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Tying the Politicians' Hands: The Optimal Limits to Representative Democracy

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Abstract

We study the optimal delegation problem which arises between the median voter (writer of the constitution) and the (future) incumbent politician when not only the state of the world and but also the politician's type (preferred policy) are the policy-maker's private information. We show that it is optimal to tie the hands of the politician by imposing him/her both a policy floor and a policy cap and delegating him/her the policy choice only in between. The delegation interval is shown to be the smaller the greater is the uncertainty about the politician's type.These results apply outside the specific problem to which our model is applied here.

Keywords: representative democracy, optimal delegation, political uncertainty.

JEL Classification: D82, H10

1 Introduction

Representative democracy may be best defined as a political system in which the power to choose public policies is delegated to elected representatives (which we will call here "politicians" for short). It is traditionally opposed to direct democracy, a system in which the general public determines itself the laws and policies.

The evaluation of the costs and benefits of delegation is then central in the old debate between the two systems. The main alleged advantage of representative democracy is that representatives are able to specialize in policy-making. This allows them to devote more time to study the state of the world than ordinary citizens and thus to make more informed decisions[1]. Benjamin Constant (1819) put it very neatly when he wrote: "The poor individuals make themselves their affairs: the rich men take bursars. It is the history of the old

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[1] The idea that elections may be a screening process allowing to delegate the policy choices to policy-makers more competent than the general population is more controversial.

nations and the modern nations. The representative system is a mandate given to a number of individuals by the mass of the people, who want that their interests are defended, and who nevertheless does not have time to defend them always themselves". Edmund Burke (1854) said the same when writing that "it is the duty of the representative to sacrifice his repose, his pleasures, his satisfactions, to those of the constituents". This idea has been revived recently after the Brexit referendum, when it has been often argued that "if consulting the public is a good thing to do", "accepting as binding the verdict of a relatively narrow majority of voters (most of whom were less informed than the average citizen about the question on which they were voting) is an entirely different matter", and more generally asking "to what extent should the citizens of a democracy be able to make decisions directly ?" (see Peter Singer (2016) among others).

Given the number and the complexity of public decisions to be taken in a modern society, the advantages of representative democracy are so important that in one way or another all modern democracies are basically representative democracies, with sometimes some elements of direct or participative democracy like referendums and/or popular initiative. It has however long be recognized that this system has costs of its own. The main drawback of representative democracy is that representatives are free, once elected, to promote their own interests. This was for instance expressed forcefully by Jean-Jacques Rousseau (1762) who thought that even if it is possible that there exists a coincidence between what the constituency and the representatives want at the time the former delegate their power to the latter, there is nothing to ensure that this coincidence will last all the time of the mandate.[2] The sources of divergence between the preferred policies of the median voter and of the elected representatives are indeed numerous, even if both share the same level information about the state of the world. In the first place, representatives cannot be bound by a binding mandate, they cannot commit to implement specific policies once elected because it is impossible to write complete contracts describing what the representatives should do in all circumstances which could occur during their mandate. In the second place, even if the representatives could commit, as assumed for instance by Downs (1957), this does not eliminate ex ante political uncertainty, i.e. the before election uncertainty about the elected politician's type. On one hand, it is indeed well-known that, even if the election is a single issue one (one-dimensional policy space), there may be multiple voting equilibria as soon as there are more than two candidates. On the other hand, when the policy space is multi-dimensional, the voters have to choose between packages: there is nothing to ensure that *on each issue* the elected representative's proposed policies are the ones preferred by a majority. Two recent examples are relevant. In France, although Président Macron was recently elected by a large majority, the suppression of the wealth tax ("Impôt de Solidarité sur la Fortune"), although present in his election platform is, according to all polls,

[2] "Le souverain peut bien dire : « Je veux actuellement ce que veut un tel homme, ou du moins ce qu'il dit vouloir » ; mais il ne peut pas dire : « Ce que cet homme voudra demain, je le voudrai encore", *Du Contrat Social*, chapter VII,1.

opposed by a majority of french voters. In the same way, US President Trump's project of repealing Obamacare is opposed by a majority of US voters. In a more general way, McKelvey (1976) has shown a long time ago that in the case of multidimensional policy spaces "it is theoretically possible to design voting procedures which, starting from any given point, will end up at any other point in the space of alternatives, even at Pareto dominated ones" (page 472).

The purpose of the present paper is to study the optimal design of a representative democracy system as a specific "delegation problem". Delegation problems deal with settings in which, according to Amador and Bagwell (2013) "a principal faces an informed but biased agent, and contingent transfers between the principal and the agent are infeasible". Indeed political settings are primarily relevant examples of this kind of problems where legal rules limit or even completely forbid transfers to elected representatives. The type of delegation problem we are interested in here is original. We do suppose that the agent (incumbent politician) has private information about the state of the world and that it is generally biased with respect to the principal's[3] preferences, but we assume that the importance and the direction of the agent's bias are not known by the principal. Instead, the representative's type as determined by the electoral process (which is not analyzed in itself but rather considered as a black box) is a random variable. We however consider throughout this paper that there is more uncertainty about the state of the world than with respect to the politician preferences, i.e. that unfettered representative democracy (i.e. full delegation to the politician) would anyway be preferred to direct democracy. For the sake of simplicity, we suppose that the voting procedure is not itself biased: the expected representative's type is the median voter's type.

