

# Generalized Glass Ceilings in the United States – A Stochastic Metafrontier Approach

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# **Generalized Glass Ceilings in the United States – A Stochastic Metafrontier Approach\***

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## **ABSTRACT**

This paper highlights the limitations inherent to the stochastic earnings frontier methodology to analyzing wage discrimination and introduces the use of the metafrontier approach as an important improvement. Using US data from the Current Population Survey, we find that white women's and black men's maximum attainable hourly earnings represent respectively 80% and 76% of those of white men on average. Furthermore, the metafrontier approach shows that male-female and white-black differences in maximum attainable earnings are observed at all levels of human capital. This innovative methodology permits the identification of a “generalized” glass ceilings against females and blacks in the US.

**Keywords:** wage differentials, discrimination, glass ceiling, stochastic frontier, stochastic metafrontier approach, sample selection correction.

**JEL classification:** J31, J71, C13

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## 1. Introduction

For many years, the literature on wage discrimination literature has constituted a major part of labor economics. The issue is of great interest to policymakers not only because it is about fairness, but also because it has serious economic consequences on the nation as a whole (see Becker 1957; Ferrant and Kolev 2016)<sup>1</sup>. Wage discrimination can be defined as the situation where given the same labor market characteristics (particularly productivity and involvement), workers from a demographic group earn lower wages relative to their counterparts from another group. Despite the rich literature on the question, identifying and assessing discriminatory practices in terms of earnings, “beyond any reasonable doubt”, remains a daunting task. Many works attempted to propose a methodology that appropriately investigates wage discrimination, one of which is the stochastic earnings frontier (SEF hereafter) approach. This new method has been presented as an advantageous alternative to the “traditional” methodology to investigating wage discrimination of the decomposition of Oaxaca (1973) and Blinder (1973). The SEF approach permits the estimation of workers' inability to capture the maximum attainable<sup>2</sup> earnings corresponding to their investment in human capital in a context of imperfect information and costly job search (Hofler and Polachek (1985)). In Robinson and Wunnava (1989), the first published SEF application to analyzing discrimination, the deviation of observed earnings from frontier earnings – referred to as “earnings inefficiency” – is said to be entirely due to direct discrimination and exclusive to the group experiencing it (females in the US). Subsequent works recognized that even the non-

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<sup>1</sup> Global economy loses about USD 12 trillion because of gender-based discrimination.

<sup>2</sup> Throughout the paper, we use interchangeably the terms “potential earnings”, “frontier earnings”, and “maximum attainable earnings”.

discriminated group may have “earnings inefficiency” for other reasons, in particular ignorance of labor market conditions (Polachek and Robst (1998) provide empirical evidence). That deviation is therefore seen as an indication of wage discrimination only when it differs statistically from one demographic group to another (Robinson and Wunnava 1989; Slottje et al. 1994; Dawson et al. 2001; Bishop et al. 2007; Diaz and Sanchez 2011). The method is argued to be superior to the traditional decomposition method because it allows one to avoid making arbitrary choices, when conducting the decomposition, about which demographic group has a discriminatory or non-discriminatory wage structure. Another argument that has been put forward is the fact that SEF – based discrimination analysis can provide estimates of an individual-specific measure of discrimination instead of average discrimination information.

Despite its advantages, the “solution” has its own limitations. While the traditional method of decomposition is criticized because of the risk of mistakenly attributing the unexplained wage gap to discrimination exclusively, the criticism extends to the SEF methodology. Indeed, differences in earnings (in)efficiency scores across demographic groups are not necessarily due to direct discrimination, and may just be an evidence of variations in non-market characteristics across the demographic groups. For instance, women and men might have different wage bargaining and/or job prospecting skills, or maybe women are more prone to compensating differentials. Women can also reduce their investment in job search if they are aware (or believe) that the associated marginal benefits are lower with the respect to those for males. Women would have therefore larger deviations from their potential earnings compared to their male counterparts, and this makes these scores an ambiguous measure for wage discrimination analysis. Furthermore, even if discrepancies in earnings (in)efficiency reflect discrimination in the labor market, the scores which are compared are either biased or not comparable. They are biased when they are obtained from a single pooled

SEF estimation which constrains workers from different groups to use the same technology to convert their investment in human capital into earnings. This is the approach adopted by Slottje et al. (1994), Bishop et al. (2007), and Diaz and Sanchez (2011). On the other hand, when the scores are obtained from the estimation of separate SEF models (Dawson et al. (2001)), they are measures relative to different group-specific earnings frontiers and hence they are not comparable across groups.

This paper proposes the use of the stochastic *metafrontier* approach to improve the reliability of SEF – based discrimination analysis while maintaining the advantages of the method. Thanks to the technology gap ratio measure, we can compare maximum attainable earnings for workers from different demographic groups whose earnings frontiers are separately estimated. A similar methodology is adopted in a recent study (Garcia-Prieto and Gómez-Costilla (2017)) which compares male-female potential earnings through the incorporation of a gender dummy in a pooled SEF estimation. However, such an approach can lead to biased estimates because of the constrained pooled estimation. Furthermore, we propose an innovative and effective method for testing and assessing the extent to which there is an artificial barrier which prevents women and/or minorities from accessing a certain level of earnings associated with management, executive or simply supervisory positions. Using US data on prime-aged individuals working full-time in the private sector (2006 Current Population Survey - Outgoing rotation group), we investigate possible wage discrimination against females and blacks. At all levels of human capital (not only for highly qualified workers), white women and black men have lower maximum attainable earnings compared to white men. This is what we call a “generalized glass ceiling” against women and blacks. Our results appear to be robust as we take into account sample selection and heteroskedasticity issues and control for job occupation and industry sectors.

