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Why Contribute to Alliances?
An Information Aggregation Approach

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Abstract

Information sharing to achieve a foreign policy objective is an important feature of modern day alliances between countries. We develop a model of strategic communication to study information aggregation within an alliance. In an alliance, i) there is public communication (cheap talk) of private information by members; ii) the actions of players are strategic substitutes; iii) there are resource constraints on actions; and, iv) members have heterogeneous preferences over final outcomes. Our analysis uncovers a novel incentive for information aggregation – the extent of resource constraints on alliance members. Specifically, truthful information sharing depends on the size of bounds on each players' action space. We show that public communication protocol can support information aggregation as long as preferences of alliance members are sufficiently *cohesive* with respect to the bounds on actions. We derive a precise characterization of cohesiveness within an alliance as a function of the biases and the resource constraints of alliance members. Further, our theory provides an informational rationale for alliance formation between countries.

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1 Introduction

Countries form alliances with each other in order to achieve common goals (e.g., military, security, and economic). Examples of such modern day alliances –military and otherwise– include NATO, The EU, ASEAN, among others. These alliances are cooperative in nature in that they consist of countries that have an overarching set of policy goals, who then act cooperatively in realizing those shared objectives[1]. At the heart of such cooperation, then, is the informational incentive of being part of an alliance[2]. Sharing information, be it external intelligence, or internal security related, is therefore a vital component of alliances between sovereign nations. Specifically, this captures an environment in which individual member nations strategically share information through diplomatic channels, and take appropriate actions that is commensurate with the information aggregated and the preferences of other members.

We develop a model of alliances that incorporates four key features: information sharing, strategic interdependency in actions, preference heterogeneity, and resource constraints. Initially, members of an alliance receive a private signal about an unknown state of the world that affects their payoff. In the communication stage, each player, publicly and simultaneously, sends a cheap-talk message about their private information to the group. After the communication stage, conditional on the private information and the messages exchanged, each player takes an action, where actions of players are strategic substitutes.

Our main finding is the full information aggregation result. Specifically, we show that all private information held by members of an alliance are revealed in equilibrium as long as players' biases are cohesive – the distance between the bias of an individual player and a weighted average of biases of the group falls within a certain bound. The intuition behind the

[1]NATO's 2010 Strategic Concept Concept (2010) document specifies this idea succinctly, and we quote - "The Alliance will engage actively to enhance international security, through partnership with relevant countries and other international organizations; by contributing actively to arms control, non-proliferation and disarmament."; further, it adds "Any security issue of interest to any Ally can be brought to the NATO table, to share information, exchange views and, where appropriate, forge common approaches.".

[2]Traditionally, in the international relations literature (Walt, 1985, Walt, 1990, and Waltz, 2010), alliance formation has primarily been studied within the purview of state capacity - either align in order to balance against a powerful state or bandwagon with a threatening state (or coalition).

result is that as long as players' biases are cohesive, each player cannot do better than fully revealing her private information. This way, as long as the players' available domain of actions is large enough, the private information held by members of the alliance is fully aggregated.

The paper closest to our work is the one by Galeotti et al. Galeotti, Ghiglino, and Squintani (2013). Though the information and communication structure are identical, a fundamental difference is that in their work, actions of players were independent of each other. As a result, the message of a player does not affect her own actions. In our setup, since actions are interdependent, a player's message also affects her beliefs about other players' actions, and therefore, affects her own action. Hagenbach and Koessler Hagenbach and Koessler (2010) study a model of strategic with multiple players and interdependent actions. However, two differences emerge. First, while they study strategic complementarities in actions, we develop a model in which actions are substitutable. Second, in their information framework, private signals of players are independent and communication is private. On the other hand, we are interested in a model where signals are correlated, but the communication protocol is public. As a result, our analysis is very distinct from either of the two papers mentioned above.

An important theoretical contribution of our work is the fact that the domain of the action set of players –resource constraint– drives truthful communication. In particular, when actions are unrestricted (no resource constraints), there is always truthful communication irrespective of the bias differences. In this sense, we find a novel interaction between bias dispersion and resource constraints, which was absent in both Galeotti et al. Galeotti et al. (2013) and Hagenbach and Koessler Hagenbach and Koessler (2010).

The rest of the paper is organized as follows. Section 2 develops the framework of the model and section 3 presents the full information aggregation result. Section 4 provides brief concluding remarks.

2 Model

A group of players, $N = \{1, 2, ..., n\}$, decide on contributions to a joint project. Each player chooses an action $x_i \in [0,1]$, where the bounds represent the resource constraint faced by every individual player. Moreover, actions of players are themselves interdependent in a way that they are strategic substitutes. The payoff of every player is dependent on an unknown common state of the world θ distributed uniformly on $[0,1]$. The state θ is not directly observable, but each player i receives a private signal $s_i \in \{0,1\}$ about the state of the world such that: $s_i = 1$ with probability θ, and $s_i = 0$ with probability $1-\theta$. Finally, each player has a bias b_i, that captures the extent to which a player cares about the outcome (without loss of generality, $0 \leqslant b_1 \leqslant b_2 \leqslant .... \leqslant b_n$).

Formally, player i's utility is given by,

$u_i(\mathbf{x};\theta,b_i)] = -[((\frac{x_i+\eta x_{-i}}{1+(n-1)\eta}) - \theta - b_i)^2]$; where $x_{-i} = \sum\limits_{j\neq i} x_j$, $\mathbf{x} = (x_1, x_2, ..., x_n)$

This utility form captures the four features described earlier. First, players' actions are interdependent in that utility depends on deviations of the (player-specific) joint contribution function from the player's ideal action, given by $\theta + b_i$. This interdependence is such that $\frac{\partial^2 u_i(.)}{\partial x_i \partial x_j} < 0$, and the degree of substitutability is not perfect, ie, $\eta \in (0,1)$. Second, there is a need for information sharing in order to aggregate each players' private information about the state θ. Third, players face a resource constraint since there are signal realizations and biases such that $\theta + b_i > 1$, but actions are bounded on $[0,1]$. Lastly, b_i captures the preference heterogeneity of players over final outcomes[3].

