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Abstract

Bridging modern macroeconomics and the economic theory of index numbers, this paper shows that real output growth as measured by National Income and Product Accounts (NIPA) is a welfare based measure. In a two-sector dynamic general equilibrium model of heterogeneous households, recursive preferences and quasi-concave technology, individual welfare depends on present and future consumption. In this context, the Bellman equation provides a representation of preferences over current consumption and investment. Applying standard index number theory to this representation of preferences, it is shown that the Fisher-Shell true quantity index is equal to the Divisia index in turn well approximated by the Fisher ideal chain index used in NIPA.

Keywords: Growth measurement, Quantity indexes, Equivalent variation, NIPA, Fisher-Shell index, Divisia index, Embodied technical change.

JEL classification numbers: C43, D91, O41, O47.

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1 Introduction

The present paper bridges modern macroeconomics and the economic theory of index numbers to show that the class of chain indexes used by National Income Product Accounts (NIPA) properly reflects changes in welfare when applied to a two-sector dynamic general equilibrium economy with heterogenous households, recursive preferences and quasi-concave technology.[1] In doing so, it evaluates the suitability of NIPA methodology for measuring real changes in GDP in a general model economy with explicit preferences and technology. In this framework, preferences are defined over consumption streams, present and future, but NIPA is constrained to use observable information at a given point in time by aggregating the main components of current final demand, encompassing consumption and investment. The Bellman equation, indeed, provides a representation of preferences, such that all information about changes in welfare is contained in current (observable) changes in consumption and investment. In particular, we show that the Fisher-Shell true quantity index when applied to this representation of preferences is equal to the Divisia index, which is known to be well approximated by the Fisher ideal chain index used by NIPA.[2] This means that the real growth rate of output in National Accounts is a welfare based measure in the very precise sense of equivalent variation –see McKenzie (1983).

Until the 90's the Bureau of Economic Analysis (BEA) featured in its NIPA a Laspeyres fixed-base quantity index to measure real GDP growth. Traditional fixed-base quantity indexes yield a reasonable good measurement of real growth provided that relative prices remain stable. However, the situation radically changed in the mid-80s when following the seminal contribution of Gordon (1990) the BEA started deflating equipment investment by a constant quality price index, which from then it is observed to permanently decline relative to the price of non-durable consumption goods and services. It was then realized that, due to the permanent decline of the relative price of

[1]For economic index number theory, see Diewert (1993), Triplett (1992), Fisher and Shell (1998) and IMF (2004, chapter 17), among many others. A renewal of interest in using money metric utility for price measurement is in Redding and Weinstein (2018).

[2]See Fisher and Shell (1968) for a definition of a Fisher-Shell index and for a discussion about the conditions of its applicability. See Triplett (1992) for a discussion on the properties of the Fisher ideal index. National accounts in Europe measure real growth by the mean of chained-type Laspeyres index following the Commission Decision 98/715/EC.

equipment, investment tends to be overweighted in the fixed-base Laspeyres index due to the so-called substitution bias. As a consequence, GDP growth was overestimated when departing from the base year. All this lead the BEA to consider alternative measures of real GDP growth, switching in the early 90s to a Fisher ideal chain index which is known to well approximate a Divisia index.[3] This paper provides a rational for this methodological change based on the idea that real output growth as measured by index numbers reflects the underlying preferences of households in a well defined technological environment.

The recognition that the price of durables were declining relative to the price of non-durables had also consequences for macroeconomics. Growth theory has been reformulated in the late 90's in order to replicate this fact. Based on the hypothesis first formulated by Solow (1960) that technical progress is embodied in capital goods, Greenwood et al (1997) proposed a simple two-sector optimal growth model with investment specific technical change where productivity grows faster in the investment than in the consumption sector.[4] Unlike the one-sector growth model, the aggregation issue needs to be raised in a two-sector economy. The general methodology suggested in this paper is then used to measure output growth in this family of models, by studing the two-sector AK model proposed by Rebelo (1991), which replicates the empirical regularities referred above in the simplest possible world –see Felbermayr and Licandro (2005). By doing so, it helps to understand the main issues behind the change in methodology referred in the previous paragraph.

This theoretical framework also sheds light on an old debate in the growth and growth accounting literature. The so-called Solow-Jorgenson controversy was revived by the differing interpretations found in Hulten (1992) and Greenwood et al (1997). The controversy can be shown to boil down to the issue of the aggregation of consumption and investment when these are measured in different units and, more importantly, when its relative price has a trend. In our conceptual framework, it becomes clear that Greenwood

[3]In the conceptual framework developed in this paper, Appendix A.2 formally explains the substitution bias inherent to the Laspeyres fixed-base index. Appendix A.3 shows in the same framework that a Fisher ideal chain index is equal to the Divisia index.

[4]Many other papers have followed. See, for example, Krusell (1998), Gort et al (1999), Greenwood et al (2000), Cummins and Violante (2002), Whelan (2003), Boucekkine et al (2003, 2005) and Fisher (2006).

et al (1997) take a path that is more consistent with the theory. However, implicitly, these authors—and others following like Oulton (2007)—also adhere to a modern version of the paradigm that consumption, and consequently its growth rate, is the relevant measure of real growth.[5] In this paper, we claim that investment growth, as reflected by the Divisia index, also matters for output growth. Notice that NIPA methodology stresses the fact that the growth rate of investment does contain information relevant to households welfare since it reflects utility gains associated with postponed consumption. This is particularly relevant in a world where technical change is embodied in durable goods, and hence where technical progress only materialize through the incorporation of new physical capital.

The issue of trends in relative prices and different sectorial growth rates is also critical for the recent literature on structural transformation, since agriculture, manufacturing and services grow at different rates during the development process.[6] In line with our findings, Duernecker et al (2017) claim that using chain indexes more accurately reflects the effects of secular changes in relative prices.

It is important to point out that the main result on this paper, that output growth in NIPA is a welfare based measure, does not require a representative household. The proof that a Fisher-Shell index is equal to the Divisia index holds true even when agents have different preferences and income, and consequently equilibrium may differ from the equilibrium of the corresponding representative agent economy. When a Fisher-Shell index is applied to a dynamic general equilibrium economy with heterogenous households, money is used as a common norm to evaluate welfare changes across individuals; money metric utility implicitly adopts an utilitarian approach weighting each households proportional to its own income. The approach leads to apply the Fisher-Shell index to individual households separately, and then aggregate their individual growth rates weighted by their shares on total income. Starting from the usual assumption that income levels may

[5]Greenwood et al (1997), in fact, is not a normative paper. It does perform the positive exercise of measuring the contribution of embodied technical change to US growth. However, in doing so, they measure output and its growth rate in units of consumption, de facto identifying real output growth with consumption growth. Cummins and Violante (2002) generalize the exercise and use standard NIPA methodology to the same objective, finding similar quantitative results. See also Greenwood and Jovanovic (2001). Sections 3 and 4 further discus these issues.

[6]For structural transformation, see Acemoglu and) (2008), Duarte and Restuccia (2010), Herrendorf et al (2013) and Ngai and Pissarides (2007), among many others.

be used to make inter-households comparisons, inter-households differences in income growth aggregate to the average growth rate of the economy. Such an approach reduces to the analysis of the income side of NIPA and, if data were available, it may be used to study the social welfare implications of observed phenomena like job polarization, where different occupations face different growth rates in earnings depending on their position in the income distribution.[7]

But, what do we mean when we say that output growth in national accounts is a welfare based measure? In order to answer this question, it is important to understand the implications of applying money metric utility in this context. Notice that when using national accounts to measure welfare gains, we face two fundamental problems. We need first to understand the relation between current production and welfare, to then understand how by subtracting inflation from nominal output growth, real output growth measures gains in welfare.

How does current production relates to households' welfare? The answer somehow relates to the seminal contribution of Weitzman (1976). Let us decompose our main argument. First, since national accounts aggregate current consumption and current investment when measuring output growth, in order to apply index number theory to a two-sector dynamic general equilibrium economy, we use the Bellman equation to represent households preferences in the consumption-investment dimension. Second, in the Bellman equation, the value function represents the equilibrium value of assets, evaluated from the point of view of households preferences, i.e., the equilibrium discounted flow of consumption utility. The value function is then a measure of households welfare defined in the domain of households assets. Third, the Bellman equation states that the return to assets, evaluated using the subjective discount rate, has to be equal to the utility of current consumption plus the value of current net investment. In this sense, when we apply index number theory to the representation of preferences emerging from the Bellman equation, we are not directly measuring welfare gains, but gains in the return to assets, using indeed households preferences to do so. Fourth, at any time, the economy uses households assets to produce output, which actually is the return to these assets. Consequently, in this context, output is nothing else than the sum of consumption utility plus the value of net investment as stated by the Bellman equation. Finally, let us stress that the return to assets, i.e. current production, is allocated not only to current con-

[7] For job polarization, see Autor and Dorn (2013) and Goos et al (2014), among others.

sumption, which generates current well-being, but also to current net investment, which will contribute through the creation of new assets to generate well-being in the future. For this reason, as pointed out by Weitzman (1976), from a welfare point of view when measuring current output we should add both consumption and net investment.

