it as follows.

$$(P'') \begin{cases} \max_{\substack{(c_T,k) \\ t=0}} \int_{t=0}^{\infty} e^{-\rho t} \left[ \xi \ln \left( c_T - \gamma_T \right) + (1 - \xi) \ln \left( \tilde{c}_N(k) - \gamma_N \right) \right] dt, \\ st \\ \vdots \\ k = T(\tilde{c}_N(k), k) - c_T - \delta k, \quad k(0) \text{ given.} \end{cases}$$

In contrast to the original problem (*P*), the small country optimal problem includes a wealth effect term  $\tilde{c}_N(k)$  in the objective function. The optimal growth problem with wealth effects was first studied by Kurz (1968).

Let us define the Hamiltonian as follows.

$$H = e^{-\rho t} \left[ \xi \ln \left( c_T - \gamma_T \right) + (1 - \xi) \ln \left( \widetilde{c}_N(k) - \gamma_N \right) \right] + \lambda \{ T(\widetilde{c}_N(k), k) - c_T - \delta k \}$$

Then, the following necessary conditions yield.

$$\frac{\partial H}{\partial \lambda} = k \Rightarrow k = T(\tilde{c}_N(k), k) - c_T - \delta k$$
(5)

$$-\frac{\partial H}{\partial k} = \dot{\lambda} \Rightarrow \dot{\lambda} = -\left\{ e^{-\rho t} (1-\xi) \left[ \frac{1}{\tilde{c}_N - \gamma_N} \right] \left( \frac{\partial \tilde{c}_N}{\partial k} \right) + \lambda \left[ T_{\tilde{c}_N} \left( \frac{\partial \tilde{c}_N}{\partial k} \right) + T_k - \delta \right] \right\}$$
(6)

$$\frac{\partial H}{\partial c_T} = 0 \Rightarrow e^{-\rho t} \xi \left[ \frac{1}{c_T - \gamma_T} \right] - \lambda = 0$$
(7)

Differentiate Eq. (7) with respect to t,

$$\dot{\lambda} = -\rho e^{-\rho t} \xi \left[ \frac{1}{c_T - \gamma_T} \right] + e^{-\rho t} \frac{-c_T}{\left(c_T - \gamma_T\right)^2}$$
(8)

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Substituting Eq. (7) into Eq. (6),

$$\dot{\lambda} = -e^{-\rho t}(1-\xi) \left[ \frac{1}{\tilde{c}_N - \gamma_N} \right] \left( \frac{\partial \tilde{c}_N}{\partial k} \right) - e^{-\rho t} \xi \left[ \frac{1}{c_T - \gamma_T} \right] \left[ T_{\tilde{c}_N} \left( \frac{\partial \tilde{c}_N}{\partial k} \right) + T_k - \delta \right]$$
(9)

Equating Eq. (8) and Eq. (9) yields

$$\begin{split} &-\rho e^{-\rho t} \xi \left[\frac{1}{c_T - \gamma_T}\right] - e^{-\rho t} \frac{c_T}{\left(c_T - \gamma_T\right)^2} \\ &= -e^{-\rho t} (1 - \xi) \left[\frac{1}{\tilde{c}_N - \gamma_N}\right] \left(\frac{\partial \tilde{c}_N}{\partial k}\right) - e^{-\rho t} \xi \left[\frac{1}{c_T - \gamma_T}\right] \left[T_{\tilde{c}_N} \left(\frac{\partial \tilde{c}_N}{\partial k}\right) + T_k - \delta\right]. \end{split}$$

Solving the above with respect to  $c_T$ , it follows

$$\begin{split} \dot{c}_{T} &= \underbrace{(1-\xi) \left(\frac{\partial \widetilde{c}_{N}}{\partial k}\right)}_{A} \left[\frac{\left(c_{T}-\gamma_{T}\right)^{2}}{\widetilde{c}_{N}-\gamma_{N}}\right] + \underbrace{\xi \left[T_{\widetilde{c}_{N}} \left(\frac{\partial \widetilde{c}_{N}}{\partial k}\right) + T_{k}-\delta\right]}_{B} \left(c_{T}-\gamma_{T}\right) \\ &= A \left[\frac{\left(c_{T}-\gamma_{T}\right)^{2}}{\widetilde{c}_{N}-\gamma_{N}}\right] + B\left(c_{T}-\gamma_{T}\right) \end{split}$$

