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Male Reproductive Health, Fairness and Optimal Policies

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Abstract

Based on epidemiological evidence, we consider an economy where agents differ through their ability to procreate. Households with impaired fertility may incur health expenditures to increase their chances of parenthood. This health heterogeneity generates welfare inequalities that deserve to be ruled out. We explore three different criteria of social evaluation in the long-run: the utilitarian approach, which considers the well-being of all households, the ex-ante egalitarian criterion, which considers the expected well-being of the worst-off social group, and the ex-post egalitarian one, which only considers the realized well-being of the worst-off. In an overlapping generations model, we propose a set of economic instruments to decentralize each solution. To correct for the externality and inequalities, both a preventive (a taxation of capital) and a redistributive policy are required.

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1 Introduction

The state of male reproductive health has been one of the main debates in medical sciences in recent years. The evidence being the growing number of descriptive studies that support the epidemiological literature from a functional perspective, using relatively easily collectible biomarkers of semen quality (like concentration, volume, number, motility and morphology). As an illustration, the seminal paper by Carlsen et al. (1992) gave birth to the famous "falling sperm counts" story and has been the object of many reappraisals henceforth (Joffe, 2010). In their meta analysis, the authors review 61 studies published between 1938 and 1991 and conclude that mean sperm concentration had fallen from 113 to 66 million/ml over the period. Meanwhile it has met with skepticism on grounds of laboratory methods, statistical issues, selection and the like, it constitutes the starting point of a whole strand of the literature that establishes a global declining quality of the spermatogenesis, at least in some places (Auger et al., 1995; Van Waeleghem et al., 1996; Irvine et al., 1996; Swan et al., 2000). An even more recent paper that goes in the same direction has struck a resounding chord within the scientific community and beyond, within western societies (Levine et al., 2017). In their large-scale study, which involves data from 185 studies and 42,000 men around the world between 1973 and 2011, the authors show that the quality of sperm in North America, Europe and Australia has dramatically declined with a 52.4 % drop in sperm concentration. In China too, this health issue has gained an increasing interest as shown by Huang et al. (2017) who confirm the declining semen quality for men in the Hunan province. To illustrate our argument, we report in Figure 1 some trends.

All these compelling evidence raise concerns that men semen quality could fall below some threshold levels that could impact fecundity, since those biomarkers seem to be suitable indicators for the chances to fatherhood. In addition and crucial to our analysis, all these studies that point out a rapid change in men reproductive health cover periods of fast economic development. Hence, we argue that post-industrial societies have created the potential for increasing the exposure to specific lifestyle factors that might impair men reproductive health. Among them, one can identify pollution that might contribute to explain the current worldwide impaired male fertility and came along with the development process. We may find in the epidemiological literature many studies
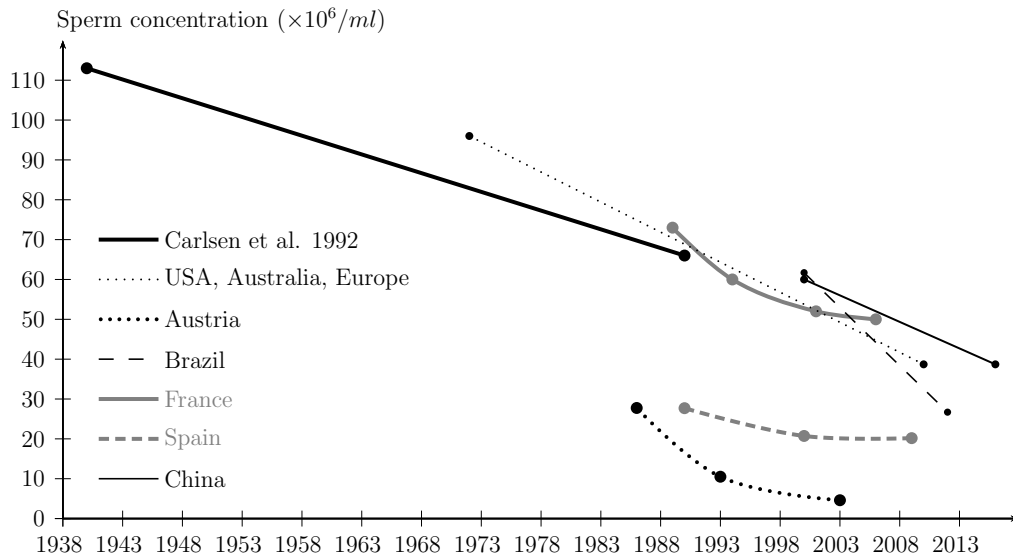
Figure 1: Evolution of sperm concentration. Data from Huang et al., 2017; Borges et al., 2015; Romero-Otero et al., 2015; Rolland et al., 2013; Carlsen et al., 1992; Levine et al., 2017; Lackner et al., 2005.

to support our view and that have long suggested adverse effects of exposure to environmental contaminants, such as persistent organic pollutants (POPs), on human reproductive health. [1]

Fatherhood can be viewed as a natural and a logical progression in the life-cycle and we should be concerned with this potential deprivation of fundamental well-being.[2] To satisfy their desire of parenthood, some agents may be compelled to use some costly Assisted Reproductive Treatments (ARTs). To that extent, we may emphasize the joint evolution of both the recent increasing use in ARTs and the decline in male reproductive health. For instance, since 2000, ART services annually grows by 5%-10% in developed countries and over 17 European countries that fully reported their ART activities in 2011, ART babies represented 2,4% of all infants born (Kupka et al., 2016). This reduced ability

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[1]See for instance De Rosa et al., 2003; Martenies and Perry, 2013; Zhou et al., 2014; Meeker et al., 2008; Tuc et al., 2007; Recio-Vega et al., 2008; Swan et al., 2003; Aneck-Hahn et al., 2007; Perry et al., 2011; Mehrpour et al., 2014 or Chiu et al., 2015.

[2]The distress associated with sub-fertility or treatments of infertility causes induces substantial socio psychological costs, like a severe degradation of self-esteem, syndromes of depression, loss of gender identity, self-assessed social pressure from families, friendships etc. (Greil, 1997; Moura-Ramos et al., 2012).

to conceive children may be costly to overcome for the society. As shown by Chambers et al. (2009), the use of ARTs represents substantial out-of-pocket health expenses. The estimated cost of a standard In Vitro Fecundation (IVF) cycle ranges from 28% of Gross National Income (GNI) *per capita* in the United States to 10% of GNI *per capita* in Japan. Moreover, before any public policy, the gross cost of a standard IVF cycle ranges from 50% of an individual's annual disposable income in the U.S., approximately 20% in the UK, Scandinavian countries and Australia, to 12% in Japan. In the same vein, Conolly et al. (2010) provide us with estimated direct costs for one fresh ART cycle going from more than 2000 euros in Belgium to around 4000 euros in G-B and more than 10000 euros in the US.

In our paper, we propose to explore the welfare inequalities induced by a health heterogeneity since impaired reproductive health entails a loss of utility. What matters to us is the unfair feature of such a health heterogeneity because agents are not responsible *a priori* for their reproductive health status. This is true if we think, for instance, to pollution exposure. We consider an overlapping generations model where the development process generates a health heterogeneity: Two types of men coexist within one generation, the fertile and infertile ones. Moreover, men with impaired fertility may incur health treatments in order to increase their chances of parenthood - if the income is sufficiently high. Hence, the decentralized economy is characterized by one externality.

In this setup, we claim that it is worth to examine what kind of stationary equilibrium would be selected by a social planner in comparison with the *laissez-faire* economy. At first, we consider the usual utilitarian social objective, where the sum of individuals' utility is maximized. But because the prevailing health heterogeneity is not the result of any actions held by agents but rather due to circumstances (let us refer for instance to pollution exposure), we also explore alternative criteria of social evaluation according to Fleurbaey (2008), Ponthiere (2016) or Fleurbaey et al. (2018). More precisely, we consider an inequality averse social planner who either maximizes the expected long term well-being of the worst-off (ex-ante egalitarian social criteria) or maximizes the long term realized well-being of the worst-off (ex-post egalitarian social criteria).

Our results drive us to formulate some policy recommendations, depending on the social criterion that is selected. To rule out the inefficiencies, capital accumulation should be taxed. We argue that this preventive policy should be favored and come along with redistribution. Again, with regards to our example

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of pollution, we claim that fighting against pollution is a relevant action since it controls the source of an altered reproductive health. In addition, and face to a risk on the success of fertility treatments, we recommend to implement a curative health policy that does not directly create incentives to invest in fertility treatments. Even more, an inequality adverse decision maker is lead to tax health expenditures but to compensate the Infertile for the loss of utility by allowing them through augmented life-time consumption levels.

Finally, our paper adds a contribution to the economic literature that investigates the prevalence of childlessness within western economies, be it voluntarily or involuntarily (Gobbi, 2013; Baudin et al., 2015) although we focus our analysis on the involuntarily motive for childlessness and the desire to father offspring which might be unsatisfied. To that extent, our paper is closer to Momota (2016) who also introduces heterogeneity among households due to the ability of having children. However, his concern is drastically different to ours: He is interested in the effects of exogenous population growth on the level of capital accumulation, whereas in our framework, population growth is endogenous. The paper also refers to the literature that explores fairness issues in the presence of health inequalities. Although most papers deals with life expectancy (Fleurbaey et al., 2014, 2016; Ponthiere, 2016), we consider an alternative health dimension which is nonetheless fundamental, that is the reproductive health status.

The paper is organized as follows: Following the Introduction, Section 2 presents the set-up of the model and Section 3 the *laissez-faire* equilibrium. Section 4 investigates the utilitarian optimal stationary solution and presents a set of optimal tools in order to decentralize it. Section 5 discusses alternative criteria of social evaluation that take into account the prevailing health inequalities. Finally Section 6 concludes. Technical proofs are relegated to Appendices.