How inflation has to be subtracted from nominal income growth in order to real output growth measure gains in welfare? Notice, first, that a utility function is a particular representation of households preferences: monotonic transformations of it change the level of utility leaving the preference map intact; so the growth rate of a particular representations is meaningless. To overcome this problem, the traditional approach to welfare measurement is to create a money metric representation of the underlying preferences by the mean of the so-called equivalent variation. Suppose an agent with a given income optimally buy a particular consumption basket at given prices, but then she receives in exchange a new preferred basket. The *equivalent variation* to this problem is the hypothetical increase in income (at the given prices) that helps the consumer reach the same level of utility as with the new consumption basket –see chapters 1 and 2 in McKenzie (1983). Since the same exercise may be repeated for all available consumption baskets, the equivalent variation can be seen as the change in a *money metric utility representiom* of the underlying preferences because a basket is at least as preferred to another if the equivalent variation is non-negative. Second, in a dynamic economy like the one described in this paper, at any time we observe a current consumption-investment choice and current prices. The next period, since household income, the shadow value of capital and equilibrium prices are different, we observe a new combination of consumption and investment. To compare these two situations the Fisher-Shell true quantity index measures changes in welfare, as represented by the Bellman equation, by the mean of an equivalent variation using the initial prices and shadow value of capital as the benchmark. This paper shows that the equivalent variation in percentage of current income turns out to be the Divisia index. Hence, the chained index used in NIPA can be seen as a measurement of welfare change. In this sense, this paper extends Witzman (1976) result to the measurement of output growth in NIPA.

This paper is organized as follows. Section 2 first describes a two-sector representative household general equilibrium economy with general recursive preferences and quasi-concave technology. It applies index number theory to the Bellman representation of these preferences and proves that the Fisher-Shell true quantity index is equal to the

Divisia index. Then, it extends the result to an economy with household heterogeneity in preferences and income. Section 3 illustrates it in the interesting case of the two-sector AK model, which replicates the observed permanent decline in the relative price of investment. In this framework, it discusses the issue of money metric utility and compare the Divisia index with a consumption equivalent measure of output growth. Finally, Section 4 discusses the main implications of our results and Section 5 concludes and suggests some possible extensions.

2 Measuring output growth

Consider a two-sector non-stochastic perfectly competitive dynamic general equilibrium economy with two goods, consumption and investment, and a general quasi-concave technology transforming capital and labor into these two goods. Firms hire capital and labor to produce them, and under the usual intertemporal budget constraint, a representative household chooses continuously consumption and savings in order to maximize intertemporal utility. All along this paper, we assume that preferences and technology are such that an equilibrium path exists and is unique.

In this framework, a National Statistical Office (NSO) would like to measure output growth consistently with individual preferences and technology. Indeed, at equilibrium the NSO only observes current nominal consumption and investment, and the corresponding prices, but has no information about individual preferences, technology and future consumption. Let us finally assume that the NSO uses this information to measure the growth rate of both consumption and investment and, then, uses index number theory to compute the growth rate of output.

The general problem in national accounts is to find an index built out of observables at t, current consumption and investment, and the associated prices, that measures changes in real output. For the fictitious economy, the NSO aggregates equilibrium consumption and investment by the mean of a Fisher-Shell true quantity index –controlling for changes in equilibrium prices. Section 2.2 shows how to construct a Fisher-Shell index in this context and proves that the Fisher-Shell index is equal to the Divisia index, which in continuous time is equal to the Fisher ideal chain index –the one used in NIPA to measure GDP growth.[8] The resulting rate of output growth is then welfare based. Section 2.3

[8]The property that in continuous time the Fisher ideal chain index is equal to the Divisia index is

generalizes the result to heterogeneous households.

2.1 Bellman equation under recursive preferences

The economy evolves in continuous time. For any date $t \geq 0$ and any consumption path $C : [0, \infty) \to \mathbb{R}_+$, ${}_tC$ denoting the restriction of C to the interval $[t, \infty)$, preferences of the representative household are represented by some recursive utility function U generated by the differential equation

$$\frac{\mathrm{d}}{\mathrm{d}t} U({}_tC) = -f(c_t, U({}_tC)). \tag{1}$$

The generating function f is assumed to be differentiable, with $f_1 > 0$ and $f_2 < 0$. Note that f_1 is the marginal utility from current consumption, lost when we move an infinitesimal period of time ahead, and so the negative sign in (1). In turn, $f_2 < 0$ is related to the implicit subjective discount rate.[9] For instance, the classical additively separable utility function is an important particular case of the general specification above in which

$$U({}_tC) = \int_t^\infty e^{-\rho(s-t)} u(c_s) ds$$

with $u'(c) > 0$, $u''(c) < 0$ and $\rho > 0$. Differentiate with respect to time t to write

$$\frac{\mathrm{d}}{\mathrm{d}t} U({}_tC) = -u(c_t) + \rho U({}_tC).$$

Hence, in this case, $f(c, U) = u(c) - \rho U$ and indeed $f_1(c, U) = u'(c) > 0$ while $f_2(c, U) = -\rho < 0$. Indeed, this clarify the interpretation given above that f_1 is the marginal utility from current consumption, lost when we move an infinitesimal period of time ahead, and f_2 is the return to household assets, which value is represented by $U({}_tC)$ and the discount rate is ρ.

Each instant t, a social planer chooses individual consumption c_t and per capita net investment $\dot{k}_t$ such that $(c_t, \dot{k}_t) \in \Gamma(k_t, \Theta_t)$ is feasible, where k_t is capital and Θ_t represents a vector of exogenous non-stochastic states. In the following, we assume that, for a given $k_t > 0$, there exists a unique consumption and investment path equilibrium $(c_s, \dot{k}_s)_{s \geq t}$ that maximizes $U({}_tC)$ subject to the technological constraint. Then, total

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shown in Appendix A3.

[9] Epstein (1987) explores conditions under which a generating function f represents a recursive utility function U. Becker and Boyd (1997, chapter 1) motivates the study of general recursive preferences.

utility is $U({}_tC)$ and the current change in welfare as measured by $U({}_tC)$ is simply given by (1).

A NSO aiming to build a true quantity index measuring output growth in this economy faces, in addition to the well known problem that preferences are not univocally represented by a utility function, the additional problem from an accounting point of view that neither preferences and technology nor foreseen consumption are observable. In this context, a NSO wishes to build a quantity index that reflects changes in welfare using only current consumption c_t and current net investment $x_t = \dot{k}_t$, both observables at instant t; and all that matters of the level of k_t is summarized in the price of investment p_t as we will argue below. To this end, however, a NSO shall need to express preferences as a function of variables observed at t. Since preferences are recursive, this amounts to express changes in welfare as a function of current consumption c_t and current net investment x_t.

In other words, a NSO needs a representation of preferences over current consumption and current net investment, and this is what the Bellman equation gives us. The original problem is to maximize $U({}_tC)$ subject to $(c_s, \dot{k}_s) \in \Gamma(k_s, \Theta_s)$ for all $s \geq t$, $k_t > 0$ given, where Θ_s is a vector of exogenous states that directly affect technology. The associated Bellman equation is

$$0 = \max_{(c,x)\in\Gamma(k_t,\Theta_t)} f(c, v(k_t, \Theta_t)) + v_1(k_t, \Theta_t)x + v_2(k_t, \Theta_t)\dot{\Theta}_t. \tag{2}$$

The intuition behind this equation becomes clear if one notes that along an optimal path $v(k_t, \Theta_t) = U({}_tC)$ so $\mathrm{d}v(k_t, \Theta_t)/\mathrm{d}t = v_1(k_t, \Theta_t)\dot{k}_t + v_2(k_t, \Theta_t)\dot{\Theta}_t = -f\big(c_t, v(k_t, \Theta_t)\big)$. Note as well that, in a sense, with all past actions summarized in k_t, the objective function in (2) is giving us the preference relation over consumption and investment at instant t.[10]

2.2 Fisher-Shell true quantity index

In this section, we show that in the dynamic general equilibrium framework developed in the previous section, the Divisia index is a true quantity index. In regard of the Bellman equation (2), preferences of the representative consumer over consumption and

[10] The planner solves a standard recursive program in which the state variable summarizes at each instant t all past information that could be relevant for today's decisions. For a brief exposition of recursive techniques in continuous time see Obstfeld (1992).

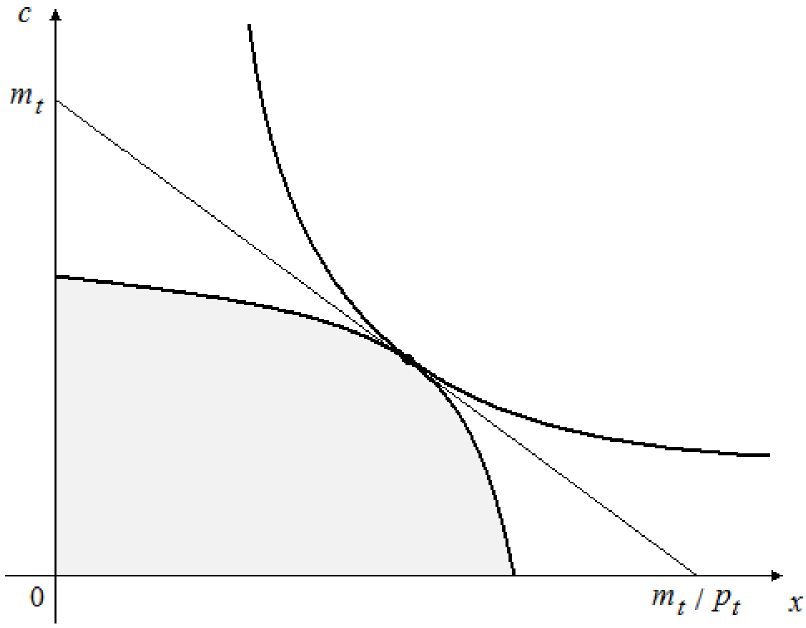

Figure 1: The production possibilities frontier and competitive prices

investment at instant t can be seen as represented by the function

$$w_t(c,x) \doteq f(c, v(k_t, \Theta_t)) + v_1(k_t, \Theta_t)x + v_2(k_t, \Theta_t)\dot{\Theta}_t.$$

To save notation, we write $w_t(c,x)$, but time enters this function only through the stock of capital k_t and the exogenous states Θ_t, both given at time t.