## 2. A stochastic metafrontier approach to wage discrimination analysis

### *(i) A stochastic earnings metafrontier model*

The metafrontier analysis allows one to take account of differences in production technologies when estimating frontier models (Battese and Rao 2002; Battese et al. 2004; O'Donnell et al. 2008). Building on that literature, we construct a stochastic earnings metafrontier (SMF hereafter) model. We follow the two-step procedure proposed by Huang et al. (2014) which has the advantage of taking account of random shocks making the metafrontier stochastic, and producing results with desirable statistical properties. In the first step, group-specific SEF models are estimated and fitted frontier earnings are obtained for each group. Then, in a second step, a new frontier is estimated, but this time over the fitted potential earnings obtained from the previous step, as the metafrontier is the envelope over group-specific frontiers.

Let  $y_{is}$  be the observed (current) hourly wage earned by individual  $i$  who belongs to the demographic group  $s$ . The individual  $i$  is assumed to convert  $x_{is}$ , a vector of general human capital variables (education, work experience and its square), following Polachek and Xiang (2006), into maximum attainable earnings (frontier or “potential” earnings) through her/his group-specific production function:  $g_s(x_{is}; \beta_s)$ :

$$\ln y_{is}^F = g_s(x_{is}; \beta_s) + v_{is}, \text{ with } i = 1, 2, 3 \dots N; s = 1, 2, 3 \dots S \quad (1)$$

Equation (1) gives group-specific stochastic earnings frontiers, and is stochastic because it takes account of pure random shocks ( $v_{is}$ ). It corresponds to the maximum earnings attainable by workers from a particular group  $s$ , given human capital endowment, taking account of random noise. Then, in equation (2), observed earnings deviate from maximum attainable earnings due to “earnings

inefficiency"  $u_{is}$  (positive or zero):

$$\ln y_{is} = \ln y_{is}^F - u_{is} \quad (2)$$

Replacing equation (2) in equation (1) we obtain:

$$\ln y_{is} = g_s(x_{is}; \beta_s) + v_{is} - u_{is} = g_s(x_{is}; \beta_s) + \varepsilon_{is} \quad (3)$$

Equation (3) corresponds to the group-specific SEF model to be estimated. Observed earnings deviate from maximum attainable earnings because of pure random shocks (normal zero-mean error term:  $v_{is} \sim N(0, \sigma_{v_s}^2)$ ) and earnings inefficiency ( $u_{is} \geq 0$ ). For the sake of presentation, we assume that  $u_{is} \sim |N(0, \sigma_{u_s}^2)|$ , which is the half-normal distribution (following the seminal paper of Aigner, Lovell and Schmidt (ALS) (1977)).

We estimate as many SEF models as there are groups. Because sample selection is a crucial issue when estimating earnings equations and particularly when analyzing discrimination (Stanley and Jarell (1998)), the recent approach proposed by Lai (2015) is used to correct for this potential bias. Given individual characteristics ( $w_{is}$ ), the probability of being employed at the time of the survey –  $\Pr(e_{is} > -w_{is}\gamma)$  – is taken into account through the following employment equation (selection):

$$d_{is}^* = w_{is}'\gamma + e_{is} \quad (3a)$$

Only the sign of the latent variable defined in equation (3a) is observed (positive when the individual is employed).

$\gamma$  is the vector of the coefficients to be estimated;

$e_{is}$  is a standard normal error term, which could be correlated with the normal zero-mean error term

that is present in the frontier ( $v_{is}$ ), with a correlation coefficient  $\rho$ .

Therefore, a simple version of the log-likelihood function for the stochastic frontier model corrected for sample selection (with a half normal specification for the inefficiency term)<sup>3</sup> is:

$$\begin{aligned} \ln L(\theta, \gamma) = \sum_{i \in \{e_{is} > -w'_{is}\gamma\}} \left[ \ln \varphi \left( \frac{\varepsilon_{is}}{\sqrt{(\sigma_v^2 + \sigma_u^2)}} \right) - \frac{1}{2} \ln(\sigma_v^2 + \sigma_u^2) - \ln \left( \frac{1}{2} \right) + \right. \\ \left. \ln \Phi_2(\Gamma_A \varepsilon_{is}; \kappa_{A,is}, \Delta_A) \right] + \sum_{i \in \{e_{is} \leq -w'_{is}\gamma\}} \ln \Phi(-w'_{is}\gamma) \end{aligned} \quad (3b)$$

Where,  $\varphi$  and  $\Phi$  denote respectively the probability density function and the cumulative density function of a standard normal variable.  $\Phi_2(\cdot; \mathbf{M}, \mathbf{V})$  is the cumulative distribution of a bivariate normal distribution with mean  $\mathbf{M}$  and variance  $\mathbf{V}$ .

The remaining elements of the log-likelihood function are:

$$\Gamma_A = \frac{1}{\sigma_v^2 + \sigma_u^2} \begin{pmatrix} \rho \sigma_v \\ -\sigma_u^2 \end{pmatrix} \quad (3c)$$

$$\kappa_{A,is} = \begin{pmatrix} -w'_{is}\gamma \\ 0 \end{pmatrix} \quad (3d)$$

$$\Delta_A = \frac{1}{\sigma_v^2 + \sigma_u^2} \begin{pmatrix} (1 - \rho^2)\sigma_v^2 + \sigma_u^2 & \rho \sigma_v \sigma_u^2 \\ \rho \sigma_v \sigma_u^2 & \sigma_v^2 \sigma_u^2 \end{pmatrix} \quad (3e)$$

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<sup>3</sup> The formulas given in Lai (2015) are given for a truncated-normal inefficiency component.

Formulas for a half-normal inefficiency component can be easily obtained replacing by zero the mean of the pre-truncated normal distribution used.

See Lai (2015) for proofs and full details.

The model can be estimated using a two-step procedure: (1) estimate the employment equation, and then (2) use the obtained predicted probability of being employed in the log-likelihood to be maximized.

