Renewable energy support: pre-announced policies and (in)-efficiency

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Abstract

This paper is essentially based on the assumption that policies supporting investment in intermittent renewable technologies cannot be contingent on meteorological events causing this intermittence. This decision was taken by most policymakers to avoid overly complex policy prescriptions. But in doing so, the first-best energy mix may be out of reach. We compare, in a unified second-best setting, the feed-in tariff, renewable premiums and tradable green certificates policy. We consider a “two-period, S-state” model. The S states reflect intermittency. Production decisions for renewable electricity are taken prior to the resolution of the uncertainty while the fossil-fuel sector adjusts its decision in each state. Retailers buy electricity on a state-dependent wholesale market which they deliver to consumers according to a fixed-tariff or a real-time-pricing contract. All these elements matter in the efficiency assessment of these policies.

Keywords: intermittency, renewables, feed-in tariff, premiums for renewable, tradable green certificates

JEL classification: D24, D61, D62, Q41, Q42, Q48

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1. Introduction

Renewable energy sources have significant benefits over fossil fuels for electricity production. Most notably, they contribute to reducing local air pollution and greenhouse gas emissions that have damaging consequences for humans and the environment (Burrett et al. 2009, Edenhofer et al. 2011, Ellabban et al. 2014, Ferroukhi et al. 2016). In order to support the deployment of renewable technologies such as solar panels and wind turbines, specific energy policies have been put in place. They aim to seek the benefits of prevailing renewables and overcome their inherent unattractiveness of producing random electricity only when the sun shines or the wind blows. Investing in these renewable technologies is less attractive for producers without government intervention. Hence, this paper is interested in studying the performance of policies for wind and solar technologies.

Many EU countries have embraced policies that directly support renewable electricity production and investment. The major policies include feed-in tariffs (FiTs), market and capacity premiums, as well as Renewable Portfolio Standards (RPS) implemented through Tradable Green Certificates (TGC) (Elie et al. 2021, Kitzing et al. 2012). In the U.S., the most popular renewable policy is the RPS (Bento et al. 2018) but mechanisms inspired by FiTs are also favored (Rickerson et al. 2007). Renewable energy policies are numerous and diversified but share a first common characteristic. They are pre-announced to incentivize long-term investments in renewable energy technologies. For example, the feed-in tariff in France for wind production of a capacity not exceeding 100 kW is fixed at 23 €ct per kWh independently of meteorological conditions. A second common characteristic of the policies is that they are financed by a levy on electricity consumption that is driven by retail tariffs. The widespread and least flexible tariff is the flat tariff that does not trigger consumers’ reactiveness to fluctuations in electricity production and wholesale price while the most flexible one is the Real-Time-Pricing (RTP). The latter is popular in countries such as Estonia, Romania, Spain Sweden, and the UK (IRENA 2019).

With intermittent electricity from solar and wind resources, the two previously mentioned features are crucial for assessing the performance of renewable policies. Firstly, pre-announced policies may not provide the right incentive to invest in renewable tech-

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1 The feed-in tariff and market premium policies are electricity generation supports. The FiT guarantees a fixed price per unit of renewable electricity sold and is above market price while the market premium for renewable electricity adds a subsidy to the wholesale price of electricity. The capacity subsidy reduces investment costs in renewable technologies. These price-driven policies are usually financed by a tax on final electricity consumption.

2 Renewable Portfolio Standards mandate that renewable sources such as solar and wind produce a specified share of electricity. A supporting mechanism for RPS is Tradable Green Certificates mandating that consumers are able to make certifiable claims that part of their consumption comes from renewable capacities. Hence, in addition to electricity, consumers buy certificates that are issued by renewable electricity producers.

3 See LEGAL (2012) on regulations of renewable energy in the EU 28 Member States.

4 Other tariffs that are more or less flexible include Time-of-Use (ToU), Critical Peak Pricing (CPP), and Variable CPP. They are described in Borenstein et al. (2002).
technologies when the wholesale electricity price fluctuates highly with solar and wind availability. An analogical example of this argument is depicted in a recent empirical study by Lamp and Samano (2023) who show the social loss from the misallocation of solar power plants. They attribute this result to uniform FiTs that are meant to incentivize electricity production driven by location-specific solar regimes. Secondly, the structure of retail electricity contracts modifies the suitable social welfare criterion according to which the efficiency of policies must be evaluated. For example, Ferrasse et al. (2022) consider the situation when retail contracts do not reflect all fluctuations in the wholesale markets. These restrictions on final electricity trades modify the efficiency criterion. Hence, this paper provides a theoretical analysis of the efficiency of renewable energy support by taking into account both the feature of pre-announced policy decisions and the structure of retail contracts.

Conceptually, we develop a partial equilibrium model of the electricity sector with two technologies: a fossil energy technology and a renewable one. The fossil energy technology produces uninterrupted electricity but causes emissions. The renewable one is emissions-free but produces electricity that depends on variable and uncertain conditions (e.g. sun or wind regime). We refer to these conditions as states of nature. Intermittency is formalized through competitive wholesale markets that are state-contingent. On the demand side, we consider that consumers are either non-reactive or reactive to price changes in the wholesale markets. In this framework, we first study the feed-in tariff, being the most popular renewable energy support (Jenner et al. 2013). Afterwards, we extend our study to other instruments namely renewable premiums and Tradable Green Certificates.

With non-reactive consumers, we use as efficiency criterion the constraint social optimum that accounts for their non-reactiveness. In the presence of intermittency, we show that the feed-in implements the constraint social optimum even with a state-independent policy rule. This is largely due to the assumption that the consumers are non-reactive. As their willingness to pay for electricity is based on an average price, we find that it is sufficient to tax them at the average marginal damage. With reactive consumers, we use as the reference for efficiency the standard social optimum that accounts for their reactiveness. We show that the feed-in implements the optimal mix if and only if the marginal damage and emissions are constant. Otherwise, we obtain a second-best energy mix where the policy may lead to either under or over-investment in renewable depending on the degree of correlation between the marginal cost change and the marginal damage.

Then, we extend our analysis to other instruments starting with two main types of renewable premiums: a market premium for renewable electricity production and an investment subsidy for renewable capacities. We demonstrate that the two policies are equivalent as they provide the same incentive to invest in renewable capacities. We even show that any premium policy is allocation equivalent to a feed-in policy with either non-reactive or reactive consumers. Finally, we consider a market-driven instrument in

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5 See, for instance, Ketterer (2014), Kyritsis et al. (2017), Rintamäki et al. (2017), and Sapio (2019) on the increase in price volatility when more electricity is produced from solar and wind resources.
the form of a Renewable Portfolio Standard implemented through Tradable Green Certificates. With non-reactive consumers, we find that the policy does not implement the constrained social optimum and is therefore outperformed by a feed-in policy. With reactive consumers, the market-based instrument still does not reach the social optimum. As the same holds for a feed-in policy, we cannot rank the instruments by comparing the second-best welfare and can only conclude that both remain inefficient.

The rest of the paper proceeds as follows. In the next section, we provide some background on renewable energy policies through a related literature review. Section 3 lays out the features of the model including those of the feed-in tariff as main policy. Section 4 defines the social welfare criterion by considering the non-reactiveness and reactivity of consumers. We study the implementability of a pre-announced feed-in tariff with non-reactive consumers in section 5 and reactive consumers in section 6. Since we show that a first-best feed-in policy is out of reach with reactive consumers, we analyze the consequences of a second-best policy choice in section 7. We also consider other renewable policies: renewable premiums in section 8 and Tradable Green Certificate as a market-based policy in section 9. Finally, section 10 concludes. The resolution of the model and proofs are relegated to the appendices.

2. Related literature

Our research is related to several strands of the literature. Firstly, it is linked to the literature on government support for solar and wind technologies. The volume of works on the latter is quite abundant with an overwhelming part of it studying the effectiveness of policies through simulations and empirical analyses. These works principally conclude an increase in solar and wind production that reduces emissions, with the magnitude of reductions depending on the electricity mix (e.g., Benitez et al. 2008, Gowrisankaran et al. 2016, Gutierrez-Martín et al. 2013, Lamont 2008, Lueken et al. 2012, Maddaloni et al. 2009, and Scorah et al. 2012). Also, a number of empirical studies assess the benefits of policies by measuring the impact of intermittency on emissions savings from renewable technologies. For instance, Dorsey-Palmateer (2019), Kaffine et al. (2020), and Wheatley (2013) typically find that intermittency reduces CO₂ emissions savings. Similarly, Novan (2015) investigates the variation in the amount of CO₂, NOₓ, and SO₂ reduced by wind production. He argues that current policies do not internalize the variation in the marginal benefits of intermittent renewable electricity. In comparison with this literature, our paper provides a theoretical framework to show how, from the lens of efficiency, intermittency affects the performance of renewable energy policies.

Theoretical studies on the deployment of intermittent renewable technologies are less common. Before the seminal paper by Ambec and Crampes (2012), several works that studied the electricity mix have abstracted from the intermittent nature of solar and wind
energy. Ambec and Crampes (2012) join the conclusion of Chao (2011) that consumers who are reactive to electricity prices help to promote investment in intermittent renewable technologies. Then, Abrell et al. (2019) and Ambec and Crampes (2019) consider environmental externalities. They analyze renewable policies both in a social optimum and in competitive electricity markets. Altogether, the two papers cover a wide range of support policies. On the one hand, Ambec and Crampes (2019) focus on non-reactive consumers where pre-announced policies are not an issue under the assumptions of their model. On the other hand, Abrell et al. (2019) study reactive consumers and propose policies that adapt to the availability of renewable resources to ensure social welfare. We complement this literature by studying both types of consumer and current renewable energy policies that are pre-announced. Hence, our results provide insights into second-best policies for the deployment of intermittent renewable technologies.

This paper is also in line with the literature on the evaluation of FiTs. Many studies adopted an observational and descriptive approach to discussing potential sources of inefficiency of FiTs (Couture and Gagnon 2010, Edenhofer et al. 2013, Fischer and Preonas 2010, Hepburn 2006, Kalkuhl et al. 2012, Mitchell et al. 2006, Schmalensee 2012). Going a step further, the validity of some of the arguments has been analyzed through economic tools. For example, FiTs are found to be second-best policies to deal with increasing shares of renewables in the energy mix (Helm and Mier 2021), market entry of renewable producers (Hirth 2013, Narita and Requate 2021), technology spillovers relating to renewable capacity (Reichenbach and Requate 2012, Andor and Voss 2016), and market power (Reichenbach and Requate 2012, García-Alamínos and Rubio 2021). Our theoretical results add to this literature as we demonstrate that, even under perfect competition, the efficiency of a FiT is not only a matter of consumers’ reactivity but also of being independent of conditions driving renewable electricity production. Also, we give a covariance expression that allows us to comment on under or over-investment in renewable technologies due to the inefficiency of the FiT. This complements the work by Lamont (2008) who provides an expression for the marginal value of an intermittent technology by considering the covariance between the renewable technology’s hourly production and the hourly system marginal cost. However, he does not consider policies and externalities that are central to our work.

Finally, this paper relates to a further branch of the literature that compares renewable policy instruments. Like the previous strands, this literature is dominated by works with a numerical and empirical approach: FiTs vs. premiums (e.g. Du and Ma 2022, Gavard 2012) and FiTs vs. TGC (Fagiani et al. 2013, Menanteau et al. 2003). They mostly conclude that FiTs are superior to other instruments for boosting renewable investments.

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7For example, Fischer and Newell (2008), Miah et al. (2012), and Tishler et al. (2008) assume that renewable energy production is deterministic and only discuss different cost structures between renewable and non-renewable technologies.

8Helm and Mier (2019) and Neerunjun (2022) study externalities from fossil-fueled electricity as well but focus on emissions pricing instruments.
Taking a more formal approach and accounting for intermittency, we share the view of Abrell et al. (2019) and Ambec and Crampes (2019) that FiTs and renewable premiums are equivalent in the presence of intermittency. We even add to this literature by showing that any second-best premium policy is allocation equivalent to a second-best feed-in policy with either non-reactive or reactive consumers. Concerning TGC, Tamás et al. (2010) find that FiTs and TGC are the same when markets are perfectly competitive and are generally different under imperfect competition. Requate (2015) share this view for the case of perfect competition. Contrary to these works but still under perfect competition, we account for intermittency and the market-based features of TGC. We show that the instrument that controls the desired ratio of renewables in the electricity mix and also the proportion of the certificate price in the total price of electricity paid by consumers leads to market certificate prices that partially internalize emissions damage. Hence, the FiT clearly dominates the TGC in the case of non-reactive consumers but the instruments cannot be ranked with reactive consumers at our degree of generality.