Main Results

When studying as a benchmark case the situations in which there is a known representative's bias, we obtain results which are in line with Amador and Bagwell((2013) general results: a policy cap when there is a rightist politician's bias and a policy floor when there is a leftist politician's bias.

Our main and more original result is when the representative's bias is random and the political process unbiased. The policy choice should then be delegated to a representative only within bounds: an upper bound (policy cap) and a lower bound (policy floor) which are to avoid extreme policy choices under delegation, the writer of the constitution always finding optimal to tie the hands of future politicians in this way. The interval of parameter values over which delegation occurs is the smaller the greater is the political uncertainty.

Related Literature

Starting from Osborne and Slivinski (1996) and Besley and Coate (1997), the economic literature has developed "an alternative theory of policy choice in representative democracies" (Besley and Coate, page 85), i.e. models in which "citizen candidates" run for election[4]. These models[5] depart from the

[3] The principal is here the writer of the constitution. Throughout the paper we call him/her the median voter though this coincidence may be true only initially.

[4] See for instance Hamlin and Hjortlund (2000) for the proportional representation case.

[5] The candidates are completely informed about the ideal policies of all others.

traditional Hotelling model of electoral competition (Downs (1957)) in two important respects: (i) the citizens-candidates have policy preferences instead of being exclusively office-motivated and (ii) cannot commit to implement another policy than their most preferred one. Under free entry, the number and types of the candidates are determined at equilibrium. Whereas the initial models assumed complete information, they have been subsequently extended to a setting where each candidate's ideal point is his/her private information (see Großer and Palfrey (2009), (2014) for theoretical analyses, and (2017) for an experimental study). While this literature analyzes the electoral process which we model instead here as a black box with random outcomes, its conclusions confirm the existence of political uncertainty: there are generally multiple equilibria and the type of the elected representative, which almost always differs from the median one[6], cannot be predicted with certainty.

Political agency models (see Besley (2006), chapter 3, for a comprehensive presentation based on a canonical model) are apparently closer to this paper since they also deal with the relationship between citizens/voters (the principals) and the politicians/government (the agent), the latter being generally better informed than the former. They encompass moral hazard models where politicians may use their office to extract rents (see Ferejohn (1986)), adverse selection models where the issue is to select the "good" politicians (see Besley and Prat (2004)) and models which combine moral hazard and adverse selection (see Besley and Case (1995), Coate and Morris (1995), Fearon (1999) among others). The basic element of these models is the idea that politicians may be made accountable[7] to the citizens via the electoral competition, i.e. their desire of being reelected combined to retrospective voting.[8] However, if concerns for reelection do act as a politicians' discipline device, it follows from these models that it is not enough to fully discipline them. Besley (2006), page 111, concludes for instance that the "reelection mechanism is imperfect" since it is unable to always eliminate "dissonant" politicians.[9] This is clearly in line with our assumptions about the existence/persistence of political uncertainty.

Finally, this paper has much in common with the papers which have studied the optimal delegation problem which Holmström (1977)[10] was the first to analyze, focusing on "interval delegation", in which the delegation set (over which the policy choice is delegated to the agent) is a single interval. Other papers have provided conditions for the optimality of interval delegation under various settings: Amador, Werning and Angeletos (2006) when money burning is allowed, Alonso and Matouschek (2008) when money burning is not allowed, Ambrus and Egorov (2009) for uniform distributions and quadratic utility functions. Amador and Bagwell (2013) paper provides necessary and also sufficient

[6]Großer and Palfrey even conclude on the contrary, both theoretically and experimentally, that " candidate entry is from the extremes of the policy space" ((2017), page 1).

[7]On political accountability see the book edited by Przeworski, Stokes and Manin (1999).

[8]The voters are supposed to hold the incumbent politician responsible for the consequences of his/her actions while in office.

[9]In the same way, Coate and Morris (1995) find that political competition does not prevent inefficient methods of redustribution to be employed.

[10]See also Holmström (1984).

conditions which encompass previous results as special cases. The present paper departs from all this literature by analyzing the case when the agent's bias (or, equivalently, what Alonso and Matouschek (2008) call the "preference divergence" between the agent and the principal) instead of being fixed is a random variable whose realized value is private information of the agent.[11] It shows that, at least when there is more state of the world uncertainty than political uncertainty and when the political process is not biased, interval delegation holds in this setting with both a policy cap and a policy floor. These results apply outside the specific problem to which our model is applied here.

Section 2 outlines the model. Section 3 analyzes two benchmark cases. Section 4 characterizes the equilibria. Section 5 concludes.

2 The Model

We consider that a policy x has to be chosen from a set $X = [-\varepsilon - \delta, \varepsilon + \delta]$ which is a closed interval on the real line. The policy-maker (agent) has a linear-quadratic welfare function

$$(\theta + t)\, x - \frac{1}{2} x^2, \tag{1}$$

where θ and t are respectively a state of the world parameter and the politician's type. Both are her private information. Equivalently $\alpha = \theta + t$ is private information of the policy-maker. The median voter has priors about θ and t such that θ is uniformly distributed over $[-\varepsilon, \varepsilon]$ and t is uniformly distributed over $[-\delta, \delta]$. The two distributions are independent[12] and there is no **expected** politician's bias, i.e. the expected politician's type is the median voter's type. We suppose throughout that $\varepsilon > \delta$, i.e. that the uncertainty about the state of the world is greater than the uncertainty about the politician's type. That means that, absent any other choice, unfettered representative democracy is preferred by the median voter to direct democracy.[13]

The median voter's (principal) has the linear-quadratic welfare function

$$\theta x - \frac{1}{2} x^2. \tag{2}$$

Notice that our basic results hold true if we substitute the same concave function $v(x)$ for $\frac{1}{2}x^2$ in (1) and (2).

