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Group Targeting under Networked Synergies*

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Abstract

A principal targets agents organized in a network of local complementarities, in order to increase the sum of agents' effort. We consider bilateral public contracts à la Segal (1999). The paper shows that the synergies between contracting and non-contracting agents deeply impact optimal contracts: they can lead the principal to contract with a subset of the agents, and to refrain from contracting with central agents.

Keywords: Multi-Agent Contracting, Network, Synergies, Aggregate Effort, Optimal Group Targeting.

JEL: C72, D85

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1 Introduction

Institutions often contract with agents to trade effort against transfer, exploiting synergies and positive externalities between them. These synergies not only exist between contracting agents but also, in many economic contexts, between contracting and non-contracting agents. To cite a few examples: in monopoly pricing with network externalities, where a firm offers network-based discounts on top of a homogeneous price, a consumer who does not receive a discount still consumes and interacts with consumers receiving a discount; in organizations where the firm offers workers a bonus, a worker not receiving any bonus still interacts with other workers; in R&D networks where a public fund provider allocates subsidies, a non-subsidized firm still spends on R&D and interacts with partner firms.[1]

In this article, we aim at understanding the relationship between the structure of the network of synergies among agents and optimal contracts. In our model, a principal maximizing the sum of agents' effort trades effort against transfer in a context where non contracting agents (called *outsiders* thereafter) exert effort and interact with contracting agents, so that reservation utilities are endogenous to offered contracts. We study bilateral contracts with public offers *à la* Segal (1999) and we specify linear-quadratic utilities with local synergies and positive externalities.

There are several forces shaping optimal contracts on networks. First, the concavity of utilities pushes the principal to contract with all agents in the society. Second, the agents with larger social influence may be offered contracts with higher effort and higher transfers. This being said, the

[1] There is a now huge empirical literature documenting the positive impact of R&D subsidies on private R&D (see for instance Czarnitzki and Fier [2002], or Görg and Strobl [2007]; see Zuniga-Vicente, Alonso-Borrego, Forcadell and Galan [2014] for a recent survey). In parallel, a literature on management and economics identified the emergence of R&D networks from the late 80s, where rival firms collaborate in the R&D phase before competition (see for instance the works of Hagedoorn and co-authors, or Goyal and Moraga [2001]).

synergies between contracting agents and outsiders matter too. Indeed, an agent rejecting an offer takes into account the reaction of outsiders to own effort reduction, which triggers a further decrease of own effort by the synergies. Hence, outsiders have a *disciplinary effect* on contracting agents. As a consequence, it may not be optimal to contract with the whole society (for convenience, we will speak about *concentration* when the targeted group is a strict subset of the whole society). Moreover, the principal may be incited to exploit the agents with a large social influence as outsiders in order to discipline contracting agents.

To isolate the role of the disciplinary effect, we first examine a simpler model without disciplinary effect, by assuming rather that agents consider outsiders' play as fixed when they reject an offer. In this benchmark model, we show that (i) when the principal deals with a fixed number of contracts, the most central agents[2] are always selected, exert higher effort and receive higher transfers, and (ii) enlarging the targeted group is always beneficial to the principal, implying that it is optimal to offer contracts with positive transfers to all agents.

We then take into account the disciplinary effect. The analysis reveals that the disciplinary effect deeply impacts the principal's strategy. We obtain two main messages. First, *the disciplinary effect can lead to concentration*, i.e. the principal may not find it optimal to contract with all agents. Concentration emerges under high intensities of interaction, where outsiders have a large disciplinary effect on contracting agents. Concentration also emerges under low budget. In particular, with zero budget, contracting with the whole society has no impact on effort, whereas the presence of outsiders stabilizes contracts with enhanced effort. Second, *the principal can find it optimal not to contract with central agents*, meaning

[2]The relevant centrality index is the so-called Bonacich centrality measure, which naturally emerges in games of linear interaction (see Ballester, Calvò-Armengol and Zenou [2006]).

that the principal prefers to exploit their large social influence to discipline contracting agents rather than to contract with them and exploit increased synergies with other agents.

The emergence of concentration obtains under two key assumptions: bilateral contracts and commitment from the principal. We then check whether concentration can emerge under alternative assumptions. We first show that contingent contracting (to others' contract acceptance) forbids concentration. Indeed, raising contingent contracts allows the principal to extract the full surplus of each agent, so it is always optimal to contract with all agents. Second, when the principal cannot commit to offered contracts, whether the budget is exogenous or endogenous matters. An endogenous budget deters concentration because, from any targeted group, the principal can always create value by contracting with an additional agent. In contrast, under exogenous budget, the budget may play as a commitment device, and concentration is still possible.

Related literature. This paper contributes to the two strands of literature on optimal intervention in presence of synergies between agents. The first strand considers optimal targeting in presence of interacting agents.[3] Allouch (2015) considers a model of a local public good under linear substitute interactions, and explores optimal transfers to improve aggregate effort. Demange (2017) studies the optimal targeting strategies of a planner aiming to increase the aggregate action of agents embedded in a social network, allowing for non-linear interaction. Galeotti, Golub and Goyal (2017) study optimal targeting in networks, where a principal aims at maximizing utilitarian welfare or minimizing the volatility of aggregate activity. In the drop-out game of Calvò-Armengol and Jackson (2004), the planner subsidizes agents' entry into the labor market. In our work, we take into account participation constraints, not addressed in the above papers.

[3] See Bloch (2015) for a recent survey.

The second strand of literature studies principal / multi-agent contracting in presence of synergies, taking into account participation constraints. A closely related literature considers coordination issues with binary actions. Bernstein and Winter (2012) study a costly participation game where participants receive positive and heterogeneous externalities from other participants, and characterize the contracts inducing full participation while minimizing total subsidies. In Sakovics and Steiner (2012), a principal subsidizes agents facing a coordination problem akin to the adoption of a network technology. Optimal subsidies target agents who impose high externalities on others and on whom others impose low externalities. With respect to this literature, we introduce continuous effort, and we cover situations where contracting with a subset of the population can be optimal. Recent studies explore optimal linear pricing with interdependent consumers (Candogan, Bimpikis and Ozdaglar [2012], Bloch and Quérou [2013] and Fainmesser and Galeotti [2016]). We contribute to this literature by enriching the set of contracts (see Appendix C for the parallel between our model and monopoly pricing).

The efficiency of contracts in presence of externalities between agents has been considered by Holmstrom (1982) and Segal (1999), but our paper differs in two main respects. First, our primary focus is the impact of the network structure on optimal contracts, and not trade efficiency. Second, we allow for synergies between non-contracting and contracting agents. Genicot and Ray (2006) - see also Galasso (2008) - study a dynamic game where outside opportunities rise with the number of non-contracting agents. Conversely, in our setting outside opportunities tend to decrease with the presence of non-contracting agents, since when an agent rejects a contract he exerts less effort.

The paper is organized as follows. Section 2 presents the model of bilateral contracting on networks. Section 3 analyzes the model without disciplinary effect, and Section 4 presents the main results of the article. Section 5 examines whether concentration emerges under contingent contracting and when the principal does not commit to offers. Section 6 concludes. All proofs are presented in Appendix A. Appendix B analyzes contingent contracting deeper, and Appendix C presents three economic applications that the model fits well.

2 Model

A principal commits to a set of contracts with a finite set of agents organized in a fixed network of local complementarities. A crucial feature of our model is that non contracting agents, that we call outsiders, still exert effort and interact with contracting agents. For instance, in R&D networks, a firm that does not accept or does not receive the offer from the principal can still spend on R&D and benefit from partners' effort. Similarly, in monopoly pricing under interdependent consumers, a consumer may not accept discounts and still consume and interact with other consumers, just as a worker who does not receive a bonus still interacts with other workers (see Appendix C for more details).