2   The Model

Let us consider a two-period overlapping generations model. During the first period, the adulthood, households work, consume, save and procreate. Each household consists of at least one man, whose health status determines the type of the household. At each date t, two types of household coexist depending on their ability to conceive children: We distinguish Fertile households (denoted by superscript F) and Infertile ones (denoted by superscript I). Even

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though individuals' choices of consumption do not directly interfere with their health determinants, in case of infertility, households can engage in medical treatments in order to improve their ability to father offspring. These fertility treatments include different types of procedures, going from basic hormonal remedies to much more sophisticated methods of assisted reproduction treatments (ART) like in vitro fertilization (IVF). They obviously aim at augmenting the chances of parenthood so that with a probability q, initially infertile couples may eventually have children. During the second period of life, households retire and consume their savings.

Demography. The proportion of fertile households within the population is denoted by π_t (the proportion of infertile ones equals $(1-\pi_t)$). This probability to be fertile is randomly distributed among the population. The total number of children born at date t equals the number of children of fertile households ($N_t n \pi_t$) plus the number of children of successfully treated infertile households ($N_t n (1-\pi_t) q_t$), where n denotes the exogenous number of children each couple may have and N_t is the size of the adult generation at date t, that is the labor force. Hence, the population evolves overtime according to

$$N_{t+1} = N_t \times n \times \left[\pi_t + (1-\pi_t) q_t \right] \tag{1}$$

Households. Households derive utility from current (c_t^i) and future consumption (d_t^i) as well as parenthood (v). They do not choose the number of children they have, although they might suffer from not being able to procreate. We do not aim at investigating fertility behaviors *per se* but consider that there is an average exogenous targeted level of fertility within the economy (or a desired number of children). This targeted level of fertility could correspond, for instance, to long-term fertility rates observed in economies which have already achieved their demographic transition and which experience now quite stable fertility rates. Since households face a health risk linked to their ability to conceive, their preferences are represented by an expected utility function, so that

$$EU^i(c_t^i, d_t^i, v) = \begin{cases} u(c_t^F) + \delta u(d_{t+1}^F) + v & \text{if } i = F \\ u(c_t^I) + \delta u(d_{t+1}^I) + q_t v & \text{if } i = I \end{cases} \tag{2}$$

Notice that the utility of parenthood, v, is constant since the number of children is exogenous. For the sake of simplicity, we consider the following specifications. On the one hand, the utility function is given by $u(z_t) = \ln z_t$. On the other hand, the probability for a treatment to be successful writes: $q(x_t) =$

$\frac{ax_t}{1+x_t}$, with $a \leq 1$ and x_t denotes the level of health care expenditures. The parameter a merely accounts for the efficiency of the available medical technology or, equivalently, the level of scientific medical knowledge.

Let us now present the budget constraints faced by households. For ease of presentation, we right now introduce a set of policy instruments that we will be useful to decentralize any optimal allocation later on. Hence, during adulthood, each household is endowed with one unit of labor inelastically offered to firms for which they receive the prevailing competitive wage, w_t.[3] In addition, they might benefit from differentiated lump-sum transfers, T_t^i. The net total income can be shared among current consumption, savings, s_t^i, and possibly for infertile households, health care expenditure. During retirement, each couple consumes the net income which equals the revenue from savings minus a transfer, θ_{t+1}. For both types of household, the first period budget constraint can be expressed as follows:

$$c_t^F + s_t^F = w_t + T_t^F \tag{3}$$

$$c_t^I + s_t^I + (1 - \sigma_t)x_t = w_t + T_t^I, \tag{4}$$

where σ_t is the health policy instrument. The second period budget constraints write:

$$d_{t+1}^i = R_{t+1}s_t^i - \theta_{t+1}, \text{ for } i = F, I \tag{5}$$

where we assume a complete depreciation of capital and we define $R_{t+1} \equiv (1 - \rho_{t+1})r_{t+1}$, with ρ_t a proportional tax on capital income.

Government. At each date t, the government provides transfers to the young generation and can subsidy health expenditure thanks to a collected tax on capital and a lump-sum tax on the old. The balanced budget constraint of the government is given by:

$$N_{t-1}[\theta_t + \rho_t r_t(\pi_{t-1}s_{t-1}^F + (1-\pi_{t-1})s_{t-1}^I)] = N_t(\pi_t T_t^F + (1-\pi_t)T_t^I) + N_t\sigma_t(1-\pi_t)x_t \tag{6}$$

Firms. One good is produced using both physical capital, K_t, and labor, L_t. We can immediately define *per capita* variables: $y_t = Y_t/L_t$, $k_t = K_t/L_t$. Again,

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[3]Notice that here the health status does not affect productivity at work. Also, to keep the analysis as simple as possible, we do not introduce rearing cost of children. Since the number of offspring is assumed to be constant, enriching the analysis with such a cost will not drastically alter our analysis.

in order to obtain tractable results, we assume a fairly standard Cobb-Douglas production function:

$$y_t = f(k_t) = k_t^\alpha \tag{7}$$

with $0 < \alpha < 1/2$. Being given the price of capital (r_t) and the competitive wage (w_t), the optimization program of firms yields:

$$r_t = \alpha k_t^{\alpha-1} \tag{8}$$
$$w_t = (1-\alpha) k_t^\alpha \tag{9}$$

Reproductive health. As documented in Introduction, the development process generates a harmful externality that negatively affects the probability of being fertile. Since GDP per capita is a well established measure of development and it increases with capital per capita, we state that $\pi_t = \pi(k_t)$ and $\pi'(k) \leqslant 0$ so that, as one economy develops, the reproductive health declines. We explicit in further details our hypothesis about this endogenous probability of being fertile in Assumption 1:

Assumption 1 *We assume that π is sufficiently close to 1 and $\epsilon_\pi \equiv \frac{\pi'(k)k}{\pi(k)}$ is close to 0 with $\pi'(k) \leqslant 0 \leqslant \pi''(k)$, $\pi(0) = \pi_0 > \pi(+\infty) > 0$.*

This assumption means that the chances of parenthood are weakly decreasing with the stock of capital and sufficiently close to 1. If we interpret the relationship between π and k as the effect of pollution on male reproductive health, Assumption 1 seems in accordance with the empirical literature.

Within this framework, we can now analyze the static choices made by households and then investigate the long-run behavior of this economy.

3   The decentralized economy

This section defines the inter-temporal equilibrium in the *laissez-faire* economy, the levels of policy instruments, T_t^i, σ_t, θ_{t+1} and ρ_{t+1}, being set to zero. The equilibrium, given the variables from the previous period, can be defined by a wage rate w_t and a gross rate of return R_t, aggregate variables K_t, L_t and Y_t and individuals variables, c_t^F, s_t^F, c_t^I, s_t^I and x_t.

Households' choices. Households maximize their expected utility (2) under the budget constraints (3)-(5) and a positivity constraint, $x_t \geq 0$. Fertile households do not expand in health and we can easily deduce their optimal level of saving, which is increasing with labor income:

$$s_t^F = \frac{\delta}{1+\delta} w_t \equiv s^F(k_t) \tag{10}$$

As for the Infertile, let us note that if $x_t = 0$, then

$$s_t^F \leq \frac{\delta}{av} \tag{11}$$

Importantly, this inequality implies that the loss of utility from a low level of consumption dominates the potential welfare gain associated with an improved reproductive health. Yet, as the utility function defined over consumption is concave, this inequality is verified all the more that consumption levels are initially low. Then, we can state that for low incomes, it is more likely that households do not invest in fertility treatments. In that configuration, $s_t^I = s_t^F$.

If equation (11) is not satisfied, $x_t > 0$ and, using equation (10), we get the optimal savings and health expenditures for infertile households that can also be expressed as functions of k_t:

$$s_t^I = s^F(k_t) - \frac{\delta}{1+\delta} x_t \tag{12}$$

$$(1 + x_t)^2 = \frac{av}{\delta} s_t^I \tag{13}$$

Solving the system (12)-(13), we deduce the expression of x_t:

$$x_t \equiv x(k_t) = \sqrt{\frac{av(1-\alpha)}{1+\delta} k_t^\alpha - 1 + A^2} - A, \ \text{with} \ A \equiv 1 + \frac{av}{2(1+\delta)}, \tag{14}$$

such that $x'(k_t) > 0$ and

$$s_t^I \equiv s^I(k_t) = \frac{\delta(1 + x(k_t))^2}{av}. \tag{15}$$

Labor market. On the labor market at date t, the supply of labor N_t being inelastic and the demand L_t being the solution to equation (9), we get that:

$$L_t = N_t = N_{t-1} \times n \times \left[\pi(k_{t-1}) + (1 - \pi(k_{t-1})) q(x(k_{t-1})) \right] \tag{16}$$

Capital market. The clearing condition on the capital market entails that the supply of savings by young individuals equals the capital used by firms:

$$K_{t+1} = N_t \left[\pi(k_t) s^F(k_t) + (1 - \pi(k_t)) s^I(k_t) \right] \tag{17}$$

Using equation (16), we derive the dynamics of capital-labor ratio:

$$k_{t+1} = \frac{\pi(k_t) s^F(k_t) + (1 - \pi(k_t)) s^I(k_t)}{n \Gamma(x(k_t), k_t)}, \tag{18}$$

where $\Gamma(x(k_t), k_t) = \left[\pi(k_t) + (1 - \pi(k_t)) q(x(k_t)) \right]$ and $n \Gamma(x(k_t), k_t)$ is the endogenous growth factor of the young population. The accumulation of physical capital induces two opposite effects on population growth: A negative direct one through the increase in the number of infertile households and an indirect positive one through health expenditure. When the negative effect dominates, we argue that physical capital accumulation entails a negative dilution effect[4], otherwise a positive one. In addition, physical capital accumulation impacts global savings, through three channels: i) it affects the distribution of infertile and fertile households within the population; ii) it increases savings for each type of household; iii) it triggers more health investments and thus involves an eviction effect on infertile savings. Therefore, the global dynamics of the economy depends on the magnitude of each mechanism.