3. The main assumptions

To capture intermittency in a parsimonious way, we consider a two-period model with $S$ states of nature in the second one. Each state $s$ occurs with probability $\pi_s \in (0, 1]$. Decisions made in the first period, i.e. prior to the resolution of uncertainty, are called \textit{ex-ante} decisions. Those made in the second period are contingent on the occurrence of a state of nature, $s$, and are called \textit{ex-post} decisions. From this point of view, any policy decision will be considered as an \textit{ex-ante} decision. For instance, in the case of a feed-in policy, the regulator commits to buy each unit of renewable electricity at an \textit{ex-ante} feed-in tariff, $\varphi$, even if she resells this electricity \textit{ex-post} on the wholesale market at price $p_s$. This operation, especially for attractive feed-in tariffs, is financed by a pre-announced tax, $\tau$, on the \textit{ex-post} total electricity consumption.

Electricity can be produced with two types of technology: one depending on an intermittent renewable resource (e.g. wind) and the other using fossil-fuel energy (e.g. coal).

\textit{The renewable energy technology.} This \textit{clean} technology depends on the availability of the renewable resource. Thus, electricity production is free of emissions but non-controllable\footnote{Renewable technologies such as wind turbines are non-controllable since their output cannot be turned on, off, or adjusted according to variations in electricity demand. In contrast, fossil-fueled generators such as coal power plants are controllable.}. It means that there is an \textit{ex-ante} investment in a production capacity, $\kappa$, that generates a state-dependent production per unit capacity given by $\varepsilon_s \in [0, 1]$. For simplicity, we assume that these unit production levels are ranked from the most to the least productive state, i.e. if $s > s'$, $\varepsilon_s < \varepsilon_{s'}$. The \textit{ex-ante} investment decision takes into account the expected returns of trading electricity on the wholesale market and the current investment cost in the renewable capacity given by $K(\kappa)$. We assume
that cost and marginal cost are increasing in capacity, \( K'(κ) > 0 \) and \( K''(κ) > 0 \) with \( K(0) = K'(0) = 0 \) and \( \lim_{κ→+∞} K'(κ) = +∞ \). The ex-post marginal cost of production is negligible mainly because the resource is “free”.

The fossil energy technology. This is an existing, fully-established, and controllable technology. It allows reliable electricity provision when the renewable energy is unavailable or insufficient. We assume that there is no capacity constraint where the existing capacity is able to provide for electricity demand reliably. This assumption also implies that electricity production from this technology is an ex-post decision and thereby state-contingent. This production is given by \( q_s \) in state \( s \). The state-contingent private production cost is described by \( c(q_s) \) which is increasing and convex \( (c'(q_s) > 0 \) and \( c''(q_s) > 0) \) with \( c(0) = c'(0) = 0 \) and \( \lim_{q_s→+∞} c(q_s) = +∞ \). The technology also creates emissions. Hence, we refer to this sector as dirty. Emissions depend on the level of production and are given by \( E(q_s) \). This function is increasing and convex \( (E'(q_s) > 0 \) and \( E''(q_s) > 0) \) with \( E(0) = E'(0) = 0 \). Emissions also generate damage. It is measured in money and given by a standard function \( D(E) \) which is twice differentiable \( (D'(E) > 0 \) and \( D''(E) > 0) \) with \( D(0) = D'(0) = 0 \).

The market structure. We introduce two competitive markets: the wholesale electricity market where producers sell their current production to retailers and the delivering market on which retailers sell delivering contracts to the consumers. The first market takes place ex-post. It is therefore organized in each state and the contingent price per unit of electricity in state \( s \) is given by \( p_s \). The retailers are assumed to be risk-neutral and buy electricity on that contingent market to fulfill the terms of the contract they have signed ex-ante with the consumers. In this work, we do not explicitly model the behavior of the retailers and only consider two polar contract structures (see Ferrasse et al. 2022 for a more general approach). In the first case, the consumers only buy contracts that deliver the same amount of electricity in each state at a flat tariff. Under perfect competition, the flat tariff corresponds to the expected electricity price on the state-contingent wholesale markets. This situation is often referred to as the case of non-reactive consumers. In the opposite case, the consumers engage themselves to be billed at the state-contingent price and to react to this price; a situation that refers to real time pricing.

The (inverse) demand for electricity. To close the model, it remains to introduce an inverse demand for electricity that describes the willingness to pay for the last unit of the current electricity consumption. This function \( P(Q) \) is decreasing \( (P'(Q) < 0) \) and verifies that \( \lim_{Q→0} P(Q) = +∞ \) and \( \lim_{Q→+∞} P(Q) = 0 \). The quantity \( Q \) basically represents the amount of electricity that is consumed by non-reactive consumers. However, when
this quantity is indexed by the state \( s \in S \), it represents reactive consumers who express their willingness to pay for the last unit of electricity in each state. Introducing an inverse demand also simplifies the definition of the consumers’ surplus.

4. Efficiency and consumer reactiveness

So far, we are concerned by a two-period and \( S \)-state model where the \textit{ex-ante} social welfare is explained by (i) the expected consumers’ surplus (ii) the full expected production costs of dirty electricity that are state-contingent and include the emissions damage, and (iii) the \textit{ex-ante} investment cost in the renewable capacity. This general definition, if we consider the point view of a regulator, implies that she is able to allocate state-dependent electricity consumption to the consumers. This definition fits well to reactive consumers who adapt their electricity consumption to the \textit{ex-post} wholesale electricity price but is not relevant for non-reactive consumers. Thus, in a second step, we introduce a constrained social welfare definition.

In a \textit{social optimum}, the regulator has the ability to adapt the dirty electricity production, state by state, and to allocate the state-contingent electricity production \( (q_s^w, \varepsilon_s \kappa^w) \) to the consumers. She can therefore select state by state an optimal \textit{ex-post} energy mix, \( (q_s^w, \varepsilon_s \kappa^w) \subseteq S \), by solving:

\[
\max_{(q_s^w)_{s \in S}, \kappa^w} \mathbb{E} \left[ \int_0^{q_s^w + \varepsilon_s \kappa} P(v) \, dv - c(q_s) - D(E(q_s)) \right] - K(\kappa) \tag{1}
\]

The solution \( (q_s^w)_{s \in S}, \kappa^w \) of this program satisfies the following the \( (S + 1) \) FOCs:

\[
\forall s \in S, P(q_s^w + \varepsilon_s \kappa^w) = c'(q_s^w) + D'(E(q_s^w)) E'(q_s^w) \tag{2}
\]

\[
K'(\kappa) = \mathbb{E} \left[ (c'(q_s^w) + D'(E(q_s^w)) E'(q_s^w)) \varepsilon_s \right] \tag{3}
\]

Since the regulator is able to adjust the energy mix state by state, the first \( S \) conditions, Eq.(2), imply that the marginal surplus or the willingness to pay for electricity, \( P(q_s^w + \varepsilon_s \kappa^w) \), is equal to the marginal social cost of dirty electricity production in each state \( s \in S \). It is the sum of the marginal private cost, \( c'(q_s^w) \), and the marginal emissions damage, \( D'(E(q_s^w)) E'(q_s^w) \). The last condition, Eq.(3), follows an equi-marginalization principle or, in the presence of uncertainty, a no-arbitrage condition. Since both sources of energy are pure substitutes for the consumers, the investment cost of the last unit of renewable capacity that provides a random flow of electricity, \( \varepsilon_s \)\( s \in S \), must be equal to the expected social cost of generating this additional flow of electricity with the flexible dirty technology. However, it should be mentioned that this definition of the social optimum reflects an optimal electricity mix across states which means that the electricity consumption is almost surely not constant across states. A situation with non-reactive consumers is therefore far from being a social optimum.

Since the social optimum is almost never reached with non-reactive consumers, we also introduce a \textit{constrained social optimum}. This implicitly follows from the fact that there
are “missing markets” in the electricity delivering system in the sense that the price of the electricity delivering contracts are not able to capture the variability of the wholesale market price (see Ferrasse et al. [2022] for further discussions). A flat tariff which is selected by non-reactive consumers induces typically a restriction on trades which says that the consumption across states must be constant. So, even if a social optimum is almost out of reach, we can consider a constrained social optimum in which, in the case of non-reactive consumers, the electricity allocation is assumed to be constant across states. In other words, we consider an optimal ex-post energy mix \((q_{cw}^s, \epsilon_s \kappa_{cw}^s)_{s \in S}\) that solves:

\[
\max_{(q_s)_{s \in S}: \kappa, Q} \mathbb{E} \left[ \int_0^{q_s + \epsilon_s \kappa} P(v) \, dv - c(q_s) - D(E(q_s)) \right] - K(\kappa) \quad \text{s.t.} \quad \forall s, \quad q_s + \epsilon_s \kappa = Q \tag{4}
\]

To obtain the two FOCs associated with this program, we substitute \(q_s\) by their value and optimize the function over the electricity consumption, \(Q\), and the investment in renewable, \(\kappa\). For interior solutions, this gives the following FOCs:

\[P(Q_{cw}^s) = \mathbb{E} \left[ c'(q_{cw}^s) + D'(E(q_{cw}^s)) E'(q_{cw}^s) \right] \tag{5}\]
\[K'(\kappa_{cw}^s) = \mathbb{E} \left[ (c'(q_{cw}^s) + D'(E(q_{cw}^s)) E'(q_{cw}^s)) \epsilon_s \right] \tag{6}\]
\[\forall s, \quad q_{cw}^s = Q_{cw}^s - \epsilon_s \kappa_{cw}^s \tag{7}\]

The difference with the social optimum is clearly reflected by Eqs.(5) and (7). The latter says that the electricity consumption is constant across the states while the first assesses the consequences of the energy mix. Since consumption does not adjust across states, the same holds for the willingness to pay for electricity. It is therefore almost impossible to equate this willingness to pay with the state-contingent marginal production of electricity including the damage contrary to Eq.(2) for an overall optimal allocation. This condition now only equates the willingness to pay to the average full marginal cost. Finally, Eq.(6) is the same as Eq.(3) and induces the same interpretation in terms of no-arbitrage.

5. Non-reactive consumers: policy implications

We now turn to the key question of this paper: is it possible to enforce an efficient ex-ante feed-in policy in our competitive economy? The answer is mitigated when consumers are non-reactive. Non-reactivity excludes the achievement of a social optimum but the inclusion of this restriction in the definition of the optimum ensures the existence of an efficient ex-ante feed-in policy.