Having maximized (3b) and thus estimated the parameters of (3) for each group, we can predict unit-specific earnings efficiency scores which are corrected for sample selection, following the formula given in Lai (2015):

$$EFF_{is} = E[\exp(-u_{is})|\{\hat{\varepsilon}_{is}|e_{is} > -w'_{is}\gamma\}] \quad (3f)$$

$$\text{with } \hat{\varepsilon}_{is} = v_{is} - u_{is} = \ln y_{is} - \hat{g}_s(x_{is}; \hat{\beta}_s)$$

Earnings efficiency is the ratio of observed earnings to group-specific frontier earnings. Thus, a worker  $i$  from the demographic group  $s$ , manages to capture  $(100\% * EFF_{is})$  of the maximum attainable earnings of *this worker's demographic group*, given her/his human capital endowment. Indeed, since we are estimating earnings frontiers separately by group, workers with the same level of human capital (HC) endowment but from different groups might have different maximum attainable earnings.

As metafrontier earnings are the envelope over group-specific frontier earnings, we can write:

$$g(x_i; \beta) = g^M(x_i; \beta^M) - U_i^M \quad (4)$$

Where the metafrontier earnings are given by  $g^M(x_i; \beta^M)$ . The subscript  $s$  is dropped because all groups (thus all workers) are pooled for the estimation of the metafrontier.  $U_i^M \geq 0$  is the deviation of an individual's frontier earnings from metafrontier earnings. When  $U_i^M = 0$ , this means, for

individual  $i$ , potential earnings equal metafrontier earnings, and therefore this individual is using the “best” technology to convert her/his HC endowment into maximum attainable earnings. The metafrontier is the “best” technology available. Because the metafrontier is the envelope over group-specific frontiers, it can be either a mixture of the various group-specific frontiers, or just one among the group-specific frontiers if this latter dominates all the remaining group-specific frontiers at all levels of HC endowment (see Figure 1).

However, we cannot estimate equation (4) because we do not observe  $g(x_i; \beta)$ , but we can obtain its fitted value  $\hat{g}(x_i; \hat{\beta})$  from the estimation of equation (3). Then, the fitted values deviate from the true values of frontier earnings as follows:

$$g(x_i; \beta) = \hat{g}(x_i; \hat{\beta}) - V_i^M \quad (5)$$

The deviation between the two values is considered asymptotically normally distributed with zero-mean:  $V_i^M \sim \mathcal{N}(0, \sigma_{V^M}^2)$  (see Huang et al. 2014; Chang et al. 2015). Therefore, equation (4) becomes:

$$\hat{g}(x_i; \hat{\beta}) - V_i^M = g^M(x_i; \beta^M) - U_i^M \quad (6)$$

Hence, the stochastic metafrontier model can be presented as follows:

$$\hat{g}(x_i; \hat{\beta}) = g^M(x_i; \beta^M) + V_i^M - U_i^M \quad (7)$$

In fact, equation (7) corresponds to a stochastic frontier model specification. The metafrontier has the same specification as the group-specific frontiers, with education, work experience and its square as input variables. For the sake of consistency, a half-normal distribution is again assumed for the inefficiency component  $U_i^M$  (distance between the metafrontier and group-specific frontiers). As the group-specific frontiers are already corrected for sample selection, there is no

need to correct again for sample selection when estimating the metafrontier. Indeed, the metafrontier is just an envelope over the group-specific frontiers; hence if the group-specific frontiers are well estimated then there is no reason that the estimated metafrontier will not be reliable. Thus, following (Aigner, Lovell and Schmidt (ALS) (1977)), the parameters of the model are estimated by maximum likelihood.

Once the estimates of equation (7) are obtained, individual-specific technology gap ratio scores (**TGR<sub>i</sub>**) are predicted using (Jondrow et al. (JLMS) (1982)):

$$TGR_i = \exp[-E(U_i^M | \hat{\varepsilon}_i^M)] \quad (7a)$$

The latter is the estimated ratio of frontier to metafrontier earnings:

$$\frac{y_i^{Frontier}}{y_i^{Metafrontier}} = \exp(-U_i^M) \quad (7b)$$

This ratio is bounded between 0 and 1. For a given individual *i*, the higher the **TGR<sub>i</sub>** is, the closer this individual's potential earnings are to metafrontier earnings. Therefore, given a level of human capital endowment, workers with a technology gap ratio of 1 could attain the maximum over all group-specific potential earnings.

“Metafrontier earnings efficiency” (**ME<sub>i</sub>**) is the ratio of current earnings to metafrontier earnings. Thus, (**100% \* ME<sub>i</sub>**) gives the percentage of the maximum possible earnings, regardless of the technology used, the individual *i* manages to attain. Basically, “metafrontier earnings efficiency” corresponds to the portion of the maximum possible earnings we would have earned, had we used the “best” technology to convert our HC endowment into maximum attainable earnings. **ME** is the product of two components: group-specific earnings efficiency and technology gap ratio (see

Huang et al. (2014) for details).

$$ME_i = EFF_i * TGR_i \quad (7c)$$

Hence, workers could fail to attain their metafrontier earnings, given the HC in which they invested, because: (a) they do not manage to capture their group-specific potential earnings and/or (b) they are in a group that does not use the “most advanced” technology to convert their HC endowment into maximum attainable earnings. It is noteworthy that metafrontier earnings efficiency (**ME**) could be equal to group-specific earnings efficiency (**EFF**) when technology gap ratio is equal to 1. In fact, the technology gap ratio (**TGR**) converts group-specific earnings efficiency into metafrontier earnings efficiency so that the latter can be compared even though workers have different earnings frontiers. This is an advantage over earlier SEF approaches.

#### *(ii) SMF-based wage discrimination analysis*

From the previous subsection, we obtain two major earnings inequality measures which can be reliably compared: the technology gap ratio and the metafrontier earnings efficiency.