Given $k_0 = K_0/L_0 \geq 0$, the inter-temporal equilibrium is a sequence $\{k_t\}$ that satisfies conditions (18) for all $t \geq 0$. A steady state with $x > 0$, if it exists, is a solution k that solves the above dynamic system (18) evaluated at the steady state so that $k_t = k_{t+1} = k$. The existence and uniqueness of such a steady state is shown in the following proposition.

Proposition 1 *Under Assumption 1 and if v sufficiently large, so that*

$$v > \tilde{v} \equiv \left[\frac{1 + \delta}{a(1 - \alpha)} \right] \times \left[\frac{(1 - \alpha)\delta}{(1 + \delta) n \pi(0)} \right]^{-\frac{\alpha}{1 - \alpha}},$$

there exists a unique steady state, k, such that health expenditure is strictly positive ($x > 0$).

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[4]Recall that the dilution effect corresponds to a decrease of *per capita* variables following an increase in the labor force or equivalently in the population growth.

Proof. See Appendix A. ∎

We show that, in the long run, the economy reaches a stationary state where infertile households do engage in health expenditures, like ARTs, if the benefit of parenthood is large enough. Indeed, investing in fertility treatments to augment the chances of parenthood is costly and induces an eviction effect on savings. Nevertheless, by increasing the share of procreating households, the demographic growth is boosted.

4   The classical utilitarian welfare analysis

The negative externality of production on the ability to naturally conceive children implies a pure loss of utility for infertile households. But beyond this, it also creates several sources of inefficiency: More physical capital means less population and more Infertile. In addition, since households may expand in health, some resources are sacrificed, which is detrimental to savings. Simultaneously, these expenditures also influence the population growth factor. Finally, because of the OLG setup, the intergenerational allocation at the equilibrium can be sub-optimal. In this set-up, we aim at exploring what would be a first-best optimal allocation. To that end, we propose a welfare analysis using a classical utilitarian criterion of social evaluation and we characterize the optimal solution. Then, we provide some policy recommendations to correct for the inefficiencies and discuss the nature of the economic policy. We may also explore welfare issues in the light of prevailing inequalities.

4.1   The utilitarian social optimum

The utilitarian social planner maximizes a social welfare objective that takes into account the sum of individuals' preferences, SW^U. In this context of endogenous population, we assume that the she does not grant any particular weight to the overall size of the population but rather the average level of utility is maximized. In order to derive clear cut results and compare them with the *laissez-faire* economy, we focus our analysis on the stationary solution. To comply with her goal, the social planner chooses the optimal levels of consumption (c^F, c^I, d^F, d^I), health expenditures (x) and physical capital (k), under the two constraints of resources and positivity for health care expenditure. The program of the utilitarian central planner can thus be written as follows:

$$
\begin{cases}
\max_{c^F,c^I,d^F,d^I,x,k} & SW^U \equiv \pi(k)\left[\ln c^F + \delta \ln d^F + v\right] \\
& +(1-\pi(k))\left[\ln c^I + \delta \ln d^I + \frac{ax}{1+x}v\right] \\
s.\,t. & k^\alpha \geq \pi(k)c^F + (1-\pi(k))(c^I + x) \\
& + \frac{\pi(k)d^F + (1-\pi(k))d^I}{n\Gamma(x,k)} + nk\Gamma(x,k) \\
& x \geq 0
\end{cases}
$$

First of all, at the optimum, we can establish that consumption levels should be equalized among heterogeneous agents, $c^F = c^I = c^*, d^F = d^I = d^*$.[5] Then we already depart from the decentralized choices of consumption and we confirm that a policy should be implemented to reach the utilitarian solution. Second, we derive the optimal trade-off between young and old consumption over the life-cycle:

$$
\delta c^* = \frac{d^*}{n\Gamma(x^*,k^*)} \tag{19}
$$

The marginal rate of substitution between young and old consumption is equal to the optimal population growth factor, $n\Gamma(x^*,k^*)$. Third, we consider the trade-off between consumption and health:

$$
\frac{av}{(1+x^*)^2} + \frac{\delta a}{(1+x^*)^2 \Gamma(x^*,k^*)} = \frac{1}{c^*} + \frac{nk^*a}{c^*(1+x^*)^2} - \frac{\mu^*}{1-\pi(k^*)}, \tag{20}
$$

where μ is the Lagrange multiplier associated to the positivity constraint on x. We claim that there exists an interior optimal solution for health expenditure when the marginal social welfare gain from investing in health equals the marginal social welfare loss. On the one hand, the marginal social welfare gain consists in a pure utility gain from parenthood plus a reallocation of resources within generations, through the reduced weight granted to old households' consumption. On the other hand, the marginal social welfare loss is the foregone consumption added to the required increase in the productive investment. Finally, we deduce the trade-off between generations, which is given by:

$$
\begin{aligned}
\alpha(k^*)^{\alpha-1} =\ & n\Gamma(x^*,k^*) - \pi'(k^*)x^* \\
& + c^*\pi'(k^*)\left[\left(1 - \frac{ax^*}{1+x^*}\right)\left(\frac{nk^*}{c^*} - \frac{\delta}{\Gamma(x^*,k^*)} - v\right)\right]
\end{aligned} \tag{21}
$$

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[5]Superscripts * indicate the utilitarian optimal solution. See Appendix B for more details.

This condition is in fact a modified golden rule. If the probability to be fertile were constant, then we would have obtained a standard golden rule, except for the presence of the growth factor. As soon as the externality occurs, additional and potentially opposite effects arise. Keeping in mind that investment and production are related, more capital means less population, lowering the cost of productive investment. Nevertheless, more capital induces more infertile households and thus lowers the social welfare. This last negative effect mitigates the former incentives to accumulate physical capital.

Using these arbitrages, we can also study the existence and the properties of the utilitarian optimal allocation. In particular, although we do not know whether the optimal level of health expenditure is higher than the *laissez-faire* one, we can show that it should be strictly positive. This is stated in Proposition 2 below:

Proposition 2 *Under Assumption 1 and v sufficiently large, there exists a unique optimal allocation (x^*, k^*) with $x^* > 0$.*

Proof. See Appendix C. ∎

Once we have described the optimal utilitarian solution, we naturally wonder how to reach it in the private economy. To do so, we derive again the decentralized optimal choices for both types of household once policy instruments are enforced and we compare them with the centralized optimal solution. We can then discuss the optimal design of the public policy to be implemented, according to the preferences of the social planner.

4.2 The utilitarian optimal policy

Comparing the *laissez-faire* solution with the first-best optimum, one sees that the social optimum can be decentralized with appropriate choices of instruments. The following proposition summarizes our results.

Proposition 3 *Consider that Assumption 1 holds, δ small and v sufficiently large, the utilitarian social optimum can be decentralized by means of the following instruments:*

(i) *A tax on capital income, $\rho^* = 1 - \frac{n\Gamma^*}{\alpha(k^*)^{\alpha-1}} \in (0,1)$;*

(ii) *A tax on health expenditure, $\sigma^* < 0$, such that $\frac{\sigma^*}{1-\sigma^*} = \frac{1}{v}\left(\frac{\delta}{\Gamma^*} - \frac{nk^*}{c^*}\right) > 0$;*

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(iii) *Differentiated lump-sum transfers between the Fertile and the Infertile,
T^{F*} and T^{I*}, satisfying $T^{F*} = 0$ and $T^{I*} = (1 - \sigma^*)x^* > 0$;*

(iv) *A positive lump-sum tax on the old, $\theta^* > 0$, to balance the government budget.*

Proof. See Appendix D. ∎

The positiveness of θ^* involves a taxation of old households' consumption. This generates fiscal resources meanwhile it incites households to save more. This capital accumulation is the source of the externality that should be corrected. Hence, the government implements a proportional tax on capital income to ensure the achievement of the modified golden rule (see equation (21)). In addition, this capital accumulation generates more consumption and allows infertile households to afford for more and more fertility treatments. It reinforces the heterogeneity between the two groups. To enforce the optimal level of health investment, we show that a tax on health expenditures is required. Nevertheless, to guarantee that consumption are identical among the two types of household, positive transfers towards the young should be differentiated. This result might be at first sight surprising and counter-intuitive; However, let us re-examine the infertile households' budget constraint: $c^I + (1 - \sigma - T^I/x)x + s^I = w$. Taking into account the redistribution effect through T^I, the health policy design can be summarized by a unique variable $t(x) \equiv \sigma + T^I/x$. At the utilitarian optimum, we easily see that $t(x) > 0$.[6] Overall, infertile couples are well subsidized by the government.

To sum-up, the optimal public policy design includes redistribution and a tax on physical capital that should be interpreted as a preventive policy tool. But a public health policy that reduces the cost of ARTs is not relevant to reach the optimum. For instance, if we follow-up with our example of pollution, the authorities should favor the reduction of polluting emissions that alter male reproductive health and ensure that infertile households can derive sufficiently high expected utility all over their life-cycle, through both consumption and parenthood.

Using a classical utilitarian criterion, the instruments to decentralize the social optimum allow to reduce inequalities among the two social groups, through the equalization of consumption and the reduction of the externality. Nonetheless, the optimal policy does not eradicate the source of health inequality but

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[6]Since $\sigma < 0$ and $T^I > 0$, $t(x)$ is strictly decreasing and there exists $\hat{x} = -T^I/\sigma > 0$ such that $t(x) > 0$ for $x < \hat{x}$ and $t(x) < 0$ for $x > \hat{x}$.

may compensate for it by subsidizing the Infertile and enforcing an identical level of consumption. We can still be concerned with such an approach since the utility of the Fertile is larger than the expected utility of the Infertile. In addition, if we consider that the risk of fertility agents are exposed to is not the consequence of any action, but rather due to circumstances, then this health heterogeneity appears to be unfair. It is typically the case of pollution exposure for which agents can not be blamed *a priori*. In the following sections, we explore alternative criteria of social evaluation such that the social planner displays inequality aversion. We then assess the optimal design of the public policy that should be implemented in comparison with the utilitarian solution.