To highlight this point, we first outline the different conditions characterizing the state-contingent competitive equilibrium. Equilibrium quantities and prices are described with superscript \(n\) for non-reactive.

\[\text{Appendix A}\] provides a complete analysis of this problem including existence and uniqueness issues.
If we start with the electricity production that is delivered on the wholesale market, it is important to distinguish the dirty sector from the clean one. In the first case, we know that the technology is controllable. It means that the producer simply adjusts, ex-post, her production, \( q^n_s \), in each state, by observing the price, \( p^n_s \), and by adopting a standard profit maximization behavior. This electricity supply is therefore obtained by simply equalizing, in each state, the wholesale price of electricity, \( p^n_s \), to the marginal cost of production:

\[
\forall s \in S, \quad p^n_s = c'(q^n_s) \tag{8}
\]

For the clean sector, the decision process is somewhat different. Since the intermittent renewable technology is not controllable, the firm chooses an ex-ante production capacity, \( \kappa^n \), that provides, at a zero cost, a random flow, \( (\varepsilon_s)_{s \in S} \), of electricity per unit of the installed capacity. This electricity, under a feed-in policy, is brought by the regulator at a state-independent feed-in price, \( \varphi \), who sells it back on the wholesale market. For a clean producer, the expected return of a unit of production capacity is therefore \( \varphi \mathbb{E}[\varepsilon_s] \) and a competitive profit-maximizing behavior requires that this return is equal to the marginal cost of investment:

\[
\varphi \mathbb{E}[\varepsilon_s] = K'(\kappa^n) \tag{9}
\]

The electricity production is bought by competitive retailers who deliver electricity to the final consumers. These last agents are billed at a flat tariff for a state-independent flow of electricity, \( Q \). The retailers’ profit maximization behavior therefore requires that this flat tariff be equal to the expected electricity price. As the feed-in policy also includes a tax, \( \tau \), per unit of electricity consumed, we assume that this one is collected by the retailers and is therefore added to this flat tariff. It means that the consumers’ willingness to pay at equilibrium satisfies:

\[
P(Q^n) = \mathbb{E}[p^n_s] + \tau \tag{10}
\]

But this also means that the retailers have to buy, on the ex-post wholesale market, a state-independent quantity of electricity, \( Q^n \), to fulfill the delivering contract. Hence, the state-contingent equilibrium of the wholesale market is given by:

\[
\forall s \in S, \quad Q^n = q^n_s + \varepsilon_s \kappa^n \tag{11}
\]

We can therefore say that a competitive electricity market equilibrium with non-reactive consumers is a set of state-contingent prices \( (p^n_s)_{s \in S} \), dirty production levels \( (q^n_s)_{s \in S} \), an investment in renewable production capacity \( \kappa^n \) and a flat electricity consumption \( Q^n \) that simultaneously solve conditions (8), (9), (10) and (11).

These conditions immediately show that there exists no feed-in policy \( (\varphi, \tau) \) which is able to reach the social optimum. Let us assume the contrary. In this case, the competitive allocation also satisfies Eqs. (2) and (3) that characterize the social optimum. The first condition states that the willingness to pay for electricity must, in each state, be equal to the full marginal cost of dirty electricity production. Thus,

\[
\forall s \in S, \quad P(Q^n) = c'(q^n_s) + D'(\mathbb{E}(q^n_s)) \mathcal{E}'(q^n_s) \tag{12}
\]
As the left side of this equation is constant across the states while the right side is increasing with $q^n_s$, these efficiency conditions applied at the competitive equilibrium imply that the dirty electricity production must be constant across states, i.e. $q^n_s = q^n$. But as long as there exist at least two states in which the intermittent electricity production, $\varepsilon_s \kappa^n$, is not the same, it is impossible to meet the equilibrium condition given by Eq.(11).

As this argument is totally independent of a particular choice of the feed-in policy, we can claim that there exists no feed-in policy $(\varphi, \tau)$ that is able to reach the social optimum.

We can even notice that the above argument also holds for a state-contingent feed-in policy. This suggests that the main source of inefficiency is not the ex-ante character of the policy but the lack of consumers’ reactivity that asks the question whether a constrained social optimum can be reached by a suitable choice of the policy instruments. The answer to this question is yes.

More precisely, let us first contrast the equi-marginalization principle given by Eq.(3) to the competitive capacity choice described in Eq.(9). If the regulator sets the feed-in tariff to:

$$\varphi^{opt} = \frac{1}{\mathbb{E}[\varepsilon_s]} \mathbb{E} \left[ c'(q^{cw}_s) + \mathcal{D}'(\mathcal{E}(q^{cw}_s)) \mathcal{E}'(q^{cw}_s) \varepsilon_s \right]$$  \hspace{1cm} (13)

it is immediate that the competitive renewable capacity corresponds, under our assumptions on $\mathcal{K}(\kappa)$, to the constrained social optimum one, i.e. $\kappa^n = \kappa^{cw}$. Let us now observe that the state-contingent electricity is, by Eq.(8), equal to the private marginal cost of dirty electricity. It follows that the competitive willingness to pay for electricity is equal to this expected marginal cost plus the tax (see Eq.(10)). So, if the regulator sets the consumption tax to the expected marginal damage of dirty electricity production, i.e.

$$\tau^{opt} = \mathbb{E} \left[ \mathcal{D}'(\mathcal{E}(q^{cw}_s)) \mathcal{E}'(q^{cw}_s) \right]$$  \hspace{1cm} (14)

she ensures that this willingness to pay is equal to the expected full marginal cost of the dirty electricity production as it is required at a constrained social optimum (see Eq.(5)). It can therefore be shown that the state-independent electricity consumption are identical, i.e. $Q^n = Q^{cw}$. Finally, since the state contingent electricity market equilibrium, Eq.(11), is identical to the constraint on the social optimum, Eq.(7), we have that $\forall s \in S, q^n_s = q^{cw}_s$.

In other words, we can assert that there exists a feed-in policy $(\varphi^{opt}, \tau^{opt})$ that implements the constrained social optimum in an economy with non-reactive consumers.

The next proposition summarizes this discussion.

**Proposition 1.** With intermittency and non-reactive consumers, we can say that:

(i) there exists no feed-in policy $(\varphi, \tau)$ that is able to reach the social optimum.

(ii) there exists nevertheless a state-independent feed-in policy $(\varphi^{opt}, \tau^{opt})$ that implements the constrained social optimum.

---

15 This intuitive proof has nevertheless a shortcoming: it ignores the specific case where the constrained social optimum leads to zero dirty electricity production in the most favorable situation for renewable. This case is discussed in the proof of Proposition 1 in the appendix.
(iii) moreover for interior solutions\(^{16}\)

(a) the optimal feed-in tariff ensures, ex-ante, the equi-marginalization at the full marginal cost of the dirty sector.

(b) the optimal tax includes the expected marginal damage of the dirty sector in the state-independent price billed to the consumers.

From a policy point of view, the policy maker has, in this case, the opportunity to announce an efficient state-independent feed-in policy as long as she targets a constrained social optimum. This result however requires a strong assumption: non-reactiveness of the demand to the wholesale electricity price. This assumption first shifts the efficiency debate to one of constrained efficiency. Secondly, due to constrained efficiency, the optimal willingness to pay for electricity must only be equal to the average full marginal cost of the dirty sector (see Eq.(3)) contrary to an efficient outcome in which this must be true state by state (see Eq.(2)). This allows to set the tax rate to the average marginal damage. One can therefore expect that the efficiency property of the policy does not extend to reactive consumers.

6. Reactive consumers: towards efficiency?

We now move to the case which is often called real time pricing in the sense that electricity demand reacts to the wholesale electricity price. This situation is therefore as if the consumers directly access the wholesale market. By using superscript \(r\) for reactive, let us first have a look at what changes in the equilibrium conditions.

On the supply side, there is nothing really new. The dirty technology remains controllable. Hence, we obtain the same profit maximization condition as in (8).

\[
\forall s \in S, \quad p_s^r = c_s'(q_s^r) \tag{15}
\]

Likewise, for the intermittent clean sector, the producer equates the expected return of a unit of production capacity to the marginal cost of investment:

\[
\varphi \mathbb{E} [\varepsilon_s] = K' (\kappa^r) \tag{16}
\]

The main difference is at the level of the consumers. Since we are in a situation as if the consumers directly access the wholesale market, we do not need to constrain the quantities to be the same in each state, i.e. condition (11) is not required. The contingent market clearing conditions simply require that the willingness to pay for electricity production is in each state equal to the state-contingent price to which the regulator adds the per unit electricity tax, i.e.

\[
\forall s \in S, \quad P(q_s^r + \varepsilon_s \kappa^r) = p_s^r + \tau \tag{17}
\]

\(^{16}\)These results only hold for interior solutions. They are extended to the boundary case in Appendix
From this, a competitive equilibrium is a contingent price vector \((p_s^r)_{s \in S}\) and a contingent energy mix \((q_s^w, \kappa^w)_{s \in S}\) that meet conditions (15) to (17).

Without restrictions on trade imposed by demand non-reactiveness, the question is now the existence of a feed-in policy that induces the social optimum. The answer is unfortunately no unless the damage and the emission function are both linear.

In order to verify this point, let us consider the case in which either the damage or the emission function is strictly convex and let us assume that there exists a feed-in policy \((\varphi, \tau)\) that induces the social optimum. It means that for a suitable choice of the policy, \((\varphi, \tau)\), the efficient energy mix \(\{(q_s^w)_{s \in S}, \kappa^w\}\) that solves Eqs. (2) and (3) is also a solution of the previous set of equations that characterize a competitive equilibrium with reactive consumers. If this assumption is true, prices, by Eq. (15), are equal to the state-contingent marginal production cost of the dirty sector evaluated at the efficient dirty production level, i.e. \(\forall s \in S, p_s^r = c'_s(q_s^w)\). It also means, by Eq. (17), that the willingness to pay for electricity at an efficient energy mix satisfies:

\[
\forall s \in S, \quad P(q_s^w + \varepsilon_s \kappa^w) = c'_s(q_s^w) + \tau
\]  

(18)

If we compare this equation with Eq. (5), the tax, \(\tau\), must be in each state equal to the damage induced by the dirty electricity production, i.e. \(\tau = D'(E(q_s^w))E'(q_s^w)\). Moreover, as \(\tau\) is state-independent and \(D'\) and/or \(E'\) are increasing, the efficient level of production of dirty electricity must be constant across states, i.e. \(q_s^w = q^w\). But this induces a contradiction. By substituting in Eq. (18), it implies that the left-hand side is state-dependent while the right-hand side is independent, i.e. \(\exists s_0, s_1 \in S\) with \(\varepsilon_{s_0} \neq \varepsilon_{s_1}\) such that \(P(q_s^w + \varepsilon_{s_0} \kappa^w) = P(q_s^w + \varepsilon_{s_1} \kappa^w)\) which is impossible with a decreasing inverse demand. This contradiction confirms that there exists no feed-in policy \((\varphi, \tau)\) that implements the social optimum.

In fact, the only case in which the social optimum is reached corresponds to that when both the damage and emissions functions are linear. Let us assume that \(D(E) = dE\) and \(E(q) = eq\). If the policy maker sets \(\varphi = \frac{\kappa'(\kappa^w)}{E'(q_s)}\), it is straightforward from Eqs. (9) and (17) that \(\kappa' = \kappa^w\). Let us observe that the ex-ante tax can be set at \(\tau = de\) which is actually the marginal damage per unit of dirty electricity, i.e. \(D'(E(q_s))E'(q_s) = de\). In this case, the definition of the willingness to pay given by Eq. (17) becomes:

\[
\forall s \in S, \quad P(q_s^r + \varepsilon_s \kappa^w) = c'_s(q_s^r) + de
\]  

(19)

As \(de\) is the state-independent marginal damage, a simple identification with the optimal willingness to pay given by Eq. (10) shows that the competitive dirty electricity production corresponds, state by state, to the optimal one, i.e. \(\forall s \in S, q_s^r = q_s^w\).

Hence, we can claim that:

**Proposition 2.** With intermittency and reactive consumers,

(i) if either the damage or the emission function is strictly convex, it is impossible to find an ex-ante feed-in policy that reaches the social optimum.

(ii) reciprocally, the case in which the damage and emission functions are linear is the only one in which there exists an ex-ante feed-in policy that realizes the social optimum.
As long as the policy maker commits to announcing her policy before the resolution of the uncertainty, her policy choice can only be second-best unless we are in the particular case of linear damage and emission. In a different but nevertheless close model, [Ambec and Crampes (2019)] who use these assumptions obtained the first-best mix with a state-independent tax rule. [Abrell et al. (2019)] who introduce a linear damage but a general emission function propose an ex-post state-dependent tax rule that yields in the social optimum. As we are mainly concerned with ex-ante policy announcement, we will now move to the second-best analysis.

7. Reactive consumers: a second-best policy

This section complete the preceding one. It can be worthwhile to study the consequences of a second-best policy choice, especially with respect to the tax-marginal damage nexus and to the distortion in full-cost equi-marginalization principle. In a preliminary step, we have however to state some properties of the previous competitive equilibrium. In particular, we need to identify the effect of the feed-in tariff and the consumption tax on the different ex-post energy mixes we get.