We consider a three-stage game. In the first stage, the principal proposes bilateral contracts as in Segal (1999). Each contract is an effort-transfer pair. In the second stage, agents simultaneously decide whether to accept or reject their respective offers. In the third stage, agents exert effort and transfers are realized. Both effort, contracts and network are assumed to be publicly observable. We study a Subgame Perfect Nash Equilibrium (SPNE). In this model, coordination is an issue and equilibrium multiplicity can arise. We analyze the SPNE of the game which maximizes the principal's objective; that is, we focus on the equilibrium such that all proposed offers are accepted.

Notations. Real numbers or integers are written in lower case, matrices (including vectors) in block letters and in boldface. We denote by $\mathbf{1}_p$ the p-dimensional vector of ones, for all $p \in \mathbb{N}$. For convenience, symbol $\mathbf{1}$ will quote for $\mathbf{1}_n$. Similarly, symbol $\mathbf{0}$ represents the n-dimensional vector of zeros. We let superscript ^{T} stand for the transpose operator. For instance, we write vector $\mathbf{X} = (x_i)_{i\in N}$, with x_i its i^{th} entry, and $x = \mathbf{1}^T\mathbf{X}$ denotes the sum of entries of vector $\mathbf{X}$.

The game in the absence of a principal. We let $N = \{1, 2, \cdots, n\}$ be the set of agents organized in a network of bilateral relationships. The network is undirected, i.e. it is formally represented by a symmetric adjacency matrix $\mathbf{G} = [g_{ij}]$, with binary element $g_{ij} \in \{0, 1\}$.[4] By abuse of language we will speak of network $\mathbf{G}$. The link between agents i and j exists whenever $g_{ij} = 1$, in which case we will say that agents i and j are neighbors. By convention, $g_{ii} = 0$ for all i. We let $\mu(\mathbf{G})$ denote the largest eigenvalue of the adjacency matrix $\mathbf{G}$.

We consider linear quadratic utilities of the form

$$u_i(q_i, Q_{-i}) = q_i - \frac{1}{2}q_i^2 + \delta\sum_{j\in N} g_{ij}q_iq_j \tag{1}$$

with $\mathbf{Q} \geq \mathbf{0}$. Parameter $\delta > 0$ measures the strength of complementarities, or intensity of interaction, between neighbors. With the above specification, utilities depend positively on neighbors' effort, and neighbors' effort levels are strategic complements.

We define Bonacich centralities, which play a prominent role in network games with linear-quadratic utilities (see Bonacich [1987]). We let the n-dimensional square matrix $\mathbf{M} = (\mathbf{I} - \delta\mathbf{G})^{-1} \geq 0$. The condition $\delta\mu(\mathbf{G}) < 1$ guarantees $\mathbf{M} \geq 0$. We let the n-dimensional vector $\mathbf{B} = \mathbf{M}.\mathbf{1}$, with entry i called b_i, denote the vector of *Bonacich centralities* of the network

[4] Our results are qualitatively robust to the introduction of non-binary and asymmetric relationships, where centralities are replaced by inward/outward centralities, and also to the introduction of a low level of strategic substitutability.

weighted by parameter δ (we avoid references to network G and parameter δ for convenience). The quantity b_i is the number of paths from agent i to others, where the weight of a path of length k from agent i to agent j is δ^k.

In the absence of contracts, agents play a unique Nash equilibrium. Any agent $i \in N$ exerts an effort equal to her Bonacich centrality b_i and obtains a utility level equal to $\frac{1}{2}\, b_i^2$.

Contracts. A contract between the principal and agent i specifies an effort $x_i \in \mathbb{R}^+$ and a monetary transfer $t_i \in \mathbb{R}$ from the principal to agent i. Assume that the principal contracts with subset $S \subset N$, with cardinality s. We let $\mathbf{X}_S = \{x_i\}_{i\in S}$ represent the corresponding profile of effort. For convenience, we will denote $\mathbf{X} = \mathbf{X}_N$. To determine the utilities of contracting agents and outsiders, we need to distinguish two cases.

Case 1: All agents in S accept their contracts. Then outsiders play a Nash equilibrium effort given $\mathbf{X}_S$. Agent j' best-response effort is given by

$$y^{BR}_{N\setminus S,j} = 1 + \delta \sum_{k\in S} g_{jk}x_k + \delta \sum_{k\notin S} g_{jk}y_k$$

where y_k represents the effort of outsider k. Let $\mathbf{G}_{N\setminus S}$ be the $(n-s)\times(n-s)$ sub-matrix of matrix $\mathbf{G}$ representing the bilateral influences between pairs of agents in $N \setminus S$, and $\mathbf{G}_{N\setminus S,S}$ the $(n-s)\times s$ sub-matrix of matrix $\mathbf{G}$ representing the bilateral influences between agents in S and agents in $N \setminus S$. The Nash equilibrium effort profile of outsiders $\mathbf{Y}^*_{N\setminus S}$, given $\mathbf{X}_S$, is written as:

$$\mathbf{Y}^*_{N\setminus S} = (\mathbf{I} - \delta\mathbf{G}_{N\setminus S})^{-1}(\mathbf{1}_{n-s} + \delta\mathbf{G}_{N\setminus S,S}\mathbf{X}_S) \tag{2}$$

The utility of a contracting agent i is given by

$$v_i(\mathbf{X}_S, \mathbf{Y}^*_{N\setminus S}, t_i) = u_i(\mathbf{X}_S, \mathbf{Y}^*_{N\setminus S}) + t_i \tag{3}$$

Case 2: An agent i in S rejects the offer while all other offers are accepted. Agent i becomes an outsider and plays a Nash equilibrium effort with other outsiders. We denote by z_i her effort, which is the entry corresponding to agent i in the following Nash profile:

$$\mathbf{Y}^*_{\{i\}\cup N\setminus S} = (\mathbf{I} - \delta \mathbf{G}_{\{i\}\cup N\setminus S})^{-1}(\mathbf{1}_{n-s+1} + \delta \mathbf{G}_{\{i\}\cup N\setminus S, S\setminus\{i\}} \mathbf{X}_{S\setminus\{i\}}) \tag{4}$$

The equilibrium utility of agent i is then given by

$$u_i(\mathbf{X}_{S\setminus\{i\}}, \mathbf{Y}^*_{\{i\}\cup N\setminus S}) = \frac{1}{2} z_i^2 \tag{5}$$

Principal's program. The principal's objective is to maximize the aggregate effort of both contracting agents and outsiders, subject to both the budget constraint and individual participation constraints. The principal's program, called program $\mathcal{P}$, is written as:

$$\max_{\{(x_i,t_i)\}_{i\in S}} \quad \mathbf{1}^T \mathbf{X}_S + \mathbf{1}^T \mathbf{Y}^*_{N\setminus S}$$

s.t.

$$\frac{1}{2} z_i^2 \leq u_i(\mathbf{X}_S, \mathbf{Y}^*_{N\setminus S}) + t_i, \ \forall i \in S \tag{6}$$

$$\sum_{i\in S} t_i \leq t \tag{7}$$

where $\mathbf{Y}^*_{N\setminus S}$ and z_i are functions of $\mathbf{X}_S$ and determined respectively by equations (2) and (4).

The above program can be part of a more general program with endogenous budget, where the principal's payoff is an increasing and concave function of the sum of agents' effort net of transfers (in Appendix C we provide three economic applications with endogenous budget). For clarity, and because the impact of network structure is essentially captured by the restricted sub-problem with a fixed budget, we abstract from optimal budget selection considerations throughout the paper and assume that the budget is fixed and not larger than the optimal budget.