5 Inequality aversion criteria

The externality generates a distribution on the two types of household and this creates different outcome prospects. Even if for the social observer the ability to fatherhood is viewed as random, the fact that it is antecedent to the decision process suggests that it should be considered as circumstances. According to the "compensation principle" (see Fleurbaey, 2008; Ponthiere, 2016), such type of inequalities due to circumstances should be eliminated as much as possible. In addition, in our set-up, the random consequence of fertility treatments generates an other type of heterogeneity among the Infertile, for a similar effort. Then, we distinguish two sources of health inequalities that deserve to be handled. To do so, we rely on two approaches developed in the economic literature: The *ex-ante* inequality aversion and the *ex-post* inequality aversion. In the first case, the social planner focuses on the differences between social groups defined by the same set of circumstances (here, the Fertile and the Infertile) while, in the second one, she considers the outcome inequalities among individuals who exert the same effort (here among the Infertile). These two approaches seem to be relevant in the case of this twofold dimension of the health inequality.

5.1 Ex-ante egalitarian criterion

Let us first consider a social planner who adopts an ex-ante egalitarian criterion of the social welfare evaluation. This social objective implies to select the allocation that maximizes the expected lifetime well-being of the worst-off social group, the infertile households. The social welfare function is:

$$SW^E \equiv \min\{EU^F(c^F, d^F, v), EU^I(c^I, d^I, v)\}$$

where $EU^F(c^F, d^F, v) = \ln c^F + \delta \ln d^F + v$ and $EU^I(c^I, d^I, v) = \ln c^I + \delta \ln d^I + \frac{ax}{1+x}v$.

It is possible to write this problem as a maximization of the expected utility of the Infertile, conditionally on the resource constraint of the economy, the positivity constraint on x and conditionally on the egalitarian constraint such that infertile households are not worse-off than fertile ones.

$$\begin{cases} \max\limits_{c^F, c^I, d^F, d^I, x, k} & \ln c^I + \delta \ln d^I + \frac{ax}{1+x}v \\ s.\,t. & k^\alpha \geq \pi(k)c^F + (1 - \pi(k))(c^I + x) \\ & + \frac{\pi(k)d^F + (1-\pi(k))d^I}{n\Gamma(x,k)} + nk\Gamma(x,k) \\ & x \geq 0 \\ & \ln c^F + \delta \ln d^F + v = \ln c^I + \delta \ln d^I + \frac{ax}{1+x}v \end{cases}$$

First of all, at the optimum,[7] we still obtain the trade-off between young and old consumption over the life-cycle:

$$\delta c^{iE} = \frac{d^{iE}}{n\Gamma(x^E, k^E)} \quad \text{for} \quad i = F, I. \tag{22}$$

As in the utilitarian solution, the marginal rate of substitution between lifetime consumption is equal to the optimal population growth factor, $n\Gamma(x^E, k^E)$. Nevertheless, contrary to the utilitarian optimal solution, consumption levels are not equalized among heterogeneous households. Indeed, we can show that:

$$c^{FE} = \frac{\xi^E}{1 - \xi^E} \frac{1 - \pi(k^E)}{\pi(k^E)} c^{IE} \quad \text{and} \quad d^{FE} = \frac{\xi^E}{1 - \xi^E} \frac{1 - \pi(k^E)}{\pi(k^E)} d^{IE}, \tag{23}$$

with ξ^E the Lagrange multiplier associated to the egalitarian constraint and $\xi^E < \pi(k^E)$. Consequently, we deduce $c^{FE} < c^{IE}$ and $d^{FE} < d^{IE}$ and thus the situation is indeed reversed compared to the *laissez-faire* economy.

Second, we obtain a similar trade-off between consumption and health compared with the utilitarian one:

$$\frac{av}{(1 + x^E)^2} + \frac{\delta a}{(1 + x^E)^2 \Gamma(x^E, k^E)} \frac{1 - \pi(k^E)}{1 - \xi^E} = \frac{1}{c^{IE}} + \frac{nk^E a}{c^{IE}(1 + x^E)^2} - \frac{\mu^E}{1 - \xi^E}, \tag{24}$$

---

[7]Superscripts ^{E} indicate the ex-ante egalitarian solution. See Appendix E for more details.

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with μ^E the Lagrange multiplier associated to the positivity constraint on x. By adapting the proof of Proposition 2, we can easily show that $\mu^E = 0$ and thus an interior solution exists, $x^E > 0$. Finally, the trade-off between generations yields an alternative modified golden rule:

$$
\begin{aligned}
\alpha(k^E)^{\alpha-1} &= n\Gamma(x^E, k^E) - \pi'(k^E)x^E \\
&+ c^{IE}\pi'(k^E)\left(1 - \frac{ax^E}{1+x^E}\right)\left[\frac{nk^E}{c^{IE}} - \frac{\delta}{\Gamma(x^E, k^E)}\left(\frac{1-\pi(k^E)}{1-\xi^E}\right)\right] \\
&+ c^{IE}\pi'(k^E)(1+\delta)\left(\frac{\xi^E}{1-\xi^E}\frac{1-\pi(k^E)}{\pi(k^E)} - 1\right)
\end{aligned}
\tag{25}
$$

First of all, we can easily show that there is a unique pair (x^E, k^E) such that $x^E > 0$ that characterizes the ex-ante egalitarian equilibrium. Second, we can compare it with the utilitarian allocation. In particular, if ξ^E were equal to $\pi(k^E)$, consumption levels would be identical and the two solutions would be exactly the same. Since $\xi^E < \pi(k^E)$, it comes that the solutions differ so that $c^{IE} > c^*$ and $d^{IE} > d^*$. Under this criterion, the weight granted to the Fertile (ξ^E) is lower. To compensate the worst-off keeping the same expected utility for all households, the social planner provides more life-time consumption to the Infertile. The consequences on the optimal level of health and physical capital are given by the following proposition.

Proposition 4 *Consider that Assumption 1 holds and v sufficiently large, under the egalitarian social criterion, the optimal level of health expenditure, $x^E > 0$, is lower than the utilitarian one, x^* and the optimal stock of capital, k^E, is larger than k^*.*

Proof. See Appendix E. ∎

The social planner cannot rule out the externality but she can reduce the inequality between the two types of household by imposing a similar expected utility. However, a risk of fertility treatment still prevails because $q(x)$ is strictly lower than 1. Hence, the Infertile are compensated through two channels: a substantial increase in consumption and a positive investment in health expenditure. Under the utilitarian criterion, consumption levels are identical and the only tool to increase the infertile expected utility is the investment in health. Then, the optimal level of health expenditure is lower in the ex-ante egalitarian solution. Finally, the eviction effect of health expenditure is smaller and global savings are augmented.

We can now discuss the design of the public policy required to decentralize the ex-ante egalitarian social optimum.[8] The expression of the proportional tax on capital is similar to the utilitarian one and one can also implement a positive lump-sum transfer to the infertile young only, $T^{FE} = 0$ and $T^{IE} > 0$. However, we can state that the tax on health expenditures is larger: $-\sigma(x^*, k^*) < -\sigma(x^E, k^E)$. Since fertility treatment is not entirely efficient, the most relevant public action consists in a more generous redistributive policy, besides the taxation of the capital, that is the source of the impaired reproductive health.

5.2   Ex-post egalitarian criterion

Let us now consider an ex-post egalitarian social planner who allocates the resources according to the level of the realized well-being of the worst-off. In our framework, infertile households suffer from not being able to procreate. This loss of utility can be overcome whenever the household chooses to expend in fertility treatments. Infertile households are identical before the success of fertility treatments is revealed and make similar decisions (c^I, d^I, x) to enjoy different well-being because some of them succeed $(i = IF)$ while some others do not $(i = II)$. Those agents are thus considered as unlucky ex-post, that is once they realized their choices. The social welfare function writes:

$$SW^P \equiv \min\{U^F(c^F, d^F, v), U^{IF}(c^I, d^I, v), U^{II}(c^I, d^I, v)\}$$

where $U^F(c^F, d^F, v) = \ln c^F + \delta \ln d^F + v$, $U^{IF}(c^I, d^I, v) = \ln c^I + \delta \ln d^I + v$ and $U^{II}(c^I, d^I) = \ln c^I + \delta \ln d^I$.