A competitive equilibrium with reactive consumers solves Eq.(15) to Eq.(17). However, a too high feed-in tariff, \( \phi \), induces a maximal willingness to pay for electricity which is lower than the minimal electricity price given by the tax rate, \( \tau \). To avoid these situations, we restrict the choice of the regulator to instruments \((\phi, \tau)\) that satisfies:

\[
P \left( (K')^{-1} \left( \phi \mathbb{E} [\varepsilon_n] \right) \right) \geq \tau
\]

Under this restriction, it can be shown that there exists a unique competitive equilibrium. Moreover, a simple application of the Implicit Function Theorem provides the effect of the feed-in tariff and the tax on the different endogenous variables. These results are given in Table 1.

<table>
<thead>
<tr>
<th>Clean capacity</th>
<th>Dirty elec.</th>
<th>Tot. production</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \kappa^r(\varphi) )</td>
<td>( q^r_s(\varphi, \tau) )</td>
<td>( Q^r_s(\varphi, \tau) )</td>
</tr>
<tr>
<td>Feed-in tariff ( \varphi )</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>Consumption tax ( \tau )</td>
<td>0</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 1: Effects of a feed-in policy

The effects of a feed-in policy are as awaited. An increase in the feed-in tariff, \( \varphi \), modifies the energy mix in favor of the clean production since investment in renewable increases while the production of dirty electricity falls in each state. We even observe that the total electricity production increases in each state. Moreover, an increase in the consumption tax, \( \tau \), reduces electricity demand and thereby the total production in each state. This adjustment is obtained by cutting back on the dirty sector’s output. This is a peculiarity of the feed-in policy. In this case, only the dirty sector is responsive to market
prices since the electricity produced by renewable is bought back at an administrated price.

The next proposition establishes these results and provides two additional properties for later use.

**Proposition 3.** For all policy parameters that satisfy \( P ( (\kappa')^{-1} (\phi \mathbb{E} [\varepsilon_s])) \geq \tau \), there exists for a feed-in policy \((\phi, \tau)\), a unique competitive equilibrium. Table 1 gives the main comparative statics. Moreover, we observe that:

(i) when the degree of efficiency, \( \varepsilon \), decreases, the state-contingent dirty production increases while total consumption decreases.

(ii) the effect of the feed-in tariff on the state-contingent electricity consumption is explicitly given by:

\[
\frac{\partial Q \theta}{\partial \tau} = \frac{\partial q_\theta}{\partial \tau} + \varepsilon_s \frac{\partial q_\theta}{\partial \varepsilon_s} = -c'' (q_\theta) \varepsilon_s \frac{\partial q_\theta}{\partial \tau}
\]

We can now move to the main question of this section: what are the distortion induced by the second-best policy choice? We mainly focus on (i) the relation between the tax, \( \tau \), and the marginal damage and (ii) the bias, \( \delta \), in the equi-marginalization principle that we measure by:

\[
\delta = \mathbb{E} [(c' (q_\theta') + D' (\varepsilon) E' (q_\theta')) \varepsilon_s] - \kappa' (k')
\]  

The optimal choice of these instruments follows from the social welfare maximization by taking into account the competitive equilibrium conditions. In other words, this policy solves:

\[
\max_{\phi, \tau} \mathbb{E} \left[ \int_0^{q_\theta (\varphi, \tau) + \varepsilon_s \kappa' (\varphi)} p (v) dv - (c (q_\theta (\varphi, \tau)) + D_s (\varepsilon_s (q_\theta (\varphi, \tau)))) \right] - \kappa (k') \tag{22}
\]

As usually, we compute the first order conditions, use the marginal conditions that characterize the competitive equilibrium (i.e. Eqs. (15) to (17)), and introduce the bias, \( \delta \), given by Eq. (21). After an algebraic manipulation, we get:

\[
\begin{cases}
\tau_{sb} = \left( \mathbb{E} \left[ \frac{\partial q_\theta}{\partial \tau} \right] \right)^{-1} \mathbb{E} \left[ D' (\varepsilon) E' (q_\theta') \frac{\partial q_\theta}{\partial \tau} \right] \\
\delta_{sb} = \mathbb{E} \left[ (\tau_{sb} - D' (\varepsilon) E' (q_\theta')) \left( \frac{\partial q_\theta}{\partial \tau} (c'' (q_\theta') \varepsilon_s) \right) \right]
\end{cases}
\]  

Finally, to simplify the reading of these equations, we introduce a new distribution of probabilities, \( \pi' \), given by \( \forall s \in S, \pi'_s = \pi_s \frac{d \pi'}{d \pi} (\mathbb{E} [\frac{d \pi'}{d \pi}])^{-1} \). This one weights the initial probabilities \( \pi_s \) by the impact of the tax on the dirty electricity production in this state relative to the average impact. In this case, the second-best tax writes:

\[
\tau_{sb} = \mathbb{E}_{\pi'} [D' (\varepsilon) E' (q_\theta')]
\]  

\[\text{17} \text{See the proof of Proposition 4 in the appendix.}\]
As with non-reactive consumers (see (iv) of Proposition 1), the optimal second-best consumption tax, $\tau_{sb}$, corresponds to an expected damage. But this average damage is now computed with an adjusted probability distribution, $(\pi'_s)_{s \in S}$, that accounts for the tax’s effect on dirty electricity production in each state. Moreover, by recalling (i) of Proposition 3, we know that the dirty production $q^*_r$ is decreasing with $\varepsilon_s$, the degree of efficiency of the renewable technology. The same therefore holds for the marginal damage as long as the damage or the emission function is strictly convex. It means that the second-best consumption tax, $\tau_{sb}$, will be below the marginal damage in the states in which renewable production is not efficient while in the opposite cases, the tax will be above the marginal damage.

The expression of the equi-marginalization gap also becomes simple with the use of the distribution $(\pi'_s)_{s \in S}$ since:

$$\delta_{sb} = \left( \mathbb{E}_{\pi'} \left[ \frac{\partial^2 c}{\partial \tau^2} \right] \right) \mathbb{E}_{\pi'} \left[ (\tau_{sb} - D'(\mathcal{E})E'(q^*_s))(\tau_{sb} - D'(\mathcal{E})E'(q^*_s)) \varepsilon_s \right]$$

(25)

Moreover, as from Eq. (24) $\mathbb{E}_{\pi'} \left[ \tau_{sb} - D'(\mathcal{E})E'(q^*_s) \right] = 0$ and since $\tau_{sb}$ is state independent, the previous equation can be viewed as a covariance. We get:

$$\delta_{sb} = \left( - \mathbb{E} \left[ \frac{\partial^2 c}{\partial \tau^2} \right] \right) \text{Cov}_{\pi'} \left[ D'(\mathcal{E})E'(q^*_s), c''(q^*_s) \varepsilon_s \right]$$

(26)

Broadly speaking, there is no distortion in the equi-marginalization rule, i.e. $\delta_{sb} = 0$, when either (i) the damage and the emission function are linear or (ii) the marginal production cost of the dirty sector is constant. The first case is not really surprising. It corresponds to a situation in which the first-best can be implemented with a state-independent policy (see our discussion in section 6). The second case is more puzzling but still logical. By combining Eq. (15) and Eq. (17), we know that the willingness to pay for electricity is, in each state, equal to the sum of the marginal production cost of the dirty sector and the tax on electricity consumption. If both are constant across states, the same holds for the willingness to pay and therefore the electricity consumption. So, regardless of the policy choice, a competitive equilibrium with reactive consumers operates as if the consumers were non-reactive. An optimal choice of the policy tools (see Proposition 1) thus results in a constrained social optimum, known to satisfy the equi-marginalization rule.

On the other hand, if this covariance is, say, positive, i.e. $\delta_{sb} > 0$, we know from the definition of $\delta$ (see Eq. (21)), that the random electricity flow obtained from the last unit of renewable capacity is less costly than the same flow produced in the dirty sector and evaluated at its expected full cost. The second-best policy therefore induces an under-investment in the renewable capacity. Vise versa, there will be over-investment in renewables if this covariance is negative. However, assessing into depth this covariance requires empirical analyses. Nevertheless, the main implication of a second-best feed-in policy is that the equi-marginalization principle is not necessarily satisfied. In other words, the total electricity production is not realized at the lowest social cost.

The next proposition summarizes our discussion of the second-best policy.
Proposition 4. The second-best feed-in policy \((ϕ^{sb}, τ^{sb})\) has the property that:
(i) the second-best tax corresponds to an average damage where the state probabilities are weighted by the state-by-state impact of the tax on dirty electricity production.
(ii) there is either under or over-investment in renewables depending whenever the covariance, \(Cov_\omega [D'(E) E'(q^*_\varepsilon) ; c''(q_s) \varepsilon_s]\), is positive or negative.
(iii) the equi-marginalization rule nevertheless holds when the damage and the emission functions are linear or when the marginal production cost of the dirty sector is constant.

8. Renewable Premiums: equivalent instruments

We now extend our analysis to other renewable policies starting with premiums. Our main point is to show that a premium system is allocation-equivalent to a feed-in policy. Their evaluation, in terms of efficiency, is therefore the same as the equivalent feed-in policy.

However, there exists several renewable premiums instruments. They are mainly of two kinds depending on whether they support investment or production, both being financed by a tax, \(τ\), on electricity consumption. In the first case, the clean producer obtains an investment subsidy, \(σκ\), proportional to her investment in renewable capacity, \(κ\), while in the second case, she benefits from an ex-post premium, \(σ\), proportional to the random electricity production. But both remain equivalent. As the decision of investment is taken ex-ante, an investment subsidy reduces the marginal investment cost by \(σκ\) while a production premium increases the expected return of last unit of capacity from \(E[p_s \varepsilon_s]\) to \(E[(p_s + σ) \varepsilon_s]\). Thus, if \(σκ = σE[\varepsilon_s]\), both premium policies provide the same incentive to invest in a renewable capacity. Furthermore, since the tax, \(τ\), plays the same role in both scenarios, both equilibria are also identical. So, if we want to compare these policies with a feed-in policy, we can only concentrate, say, on production premiums.

Now, let us consider the production premium policy given by \((σ, τ^σ)\) and spell out the equilibrium conditions. On the supply side, the marginal cost of the dirty production remains, state by state, equal to the price. What essentially changes is the incentive to invest in the clean technology. With a premium \(σ\) per unit of clean electricity, the expected return of the last unit of capacity is now of \(E[(p_s + σ) \varepsilon_s]\). The equilibrium conditions on the supply side therefore become:

\[
∀s ∈ S, \quad p_s = c'(q_s) \quad \text{and} \quad K'(κ) = E[(p_s + σ) \varepsilon_s] \quad (27)
\]

On the demand side, the willingness to pay of consumers integrates the tax, \(τ^σ\). However, this part of the equilibrium conditions differs whenever consumers are reactive or not. With non-reactive consumers, total consumption must be constant across the states and the willingness to pay is equal to an average price. In the other case, reactive consumers freely adjust their electricity consumption according to the price signal. We therefore have the respective equilibrium condition:

\[
P(Q) = E[p_s] + τ^σ \quad \text{and} \quad ∀s, Q = q_s + \varepsilon_sκ \quad (28)
\]
∀s ∈ S, \quad P(q_s + ε_s) = p_s + τσ \quad (29)

We now show that an allocation induced by a premium policy \((\sigma, \tau)\) can also be obtained from a suitable choice of a feed-in mechanism \((\varphi, \tau)\). To construct this equivalence mechanism, let us start with the supply side. The equilibrium conditions when the market is regulated by the respective policies are given by Eqs. (27), (8) and (9). A quick look at these equations suggest that the incentives to invest in renewables are the same under both policies if the marginal returns of the investment in renewables are the same, i.e. \(E[(p_s + \sigma)\varepsilon_s] = \varphi E[\varepsilon_s]\). The behaviors of the producers are identical for both policies if we set \(\varphi = \sigma + E[p_s\varepsilon_s]\). If we now move to the demand side, we have to care of the consumers’ reactiveness and compare Eq. (28) with Eq. (10) for a flat tariff and Eq. (29) with Eq. (17) for flexible ones. But, for both cases, if the tax is the same, \(\tau = \tauσ\), the consumers’ willingness to pay is respectively the same. To conclude, an allocation induced by a premium policy \((\sigma, \tauσ)\) is the same as one induced by the feed-in policy \((\varphi, \tau) = (\sigma + E[p_s\varepsilon_s], \tauσ)\). A similar argument, of course, holds if we fix the feed-in policy and compute an equivalent premium mechanism.