For a given state of the system, as represented by k_t and Θ_t, the function $w_t(c,x)$ can then be seen as a representation of individual preferences over consumption and net investment, the last summarizing postponed consumption. To the extent that the exogenous states and the stock of capital will change along an equilibrium path, these preferences are time-dependent. This is precisely the building block of the true quantity index introduced by Fisher and Shell (1968). Since welfare comparisons must be done within the same preference map, the Fisher-Shell true quantity index proposes to fix not only prices but also preferences. In particular, *it compares income today with the hypothetical level of income that would be necessary to attain the level of utility associated with tomorrow's income and prices with today's prices and today's preferences –as evaluated by $w_t(c,x)$.* The remain of this section elaborates this idea.

In the following, the NSO adopts the consumption good as numeraire. Of course, the choice of the numeraire is inconsequential. Let us define equilibrium nominal net income

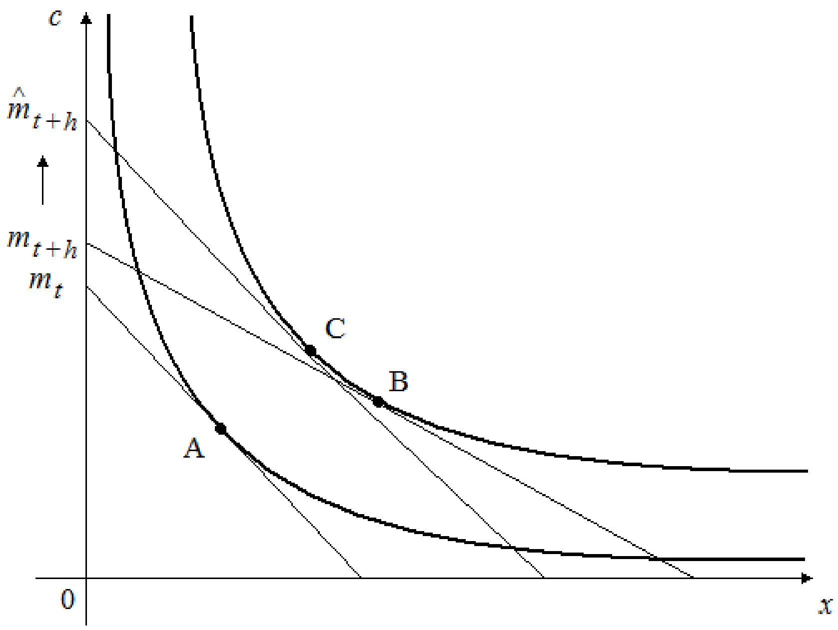

Figure 2: The Fisher-Shell true quantity index

at time t, along an equilibrium path for (c_t, x_t, p_t), as $m_t \doteq c_t + p_t x_t$. Under standard assumptions, optimal choices will lie in the boundary of $\Gamma(k_t, \Theta_t)$ so that there is a well-defined equilibrium price of net investment $p_t > 0$ expressed in units of consumption (see Figure 1). The constraint $(c, x) \in \Gamma(k_t, \Theta_t)$ can be replaced by the linear constraint $c + p_t x \leq m_t$ in the problem of the Bellman equation (see Figure 1). Hence, the associated indirect utility function of the representative household problem is defined as

$$u_t(m_t, p_t) \doteq \max_{c + p_t x \leq m_t} w_t(c, x)$$

while the expenditure function is

$$e_t(u_t, p_t) \doteq \min_{w_t(c,x) \geq u_t} c + p_t x.$$

The fundamental idea behind *money metric utility* is to use the expenditure function to make welfare comparisons, by associating utility u_t to observed income $m_t = c_t + p_t x_t$. Notice that at the equilibrium of the fictitious economy, the NSO observes $\{c_t, x_t, p_t\}$ at both the current period t and the future period $t + h$, $h > 0$.[11] Since comparisons must be done within the same preference map, the Fisher-Shell true quantity index fixes both prices and preferences. In particular, it compares income today m_t with the hypothetical

[11] Of course, the NSO produces these measures after period $t + h$.

level of income $\hat{m}_{t+h}$ that would be necessary to attain the level of utility $u_t(m_{t+h}, p_{t+h})$ associated with tomorrow's income and prices m_{t+h}, p_{t+h} with today's prices p_t and today's preferences as represented by functions $e_t(u, p)$ and $u_t(m, p)$. This artificial level of income is given by

$$\hat{m}_{t+h} = e_t\big(u_t(m_{t+h}, p_{t+h}), p_t\big).$$

The idea is illustrated in Figure 2 in a situation where nominal income increases and the price of investment declines. The preference map corresponds to instant t preferences as represented by w_t. Point A is the observed situation at instant t. Point B is the hypothetical choice using instant t preferences when facing observed prices p_{t+h} and income m_{t+h}. Point C represents the choice that maintains such level of utility but with prices p_t. In the end, it does compare two levels of income that correspond to the same price vector so it is clear that the index is extracting price changes. In this particular case, the true quantity index is just reflecting the fact that the true output deflator is dropping with the price of investment, that is to say that income in real terms is growing more than nominal income m_{t+h}/m_t.[12] Note that the difference $\hat{m}_{t+h} - m_t$ is the *equivalent variation* which can be seen as the composition of nominal growth $m_{t+h} - m_t$ and the *compensating variation* $\hat{m}_{t+h} - m_{t+h}$. The equivalent variation measures by how much income would have to increase beyond its nominal increase to compensate for not having the price of investment dropping.

In continuous time, the reasoning is the same but the time gap h tends to zero. The instantaneous Fisher-Shell index is defined as[13]

$$g_t^{\mathrm{FS}} \doteq \frac{\mathrm{d}}{\mathrm{d}h} \left.\frac{\hat{m}_{t+h}}{m_t}\right|_{h=0} = \frac{1}{m_t} \left.\frac{\mathrm{d}\hat{m}_{t+h}}{\mathrm{d}h}\right|_{h=0},$$

that is, the instantaneous growth rate of the factor $\frac{\hat{m}_{t+h}}{m_t}$ when h gets arbitrarily small. To compute this index note that

$$\left.\frac{\mathrm{d}\hat{m}_{t+h}}{\mathrm{d}h}\right|_{h=0} = e_{1,t}\big(u_t(m_t, p_t), p_t\big)\Big(u_{1,t}(m_t, p_t)\dot{m}_t + u_{2,t}(m_t, p_t)\dot{p}_t\Big)$$

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[12] If alternatively, the investment good were the numeraire, nominal income would be growing faster than the hypothetical income $\hat{m}$ and consumption price changes should be subtracted from nominal income growth to get real income growth. Indeed, the real growth rate will remain unchanged, since it does not depend on the choice of the numeraire.

[13] See Appendix A1 for a rationale of this definition.

where subscripts denote the partial derivatives with respect to the corresponding arguments. To obtain an expression for all these derivatives let us go back to the dual and primal problems discussed above. Let μ be the Lagrange multiplier of the maximization problem in the definition of the indirect utility function, measuring the marginal contribution of income m to welfare w. We have, from the the primal problem

$$u_{1,t}(m_t, p_t) = \mu$$
$$u_{2,t}(m_t, p_t) = -\mu x_t,$$

and, since the expenditure function is the inverse of the indirect utility function,

$$e_{1,t}(u_t, p_t) = \frac{1}{\mu}.$$

As expected, the marginal contribution of income to welfare, $\partial u/\partial m = \mu$, is equal to the inverse of the marginal contribution of utility u to total expenditure, $\partial e/\partial u = 1/\mu$. Moreover, the negative marginal contribution of prices to welfare is $\partial u/\partial p = -\mu x$, since an increase in prices reduces income by x units. These properties are critical for the result below and they are directly related to the *money metric utility* nature of the Fisher-Shell index, which defines the hypothetical income $\hat{m}$ using the expenditure function to valuate changes in utility after controlling for changes in prices.

Using the three conditions above in the definition of the Fisher-Shell index, we conclude that

$$g_t^{\mathrm{FS}} = \frac{\dot{m}_t - x_t \dot{p}_t}{m_t} = \frac{\dot{m}_t}{m_t} - \frac{p_t x_t}{m_t}\frac{\dot{p}_t}{p_t}.$$

Notice that the marginal terms e_1, u_1 and u_2 in the definition of the Fisher-Shell index simplify as a direct consequence of the properties discussed in the paragraph above; all three are related to the marginal value of income μ. It is in this sense that money metric utility operates in the Fisher-Shell index. Since gains in welfare are measured as a compensating variation by comparing the artificial level of income $\hat{m}_{t+h}$ with the nominal income m_t, and prices enter linearly in the budget constraint, gains in welfare are equal to the change in nominal income (arbitrarily measured here in units of the consumption good) minus the contribution of prices to it (which comes only from the change of investment prices, weighted by the equilibrium (net) investment share).

Finally, differentiate the definition of nominal income $m_t = c_t + p_t x_t$ with respect to time and define the equilibrium share of net investment to net income as $s_t \doteq p_t x_t / m_t$ to write

$$\frac{\dot{m}_t}{m_t} = (1 - s_t)\,\frac{\dot{c}_t}{c_t} + s_t \frac{\dot{x}_t}{x_t} + s_t \frac{\dot{p}_t}{p_t},$$

which implies that

$$g_t^{\text{FS}} = (1 - s_t)\,\frac{\dot{c}_t}{c_t} + s_t\frac{\dot{x}_t}{x_t} \doteq g_t^{\text{D}}$$

where g_t^{D} denotes the Divisia index. We have then shown that, for all t, the Fisher-Shell index g_t^{FS} is equal to the Divisia index g_t^{D}. In this framework, by definition, the Divisia index is the average of the growth rates of consumption and net investment, weighted by their corresponding equilibrium shares in total net income.