The analysis calls for intuitive preliminary observations. First, individual participation constraints are binding at optimum, otherwise the principal could save on transfers for the same objective, and use the saved budget to increase effort. Second, at optimum, transfers are non-negative; otherwise agents are better off rejecting the offer and playing their best-responses. That is, the principal should only be rewarding agents. Last, such a program should admit optimal contracts for all selected groups $S \in N$. We note that the condition $\delta < \frac{1}{2\mu(\mathbf{G})}$ is sufficient.[5]

3 A benchmark: no disciplinary effect

This section studies optimal contracting in a model without disciplinary effect. This model depicts a useful backdrop to assess the role of the disciplinary effect. As for the above model, the principal cannot tax an agent, otherwise the agent would be better off rejecting the offer and playing her best-response effort.

We assume that an agent, receiving an offer and contemplating the opportunity of rejecting the offer, considers outsiders' play as fixed[6] (in contrast, the agent takes into account outsiders' reactions to own deviation in the model). To simplify notations, we denote by y_j be the effort of outsider j under acceptance of all contracts in S (see equation (2)). The best-response effort of an agent $i \in S$ is given by $x_i^{BR} = 1 + \delta \sum\limits_{j \in S} g_{ij} x_j + \delta \sum\limits_{j \in N \setminus S} g_{ij} y_j$. Agent i's individual participation constraint becomes:

$$\frac{1}{2}(x_i^{BR})^2 \leq u_i(\mathbf{X}_S, \mathbf{Y}^*_{N \setminus S}) + t_i \tag{8}$$

We obtain:

Lemma 1. *Suppose that the principal contracts with a set $S \subset N$, so that there is no transfer for agents in the set $N \setminus S$. The performance of the optimal contracts is equal to $\sum\limits_{i \in N} b_i + \sqrt{2t} \sqrt{\sum\limits_{j \in S} b_j^2}$.*

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[5] As will be shown thereafter, this condition guarantees the existence of the optimal contingent contract, and no set of bilateral contracts with any group of agents can do better.

[6] This is equivalent to assume that outsiders play before contracting agents.

This simple lemma admits two immediate implications. First, for a fixed number of contracts, the most performing group maximizes the sum of squared Bonacich centralities, meaning that agents with the highest centralities are selected. In particular, on the star network, the central agent belongs to the group of largest performance; on the line network, all members of the best group are positioned at the center of the line; on any regular network, like the circle or the complete network, all groups of same size generate the same performance. Second, including an additional agent in a group strictly increases the principal's objective. This leads to an unambiguous prediction about the possible emergence of concentration in group targeting. Defining $\|\mathbf{B}\|$ as the euclidian norm of vector $\mathbf{B}$, we obtain:

Proposition 1. *When there is no disciplinary effect, the optimal group is the whole society and all transfers are positive. Optimal contracts* $\{(\hat{x}_i, \hat{t}_i)\}_{i\in N}$ *satisfy:*

$$\begin{cases} \hat{x}_i = b_i + \frac{\sqrt{2t}}{\|\mathbf{B}\|} \cdot b_{\mathbf{B},i} \\ \hat{t}_i = \frac{t}{\|\mathbf{B}\|^2} \cdot b_i^2 \end{cases} \tag{9}$$

Proposition 1 indicates that both supplementary effort, $\hat{x}_i - b_i$, and transfer $\hat{t}_i$ are increasing with centrality measures. Increased effort is proportional to weighted Bonacich centrality, with weights themselves equal to un-weighted Bonacich centrality. Note that the two centrality measures $b_{B,i}$ and b_i may not be aligned. Transfers are positive[7] and proportional to squared centralities, meaning that there is a bonus for central agents. We also note that the network structure affects the transfer per unit of increased effort; i.e., $\frac{t_i}{\hat{x}_i - b_i}$ is proportional to $\frac{b_i^2}{b_{\mathbf{B},i}}$. Moreover, budget-sharing among agents, $\frac{\hat{t}_i}{t} = \frac{b_i^2}{\|\mathbf{B}\|}$, is independent of budget level and is concentrated

[7] It can be shown that transfers are positive for any concave utilities satisfying complementarities.

in favor of central agents. The optimal aggregate effort is written

$$\hat{x} = b + \sqrt{2t} \cdot \|\mathbf{B}\| \tag{10}$$

Equation (10) is useful for a comparative analysis of network structures from the perspective of the principal's payoff. From the above formula, optimal contracts perform better for networks with higher values of b and $\|\mathbf{B}\|$. We deduce that, *when it is optimal to target the whole society, adding links from any network structure always increases the aggregate effort.* It is more difficult to compare the respective aggregate effort over networks with the same number of links. The analysis highlights Nested-Split graphs (NSGs for short).[8] Applying Lemma 1 in Belhaj, Bervoets and Deroïan (2016), we can state that *when it is optimal to target the whole society, of all networks with the same number of links, the network that maximizes the sum of agents' effort is a Nested-Split graph.*

4 Optimal bilateral contracts

In this section, we study the optimal bilateral contracts in presence of disciplinary effect. This means that, when agents reject an offer, they take into account that outsiders adjust their play. To get some insights, we first examine the two-agent case, then we analyze optimal group selection for general networks in the polar cases of low and high intensities of interaction, and finally we explore general intensities of interaction on specific network structures.

Two-agent society. We consider a society composed of two connected agents, say agents 1 and 2. If the principal contracts with both agents, the reservation utility of an agent takes as given the effort prescribed in

[8]NSGs are such that, for any pair of agents on the network, one neighborhood is nested in the other. Although this property is very demanding, NSGs cover a wide variety of structures, such as the complete network and star-like networks (see Mahadev and Peled [1995] for more details).

the contract of the other agent. In contrast, when the principal contracts with, say, agent 1, agent 2 always plays a best-response to agent 1's effort; so, when agent 1 rejects her offer, agent 2's effort is lower than her effort under offer acceptance. Hence, agent 1's reservation utility is lower when the principal contracts with agent 1 than when the principal contracts with both agents. This decrease in the outside option of agent 1 allows the principal to increase agent 1's effort, and, ultimately, it may be profitable for the principal to exclude agent 2. That said, by concavity of utilities, distributing transfers among a large group is attractive. Which effect dominates is highly dependent on parameters and network structure.

We let $t_c(\delta) = \frac{2\delta^2(1+\delta)}{1-2\delta^2-\delta^4}$ for $\delta \in [0, \kappa[$ with $\kappa = \sqrt{\sqrt{2}-1}$. We obtain:

Proposition 2. *In the two-agent society, it is optimal to contract with a single agent if and only if $\delta \in [\kappa, \frac{1}{\sqrt{2}}[$, or $\delta < \kappa$ and $t < t_c(\delta)$.*

Proposition 2 essentially provides three messages. First, it may not be optimal to select the whole society, confirming that the disciplinary effect can dominate. Second, when $\delta < \kappa$, the disciplinary effect dominates under very low budget. Indeed, even with null budget, the principal can increase aggregate effort by contracting with a single agent, exploiting both the commitment feature of the contract (from the agent) and the disciplinary effect. In contrast, contracting with the two agents with null budget yields no increase in effort. Last, the disciplinary effect is prevailing for high intensities of interaction; even more, under very high intensities of interaction (i.e., for $\delta \geq \kappa$ on the figure), concentration emerges for all budgets. These messages are illustrated in Figure 1, which presents the optimal group size as a function of the budget and of the intensity of interaction. The range of possible intensities of interaction is $[0, \frac{1}{\sqrt{2}}[$. The upper bound corresponds to the threshold above which the optimum no longer exists (i.e. effort would escalate to infinity):