Comparing the **TGR** across different groups allows us to know which group defines the metafrontier or is the closest to the metafrontier, depending on the form of the latter<sup>4</sup>. Therefore, we could know, given the same human capital endowment, whether the maximum wages attainable by workers from a specific demographic group (females or minorities) are lower than those of their counterparts from another demographic group. Consequently, the group with the lowest

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<sup>4</sup> Either the domination of a group’s earnings frontier over that of all the other groups, or the mixture of the various group-specific earnings frontiers (see Figure 1).

technology gap ratios has the lowest maximum attainable hourly wage and is considered to be discriminated against. Indeed, differences in technology gap ratios are entirely due to differences in frontier earnings, hence due solely to belonging to different demographic groups, and are therefore plainly the consequence of wage discrimination against a group.

Nevertheless, unlike with the *TGR*, we cannot reliably draw conclusions about wage discrimination only by comparing metafrontier earnings efficiency scores. Although *ME* scores are numerically comparable (contrary to what has been used in earlier studies), they do not capture solely wage discrimination. By definition *ME* is the product of the *TGR* with group-specific earnings efficiency (*EFF*) which is considered as a measure of workers' information (or wage bargaining skills<sup>5</sup>) in the labor market. *EFF* could also include some forms of compensating differentials. Hence, differences in *ME* cannot reliably be attributed to discrimination, although disparities in wage bargaining or job prospecting skills and compensating differentials might be the consequence of pre-market discrimination, due to the role played by education and society with regard to women or minorities.

For all these reasons, our preferred wage discrimination measure is the *TGR*. The *ME* measure will be used to provide an indication of whether workers from a given demographic group manage better than those from another group to capture their metafrontier earnings (their maximum

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<sup>5</sup> For instance: "Women often do not get what they want and deserve because they do not ask for it". "Men are more likely than women to negotiate for what they want". (see Nice Girls Don't Ask by Babcock, L., Laschever, S., Gelfand, M. and Small, D. (2003), Harvard Business Review, pp.1).

possible earnings regardless of the group they belong to). This new method to analyzing wage discrimination has many advantages over previous methodologies, particularly the decomposition of Oaxaca (1973) and Blinder (1973). The method does not require any a priori assumption about the discriminatory wage structure. In addition, the *TGR* provides an individual-specific measure of discrimination. More importantly, discrimination analysis with the *TGR* has the advantageous feature of controlling for part of any unobserved heterogeneity even in a cross-section context. As argued by Greene (2005), in a cross sectional stochastic frontier model, the inefficiency component is “forced” to capture individual unobserved heterogeneity, possibly biasing group-specific earnings efficiency. Consequently, because the discrimination analysis introduced in this paper consists in comparing group-specific maximum attainable hourly wages and not group-specific efficiency measures, this issue appears to be, in fact, an advantage for the methodology.

***(iii) The stochastic earnings metafrontier approach and the glass ceiling phenomenon***

The definition of the technology gap ratio suggests that if those scores are higher for male workers (at least on average) relative to those of female workers, then this simply means maximum earnings attainable by females are lower (at least on average) than those attainable by males, given HC endowments. Such a scenario is reminiscent of the concept of the “glass ceiling”. Indeed, the “glass ceiling” concept comes from the idea that there exist discriminatory barriers which prevent female workers (or workers belonging to some minorities) from reaching top job positions<sup>6</sup>. This leads to a situation where, given the same HC endowment, women do not earn as much as their men counterparts because they are less likely to get promoted. Therefore, the maximum earnings

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<sup>6</sup>The concept “glass ceiling” was originally used by Hymowitz, C., and Schellhardt, T. D. (1986), in The Glass Ceiling Special Report on the Corporate Woman, The Wall Street Journal, 24.

women could attain would lie below those attainable by males. And this is exactly why we advocate the use of the technology gap ratio to analyze the glass ceiling phenomenon.

The technology gap ratio is an appropriate and quite intuitive tool to conducting glass ceiling analysis, and provides a different approach to the existing methods in the literature. Earlier works have in part been concentrated on observing whether gender wage gap increases and accelerates at the top of the wage distribution. To do so, quantile regression (conditional) is used to estimate wage gap at different percentiles along the wage distribution. If there is a glass ceiling, the wage gap will be much greater at the top of earnings distribution (Albrecht et al. 2003, 2015; Arulampalam et al. 2007; Le and Miller 2010). On the other hand, a different approach examines gender-specific promotion possibilities (Powell and Butterfield 1994; Groot and van den Brink 1996; McDowell et al. 1999; Sabatier 2010; Busch and Holst 2011).

The approach we propose, based on the technology gap ratio, goes further than these approaches to investigating the glass ceiling phenomenon. As we have seen above, those already existing approaches have in common to verifying whether women (or some minorities) cannot get access to some top job positions. However, by comparing group-specific potential earnings with the *TGR*, we can examine whether there exist discriminatory barriers not only for top job positions, but at all levels of the professional hierarchy. In fact, even at the bottom, workers from a given demographic group could be denied access to simple supervisory positions. This is the case, for instance, when in a factory, male workers are much more likely to be appointed as foreman or team leader compared to their female counterparts.

### **3. Data**

#### ***(i) Data source and selection of the sample***

Our interest in the U.S is not only motivated by the availability of rich sets of racial and ethnic data, but also because the issue of wage discrimination has garnered considerable attention in the U.S over the last years. An example of this is President Obama signing his first bill into law, the “Lilly Ledbetter Fair Pay Act”, which shows how important the issue is to Americans. Furthermore, the specificity of access to higher education in the U.S drew our attention about the possible serious consequences that wage discrimination might have on discriminated groups in that country. Higher education is expensive in the U.S and students generally need loans to get to college. Here, the problem lies in the fact that female (or minorities) college graduates are going to be as indebted as their male counterparts; however, females (or minorities) will be less able to pay off their student loans if they are discriminated against. In fact, besides suffering the consequences of discrimination over their whole working life, those female (or minorities) graduates will have lower pensions, even if they participated in labor market to the same extent as their counterparts.