We have then shown that in this framework the Divisia index is a true quantity index, and as such it is a welfare measure. The interpretation is straightforward. It is clear that g_t^{FS} is a measure of real growth since it is constructed as the growth rate of nominal income subtracting pure price changes, in this case the change of the relative price of investment p_t. The index only keeps changes in quantities. It is also clear that it is a true index because it is constructed from the representative household's preferences using standard theory.[14] The beauty of the result is that the NSO does not need to know people preferences and production technology, but current quantities and prices.

2.3 On household heterogeneity

The argument above was built under the assumption of a representative household. In this section, we show that the same reasoning applies to a heterogeneous agents economy where households have both heterogeneous preferences and heterogeneous income. Critical in the result is the fact that the utility representation of preferences derived from the Bellman equation is quasilinear, belonging to the Gorman family.[15]

First, let us assume that there is a continuum of heterogeneous households of unit mass with recursive preferences represented by the utility U_i generated by the differential equation

$$\frac{1}{dt}U_i({}_tC_i) = -f_i(c_{i,t}, U_i({}_tC_i)),$$

where ${}_tC_i$ represents the consumption path of household i and function f_i has the same

[14] This equivalence would come as no surprise to index number theorists. The Fisher ideal chain index is known to approximate in general some sort of true quantity index because both are bounded from above and below by the Laspeyres and Paasche indexes respectively. In continuous time, these indexes tend to each other as the time interval h tends to zero. Further, in general, the Divisia index coincides with the Fisher ideal chain index if the growth rates of consumption and investment are constant.

[15] See Gorman (1953, 1961).

properties as above. Second, at equilibrium capital is distributed across households according to φ_t, which maps any individual i at any instant t into a quantity of capital. Let us finally assume that, for this economy, an equilibrium exists and is unique. Notice that equilibrium may likely be different from the corresponding equilibrium with a representative household, implying that the distribution of preferences and capital across individuals matters.

In the recursive competitive equilibrium representation of this economy, with exogenous state Θ_t and an equilibrium distribution of capital φ_t, the problem of a household i with capital $k_{i,t}$ can be written as

$$\begin{aligned} 0 = \quad \max \quad & f_i(c_i, v_i(k_{i,t}, \Theta_t, \varphi_t)) + v_{i,1}(k_{i,t}, \Theta_t, \varphi_t)x_i + \pi_{i,t} \\ \text{s.t.} \quad & c_i + p_t x_i = m_{i,t} \end{aligned}$$

where c_i and x_i are household's current consumption and net investment, respectively, p_t is the equilibrium price, common to all households, and $m_{i,t}$ is the equilibrium net income of individual i. $\pi_{i,t}$ represents the differential terms of $v_i(k_{i,t}, \Theta_t, \varphi_t)$ with respect to time that are exogenous to the problem of the consumer, i.e., those corresponding to Θ_t and φ_t.

As in section 2.2, the optimization problem of household i is associated with the instantaneous utility function over consumption and net investment

$$w_{i,t}(c_i, x_i) \doteq f_{i,t}(c_i) + x_i,$$

where $f_{i,t}(c_i) \doteq f_i(c_i, v_i(k_{i,t}, \Theta_t, \varphi_t))/v_{i,1}(k_{i,t}, \Theta_t, \varphi_t)$. Notice that we have subtracted $\pi_{i,t}$ from the right hand side of the Bellman equation and then divided it by $v_{i,1}(k_{i,t}, \Theta_t, \varphi_t)$. Since non of these two terms depend on c or x, such a transformation has no effect on the households program. Function $w_{i,t}(c_i, x_i)$ is maximized under the budget constraint $c_i + p_t x_i = m_{i,t}$, where, as said above, p_t is the equilibrium price and $m_{i,t}$, for all i, represents equilibrium household net income. Since this utility representation is quasilinear, it belongs to the Gorman family. It is easy to show that the indirect utility and expenditure functions become

$$\begin{aligned} u_{i,t}(m_{i,t}, p_t) &= A_{i,t}(p_t) + m_{i,t}/p_t \\ e_{i,t}(u_{i,t}, p_t) &= p_t\big(u_{i,t} - A_{i,t}(p_t)\big), \end{aligned}$$

where $A_{i,t}(p_t)$ is defined below. In fact, from the household problem, optimal consumption c_i solves

$$f'_{i,t}(c_i) = 1/p_t.$$

By denoting the implicit solution for c_i as $c_{i,t}(p_t)$, it is then easy to show that

$$A_{i,t}(p_t) = f_{i,t}\big(c_{i,t}(p_t)\big) - c_{i,t}(p_t)/p_t.$$

Let us define the artificial level of household i income as in section 2.2, i.e.,

$$\hat{m}_{i,t+h} = e_{i,t}\big(u_{i,t}(m_{i,t+h}, p_{t+h}), p_t\big) = p_t\Big(A_{i,t}(p_{t+h}) - A_{i,t}(p_t)\Big) + p_t/p_{t+h}\, m_{i,t+h},$$

which is linear on income due to the fact that preferences are quasilinear. Consistently with national accounts, aggregate income is defined as $m_t = \int_i m_{i,t}\mathrm{d}i$, which also measures per capita income since population has been normalized to unity. Let us now define the aggregate hypothetical income consistently with the definition of per capita income as $\tilde{m}_t = \int_i \hat{m}_{i,t}\mathrm{d}i$. Using the results just above,

$$\tilde{m}_{t+h} = p_t\Big(\bar{A}_t(p_{t+h}) - \bar{A}_t(p_t)\Big) + p_t/p_{t+h}\, m_{t+h},$$

where

$$\bar{A}_t(p_t) = \int_i A_{i,t}(p_t)\mathrm{d}i.$$

Notice that, in general, the average hypothetical income $\tilde{m}_{t+h}$ at the equilibrium of the heterogeneous household economy, may be different from the hypothetical income $\hat{m}_{t+h}$ at equilibrium of the representative household economy, since these two economies may likely have different equilibrium paths.

As in section 2.2, let us define the Fisher-Shell index for the economy with heterogeneous households as

$$\tilde{g}_t^{\mathrm{FS}} \doteq \frac{1}{m_t}\left.\frac{d\tilde{m}_{t+h}}{dh}\right|_{h=0}.$$

Operating on the definition of $\tilde{m}_{it+h}$ above

$$\left.\frac{d\tilde{m}_{t+h}}{dh}\right|_{h=0} = \dot{m}_t + \Big(p_t\bar{A}_t'(p_t) - m_t/p_t\Big)\dot{p}_t.$$

Notice that

$$\bar{A}_t'(p_t) = \int_i A_{i,t}(p_t)di = \int_i \big(\underbrace{f_{i,t}'c_{i,t}' - 1/p_t c_{i,t}'}_{=0,\text{ since } f_{it}'=1/p_t} + c_{i,t}/p_t^2\big)di = c_t/p_t^2,$$

where $c_t = \int_i c_{it}\mathrm{d}i$ is per capita consumption. Then

$$\tilde{g}_t^{\mathrm{FS}} = \frac{\dot{m}_t}{m_t} - s_t\frac{\dot{p}_t}{p_t} = (1-s_t)\frac{\dot{c}_t}{c_t} + s_t\frac{\dot{x}_t}{x_t},$$

where $s_t \doteq p_t x_t / m_t$ as before and $x_t = \int_i x_{it} \mathrm{d}i$ is per capita net investment. The Fisher-Shell index is, indeed, equal to the Divisia index, meaning that the growth rate in NIPA is a welfare measure irrespective of households being either homogeneous or heterogeneous. Of course, at equilibrium, consumption and investment may be growing at different rates than in the corresponding representative household model, and the saving rate may also be different. Consequently, even when the growth rate, as measured by the Divisia index is a welfare measure in both economies, these two economies may be growing at different rates.

Two assumptions are critical for the main result in this section, i.e., that the Fisher-Shell index is equal to the Divisia index under heterogeneous households. First, as in the case of homogeneous households, under money metric utility, nominal income is the metric used to measure households' utility, implying that gains in welfare are measured as gains in nominal income minus inflation; the main principle used by national accounts. The second critical assumption is the use of the quasilinear representation of preferences that emerges from the Bellman equation representation of household preferences in the space of current consumption and current net investment. This assumption is not critical at all in the case of a representative household; in facts, in Section 2.2, we show that the Fisher-Shell index is equal to the Divisia index for a general function $w(c, x)$. Indeed, it is critical in this section, since we profit from the quasi linearity representation of preferences to show that aggregate utility gains, as measured by the Fisher-Shell index, are equal to gains in nominal per capita income minus inflation.

This result comes at non surprise. By adopting aggregate nominal income as a norm for measuring aggregate output, the Fisher-Shell index implicitly assumes that the aggregate welfare function is utilitarian, giving to each household a weight proportional to its income. This clearly reflects in the definition of the artificial income measure $\tilde{m}_t$.

3 Embodied technical progress

As referred in the Introduction, following Gordon (1990)'s observation that quality adjusted equipment investment prices were permanently declining relative to the price of non-durable consumption goods and services, the Bureau of Economic Analysis (BEA) moved first to control for quality improvements in the measurement of investment prices, and second to a Fisher ideal chain index to measure output growth. The first change made investment to grow faster than non-durable consumption. As an undesirable consequence of trends in relative prices, the fixed-base quantity index used to measure GDP growth became obsolete fast enough to provided appropriate growth figures. In facts, in this case, fixed-base quantity indexes suffer from the well known substitution bias problem that tends to overestimate the weight of the fast growing items (see Appendix 2). The second change addresses this last problem by making the NIPA measure of output growth to be approximately equal to the Divisia index.