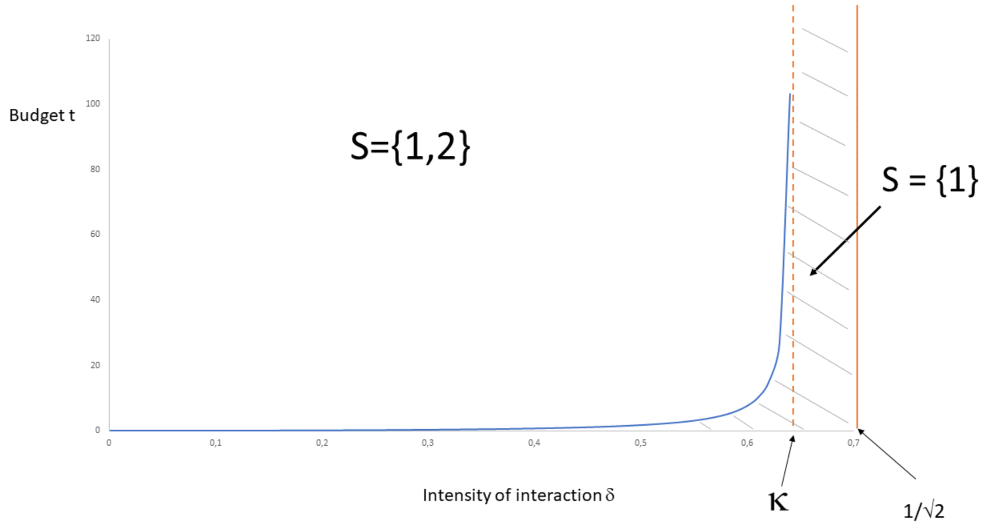

Figure 1: $n = 2$; Optimal group as function of parameters δ, t.

General case. The study of optimal contracts in general network structures confirms these intuitive messages. The next proposition clarifies the issue in the limit cases of low and high intensity of interaction:

Proposition 3. *Fix $t > 0$. (i) When the intensity of interaction is high enough, the optimal group is a strict subgroup of the society. (ii) When the intensity of interaction is low enough, the optimal group is the whole society. Optimal contracts $\{(\hat{x}_i, \hat{t}_i)\}_{i \in N}$ are then identical to the optimal contracts under no disciplinary effect, as given by equation (9).*

Proposition 3 indicates that, under high intensity of interaction, the disciplinary effect dominates. In the opposite, under low intensity of interaction the concavity of utilities dominates, so that the principal contracts with the whole society and the solution of the program coincides with the optimal contract of the simplified model without disciplinary effect.

Interestingly, concentration not only arises under high interaction, but also under low budget level:

Proposition 4. *Fix $\delta > 0$. When the budget is small enough, the optimal group is a strict subset of the society.*

Proposition 4 suggests that the disciplinary effect dominates for low budget. The intuition is the same as in the two-agent case (commitment plus disciplinary effect).

Propositions 3 and 4 confirm the messages obtained in the two-agent case, by providing conditions under which it is optimal to concentrate the budget over a subset of the whole society. However, these propositions are silent about the best group to target. The performance of any targeted group can be computed, but in general sorting groups by their performance can hardly be done analytically, because there is no monotonic relationship between group composition and parameters t or δ. To illustrate further how network structure affects optimal group selection, we explore specific network structures by means of numerical computations.[9] We explore in the order the complete network, the circle, the star and line. The numerical analysis illustrates the relationship between the size of the group and parameters δ and t and the relationship between centrality and optimal targeting.

The complete network. The complete network is such that there is a link between all pairs of agents. In the complete network, all agents have same centrality. Furthermore, once the principal targets a group, all agents inside the group have the same positions and are offered the same contract. As well, all outsiders have the same positions. Hence, with this structure,

[9] An additional on-line appendix is available at `https://sites.google.com/view/fredericderoian/recent-working-papers`. In this appendix, we present the performance of any targeted group through standard Lagrangian method on any networks, and we provide a numerical program determining the optimal group.

we only study the link between group size and optimal targeting. Let

$$\begin{cases} A = -\frac{1}{2} + \delta(s-1) + \delta^2 \frac{s(n-s)}{1-\delta(n-s-1)} - \frac{\delta^2}{2}\frac{(s-1)^2}{(1-\delta(n-s))^2} \\ B = 1 + \delta\frac{n-s}{1-\delta(n-s-1)} - \delta\frac{s-1}{(1-\delta(n-s))^2} \\ C = \frac{t}{s} - \frac{1}{2(1-\delta(n-s))^2} \end{cases}$$

We have $A < 0$ for low intensity of interaction, and we consider by δ_A the smallest intensity of interaction such that $A = 0$ (this ensures the convexity of the participation constraint). Let $x(s) = \frac{B+\sqrt{B^2-4AC}}{-2A}$. We obtain:

Proposition 5. *Let* $\delta < \min(\delta_A, \frac{1}{n-1})$ *and* $t \geq 0$. *The optimal group size* s^* *maximizes*

$$\frac{(1+\delta)s \cdot x(s) + (n-s)}{1-\delta(n-s-1)}$$

Figure 2 illustrates optimal group targeting on the complete network with $n = 8$ for various parameters δ and t. All three messages given

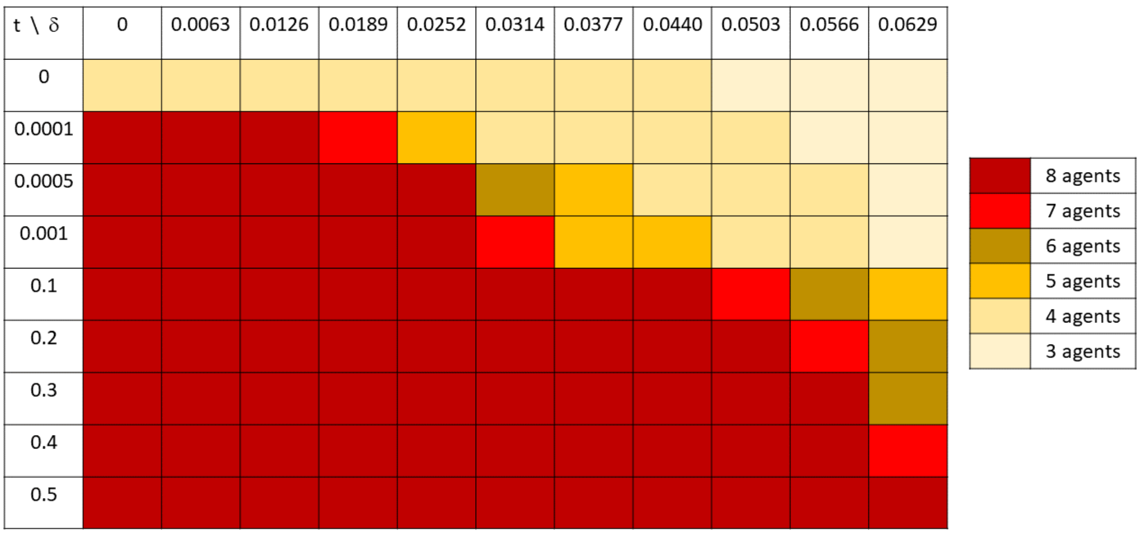

Figure 2: Optimal group size on the complete network ($n = 8$), as a function of the budget and of the intensity of interaction.

for $n = 2$ are confirmed in this figure, and also emerge from a large set of numerical computations. First, the disciplinary effect can lead the principal to select a group of intermediary size.[10] The second message is that fixing the intensity of interaction and increasing the budget can only increase the size of the optimal set. Third, fixing the budget and increasing the intensity of interaction can only reduce the size of the optimal set.