Following the ex-post egalitarian criterion, the objective of the social planner is to maximize the utility of infertile households who have invested in health but for whom the treatment was unsuccessful. We can write the optimization program conditionally on the resource constraint of the economy, the positivity constraint on x and the egalitarian constraint such that unlucky infertile households are not worse-off than fertile ones:

---

[8]See Appendix F.

$$
\begin{cases}
\max\limits_{c^F,c^I,d^F,d^I,x,k} & \ln c^I + \delta \ln d^I \\[2mm]
s.\,t. & k^\alpha \geq \pi(k)c^F + (1-\pi(k))(c^I+x) \\[1mm]
& + \frac{\pi(k)d^F + (1-\pi(k))d^I}{n\Gamma(x,k)} + nk\Gamma(x,k) \\[1mm]
& x \geq 0 \\[1mm]
& \ln c^F + \delta \ln d^F + v = \ln c^I + \delta \ln d^I
\end{cases}
$$

First of all, at the ex-post egalitarian optimum,[9] we obtain the usual trade-off between consumption over the life-cycle:

$$
\delta c^{iP} = \frac{d^{iP}}{n\Gamma(x^P,k^P)} \quad \text{for } i = F,I. \tag{26}
$$

In addition, when the social planner is averse to inequalities (as in the ex-ante egalitarian case), we establish that consumption levels for young and old infertile households are larger than the ones of the Fertile: $c^{IP} > c^{FP}$ and $d^{IP} > d^{FP}$. We can also derive the trade-off between consumption and health:

$$
\frac{\delta a}{(1+x^P)^2 \Gamma(x^P,k^P)}\frac{1-\pi(k^P)}{1-\xi^P} = \frac{nk^P a}{c^{IP}(1+x^P)^2} + \frac{1}{c^{IP}} - \frac{\mu^P}{1-\xi^P}, \tag{27}
$$

where ξ^P and μ^P are the Lagrange multipliers associated to the egalitarian and positivity constraints. Compare to the ex-ante egalitarian solution, the pure utility gain from parenthood vanishes and thus the marginal social benefit of investing in health is reduced (see equation (24)). In this configuration, we show that $\mu^P > 0$ and thus $x^P = 0$. Finally, we obtain the same trade-off between generations as in the ex-ante egalitarian solution:

$$
\begin{aligned}
\alpha(k^P)^{\alpha-1} =\ & n\Gamma(x^P,k^P) - \pi'(k^P)x^P \\
& + c^{IP}\pi'(k^P)\left(1 - \frac{ax^P}{1+x^P}\right)\left[\frac{nk^P}{c^{IP}} - \frac{\delta}{\Gamma(x^P,k^P)}\left(\frac{1-\pi(k^P)}{1-\xi^P}\right)\right] \\
& + c^{IP}\pi'(k^P)(1+\delta)\left(\frac{\xi^P}{1-\xi^P}\frac{1-\pi(k^P)}{\pi(k^P)} - 1\right)
\end{aligned} \tag{28}
$$

We can establish the following proposition:

Proposition 5 *Consider that Assumption 1 holds, δ is low enough and v sufficiently large, under the ex-post egalitarian social criterion, the optimal level of health expenditure, x^P, is nil and the optimal stock of capital, k^P, is larger than k^*.*

---

[9]Superscripts ^{P} indicate the ex-post egalitarian solution. See Appendix H for more details.

Proof. See Appendix G. ∎

Comparing this result with the two previous criteria of social evaluation, one sees that compensating the unlucky infertile involves no investment in fertility treatments, $x^P = 0$. Health expenditure are not used by the social planner to reduce inequalities. The Infertile are compensated only through larger levels of consumption, so that $c^{IP} > c^*$. We can also emphasize that the well-being of unlucky and lucky infertile households become identical. Thanks to this ex-post criterion, all households display the same utility although only the expected utilities were equalized in the ex-ante egalitarian solution. Let us turn now to the design of the public policy to decentralize the ex-post egalitarian social optimum.[10] We can state that the tax on fertility treatment is sufficiently large to incite households not to invest in health expenditures. The expression of the other instruments are similar to the previous cases. We can compare the two egalitarian allocations. First, we can show that $x^E > x^P = 0$. Second, since k is a decreasing function of x, it comes that $k^P > k^E$.

6   Concluding remarks

Based on epidemiological evidence, we assume that the development process impacts negatively fertility. We analyze the implications of such a feature considering an OLG economy where households with impaired fertility may incur health expenditures in order to increase their chances of parenthood. We compare three long-run optimal allocations, depending on the social criteria. We claim that when the social planner does not exhibit any aversion to inequality, the optimal level of health expenditure is the highest one. When she displays inequality aversion, optimal health expenditure diminishes and is equal to zero when she focuses on the realized worst-off well-being. On the contrary, the utilitarian optimal level of capital is the lowest one and the ex-post egalitarian one is the largest. Then, we determine the optimal policy to decentralize each allocation. We underscore that to correct for the prevailing externality and the health inequality it induces, it is necessary to implement both preventive and redistributive policies, to globally subsidy mostly harmed agents within the economy. More precisely, the tax on health expenditure is a crucial tool to reduce the heterogeneity and thus inequalities. Indeed, when the social planner is inequality averse, the optimal tax on health expenditure is larger than

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[10]See Appendix H.

the utilitarian one. Opposite to public policies implemented in many countries which consist in subsidizing ARTs, we argue that to reduce inequalities, it is more appropriate to enforce redistribution and prevention. For instance, if the impaired reproductive health comes from pollution exposure, a fair health policy could be a taxation of polluting emissions. Our strong conclusions could be mitigated if we consider income heterogeneity besides health heterogeneity. In this case, we may justify a positive subsidize to health expenditure that comes along with a preventive and redistributive policy. This paves the way to future research.

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Appendices

A Proof of Proposition 1

Let us consider the regime in which $x_t = 0$. If a solution exists, then condition (11) should be satisfied. This implies that:

$$k_t \le \left[\frac{1+\delta}{av(1-\alpha)}\right]^{\frac{1}{\alpha}} \equiv \tilde{k} \tag{A.1}$$

When $x_t = 0$ and using (18), total savings are given by

$$k_{t+1} = \frac{(1-\alpha)\delta k_t^\alpha}{(1+\delta)n\pi(k_t)} \tag{A.2}$$

The steady-state is defined as a fixed point such that $k_{t+1} = k_t = k$ and is a solution if:

$$k = \frac{(1-\alpha)\delta k^\alpha}{(1+\delta)n\pi(k)} \tag{A.3}$$

We can easily see that the $RHS(k)$ of the equation (A.3) has the following properties: $RHS(0) = 0$ and $\lim_{k\to+\infty} RHS(k) = +\infty$. Then, k cannot be a solution if $RHS(\tilde{k}) > \tilde{k}$. Since, π is a decreasing function of k, this is satisfied under the following sufficient condition:

$$\frac{(1-\alpha)\delta}{(1+\delta)n} > \pi(0)\left[\frac{1+\delta}{av(1-\alpha)}\right]^{\frac{1-\alpha}{\alpha}} \tag{A.4}$$

This is true for a sufficiently large value of v:

$$v > \tilde{v} \equiv \left[\frac{1+\delta}{a(1-\alpha)}\right] \times \left[\frac{(1-\alpha)\delta}{(1+\delta)n\pi(0)}\right]^{-\frac{\alpha}{1-\alpha}} \tag{A.5}$$

Let us now consider the regime in which $x_t > 0$. Putting (**??**), (14) and (15) into (18), we have:

$$k_{t+1} = \frac{\frac{\delta}{1+\delta}(1-\alpha)k_t^\alpha - (1-\pi(k_t))\frac{\delta}{1+\delta}x(k_t)}{n\left[\pi(k_t) + (1-\pi(k_t)q(x(k_t))\right]} \equiv g(k_t) \tag{A.6}$$

It immediately comes that

25

$$k_{t+1} = g(k_t) < \frac{\frac{\delta}{1+\delta}(1-\alpha)k_t^\alpha}{n\pi(k_t)} \tag{A.7}$$

Since $\frac{g(k_t)}{k_t} < \frac{\frac{\delta}{1+\delta}(1-\alpha)k_t^{\alpha-1}}{n\pi(k_t)}$, we deduce that $\lim\limits_{k_t \to +\infty} \frac{g(k_t)}{k_t} = 0$ and there exists a steady state, k, such that $x > 0$. We can easily show that this steady state is unique.

B   The optimal program

To solve the optimal program, let us write the Lagrangian as:

$$
\begin{aligned}
\mathscr{L} &= \pi(k)\left[\ln c^F + \delta \ln d^F + v\right] + (1-\pi(k))\left[\ln c^I + \delta \ln d^I + \frac{ax}{1+x}v\right] \\
&+ \lambda\left[k^\alpha - \pi(k)c^F - (1-\pi(k))(c^I+x) - \frac{\pi(k)d^F + (1-\pi(k))d^I}{n\Gamma(x,k)} - nk\Gamma(x,k)\right] \\
&+ \mu x
\end{aligned}
$$

We obtain the following First Order Conditions:

$$\frac{\partial \mathscr{L}}{\partial c^i} = 0 \Leftrightarrow \frac{1}{c^i} = \lambda, \quad i = F, I \tag{B.8}$$

$$\frac{\partial \mathscr{L}}{\partial d^i} = 0 \Leftrightarrow \frac{\delta n\Gamma(x,k)}{d^i} = \lambda, \quad i = F, I \tag{B.9}$$

$$
\begin{aligned}
\frac{\partial \mathscr{L}}{\partial x} &= \frac{(1-\pi(k))av}{(1+x)^2} + \lambda\Gamma_x(x,k)\left\{\frac{[\pi(k)d^F + (1-\pi(k))d^I]}{n\Gamma(x,k)^2} - nk\right\} \\
&\quad -\lambda(1-\pi(k)) + \mu = 0 \tag{B.10}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial \mathscr{L}}{\partial k} &= \pi'(k)[\ln c^F + \delta \ln d^F + v - \ln c^I - \delta \ln d^I - \frac{ax}{1+x}v] \tag{B.11} \\
&+ \lambda\left[\alpha k^{\alpha-1} - n\Gamma(x,k) - nk\Gamma_k(x,k)\right] - \lambda\pi'(k)(c^F - (c^I+x)) \\
&- \lambda\left[\frac{\pi'(k)(d^F - d^I)\Gamma(x,k) - \Gamma_k(x,k)[\pi(k)d^F + (1-\pi(k))d^I]}{n\Gamma(x,k)^2}\right] = 0
\end{aligned}
$$