This set-up lends itself naturally to situations in international affairs that involve countries cooperating with each other to resolve a common foreign policy objective - like engaging in conflicts, or providing assistance to peacekeeping, among others. In such scenarios, each country in the alliance has potentially varying degrees of information and interest in the cause.

[3]These features are perfectly encapsulated by the NATO Strategic Concept 2010 document. Specifically, it identifies three key objectives of the security alliance between NATO countries - Collective Defence, Crisis Management and Cooperative Security. The emphasis is on 'cooperation' and 'collective', and achieving this requires a way to deal with preference heterogeneity within the alliance and structure to communicate information among the members.

Sharing private information enables countries in an alliance to target resources in an efficient way, and doing so is a vital component of successful cooperation[4].

2.1 Communication Round

With no communication, player types are correlated and there is complementarity in players' signals[5]. Each player, before the communication stage, can be classified into one of two types - high ($s_i = 1$) and low ($s_i = 0$). We allow for communication in the following way: after each player receives their signal s_i, they publicly and simultaneously communicate their information through cheap-talk messages $m_i \in M_i$ to each of the other $n-1$ players.

In this paper, we focus on pure messaging strategies and a public communication protocol, in which each player simultaneously sends a public message $m_i(s_i)$ to every other player in the group[6]. Player i's messaging strategy is given by,

$m_i : \{0,1\} \longrightarrow \{0,1\}$

A truthful message by i to the group implies $m_i(s_i) = s_i$ for $s_i = 0$ and 1, and babbling message is one where $m_i(s_i) = m_i(1-s_i)$. Let $\mathbf{m} = (m_1, m_2, ..., m_n)$ be the communication strategy of the n players[7]. Through out this paper, we abstract away from other more complicated forms of messaging strategies[8], and focus on pure communication strategies for reasons of tractability and clearer exposition of the trade-off's.

[4]For example, consider NATO's Partnership Action Plan against Terrorism, drafted post the September 2011 attacks. It clearly delineates the vital element of information sharing as one of the key requirements for effectively fighting terrorism and other security related challenges. For more, see http://www.nato.int/cps/en/natohq/official_texts_19549.htm.

[5]In our model, player signals are conditionally independent. However, there is signal correlation and complementarity in the following way: $\Pr(s_j = 1 \mid s_i = 1) = \frac{2}{3}, \Pr(s_j = 0 \mid s_i = 1) = \frac{1}{3}$ and $\Pr(s_j = 0 \mid s_i = 0) = \frac{2}{3}, \Pr(s_j = 1 \mid s_i = 0) = \frac{1}{3}$.

[6]Public communication protocols are very common in the real-world. For example, forums like UN, NATO and other regional alliances often get together and share private information about a common issue. Public diplomacy remains a main feature of such organizations.

[7]Both the signaling structure and messaging strategies are similar to Galeotti et al. Galeotti et al. (2013) In fact, to be more precise, they use a more general communication protocol, placing no restriction on messages being public or private.

[8]For example, one such mixed strategy would be a partially separating strategy under which player i babbles (or reveals) for one signal type and mixes between truth-telling and babbling for the other signal type.

2.2 Action round

Once messages have been exchanged, each player decides on their individual contribution x_i. We will once again focus on pure second-stage strategies. Since the utility function is strictly concave in x_i, best-responses exist and are unique. A second stage strategy can be defined as follows:

$\tau_i : S_i \times (M_i \times M_{-i}) \to [0, 1]$

Therefore, $\tau_i(s_i, (m_i, m_{-i}))$ is the strategy of player i with signal s_i, having sent message m_i and received messages $m_{-i} = (m_j)_{j \in N \setminus \{i\}}$ from the group. Let $\tau(\mathbf{s}, \mathbf{m}) = (\tau_i(s_i, (m_i, m_{-i})))_{i \in N}$ be the strategy profile of the players.

2.3 Equilibrium Definition

Given the above structure of messaging, the players can be grouped post the communication round into two sets (according to equilibrium beliefs) - truthful set and babbling set. We define them in the following way:

Definition 1 *Truthful set,* $T = \{i : m_i(0) = 0, m_i(1) = 1\}$

Definition 2 *Babbling set,* $B = \{j : m_j(0) = m_j(1)\}$

The first is just the set of players whose messages are believed in equilibrium as informative, and messages from the second are ignored as uninformative (note that all this is based on equilibrium beliefs). Given this, the vector of messages after communication consists of $|T|$ truthful messages $m_T = \{m_i : i \in T\}$ and $|B|$ babbling messages $m_B = \{m_j : j \in B\}$. Note that any off-equilibrium path messages are believed and treated as if they were equilibrium messages. This gives rise to an IC constraint for truth-telling such that, in equilibrium, each player's beliefs about other players' messages are updated using Bayes' rule.

The equilibrium concept is sequential equilibrium in pure strategies. An equilibrium is defined as a strategy profile $(m, \tau) = ((m_i)_{i \in N}, (\tau_i)_{i \in N})$ such that,

1. Actions are sequentially rational, given messages and beliefs:

   $\forall i \in N,\, m_{-i} \in M_{-i}$:

$$\tau_i(s_i,(m_i,m_{-i})) \in \arg\max_{x_i} \int_0^1 \sum_{s_{-i}\in\{0,1\}^{n-1}} u_i(x_i,(\tau_j(s_j,(m_j,m_{-j})))_{j\neq i};\theta,b_i) \Pr(s_{-i}\mid\theta) f(\theta|m_{-i},s_i)d\theta$$

2. Messages are truthful iff they satisfy the IC for truth-telling:

   $\forall i \in N, s_i \in \{0,1\}$:

$$-\int_0^1 \sum_{s_{T-1}\in\{0,1\}^{t-1}} \sum_{s_B\in\{0,1\}^{n-t}} u_i(\tau_i(s_i,(s_i,m_{-i})),(\tau_j(s_j,(s_i,m_{-i})))_{j\in T-1}, (\tau_k(s_B(k),(s_i,m_{-i})))_{k\in B};\theta,b_i) f(\theta,s_{T-1},s_B|s)d\theta$$
$$\geq$$
$$-\int_0^1 \sum_{s_{T-1}\in\{0,1\}^{t-1}} \sum_{s_B\in\{0,1\}^{n-t}} u_i(\tau_i(s_i,(1-s_i,m_{-i})),(\tau_j(s_j,(1-s_i,m_{-i})))_{j\in T-1}, (\tau_k(s_B(k),(1-s_i,m_{-i})))_{k\in B};\theta,b_i) f(\theta,s_{T-1},s_B|s)d\theta$$

   where s_{T-1} is the set of $(T-1)$ truthful signals, apart from player i and s_B is the set of babbling signals

3 Equilibrium Characterization

We proceed by first characterizing the optimal best responses in the contributions stage of the game. When deciding on how much to contribute given the information generated by communication, each player takes into account the interdependence in actions. Let $t = |T|$ and $b = |B|$ be the number of truthful players and babbling players respectively, after the communication stage.