The circle network. The circle network contains n links and each agent has two neighbors. All agents have the same structural positions on the circle network. However, in contrast with the complete network, the disposition of contracting agents is a matter. Numerical computations on the circle network confirm the huge impact of the disciplinary effect. Figure

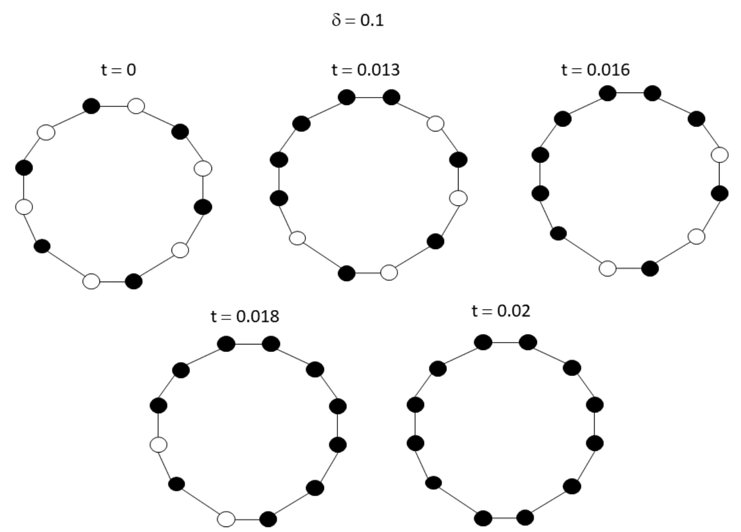

Figure 3: Optimal group on the circle network; targets are black.

3 presents the optimal group for a fixed intensity of interaction and varied budgets on the 12-agent circle network. It not only shows that intermediary groups can emerge at optimum, but also that both contracting agents and

[10]It is interesting to note the difference from Zhou and Chen (2015), who examine optimal split by a central planner into two groups, a leader group and a follower group, on the complete network. In their model, for low intensity of interaction, it is always optimal to divide the population into two equal parts (Proposition 5, p. 222 in their article). In our setting, even with zero budget other solutions can emerge, due to participation constraints.

outsiders can be irregularly distributed on the circle, which stands in sharp contrast with the benchmark without disciplinary effect (where, by Lemma 1, all targeted groups of same size yield the same performance).

The star network. The star structure contains $n-1$ links and one agent is involved in all links. Here, agents have heterogeneous positions on the network, and in particular the agent with $n-1$ links is unambiguously more central than other agents. We call her the *central agent* and the other agents are called *peripheral agents*. Actually, the central agent needs not belong to the optimal group. Again, this stands in sharp contrast with the model without disciplinary effect. Figure 4 illustrates this finding on the 10-agent star network. Figure 4 shows that (i) the disciplinary effect can

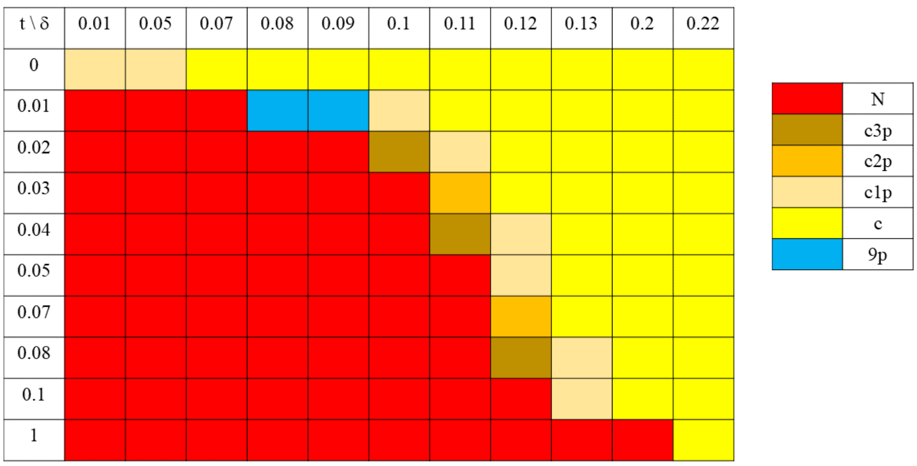

Figure 4: Optimal group size on the 10-agent star network, as a function of the budget and of the intensity of interaction. We use the following symbols: c for the central agent; $9p$ for the nine peripheral agents; cip for the central agent and a number $i=\{1,2,3\}$ of peripheral agents; N for the whole society. In blue cells, the principal does not contract with the central agent.

lead to exclude the central agent from the optimal group, and (ii) there is a non-monotonic effect in the composition of the optimal group with respect to both budget level and intensitiy of interaction. I.e., fixing the intensity of interaction and increasing the budget, or fixing the budget and increasing the intensity of interaction, it can be that the central agent belongs to the optimal group, then is excluded, and then belongs again to the target.

The line network. The line network contains $n-1$ links and no agent has more than two neighbors. Like the star network, agents closer to the middle of the line are unambiguously more central. Figure 5 illustrates

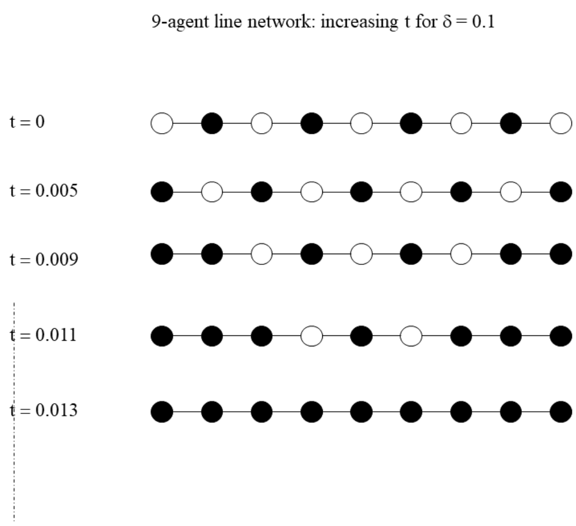

Figure 5: Optimal group on the line network; targets are black.

that the principal may not contract with agents of intermediary centrality on the 9-agent line network even when the most peripheral agents receive an offer. By contrast, in the model without disciplinary effect, the best target of fixed size contains the most central agents. Increasing the budget enlarges the optimal group, but does not induce a systematic concentration of targets toward the center of the line. Note that, as budget increases, a given agent can be included in the targeted group, then excluded, and then included again.

5 Discussion

One main insight of the above analysis is that contracting with the whole society may not be optimal under high intensities of interaction and under low budget level. This is because outsiders discipline contracting agents. This result obtains under two key assumptions in the model: we assumed bilateral contracts, and we also assumed that the principal committed to proposed contracts. In this section, we examine whether concentration can emerge under contingent contracts, and when the principal does not commit to her offers.

Contingent contracts. So far, we have restricted attention to simple bilateral contracts. The principal can improve his payoff by proposing contingent contracts. A set of contingent contracts $(x_i, t_i)_{i \in N}$ is defined here as a collection of individual take-it-or-leave-it offers made to all members of the society simultaneously and such that when one agent rejects the offer, no contract is executed. In this case, each agent i exerts the effort b_i corresponding to the Nash equilibrium played in the absence of contracts and obtains a utility level equal to $\frac{1}{2}b_i^2$.[11] This means that reservation utilities are exogenous to offered contracts. Since reservation utilities are exogenous, it is optimal to contract with all agents (see Appendix B for a characterization of optimal contingent contracts).

No principal's commitment. We examine whether concentration can still emerge when the principal does not commit to her offers. The emergence of concentration depends on whether the budget is exogenous or

[11] It is assumed here that the principal cannot threaten agents with a utility level below that obtained in the absence of contracts. Hence, in the principal's program, agent i's participation constraint is given by $\frac{1}{2}b_i^2 \leq x_i - \frac{x_i^2}{2} + \delta \sum_{j \in N} g_{ij} x_i x_j + t_i$.

endogenous. Under fixed budget, the budget may play as a commitment device (by the principal), and concentration is still possible. For instance, in the two agent case, if concentration is optimal under commitment, the corresponding optimal contract will be still optimal under no commitment. Under endogenous budget, once an initial subset of agents accept their contracts, the principal always finds it profitable to propose a contract to an outsider with a transfer ϵ and increased effort with respect to the initial play equal to $\sqrt{2\epsilon}$ (this contract will be accepted by the agent - see equation (11)). This results in an increase of the principal's objective for ϵ sufficiently low. Assuming that the outsider does not anticipate a further deviation of the principal, she is better off accepting the offer.