26

where

$$\Gamma_x(x,k) = (1-\pi(k))\frac{a}{(1+x)^2} > 0 \qquad \text{(B.12)}$$

$$\Gamma_k(x,k) = \pi'(k)(1-\frac{ax}{1+x}) < 0 \qquad \text{(B.13)}$$

and with complementary slackness :

$$k^\alpha \geq \pi(k)c^F + (1-\pi(k))(c^I + x) + \frac{\pi(k)d^F + (1-\pi(k)d^I}{n\Gamma(x,k)}$$

$$+ \quad nk\Gamma(x,k), \ \lambda \geq 0 \qquad \text{(B.14)}$$

$$\mu x = 0, \ \mu \geq 0 \qquad \text{(B.15)}$$

C Proof of Proposition 2

C.1 Existence

If $x^* = 0$, we have $\Gamma(0,k) = \pi(k)$, $\Gamma_x(0,k) = a(1-\pi(k))$ and $\Gamma_k(0,k) = \pi'(k)$. Using (19)-(21), an allocation with $x^*=0$ is defined by:

$$\alpha k^{\alpha-1} - n\pi(k) = c^*\pi'(k)\left[-v - \frac{\delta}{\pi(k)} + \frac{nk}{c^*}\right] \quad \text{(C.16)}$$

$$k^\alpha = (1+\delta)c^* + nk\pi(k) \quad \text{(C.17)}$$

$$av - \frac{1}{c^*} + \frac{a\delta}{\pi(k)} - \frac{nak}{c^*} + \frac{\mu}{1-\pi(k)} = 0 \quad \text{(C.18)}$$

From (C.17), we define c^* as a function of k, such that $c^* = \frac{k^\alpha - \pi(k)nk}{(1+\delta)} \equiv c(k)$. Then using (C.16) and (C.18), we obtain the following system:

$$\alpha k^{\alpha-1} - n\pi(k) - \pi'(k)nk = -c(k)\pi'(k)\left[v + \frac{\delta}{\pi(k)}\right] \quad \text{(C.19)}$$

$$\frac{\mu}{1-\pi(k)} = \frac{1}{c(k)} + \frac{a}{c(k)}\left[\frac{\alpha k^{\alpha-1} - n\pi(k)}{\pi'(k)}\right] \quad \text{(C.20)}$$

Let us consider equation (C.20). We see that $\mu^* > 0$ if and only if:

$$\epsilon_\pi < -ak\frac{\left[\alpha k^{\alpha-1} - n\pi(k)\right]}{\pi(k)} \quad \text{(C.21)}$$

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We also notice that (C.19) rewrites:

$$\frac{\left[\alpha k^{\alpha-1} - n\pi(k)\right]k}{\pi(k)} = c(k)\epsilon_\pi \left[\frac{nk}{c(k)} - v - \frac{\delta}{\pi(k)}\right] \qquad \text{(C.22)}$$

Using (C.22), the inequality (C.21) reduces to:

$$v < \frac{1}{ac(k)} + \frac{nk}{c(k)} - \frac{\delta}{\pi(k)} \qquad \text{(C.23)}$$

From (C.22), we also deduce that $\alpha k^{\alpha-1}$ is close to $n\pi(k)$ when ϵ_π is close to 0. This implies that both k and $c(k)$ have positive and finite values. Therefore, the inequality (C.23) is violated for v high enough, which means that $x^* = 0$ is not possible.

Using (19), the system of equations (20), (21) and (B.14) with $x^* > 0$ and $\mu^* = 0$, satisfied by such an allocation, can be written:

$$c^*\pi'(k)\left[(1 - \frac{ax}{1+x})\left(\frac{nk}{c^*} - \frac{\delta}{\Gamma(x,k)} - v\right)\right] - \pi'(k)x \;=\; \alpha k^{\alpha-1} - n\Gamma(x,k) \qquad \text{(C.24)}$$

$$\frac{\delta}{\Gamma(x,k)} - \frac{nk}{c^*} + v \;=\; \frac{(1+x)^2}{ac^*} \qquad \text{(C.25)}$$

$$(1+\delta)c^* + (1-\pi(k))x + n\Gamma(x,k)k \;=\; k^\alpha \qquad \text{(C.26)}$$

From (C.24), we deduce:

$$c^* = \frac{1}{1+\delta}[k^\alpha - (1-\pi(k))x - n\Gamma(x,k)k] \equiv c(x,k) \qquad \text{(C.27)}$$

Using this equation, the system becomes:

$$c(x,k)\pi'(k)\left[(1 - \frac{ax}{1+x})\left(\frac{nk}{c(x,k)} - \frac{\delta}{\Gamma(x,k)} - v\right) - \frac{x}{c(x,k)}\right] = \alpha k^{\alpha-1} - n\Gamma(x,k) \qquad \text{(C.28)}$$

and

$$c(x,k)\left[\frac{\delta}{\Gamma(x,k)} + v - \frac{nk}{c(x,k)}\right] = \frac{(1+x)^2}{a} \qquad \text{(C.29)}$$

An optimal allocation is a solution (x^*, k^*) to the system (C.28) and (C.29). Now, substituting (C.29) into (C.28), we get:

$$G(x,k) \;\equiv\; \alpha k^{\alpha-1} - n\Gamma(x,k) + \frac{\pi'(k)}{a}[1 + 2x + x^2(1-a)] = 0 \qquad \text{(C.30)}$$

It implicitly defines x as a function of k, i.e. $x = x(k)$, if $G_x \equiv \partial G/\partial x \neq 0$. Differentiating (C.30) with respect to x, we obtain:

$$G_x \;=\; \frac{-na(1-\pi(k))}{(1+x)^2} + \frac{2\pi'(k)(1+x(1-a))}{a} \qquad \text{(C.31)}$$

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We can immediately see that $G_x < 0$ meaning that (C.31) implicitly defines $x = x(k)$. Hence, an optimal allocation is a solution $k^* > 0$ to equation (C.29) with $x^* = x(k^*)$.

From (C.28), we also have $\alpha k^{\alpha-1} > n\Gamma(x(k), k)$, where $\Gamma(x(k), k) \geqslant \pi(k) > \pi(+\infty)$. Therefore, there exists $\bar{k} > 0$ defined by $\alpha \bar{k}^{\alpha-1} = n\Gamma(x(\bar{k}), \bar{k})$ such that $\alpha k^{\alpha-1} > n\Gamma(x, k)$ for all $k < \bar{k}$. Hence, k^* belongs to $(0, \bar{k})$.

Let us note $LHS(k)$ the left-hand side and $RHS(k)$ the right-hand side of (C.29), respectively. When k tends to 0, we deduce, using (C.26), that $x(k)$ tends to 0 too. We get $RHS(0) = 1/a > 0 = LHS(0)$. Moreover,

$$LHS(\bar{k}) = \frac{(1-\alpha)\bar{k}^\alpha - (1-\pi(\bar{k}))x(\bar{k})}{(1+\delta)} \left[\frac{\delta}{\Gamma(x(\bar{k}), \bar{k})} + v \right] - n\bar{k}$$

$$RHS(\bar{k}) = \frac{(1 + x(\bar{k}))^2}{a}$$

with $(1-\alpha)\bar{k}^\alpha > (1-\pi(p(\bar{k})))x(\bar{k})$. Since $\bar{k}$ has a bounded value and (C.26) is satisfied, $x(\bar{k})$ is bounded above. This implies that $LHS(\bar{k}) > RHS(\bar{k})$ if v is sufficiently large. Then, there exists a solution $k^* \in (0, \bar{k})$ to equation (C.29).

C.2 Unicity and 2nd order conditions

Let us consider that $\pi(k) = \pi$ is constant, i.e. $\epsilon_\pi = 0$. Since $x > 0$, the social planner solves:

$$\begin{cases} \max_{c^F, c^I, d^F, d^I, x, k} & \pi(\ln c^F + \delta \ln d^F + v) + (1-\pi)(\ln c^I + \delta \ln d^I + \frac{ax}{1+x}v) \\ s.\, to & k^\alpha = \pi c^F + (1-\pi)(c^I + x) + \frac{\pi d^F + (1-\pi)d^I}{n\Gamma(x)} + nk\Gamma(x) \end{cases}$$

with $\Gamma(x) \equiv \pi + (1-\pi)\frac{ax}{1+x}$. Maximizing this objective function is equivalent to maximize:

$$\ln(c^F)^\pi (c^I)^{1-\pi} + \delta \ln(d^F)^\pi (d^I)^{1-\pi} + (1-\pi)\frac{ax}{1+x}v \qquad (C.32)$$

This program can be solved in two steps. In a second step, we maximise $\ln C = \ln(c^F)^\pi (c^I)^{1-\pi}$ under the constraint $\pi c^F + (1-\pi)c^I = P^c C$ with respect to c^F and c^I, taking the level of consumption expenditures $P^c C$ as given. We perform the same exercise for $\ln D = \ln(d^F)^\pi (d^I)^{1-\pi}$ under the constraint $\pi d^F + (1-\pi)d^I = P^d D$ with respect to d^F and d^I, taking the level of consumption expenditures $P^d D$ as given. Using the first order conditions, we deduce that $P^c = 1$ and $P^d = 1$.