[11] Koessler and Martimort (2012) have instead studied the case when the decision space is two-dimensional. They assume quadratic utility functions and uniform distribution of the agent's type.

[12] We don't see any good reason pointing to some correlation between the state of the world and the politician bias since, absent any commitment problem, the politician's type preferred by the median voter is always $t = 0$. This is equivalent in our framework to the assumption in Amador and Bagwell (2013) Proposition 3 of an independence of the agent's bias wrt the state of the world;

[13] By unfettered representative democracy we mean a system in which the policy choice is always delegated to the politician.

Given that t is uniformly distributed over $[-\delta, \delta]$, the median voter's type corresponds to the median (and average) politician's type $t = 0$.[14] The median voter is uninformed about the realized values of θ and t.

The politician's preferred or flexible policy is given by

$$x^f(\alpha) = \alpha. \tag{3}$$

Notice that, given the above assumptions on the distributions of θ and t, the density[15] and cumulative density functions of the variable $\alpha = \theta + t$ are respectively obtained as

$$f(\alpha) = \begin{cases} \frac{\varepsilon+\delta+\alpha}{4\varepsilon\delta}, \ \forall\alpha \in [-\varepsilon-\delta, \delta-\varepsilon] \\ \frac{1}{2\varepsilon}, \ \forall\alpha \in [\delta-\varepsilon, \varepsilon-\delta] \\ \forall\alpha \in [\varepsilon-\delta, \varepsilon+\delta] \end{cases}, \tag{4}$$

and

$$F(\alpha) = \begin{cases} \frac{(\varepsilon+\delta+\alpha)^2}{8\varepsilon\delta}, \ \forall\alpha \in [-\varepsilon-\delta, \delta-\varepsilon] \\ \frac{\alpha+\varepsilon}{2\varepsilon}, \forall\alpha \in [\delta-\varepsilon, \varepsilon-\delta] \\ \frac{2\varepsilon(\alpha+3\delta)-\varepsilon^2-(\alpha-\delta)^2}{8\varepsilon\delta}, \ \forall\alpha \in [\varepsilon-\delta, \varepsilon+\delta] \end{cases}. \tag{5}$$

An allocation is a function $\mathbf{x} : [-\varepsilon-\delta, \varepsilon+\delta] \longrightarrow X$, that represents the policy as a function of the (aggregate) private information. An equilibrium is an allocation which maximizes the median voter's welfare function

$$\max W_{Med} = \int_{-\delta}^{\delta}\int_{-\varepsilon}^{\varepsilon}\left(\theta x(\alpha) - \frac{1}{2}x(\alpha)^2\right)\frac{1}{4\varepsilon\delta}d\theta dt, \text{ subject to:} \tag{6}$$

$$\alpha = \arg\max_{\widehat{\alpha}}\{\alpha x(\widehat{\alpha}) - \frac{1}{2}x(\widehat{\alpha})^2\}, \text{ for all } \alpha \in [-\varepsilon-\delta, \varepsilon+\delta], \tag{7}$$

where the last equation (7) is an incentive-compatibility constraint which arises from the fact that the politician is privately informed of the value of $\alpha = \theta + t$.

Notice that the maximand in (6) may be usefully rewritten as

$$W_{Med} = \int_{-\varepsilon-\delta}^{\varepsilon+\delta}\left(\int_D\left(\theta x(\alpha) - \frac{1}{2}x(\alpha)^2\right)\frac{1}{4\varepsilon\delta}d\theta\right)d\alpha = \tag{8}$$
$$\int_{-\varepsilon-\delta}^{\varepsilon+\delta}\left(x(\alpha)\left(\int_D\frac{\theta}{4\varepsilon\delta}d\theta\right) - \frac{1}{2}x(\alpha)^2 f(\alpha)\right)d\alpha,$$

where $D = \{\theta \in [-\varepsilon, \varepsilon] \cap [\alpha-\delta, \alpha+\delta]$ is the range of possible values of θ given that $\theta + t = \alpha$ and $f(\alpha)$ is given by (4).

[14] The importance of what Alonso and Matouschek (2008) call the "preference divergence" is then simply measured here by the (absolute) value of t.

[15] Since θ and t are independently distributed, $f(\alpha) = \int_D \frac{1}{4\epsilon\delta}d\theta$ where $D = \{\theta \in [-\varepsilon, \varepsilon] \cap [\alpha-\delta, \alpha+\delta]$.