Another intuitive way of thinking about the action stage is to abstract away from communication, and assume the following. Suppose that all agents were exogenously given information m_T, and a sub-group of b agents were additionally provided with a private signal - 0 or 1. Given this exogenous information structure, what are the optimal actions of each player given their information, and the interdependence in actions. The solution to the problem is then a Bayesian Nash equilibrium (BNE) in which there are b players who can be either of two types, and t truthful players.

The maximization problem of each of the t truthful players is given by,

$$\forall i \in T : \max_{x_i} E_{\theta, s_B}[u_i((x_i, x_{T\setminus\{i\}}, x_B(s_B)); \theta, b_i) \mid m_T] \tag{1}$$

where, $x_{T\setminus\{i\}} = \{x_j : j \in T\setminus\{i\}\}$ and $x_B(s_B) = \{x_j(s_B(j)) : j \in B\}$ are the vector of actions by the $t-1$ truthful players other than i, and b babbling players respectively.

Analogously, the maximization problem for a babbling player j with private signal s_j is given by,

$$\forall j \in B, s_j \in \{0,1\} : \max_{x_j(s_j)} E_{\theta, s_B}[u_j((x_j(s_j), x_T, x_{B\setminus\{j\}}(s_{B\setminus\{j\}})); \theta, b_j) \mid m_T, s_j] \tag{2}$$

where, $s_{B\setminus\{j\}} \in \{0,1\}^{n-t-1}$ is the vector of all possible signals of the remaining $(n-t-1)$ babbling players, x_T is the vector of actions of all the truthful players, and $x_{B\setminus\{j\}}(s_{B\setminus\{j\}})$ is the vector of actions of remaining $(n-t-1)$ babbling players aside j. Hence, players choose an action that solves a system of equations $(t+2b)$ given by their maximization problem[9], as stated above.

[9] The usefulness of looking at public communication can be seen from the above equations. Every player in the group knows precisely who the set of truthful players are (in equilibrium beliefs), their messages, as well as the set of babbling players. Moreover, given the beta-binomial distribution, each player can then form expectations of what private information any babbling player holds. The babbling player is then one of two types - low signal or signal type - and every player in the group has the same posterior about the conditional expectation over babbling types.

3.1 Characterization of equilibrium contributions

3.1.1 No Resource constraints

We proceed by first characterizing the best responses of each type of player and their equilibrium actions, as if there were no resource constraints on the actions of players. The reason to do this is twofold. First, it abstracts away from the difficulty of thinking about interdependent actions with bounds, and allows us to find closed-form solutions to the action stage problem. Second, the exact form of equilibrium actions, as will be made clearer later, provides important intuition to think about the messaging strategies of players and characterize the messaging equilibrium.

We also impose a bound on the dispersion of biases so that the alignment of any player i remains within certain limits[10]. Specifically, we define the following:

Definition 3 *Let $A_i = [b_i - \frac{\eta}{(1+(n-1)\eta)} \sum_{j \in N} b_j]$, be a measure of dispersion of the alignment of interests of player i from that of the group. Further, assume that $\forall i \in N$, $\frac{(1+(n-1)\eta)}{1-\eta}.A_i \in (-1, 1)$.*

Lemma 1 *Under unrestricted domain ($x_i \in \mathbb{R}$) and public communication, the players' sequentially rational action after receiving t truthful messages and $(n-t)$ babbling messages is given by:*

Truthful player:

$$x_{i \in T} = \frac{(1+(n-1)\eta)}{1-\eta}.A_i + \frac{(k+1)}{(t+2)}$$

Babbling player with low signal:

$$x_{(i \in B, s_i=0)} = \frac{(1+(n-1)\eta)}{1-\eta}.A_i + \frac{k+1}{t+2}.\frac{h(t)}{1+h(t)}$$

[10]Since we consider communication with resource constraints, it is necessary to limit the values a player's bias can take. For example, we cannot have one player to have an extremely high bias, like $b_i = 10$ or so. This trivializes the problem when bounds are introduced, since, irrespective of the communication, player i always takes the maximum action within the bound, $x_i = 1$. Moreover, every such player with extreme biases always misreport their signal in equilibrium.

Babbling player with high signal:

$$x_{(i \in B, s_i=1)} = \frac{(1+(n-1)\eta)}{1-\eta}.A_i + \frac{k+1}{t+2}.\frac{h(t)}{1+h(t)} + \frac{1}{1+h(t)}$$

where $h(t) = \frac{\frac{(2+t(1-\eta))}{(1+(n-1)\eta)}}{\left(1+\frac{(2+t(1-\eta))}{(1+(n-1)\eta)}\right)}$

Notice that the actions post communication is dependent on A_i, the difference in bias of the player from the weighted average of biases of all players in the group. We construe this difference as a measure of alignment of interests in the group, or alternatively, as a measure of dispersion in the biases.

When players' actions have unrestricted domain (meaning $x_i \in \mathbb{R}$), players are able to exactly compensate for the messages in equilibrium and possibly 'undo' the effects of communication by choosing an optimal action that exactly matches their ideal state, given the set of equilibrium messages. Under unrestricted domain, there always exists a fully revealing equilibrium in which every player reveals her private information to the group[11].