6 Conclusion

This paper considered agents organized in a network of local complementarities, and a principal trading effort through bilateral contracts. We found that the synergies between contracting and non-contracting agents strongly impacts the relationship between network structure and optimal contracting. The main message of this paper was that such synergies can lead the principal to target a strict subgroup of the society, and possibly to refrain from contracting with central agents.

The issues raised in this paper merit further related investigation. First, in some circumstances the principal could design the network to increase the sum of agents' effort. It would be challenging to study the optimal principal's policy if the principal could use his budget to subsidize the formation or deletion of links as well as effort. Second, effort is often imperfectly observable in many economic applications, like in teams, and it would be interesting to investigate this issue further.

7 Appendix A: Proofs

This appendix gathers all proofs.

Proof of Lemma 1. Suppose that the principal contracts with all agents in the set $S = \{1, \cdots, s\}$ (labeling is without loss of generality). Define the n-dimensional vector $\mathbf{\Phi} = (\phi_i)_{i\in N}$ such that $\phi_i = \sqrt{2t_i}$ for all $i \in S$ and $\phi_i = 0$ otherwise. Then, we observe that $u_k(x_k, x_{-k}) = x_k x_k^{BR} - \frac{1}{2}x_k^2$, i.e. $\frac{1}{2}\left(1+\delta\sum\limits_j g_{kj}x_j\right)^2 - u_k(x_k, x_{-k}) = \frac{1}{2}\left(x_k - x_k^{BR}\right)^2$. The participation constraint of an agent $i \in S$ can then be written as

$$x_i - x_i^{BR} = \phi_i \tag{11}$$

Also, the play of an outsider $j \in N \setminus S$ basically satisfies $y_j - y_{N\setminus S,j}^{BR} = \phi_j (= 0)$. For convenience define the n-dimensional vector $\mathbf{V} = (v_i)_{i\in N}$ where $v_i = x_i$ for $i \in S$ and $v_j = y_j$ for $j \in N \setminus S$. The whole system of the n equations is then written as $\mathbf{V} = (\mathbf{I} - \delta\mathbf{G})^{-1}(\mathbf{1} + \mathbf{\Phi})$, from which we deduce by summation over all entries the aggregate effort in the society:

$$\sum_{i\in S} x_i + \sum_{j\in N\setminus S} y_j = \sum_{k\in N} b_k + \sum_{i\in S} b_i\sqrt{2t_i} \tag{12}$$

Since $\sum\limits_{k\in N} b_k$ is independent of the set S, the optimal contracts in the set S maximize the quantity $\sum\limits_{i\in S} b_i\sqrt{2t_i}$ under the budget constraint $\sum\limits_{i\in S} t_i = t$. Basic optimization through Lagrangian method yields the optimal transfer

$$t_i = \frac{t}{\sqrt{\sum\limits_{j\in S} b_j^2}} b_i^2 \tag{13}$$

Plugging all transfers in equation (12), aggregate effort reaches the value $\sum\limits_{k\in N} b_k + \sqrt{2t}\sqrt{\sum\limits_{j\in S} b_j^2}$. □

Proof of Proposition 1. By Lemma 1, enlarging a set S to $S' = S \cup \{j\}$ for any agent $j \in N \setminus S$ induces a strict increase in the principal's objective.

This implies that the principal finds it optimal to contract with the whole society. We then characterize the optimal contracts. Transfers are given by applying equation (13) to the case $S = N$, so we get $t_i = \frac{t}{\|\mathbf{B}\|^2} b_i^2$. Remembering that $x_i = x_i^{BR} + \sqrt{2t_i}$, that $\mathbf{X}^{BR} = \mathbf{1} + \delta \mathbf{G} \mathbf{X}$ and plugging the value of the transfer into the above expression, we obtain $\mathbf{X} = \mathbf{B} + \frac{\sqrt{2t}}{\|\mathbf{B}\|} \mathbf{B}_{\mathbf{B}}$ (where by convention $\mathbf{B}_{\mathbf{B}} = \mathbf{M}\mathbf{B}$) and $x = b + \sqrt{2t}\ \|\mathbf{B}\|$.

To finish, we note that the objective of the planner, $\hat{x}$, is increasing in the budget, confirming that all constraints are binding. □

Proof of Proposition 2. We compare the increase of aggregate effort in the two situations.

- Performance of the target $S = \{1, 2\}$:

We have $P_{\{1,2\}} = x - b = \sqrt{2t}\|B\|$ with $b_i = \frac{1}{1-\delta}$. We find

$$P_{\{1,2\}} = \frac{2\sqrt{t}}{1-\delta}$$

- Performance of the target $S = \{1\}$:

We have $P_{\{1\}} = 1 + (1+\delta)x - \frac{2}{1-\delta}$, with x solving the participation constraint given by $x - \frac{x^2}{2} + \delta x(1 + \delta x) + t - \frac{1}{2}\frac{1}{(1-\delta)^2} = 0$. I.e., for $\delta < \frac{1}{\sqrt{2}}$, $x = \frac{1}{1-2\delta^2}\left[1 + \delta + \frac{1}{1-\delta}\sqrt{\delta^4 + 2(1-2\delta^2)(1-\delta)^2 t}\right]$. We deduce that $P_{\{1\}} = \frac{(1+\delta)}{(1-2\delta^2)(1-\delta)}\left[\delta^2 + \sqrt{\delta^4 + 2(1-2\delta^2)(1-\delta)^2 t}\right]$.

We have $P_{\{1\}} > P_{\{1,2\}}$ if and only if

$$(1+\delta)\left[\delta^2 + \sqrt{\delta^4 + 2(1-2\delta^2)(1-\delta)^2 t}\right] > 2(1-2\delta^2)\sqrt{t}$$

We do the squaring, then isolate the square root and then do the squaring again. This gives in total $at^2 + bt < 0$ with $a = (1-2\delta^2)^2(1-2\delta^2-\delta^4)^2$ and $b = -4\delta^4(1+\delta)^2(1-2\delta^2)^2$. Then $P_{\{1\}} > P_{\{1,2\}}$ when $t < t_c(\delta)$ such that

$$t_c(\delta) = \frac{2\delta^2(1+\delta)}{1-2\delta^2-\delta^4}$$

This is an increasing and convex function, which tends to infinity when δ tends to $\kappa = \sqrt{\sqrt{2}-1}$. When $\delta \in [\kappa, \frac{1}{\sqrt{2}}[$, we have $a \leq 0, b < 0$ thus it is always true that $at + b < 0$. □

Proof of Proposition 3. Assume $t > 0$. We show that, when δ is sufficiently low, the optimal group is the whole society. Assume $\delta = 0$. Suppose that the principal contracts with a set S of agents of cardinality s. Then the principal proposes each an homogeneous transfer $\frac{t}{s}$. From binding participation constraints, we get $x_i = 1+\sqrt{\frac{2t}{s}}$ for all $i \in S$, and the optimal aggregate effort is equal to $n + \sqrt{2st}$. This quantity being increasing in s, the principal finds profitable to contract with the whole society when $\delta = 0$ and $t > 0$. By continuity on parameter δ, the result follows for low enough intensities of interaction.