Let us from now on denote

$$W_{pol}(\alpha) = \max_{\widehat{\alpha}} \{\alpha x(\widehat{\alpha}) - \frac{1}{2} x(\widehat{\alpha})^2\}.$$

By a standard revealed preference argument, $x(\alpha)$ is non-decreasing and a.e differentiable for all $\alpha \in [-\varepsilon - \delta, \varepsilon + \delta]$. $W_{pol}(\alpha)$ is continuous as a supremum of convex functions and

$$\begin{aligned} W_{pol}(\alpha) &= \alpha x(\alpha) - \frac{1}{2} x(\alpha)^2 = W_{pol}(\underline{\alpha}) + \int_{\underline{\alpha}}^{\alpha} x(s) ds & (9) \\ &= W_{pol}(\overline{\alpha}) - \int_{\alpha}^{\overline{\alpha}} x(s) ds. & (10) \end{aligned}$$

Notice that substituting in (8) for $\frac{1}{2}x(\alpha)^2$ its value from (9) and integrating by parts, we obtain for any incentive-compatible policy $x(\alpha)$, median voter's expected welfare as

$$W_{Med} = \int_{-\varepsilon-\delta}^{\varepsilon+\delta} x(\alpha) \left[\Theta(\alpha) - \alpha + h(\alpha)\right] f(\alpha) d\alpha + W_{pol}(\underline{\alpha}), \tag{11}$$

where $\Theta(\alpha) = \frac{\int_D \frac{\theta}{4\varepsilon\delta} d\theta}{f(\alpha)}$ is the expected value of θ conditional on the sum $\theta + t$ being equal to α, $h(\alpha) = \frac{1-F(\alpha)}{f(\alpha)}$ is the inverse hazard rate, and $f(\alpha)$ and $F(\alpha)$ are respectively defined by (4) and (5).

In the following, for the sake of convenience, let us denote $\left[\Theta(\alpha) - \alpha + h(\alpha)\right] f(\alpha)$ by $g(\alpha)$.

Notice that, equivalently,

$$W_{Med} = \int_{-\varepsilon-\delta}^{\varepsilon+\delta} x(\alpha) \left[\Theta(\alpha) - \alpha - H(\alpha))\right] f(\alpha) d\alpha + W_{pol}(\overline{\alpha}),$$

where $H(\alpha) = \frac{F(\alpha)}{f(\alpha)}$.

From (9), at any point of differentiability, either an incentive-compatible policy $x(\alpha)$ corresponds to the flexible policy $x(\alpha) = \alpha$ or $x'(\alpha) = 0$ (constant policy). Incentive-compatible policies may exhibit discontinuities. At a point $\widetilde{\alpha}$ of discontinuity where $\lim_{\alpha \downarrow \widetilde{\alpha}} x(\alpha) = x^+(\widetilde{\alpha}) > \lim_{\alpha \uparrow \widetilde{\alpha}} x(\alpha) = x^-(\widetilde{\alpha})$, $W_{pol}(\alpha)$ remains continuous, namely $\widetilde{\alpha} x^+(\widetilde{\alpha}) - \frac{1}{2} x^+(\widetilde{\alpha})^2 = \widetilde{\alpha} x^-(\widetilde{\alpha}) - \frac{1}{2} x^-(\widetilde{\alpha})^2$.[16]

It is then possible to distinguish the different types of incentive compatible policies

1. Fully flexible policies such that $x(\alpha) = \alpha$, $\forall \alpha \in [-\varepsilon - \delta, \varepsilon + \delta]$;

2. Semi-flexible policies such that $x(\alpha) = \alpha$, $\forall \alpha \in [\alpha_0, \alpha_1] \subset [-\varepsilon - \delta, \varepsilon + \delta]$, which may include a policy cap ($\alpha_1 < \varepsilon + \delta$), a policy floor ($\alpha_0 > -\varepsilon - \delta$) or both;

[16] From what it directly follows that $\widetilde{\alpha} = \frac{1}{2}(x^+(\widetilde{\alpha}) + x^-(\widetilde{\alpha}))$.

3. Step policies made of $n \geq 1$ intervals (steps) over which types are pooled (i.e. policies are constant);

4. Hybrid policies which include both flexible segments and discontinuities from a higher step to a lower one.

3 Benchmarks

3.1 Direct democracy versus (unfettered) representative democracy

Let first us consider the case when the only possible alternative for the median voter is either to delegate the policy choice to the politician for all $\alpha \in [-\varepsilon - \delta, \varepsilon + \delta]$ (a fully flexible policy) or to choose directly a constant policy x (a one step-policy). The first case corresponds to unfettered representative democracy while the second one is direct democracy.

In the case of a fully flexible policy, $x(\alpha) = \alpha$, $\forall \alpha \in [-\varepsilon - \delta, \varepsilon + \delta]$. The median voter's expected payoff is easily obtained from (6) as $\frac{1}{6}(\varepsilon^2 - \delta^2)$.

In the case of a one-step policy with $x(\alpha) = x$, $\forall \alpha \in [-\varepsilon - \delta, \varepsilon + \delta]$. The median voter's expected payoff is easily obtained from (6) as $-\frac{1}{2}x^2$. The best one-step policy is then $x = 0$, leading to a zero expected payoff.

Claim 1 *Unfettered representative democracy is better than direct democracy iff there is a more uncertainty about the state of the world than about the politician's type, i.e. iff* $\varepsilon^2 > \delta^2$.