In a fully revealing equilibrium with unrestricted domain of actions, $T = N$ and every player plays the following action, post the communication round:

$$x_{i \in N} = \frac{(1+(n-1)\eta)}{1-\eta}.A_i + \frac{(k+1)}{(t+2)} \tag{3}$$

3.1.2 Resource constraints

Intuitively, introducing resource constraints by restricting the set of actions to $[0, 1]$ changes the nature of information revelation for the players by reintroducing trade-off's between providing more information and concerns of under (or over) provision. This arises because with bounded actions, players are unable to completely compensate for the effect of their message on the

[11]This point has been made in Venkatesh (2016). Specifically, it shows that, for a specific class of utility functions, when players' actions are unrestricted, there is completely truthful communication as players can undo the effects of communication in the subsequent action stage. This ability to compensate for the messages ex-post precludes the incentives to lie and ensures truthful communication.

actions of others.

To make this point clearer, let us consider the action of n - the player with the highest bias. In the fully revealing equilibrium described above, under a restricted action set $[0, 1]$, $x_n = \min\{1, \frac{(1+(n-1)\eta)}{1-\eta}.A_i + \frac{(k+1)}{(t+2)}\}$. This implies that $x_n = 1$ whenever the other expression exceeds the bound. This leads to under-provision as other players would not substitute completely (since $\eta < 1$), leaving player n to suffer a loss from under-provision. The problem of under-provision is exacerbated when player n's signal is $s_n = 0$. Fearing under-provision, player n can do better by exaggerating her private information and sending a message $m_n = 1$ to the group instead. This type of exaggeration has two effects. First, there is a pure information effect that pushes every players' actions up. Second, there is a countervailing free-riding effect in that players now also understand that n is also going to take a higher action, and hence will adjust their actions accordingly. Nevertheless, player n benefits from misreporting since even in equilibrium, each of the other players' action are higher (in expectations), and this reduces the loss from under-provision. In equilibrium, though, player n's message is never credible for precisely the above reasons, and thus will not be believed.

Players with very low biases in the group face the opposite problem - that of over-provision. Without loss of generality, take the case of player 1 with bias b_1. Again, when each of the other $(n-1)$ players are revealing truthfully, say, player 1 may have an incentive to deviate from reporting a high signal ($s_1 = 1$) truthfully. Player 1's optimal action as dictated by the previous lemma is $x_1 = \max\{0, \frac{(1+(n-1)\eta)}{1-\eta}A_i + \frac{(k+1)}{(t+2)}\}$. When, however, for any possible signal realization $s_{N\setminus\{1\}}$, the optimal action of player 1 is below zero, then the bounds kick in and $x_1 = 0$, leading to over-provision concerns. That is, player 1 can benefit from under-reporting her high signal and instead send a message $m_1 = 0$. This would push the actions of the rest of the players in group down, thereby decreasing losses from over-provision.

Therefore, with resource constraints, two types of problems arise with communication. Players with a higher preference, in order to avoid under-provision, may tend to exaggerate their private information and those with lower biases, fearing over-provision, may end up under-reporting their signals. One question that arises naturally in this context is who exactly are the players with under (over) provision concerns. A closer look at the equilibrium actions under unrestricted domain provides crucial intuition for answering this question.

For every player i such that $A_i < 0$, actions in any equilibrium can never hit the upper bound. However, there may be truthful signal realizations under which the optimal action may be less than 0, but because of the lower bound on actions, player i would constrained to play $x_i = 0$. This implies for these players, there are over-provision concerns, meaning they would always report their low signal truthfully, but under-report the high one. Similarly, the players for whom $A_i > 0$ would worry about the upper bound as their actions are always positive for any set of signal realizations. These players are ones who fear under-provision and therefore, have an incentive to exaggerate their low signals.

Therefore, players themselves can be separated into two types - $0 - type$ and $1 - type$. We define them in the following way:

Definition 4 $0 - type = \{i \in N : A_i < 0\}$

Definition 5 $1 - type = \{i \in N : A_i > 0\}$

The players in the set $0 - type$ always reveal their low signal, but face incentives to misrepresent $s_i = 1$. Vice versa, the players in the set $1 - type$ always reveal their high signal, but may misrepresent their low signal, $s_i = 0$.

3.2 Full Information aggregation with resource constraints

In the previous subsection, we put forth some of the trade off's involved in information revelation with resource constraints. Particularly, the set of players can be (ex-ante) partitioned into either a $0 - type$ or $1 - type$, depending on the initial distribution of biases $\mathbf{b}$. Further, we know that with public communication and pure messaging strategies, information revelation takes the form of a partition of truthful players T and babbling player B. The natural question that arises is, under what conditions does there exists full information aggregation with resource constraints. The following Lemma provides the necessary and sufficient conditions required for complete information aggregation ($|T| = n$) under resource constraints:

Theorem 1 *Under public communication protocol with given bias distribution* $\mathbf{b}$ *and resource constraints, there is a n-player equilibrium such that every player in the group reveals truthfully if and only if:*

$$\forall i \in N : \qquad |A_i| \leqslant \frac{2}{n+2} . \frac{(1-\eta)}{(1+(n-1)\eta)}$$

Proof. See Appendix A ■

We provide brief intuition for the above result. To do so, we borrow from earlier arguments as well as from Austen-Smith Austen-Smith (1993). For a player in the group to reveal information truthfully, it must be that for every possible signal type, and every possible signal realization of the other $(n-1)$ players, her action must be within the bounds. For a $0-type$ player, the relevant one is the lower bound of 0, and for the $1-type$, upper bound of 1.

Let a player, say i, from the set $0-type$ hold a signal $s_i = 1$. In any equilibrium where t players reveal truthfully, it must hold that the equilibrium action of player i is greater than zero, for every possible signal realization[12] of the remaining $(t-1)$ truthful players. Otherwise, there is an incentive to lie for player i in the communication stage. Moreover, what matters is the sufficient statistic (given by $\sum\limits_{j \in T \setminus \{i\}} s_j$) of the $(t-1)$ signals. Given this formulation, truth-telling for a $0-type$ player has to satisfy the tightest IC constraint - meaning her action under the tightest constraint has to be within the bounds. If this was not so, an equilibrium with $i \in T$ violates the IC for truth-telling. The tightest constraint for a $0-type$ player occurs when $k = 1$, meaning all other $(t-1)$ signals are 0 ($\sum\limits_{j \in T \setminus \{i\}} s_j = 0$) and $s_i = 1$. Once this is satisfied, all other IC constraints are satisfied automatically, by single crossing property of the utility function ($\frac{\partial^2 u_i}{\partial x_i \partial \theta} > 0$).