As any allocation obtained by a premium policy can be achieved by an equivalent feed-in policy, and reciprocally, we can even claim that the set of reachable allocations are the same whether we implement a premium or feed-in policy. These sets may however be different depending on whether consumers are reactive or not. Moreover, as any social optimum is the selection of the best reachable allocation, this one will be the same for respectively non-reactive and reactive consumers. Hence, there exist in both cases, equivalent optimal premium and feed-in policies. This even remains true if we implement a second-best policy.

We summarize this discussion through the following:

**Proposition 5.** If we consider premium policies in favor of renewables investment or production, we can assert that:

(i) both policies are equivalent if the investment subsidy, \(\sigmaκ\), and the production premium, \(\sigma\), satisfy: \(\sigmaκ = \sigma E[\varepsilon_s]\).

(ii) any premium policy is even allocation equivalent to a feed-in policy with either non-reactive or reactive consumers.

(iii) any optimal premium policy can therefore be obtained by an equivalent optimal feed-in policy \((\varphi, \tau) = (\sigma + E[p_s\varepsilon_s], \tauσ)\).

9. Tradable Green Certificates

This policy, unlike the previous one, is market-based. The regulator delivers green certificates to the producer of renewable electricity in proportion to her clean electricity production. For simplicity, we set this proportion to one. The quantity of certificates is therefore only known \(ex-post\) and the certificates are sold on competitive markets. It means that their price, \(gs\), is state-contingent and from an \(ex-ante\) point of view, we have to consider as many markets as states of nature. The demand for certificates comes from
the *ex-post* final electricity consumption. Each consumer has the obligation to hold a share of \( \gamma \leq 1 \) of their electricity consumption in the form of green certificates. This share \( \gamma \) is in fact the policy instrument and, as usual, we assume that the announcement of this policy must be done before the intermittent production level is known.

Let us now move to the competitive condition. On the production side, there is nothing really new. The *dirty* producer still equates, in each state, the electricity price to the marginal cost while, under perfect competition, the certificate price is akin to a production premium for the *clean* sector. The main difference is now that this subsidy is given by the certificate market price and is state-contingent. But in any case, the *clean* producer equates the marginal cost of her investment in renewables to the expected returns of selling electricity and the associated certificates. In other words, we satisfy on the production side when:

\[
\forall s, \quad p_s^\gamma = c'(q_s^\gamma) \quad \text{and} \quad K'(\kappa^\gamma) = \mathbb{E}[(p_s^\gamma + g_s) \varepsilon_s]
\]

(30)

On the consumption side, the willingness to pay for one unit of electricity should now integrate the cost of purchasing a share \( \gamma \) of certificates at price \( g_s \). But as usual, these equilibrium conditions differ according to the reactiveness of the consumers. With a flat tariff, consumption is constant across the states and the willingness to pay equates the expected electricity price including the one of the certificates. In the other case, the willingness to pay is equal to the total state-contingent cost of a unit of electricity for consumers. In other words, we have either:

\[
P(Q^\gamma) = \mathbb{E}[p_s^\gamma + \gamma g_s] \quad \text{and} \quad \forall s, \quad Q^\gamma = q_s^\gamma + \varepsilon_s \kappa^\gamma
\]

or

\[
\forall s, \quad P(Q^\gamma_s) = p_s^\gamma + \gamma g_s
\]

(31)

(32)

The real difference with administered instruments lies in the fact that we now have to consider the contingent equilibrium of the certificate market. We model this condition by a state-contingent free disposal equilibrium given by:

\[
\forall s, \quad g_s(\varepsilon_s \kappa^\gamma - \gamma Q^\gamma_s) = 0 \quad \text{and} \quad \varepsilon_s \kappa^\gamma - \gamma Q^\gamma_s \geq 0
\]

(33)

with of course \( Q^\gamma_s \) independent of \( s \) for non-reactive consumers. This equilibrium condition simply states that a strictly positive certificates price, \( g_s > 0 \), requires that the quantity of certificates distributed to the clean producer, \( \varepsilon_s \kappa^\gamma \), equates the proportion of certificate that should be held by the consumers, \( \gamma Q^\gamma_s \). But if there is an excess supply, \( \varepsilon_s \kappa^\gamma > \gamma Q^\gamma_s \), the certificate price will be zero.

It is also this last equilibrium condition associated with a pre-announcement of the share, \( \gamma \), that restricts the efficiency of this policy. Indeed, if the certificate price aims to convey information on the marginal damage of the *dirty* sector, it has to be at least positive in each state. This requires, for reactive consumers, that the share of renewables in the total electricity consumption is constant across states, i.e. from Eq.(33), if \( \forall s, \ g_s \).
then \( \forall s, \gamma = \frac{\varepsilon s \kappa^c}{Q_s^c} \). For non-reactive consumers, the situation is even worse since electricity consumption is state-independent. Eq. (33) then implies that a positive certificate price is obtained in at most one state while \( g_s = 0 \) in the others. As efficiency is out of reach due to the pre-announcement of the policy, we can nevertheless ask the question whether this market-based policy perform better or not than a feed-in or its equivalent subsidy policy.

With non-reactive consumers, the conclusion is non-ambiguous. We show that there exists no choice of \( \gamma \) for which the constrained social welfare allocation can be implemented contrary to a feed-in policy.

To get the intuition of the argument, we take a green certificate equilibrium that solves Eqs. (30), (31) and (33) and assume that this allocation also satisfies the first-order conditions of a constraint social optimum (see Eqs. (5) to (7)). By a basic identification, we get:

\[
\mathbb{E} [g_s^2 \varepsilon_s] = \mathbb{E} [D'(E(q_s^{cw})) E'(q_s^{cw}) \varepsilon_s] \quad \text{and} \quad \mathbb{E} [\gamma g_s^2] = \mathbb{E} [D'(E(q_s^{cw})) E'(q_s^{cw})] \quad (34)
\]

From our previous discussion on the certificate market and efficiency, we also know that \( g_s^2 = 0 \) except, perhaps, for one state, for instance, \( s_0 \). Moreover, \( g_s^2 > 0 \) requires to set \( \gamma \) at \( \gamma = \varepsilon_{s_0} \frac{Q_s^{cw}}{Q_s^c} \). By substitution in Eq. (34), we get two different definition of the price \( g_{s_0} \), i.e.:

\[
g_{s_0}^2 = \mathbb{E} [D'(E(q_s^{cw})) E'(q_s^{cw}) \varepsilon_s] \quad \text{and} \quad g_{s_0}^2 = Q_s^{cw} \mathbb{E} [D'(E(q_s^{cw})) E'(q_s^{cw})] \quad (35)
\]

This requires that:

\[
\mathbb{E} [D'(E(q_s^{cw})) E'(q_s^{cw}) \left( \frac{Q_s^{cw}}{Q_s^c} - \varepsilon_s \right)] = 0 \quad (36)
\]

As \( D', E' > 0 \), this also implies that there exists a state \( s_1 \) with the property that \( \left( \frac{Q_s^{cw}}{Q_s^c} - \varepsilon_{s_1} \right) < 0 \). But it means that the total electricity production, \( Q^{cw} \), is lower than the clean production, \( \varepsilon_{s_1} \kappa^{cw} \), in state \( s_1 \): a contradiction.

From a policy point of view, this result clearly implies that a feed-in or equivalent subsidy policies that implements a constrained social welfare allocation dominate a market-based instrument like green certificates when consumers are non-reactive.

With reactive consumers, the result is slightly different. It can of course be shown that a green certificate equilibrium does not reach the first-best. In fact, basic identification with the welfare first-order conditions (see Eqs. (2) and (3)) now suggests that:

\[
\mathbb{E} [g_s^2 \varepsilon_s] = \mathbb{E} [D'(E(q_s^{cw})) E'(q_s^{cw}) \varepsilon_s] \quad \text{and} \forall s, \quad \gamma g_s^2 = D'(E(q_s^{cw})) E'(q_s^{cw}) \quad (37)
\]

Moreover, a positive marginal damage in each state requires positive certificate prices, \( \forall s, g_s > 0 \). The market clearing conditions of green certificates thus imply that \( \forall s, \quad \gamma = \frac{Q_s^{cw}}{Q_s^c} \). By replacing in the second set of equation of Eq. (37) and taking the average, we get \( \mathbb{E} [g_s \varepsilon_s] = \mathbb{E} \left[ \frac{Q_s^{cw}}{Q_s^c} D'(E(q_s^{cw})) \left( \frac{Q_s^{cw}}{Q_s^c} - \varepsilon_s \right) \right] \). Hence, we again obtain:

\[
\mathbb{E} \left[ D'(E(q_s^{cw})) E'(q_s^{cw}) \left( \frac{Q_s^{cw}}{Q_s^c} - \varepsilon_s \right) \right] = 0 \quad (38)
\]
that induces the same contradiction.\footnote{The reader should notice that this argument can be replicated in the case in which $\gamma$ is state-dependent. This means that even with a flexible policy tool, $\gamma_s$, the introduction of green certificates cannot reach the first-best energy mix.}

The main difference is that a feed-in equilibrium is also inefficient. It means that the respective performance of these policy tools can only be assessed by comparing the welfare achieved at the second-best. In addition, the two explanations for inefficiency are completely different. Under a feed-in policy, the \textit{ex-ante} tax does not equate the marginal damage but only its “average” value (see Eq.\eref{eq:24}), while under a green certificates policy, the activation of these markets, i.e. $g_s > 0$, induces a constant energy mix across the states given by $\gamma = \frac{2\kappa^2}{Q_s}$. This clearly suggests that the instruments cannot be ranked at our degree of generality and the choice of one of these tools requires econometric investigations.

To convince the reader, we consider a very simple linear quadratic example with only two states: renewables are either switched on or off. In fact, we set: $P(Q) = 10 - Q$, $c(q) = q^2$, $D(E) = dE$ with $d \in [2, 3]$, $E(q) = q$ and $K'(\kappa) = 2\kappa^2$. We say that the renewables are switched on with a probability of 0.75. We compute, under both policies, the market equilibrium, study the optimal choice of the instruments and evaluate the social welfare. In Figure \ref{fig:1}, we display the optimal social welfare reached by a feed-in and green certificates policy. As the slope of the marginal damage changes, the feed-in policy first perform less than the certificate policy. But after $d = 3.675$, the relation is reversed.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1.png}
\caption{Social Welfare: Feed-in vs Certificate policy}
\end{figure}

The next proposition summarizes this discussion.

\begin{proposition}
If we introduce market-based instruments, like green certificates, we ob-
\end{proposition}
serve that:

(i) with non-reactive consumers, the market-based instrument does not implement the constrained social optimum. This instrument is therefore dominated by a feed-in policy that reaches the constrained social optimum.

(ii) with reactive consumers, the market-based instrument does not reach the social optimum. As the same holds for a feed-in policy, it is impossible to assess which instrument is the most efficient by comparing the second-best welfare.

10. Concluding remarks

This paper is essentially based on the assumption that policies supporting investment in intermittent renewable energy like solar or wind resources cannot be contingent on meteorological events causing intermittence (e.g. sun or wind regime). This decision was taken by most policy-makers to avoid overly complex policy prescriptions. But the main consequence in doing so is that the first-best energy mix may be out of reach since the marginal damage induced by fossil-fuel electricity sector remains state-dependent. The main point of the paper was to compare the performance of the feed-in, premiums and tradable green certificates policy in this second-best setting. We even introduce the notion of constrained welfare to account for consumers non-reactiveness.