Almost contemporaneously, a new literature developed in macroeconomics aimed to accommodate growth theory to this new evidence. Greenwood et al (1997), in their seminal paper, extend the Ramsey model to a two sector (consumption and investment) growth model with two sources of technical progress, consumption and investment specific technical change (disembodied and embodied in capital goods, respectively). This model is able to replicate the permanent decline in the relative price of equipment investment, as well as the fact that investment grows faster than consumption (implying that the investment to output ratio is permanently growing). In this context, it is particularly clear that the aggregation issue is far from trivial since consumption and investment grow at different rates.

This section describes a simple version of the two-sector AK model proposed by Rebelo (1991) and apply to it the Fisher-Shell index proposed in Section 2.2 to show that the BEA had good fundamental reasons to use a Fisher ideal chain index to measure output growth. As shown in Felbermayr and Licandro (2005), the two-sector AK model is the simplest endogenous growth model that replicates the observed permanent decline in the relative price of equipment and the permanent increase in the investment to output ratio.[16] We have preferred to use it instead of the original Greenwood et al (1997) model, since the AK model has the advantage of jumping to the balanced growth path from the initial time, which allows for an explicit solution of the value function. This is very useful to understand the role of money metric utility in the main statement of this paper that the growth rate in NIPA is a welfare measure.

[16]See also Acemoglu (2009), chapter 11.3.

3.1 The two-sector AK model

The model in this section is based on Rebelo (1991), follows Felbermayr and Licandro (2005) closely, and entails all the characteristics that are relevant to the present discussion in the simplest possible framework. The stock of capital at each instant t is k_t, from which a quantity $h_t \leq k_t$ is devoted to the production of the consumption good. Consumption goods technology is

$$c_t = h_t^{\alpha},$$

where $\alpha \in (0,1)$. The remaining stock $k_t - h_t \geq 0$ is employed in the production of new capital goods with a linear technology

$$\dot{k}_t = A(k_t - h_t)$$

where $A > 0$ is the marginal product of capital in the investment sector net of depreciation. There is a given initial stock of capital $k_0 > 0$. Again, we will write $x_t = \dot{k}_t$ for net investment.

The representative household has preferences over consumption paths represented by[17]

$$\int_0^{\infty} \frac{c_t^{1-\sigma}}{1-\sigma} \, \mathrm{e}^{-\rho t} \, \mathrm{d}t, \tag{3}$$

that is, the additive case mentioned above, where $\rho > 0$ is the subjective discount rate and $\sigma \geq 0$ the inverse of the intertemporal elasticity of substitution.

In the absence of market failures, equilibrium allocations are solutions to the problem of a planner aiming at maximizing household's utility subject to the technological constraints. The Bellman equation associated with the planner's problem is

$$\rho v(k_t) = \max_{x = A(k_t - c^{1/\alpha})} \frac{c^{1-\sigma}}{1-\sigma} + v'(k_t)x. \tag{4}$$

The value function $v(k_t)$, which is (3) evaluated at equilibrium, represents the value of capital measured as the discounted flow of consumption utility. The return to assets, as measured by the subjective discount rate ρ, is equal to the utility of current consumption plus the value of net investment, representing the utility of postposed consumption –the future consumption that this additional capital will produce in the future.

[17]This is a particular case of the general preferences in Section 2.1. Here the correspondence Γ is defined for every $k \geq 0$ as the set $\Gamma(k)$ of pairs $(c, \dot{k})$ such that there exists h with $0 \leq h \leq k$, $c \leq h^{\alpha}$, and $\dot{k} \leq A(k-h)$.

The equilibrium growth rate of capital is

$$\gamma = \frac{A-\rho}{1-\alpha(1-\sigma)}. \tag{5}$$

From the feasibility constraints, it is clear that the growth rate of investment is also γ, and that $\alpha\gamma$ is the growth rate of consumption. Competitive equilibrium allocations are balanced growth paths from $t \geq 0$.

Returns to scale differ between sectors. Since $\alpha < 1$, as the stock of capital grows the investment sector becomes more productive with respect to the consumption goods sector. Differences in productivity causes the decline in investment prices relative to consumption goods prices. This difference in returns to scale can be interpreted, as put forward by Boucekkine et al (2003), as a consequence of strong spillovers in the production of investment goods.[18] From the feasibility constraints, we can obtain the competitive equilibrium price of investment in terms of consumption units as the marginal rate of transformation:

$$p_t = -\frac{dc_t}{dx_t} = -\frac{dc_t}{dh_t}\frac{dh_t}{dx_t} = \frac{\alpha h_t^{\alpha-1}}{A}.$$

Since the stock of machines used in the consumption goods sector grows at the constant rate γ, the price of investment relative to consumption decreases at rate $(\alpha-1)\gamma < 0$.

The competitive equilibrium allocation displays the regularities observed in actual data. Investment grows faster than consumption since $\gamma > \alpha\gamma$. The relative price of investment decreases at rate $(\alpha-1)\gamma < 0$. Indeed, the nominal share of investment in income remains constant. To see this, let us take the consumption good as numeraire and define nominal income as in the general case as $m_t = c_t + p_t x_t$. From the equilibrium equations, one can show after some simple algebra that the investment share

$$s_t = \frac{p_t x_t}{m_t} = \frac{p_t x_t}{c_t + p_t x_t} = \frac{\alpha(A-\rho)}{\rho(1-\alpha)+\alpha\sigma A} \doteq s$$

for all $t \geq 0$. To be precise, s is the equilibrium share of investment in total income.[19]

[18] Cummins and Violante (2002) observe that their measure of investment-specific technical change occurs first in information technology and then accelerates in other industries. They conclude that information technology is a "general purpose" technology, an interpretation that matches well with the spillovers' interpretation. See also Boucekkine et al (2005).

[19] It is interesting to notice that in the two-sector AK model, as well as in the balanced growth path of the Greenwood et al (1997) model, the investment share is constant. Consequently, measuring output growth by the mean of a fixed-base or a chained index does not make any difference.

At this point it may be worth stressing that the choice of the consumption good as numeraire is inconsequential. The argument above follows equally if we choose to measure income in units of investment, $p_t^{-1}c_t + x_t$, or, for that matter, in any other arbitrary monetary unit provided that relative prices are respected –that is, that the price of investment relative to consumption is p_t. This is important because identifying real growth with growth of nominal income is as arbitrary as the choice of the numeraire in which nominal income is expressed.

3.2 Measuring output real growth

In this section, we apply the general theory proposed in Section 2 to the two-sector AK model. As in the general case, in regard of the Bellman equation (4), the function

$$w_t(c, x) = \frac{c^{1-\sigma}}{1-\sigma} + v'(k_t)x$$

can be seen as representing preferences over current consumption and current net investment. Again, the constraint in the Bellman equation (4) can be replaced by the budget constraint $c + p_t x \leq m_t$ because the budget line is tangent to the production possibilities frontier locally at the optimum. Notice that in this example the utility representation $w_t(c, x)$ changes over time only because the marginal value of capital does. In this sense, applying the Fisher-Shell index in this context is equivalent to compute a quantity index that *compares income today with the hypothetical level of income that would be necessary to attain the level of utility associated with tomorrow's income and prices, with today's prices and today's shadow value of capital.*

Let us define the indirect utility $u_t(m_t, p_t)$ and the expenditure function $e_t(u_t, p_t)$ as in Section 2. Recall that the Fisher-Shell true quantity index compares income today m_t with the hypothetical level of income $\hat{m}_{t+h}$ that would be necessary to attain the level of utility associated with tomorrow's income and prices m_{t+h}, p_{t+h} with today's prices p_t and today's preferences as evaluated by e_t, u_t. It is important to notice that, since $v'(k_t)$ is the shadow price of capital, applying the Fisher-Shell index in this context reduces to compare current income with an hypothetical income using today's market and shadow prices of investment. Nothing else changes with t in the preference representation $w_t(c, x)$ than $v'(k_t)$.

Denote again this artificial level of income as

$$\hat{m}_{t+h} = e_t\big(u_t(m_{t+h}, p_{t+h}), p_t\big).$$

From the definition of g_t^{FS} in Section 2, we conclude that, for all $t \geq 0$,

$$g_t^{\mathrm{FS}} = (1-s)\alpha\gamma + s\gamma = \frac{\alpha A(A-\rho)}{\rho(1-\alpha)+\alpha\sigma A}$$

As already said, the Fisher-Shell quantity index is equal to the Divisia index. As in the general case, the interpretation is straightforward: g^{FS} is a measure of real growth because it is constructed as the growth rate of nominal income substracting pure price changes, in this case the change of the relative price of investment p_t. The index only keeps changes in quantities. It is also clear that it is a true index because it is constructed from the representative household's preferences.

3.3 On money metric utility

What are the implications of money metric utility for the measurement of output growth in the case of the two-sector AK model? This section argues, that money metric utility, implicit in the Fisher-Shell index, selects a particular representation of preferences that makes welfare to grow at the rate g^{FS}. This particular representation depends crucially on preferences and technology. Let us develop this argument.

Since the two-sector AK model jumps to the balanced growth path at the initial time, a constant fraction of total capital will be permanently allocated to the production of consumption goods and capital will be permanently growing at the endogenous rate γ. After substituting the optima consumption path in (3), the value function reads

$$v(k_t) = Bk_t^{\alpha(1-\sigma)}, \quad \text{with} \quad B = \frac{(A-\gamma)^{\alpha(1-\sigma)}}{(1-\sigma)\big(\rho - \alpha\gamma(1-\sigma)\big)}. \tag{6}$$

Notice that both the exponent of k_t and B depend on preferences and technology.

The argument is the following. The utility function in (3) is one among many representations of the same preference order –constant intertemporal elasticity of substitution preferences. The Fisher-Shell index arbitrarily choses another, the one that at equilibrium grows at rate g^{FS} and adopts nominal income at some reference time as its benchmark. We build the argument in two steps.