Thus, we finally obtain the following two differential equations:

$$\begin{cases} \mathbf{\dot{k}} = \phi_1(k, c_T) = T(\tilde{c}_N(k), k) - c_T - \delta k \\ \mathbf{\dot{c}}_T = \phi_2(k, c_T) = A \left[ \frac{\left(c_T - \gamma_T\right)^2}{\tilde{c}_N(k) - \gamma_N} \right] + B\left(c_T - \gamma_T\right) \end{cases}$$

## 2.2 Optimal steady state

Note that Assumption 1 implies that  $r^* = T_k \Big|_{\binom{k^*, c_T^*}{k}}$ . The optimal steady state will be obtained by solving the following system:

$$\left(\phi_{1}\left(k^{*}, c_{T}^{*}\right) = T\left[\tilde{c}_{N}\left(k^{*}\right), k^{*}\right] - c_{T}^{*} - \delta k^{*} = 0$$
(10)

$$\begin{cases} \phi_2(k^*, c_T^*) = A\left[\frac{\left(c_T^* - \gamma_T\right)^2}{\widetilde{c}_N(k^*) - \gamma_N}\right] + B\left(c_T^* - \gamma_T\right) = 0 \end{cases}$$
(11)

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From Eq. (11), it follows that

$$A\left[\frac{\left(c_{T}^{*}-\gamma_{T}\right)}{\widetilde{c}_{N}(k^{*})-\gamma_{N}}\right]+B=0 \Rightarrow c_{T}^{*}=-\frac{B}{A}\left(\widetilde{c}_{N}(k^{*})-\gamma_{N}\right)+\gamma_{T}$$
(12)

where from Eq. (4),

$$\tilde{c}_{N}(k^{*}) = \Gamma k^{*} + \Pi \Rightarrow \left(c_{T}^{*} - \gamma_{T}\right) = -\frac{B}{A}\left(\tilde{c}_{N}(k^{*}) - \gamma_{N}\right) = -\frac{B}{A}\left(\Gamma k^{*} + \Pi - \gamma_{N}\right)$$
(13)

Furthermore, from Eq. (1) and Eq. (3) it follows that

$$T\left[\tilde{c}_{N}(k^{*}), k^{*}\right] = \left[\frac{\beta(1-\alpha)}{\Delta(\tilde{c}_{N}(k^{*}), k^{*})}\right]^{1-\alpha}(k^{*}-k_{N}^{*})$$
$$= \left[\frac{\beta(1-\alpha)}{\left(\frac{\alpha}{r^{*}}\right)^{\frac{1}{1-\alpha}}\left[\beta(1-\alpha)\right]}\right]^{1-\alpha}\left\{k^{*}+\left[\frac{\beta(1-\alpha)}{\alpha-\beta}\right]k^{*}-\left(\frac{\alpha}{r^{*}}\right)^{\frac{1}{1-\alpha}}\left[\frac{\beta(1-\alpha)}{\alpha-\beta}\right]\right\}.$$
(14)

Substituting Eq. (12) and Eq. (13) into Eq. (10), we can uniquely solve with respect to  $k^*$  and substituting the result into Eq. (12),  $c_T^*$  is also uniquely determined.