We use data from the Current Population Survey (CPS) which is the main labor statistics source in the US. The data are collected by the United States Census Bureau. We use the Outgoing Rotation Group (ORG) files made publicly available by the Center for Economic and Policy Research (CEPR). The advantage of the ORG extract lies in the fact that it is the largest and the most representative data on the US labor market, providing detailed information on earnings, education, demographic characteristics, etc. The 2006 annual extract of the CEPR ORG is used, a choice motivated by the fact that 2006 is far enough from any economic upheaval (see Figure A1 in

**Appendix**) in order to avoid misleading results and conclusions. In addition, we are able to examine the glass ceiling effect in the US 10 years after the Glass ceiling commission's (1991 – 1996) important recommendations to the US Government and businesses about removing the phenomenon.

The analysis focuses on prime-aged workers (25 – 55 years). We exclude workers from fishing, farming, hunting and agriculture sectors. Workers from the public sector are also excluded as there might exist worker selection into the public or the private sector according to personal characteristics (see Bellante and Link 1981; Blank 1985). Students, self-employed and part-time (working less than 35 hours) workers are also removed from our analysis for the purpose of analyzing wage discrimination on workers with roughly the same degree of involvement on the labor market. Finally, we excluded workers whose hourly wage is below \$1 or higher than \$100 to remove the outliers.

### ***(ii) Descriptive statistics***

Table 1 gives the composition of the sample used for the analysis. The figures show that the overall sample is balanced according to gender. However, among employed individuals, there are more males than females (56% for the former and 44% for the latter). This is due to lower labor market participation for women relative to that of men. While they have similar unemployment rates, non-participation in labor force for women is more than double that of men (see Figure 2). It is interesting to see that female workers are slightly more educated (hold a Bachelor or higher) than male workers. In fact, the difference comes mainly from gender educational attainments among blacks, as there is no difference among whites.

Black workers constitute a minority which represents about more than one-tenth of the employed

individuals. Figure 2 suggests that unemployment for blacks is around the double that of whites, among both females and males. Among men, non-participation in labor market for whites is around the half of that of blacks, while there is no difference among women.

Descriptive statistics on the variables used in the estimation of stochastic earnings frontier models are given in Table 2. The natural logarithm of hourly wage is used as dependent variable. The hourly wage in question here, includes overtime, tips and commissions, and is the one recommended by the CEPR to be the most reliable and consistent among the hourly wage variables available. The explanatory variables, which determine the group-specific stochastic earnings frontier, consist of general human capital variables: years of education and potential work experience (defined by age minus years of education minus five years, assuming the minimum age of schooling to be five).

#### **4. Empirical results**

##### ***(i) Estimated group-specific frontiers***

Group-specific earnings frontiers are first estimated before fitting the stochastic earnings metafrontier model, following the procedure presented in Section 2. The technology used by female workers to convert their human capital into earnings is allowed to be different from that of male workers and the same is done by race.

Table 3 gives the parameter estimates of earnings frontiers corrected for sample selection, for women and men. As recommended by Lai (2015) and Greene (2010), we use the Murphy-Topel variance estimator to adjust the variance-covariance matrix of the parameters estimated in the sample selection stochastic frontier model. A likelihood ratio (LR) test that the two genders use

the same technology (null hypothesis:  $H_0$ ) to convert their human capital into earnings confirms at the 1% significance level<sup>7</sup> that two models are more appropriate than a single pooled one. This is an important finding which makes the estimation of a metafrontier relevant.

Within each gender group, the hypothesis that whites and blacks have the same technology is rejected by a LR test at the 1% significance level<sup>8</sup>. In addition, we tested whether one earnings frontier for the whole sample (null hypothesis:  $H_0$ ) is a better fit compared to four earnings frontiers, one for each gender and race (black women, white women, black men and white men). This hypothesis is also rejected at the 1% significance level. Thus, four earnings frontiers are estimated: for black women, white women, black men and white men.

Table 4 presents the parameter estimates for the race-specific earnings frontiers (within each gender group), corrected for sample selection. It is noteworthy that all the correlation coefficients (“rho”) between the unobservables for the selection model and those for the SEF model (the noise)

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<sup>7</sup> The likelihood ratio statistic is:

$$LR = -2 \times \{Ln[H_0] - Ln[H_1]\} = -2 \times \{(-64,551.95) - (-62,839.94)\} = 3,424.02 \sim \chi^2_{(8)}$$

<sup>8</sup> The likelihood ratio statistics are:

- Women:  $LR = -2 \times \{Ln[H_0] - Ln[H_1]\} = -2 \times \{(-31,487.76) - (-31,328.68)\} = 3,424.02 \sim \chi^2_{(8)}$
- Men:  $LR = -2 \times \{Ln[H_0] - Ln[H_1]\} = -2 \times \{(-31,352.18) - (-31,121.56)\} = 461.23 \sim \chi^2_{(8)}$
- All sample:  $LR = -2 \times \{Ln[H_0] - Ln[H_1]\} = -2 \times \{(-64,551.95) - (-62,450.25)\} = 4,203.40 \sim \chi^2_{(22)}$

are significant. They are positive for women and negative for men, indicating the necessity of correcting sample selection. Individuals excluded from the estimation are those with no observable salary. These persons do not participate in the labor market or are unemployed. Among women, absence of sample selection correction leads to overestimated maximum attainable wages and underestimated returns to frontier earnings (see **Appendix**, Table A2). Observed female workers (full-time, in the private sector) are those who have an advantage in participating in the labor market compared to a domestic work, and those who display better productivity signals to get hired. Therefore, women in the selected sample are likely the most competitive and the most involved in the labor market, and this is why earnings efficiency decreases when selection is taken into account among women. The positive selection effect is even more pronounced among black women, probably because the gains from labor market participation are lower and because selection to employment is more demanding, especially if they are discriminated against. On the other hand, the story is different with male workers. Exclusion of unemployed and non-participants leads to the underestimation of the frontier and an overestimation of returns to labor market characteristics. This negative selection can be explained in relation with job search theory. Men set a reservation wage, which is linked to their human capital (their productivity), they will accept the first job offering a pay higher than their reservation wage. In such a context, the higher is the reservation wage, the lower is the probability of finding an offer that matches expectations, but the higher will be the likelihood of reaching his maximum attainable wage. Thus, those who are observed in employment have lower earnings efficiency.