This conclusion is modified in favor of direct democracy if the median voter expects the politician to be biased in average, namely if the expected politician's type differs from the median voter's type. Let's for instance suppose that t is uniformly distributed between $-\delta + \mu$ and $\delta + \mu$ so that the expected value of t equals $2\delta\mu$. The median voter's expected payoff under representative democracy becomes $\frac{1}{6}(\varepsilon^2 - \delta^2 - 3\mu^2)$: the greater the expected politician's bias, as measured by μ^2, the better is the case for direct democracy versus unfettered representative democracy.

3.2 Known politician's bias

Let us now consider the case when the median voter knows the politician's type t and there is a politician's bias, i.e. the politician's type differs from the median voter's ($t \neq 0$). We simply assume that the politician's bias is not so important as to lead the politician to possibly choose policies which the median voter would not select whatever the state of the world, i.e. $t \in (-\varepsilon, \varepsilon)$.[17] This yields a model which is encompassed by Amador and Bagwell (2013) general analysis of optimal delegation.

[17] In the opposite case, it is quite easy to show that direct democracy with $x = 0$ is the best option from the median voter's point of view.

Claim 2 *(i) if the politician has a rightist bias, i.e. $t > 0$, it is optimal to delegate the policy choice (i.e. to select $x(\theta + t) = \theta + t$) to the politician for all $\theta \in [-\varepsilon, \varepsilon - 2t]$ and to impose a policy cap $x = \varepsilon - t$ for θ greater than $\varepsilon - 2t$;*

(ii) if the politician has a leftist bias, i.e. $t < 0$, it is optimal to delegate the policy choice ($x(\theta + t) = \theta + t$) to the politician for all $\theta \in [-\varepsilon - 2t, \varepsilon]$ and to impose a policy floor $x = -\varepsilon - t$ for θ smaller than $-\varepsilon - 2t$.

Proof. See Appendix. ■

In the case of a known rightist politician's bias, imposing a policy cap rules out extreme-right policies. This cap is the lower the greater the politician's bias. Symmetrically, in the case of a known leftist politician's bias, imposing a policy floor rules out extreme left policies. The floor is the higher the greater the politician's bias.

Claim 1 may be somewhat generalized to the case where θ is not necessarily uniformly distributed. Assume simply a strictly positive continuous density function $f(\theta)$. It may be shown from Amador and Bagwell (2013) than,if $F(\theta) + tf(\theta)$ is non decreasing,[18]

(i) if $t > 0$, a flexible policy $x(\alpha) = \alpha$ for all $\theta \in [-\varepsilon, x - t]$ plus a policy cap $x(\alpha) = x < \varepsilon + t$ for all $\theta \in [x - t, \varepsilon]$ is optimal where x solves $\int_{x-t}^{\varepsilon}(\theta - x)f(\theta)d\theta = 0$;

(ii) if $t < 0$, a policy floor $x > -\varepsilon + t$ for all $\theta \in [-\varepsilon, x - t]$ plus a flexible policy $x(\alpha) = \alpha > -\varepsilon - \delta$ for all $\theta \in [x - t, \varepsilon]$ is optimal where x solves $\int_{-\varepsilon}^{x-t}(\theta - x)f(\theta)d\theta = 0$.

4 Equilibria

Remember that we consider the case where the variance of the political uncertainty parameter is smaller than the variance of state of the world parameter, i.e. $\varepsilon > \delta$.

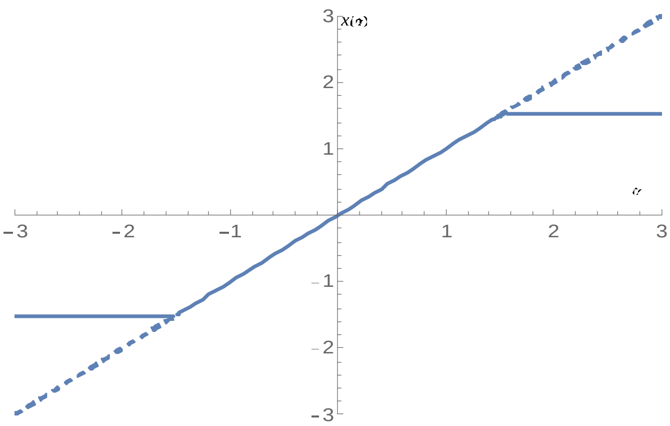
Proposition 1 *It is optimal for the median voter to set a policy cap $x(\alpha) = \varepsilon - \frac{\delta}{2}$ for all $\alpha \in \left[\varepsilon - \frac{\delta}{2}, \varepsilon + \delta\right]$, to delegate the policy choice to a politician, i.e. to select a flexible policy $x(\alpha) = \alpha$, for all $\alpha \in \left[-\varepsilon + \frac{\delta}{2}, \varepsilon - \frac{\delta}{2}\right]$ and to set a policy floor $x(\alpha) = -\varepsilon + \frac{\delta}{2}$ for all $\alpha \in \left[-\varepsilon - \delta, -\varepsilon + \frac{\delta}{2}\right]$.*

Proof. See Appendix. ■

According to Proposition 1, for a given level ε of the state-of-the-world uncertainty, the length of the delegation interval $\left[-\varepsilon + \frac{\delta}{2}, \varepsilon - \frac{\delta}{2}\right]$ (over which the policy choice is fully delegated to a politician) is the greater the smaller is the level δ of political uncertainty. Full delegation, i.e. unfettered representative democracy, is optimal iff there is no political uncertainty at all ($\delta = 0$). The optimal policy is pictured in Figure 1 below for $\varepsilon = 2$ and $\delta = 1$. The cap $x(\alpha) = 1.5$ applies for $\alpha \in [1.5, 3]$ and the floor $x(\alpha) = -1.5$ applies for

[18] Condition (c1) page 1550. An alternative and simpler proof is possible along the lines in Laussel and Resende (2017).