An analogous logic ensues for any truth-telling player belonging to the set $1-type$. For a player $i \in 1-type$ to separate her messages in equilibrium, her equilibrium actions have to

[12]Given the nature of beta binomial distribution, signals are ex-ante correlated, and that given a signal, all possible contingencies of remaining $(t-1)$ signals occur with a positive probability.

to be < 1 for every possible realization of the remaining $(t-1)$ signals. This further implies that the tightest IC in this case is one where the sufficient statistic of the rest of the truthful messages is the highest possible, ie, $\sum_{j \in T \setminus \{i\}} s_j = (t-1)$, and $s_i = 0$. Again, as before, if this IC is satisfied, every other constraint would also be satisfied because of single crossing property.

Theorem 1 clearly shows the importance of alignment of interests for information transmission within an alliance. Despite varying degrees of interest about the ideal state of the world, it is possible for alliances to aggregate information as long as the dispersion in the biases is within a bound. To this effect, players in the group must be 'closely' aligned. A_i provides a measure of this alignment that ensures aggregation of information.

4 Conclusion

The full information aggregation result is useful for two reasons. Firstly, we introduce a novel methodology to obtain full information aggregation equilibrium with interdependent actions. The technique of using bounds on actions as IC constraints is, to the best of our knowledge, a first in the wider theoretical literature. Secondly, full aggregation result highlights the importance of alignment of interest for successful information sharing within members of an alliance. Specifically, we provide an intuitive characterization for cohesiveness of an alliance.

A Full Information Aggregation

Before proceeding to prove Lemma 1, we provide some basic insights into the nature of maximization problem that each type of players face, and in general, lay out some important properties of the Beta-Binomial distribution that we employ in our paper.

We start by reformulating the maximization problem faced by a truthful player, given in equation 1, as follows:

$$\max_{x_i} \int_0^1 \sum_{s_B \in \{0,1\}^{n-t}} u_i((x_i, x_{T\setminus\{i\}}, x_B(s_B)); \theta, b_i) \Pr(s_B|\theta) f(\theta|m_T) d\theta$$

The conditional density $f(\theta|m_T)$ belongs to a standard beta-binomial distribution. Letting $k = \sum_{i \in T} s_i$, the number of signals s_i with $i \in T$ that are equal to one, the posterior distribution of θ with uniform prior on $[0,1]$, given k successes in t trials, is a Beta distribution with parameters $k+1$ and $t-k+1$. As a consequence, $f(\theta|m_T) = \frac{(t+1)!}{k!(t-k)!}\theta^k (1-\theta)^{t-k}$ and $E[\theta|m_T] = [k+1]/[t+2]$. Further, for any s_B, letting $\ell(s_B) = \sum_{q \in B} s_q$, it is the case that $\Pr(s_B|\theta) = \theta^{\ell(s_B)} (1-\theta)^{n-t-\ell(s_B)}$.

In a similar way, the problem of every babbling player $j \in B$ with a private signal s_j, stated in equation 2, can be expanded as the following:

$$\max_{x_j(s_j)} \int_0^1 \sum_{s_{B\setminus\{j\}} \in \{0,1\}^{n-t-1}} u_j((x_j(s_j), x_T, x_{B\setminus\{j\}}(s_{B\setminus\{j\}})); \theta, b_j) \Pr(s_{B\setminus\{j\}}|\theta)$$
$$f(\theta|m_T, s_j) d\theta$$

Again, the posterior density $f(\theta|m_T, s_j)$ belongs to the beta family, with $k + s_j$ successes in $t+1$ signals, and is a Beta distribution with parameters $k + s_j + 1$ and $(t - k - s_j + 2)$. Consequently, $f(\theta|m_T, s_j) = \frac{(t+2)!}{(k+s_j)!(t+1-k-s_j)!}\theta^{k+s_j} (1-\theta)^{t+1-k-s_j}$ and $E[\theta|m_T, s_j] = [k + s_j + 1]/[t+3]$. As before, for any $s_{B\setminus\{j\}}$, $\Pr(s_{B\setminus\{j\}}|\theta) = \theta^{\ell(s_{B\setminus\{j\}})} (1-\theta)^{n-t-\ell(s_{B\setminus\{j\}})}$.

A.1 Characterization of second-stage contributions with unrestricted domain

We begin the characterization by first solving the best responses of each of the three types of players from equations 1 and 2.

i) Truthful player's problem:

$$E_\theta[u_i(\mathbf{x},\mathbf{m})] =$$
$$-\int_0^1 \sum_{s_B\in\{0,1\}^{n-t}} \left(\frac{x_i+\eta\sum_{j\in T\setminus\{i\}} x_j+\eta\sum_{j\in B} x_j(s_j)}{1+(n-1)\eta}-\theta-b_i\right)^2 \Pr(s_B|\theta)f(\theta|m_T)d\theta$$

where $f(\theta|m_T)=\frac{(t+1)!}{k!(t-k)!}\theta^k\left(1-\theta\right)^{t-k}$, iff $0\le\theta\le1$.

Differentiating the above with respect to x_i, we get the following FOC:

$$\int_0^1 \sum_{s_B\in\{0,1\}^{n-t}} \left(\frac{x_i+\eta\sum_{j\in T\setminus\{i\}} x_j+\eta\sum_{j\in B} x_j(s_j)}{1+(n-1)\eta}-\theta-b_i\right)$$
$$\Pr(s_B|\theta)f(\theta|m_T)d\theta = 0$$

Simplifying, we obtain:

$$x_i+\eta\left[\sum_{j\in T\setminus\{i\}} x_j+\int_0^1 \sum_{s_B\in\{0,1\}^{n-t}}\sum_{j\in B} x_j(s_j)\Pr(s_B|\theta)f(\theta|m_T)d\theta\right] = \left(b_i+E[\theta|m_T]\right)\left[1+(n-1)\eta\right] \quad (4)$$

ii) Babbling player's problem:

With analogous procedures, the expected utility of a babbling player i with signal s_i is:

$$\begin{aligned}
E_\theta[u_i(\mathbf{x},\mathbf{m})] &= -E_{\theta,s_{-i}}\left[\left(\tfrac{x_i(s_i)+\eta\sum_{j\neq i}x_j(s_j)}{1+(n-1)\eta}-\theta-b_i\right)^2 \mid m_T,s_i\right] \\
&= -E_{\theta,s_{B\setminus\{i\}}}\left[\left(\tfrac{x_i(s_i)+\eta\sum_{j\in T}x_j+\eta\sum_{j\in B\setminus\{i\}}x_j(s_j)}{1+(n-1)\eta}-\theta-b_i\right)^2 \mid m_T,s_i\right] \\
&= -\int_0^1\sum_{s_{B\setminus\{i\}}\in\{0,1\}^{n-t-1}}\left(\tfrac{x_i(s_i)+\eta\sum_{j\in T}x_j+\eta\sum_{j\in B\setminus\{i\}}x_j(s_j)}{1+(n-1)\eta}-\theta-b_i\right)^2\Pr(s_{B\setminus\{i\}}|\theta) \\
&\qquad f(\theta|m_T,s_i)d\theta
\end{aligned}$$

Again, the density $f(\theta|m_T, s_i)$ belongs to the beta family such that $f(\theta|m_T, s_i) = \frac{(t+2)!}{(k+s_i)!(t+1-k-s_i)!}\theta^{k+s_i}(1-\theta$ iff $0 \leq \theta \leq 1$. Differentiating the above equation, we derive the following

FOC:

$$\int_0^1 \sum_{s_{B\setminus\{i\}} \in \{0,1\}^{n-t-1}} \left(\frac{x_i(s_i) + \eta \sum_{j \in T} x_j + \eta \sum_{j \in B\setminus\{i\}} x_j(s_j)}{1+(n-1)\eta} - \theta - b_i \right) \Pr(s_{B\setminus\{i\}}|\theta)$$

$$f(\theta|m_T, s_i) d\theta = 0$$

Simplifying yields,

$$x_i(s_i) + \eta \left[\sum_{j \in T} x_j + \int_0^1 \sum_{s_{B\setminus\{i\}} \in \{0,1\}^{n-t-1}} \sum_{j \in B\setminus\{i\}} x_j(s_j) \Pr(s_{B\setminus\{i\}}|\theta) f(\theta|m_T, s_i) d\theta \right] = \tag{5}$$

$$(b_i + E[\theta|m_T, s_i]) \left[1 + (n-1)\eta\right]$$

We focus on linear equilibrium strategies of the form: $x_i = A.(b_i + E[\theta|m_T]) + B$ for truthful players, and $x_i(s_i) = A_{s_i}.(b_i + E[\theta|m_T, s_i]) + B_{s_i}$ for babbling players. Plugging the linear forms into expression (4), we get the following,

$$A(b_i + E[\theta|m_T]) + B + \eta \sum_{j \in T\setminus\{i\}} \left[A(b_j + E[\theta|m_T]) + B\right]$$

$$+\eta \int_0^1 \sum_{s_B \in \{0,1\}^{n-t}} \sum_{j \in B} \left[A_{s_j}(b_j + E[\theta|m_T, s_j]) + B_{s_j}\right] \Pr(s_B|\theta) f(\theta|m_T) d\theta =$$

$$(b_i + E[\theta|m_T]) \left[1 + (n-1)\eta\right]$$

Using linearity of the strategies $x_i(s_i)$, above expression can be rewritten as:

$$[A(b_i + E[\theta|m_T]) + B] + \eta \sum_{j \in T \setminus \{i\}} \left[A(b_j + E[\theta|m_T]) + B\right]$$
$$+\eta \int_0^1 \sum_{j \in B} \sum_{s_j \in \{0,1\}} \left[A_{s_j}(b_j + E[\theta|m_T, s_j]) + B_{s_j}\right] \Pr(s_j|\theta) f(\theta|m_T) d\theta =$$
$$(b_i + E[\theta|m_T]) \left[1 + (n-1)\eta\right]$$

Substituting in the functional forms of $\Pr(s_j|\theta)$ and $f(\theta|m_T, s_i)$, we obtain:

$$(b_i + E[\theta|m_T]) \left[1 + (n-1)\eta\right] =$$
$$[A(b_i + E[\theta|m_T]) + B] + \eta \textstyle\sum_{j \in T \setminus \{i\}} \left[A(b_j + E[\theta|m_T]) + B\right]$$
$$+\eta \textstyle\int_0^1 \sum_{j \in B} \left[A_0(b_j + E[\theta|m_T, s_j = 0]) + B_0\right] (1-\theta) \frac{(t+1)!}{k!(t-k)!} \theta^k (1-\theta)^{t-k} d\theta$$
$$+\eta \textstyle\int_0^1 \sum_{j \in B} \left[A_1(b_j + E[\theta|m_T, s_j = 1]) + B_1\right] \theta \frac{(t+1)!}{k!(t-k)!} \theta^k (1-\theta)^{t-k} d\theta$$

which, because $\int_0^1 (1-\theta) \frac{(t+1)!}{k!(t-k)!} \theta^k (1-\theta)^{t-k} d\theta = 1 - E\left[\theta|m_T\right]$ and $\int_0^1 \theta \frac{(t+1)!}{k!(t-k)!} \theta^k (1-\theta)^{t-k} d\theta = E[\theta|m_T]$ is further simplified as:

$$\begin{aligned}
(b_i + E[\theta|m_T]) \left[1 + (n-1)\eta\right] &= A(b_i + E[\theta|m_T]) + B \\
&+ \eta \sum_{j \in T \setminus \{i\}} \left[A(b_j + E[\theta|m_T]) + B\right] \\
&+ \eta \sum_{j \in B} \left[A_0(b_j + E[\theta|m_T, s_j = 0]) + B_0\right] \left(1 - E\left[\theta|m_T\right]\right) \\
&+ \eta \sum_{j \in B} \left[A_1(b_j + E[\theta|m_T, s_j = 1]) + B_1\right] E[\theta|m_T]
\end{aligned}$$

Substituting back the linear strategies x_i and $x_j\left(s_j\right)$ gives the best response for the truthful players,

$$(b_i + E[\theta|m_T])\left[1 + (n-1)\eta\right] = x_i + \eta \sum_{j \in T\setminus\{i\}} x_j + \eta\left(1 - E\left[\theta|m_T\right]\right) \sum_{j\in B} x_j(0) + \eta E[\theta|m_T] \sum_{j \in B} x_j(1)$$