First, let us denote by $\hat{v}_t$ the equilibrium welfare of the representative agent at time t measured on an arbitrary unit. Let us then make two assumptions concerning $\hat{v}_t$, consistently with the main implicit assumptions of the Fisher-Shell index. We assume first that at the initial time, $t = 0$, $\hat{v}_0 = (c_0 + p_0 x_0)/\rho$. This is the money metric utility assumption that the return to assets is equal to nominal income at the reference time, here $t = 0$. The second assumption is that $\hat{v}_t$ grows at the rate $g = g^{\mathrm{FS}}$, meaning that $\dot{\hat{v}}_t = g\hat{v}_t$. Then, for all $t \geq 0$,

$$\hat{v}_t = \hat{v}_0 \, \mathrm{e}^{gt}.$$

Consequently, if a utility representation of household preferences consistent with the Fisher-Shell index exists, it has to be that at equilibrium welfare is a potential function of k_t with exponent g/γ. Let us now show that such a representation exists.

Second, we adopt the following alternative representation of the constant intertemporal elasticity of substitution preferences in (3)

$$\hat{v}(k_t) = \max C \left(\int_t^{\infty} \frac{c_s^{1-\sigma}}{1-\sigma} \, \mathrm{e}^{-\rho(s-t)} \, \mathrm{d}t \right)^{\beta},$$

which is maximized subject to the technological constraints in Section 3.1; with $\beta C > 0$. Since this new utility function represents the same preferences as those of the original two-sector AK model, the equilibrium path is the same. Consequently, we can easily show that at equilibrium

$$\hat{v}(k_t) = C \, v(k_t)^{\beta} = \hat{v}_0 \, \mathrm{e}^{gt},$$

where $C = \hat{v}_0 B^{-\beta} k_0^{-\alpha(1-\sigma)\beta}$ and $\beta = \frac{g}{\alpha\gamma(1-\sigma)}$, both depending on preferences and technology parameters, and C additionally dependign on both the initial capital stock and the initial nominal income.[20] We have then proved that the growth rate as measured by the Fisher-Shell index is a welfare measure in the sense that it is equal to the growth rate of a particular representation of household preferences. The choice of this representation directly results from the key assumptions in money metric utility that welfare is measured in units of nominal income at some reference time.

3.4 On consumption equivalence

When measuring welfare gains, consumption equivalence is a usual way of dealing with the previous referred problem of the representation of preferences. Notice that consumption equivalence is a compensating variation measure, in terms of the entire consumption path instead of current income. In our context, the problem reduces to measure the hypothetical increase in the consumption path that makes an individual evaluating her welfare at time t indifferent between staying at t or jumping directly to time to $t + h$.

[20]Notice that β and C may be both positive or negative depending on σ being smaller or larger than one, respectively. The constraint $\beta C > 0$ then holds for any $\sigma > 0$.

Let us formulate the problem formally in the case of the two-sector AK model developed in this secction.

Let us assume that preferences are separable and constant intertemporal elasticity of substitution, like in (3). The hypothetical increase in the consumption path λ_h that makes an individual indifferent between staying at t or jumping to $t+h$ has then to verify the condition

$$\lambda_h^{1-\sigma} v(k_t) = v(k_{t+h}),$$

where $v(k_t)$ and $v(k_{t+h})$ refer to the equilibrium welfare of the representative household at times t and $t+h$ as measured by the value function $v(k)$. Indeed, the compensating variation λ_h directly depends on the length of the interval h.

From (6), the previous condition becomes

$$\lambda_h = \left(\frac{v(k_{t+h})}{v(k_t)} \right)^{\frac{1}{1-\sigma}} = \mathrm{e}^{\alpha\gamma h}.$$

The growth rate of welfare consistent with consumption equivalence is then the derivate of λ_h with respect to h evaluated at $h = 0$, which in the case of the two-sector AK model reads

$$g^{\mathrm{ce}} = \frac{1}{\lambda_h} \frac{\mathrm{d}\lambda_h}{\mathrm{d}h} \bigg|_{h=0} = \alpha\gamma.$$

The consumption equivalence measure of the growth rate of the the two-sector AK economy is equal to the growth rate of consumption. The result comes at no surprise, since the consumption path implicit in $v(k_{t+h})$ is the same as the consumption path in $v(k_t)$ multiplied by the factor $\mathrm{e}^{\alpha\gamma t}$. We have then to conclude that in the case of the two-sector AK model, measuring growth by the mean of the growth rate of consumption is also welfare based. The simplicity of the results relies on the fact that the two-sector AK economy is always at its balanced growth path. In the general case of a concave technology, like in Greenwood et al (1997), if the economy is not at its balanced growth path, the calculation of the consumption equivalent growth rate is not straightforward and this results cannot be easily extended.

4 Discussion

In the framework of two-sector dynamic general equilibrium models, Section 2 shows that the Divisia index is, in fact, a true quantity index. This is of substantive interest since the Fisher ideal chain index used in actual national accounts approximates well the Divisia index, implying that the growth rate of output in NIPA is welfare based. This section discusses the implications of this result. To make our main point clear, this section refers to representation of preferences like in equation (4).

More on money metric utility. Notice that at equilibrium the welfare of the representative household, $v(k)$ in the Bellman equation (4), measures the value of assets, represented here by the capital stock. Then, $\rho\, v(k)$ is the return to these assets as evaluated using the subjective discount rate ρ. From (4), at equilibrium the return to assets is equal to the utility of current consumption plus the value of current investment, the latter being assessed at the marginal value of capital $v'(k)$. Of course, welfare as measured by $v(k)$ is defined in an arbitrary unit: monotonic transformations of preferences will change the level of utility leaving the preference map intact; consequently, the growth rate of different representations will not be necessarily the same. To overcome this problem, as discussed in the introduction, the literature on welfare measurement adopts current income as a sensible norm to measure changes in welfare; it does by using an equivalent variation measure (i.e., money metric utility). Since income as measured by national accounts represents the return to the stock of assets, the Fisher-Shell quantity index and then the Divisia index are equivalent variation measures quantifying changes in the return to capital. Since the subjective discount rate in (4) is time independent, the Divisia index also measures changes in welfare.

Investment matters. The following example makes it more clear why investment matters in the definition of output growth. Consider a world with embodied technical progress –as the one in Greenwood and Yorukoglu (1997), for example. Let the consumption path in this economy be depicted as in Figure 3. In period T there is an unexpected permanent technology shock to the investment sector: embodied technical progress accelerates. New machines, if produced and added to the capital stock, can make the productivity in the consumption goods sector grow faster indefinitely. In our example, hence, after observing the unexpected acceleration of investment specific technical change in T, the consumer finds optimal to initially reduce consumption in order to increase investment and, then, profit from technical progress. In this world, at time T households welfare increases: the drop in consumption reflects the interest of the consumer in benefiting from faster growth thereon; if this move would have not increased her welfare, she would have chosen not to increase investment and remain in the original

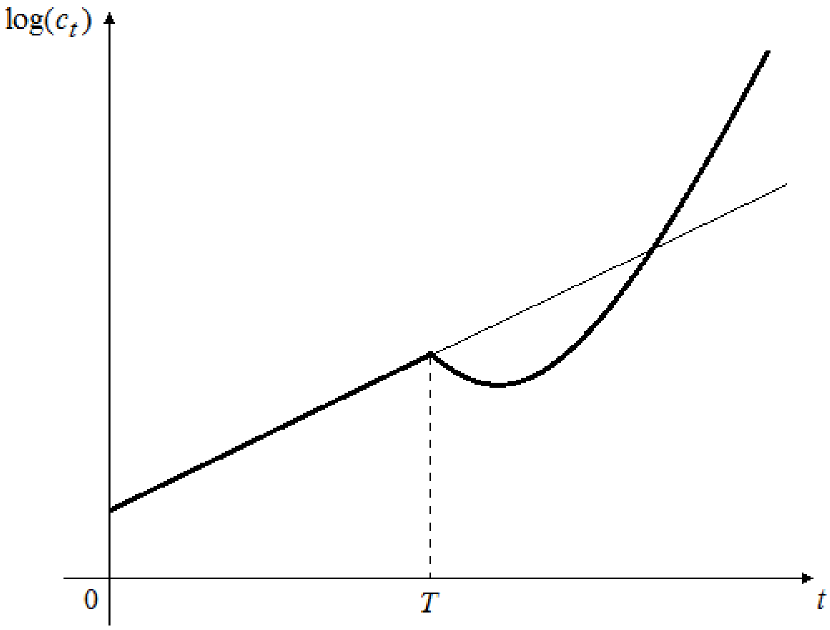

Figure 3: Investment matters

path with lower growth. Then, the consumption growth rate at time T does not measure welfare correctly. In fact, it has the opposite sign! However, the growth rate of output as measured by the Divisia index does, since it captures well the gains in welfare coming from the acceleration of technical progress and the associated optimal increase in investment. Remind that technical progress is assumed to be investment specific. Then, gains in productivity require new investments. The discussion above helps to illustrate why the growth rate of investment matters for output growth measurement. Faster growing investment today represents our best proxy for the preference for faster consumption growth tomorrow.

Net National Product. In connection with these considerations, the use of the Bellman equation makes it clear why production in national accounts is measured as final demand. Since present and future consumption is all that matter for welfare, and net investment measures the value of the future consumption it will produce, a welfare measure of output growth has to weight the growth rate of both final demand components, consumption and net investment. This interpretation is consistent with Weitzman (1976)'s claim that "net national product is a proxy for the present discounted value of future consumption."[21] In fact, his equation (10) is in spirit equivalent to the Bellman equations (2) and (4), which rationalize our choice of taking current net income as the proper norm in the Fisher-Shell true quantity index. It is important to point out that Weitzman (1976) is not about output growth and its relation to welfare gains in the growth process, but about the level of output and its relation to the level of welfare. In this sense, the non trivial question of the appropriate measurement of output growth has remained open until our days. The best result in this direction is in a subsequent paper by Asheim and Weitzman (2001). That paper builds a measure of the level of output and shows that output growth is a necessary and sufficient condition for welfare growth, but without providing any specific insight on how output growth should be measured. This papers gives a fundamental step ahead in this direction: by applying standard index number theory, we show that the precise way NIPA measures growth is welfare based.