Therefore, in a first step, we have to solve:

$$\begin{cases} \max\limits_{C,D,x,k} & \ln C + \delta \ln D + (1-\pi)\frac{ax}{1+x}v \\ s.\ to & k^\alpha = C + \frac{D}{n\Gamma(x)} + (1-\pi)x + nk\Gamma(x) \end{cases}$$

Note that this program above, using the constraint, can be rewritten $\max_{D,x,k} V$, with:

$$V \equiv \ln\left[k^\alpha - \frac{D}{n\Gamma(x)} - (1-\pi)x - nk\Gamma(x)\right] + \delta \ln D + (1-\pi)\frac{ax}{1+x}v \qquad \text{(C.33)}$$

where $C = k^\alpha - \frac{D}{n\Gamma(x)} - (1-\pi)x - nk\Gamma(x)$. We can then derive the following first order conditions:[11]

$$V_D = -\frac{1}{n\Gamma(x)C} + \frac{\delta}{D} = 0 \qquad \text{(C.34)}$$

$$V_x = \frac{D\Gamma'(x)/[n\Gamma(x)^2] - (1-\pi) - nk\Gamma'(x)}{C} + \frac{(1-\pi)av}{(1+x)^2} = 0 \qquad \text{(C.35)}$$

$$V_k = \frac{\alpha k^{\alpha-1} - n\Gamma(x)}{C} = 0 \qquad \text{(C.36)}$$

We easily deduce that:

$$D = \delta n\Gamma(x)C \qquad \text{(C.37)}$$

$$D\Gamma'(x)/[n\Gamma(x)^2] - (1-\pi) - nk\Gamma'(x) = -C\frac{(1-\pi)av}{(1+x)^2} \qquad \text{(C.38)}$$

$$\alpha k^{\alpha-1} = n\Gamma(x) \qquad \text{(C.39)}$$

Establishing the second order conditions for this last program gives us the second order conditions for the program (C.32). Hence, we differentiate (C.34)-(C.36) and use (C.37)-(C.39), $\Gamma'(x) = (1-\pi)\frac{a}{(1+x)^2}$ and $\Gamma''(x) = -2(1-\pi)\frac{a}{(1+x)^3}$ to compute the following Hessian matrix:

$$H \equiv \begin{bmatrix} V_{DD} & V_{Dx} & V_{Dk} \\ V_{xD} & V_{xx} & V_{xk} \\ V_{kD} & V_{kx} & V_{kk} \end{bmatrix}$$

---

[11]In the following, we note $V_u \equiv \partial V/\partial u$ and $V_{uv} \equiv \partial^2 V/\partial v \partial u$, with $\{u,v\} = \{D,x,k\}$.

with

$$V_{DD} = -\frac{1+\delta}{n^2\Gamma(x)^2 C^2\delta} < 0 \tag{C.40}$$

$$V_{Dx} = \frac{\Gamma'(x)}{nC\Gamma(x)^2}[1-\Gamma(x)v] = V_{xD} \tag{C.41}$$

$$V_{Dk} = 0 = V_{kD} \tag{C.42}$$

$$V_{xx} = -\frac{2(1-\pi)}{C(1+x)} - \frac{2\delta\Gamma'(x)^2}{\Gamma(x)^2} - \Gamma'(x)^2 v^2 < 0 \tag{C.43}$$

$$V_{xk} = -\frac{n\Gamma'(x)}{C} = V_{kx} \tag{C.44}$$

$$V_{kk} = \frac{(\alpha-1)n\Gamma(x)}{Ck} < 0 \tag{C.45}$$

To prove that an optimal allocation is a maximum, we have to show that $\mathcal{H}_1 \equiv V_{DD} < 0$, $\mathcal{H}_2 \equiv V_{DD}V_{xx} - V_{Dx}V_{xD} > 0$ and $\mathcal{H}_3 \equiv det H < 0$.

$\mathcal{H}_1 < 0$ is obvious. Let us now determine the sign of $\mathcal{H}_2$. Using (C.40), (C.41) and (C.43), we get:

$$\mathcal{H}_2 n^2 C^2 \Gamma(x)^4 = \frac{1+\delta}{\delta}\Gamma(x)^2\frac{2(1-\pi)}{C(1+x)} + 2\Gamma'(x)^2\delta$$
$$+ 2\Gamma(x)\Gamma'(x)^2 v + \Gamma'(x)^2\left[\frac{1+\delta}{\delta} - \Gamma(x)^2\right]v^2 \tag{C.46}$$

We observe that $\mathcal{H}_2 > 0$ because $\Gamma(x) \leqslant 1$. Finally, let us investigate the properties of $\mathcal{H}_3$. Using (C.42), we have $\mathcal{H}_3 = V_{DD}V_{xx}V_{kk} - V_{xD}^2 V_{kk} - V_{xk}^2 V_{DD}$. Then, using (C.40), (C.41) and (C.43)-(C.45), we obtain after some computations:

$$\mathcal{H}_3 n^2 C^3 k\Gamma(x)^4 = -n\Gamma(x)^2\frac{1+\delta}{\delta}\frac{1-\pi}{C}\left[\frac{2(1-\alpha)\Gamma(x)}{1+x} + \Gamma'(x)\right]$$
$$- n\Gamma'(x)^2\Gamma(x)[1-\alpha+\delta(1-2\alpha)]$$
$$- n\Gamma'(x)^2\Gamma(x)^2\left[1-2\alpha-\frac{1}{\delta}\right]v - \frac{(1-\alpha)n\Gamma'(x)^2\Gamma(x)^3}{\delta}v^2$$

We deduce that $\mathcal{H}_3 < 0$ if v is sufficiently large.

By a continuity argument, our result still holds if π weakly depends on k, i.e. ϵ_π is close to 0. Therefore, any optimal allocation is a maximum if v sufficiently large and ϵ_π close to 0. Note also that since this last result holds for any optimal allocation, such an allocation is unique.

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D   Proof of Proposition 3

Each household maximises the utility (2) under the budget constraints (3)-(4). The first order conditions and the budget constraints allow us to derive the stationary levels of consumption, savings for both types of household and health expenditure:

$$c^F = \frac{1}{1+\delta}\left(w + T^F - \frac{\theta}{R}\right) \tag{D.47}$$

$$d^F = \frac{R\delta}{1+\delta}\left(w + T^F - \frac{\theta}{R}\right) \tag{D.48}$$

$$s^F = \frac{\delta}{1+\delta}\left(w + T^F\right) + \frac{\theta}{R(1+\delta)} \tag{D.49}$$

and

$$c^I = \frac{1}{1+\delta}\left[w + T^I - \frac{\theta}{R} - (1-\sigma)x\right] \tag{D.50}$$

$$d^I = \frac{R\delta}{1+\delta}\left[w + T^I - \frac{\theta}{R} - (1-\sigma)x\right] \tag{D.51}$$

$$s^I = \frac{\delta}{1+\delta}\left[w + T^I - (1-\sigma)x\right] + \frac{\theta}{R(1+\delta)} \tag{D.52}$$

$$(1+x)^2 = \frac{av}{(1-\sigma)\delta}\left(s^I - \frac{\theta}{R}\right) \tag{D.53}$$

Finally, the government that perceives the different taxes balances its budget at each period of time. Taking into account the population size, this means that[12]:

$$\frac{\theta}{n\Gamma} + \rho\alpha k^\alpha = \pi T^F + (1-\pi)T^I + \sigma(1-\pi)x \tag{D.54}$$

Using the previous section, we recall that an optimal allocation is characterised by equations (19), (C.24), (C.25) and (C.26).

We are now able to derive the appropriate policy design that allows for decentralising the stationary optimal allocation. Using (D.47), (D.48), (D.50) and (D.51), the condition (19) is, partly, satisfied for:

$$T^F = T^I - (1-\sigma)x^* \tag{D.55}$$

---

[12]When this is not a source of confusion, we skip the arguments of the functions.

Obviously, we can set T^F to zero and thus, $T^I = (1-\sigma)x^*$. Then, the hetero-geneity in consumption among the two types of household is eliminated.

Comparing (19) with the FOCs of the decentralized economy, we should have that $R = (1-\rho)\alpha k^{\alpha-1} = n\Gamma(x,k)$, *i.e.*

$$\rho = 1 - \frac{n\Gamma(x^*,k^*)}{\alpha(k^*)^{1-\alpha}} \in (0,1) \tag{D.56}$$

Using (C.25), we obtain:

$$-\frac{\sigma}{1-\sigma} = \frac{1}{\nu}\left(\frac{nk^*}{c^*} - \frac{\delta}{\Gamma(x^*,k^*)}\right) \tag{D.57}$$

It is straightforward that $\sigma < 1$. As we have seen in the proof of Proposition 3, k^* has a finite value. Moreover, the consumption is bounded above, $c^* < (k^*)^{\alpha}/(1+\delta)$ and $\Gamma(x^*,k^*) > \pi(+\infty)$. We deduce that for δ low enough, we have $\sigma < 0$.

Note that this last inequality requires that $nk^*\Gamma^* > \delta c^*$. Using the FOCs of the decentralized economy and the optimality condition (19), we deduce that $s^F = s^I = s^*$. Hence, the equilibrium on the capital market writes $nk^*\Gamma^* = s^*$. We deduce $s^* > \delta c^*$. Using (D.47) and (D.49), we obtain that $\theta > 0$.

E   Proof of Proposition 4

Let us begin by showing that $\xi^E < \pi(k^E)$. By using the egalitarian constraint and (23), we obtain:

$$\ln\left(\frac{\xi^E}{1-\xi^E}\frac{1-\pi(k^E)}{\pi(k^E)}c^{IE}\right) + \delta\ln\left(\frac{\xi^E}{1-\xi^E}\frac{1-\pi(k^E)}{\pi(k^E)}d^{IE}\right) + \nu = \ln c^{IE} + \delta\ln d^{IE} + \frac{ax^E}{1+x^E}\nu$$

$$\Leftrightarrow (1+\delta)\ln\left(\frac{\xi^E}{1-\xi^E}\frac{1-\pi(k^E)}{\pi(k^E)}\right) = \left(\frac{ax^E}{1+x^E}-1\right)\nu$$

which is negative. Consequently, $\frac{\xi^E}{1-\xi^E}\frac{1-\pi(k^E)}{\pi(k^E)} < 1$ and thus $\xi^E < \pi(k^E)$.