We started our study with the feed-in policy. In the presence non-reactive consumers, we showed that this policy implements the constraint social optimum even with a state-independent policy rule. But with reactive consumers for which the standard social optimum applies, we showed that the feed-in policy implements the optimal mix if and only if the marginal damage and emissions are constant. Otherwise, we obtained a second-best energy mix where the policy may lead to either under or over-investment in renewables depending on the degree of correlation between the marginal cost change and the marginal damage. Then, we extended our analysis to the two main types of renewable premiums: a market premium for renewable electricity production and an investment subsidy for renewable capacities. We first verified that these two premium policies are equivalent as they provide the same incentive to invest in renewable capacities. We even showed that any premium policy is allocation equivalent to a feed-in policy with either non-reactive or reactive consumers. It means that any result characterizing an optimal feed-in policy holds for a premium policy up to an adjustment of the value of the instruments. Finally, we considered a market-driven instrument: Tradable Green Certificates. With non-reactive consumers, we found that the policy does not implement the constrained social optimum and is therefore outperformed by a feed-in policy. With reactive consumers, the market-based instrument still did not reach the social optimum. As the same holds for a feed-in policy, we were not able to rank the instruments by comparing the second-best welfare. A counter-example showing that the feed-in can dominate the green certificates policy and vice-versa was provided.

We now discuss some limitations and potential extensions of our policy analysis for renewables. Firstly, we have abstracted from energy storage technologies. These are
presently costly but will play an important role in the future in ensuring reliable electricity supply in the presence of intermittent renewables. However, ignoring storage capacities in our model does not necessarily undermine our assessment of the second-best aspect of \textit{ex-ante} policies, especially in the case of reactive consumers. To make our point, let us remember that in the latter case, the first-best feed-in policy requires the consumption tax to be equal, in each state of nature, to the marginal damage induced by the dirty electricity production. It would be highly fortuitous to expect that with storage capacities, the marginal damage in all states of nature be exactly the same so as to be efficiency internalized by a single consumption tax. In fact, even in the particular case of linear damage and emission, [Helm and Mier (2021)] discuss the second-best characteristic of the consumption tax when it is determined by the aim to finance renewable subsidies. Nevertheless, studying storage capacities through the lens of a dynamic stochastic model is a useful extension to consider several storage technologies that respond to different renewable availabilities. For example, batteries are used primarily for intra-day storage and long-duration technologies in the form of pumped hydro storage or power-to-gas are used for inter-season storage [Dowling et al. (2020)].

Another extension of the study could be to make the policies more state dependent. Of course, its design can be quite complicated if it is contingent on the various weather events. That is why policymakers have ignored this option. But these ex-ante policies can be built on the ex-post observation of a variable that is correlated to these weather events, such as the wholesale price of electricity. Since we know that competitive prices reveal information in almost all cases (?), it might be possible to improve efficiency by modifying the policy rules along these lines. We however leave this problem to further work.

References


Appendix A. The constrained welfare problem

To simplify notations, we define \( C(q) = c(q) + D(\mathcal{E}(q)) \), the full cost of the dirty sector including the damage. This one satisfies, under our assumptions, \( C''(q) > 0 \), \( C''(q) < 0 \), and \( C(0) = C'(0) = 0 \). Now let us study the optimization problem given by Eq. (4). If we substitute all \( q_s \) by their value, the problem writes:

\[
\max_{Q,\kappa_s \geq 0} \left\{ \int_0^Q P(v) dv - E[C(Q - \varepsilon_s\kappa)] - K(\kappa) \quad \text{s.t.} \forall s \in S, Q - \varepsilon_s\kappa \geq 0 \right\} \tag{A.1}
\]

Now, remember that for \( s > s', \varepsilon_s < \varepsilon_{s'} \). So, if a constraint \( Q - \varepsilon_s\kappa \geq 0 \) with \( s > 1 \) is binding, it is impossible to satisfy the first \( s - 1 \) constraints and to obtain a solution. We can therefore reduce the set of constraints to a unique one \( Q - \varepsilon_1\kappa \geq 0 \) and the Kuhn-Tucker conditions become:

\[
\begin{align*}
\{ & P(Q) - E[C'(Q - \varepsilon_s\kappa)] + \lambda_Q + \lambda = 0 \\
& E[\varepsilon_s C'(Q - \varepsilon_s\kappa)] - K'(\kappa) + \lambda_s - \lambda_1 = 0 \}
\end{align*}
\]

\[
\begin{align*}
\{ & \lambda_Q = 0, \quad \lambda_Q, Q \geq 0 \\
& \lambda_s = 0, \quad \lambda_s, \kappa \geq 0 \\
& \lambda(Q - \varepsilon_1\kappa) = 0, \quad \lambda_s(Q - \varepsilon_1\kappa) \geq 0
\end{align*}
\] \tag{A.2}

where \( \lambda_Q, \lambda_s \) and \( \lambda \) denote the Lagrangian multipliers.

1/ The boundary solution

Let us first observe that \( \lambda_Q = 0 \). If not, the exclusion condition ensures that \( Q = 0 \) and the unique constraint leads to \( \kappa = 0 \). But in this case, the first optimality condition says that \( \lambda_Q + \lambda = -\lim_{Q \to 0} P(Q) + E[C'(0)] < 0 \), a contradiction. Secondly, we also observe that \( \lambda_s = 0 \). If not, \( \kappa = 0 \). To obtain the desired contradiction, two cases must be considered depending whenever \( Q > 0 \) or \( Q = 0 \). In the first one, the third slackness condition implies that \( \lambda = 0 \) and, since \( K'(0) = 0 \), the second optimality condition becomes \( \lambda_s = -E[C'(Q, \varepsilon_s)] < 0 \). In the second case, since \( C'(0) = 0 \), the first optimality condition becomes \( \lambda = -\lim_{Q \to 0} P(Q) < 0 \).

So, if a boundary solution exists, we should have \( \lambda > 0 \), and \( Q = \varepsilon_1\kappa \), i.e., a situation in which there is no conventional electricity production in the most renewable-friendly state. To characterize this situation, let us set \( \lambda_Q = \lambda_s = 0 \) and \( Q = \varepsilon_1\kappa \) in Eq. (A.2). By elimination \( \lambda \) from the two optimality conditions, we get:

\[
f(\kappa) = P(\varepsilon_1\kappa) - E \left[ \frac{\varepsilon_1-\varepsilon_s}{\varepsilon_1} C'(\kappa (\varepsilon_1 - \varepsilon_s)) \right] - \frac{1}{\varepsilon_1} K'(\kappa) = 0 \tag{A.3}
\]

Moreover, we observe that:

- \( f'(\kappa) = \varepsilon_1 P'(\varepsilon_1\kappa) - E \left[ \frac{(\varepsilon_1-\varepsilon_s)^2}{\varepsilon_1} C''(\kappa (\varepsilon_1 - \varepsilon_s)) \right] - K''(\kappa) < 0 \),
- \( \lim_{\kappa \to 0} f(\kappa) = \lim_{\kappa \to 0} P(\varepsilon_1\kappa) = +\infty \) since \( C'(0) = K'(0) = 0 \)
- \( \lim_{\kappa \to +\infty} f(\kappa) = -\infty \) since \( \lim_{\kappa \to +\infty} P(\varepsilon_1\kappa) = 0 \) and \( \lim_{\kappa \to +\infty} K'(\kappa) = \lim_{\kappa \to +\infty} C'(\kappa (\varepsilon_1 - \varepsilon_s)) = +\infty \).

It follows that there is a unique \( \kappa_0 \) solution to Eq. (A.3). If we also verify that

\[
\lambda = -\left( P(\varepsilon_1\kappa_0) - E[C'((\varepsilon_1 - \varepsilon_s)\kappa_0)] \right) > 0 \tag{A.4}
\]

we know that the constrained first best is given by \( \kappa_cw = \kappa_0, Q_cw = \varepsilon_1\kappa_0 \) and \( \forall s, q_csw = (\varepsilon_1 - \varepsilon_s)\kappa_0 \geq 0 \). If we solve Eq. (A.3) we can therefore say:

**Lemma 1.** Let \( \kappa_0 \) be the unique solution to Eq. (A.3). If \( P(\varepsilon_1\kappa_0) - E[C'((\varepsilon_1 - \varepsilon_s)\kappa_0)] \leq 0 \), then the constrained social optimum is given by \( \kappa_cw = \kappa_0, Q_cw = \varepsilon_1\kappa_0 \) and \( \forall s, q_csw = (\varepsilon_1 - \varepsilon_s)\kappa_0 \geq 0 \)

2/ The interior solution
Lemma 2. Let \( \kappa_0 \) be the unique solution to Eq.\ref{A.3}. If \( P ( \varepsilon_1 \kappa_0 ) - \mathbb{E} [ C' \left( (\varepsilon_1 - \varepsilon_s) \kappa_0 \right) ] > 0 \), then the constrained social optimum is given by the unique solution to:

\[
\begin{cases}
  g_1 (Q, \kappa) = P (Q) - \mathbb{E} [ C' (Q - \varepsilon_s \kappa) ] = 0 \\
  g_2 (Q, \kappa) = \mathbb{E} [ \varepsilon_s C'' (Q - \varepsilon_s \kappa) ] - K''(\kappa) = 0
\end{cases}
\]

in the interior of \( D = \{(Q, \kappa) \geq 0 : Q \geq \varepsilon_1 \kappa\} \).

To prove this Lemma, let us start with two helpful observations:

(i) The function \( \gamma_1 (\kappa) = g_1 (\varepsilon_1 \kappa, \kappa) \) satisfies:

- \( \gamma'_1 (\kappa) = \varepsilon_1 P' (\varepsilon_1 \kappa) - \mathbb{E} [ (\varepsilon_1 - \varepsilon_s) C'' (\varepsilon_1 - \varepsilon_s \kappa) ] < 0 \),
- \( \lim_{\kappa \to 0} \gamma_1 (\kappa) = +\infty \) since \( \lim_{\kappa \to 0} P(Q) = +\infty \) and \( C'(0) = 0 \),
- \( \lim_{\kappa \to +\infty} \gamma_1 (\kappa) = -\infty \) since \( \lim_{\kappa \to +\infty} C''(Q) = +\infty \) and \( \forall s > 1, \varepsilon_1 - \varepsilon_s > 0 \).

There is therefore a \( \bar{\kappa} \) unique satisfying \( \gamma_1 (\bar{\kappa}) = 0 \) and this immediately implies that \( \forall Q = \varepsilon_1 \kappa \) and \( \kappa > \bar{\kappa} \), \( g_1 (Q, \kappa) < 0 \). We now extend this observation for all \( Q > \varepsilon_1 \max \{\bar{\kappa}, \kappa\} \). For that purpose, note that \( \partial_Q g_1 (Q, \kappa) = P' (Q) - \mathbb{E} [ C'' (Q - \varepsilon_s \kappa) ] < 0 \) and \( \partial_\kappa g_1 (Q, \kappa) = \mathbb{E} [ \varepsilon_s C'' (Q - \varepsilon_s \kappa) ] > 0 \). It follows:

\[
g_1 (Q, \kappa) < g_1 (\varepsilon_1 \max \{\bar{\kappa}, \kappa\}, \kappa) \leq g_1 (\varepsilon_1 \max \{\bar{\kappa}, \kappa\}, \max \{\bar{\kappa}, \kappa\}) = \gamma_1 (\max \{\bar{\kappa}, \kappa\}) \leq 0 \tag{A.6}
\]

We can therefore exclude these values of \( (Q, \kappa) \) from the search of a solution to Eq.\ref{A.5} and restrict our attention to the triangle \( T = \{(Q, \kappa) \in [0, \varepsilon_1 \bar{\kappa}] \times [0, \bar{\kappa}] : Q \geq \varepsilon_1 \bar{\kappa}\} \) instead of \( D \).

(ii) Now define \( \gamma_2 (\kappa) = g_2 (\varepsilon_1 \kappa, \kappa) \) and observe from Eq.\ref{A.3} that \( f (\kappa) = \gamma_1 (\kappa) + \frac{1}{\varepsilon_1} \gamma_2 (\kappa) \). As \( \gamma_1 (\kappa_0) > 0 \) under the assumption with delineate the interior case, we must have \( \kappa_0 < \bar{\kappa} \) since \( \gamma'_1 (\kappa) < 0 \) (see (i)). But from our early discussion \( f' (\kappa) < 0 \), so that \( f (\kappa_0) > f (\bar{\kappa}) = \frac{1}{\varepsilon_1} \gamma_2 (\bar{\kappa}) \). As \( \gamma_2 (0) = \mathbb{E} [ \varepsilon_s C'' (\varepsilon_1 - \varepsilon_s \kappa) ] > 0 \), we can define \( \bar{k} \in (0, \bar{\kappa}) \) as the largest \( \kappa \) for which \( \gamma_2 (\kappa) \geq 0 \).