We show that when δ is high enough, the optimal group is a strict subgroup of the society. It is sufficient to show that targeting a unique agent is better than targeting the whole society. Indeed, by selecting a unique agent i, it is easily shown that the principal induces an increase in aggregate effort equal to

$$\frac{b_i}{m_{ii}} \cdot \frac{\sqrt{(m_{ii}-1)^2 b_i^2 + 2m_{ii}(2-m_{ii})\ t} + (m_{ii}-1)b_i}{2-m_{ii}} \tag{14}$$

Then observe that effort goes to infinity as soon as δ tends to δ_c (which is the intensity of interaction such that $m_{ii} = 2$). At this value δ_c, the increase in aggregate effort, when the principal contracts with the whole society, is finite and equal to $\sqrt{2t}\|\mathbf{B}\|$. □

Proof of Proposition 4. Fix $\delta > 0$. With a null budget, the principal cannot modify effort when dealing with the whole society, as shown in equation (10). In contrast, with $t = 0$, dealing with a single agent i allows the principal to strictly increase aggregate effort, as shown in equation (14) by setting $t = 0$. □

Proof of Proposition 5. Consider any set S selected by the principal. By symmetry of the problem, the pairs effort - transfer are identical for each agent in S. We let x denote the optimal effort of any agent in S. As well, we let y be the representative effort of any non-contracting agent in $N \setminus S$. Since non-contracting agents play a best-response, taking as given the effort of contracting agents, we get $y = \frac{1+\delta sx}{1-\delta(n-s-1)}$. Hence, the principal maximizes $P(s,x) = sx + (n-s)\frac{1+\delta sx}{1-\delta(n-s-1)} = \frac{(1+\delta)s\ x(s)+n-s}{1-\delta(n-s-1)}$ over individual effort $x \geq 0$ under the participation constraint:

$$x - \frac{1}{2}x^2 + \delta x(s-1)x + \delta x(n-s)\frac{1+\delta sx}{1-\delta(n-s-1)} + \frac{t}{s} = \frac{1}{2}\Big(\frac{1+\delta(s-1)x}{1-\delta(n-s)}\Big)^2$$

The principal has to determine the optimal group size s^*. To solve this problem, the principal has to find the maximal nonnegative number $x(s)$ solving the participation constraint, and then determine the number s^* maximizing $P(s, x(s))$. The optimal effort $x(s)$ solves a second-order polynomial equation, and it is expressed as $x(s) = \frac{-B-\sqrt{B^2-4AC}}{2A}$, where

$$\begin{cases} A = -\frac{1}{2} + \delta(s-1) + \delta^2\frac{s(n-s)}{1-\delta(n-s-1)} - \frac{\delta^2}{2}\frac{(s-1)^2}{(1-\delta(n-s))^2} \\ B = 1 + \delta\frac{n-s}{1-\delta(n-s-1)} - \delta\frac{s-1}{(1-\delta(n-s))^2} \\ C = \frac{t}{s} - \frac{1}{2(1-\delta(n-s))^2} \end{cases}$$

The solution is well-defined when δ is sufficiently low so that the inequality $A < 0$ holds (which ensures convexity of the participation constraint). □

8 Appendix B: Contingent contracts

In this appendix, we explore contingent contracts in more details.

Contingent contracting allows the principal to extract the full surplus from agents. A set of contingent contracts $(x_i, t_i)_{i \in N}$ is defined here as a collection of individual take-it-or-leave-it offers made to all members of the society simultaneously. Hence, when one agent rejects the offer, no contract is executed; each agent i exerts the effort corresponding to the

Nash equilibrium played in the absence of contracts, i.e. b_i, and obtains a utility level equal to $\frac{1}{2}b_i^2$.[12] This means that reservation utilities are exogenous to offered contracts.

Since reservation utilities are exogenous, optimal group targeting is not an issue here, i.e. the principal always finds it optimal to propose contracts to every agent in the society. Agent i's participation constraint is given by:

$$\frac{1}{2}b_i^2 \leq x_i - \frac{x_i^2}{2} + \delta \sum_{j\in N} g_{ij}x_ix_j + t_i$$

We characterize the optimal contingent contracts. Setting $X = X_N$ for convenience, and taking care that both the budget constraint and participation constraints are binding at optimum, the principal's program is written:

$$\max_{\{(x_i,t_i)\}_{i\in N}} \quad \sum_{i\in N} x_i$$

$$\text{s.t.} \begin{cases} \frac{b_i^2}{2} = x_i - \frac{x_i^2}{2} + \delta \sum_{j\in N} g_{ij}x_ix_j + t_i, \ \forall i \in N \\ \sum_{i\in N} t_i = t \end{cases}$$

For convenience, we write $\mathbf{B}' = \mathbf{B}(\mathbf{G}, 2\delta)$ and $b' = b(\mathbf{G}, 2\delta)$. We define $\kappa(t) = \sqrt{1 + \frac{2t - \|\mathbf{B}\|^2}{b'}}$.[13] We let $\{(\tilde{x}_i, \tilde{t}_i)\}_{i\in N}$ be the set of optimal contingent contracts. We obtain:

Proposition 6. *The optimal contingent contract is written for all $i \in N$:*

$$\begin{cases} \tilde{x}_i = \big(1 + \kappa(t)\big)b'_i \\ \tilde{t}_i = \frac{1}{2}\Big[b_i^2 + (\kappa(t)^2 - 1)b'_i\Big] \end{cases}$$

[12] It is assumed here that the principal cannot threaten agents with a utility level below that obtained in the absence of contracts.

[13] The member under the square root is positive. Indeed, the positiveness is equivalent to budget t being larger than the difference between aggregate initial equilibrium utilities and aggregate utilities of the efficient allocation in the absence of contracts. This latter difference is negative and the budget is nonnegative.

Proof of Proposition 6. We first suppose that both participation constraints and the budget constraint are binding at optimum, and second, we check that these constraints are binding.

The reservation utility of every agent k is exogenous to contracts and equal to $\frac{b_k^2}{2}$, and the sum of all reservation utilities is thus equal to $\frac{\|\mathbf{B}\|^2}{2}$. The derivative of the Lagrangian with respect to x_i entails $x_i - 2\delta \sum_{j \in N} g_{ij} x_j = 1 + \frac{1}{\lambda}$. Recalling that $b'_i = b_i(\mathbf{G}, 2\delta)$ (this centrality are well-defined as $2\delta < \mu(\mathbf{G})$), we get $x_i = \left(1 + \frac{1}{\lambda}\right) b'_i$. Agent i's binding participation constraint is written $t_i = \frac{b_i^2}{2} + \frac{1}{2}\left(x_i - 2\delta \sum_{j \in N} g_{ij} x_j - 2\right) x_i$. Plugging effort in this latter equation, we get $t_i = \frac{b_i^2}{2} + \frac{1}{2}\left(\frac{1}{\lambda} - 1\right)\left(1 + \frac{1}{\lambda}\right) b'_i$. Summing transfers over all agents and remembering that the budget constraint is binding, we obtain $2t = \|\mathbf{B}\|^2 + \sum_i \left(\frac{1}{\lambda} - 1\right)\left(1 + \frac{1}{\lambda}\right) b'_i$. Rearranging, we obtain $\lambda = \sqrt{\frac{b'}{2t + b' - \|\mathbf{B}\|^2}}$, and we are done.

To finish, we note that the objective of the planner, $\tilde{x}$, is increasing in the budget. This implies that all constraints are binding. □

Optimal contingent effort is always well-defined. Effort increases with Bonacich centrality of decay parameter equal to 2δ; that is, it takes into account both received and generated externalities. Agent i's optimal effort can be decomposed into a zero-budget component and a pure budget component:

$$\tilde{x}_i = \underbrace{\left(1 + \sqrt{1 - \frac{\|\mathbf{B}\|^2}{b'}}\right) b'_i}_{\text{zero-budget component}} + \underbrace{\left(\kappa(t) - \sqrt{1 - \frac{\|\mathbf{B}\|^2}{b'}}\right) b'_i}_{\text{pure budget component}}$$

With null budget, the optimal effort $\tilde{x}_i$ is still enhanced and proportional to the Bonacich centrality b'_i. This is in sharp contrast with bilateral contracts, where the principal cannot increase effort when $t = 0$.