Applying the same principles to equation (5), we obtain the expression:

$$\begin{aligned}
&(b_i + E[\theta|m_T, s_i])\left[1 + (n-1)\eta\right] = \\
&A_{s_i}(b_i + E[\theta|m_T, s_i]) + B_{s_i} + \eta \sum_{j\in T}\left[A(b_j + E[\theta|m_T]) + B\right] \\
&+ \eta \int_0^1 \sum_{s_{B\setminus\{i\}} \in \{0,1\}^{n-t-1}} \sum_{j \in B\setminus\{i\}} \left[A_{s_j}(b_j + E[\theta|m_T, s_j]) + B_{s_j}\right] \Pr(s_{B\setminus\{i\}}|\theta) f(\theta|m_T, s_i) d\theta
\end{aligned}$$

The manipulations on this equation are analogous in that we did previously. Hence, performing similar substitutions, we obtain the expression:

$$\begin{aligned}
(b_i + E[\theta|m_T, s_i])\left[1 + (n-1)\eta\right] = {} & A_{s_i}(b_i + E[\theta|m_T, s_i]) + B_{s_i} \\
& + \eta \sum_{j\in T}\left[A(b_j + E[\theta|m_T]) + B\right] \\
& + \eta \sum_{j \in B\setminus\{i\}} \left[A_0(b_j + E[\theta|m_T, s_j = 0]) + B_0\right]\left(1 - E\left[\theta|m_T, s_i\right]\right) \\
& + \eta \sum_{j \in B\setminus\{i\}} \left[A_1(b_j + E[\theta|m_T, s_j = 1]) + B_1\right] E[\theta|m_T, s_i]
\end{aligned}$$

which, again, gives us the following FOC for babbling players with private signal s_i ($= 0$ or 1)

$$(b_i + E[\theta|m_T, s_i])\left[1 + (n-1)\eta\right] = x_i(s_i) + \eta \sum_{j \in T} x_j + \eta\left(1 - E\left[\theta|m_T, s_i\right]\right) \sum_{j\in B\setminus\{i\}} x_j(0) + \eta E[\theta|m_T, s_i] \sum_{j \in B\setminus\{i\}} x_j(1)$$

Together, we can sum up the best responses for the three types of players as the following:

Truthful player $i \in T$ -

$$x_i = (b_i + E[\theta|m_T])\left[1 + (n-1)\eta\right] - \eta \sum_{j \in T \setminus \{i\}} x_j - \eta\left(1 - E\left[\theta|m_T\right]\right) \sum_{j \in B} x_j(0) \\ - \eta E[\theta|m_T] \sum_{j \in B} x_j(1) \tag{6}$$

Babbling player with low signal $i \in B$, $s_i = 0$ -

$$x_i(0) = (b_i + E[\theta|m_T, 0])\left[1 + (n-1)\eta\right] - \eta \sum_{j \in T} x_j - \eta\left(1 - E\left[\theta|m_T, 0\right]\right) \sum_{j \in B \setminus \{i\}} x_j(0) \\ - \eta E[\theta|m_T, 0] \sum_{j \in B \setminus \{i\}} x_j(1) \tag{7}$$

Babbling player with high signal $i \in B$, $s_i = 1$ -

$$x_i(1) = (b_i + E[\theta|m_T, 1])\left[1 + (n-1)\eta\right] - \eta \sum_{j \in T} x_j - \eta\left(1 - E\left[\theta|m_T, 1\right]\right) \sum_{j \in B \setminus \{i\}} x_j(0) \\ - \eta E[\theta|m_T, 1] \sum_{j \in B \setminus \{i\}} x_j(1) \tag{8}$$

To verify if the equilibrium actions dictated by Lemma 1 is indeed right, we substitute them into the RHS of each of the above three equations 6, 7 and 8.

Take equation 6 :

$x_i \;\; = (b_i + E[\theta|m_T])\left[1 + (n-1)\eta\right] - \eta \sum_{j \in T \setminus \{i\}} x_j - \eta\left(1 - E\left[\theta|m_T\right]\right) \sum_{j \in B} x_j(0) - \eta E[\theta|m_T] \sum_{j \in B} x_j(1)$

$$\begin{aligned} x_i = (b_i + E[\theta|m_T])\left[1 + (n-1)\eta\right] - \eta \sum_{j \in T \setminus \{i\}} \frac{(1 + (n-1)\eta)}{1 - \eta} b_j + \frac{(t-1)\eta^2}{(1-\eta)} \sum_{g \in N} b_g \\ - \eta.(t-1)\frac{(k+1)}{(t+2)} - \eta \sum_{j \in B} \frac{(1 + (n-1)\eta)}{1 - \eta} b_j + \frac{b.\eta^2}{(1-\eta)} \sum_{g \in N} b_g \\ - \eta.b.\frac{(k+1)}{(t+2)}.\frac{h(t)}{1 + h(t)} - \eta.b.E[\theta|m_T].\frac{1}{1 + h(t)} \end{aligned}$$

Making the substitution that $E[\theta|m_T] = \frac{(k+1)}{(t+2)}$, we get,

$$x_i = [1 + (n-1)\eta]\, b_i + \frac{(k+1)}{(t+2)} + (n-1)\eta \frac{(k+1)}{(t+2)} - \eta \sum_{j \in N \setminus \{i\}} \frac{(1+(n-1)\eta)}{1-\eta} b_j + \frac{(n-1)\eta^2}{(1-\eta)} \sum_{g \in N} b_g - \eta.(n-1)\frac{(k+1)}{(t+2)}$$

$$x_i = [1 + (n-1)\eta]\, b_i + \frac{(k+1)}{(t+2)} - \frac{\eta}{1-\eta} \sum_{j \in N \setminus \{i\}} b_j - \frac{(n-1)\eta^2}{1-\eta} \sum_{j \in N \setminus \{i\}} b_j + \frac{(n-1)\eta^2}{(1-\eta)} b_i + \frac{(n-1)\eta^2}{(1-\eta)} \sum_{j \in N \setminus \{i\}} b_j$$

$$\implies x_i = \frac{(1+(n-2)\eta)}{1-\eta} b_i - \frac{\eta}{1-\eta} \sum_{j \in N \setminus \{i\}} b_j + \frac{(k+1)}{(t+2)}$$

The above equation can be rewritten as,

$$x_i = \frac{(1+(n-1)\eta)}{1-\eta} [b_i - \frac{\eta}{(1+(n-1)\eta)} \sum_{j \in N} b_j] + \frac{(k+1)}{(t+2)}$$

Similar substitutions and simplification yields the equilibrium actions of the babbling types from their best response equations 7 and 8. This completes the proof.