At this point it may be worth clarifying that, as pointed out by Weitzman (1976), it is not GDP but Net National Product (NNP) what matters for welfare.[22] Depreciated capital is a lost resource that does not contribute to welfare. It is in this sense that some authors claim that NNP is relevant for welfare and GDP for productivity –see the discussion in Oulton (2004). If the depreciation rate is constant, however, net and gross investment grow at the same rate. Indeed, when investment growth faster than consumption, NNP grows slower than GDP since the share of net investment on net income is smaller than the corresponding share of gross investment.

Paradox of endowment vs production economies. It is very important to understand that a true quantity index of output growth is a welfare based measure conditional on both preferences and technology. In other words, it does not reflect changes in welfare independently of the possibilities allowed by technology. The example below shows the interplay between technology and preferences in the definition of output growth emerging from index number theory applied to this family of problems.

Consider the following example that clarifies further the meaning of measuring welfare changes. For the two-sector AK model in Section 3, take any configuration of parameters such that, for example, the growth rate of investment at equilibrium is 6% and the

[21]Weitzman's argument is developed in a simple optimal growth model with linear utility and the proof is based on the assumption that current income remains constant over time. In its own words, he gets "the right answer, although for the wrong reason."

[22]Since the model economy in Section 2 is closed, GDP and GNP are equal, as well as NDP and NNP.

investment share is 20%. Let α be equal to 1/3. The Divisia index tells us that this economy will be growing at 2.8%, since consumption represents 80% of output and will grow at 2%. Alternatively, consider an *endowment* economy with exactly the same preferences and the same equilibrium consumption flow. In this economy, consumption is mana from haven. Indeed, a household would be indifferent between living in the AK or in the endowment economy, since she will get the same consumption path, that she will evaluate using the same preference map. In the endowment economy, indeed, index number theory will associate income with current consumption; the Divisia index will then measure output grow as consumption growth; 2% in our example. Why is it the case that two economies where people have identical preferences and face exactly the same consumption path do not grow at the same rate? The reason is that a true quantity index takes current income as a norm and current income is defined differently; at any time, both economies share the same consumption utility, but investment goods are produced only in the production economy. These seemingly paradoxical example illustrate well the intimate relation between preferences (what we want to do) and technology (what we can do) when measuring output growth. Indeed, in this particular example, both measures of output growth are welfare based and consistent with NIPA methodology. The example makes also clear the implications of measuring production as final demand: since there is no investment in the endowment economy, output growth becomes identical to consumption growth.

Growth accounting. To end this discussion, let us review the implications for growth accounting. In terms of model representations of actual economies, the introduction of more than one sector with different growth rates raises the practical and conceptual issue of how output growth has to be measured. The choice of the appropriate output growth rate affects every quantitative exercise based on the measurement of growth. This is the case in the literature on growth accounting under embodied technical change, the so-called Solow-Jorgenson controversy.[23] To measure the contribution of investment specific technical change to growth, Hulten (1992) measures growth (his equation (7)) following Jorgenson (1966). He suggests a raw addition of consumption and investment units, calling the outcome quality-adjusted output. Using our notation, this strategy amounts to $c_t + x_t$. Greenwood et al (1997) note that, in their setting, adding consumption and effective investment turns the economy into a standard Solow (1960) growth model

[23]See Hercowitz (1998) for a review of the Solow-Jorgenson controversy.

with no embodied technical change. Greenwood et al (1997) correctly state that any aggregation requires the different quantities to be expressed in a common unit and they adopt the consumption good as their standard. For this purpose, investment has to be multiplied by its relative price, in our notation their choice of output level would be $y_t = c_t + p_t x_t$.[24] Oulton (2007) generalizes the argument and suggests that output components have to be deflated by the consumption price index in order to measure growth. But this is indeed what Greenwood et al (1997) suggest when they identify non-durable production with real output and the real growth rate with the growth rate of consumption. What the present paper clarifies is that the issue is not the units used to measure real output *levels* but the choice of the right index of real output *growth*. In this sense, we follow Licandro et al (2002) and conclude that the "true" contribution of ETC to output growth, reflecting welfare changes, has to be measured using NIPA methodology as in Cummins and Violante (2002).[25]

A word of caution. We have to be careful in the way we interpret the output growth rate in this framework. Since raising the growth performance of an economy is costly, it is well-known in endogenous growth theory that there exists an optimal growth rate. In the case of the two-sector AK model above, the optimal growth rate of capital is γ as shown in equation (5). Let us then assume, for example, that the two-sector AK model is at equilibrium growing at its optimal growth rate, but at time $t = 0$ an uninformed government decides to introduce some incentives to promote growth, for example by subsidizing capital production and then distorting the private return to capital. At the time of the reform, $t = 0$, the economy starts growing faster at the cost of a reduction in welfare. From this time on ahead, the growth rate of output in the distorted economy, like in Section 3.3, will measure welfare gains, which will be larger than in the efficient economy. Unfortunatelly, the initial welfare losses will not be captured by national accounts, since changes in the value of assets are in general not registered.

Let us formalize the previous statement by following the same steps as in Section 3.3.

[24] In their setting, this choice looks somewhat natural because the investment sector uses as input the consumption good. In their notation $y_t = c_t + p_t x_t$ is total output in the non-durable sector, even if only c_t is consumed and the remaining production $p_t x_t$ is allocated to the investment sector.

[25] Indeed, as suggested in section 3.4, applying consumption equivalence to the balanced growth path of the Greenwood et al (1997) economy makes the growth rate of consumption an alternative welfare based measure of output growth.

The value function of the distorted economy reads, for $t \geq 0$,

$$v_d(k_t) = B_d\, k_{d,t}^{\alpha(1-\sigma)}, \quad \text{with} \quad B_d = \frac{(A-\gamma_d)^{\alpha(1-\sigma)}}{(1-\sigma)\big(\rho - \alpha\gamma_d(1-\sigma)\big)}.$$

where

$$\gamma_d = \frac{\tau A - \rho}{1-\alpha(1-\sigma)} \quad \text{and} \quad k_{d,t} = k_0 \mathrm{e}^{\gamma_d}.$$

The distortion introduced by the subsidy is represented by the wedge $\tau > 1$. It is easy to see that $|B_d| < |B|$ and decreasing with $\tau > 1$, meaning that at $t = 0$ the policy generates welfare losses, which are larger the larger is the distortion. Paradoxically, welfare in the distorted economy is growing faster; reflecting the fact that there exists a finite time $t_d > 0$ from which $v_d(k_{d,t})$ becomes larger than $v(k_t)$.

5 Conclusions and extensions

This paper shows that a Fisher-Shell true quantity index is equal to the Divisia index when applied to a two-sector dynamic general equilibrium economy with heterogeneous households, general recursive preferences and general technology transforming production factors (capital and labor) into consumption and investment. Indeed, it turns out that the chained-type index used by national accounts to compute real output growth is well approximated by the Divisia index. Consequently, real GDP growth in NIPA is welfare based in the precise sense of equivalent variation. This result is illustrated in the framework of the two-sector AK model, which replicates the well-know stylized facts that investment grows faster than consumption and that the relative price of investment permanently declines. More important, changes in the growth rate of investment induced by changes in embodied technical progress turn out to be a relevant part of welfare increases along an equilibrium path. Investment then matters in the measurement of output growth. In general, this paper can be seen as a recall that index number theory has an important role to play clarifying the criteria with which we construct our indexes. In particular, this approach may be of great relevance for the debate on the use on index number theory to rationalize the use of the Penn World Tables (see Neary (2004) and van Veelen and van der Weide (2008)).

Let us finally comment on those dimensions in which this approach could be extended and those in which it will be hard to do. Broaden it to many durable and non-durable goods seems straightforward. The approach could also be applied to many forms of non-optimal equilibria. Notice that, in this case, the production possibility frontier will not be tangent to an indifference curve at equilibrium, and hence the generalization will not be straightforward. However, if the representative household is price taker in all markets, irrespective of the fact that prices are distorted, at equilibrium the budget constraint will be tangent to an indifference curve. Under theses circumstances, index number theory could be applied to compare different points in the equilibrium path in a similar way we did in Section 2. In particular, for a stationary economy moving from a distorted to a non distorted equilibrium, the Divisia index could be measuring the welfare gains period by period.

Appendix: Quantity indexes in continuous time

A1. Quantity indexes in continuous time

In continuous time, let us define a growth factor Γ_{t+h}^{t} as the gross rate of growth of an arbitrary variable between a base time t and a current time $t+h$, $h \geq 0$. When $h \leq 0$, Γ_{t+h}^{t} measures the gross rate of growth between the base time $t+h$ and the current time t. In the jargon of national accounts, Γ_{t+h}^{t} is referred as a volume index. Let us then define the instantaneous growth rate of the underline variable at time $t+h$ when the base time is t as

$$g_{t+h}^{t} = \frac{\mathrm{d}\Gamma_{t+h}^{t}}{\mathrm{d}h} \frac{1}{\Gamma_{t+h}^{t}}. \tag{7}$$

Notice that in continuous time, $h \geq 0$, the derivate of a growth factor at any time $t+h$ is equal to the growth rate of the variable itself at $t+h$. Let z_t be a continuous-time variable and $\Gamma_{t+h}^{t} = z_{t+h}/z_t$ the growth factor. Apply (7) to get

$$\frac{\mathrm{d}\Gamma_{t+h}^{t}}{\mathrm{d}h} \frac{1}{\Gamma_{t+h}^{t}} = \frac{\dot{z}_{t+h}}{z_{t+h}}.$$

This way of defining the instantaneous growth rate may look odd but it may be useful in those cases in which we have an index like Γ_{t+h}^{t} but no explicit variable giving rise to it like z_t in this example.