The relationship between transfer and centrality, or effort, is not monotonic, and budget matters. for sufficiently large budget, i.e. for $t > \frac{\|\mathbf{B}\|^2}{2}$, transfer increases with centralities b_i and b'_i. However, for $t < \frac{\|\mathbf{B}\|^2}{2}$, i.e. when the budget does not cover the sum of utilities in the absence of contracts, the relationship between transfer and centralities is ambiguous: it increases with b_i^2 and decreases with b'_i. Furthermore, the principal can even tax the agents with the smallest indexes $\frac{b_i^2}{b'_i}$. Precisely an agent is taxed whenever $\frac{b_i^2}{b'_i} < \frac{b'}{\|\mathbf{B}\|^2 - 2t}$. In general, the emergence of taxation depends on the network structure. Taxation is more likely to occur under high dispersion in Bonacich centralities. Moreover, as budget t tends to zero, there is always a taxed agent on the network, for all positive intensities of interaction (except on a regular network where all centralities are identical).

Finally, the performance of the aggregate optimal effort is measured by $\tilde{x} = b' + \sqrt{b'(2t + b' - \|\mathbf{B}\|^2)}$. It is easily shown that the aggregate optimal effort $\tilde{x}$, as well as the gap between contingent and bilateral contract, $\tilde{x} - \hat{x}$, increase with link addition, with intensity of interaction δ, and with budget level.

9 Appendix C: Economic applications

In this appendix, we show that our model fits with three economic applications: a monopolist setting discounts in presence of interdependent consumers, a firm's owner distributing a bonus to workers, and a public funding provider subsidizing an R&D network. For the sake of simplicity, in all of these applications, the principal is assumed to contract with the whole society.

Monopoly pricing with discounts under interdependent consumers. We consider a model à la Candogan et al (2012), where consumers' gross utilities from consuming q_i units of the good are given by:

$$u_i(q_i, x_{-i}) = q_i - \frac{1}{2}q_i^2 + \delta\sum_{j\in N} g_{ij}q_iq_j$$

The monopolist incurs a constant marginal production cost $c > 0$. For the sake of simplicity, we assume that it is optimal for the monopolist to enter into contracts with positive quantities sold to all consumers. This requires production costs to be sufficiently low.

The monopolist proposes to each consumer i a fixed homogeneous unit price p, and a discount d_i. We consider p to be already set by the monopolist, and focus on discounts. A contract is a pair (q_i, d_i). Agent i is charged $pq_i - d_i$ if she accepts the offer, otherwise she is charged pq_i^{BR}, where $q_i^{BR} = 1 + \delta \sum_{j\in N} g_{ij}q_j$ is the quantity purchased by agent i under offer rejection.

The program of the monopolist is given by:

$$\max_{\{(q_i,d_i)\}_{i\in N}} \quad (p-c)\sum_{i\in N} q_i - \sum_{i\in N} d_i \tag{15}$$

$$\text{s.t. } u_i(q_i^{BR}, q_{-i}) - pq_i^{BR} \leq u_i(q_i, q_{-i}) - pq_i + d_i, \ \ \forall i \in N$$

This problem can be solved in two steps: first, find for a given total discount d the optimal offers $\hat{\mathbf{Q}}(d), \hat{\mathbf{D}}(d)$ under the constraint that $d = \sum_i d_i$, and then determine the optimal total discount. The first step gives:

$$\max_{\{(q_i,d_i)\}_{i\in N}} \quad \sum_{i\in N} q_i$$

$$\text{s.t.} \begin{cases} u_i(q_i^{BR}, q_{-i}) - pq_i^{BR} = u_i(q_i, q_{-i}) - pq_i + d_i, \ \ \forall i \in N \\ \sum_i d_i = d \end{cases}$$

This problem replicates subprogram $(\mathcal{P})$.

Company bonus distribution. We enrich the classic team production model (Holmstrom [1982]), where the firm is composed of n workers and an owner distinct from the workers, by adding a network aspect as follows.

Workers are organized in a network representing local complementarities. In particular, a worker's effort generates quadratic costs, but synergies with neighbors help reduce effort cost[14] as follows:

$$c_i(x_i, x_{-i}) = \frac{x_i^2}{2} - \delta\Big(\sum_{j\in N} g_{ij} x_i x_j\Big)$$

The larger the sum of the effort by agent i's neighbors, the lower the cost. Furthermore, the cost function shows complementarities in neighbors' effort: for a fixed level of neighbors' effort, a higher level of own effort entails a larger impact by neighbors' effort on own effort cost. Workers are paid by the firm's owner a wage given by $v + x_i$, $v \geq 0$, where v is a fixed common fee, and x_i is a linear compensation normalized to unity in individual output - which is assumed for simplicity to exactly reflect effort.[15] Our model allows the firm's owner, on top of initial wages, to give each agent i an individual reward t_i conditional on effort level. The owner wants to maximize output minus wages, and the budget is endogenous to workers' effort. In the simple setup where the firm's output depends on the sum of effort, the owner's program is written:

$$\max_{\{(x_i,t_i)\}_{i\in N}} \quad F\Big(\sum_{i\in N} x_i\Big) - \sum_{i\in N} t_i$$

$$\text{s.t.} \begin{cases} \sum_{i\in N} t_i \leq t \\ \forall i \in N, v + \frac{1}{2}\big(1 + \delta \sum_{j\in N} g_{ij} x_j\big)^2 \leq v + x_i + t_i - c_i(x_i, x_{-i}) \end{cases}$$

Again, the subprogram with fixed budget replicates subprogram $(\mathcal{P})$.

Research activity and science parks. The world of research is a world of synergies. A huge literature documents the role played by collaborative research among firms or academics (see Goyal and Van der Leij (2006) for an

[14] See Mas and Moretti (2009) for some empirical evidence of local synergies at the workplace.

[15] The linear compensation is not linked to the firm's profits. When a firm's profits are uncertain, linear compensation constitutes a kind of insurance against that risk. The value of the firm's output is an increasing function of the sum of effort by employees. Due to production costs, the value function is usually concave.

empirical analysis of the properties of the networks of collaboration among academics, and Hagedoorn (2002) for a description of R&D networks among firms). As in Goyal and Moraga (2001), consider independent markets with linear demand $d-p$, $d>0$, and x_i as firm i's R&D effort. There is no fixed cost, and marginal costs are related to partners' effort levels through the relation $c_i = c - x_i - \sum_j g_{ij}x_j$. The Cournot equilibrium profit is therefore written

$$u_i(\mathbf{X}) = (d - c + x_i + \sum_j g_{ij}x_j)^2 - \gamma x_i^2$$

where γx_i^2 is the cost of R&D effort and c the constant marginal production cost, $\gamma > 1$. Define for convenience $z_i = \sum\limits_{j \in N} g_{ij}x_j$. This profit function corresponds to a modified version of utility (**??**), where agent i's utility is of type $u_i(x_i, z_i) + v_i(z_i)$, with $v_i()$ a non-decreasing function. In this context, the principal can be a public institution, like a regional institution, a state or a union of states, seeking to foster research. The public funding provider may be interested in maximizing the sum of R&D effort.[16] Formally, the public institution's program is written:

$$\max_{\{(x_i,t_i)\}_{i\in N}} \sum_{i\in N} x_i$$

$$\text{s.t.} \begin{cases} \sum\limits_{i\in N} t_i \leq t \\ \forall i \in N, \frac{1}{\gamma-1}\big(d-c+z_i\big)^2 \leq (d-c+x_i+z_i)^2 - \gamma x_i^2 + t_i \end{cases}$$

Note that, for a given total budget t, and denoting $x_i^{BR} = \frac{d-c+z_i}{\gamma-1}$, agent i's participation constraint is written $(\gamma-1)(x_i - x_i^{BR})^2 \leq t_i$. The above program therefore replicates subprogram $(\mathcal{P})$.

[16]Minimizing the sum of industry costs may also be desirable. In that case, the objective is a weighted sum of effort where weights are proportional to the agent's degree. Our analysis can easily be extended to this case.

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