The rest of this paper will focus only on results corrected for selectivity. Returning to Table 4, all human capital variables are significant at the 1% significance level for all subgroups. Within both racial groups, women's returns to education are higher than those of men's. Comparing returns to

potential experience, those of white women are lower than men's. The opposite is true among black workers. All returns on frontier earnings are higher for white males compared to black males. However, within the females group, returns on education are higher for blacks, while the latter have lower returns on potential experience. Furthermore, there is little difference in subgroup-specific earnings efficiency between white and black female workers. Unexpectedly, there is no evidence of an earnings inefficiency component for black male workers (its variance is not significant), while white male workers manage to obtain on average 83% of their potential earnings. Based on the earlier approaches to investigating wage discrimination with SEF models (Robinson and Wunnava 1989; or Dawson et al. 2001), such results would have suggested that there is no discrimination against black workers, and white males are discriminated against. However, these group-specific earnings efficiency measures are not comparable. White males might have much higher potential earnings compared to those of black males. Basically, trying to establish which demographic group has the lowest earnings efficiency using the group-specific scores obtained from the estimation of separate SEF models (one for each group) is similar to comparing different computers' prices, expressed in different currencies. The unique way to conduct such a comparison properly is to convert those prices into the same currency (the strongest one). That is what the stochastic metafrontier approach does.

***(ii) Stochastic earnings metafrontier estimation and discrimination analysis***

The four group-specific stochastic earnings frontier (SEF) models estimated above are used to predict fitted earnings frontier values for all individuals in each group. Fitted potential earnings constitute the dependent variable for the estimation of the stochastic earnings metafrontier (SMF) model (see equation (7) in Section 2). A SMF over all four groups (black women, white women, black men and white men) is estimated and the results are reported in Table 5. There is no

difference for the parameters estimates between white men's specific-frontier and the estimated SMF, suggesting that white men's earnings frontier is the metafrontier and is above the remaining three groups' frontiers. There is a dominance of one group's frontier over that of the other groups (see  $SMF_A$  on Figure 1).

Having estimated the metafrontier, technology gap ratios (*TGR*) and metafrontier earnings efficiency scores (*ME*) can be estimated for each worker in each group as presented in equations (7a), (7b) and (7c). Table 6 depicts some descriptive statistics for those measures.

The most convenient and relevant comparison of earnings efficiency across the four demographic groups is the one comparing *ME* scores obtained from the SMF (estimated over all four groups). In this case each worker's earnings efficiency is predicted with the respect to the same maximum possible earnings, given a human capital endowment. The results for *ME* in Table 6 suggest that black women manage to capture on average only 41% of *the maximum potential earnings corresponding to their human capital endowment, available on the labor market without any distinction as to race or gender*. White women, black men and white men capture respectively, 51%, 66% and 83% of their metafrontier earnings. Black males' metafrontier efficiency is now much lower than that of white males since their potential earnings are much lower than those of white males. This shows how the comparison of group-specific earnings efficiency can be misleading, especially when investigating wage discrimination. However, as argued in Section 2, these group differences in *ME* scores cannot be attributed exclusively to wage discrimination, and we will therefore only rely on the *TGR* for such an analysis.

The *TGR*, is for a given human capital endowment, the ratio of the worker's group-specific maximum attainable earnings to the metafrontier earnings (maximum possible earnings regardless

of the group). White men have a *TGR* of 100%. This is in line with what has been said before: white men's earnings frontier is the envelope over all four groups (the metafrontier). Comparing the remaining three groups' technology gap ratios to those of white males, it is striking how far black women's earnings frontier is from that of white men. Black females' potential earnings constitute only 53% of those of white men, while white females' and black males' maximum attainable earnings constitute respectively 65% and 66% of the maximum earnings attainable by white males. To put it more clearly, if the maximum attainable hourly wage for white male workers was \$100, a black female worker, a white female worker, and a black male worker could aspire at the most, respectively to only \$53, \$65 and \$66, given the same labor market characteristics. In Figure 3, differences in potential earnings in the US is well illustrated: fitted values of group-specific earnings frontiers are compared to the metafrontier at different levels of education. The graph shows that white men's specific-frontier earnings are confounded with the metafrontier earnings which are well above the other remaining group-specific earnings frontiers. Black males' potential earnings are above those of females, except for high levels of education where the latter catch up with the former and even surpass them. At all levels of education, black females' frontier earnings are clearly beneath those of white females. In Figure 4, where the graph in Figure 3 is replicated for different levels of work experience, we can see that the maximum earnings attainable by whites are above those attainable by blacks, for both men and women. However, while at all levels of work experience white males' frontier earnings are above those of white females, we note at the beginning of working life (less than 5 years of work experience), black males' potential earnings are below those of black females.

Generally, white-black wage discrimination is investigated among men, and gender-based wage discrimination among whites. Therefore, the focus is now solely on white females, white males,

and black males. Comparing women to men among whites on the one hand, and whites to blacks among men on the other, there is a clear dominance of white men's potential earnings over those of white women and black men at all levels of education, or work experience. White women's and black men's maximum attainable hourly earnings represent on average about two-thirds of those of white men. This is evidence of wage discrimination against females and blacks. The *TGR* gives a worker-specific indication about the extent of the discrimination. The more it deviates from 100% which is the value for all white men, the larger the magnitude of the discrimination.

We used paired t-tests of the null hypothesis specifying that the respective compared efficiency measures are, on average, the same for the different groups considered. Table 7 gives the testing results. In all cases, the null hypothesis is rejected at the 1% significance level. However, white men's measures are not compared to those of white women and black women, as the differences are too strong.