A.2 Proof of Theorem 1

Necessity: As argued in Section 3, a $0 - type$ player always reveals the low signal and the $1 - type$ player never misreports a high signal. The only cases of relevance then is one where $0 - type$ ($1 - type$) gets a high (low) signal.

Take the case of a $0 - type$ player. For i to reveal a high signal $s_i = 1$, it must be that, for any possible realization of the other $(n-1)$ players' signals, sending a truthful message $m_i = s_i = 1$ must be optimal. This means that the equilibrium action of i, $x_i(1, 1, m_{-i}) \geqslant 0$ for any set of (truthful) messages from the other players, m_{-i}. Since the posterior on the state θ is a beta-binomial distribution, what matters is the sufficient statistic k, the number of 1's in the set of messages (m_i, m_{-i}).

Therefore, for i to reveal $s_i = 1$, a set of n constraints (corresponding to $k = 1$ to n). However, the tightest constraint that would ensure this is when every other player reveals 0, meaning that $\sum m_{-i} = 0$. In this case, if $m_i = 1$, then $k = \sum\limits_{j \in N} m_j = 1$ and therefore the expected value of θ, $E[\theta \mid m] = \frac{2}{n+2}$. Once this constraint is satisfied, every other IC for player i must be satisfied. From the equation 3, it must be that,

$$\frac{(1+(n-1)\eta)}{1-\eta}.A_i + \frac{2}{(n+2)} \geqslant 0$$

$$A_{i\in 0-type} \geqslant -\frac{2}{(n+2)}.\frac{(1-\eta)}{(1+(n-1)\eta)} \tag{9}$$

A similar argument ensues for a player $j \in 1-type$. For i to reveal a low signal truthfully, it must be that for any other order of $(n-1)$ truthful signals from the other players, player i's optimal action upon sending the message $m_j = s_j = 0$ must be within the upper bound of the action set. As before, we only need to concentrate on the tightest IC that satisfies this condition. In the case of j, this is the constraint when $\sum m_{-j} = (n-1)$, that is, every other player reveals a high signal. In this case, if $m_j = 0$, then $k = \sum\limits_{N} m = (n-1)$ and therefore the expected value of θ, $E[\theta \mid m] = \frac{n}{n+2}$. Once this constraint is satisfied, every other IC for player j must be satisfied. From the equation 3, it must be that,

$$\frac{(1+(n-1)\eta)}{1-\eta}.A_j + \frac{n}{(n+2)} \leqslant 1$$

$$A_{j\in 1-type} \leqslant (1-\frac{n}{(n+2)}).\frac{(1-\eta)}{(1+(n-1)\eta)} = \frac{2}{(n+2)}.\frac{(1-\eta)}{(1+(n-1)\eta)} \tag{10}$$

Since $A_{i\in 0-type} < 0$ and $A_{j\in 1-type} > 0$ by definition, by combining equations 9 and 10, we conclude that there is full information aggregation if:

$$A_{i\in N} \leqslant \frac{2}{(n+2)}.\frac{(1-\eta)}{(1+(n-1)\eta)} \tag{11}$$

Sufficiency:

We prove this by contradiction. Suppose there is a $n-$player equilibrium and also that for some player $i \in N$, condition 11 is violated. Without loss of generality, let the condition be violated for player n, with conflict of interest b_n. Then, given that each of remaining $(n-1)$ players are being truthful, it requires to be checked if n has an incentive to report her signal. Since $b_n = \sup\{b_i : i \in N\}$, n is a $1 - type$ player. Further, as before, $s_n = 0$ and n reports truthfully. Then, if each of the other signals are such that $\sum m_{-n} = (n-1)$, then the equilibrium action of n is $x_n = \min\{1, \frac{(1+(n-1)\eta)}{1-\eta}.A_j + \frac{n}{(n+2)}\} = 1$, since condition 11 is violated by our construction. This implies there is under-provision from n's point of view.

Now instead, if n misreports her signal and sends a message $m_n = 1 - s_n = 1$, then the actions of every other player is increased in equilibrium to the following:

$$x_{j\in N\setminus\{n\}}(s_j, m_{-n}, 1) = \frac{(1+(n-1)\eta)}{1-\eta}.A_j + \frac{(n+1)}{(n+2)}$$

The above is the equilibrium action of every player other than n, who received a signal s_j, received truthful messages from every other player apart from n, m_{-n}, and receive the message $m_n = 1$ from n. Letting the above expression to be within the bounds, meaning $0 \leqslant x_{j\in N\setminus\{n\}}(s_j, m_{-n}, 1) \leqslant 1$, this implies x_n is also modified according to n's best response equation, given in 6. Specifically,

$$x_n(s_n, m_{-n}, 1) \quad = (b_n + E[\theta|m_{-n}, s_n])\,[1+(n-1)\eta] - \eta \sum_{j\in N\setminus\{n\}} x_j(s_j, m_{-n}, 1)$$

Substituting and simplifying yields the following revised action,

$$x_n(s_n, m_{-n}, 1) \quad = \frac{(1+(n-1)\eta)}{1-\eta}.A_j + \frac{n(1-\eta)+\eta}{(n+2)}$$

The optimal action is therefore $x_n = \min\{1, \frac{(1+(n-1)\eta)}{1-\eta}.A_j + \frac{n(1-\eta)+\eta}{(n+2)}\}$. It is easy to conclude that irrespective of whether $x_n(s_n, m_{-n}, 1) \leqslant 1$ or not, n is better off since the actions of other players have unequivocally risen. Thus, n benefits from deviating to $m_n = 1$ when $s_n = 0$. But if this is true, then a $n-$player equilibrium ceases to exist, contradicting the starting assumption. This concludes the proof.

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