Using the notation introduced in Section 2, the starting point in index number theory is some nominal aggregate income $c_t + p_t x_t$. Remind that we have adopted consumption as the numeraire so that its price is normalized to one while the price of investment in consumption units is p_t. Laspeyres quantity indexes use time t (the base time) prices as weights based on the following growth factor

$$\mathcal{L}^t_{t+h} = \frac{c_{t+h} + p_t x_{t+h}}{c_t + p_t x_t}.$$

It does allow to compute the growth rate of output by putting all nominal values at base time prices. Notice that in this framework the *real* unit in which quantities are measured is nominal income $c_t + p_t x_t$ at the base time. Paasche indexes take current prices as weights by defining the growth factor as

$$\mathcal{P}^t_{t-h} = \frac{c_t + p_t x_t}{c_{t-h} + p_t x_{t-h}},$$

$h \geq 0$. Real output growth is measured at current t prices.

The Fisher ideal growth factor with time base t and current time $t+h$, $h \geq 0$, is defined as

$$\mathcal{F}^t_{t+h} = \left(\mathcal{L}^t_{t+h} \mathcal{P}^{t+h}_t\right)^{\frac{1}{2}}. \tag{8}$$

The definition in equation (7) is also applied in Section 2.2 to the Fisher-Shell quantity index since we have a well-defined factor $\hat{m}_{t+h}/m_t$. Notice that in the definition of g^{FS}_t, we use the property that $\lim_{h\to 0} \hat{m}_{t+h} = m_t$.

A2. Fixed-base quantity indexes in continuous time

Traditional measures of real growth stem from fixed-base quantity indexes. The most common among them are the Laspeyres and Paasche indexes referred in Appendix A1. From Appendix A1, the Laspeyres factor of change between t and $t+h$ is

$$\mathcal{L}^t_{t+h} = \frac{c_{t+h} + p_t x_{t+h}}{c_t + p_t x_t},$$

for $h \geq 0$, where t represents the base time and $t+h$ the current time. In continuous time, the Laspeyres index $g^{\mathcal{L}\,t}_{t+h}$ is the instantaneous growth rate of factor $\mathcal{L}^t_{t+h}$ as a function of h –see equation (7). That is,

$$g^{\mathcal{L}\,t}_{t+h} = \frac{\mathrm{d}\mathcal{L}^t_{t+h}}{\mathrm{d}h} \frac{1}{\mathcal{L}^t_{t+h}} = \frac{\dot{c}_{t+h} + p_t \dot{x}_{t+h}}{c_{t+h} + p_t x_{t+h}},$$

which measures the instantaneous real growth rate at $t+h$ for the given base time t. The Laspeyres index is popular because it is conceptually simple.

However, if the relative price of investment permanently declines and substitution makes real investment permanently grow faster than real consumption, as observed in the data, the Laspeyres index tends to give too much weight to investment as we depart from the base time t. In order to illustrate it, let us assume the economy is at a balanced growth path with constant investment and consumption shares, s and $1-s$ respectively, $s \in (0,1)$, the relative price of investment goods p_t declining at a constant rate, and investment and consumption growing at the constant rates g_x and g_c, respectively, $g_x > g_c > 0$.

Note, indeed, that the Laspeyres fixed-base index reads

$$g_{t+h}^{\mathcal{L}\,t} = \frac{c_{t+h}}{c_{t+h} + p_t x_{t+h}} g_c + \frac{p_t x_{t+h}}{c_{t+h} + p_t x_{t+h}} g_x. \tag{9}$$

Since x_{t+h} grows relative to c_{t+h}, it is easy to see that along a balanced growth path the weight of consumption in the Laspeyres fixed-base index decreases and the weight of investment increases with h. This effect is known in the index numbers literature as the *substitution bias*. Fast growing items when weighted using past (relatively high) prices are overweighted, overstating the real growth rate of output. The effect is larger the farther we are from the base time, converging to the growth rate of investment as h goes to infinity.

The Paasche index uses current prices as a base, instead of past prices, and hence tends to understate real growth as we go back in time. The Passche factor is

$$\mathcal{P}_{t-h}^{t} = \frac{c_t + p_t x_t}{c_{t-h} + p_t x_{t-h}}$$

for all $h \geq 0$ and the growth rate

$$g_{t-h}^{\mathcal{P}\,t} = \frac{\mathrm{d}\mathcal{P}_{t-h}^{t}}{\mathrm{d}h} \frac{1}{\mathcal{P}_{t-h}^{t}} = \frac{c_{t-h}}{c_{t-h} + p_t x_{t-h}} g_c + \frac{p_t x_{t-h}}{c_{t-h} + p_t x_{t-h}} g_x, \tag{10}$$

under the assumption that the growth rates of both consumption and investment are constant. As h grows, so $t-h$ decreases, the weight of consumption increases because x_{t-h}/c_{t-h} decreases, converging to the growth rate of consumption as h goes to infinity.

For the arguments developed above, both Laspeyres and Paasche fixed-base indexes yield poor measures of real growth when output components grow at different rates because of changing relative prices. The farther we are from the base time, the more the Laspeyres index overstates growth, and the more the Paasche index underesates it.[26]

[26] Updating regularly the base is not a solution because it would imply a permanent revision of past growth performance. It posses the additional problem of multiple real growth measures for each period, each of them affected differently for the substitution bias depending on the associated base period.

Indeed, it is easy to see that in continuous time both Laspeyres and Paasche quantity indexes are equal to the Divisia index when evaluated at t:

$$\left.\frac{\mathrm{d}\mathcal{L}_{t+h}^{t}}{\mathrm{d}h}\frac{1}{\mathcal{L}_{t+h}^{t}}\right|_{h=0}=\left.\frac{\mathrm{d}\mathcal{P}_{t-h}^{t}}{\mathrm{d}h}\frac{1}{\mathcal{P}_{t-h}^{t}}\right|_{h=0}=(1-s_t)g_{ct}+s_t g_{xt},$$

where $s_t=\frac{p_t x_t}{c_t+p_t x_t}$ is the investment share, $g_{ct}=\frac{\dot{c}_t}{c_t}$ the growth rate of consumption and $g_{xt}=\frac{\dot{x}_t}{x_t}$ the growth rate of investment.[27] Given that in continuous time, both Laspeyres and Paasche quantity indexes are equal to the Divisia index at t, it is easy to show that the Fisher ideal index is equal too. It is trivial to see that this property also applies to the Fisher ideal index.

A3. Chained-type quantity indexes in continuous time

In this appendix, we use our simple framework to review the BEA methodology.[28] The introduction by the BEA of quality corrections in equipment prices in the mid-eighties revealed a persistent declining pattern in the price of equipment relative to the price of non-durable consumption goods. Since then, real investment appears to be growing much faster than real non-durable consumption. In this new scenario, fixed-base quantity indexes face the severe substitution bias problem explained in Appendix A2 above. For this reason, the BEA moved to a chained-type index based on a Fisher ideal index computed for contiguous periods.[29] Let us first define a Fisher ideal index to them define a Fisher ideal chain index both in continuous time.

Let us now define a Fisher ideal chain (factor) index for the time interval $(0,T)$, where $t=0$ represents now the *reference time* (in contraposition to the *base time*). The key assumption of chained indexes is that the base time moves with t, by taking t as the base time when computing the growth rate at time t. From Appendix A2, for any time

[27]In discrete time, the weights of consumption and investment growth rates in the Laspeyres and Paasche indexes are different from current income shares.

[28]Young (1992) is a non-technical presentation of the methodological changes introduced in NIPA. Whelan (2002, 2003) provides a more detailed guide into the new methods in use at BEA to measure real growth. For economic index number theory see Diewert (1993), Triplett (1992), Fisher and Shell (1998) and IMF (2004, chapter 17).

[29]Diewert (1993) provides a clear explanation of the index suggested by Fisher (1922).

$t \in (0, T)$, the instantaneous growth rate of the Fisher ideal index is

$$g_t^{\mathcal{F}} = \frac{\mathrm{d}\mathcal{F}_{t+h}^t}{\mathrm{d}h} \frac{1}{\mathcal{F}_{t+h}^t}\bigg|_{h=0} = (1 - s_t) g_{ct} + s_t g_{xt}.$$

Even if there is a trend in relative prices, inducing the substitution of one good for another, the chained-type index allows weights to change continuously to avoid the emergence of any substitution bias.

Let us assume that s_t, g_{ct} and g_{xt} are continuous function of t, then the Fisher ideal index $g_t^{\mathcal{F}}$ is continuous too. A Fisher ideal chain (factor) index $\mathcal{C}_t^{\mathcal{F}}$ is defined by the differential equation

$$\dot{\mathcal{C}}_t^{\mathcal{F}} = g_t^{\mathcal{F}} \mathcal{C}_t^{\mathcal{F}},$$

$\mathcal{C}_0^{\mathcal{F}} = 1$, which solution is

$$\mathcal{C}_t^{\mathcal{F}} = \mathrm{e}^{\int_0^t g_s^{\mathcal{F}} \mathrm{d}s}.$$

A chained factor index for a time interval $t \in (0, T)$ is build in two stages. First, at any time $t \in (0, T)$ a growth rate is computed using t as the base time. Second, the time t growth rates computed at the first stage are chained in order to build growth factors in an interval of time $t \in (0, T)$. Notice that fixed-base factor indexes are equal to one at the base time. In the case of chained indexes base times are changing. For this reason, the time at which the factor index is set equal to one is now called the reference time.

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