## 3. Balassa-Samuelson effect

To derive the BS effect, we need to go back to Eq. (1). Since the world interest rate  $r^*$  is given in the world market, the following relation holds in the non-efficiency unit on the optimal steady state.

$$T_{k}^{*} = A_{T} \alpha \left[ \frac{\beta(1-\alpha)}{\Delta^{*}} \right]^{1-\alpha} = r^{*} \Rightarrow \Delta^{*} = \beta(1-\alpha) \left[ \frac{A_{T} \alpha}{\delta + \rho} \right]^{\frac{1}{1-\alpha}}$$

Also note that non-efficient unit prices are derived from Eq. (2) as follows.

$$\begin{split} T^*_{\widetilde{c}_N} &= -\frac{r^* k_N^*}{\beta A_N c_N^*} = -\frac{r^*}{A_N \beta} (\Delta^*)^{1-\beta} \Big[ \alpha (1-\beta) \Big]^{-(1-\beta)} \\ &= - \left( \frac{r^*}{\beta A_N} \right) \! \left[ \left( \frac{A_T \alpha}{\delta + \rho} \right)^{\frac{1}{1-\alpha}} \frac{\beta (1-\alpha)}{\alpha (1-\beta)} \right]^{1-\beta} \\ &\Rightarrow p^* = \! \left( A_T \right)^{\frac{1-\beta}{1-\alpha}} \! \left( \frac{\delta + \rho}{\beta A_N} \right) \! \left( \frac{\alpha}{r^*} \right)^{\frac{1-\beta}{1-\alpha}} \! \left[ \frac{\beta (1-\alpha)}{\alpha (1-\beta)} \right]^{1-\beta} \quad \left( \because p^* = -T^*_{\widetilde{c}_N} \right) \end{split}$$

Note that  $A_T$  and  $A_N$  denote the technical progresses and the functions of time. Taking the logarithm of the above relation on the optimal steady state and differentiating it by time yields

$$\begin{split} &\ln p^* = \ln \left\{ \left( \frac{\delta + \rho}{\beta} \right) \left( \frac{\alpha}{r^*} \right)^{\frac{1 - \beta}{1 - \alpha}} \left[ \frac{\beta (1 - \alpha)}{\alpha (1 - \beta)} \right]^{1 - \beta} \right\} + \left( \frac{1 - \beta}{1 - \alpha} \right) \ln A_T(t) - \ln A_N(t) \\ &= \Psi + \left( \frac{1 - \beta}{1 - \alpha} \right) \ln A_T(t) - \ln A_N(t) \\ &\vdots \\ &\Rightarrow \frac{p^*}{p^*} = \left( \frac{1 - \beta}{1 - \alpha} \right) \frac{A_T}{A_T} - \frac{A_N}{A_N} \end{split}$$

The final equation is exactly the same as the BS effect equation given as (I). And it still depends only on the production technology parameters.

Furthermore, we show that the optimal steady state is saddle-point stable. The stability argument requires the following additional assumption.

Assumption 2. The interest rate is given in the world market such that  $r^* > \delta$ .

It implies that the world interest rate is greater than the depreciation rate.

To study the stability of the system, we need to calculate partial derivatives of functions;  $\phi_1, \phi_2$ .

$$\begin{cases} \left. \frac{\partial \phi_1}{\partial k} \right|_{\left(k^*, c_T^*\right)} = T^*_{\widetilde{c}_N} \left( \frac{\partial \widetilde{c}_N}{\partial k} \right) \right|_{\left(k^*, c_T^*\right)} + r^* - \delta, \\ \left. \frac{\partial \phi_1}{\partial c_T} \right|_{\left(k^*, c_T^*\right)} = -1. \end{cases}$$

$$\Rightarrow \left. \frac{\partial c_T}{\partial k} \right|_{\phi_1 = 0} = - \left[ T^*_{\widetilde{c}_N} \left( \frac{\partial \widetilde{c}_N}{\partial k} \right) \right|_{\left(k^*, c_T^*\right)} + r^* - \delta \right] < 0.$$