*(iii) Is there a glass ceiling phenomenon in the US?*

The findings presented above, concerning differences in maximum attainable earnings, evoke inevitably the idea of "glass ceiling". Although our approach is different from the classical approach to analyzing the glass ceiling phenomenon, it is clear that the idea of maximum earnings attainable by a given demographic group being below those attainable by another demographic group, is related to the concept of "glass ceiling". Indeed, our results clearly show that, at all levels of HC endowment, the maximum earnings black male workers could attain are much lower than those white male workers could attain. The same is true when we compare white women to white men. This implies that there are unbridgeable earnings limits for women and blacks compared to men and whites respectively. This could happen if they do not have access to the same job

responsibilities and/or their potentials are not valued in the same way as their counterparts. Those limits may correspond to what has been called a "glass ceiling".

Using the same approach as Albrecht et al. (2003) with Quantile Regressions, we did not find any pattern of increasing and accelerating wage gaps neither between women and men (among whites) nor between blacks and whites (among men) (see Table 8). The results do not provide strong evidence of the existence of a glass ceiling in the US, neither against women nor against blacks. Using US data from March 1999 CPS, Albrecht et al. (2003) found similar results, and concluded that there was a weak glass ceiling effect against females in the US. This contrasts with the findings from the metafrontier analysis. In fact, as argued in Section 2, this can be explained by the fact that the traditional method considers that the glass ceiling phenomenon can only be present at the top of the professional ladder. On the other hand, the method we propose allows us to analyze whether there are invisible and artificial barriers against a group at all levels of the professional hierarchy, and not only at the top of the ladder among highly qualified individuals. Hence, the use of the metafrontier approach to investigating the glass ceiling phenomenon could be regarded as being more illuminating than the traditional method. We did find that at all levels of education and work experience (Figures 3 and 4), potential earnings are larger for whites and men relative to blacks and women respectively. Therefore, artificial barriers occur not only at the top of the ladder. We refer to this as a “generalized” glass ceiling phenomenon.

## **5. Robustness**

Although the approach this paper proposes to investigate wage discrimination overcomes many of the limitations of the traditional approach, we should be cautious and check the reliability of our results. Our metafrontier-based discrimination analysis is introduced with the simplest form of

group-specific stochastic earnings frontier. Our simple specification choices allow us to focus on the purpose of this paper but might raise some questions. First, we treat potential earnings as being determined by general human capital endowment which can be defined in terms of education and work experience (and its square). Such a choice based on Mincer's earnings specification (Mincer (1974)), has been adopted before by a number of studies in the SEF literature (see Robinson and Wunnava 1989; Polachek and Yoon 1996; Polachek and Xiang 2006; Bishop et al. 2007). However, one might argue that certain variables might shift earnings frontiers downward or upward and this could change significantly our results. For instance, women and men might not work in the same Industry Sectors and/or might not have the same job occupations. Second, career-interruption due to childbirth is another issue that may be relevant here since withdrawal from the labor force is associated with lower wages (Mincer and Ofek 1982; Kim and Polachek 1994). Thus, women's potential earnings might have depreciated due to such careers-interruptions, which might also alter our findings. Third, Kumbhakar and Lovell (2000) argue that ignoring heteroscedasticity when estimating stochastic frontier models, especially for the inefficiency component, biases frontier and efficiency estimates. This could therefore bias our new discrimination measure, the *TGR*.

To check the robustness of our results, dummy variables to control for job occupations and industry sectors are introduced in the group-specific SEF models. In a second specification, along with the additional controls, a heteroscedasticity parametrization of the variances of the error terms is adopted. The variances are expressed as a function of years of education, work experience, and individual characteristics which include the number of children, being married, living in a major city, and being a foreign citizen. Estimated efficiency measures are given in Table 9 (the parameter estimates are given in the **Appendix**). These measures are compared to those obtained with our baseline model which is specified with only human capital variables. Based on group-specific

earnings efficiency (*EFF*), discrimination against black males cannot be demonstrated in any of the robustness estimations. The results controlling for heteroscedasticity would have even suggested discrimination against white males compared to white females or black males. This confirms that the *EFF* measure is not to be used for discrimination analysis. However, based on the *TGR* scores, none of our conclusions change in any of the robustness specifications. Comparing white females to white males on the one hand, and black males to white males on the other, there is still evidence of wage discrimination against females and blacks. In both cases, dominance of white males' group specific earnings frontier is evident (*TGR* of 100%) and supports the idea that there is a "generalized" glass ceiling against females and blacks. Only the magnitude of the discrimination is reduced. For the heteroscedasticity specification, which we believe to be the least biased, the maximum attainable hourly earnings for white females and black males represent now on average respectively 80% and 76% of those for white males (compared to respectively 65% and 66% for the baseline model).

Stochastic frontier models can be specified to accommodate unobserved heterogeneity through the incorporation of fixed effects when panel data is used. However, despite all its advantages described in Section 3, the data used in this study are not of longitudinal nature. Even if panel data were available, group-specific earnings frontiers would be determined only by work experience, while education returns would be captured by fixed-effects. This is not convenient for a discrimination analysis approach which is based on the comparison of group-specific earnings frontiers with *TGR* scores. Nevertheless, as argued in Section 2, it is legitimate to believe that unobserved heterogeneity will have limited impact in our case, since it is in part captured by the earnings inefficiency component when group-specific SEF are estimated.

## 6. Conclusion

In this paper, we have addressed the serious issue of wage discrimination in the U.S. We introduce a novel and reliable method to examining wage discrimination with stochastic earnings metafrontier models. Such an approach overcomes the limitations of the previous approaches. Indeed, as we have seen, previous approaches to investigating wage discrimination with stochastic earnings frontier models may lead to questionable results and conclusions. Thus, based on the stochastic earnings *metafrontier* methodology, we have predicted and compared earnings efficiency scores across groups. These efficiency scores are comparable and have the advantage of being estimated while relaxing the strong hypothesis of a unique technology to convert investment in human capital into earnings. The technology gap ratio appears to be a useful measure for investigating wage discrimination, while the metafrontier earnings efficiency allows one to know which group of workers succeeds better in attaining the maximum available hourly wages in the labor market, given a level of human capital. We find evidence of wage discrimination against women and blacks since men and whites have the highest maximum attainable hourly wages, given a human capital endowment. In addition, we observe that whites and males exhibit the lowest deviation of their observed earnings from the maximum possible earnings, regardless of the group (gender or race), given a level of human capital.