$\alpha \in [-3, -1.5]$. The flexible policy $x(\alpha) = \alpha$ is preferred when $\alpha \in [-1.5, 1.5]$. The dashed line indicates what would be a flexible policy outside the range in which it is optimal.



The optimal policy

The proof of Proposition 1 is in several steps. The first is to show that a flexible policy is always better than any discontinuous incentive-compatible policy when $\alpha \in \left[-\varepsilon - \frac{\delta}{2}, \varepsilon + \frac{\delta}{2}, \right]$. The second one is to prove that (i) it is better to have some policy cap (resp. policy floor) and that the optimal policy cap (resp. policy floor) is such that $x(\alpha) = \varepsilon - \frac{\delta}{2}$ for all $\alpha \in \left[\varepsilon - \frac{\delta}{2}, \varepsilon + \delta\right]$ (resp. such that $x(\alpha) = -\varepsilon + \frac{\delta}{2}$ for all $\alpha \in \left[-\varepsilon - \delta, -\varepsilon + \frac{\delta}{2}\right]$). The we prove that discontinuous policies are also dominated at the two extremes of the interval.

5 Conclusion

In this paper we have analyzed an original optimal delegation game between the median voter (writer of the constitution) and the elected representative (incumbent politician) in which the latter is privately informed of the state of the world and the former does not know either the (future) politician's type. This type is seen as the exogenous random outcome of an electoral process which is not analyzed here. Our game may be considered to take place *before* the politician is elected, for instance when the Constitution or, more generally the fundamental laws, are written. The main result is that, in that setting, it is optimal for the writer of the constitution, presumably the median voter, to impose an upper and a lower bound to the policies which the incumbent politician is allowed to implement, i.e. to tie the hands of future politicians.

This "interval delegation" result was already obtained in delegation models with known agent's type but is new in a model where the politician's type is private information. Moreover, the form taken here by the interval delegation, with an upper *and a lower* limit, is itself new compared with the results obtained in applied delegation models with know agent's type. The most prominent example is optimal trade agreements between governments with private political pressures as studied by Bagwell and Staiger (2005) or Amador and Bagwell ((2013), Section 4) where the main result is a simple tariff *cap*.[19] Another one is the existence of a simple *floor* (minimum) equilibrium output equilibrium in public intrinsic common agency games (Martimort, Semenov and Stole (2016)). More generally, Amador and Bagwell (2013) Proposition 3 (page 1552) specializes their general results on interval delegation to the case where the agent's bias is not affected by the state of the world and give sufficient conditions for the interval allocation to take the form of a simple cap. On the contrary, from the present model it seems to be that, when not only the state of the world but also the agent's type are private information, the interval allocation is taking the form of a delegation interval between a floor and a cap.

We are rather confident that these results would extend to the case of more general distribution functions. More specifically, we conjecture that three natural assumptions may be sufficient to entail the optimality of an interval delegation with a cap and a floor: (i) a non increasing hazard rate, $h'(\alpha) \leq 0$, (ii) a derivative $\Theta'(\alpha) \leq 1$ and a density function $f(\alpha)$ symmetric, non-decreasing between $-\varepsilon - \delta$ and 0 and non-increasing between 0 and $\varepsilon + \delta$. Obviously, these conditions are satisfied when the distributions of θ and t are uniform as assumed here. We leave the proof for a future technical note.

Finally, it would certainly be interesting to consider the case where the political process is not only random but also biased, i.e. when the writer of the constitution rationally expects the expected politician's politician type to differ from his/her own one for a variety of reasons.[20] Very likely, the symmetry between the floor and the cap would be lost, the writer of the constitution willing to tie more the hands of future politicians in the direction in which they are more likely to be biased.

[19] See also Beshkar, Bond and Ro (2011), Amador and Bagwell (2012).

[20] One of them being the influence of pressure groups, another (often the same) the possibly greater influence of a- uent people on the politicians' decisions.(see for instance Gilens (2014)).

APPENDIX

Proof of Claim 1

The proof is given for the case of a rightist bias ($t > 0$). A symmetric argument applies in the case of a leftist bias. The proof is in three steps.

Step 1: A discontinuous incentive-compatible policy is never optimal.

Consider indeed an incentive-compatible policy which includes an interval $[\alpha_0, \alpha_1]$ such that $x(\alpha) = \alpha_0$, $\forall \alpha \in [\alpha_0, \frac{\alpha_0+\alpha_1}{2})$ and $x(\alpha) = \alpha_1$, $\forall \alpha \in (\alpha_0, \frac{\alpha_0+\alpha_1}{2}]$.[21] Consider then the incentive-compatible policy identical the previous one, except that $x(\alpha) = \alpha$, $\forall \alpha \in [\alpha_0, \alpha_1]$. The difference in expected median voter's utilities between the latter and the former is easily computed as $\frac{(\alpha_1-\alpha_0)^3}{48\varepsilon} > 0$.