Let us now move to the existence and uniqueness of a solution to Eq.\ref{A.5} in the interior of \( T = \{(Q, \kappa) \in [0, \varepsilon_1 \bar{\kappa}] \times [0, \bar{\kappa}] : Q \geq \varepsilon_1 \bar{\kappa}\} \). This proof is based on a homotopy argument. An intuitive presentation can be found in [Eaves and Schmedders 1999].

(i) Construction of \( h(Q, \kappa) \).

Let us define \( h(Q, \kappa) = \left( \frac{\bar{Q} - Q}{\bar{\kappa} - \kappa} \right) \) with \( (\bar{Q}, \bar{\kappa}) \in \{(Q, \kappa) \in (\varepsilon_1 \bar{\kappa}, \varepsilon_1 \kappa) \times (\bar{\kappa}, \kappa) : Q > \varepsilon_1 \kappa\} \). The system \( h(Q, \kappa) = 0 \) admits obviously a unique solution \( (Q, \kappa) = (\bar{Q}, \bar{\kappa}) \) which is regular since \( \det (\partial h) = 1 \).

We can now construct \( H(Q, \kappa, \lambda) = (1 - \lambda) g(Q, \kappa) + \lambda h(Q, \kappa) \). Since this function takes as parameters \( (\bar{Q}, \bar{\kappa}) \) and \( \forall \lambda \in (0, 1), \text{rank} \left( \frac{\partial H}{\partial (Q, \kappa)} \right) = 2 \), the generic transversality theorem (see [Eaves and Schmedders 1999]) gives us the opportunity to choose \( (Q, \kappa) \) such that 0 is a regular value of \( H \).

(ii) \( H^{-1}(0) \) is a compact subset of \( \text{int}(T) \times [0, 1] \).

Let us take any sequence \( (Q_n, \kappa_n, \lambda_n) \to (Q_\infty, \kappa_\infty, \lambda_\infty) \) with the property that \( \forall n, (Q_n, \kappa_n, \lambda_n) \in H^{-1}(0) \) and show that is impossible that \( (Q_\infty, \kappa_\infty, \lambda_\infty) \in \partial T \times [0, 1] \). First assume that \( \kappa_\infty = 0 \) and \( Q_\infty > 0 \). In this case \( g_2 (Q_\infty, \kappa_\infty) = \mathbb{E} [ \varepsilon_s C'' (Q_\infty) ] > 0 \) since \( K'(0) = 0 \) and \( h_2 (Q_\infty, \kappa_\infty) = \bar{k} > 0 \),
hence $H_2(Q_\infty, \kappa_\infty, \lambda_\infty) > 0$. It must therefore exist a rank $N$ in the sequence $(Q_n, \kappa_n, \lambda_n)$ such that \(\forall n \in N, H_2(Q_n, \kappa_n, \lambda_n) = 0\) which is the desired contradiction. Secondly assume that $Q_\infty = \varepsilon_1 \bar{\kappa}$ and $\kappa_\infty < \bar{\kappa}$. In this case, $g_1(\varepsilon_1 \bar{\kappa}, \kappa_\infty) < g_1(\varepsilon_1 \bar{\kappa}, \bar{\kappa}) = 0$ since $\bar{\kappa} > 0$ and by construction $g_1(\varepsilon_1 \bar{\kappa}, \bar{\kappa}) = 0$. Moreover $h_1(Q_\infty, \kappa_\infty) = \bar{Q} - \varepsilon_1 \bar{\kappa} < 0$ by the choice of $\bar{Q}$, hence $H_1(Q_\infty, \kappa_\infty, \lambda_\infty) < 0$, which provides the contradiction. Finally, assume that $Q_\infty = \varepsilon_1 \bar{\kappa}$ with $\kappa_\infty \in [0, \bar{\kappa}]$. Two subcases must be considered depending on whether $\kappa_\infty \in (\bar{\kappa}, \bar{\kappa})$ or $\kappa_\infty \in [0, \bar{\kappa}]$. In the first subcase, $\kappa_\infty > \bar{\kappa}$ and, by construction of $\bar{g}$ (see preliminary remark (ii)), $\gamma_2(\kappa_\infty) = g_2(\kappa_\infty, \kappa_\infty) < 0$. As $\kappa_\infty > \bar{\kappa}$, we also have that $h_2(Q_\infty, \kappa_\infty, \lambda_\infty) < 0$, hence $H_2(Q_\infty, \kappa_\infty, \lambda_\infty) < 0$, the usual contradiction. If $\kappa_\infty \in [0, \bar{\kappa}]$, we have $\kappa_\infty \leq \bar{\kappa} < \bar{\kappa}$ and, by the preliminary remark (i), $0 = \gamma_1(\kappa) < \gamma_1(\kappa_\infty) = g_1(Q_\infty, \kappa_\infty, \lambda_\infty)$. Moreover $Q_\infty = \varepsilon_1 \kappa_\infty \leq \varepsilon_1 \bar{\kappa} < \bar{Q}$, by construction of $(\bar{Q}, \bar{\kappa})$ hence $h_1(Q_\infty, \kappa_\infty) > 0$ and so $H_1(Q_\infty, \kappa_\infty, \lambda_\infty) > 0$.

(iii) $\text{det} (\partial Q, g) > 0$.

By computation

$$\text{det} (\partial Q, g) = \text{det} \left( \begin{array}{cc} \partial (\bar{Q}) (Q) - E [C'' (Q - \varepsilon_s \kappa)] & - E [\varepsilon_s C'' (Q - \varepsilon_s \kappa)] \\ E [\varepsilon_s C'' (Q - \varepsilon_s \kappa)] & 0 \end{array} \right)$$

$$= \left( \begin{array}{c} -P'' (Q) \left( E [\varepsilon_s C'' (Q - \varepsilon_s \kappa)] \right) + K'' (\kappa) \end{array} \right)$$

$$> 0,$$ under our assumptions

$$+ E [C'' (Q - \varepsilon_s \kappa)] \left( [\varepsilon_s C'' (Q - \varepsilon_s \kappa)] \right) - (E [\varepsilon_s C'' (Q - \varepsilon_s \kappa)])^2$$

As $C'' (Q - \varepsilon_s \kappa)$ is a positive random variable, we observe that:

$$E [C'' (Q - \varepsilon_s \kappa)] \left( [\varepsilon_s C'' (Q - \varepsilon_s \kappa)] \right) > (E [\varepsilon_s C'' (Q - \varepsilon_s \kappa)])^2$$

and it follows from the Cauchy-Schwarz inequality\footnote{Remember that for two random variables $X, Y$ we have $(E(XY))^2 \leq E(X^2)E(Y^2)$} that:

$$E [C'' (Q - \varepsilon_s \kappa)] \left( [\varepsilon_s C'' (Q - \varepsilon_s \kappa)] \right) \geq (E [\varepsilon_s C'' (Q - \varepsilon_s \kappa)])^2$$

We can therefore conclude that $\text{det} (\partial Q, g) > 0$.

Appendix B. Proof of Proposition 1

Let us first observe that the definition of a competitive equilibrium with non reactive consumers (see Eqs. (8), (9), (10) and (11)) can be reduced to:

$$\varphi E [\varepsilon_s] = K' (\kappa^n) \quad \text{and} \quad P(Q^n) - E [c' (Q^n - \varepsilon_s \kappa^n)] = \tau = 0$$

From the condition, it is immediate that the feed-in tariff must be set at $\varphi^{opt} = \frac{1}{E[\varepsilon_s]} K' (\kappa^{cw})$ in both the interior or the boundary case. It ensures that $\kappa^n = \kappa^{cw}$. Moreover a quick look at the first order conditions of the constrained social optimum (see Eq. (A.2)) shows that for an interior solution, i.e. $\lambda_Q = \lambda_c = \lambda = 0$,

$$\varphi^{opt} = \frac{1}{E[\varepsilon_s]} K' (\kappa^{cw}) = \frac{1}{E[\varepsilon_s]} E [\varepsilon_s C' (Q^{cw} - \varepsilon_s \kappa^{cw})]$$

But for the boundary solution $\kappa^{cw} = k_0$ and $Q^{cw} = \varepsilon_1 k_0$ with $\lambda_Q = \lambda_c = 0$ but $\lambda > 0$, this is not true since:

$$\varphi^{opt} = \frac{1}{E[\varepsilon_s]} K' (\kappa^n) < E [\varepsilon_s C' (\varepsilon_1 - \varepsilon_s) k_0^n]$$

There is therefore, at the constrained social optimum, under-investment in renewable capacity with respect to the equi-marginalization rule.
Let us now move to the setting of the tax, $\tau$. For a interior constrained social optimum, it is obvious to set $\tau^{opt} = E[D'(\epsilon(q_1^cw))\epsilon'(q_1^cw)]$ because in this case the willingness to pay for electricity is equal to the average full marginal cost. For a boundary solution the question is somewhat different. We need to set $\tau^{opt}$ such that $Q^* = \epsilon_1\kappa_0$ is the unique solution to:

$$P(Q^*) - E[c'(Q^* - \epsilon_s\kappa^cw)] - \tau = 0$$

(B.4)

This requires to set:

$$\tau^{opt} = E \left[ \left( \frac{\epsilon_s - \epsilon_1}{\epsilon_1} \right) D'(\epsilon((\kappa_0(\epsilon_1 - \epsilon_s))))\epsilon'((\kappa_0(\epsilon_1 - \epsilon_s))) \right] - E \left[ \frac{\epsilon_s - \epsilon_1}{\epsilon_1} C'((\kappa_0(\epsilon_1 - \epsilon_s))) \right] + \frac{1}{\epsilon_1} K'(\kappa_0)$$

(B.5)

By doing so, the left-hand side of Eq.(B.4) evaluated at $Q^* = \epsilon_1\kappa_0$ writes:

$$P(\epsilon_1\kappa_0) - E[c'(\epsilon_1 - \epsilon_s)\kappa_0] - \tau^{opt} = P(\epsilon_1\kappa_0) - E \left[ \left( \frac{\epsilon_s - \epsilon_1}{\epsilon_1} \right) C'((\kappa(\epsilon_1 - \epsilon_s))) \right] - \frac{1}{\epsilon_1} K'(\kappa_0)$$

(B.6)

and must be equal to 0 by the definition of $\kappa_0$ (see Eq.(A.3)).

Appendix C. Proof of Proposition 3

1/ Existence and uniquness of a competitive equilibrium with reactive consumers

The equilibrium quantities $((\epsilon_1))_{s \in S}, \kappa$ are given by the following set of equations:

$$\forall s \in S, \; P(\epsilon_1 + \epsilon_s, \kappa) - c'(\epsilon_1) - \tau = 0 \; \text{and} \; K'(\kappa) = \varphi E[\epsilon_s]$$

(C.1)

Since the range of $K'(\kappa)$ is $\mathbb{R}_+$, we can say that $\kappa = (K')^{-1} (\varphi E[\epsilon_s])$. By plugging this quantity into the $S$ first equations, we obtain:

$$\forall s \in S, \; f_s(\epsilon_1) = P \left( \epsilon_1 + \epsilon_s, (K')^{-1}(\varphi E[\epsilon_s]) \right) - c'(\epsilon_1) - \tau = 0$$

(C.2)

Now, let us observe that $\forall s \in S$ (i) $f_s'(\epsilon_1) < 0$ since $P'(Q) < 0$ and $c''(Q) > 0$, (ii) $\lim_{\epsilon_1 \to -\infty} f_s(\epsilon_1) < 0$ since $\lim_{Q \to +\infty} P(Q) = 0$ and (iii)

$$\lim_{\epsilon_1 \to 0} f_s(\epsilon_1) = P \left( (K')^{-1}(\varphi E[\epsilon_s]) \right) - \tau$$

$$\geq P \left( (K')^{-1}(\varphi E[\epsilon_s]) \right) - \tau \text{ since } \epsilon_s \leq 1\text{ and } P'(Q) < 0$$

$$> 0 \text{ under our policy restriction Eq.}(20)$$

These three observations concludes the existence and uniqueness part of the proof.