Now, we turn to Proposition 4. Using (20) and (21), an optimal utilitarian allocation is defined by the following two equations:

$$H(x,k) \equiv (1-\pi(k))\left[\frac{a\nu}{(1+x)^2} + \frac{a\delta}{\Gamma(x,k)(1+x)^2} - \frac{ank}{c(x,k)(1+x)^2} - \frac{1}{c(x,k)}\right] = 0$$

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$$J(x,k) \equiv \frac{1}{c(x,k)} \left[\alpha k^{\alpha-1} - n\Gamma(x,k) \right] + \pi'(k) \left(1 - \frac{ax}{1+x} \right) \left[v - \frac{nk}{c(x,k)} + \frac{\delta}{\Gamma(x,k)} \right] + \frac{\pi'(k)x}{c(x,k)} = 0$$

whereas using (24) and (25), the egalitarian ex-ante criterion satisfies:

$$H^E(x,k) \equiv (1-\xi) \left[\frac{av}{(1+x)^2} + \frac{a\delta}{\Gamma(x,k)(1+x)^2} \frac{1-\pi(k)}{1-\xi} - \frac{ank}{c^I(x,k)(1+x)^2} - \frac{1}{c^I(x,k)} \right] = 0$$

$$\begin{aligned} J^E(x,k) \quad \equiv \quad & \frac{1}{c^I(x,k)} \left[\alpha k^{\alpha-1} - n\Gamma(x,k) \right] + \pi'(k)\left(1 - \frac{ax}{1+x}\right)\left[-\frac{nk}{c^I(x,k)} + \frac{\delta}{\Gamma(x,k)}\left(\frac{1-\pi(k)}{1-\xi}\right) \right] \\ & + \frac{\pi'(k)x}{c^I(x,k)} - \pi'(k)(1+\delta)\left(\frac{\xi}{1-\xi}\frac{1-\pi(k)}{\pi(k)} - 1\right) = 0 \end{aligned}$$

where using (22), the constraint on the good market and (C.27), we have $c^{IE} = \frac{1-\xi^E}{1-\pi(k)} c(x,k) \equiv c^I(x,k)$.

Let us consider that π is constant, thus $J(x,k) = 0$ and $J^E(x,k) = 0$ are both equivalent to $\alpha k^{\alpha-1} = n\Gamma(x)$, with $\Gamma(x) = \pi + (1-\pi)\frac{ax}{(1+x)}$. We deduce a negative relationship $k(x)$ between k and x, i.e. $k'(x) < 0$. This still holds when π weakly depends on k.

Before comparing $H(x,k)$ and $H^E(x,k)$, let us derive the following result. Substituting the expressions of the consumption in the equality $\ln c^{FE} + \delta \ln d^{FE} + v = \ln c^{IE} + \delta \ln d^{IE} + \frac{ax^E}{1+x^E} v$, we obtain:

$$(1+\delta)\ln\left[\frac{\xi^E}{1-\xi^E}\frac{1-\pi(k(x^E))}{\pi(k(x^E))}\right] = \left(\frac{ax^E}{1+x^E} - 1\right)v$$

Since π is decreasing in k and k decreasing in x^E, this equation defines a positive relationship between x^E and ξ^E.

Now, using the expression of $c^I(x,k)$, we get:

$$H^E(x,k) \equiv (1-\pi(k)) \left[\frac{1-\xi^E}{(1-\pi(k))} \frac{av}{(1+x)^2} + \frac{a\delta}{\Gamma(x,k)(1+x)^2} - \frac{ank}{c(x,k)(1+x)^2} - \frac{1}{c(x,k)} \right]$$

Since $\xi^E < \pi(k)$, we deduce that $H^E(x,k(x)) > H(x,k(x))$ for all $x > 0$. We also see that $H^E(x,k(x))$ is decreasing in ξ^E and tends to $H(x,k(x))$ when ξ^E tends to $\pi(k)$. Therefore, to be consistent with the fact that x^E increases with

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respect to ξ^E, the unique solutions x^E and x^* solving respectively $H^E(x, k(x)) = 0$ and $H(x, k(x)) = 0$ are such that these two functions of x are increasing in x. Therefore, we deduce that $x^E < x^*$. Since $k'(x) < 0$, we also have $k^* < k^E$.

Finally, at the optimum, $c^{IE} = \frac{1-\xi^E}{1-\pi(k^E)} c(x^E, k^E) > \frac{1-\xi^E}{1-\pi(k^E)} c(x^*, k^*)$ since $c(x, k)$ is an increasing function of k and decreasing in x. So that, $c^{IE} > c(x^*, k^*) = c^*$.

F Optimal policy in the case of an ex-ante egalitarian criterion

We are able to derive the appropriate policy design that allows for decentralising the stationary optimal allocation. Using (D.47), (D.48), (D.50) and (D.51), the egalitarian constraint is, partly, satisfied for:

$$
\ln\left(w + T^{IE} - \frac{\theta}{R} - (1-\sigma)x^E\right) = \ln\left(w + T^{FE} - \frac{\theta}{R}\right) + \left(1 - \frac{ax^E}{1+x^E}\right)\frac{v}{1+\delta}
$$

As $c^{IE} > c^{FE}$, we have $T^{FE} < T^{IE} - (1-\sigma)x^E$. Obviously, we can set T^{FE} to zero and thus

$$
T^{IE} = (1-\sigma)x^E + \left(w - \frac{\theta}{R}\right)\left(e^{\left(1 - \frac{ax^E}{1+x^E}\right)\frac{v}{1+\delta}} - 1\right) \tag{F.58}
$$

Comparing (22) with the FOCs of the decentralized economy, we should have that $R = (1-\rho)\alpha k^{\alpha-1} = n\Gamma(x, k)$, *i.e.*

$$
\rho^E = 1 - \frac{n\Gamma(x^E, k^E)}{\alpha(k^E)^{1-\alpha}} \in (0, 1) \tag{F.59}
$$

Using (24), we obtain:

$$
1 - \sigma^E = \frac{avc^{IE}}{(1+x^E)^2} \tag{F.60}
$$

Since $c^{IE} > c^*$ and $x^E < x^*$, we have $\sigma^E < \sigma^* < 0$.

G Proof of Proposition 5

This proof uses a methodology similar to the proof of Proposition 4. Using (27) and (28), we get the two following implicit functions:

$$
H^P(x, k) \equiv (1-\xi)\left[\frac{a\delta}{\Gamma(x, k)(1+x)^2}\frac{1-\pi(k)}{1-\xi} - \frac{ank}{c^I(x, k)(1+x)^2} - \frac{1}{c^I(x, k)}\right] + \mu = 0 \tag{G.61}
$$

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$$J^P(x,k) \equiv \frac{1}{c^I(x,k)}\left[\alpha k^{\alpha-1} - n\Gamma(x,k)\right] + \pi'(k)\left(1 - \frac{ax}{1+x}\right)\left[-\frac{nk}{c^I(x,k)} + \frac{\delta}{\Gamma(x,k)}\left(\frac{1-\pi(k)}{1-\xi}\right)\right]$$

$$+ \frac{\pi'(k)x}{c^I(x,k)} - \pi'(k)(1+\delta)\left(\frac{\xi}{1-\xi}\frac{1-\pi(k)}{\pi(k)} - 1\right) = 0 \tag{G.62}$$

where $c^I(x,k) = \frac{1-\xi}{1-\pi(k)}c(x,k) = c^{IP}$.

When π is constant, equation (G.62) writes $\alpha k^{\alpha-1} = n\Gamma(x)$, where $\Gamma(x) = \pi + (1-\pi)\frac{ax}{(1+x)}$. This equality implicitly defines a negative relationship $k(x)$ between k and x, i.e. $k'(x) < 0$. Since $\Gamma(x) \in (\pi(+\infty), 1)$, we also deduce that k^P has a finite and strictly positive value. This still holds when π weakly depends on k.

Using now (G.61),

$$\mu^P = (1-\xi^P)\left[-\frac{a\delta}{\Gamma(x^P,k^P)(1+x^P)^2}\frac{1-\pi(k^P)}{1-\xi} + \frac{ank^P}{c^I(x^P,k^P)(1+x^P)^2} + \frac{1}{c^I(x^P,k^P)}\right]$$

Thus,

$$\mu^P > \frac{(1-\pi(k^P))a}{(1+x^P)^2}\left(\frac{nk^P}{c^I(x^P,k^P)}\frac{1-\xi^P}{1-\pi(k^P)} - \frac{\delta}{\Gamma(x^P,k^P)}\right)$$

The right-hand side of this inequality is strictly positive if $nk^P\Gamma(x^P,k^P) > \delta c(x^P,k^P)$. Since $c(x^P,k^P)$ is bounded above by the production $(k^P)^\alpha$ and k^P has a finite and strictly positive value, this last inequality is satisfied if δ is sufficiently low.

We deduce that $x^* > x^P = 0$. Since $k'(x) < 0$, we also get $k^* < k^P$.

H   Optimal policy in the case of an ex-post egalitarian criterion

We are able to derive the appropriate policy design that allows for decentralising the stationary optimal allocation. Using (D.47), (D.48), (D.50) and (D.51), the egalitarian constraint is, partly, satisfied for:

$$\ln\left(w + T^{IP} - \frac{\theta}{R}\right) = \ln\left(w + T^{FP} - \frac{\theta}{R}\right) + \frac{v}{1+\delta}$$

We can set T^{FP} to zero and thus

$$T^{IP} = \left(w - \frac{\theta}{R}\right)\left(e^{\frac{v}{1+\delta}} - 1\right) \tag{H.63}$$

Comparing (22) with the FOCs of the decentralized economy, we should have that $R = (1-\rho)\alpha k^{\alpha-1} = n\Gamma(x,k)$, *i.e.*

$$\rho^P = 1 - \frac{n\Gamma(0,k^P)}{\alpha(k^P)^{1-\alpha}} \in (0,1) \tag{H.64}$$

Using the condition under that households choose $x = 0$, we obtain:

$$1 - \sigma^P = avc^{IP} \tag{H.65}$$

Since $c^{IP} > c^*$ and $x^* > 0$, we have $\sigma^P < \sigma^* < 0$.