Step 2: if $t > 0$, a policy-floor, i.e. $x(\alpha) = \alpha_0$, $\forall \alpha \in [-\varepsilon + t, \alpha_0]$ is never optimal. Consider instead having a fully flexible policy $x(\alpha) = \alpha$ over the same interval. The expected median voter's utility under the latter policy exceed the one under the former by $\frac{1}{12\varepsilon}(\alpha_0 + \varepsilon - t)^2(2t + \varepsilon + \alpha_0) > 0$.

Step 3: Given the results in the two previous steps, the optimal policy is continuous and corresponds either to the fully flexible policy or to a semi-flexible policy with policy cap, $x(\alpha) = \alpha$, $\forall \alpha \in [-\varepsilon + t, \alpha_1]$ and $x(\alpha) = \alpha_1$, $\forall \alpha \in [\alpha_1, \varepsilon + t]$. The expected median voter's utility under a policy cap is obtained as the following third-order polynomial:

$$\frac{1}{12\varepsilon}\left[\alpha_1^3 - 3\varepsilon\ \alpha_1^2 - 3(\varepsilon^2 - t^2)\alpha_1 + (t-\varepsilon)^2(2t+\varepsilon)\right],$$

which takes its maximum value over the interval $[-\varepsilon + t, \varepsilon + t]$ at $\alpha_1 = \varepsilon - t$.[22]

Proof of Proposition 1

The proof is in several steps.

Step 1: Let us show that a discontinuous incentive-compatible policy over an interval $[\alpha_0, \alpha_1] \subseteq \left[-\varepsilon - \frac{\delta}{2}, \varepsilon + \frac{\delta}{2}, \right]$ is dominated by choosing the flexible policy $x(\alpha) = \alpha$ over this interval.

Consider indeed an incentive-compatible policy which includes an interval $[\alpha_0, \alpha_1] \subseteq \left[-\varepsilon - \frac{\delta}{2}, \varepsilon + \frac{\delta}{2}, \right]$ such that $x(\alpha) = \alpha_0$, $\forall \alpha \in [\alpha_0, \frac{\alpha_0+\alpha_1}{2})$ and $x(\alpha) = \alpha_1$, $\forall \alpha \in (\alpha_0, \frac{\alpha_0+\alpha_1}{2}]$.

Let us consider the difference between the expected median voter's utility with the above discontinuous policy and the flexible policy $x(\alpha) = \alpha$. Given (11) it is given by

[21] Notice that the politician's utility function is continuous at the discontinuity point $\widetilde{\alpha} = \frac{\alpha_0+\alpha_1}{2}$.

[22] $\varepsilon + t$ corresponds to a local minimum.

$$\Delta(\alpha_0,\alpha_1) = \int_{\alpha_0}^{\alpha_1} \alpha g(\alpha)d\alpha - [\int_{\frac{\alpha_0+\alpha_1}{2}}^{\alpha_1} \alpha_1 g(\alpha)d\alpha + \int_{\alpha_0}^{\frac{\alpha_0+\alpha_1}{2}} \alpha_0 g(\alpha)d\alpha] =$$
$$\int_{\alpha_0}^{\alpha_1} \alpha g(\alpha)d\alpha - [\alpha_1(G(\alpha_1) - G(\frac{\alpha_0+\alpha_1}{2})) + \alpha_0(G(\frac{\alpha_0+\alpha_1}{2}) - G(\alpha_0))],$$

where $G(\alpha) = \int_{\alpha}^{\varepsilon+\delta} g(s)ds$.

Differentiating with respect to α_1, we obtain:

$$\frac{\partial\Delta(\alpha_0,\alpha_1)}{\partial\alpha_1} = G(\frac{\alpha_0+\alpha_1}{2}) - G(\alpha_1) + g(\frac{\alpha_0+\alpha_1}{2})(\alpha_1 - \frac{\alpha_0+\alpha_1}{2}).$$

If $g'(\alpha) < 0$ this expression is positive so that $\Delta(\alpha_0,\alpha_1) > 0$ for all $\alpha_1 > \alpha_0$. Now, from the definition of $g(\alpha)$, (4) and (5), we have

$$g(\alpha) = \begin{cases} -\frac{\alpha^2+\alpha(\delta+2\varepsilon)-3\delta\varepsilon+\varepsilon^2}{4\varepsilon\delta}, \ \forall\alpha \in [-\varepsilon-\delta, \delta-\varepsilon] \\ \frac{\varepsilon-\alpha}{2\varepsilon}, \ \forall\alpha \in [\delta-\varepsilon, \varepsilon-\delta] \\ \frac{(\varepsilon-\alpha)(\varepsilon+\delta-\alpha)}{4\varepsilon\delta}, \ \forall\alpha \in [\varepsilon-\delta, \varepsilon+\delta] \end{cases}. \quad (12)$$

It is easy to see now that $g'(\alpha) < 0$, $\forall\alpha \in (-\varepsilon - \frac{\delta}{2}, \varepsilon + \frac{\delta}{2})$.

Step 2: Let us show that a policy cap is always optimal[23] and that the best policy cap is $x(\alpha) = \varepsilon - \frac{\delta}{2}$ for all $\alpha \in \left[\varepsilon - \frac{\delta}{2}, \varepsilon + \delta\right]$.