2/ The comparative statics results of Table 1

Since $\kappa = (K')^{-1} (\varphi E[\epsilon_s])$, it is immediate that $\frac{\partial \kappa}{\partial \tau} = 0$ and $\frac{\partial \kappa}{\partial \varphi} = \frac{1}{K''(K')^{-1}(\varphi E[\epsilon_s])} > 0$. For the effects on $q_s$, the implicit function theorem applied to Eq.(C.2) shows that:

$$\forall s \in S, \; \frac{\partial \epsilon_1}{\partial \tau} = \frac{1}{P'(q_s + \epsilon_s, \kappa) - c'(q_s)} < 0 \text{ and } \frac{\partial \epsilon_1}{\partial \varphi} = \frac{\epsilon_s P'(q_s + \epsilon_s, \kappa)}{P'(q_s + \epsilon_s, \kappa) - c'(q_s)} < 0$$

(C.3)

Finally, since $\forall s \in S$, $Q_s = \epsilon_1 + \epsilon_s, \kappa$, we have $\frac{\partial Q_s}{\partial \tau} = \frac{\partial \epsilon_1}{\partial \tau} < 0$ since $\frac{\partial \epsilon_1}{\partial \varphi} = 0$ and

$$\frac{\partial Q_s}{\partial \varphi} = \frac{\partial \epsilon_1}{\partial \varphi} + \epsilon_s \frac{\partial \epsilon_1}{\partial \tau} = \epsilon_s \frac{\partial \epsilon_1}{\partial \varphi} \left( 1 - \frac{P'(q_s + \epsilon_s, \kappa)}{P'(q_s + \epsilon_s, \kappa) - c'(q_s)} \right) = -\epsilon_s \frac{\partial \epsilon_1}{\partial \varphi} \frac{c'(q_s)}{P'(q_s + \epsilon_s, \kappa) - c'(q_s)} > 0$$

(C.4)

3/ Points (i) and (ii) of the proposition

(i) $\forall s, \epsilon_s > \epsilon_s' \Rightarrow q_s^* < q_s'^* \text{ and } Q_s^* > Q_s'^*$
Assume the contrary and let us first consider the case in which \( \exists \varepsilon_s, \varepsilon_s' \) such that \( \varepsilon_s > \varepsilon_s' \) and \( q_s \geq q_s' \). Since \( P'(Q) < 0 \) and \( c'(q_s) > 0 \), this implies that for all \( \kappa \):

\[
P(q_s' + \varepsilon_s \kappa) - c'(q_s') \leq P(q_s + \varepsilon_s \kappa) - c'(q_s) < P(q_s' + \varepsilon_s \kappa) - c'(q_s')
\]

But from the equilibrium condition (see Eq. (C.1)), the left-hand and the right-hand side of the previous inequalities must both be equal to the tax rate, \( \tau \), a contradiction.

Now assume \( \exists \varepsilon_s, \varepsilon_s' \) such that \( \varepsilon_s > \varepsilon_s' \) and \( Q_s' \leq Q_s'' \). It follows that \( P(Q_s') \geq P(Q_s'') \). But, from our previous result, we know that \( q_s' < q_s'' \) and therefore \( -c'(q_s') > -c'(q_s'') \). The addition of these two inequalities induces the same contradiction as previously.

(ii) \( \frac{\partial Q_s'}{\partial s'} = \frac{\partial q_s'(\delta, \kappa)}{\partial s'} + \varepsilon_s \frac{\partial q_s'(\delta, \kappa)}{\partial \kappa} = -c''(q_s') \varepsilon_s \frac{\partial q_s'(\delta, \kappa)}{\partial \kappa} \)

This result follows directly from Eq. (C.4) since by Eq. (C.3) \( \frac{\partial q_s}{\partial \kappa} = \frac{1}{\tau(q_s + \varepsilon_s \kappa) - c'(q_s)} \).

**Appendix D. Proof of Proposition 4**

To complete the proof of this proposition, it remains to come back on the transformation of the optimality conditions in order to identify the consumption the \( s^{ab} \) and the distortion \( d^{ab} \) in the equi-marginalization rule. Starting with the second best optimization problem (see Eq. (22), the first order optimality conditions in order to identify the consumption the \( \phi \) and Eq. (D.2) becomes:

\[
\frac{\partial q_s}{\partial \kappa} = \frac{\partial q_s(\delta, \kappa)}{\partial \kappa} + \varepsilon_s \frac{\partial q_s(\delta, \kappa)}{\partial \kappa} = -c''(q_s') \varepsilon_s \frac{\partial q_s(\delta, \kappa)}{\partial \kappa} 
\]

From the competitive equilibrium (see Eq. (18), we know that \( \tau = P(Q_s') - c'(q_s') \). Hence:

\[
\begin{cases}
\mathbb{E} \left[ (P(Q_s') - (c'(q_s') + D'(\varepsilon') (q_s'))) \frac{\partial q_s'}{\partial \varepsilon'} \right] = 0 \\
\mathbb{E} \left[ (P(Q_s') - (c'(q_s') + D'(\varepsilon') (q_s'))) \frac{\partial q_s'}{\partial \kappa} \right] + (\mathbb{E}[\tau + c'(q_s')] \varepsilon_s) - K'(\varepsilon') \frac{\partial q_s'}{\partial \varepsilon'} = 0
\end{cases}
\]

By definition of the distortion \( \delta \) in the equi-marginalization rule (see Eq. (21)) we have:

\[
\mathbb{E} [c'(q_s')] \varepsilon_s - K'(\varepsilon') = -\mathbb{E} [D'(\varepsilon) \varepsilon'(q_s')] \varepsilon_s
\]

and Eq. (D.2) becomes:

\[
\begin{cases}
\mathbb{E} \left[ (\tau - D'(\varepsilon') (q_s')) \frac{\partial q_s}{\partial \varepsilon'} \right] = 0 \\
\mathbb{E} \left[ (\tau - D'(\varepsilon') (q_s')) \frac{\partial q_s}{\partial \kappa} \right] + (\mathbb{E}[(\tau - D'(\varepsilon') (q_s')) \varepsilon_s] + \delta) \frac{\partial q_s}{\partial \varepsilon'} = 0
\end{cases}
\]

\[
\iff \begin{cases}
\mathbb{E} \left[ (\tau - D'(\varepsilon') (q_s')) \frac{\partial q_s}{\partial \varepsilon'} \right] = 0 \\
\mathbb{E} \left[ (\tau - D'(\varepsilon') (q_s')) \left( \frac{\partial q_s}{\partial \kappa} + \varepsilon_s \frac{\partial q_s'}{\partial \kappa} \right) \right] + \delta \frac{\partial q_s}{\partial \varepsilon'} = 0
\end{cases}
\]

Finally from (ii) of proposition 3 we know that \( \frac{\partial q_s}{\partial \varepsilon} + \varepsilon_s \frac{\partial q_s'}{\partial \varepsilon'} = -c''(q_s') \varepsilon_s \frac{\partial q_s'}{\partial \kappa} \). It follows:

\[
\begin{cases}
\tau^{ab} = \left( \mathbb{E} \left[ \frac{\partial q_s}{\partial \varepsilon} \right] \right)^{-1} \mathbb{E} \left[ D'(\varepsilon) \varepsilon'(q_s') \frac{\partial q_s}{\partial \varepsilon'} \right] \\
\delta^{ab} = \mathbb{E} \left[ (\tau^{ab} - D'(\varepsilon) \varepsilon'(q_s')) \left( \frac{\partial q_s}{\partial \kappa} \right) \right] \left( \frac{\partial q_s'}{\partial \kappa} \right) \varepsilon_s
\end{cases}
\]

\[\text{To simplify notation, we omit the argument } \tau \text{ and } \varphi \text{ in the equations.}\]
Appendix E. The linear-quadratic case of section 9

Let us assume that \( P(Q) = a - bQ, \ c(q) = \frac{q^2}{2}, \ D(E) = \frac{q^2}{2} \), \( E(q) = q \) and \( \mathcal{K}(\kappa) = \frac{\kappa}{2} \) and let us only introduce two states in which renewable are either switched on or off, i.e. \( \varepsilon_1 = 1 \) and \( \varepsilon_2 = 0 \) with probability \( \pi \).

1/ The feed-in case

With reactive consumers, the market equilibrium equations (see Eqs. (15) to (17)) becomes:

\[
\begin{align*}
\begin{cases}
  cq_1 = p_1, \quad k\kappa = \pi, \quad \text{and} \\
  cq_2 = p_2, \quad a - b(q_1 + \kappa) = p_1 + \tau \\
  a - bq_2 = p_2 + \tau
\end{cases}
\end{align*}
\]

and a simple computation shows that:

\[
\begin{align*}
\begin{cases}
  \kappa = \frac{\pi}{b+\pi} \varphi \\
  q_1 = \frac{1}{b+\pi}(a - b\kappa \varphi - \tau) \\
  q_2 = \frac{1}{b+\pi}(a - \tau)
\end{cases}
\end{align*}
\]

After some tedious computations, the total surplus is given by:

\[
SW(\tau, \varphi) = A + B \left( \frac{\tau}{\varphi} \right) + \frac{1}{2} \left( \frac{\tau}{\varphi} \right) C \left( \frac{\tau}{\varphi} \right)
\]

with

\[
A = \frac{a^2(c-d)}{2(b+c)^2}, \quad B = \left( \frac{a(b+d)}{(b+c)^2} - \frac{\pi^2(b+c)}{b+\pi} \right), \quad C = \left[ -\frac{2b+c+d}{(b+c)^2} \pi^2(b+c) \right] - \frac{\pi^2(b+c)}{k(b+c)^2} \left( 1 + \frac{\pi b(b+c+2c^2)}{b(b+c)^2} \right)
\]

The choice of the second best policy therefore simply reduces to the optimization of a linear-quadratic problem. Hence:

\[
\left( \varphi^{\text{opt}} \right) = -C^{-1}B' \quad \text{and} \quad SWMax \ = \ A - \frac{1}{2} BC^{-1} B'
\]

2/ The green certificate case

As no certificates are distributed in state 2, we assume that the market is not organized and set \( g_2 = 0 \). It follows that the market equilibrium conditions given by Eqs. (30) to (33) become:

\[
\begin{align*}
\begin{cases}
  cq_1 = p_1, \quad \text{and} \\
  cq_2 = p_2, \quad k\kappa = \pi(p_1 + g_1) + (1 - \pi)p_2, \quad \text{and} \\
  a - bq_2 = cq_2
\end{cases}
\end{align*}
\]

To solve this set of equations, two cases must be considerate depending whenever \( g = 0 \) or \( g > 0 \). By investigating these cases helps to define a threshold \( \bar{\gamma} = \frac{c(b+c)}{k(b+c)} \) below which \( g = 0 \). However to ensure that the equilibrium quantities are positive for all \( \gamma \in [0,1] \), we need a restriction on the set of parameter which says that \( k(b+c) > (1-\pi)bc \). Under this restriction, the equilibrium quantities are piece-wise continuous function over \( \gamma \in [0,1] \) given by:

\[
\begin{align*}
\begin{cases}
  q_1 = \frac{a(bk-bc+ck+bc\gamma)}{bk+ck+bc\gamma}, \\
  q_2 = \frac{a}{bk+ck+bc\gamma}, \\
  \kappa = \frac{bk+ck+bc\gamma}{bk+ck+bc\gamma}
\end{cases}
\end{align*}
\]

for \( \gamma \leq \bar{\gamma} \)

\[
\begin{align*}
\begin{cases}
  q_1 = \frac{a}{bk+ck+bc\gamma}, \\
  q_2 = \frac{a}{bk+ck+bc\gamma}, \\
  \kappa = \frac{bk+ck+bc\gamma}{bk+ck+bc\gamma}
\end{cases}
\end{align*}
\]

for \( \gamma > \bar{\gamma} \)

It remains to replace these quantities in the social welfare function, to compute the maximum over \( \gamma \in [0,1] \) and to compare this quantity to the social welfare \( SWMax \) obtain at an optimal choice of the feed-in policy. Due to the complexity of the expressions, we leave this job to MATLAB. We simply exhibit a set of parameters for which, by changing the coefficient associated to the marginal damage, first the optimal certificate policy dominates the feed-in policy and latter the reverse occurs.