Furthermore,

$$\begin{cases} \left. \frac{\partial \phi_2}{\partial k} \right|_{\left(k^*, c_T^*\right)} = A\left(c_T^* - \gamma_T\right)^2 \left(\frac{1}{\tilde{c}_N(k^*) - \gamma_N}\right)^2 \left(-\frac{\partial \tilde{c}_N}{\partial k}\right) \right|_{\left(k^*, c_T^*\right)} \\ = (1 - \xi) \left(\frac{c_T^* - \gamma_T}{\tilde{c}_N(k^*) - \gamma_N}\right)^2 \cdot - \left(\frac{\partial \tilde{c}_N}{\partial k}\right)^2 \right|_{\left(k^*, c_T^*\right)} < 0 \\ \left. \frac{\partial \phi_2}{\partial c_T} \right|_{\left(k^*, c_T^*\right)} = 2A \left(\frac{c_T^* - \gamma_T}{\tilde{c}_N(k^*) - \gamma_N}\right) + B = A \left(\frac{c_T^* - \gamma_T}{\tilde{c}_N(k^*) - \gamma_N}\right) \text{ from Eq. (20).} \end{cases}$$

$$\Rightarrow \left. \frac{\partial c_T}{\partial k} \right|_{\phi_1 = 0} = \frac{-(1-\xi) \left( \frac{c_T^* - \gamma_T}{\widetilde{c}_N(k^*) - \gamma_N} \right)^2 \left( \frac{\partial \widetilde{c}_N}{\partial k} \right)^2 \right|_{\left(k^*, c_T^*\right)}}{A \left( \frac{c_T^* - \gamma_T}{\widetilde{c}_N(k^*) - \gamma_N} \right)} > 0.$$

Note that A < 0.

Based on the signs of the partial derivatives, we can draw the phase diagram Figure 1 as depicted below. It is clear that the optimal steady state exhibits the saddle-point stability.

We have concluded the following proposition.

**Proposition.** There exists a unique saddle-point stable optimal steady state;  $(k^*, c_T^*)$  and exhibits the BS effect under Assumptions 1 and 2.

## 4. Conclusion

We demonstrate that the BS effect is established along the long-run optimal steady state. Similar to its original nature, the BS effect is independent of the demand side. Only stability depended on demand-side parameters. Therefore, this study strongly supports the application of the BS effect to explain phenomena such as the long-run depreciation of the real effective exchange rate.

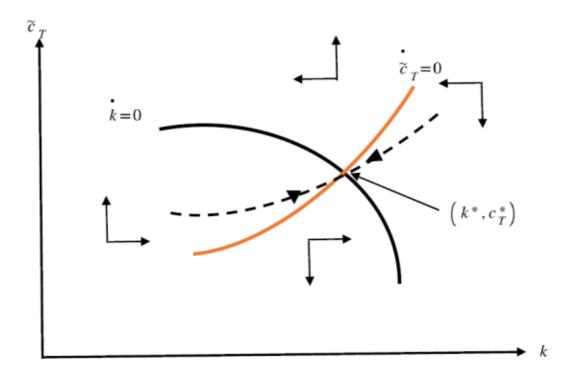


Figure 1. Saddle-point stability in small countries

### References

1. Asea, Patrick K., and W. Max Corden. "The Balassa-Samuelson Model: An

Overview." Review of International Economics 2.3 (1994): 191-200.

- 2. Balassa, Bela. "The purchasing-power parity doctrine: a reappraisal." *Journal of Political Economy* 72.6 (1964): 584-596.
- 3. Baierla, Gary, Kazuo Nishimura, and Makoto Yano. "The role of capital depreciation in multi-sectoral models." *Journal of Economic Behavior & Organization* 33.3-4 (1998): 467-479.
- 4. Couharde, Cécile, et al. "Measuring the Balassa-Samuelson effect: A guidance note on the RPROD database." *International Economics* 161 (2020): 237-247.
- 5. Samuelson, Paul A. "Theoretical notes on trade problems." *The Review of Economics and Statistics* (1964): 145-154.
- 6. Urzúa, Carlos M. "The Balassa-Samuelson and the capital-intensity hypotheses in a nutshell." *Research in Economics* 74.4 (2020): 336-343.
- 7. Kurz, Mordecai. "Optimal economic growth and wealth effects." *International Economic Review* 9.3 (1968): 348-357.

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