Let us consider an arbitrary policy cap $x(\alpha) = \alpha_1$ for all $\alpha \in [\alpha_1, \varepsilon+\delta]$. According to (11), the difference in expected median voter's utility with the same incentive compatible policy, where instead the policy selected for all $\alpha \in [\alpha_1, \varepsilon+\delta]$ is the flexible policy $x(\alpha) = \alpha$, is given by

$$\Delta W(\alpha_1) = \int_{\alpha_1}^{\varepsilon+\delta} (\alpha_1 - \alpha)g(\alpha)d\alpha, \quad (13)$$

where $g(\alpha)$ follows from (12). We obtain

$$\Delta W(\alpha_1) = \begin{cases} \frac{\delta^2-\varepsilon^2-3\alpha_1^2}{6} - \frac{(\alpha+\delta+\alpha_1)^3(\delta-\varepsilon-\alpha_1)}{48\varepsilon\delta} < 0, \ if \ \alpha \in [-\varepsilon-\delta, \delta-\varepsilon] \\ \frac{(\alpha_1-\varepsilon)(\alpha_1-\delta-\varepsilon)(\alpha_1+\delta-\varepsilon)}{12\varepsilon} \leq 0, \ if \ \alpha_1 \in [\delta-\varepsilon, \varepsilon-\delta] \\ \frac{1}{48\delta\varepsilon}(\alpha_1+\delta-\varepsilon)(\delta+\varepsilon-\alpha_1)^3, \ if \ \alpha_1 \in [\varepsilon-\delta, \varepsilon+\delta] \end{cases}.$$

$\Delta W(\alpha_1)$ is equal to 0 when $\alpha_1 = \varepsilon+\delta$ and when $\alpha_1 = \varepsilon-\delta$, strictly positive for all $\alpha_1 \in (\varepsilon-\delta, \varepsilon+\delta)$ and strictly negative elsewhere. It takes its maximum value for $\alpha_1 = \varepsilon - \frac{\delta}{2}$. It follows that some policy cap is always optimal and that the best policy cap is to set $x(\alpha) = \varepsilon - \frac{\delta}{2}$ for all $\alpha \in \left[\varepsilon - \frac{\delta}{2}, \varepsilon+\delta\right]$.

Step 3: Setting a policy cap $x(\alpha) = \alpha_1$ for all $\alpha \in (\frac{\alpha_0+\alpha_1}{2}, \varepsilon+\delta]$ and then jumping down to $x(\alpha) = \alpha_0 < \alpha_1$ for all $\alpha \in [\alpha_0, \frac{\alpha_0+\alpha_1}{2})$[24] is always

[23] The proof of the optimal policy floor is omitted since it follows the same lines.

[24] Notice that this is an incentive-compatible policy.

dominated by choosing instead a policy cap $x(\alpha) = \alpha_0$ for all $\alpha \in [\alpha_0, \varepsilon + \delta]$ iff $\frac{\alpha_0+\alpha_1}{2} > \varepsilon - \frac{\delta}{2}$. Indeed the difference between the expected median voter's utilities under the latter and the former equals

$$\begin{aligned}\int_{\frac{\alpha_0+\alpha_1}{2}}^{\varepsilon+\delta} (\alpha_0 - \alpha_1)\, g(\alpha) d\alpha &= \\ \frac{1}{96\varepsilon\delta}(\alpha_1 - \alpha_0)(\alpha_0 + \alpha_1 - 2\varepsilon + \delta)(\alpha_1 + \alpha_0 - 2(\varepsilon + \delta))^2 &> 0.\end{aligned}$$

Step 4: By an argument symmetrical to Step 2, a policy floor is always optimal and that the best policy floor is $x(\alpha) = -\varepsilon + \frac{\delta}{2}$ for all $\alpha \in \left[-\varepsilon - \delta, -\varepsilon + \frac{\delta}{2}\right]$. By an argument symmetrical to Step 3, one can show that, when $\frac{\alpha_0+\alpha_1}{2} < -\varepsilon + \frac{\delta}{2}$, a simple policy floor at α_1 is better than a policy floor $x(\alpha) = \alpha_0$ between $-\varepsilon - \delta$ and $\frac{\alpha_0+\alpha_1}{2}$ followed by an upward jump to $x(\alpha) = \alpha_1$ between $\frac{\alpha_0+\alpha_1}{2}$ and α_1.

Conclusion: Discontinuous equilibria are ruled out by Step 1 when $\frac{\alpha_0+\alpha_1}{2} \in \left(-\varepsilon - \frac{\delta}{2}, \varepsilon + \frac{\delta}{2}\right)$ and by Steps 3-4 when $\frac{\alpha_0+\alpha_1}{2} > \varepsilon - \frac{\delta}{2}$ or $\frac{\alpha_0+\alpha_1}{2} < -\varepsilon + \frac{\delta}{2}$. Since $\left[-\varepsilon + \frac{\delta}{2}, \varepsilon - \frac{\delta}{2}\right] \subset (-\varepsilon - \frac{\delta}{2}, \varepsilon + \frac{\delta}{2})$, it follows that discontinuous incentive-compatible policies never maximize the median voter's expected utility. The only possible equilibria are then flexible or semi-flexible incentive compatible policies. From Steps 2 an 4, the best of them is the semi-flexible policy described in Proposition 1.

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