The second contribution of this paper is an illuminating methodology for investigating the glass ceiling phenomenon. We observe that, at all levels of human capital, white males' maximum attainable earnings are well above those of black males. The same applies when we compare white men with white women. We call this phenomenon a "generalized" glass ceiling against blacks, and women.

We take account of sample selection bias when estimating the metafrontiers. This is a novelty in the literature to the best of our knowledge. We also verified the robustness of our results by taking account of various controls, and trying different specifications: we did not find anything which challenged our conclusions, although the magnitudes are slightly modified. Nevertheless, we should be cautious and we must recognize that a more precise analysis could be achieved when taking account of possible unobserved heterogeneity. While a fixed effects approach is not appropriate in the context of earnings metafrontier analysis, another panel-based approach might be possible. Tsionas and Kumbhakar (2012), proposed a four components panel data stochastic frontier model which includes standard noise, random effects, time invariant inefficiency and time varying inefficiency. With such a model, unobserved heterogeneity can be captured either by random effects or by the time invariant inefficiency component. This could represent an interesting extension of the approach presented here.

## Tables and Figures

**Table 1** Composition of the sample (proportions in %)

Variables	All sample		Women (employed)		Men (employed)	
	all job statuses	employed	Blacks	Whites	Blacks	Whites
Females	50.86	44.25	100	100	0	0
Blacks	12.93	11.47	100	0	100	0
Whites	87.07	88.53	0	100	0	100
Age: between 25 - 30	17.35	17.80	20.26	17.91	19.73	17.21
Age: between 30 - 40	30.11	30.44	33.40	28.49	33.57	31.22
Age: between 40 - 50	36.20	36.63	34.01	37.60	33.86	36.51
Age: between 50 - 55	16.33	15.13	12.33	16.01	12.85	15.05
Bachelor's degree or higher	28.59	31.69	23.19	32.89	21.38	32.89
Married	60.88	62.03	37.08	60.67	52.90	67.08
With child under 5	18.17	17.22	17.12	14.09	16.96	19.62
Lives in a central city	21.36	20.87	43.68	18.07	42.91	17.88
Foreigner (not U.S citizen)	2.93	2.74	5.38	2.13	7.68	2.35
Employed	72.18	100	100	100	100	100
Unemployed	3.92	0	0	0	0	0
Out of labor market	23.90	0	0	0	0	0
Job occupation <sup>1</sup>	-	37.51	31.67	42.65	23.12	35.85

Goods producing <sup>2</sup>	-	26.49	11.27	13.79	28.55	37.72
Service producing: Wholesale and retail trade	-	16.49	10.90	15.85	15.04	17.82
Service producing: Educational and health	-	17.30	36.84	29.04	11.22	6.67
Service producing: Financial, Professional and Business	-	21.33	23	25.38	18.87	18.33
Service producing: Other	-	18.39	18	15.94	26.32	19.46
Observations	93,495	67,485	4,240	25,624	3,503	34,118

Notes: author's calculation from CPS ORG 2006.

<sup>1</sup> Management, Business, Financial, Professional and related occupations

<sup>2</sup> Mining, Construction, Manufacturing

**Table 2** Earnings and Human Capital variables

<b>Panel A: By gender and race</b>								
Variables	Women		Men		Blacks		Whites	
	Mean	St.D	Mean	St.D	Mean	St.D	Mean	St.D
Hourly wage	21.67	12.46	27.32	15.45	19.78	11.33	25.47	14.72
Years of schooling	14.23	2.06	14.18	2.18	13.62	2.18	14.29	2.09
Potential experience	21.44	9.16	21.47	9.07	21.26	9.17	21.48	9.11
Observations	29,864		37,621		59,742		7,743	

<b>Panel B: By race (Table 2 continued)</b>								
Variables	Women				Men			
	Blacks		Whites		Blacks		Whites	
	Mean	St.D	Mean	St.D	Mean	St.D	Mean	St.D
Hourly wage	18.86	10.96	22.13	12.64	20.90	11.66	27.98	15.64
Years of schooling	13.68	2.14	14.32	2.03	13.54	2.24	14.26	2.15
Potential experience	21.07	9.21	21.50	9.15	21.51	9.12	21.46	9.07
Observations	4,240		25,624		3,503		34,118	

Notes: author's calculation from CPS ORG 2006.

**Table 3** Gender-specific stochastic earnings frontier (selection correction)

Frontier	Women			Men		
	Coef.	St. Errors	M.Topel	Coef.	St. Errors	M.Topel
Years of education	0.1545	(0.0018)***	[0.0019]***	0.0972	(0.0014)***	[0.0028]***
Experience	0.0237	(0.0014)***	[0.0014]***	0.0287	(0.0013)***	[0.0030]***
Experience2	-0.0004	(0.0000)***	[0.0000]***	-0.0004	(0.0000)***	[0.0001]***
Constant	0.5231	(0.0304)***	[0.0312]***	1.6211	(0.0618)***	[0.1546]***
Sigma-u	0.3626	(0.0171)***	***	0.1822	(0.0912)***	
Sigma-v	0.4548	(0.0050)***	***	0.4908	(0.0151)***	***
Rho	0.7181	(0.0363)***	***	-0.6690	(0.0129)***	***
Log Likelihood	-31,487.76			-31,352.18		
Mean Eff (%)	78.16			88.61		
Observations	29,864			37,621		

Notes: author's calculation from CPS ORG 2006.

\*\*\*, \*\*, \* represent, respectively, significance at the level of 1, 5, and 10%

The estimated Probit model for selection of being employed includes as explanatory variables: education, potential experience (and its square), U.S citizenship, race, living in a central city, being responsible of dependent child under 5, and being married.

M.Topel: Significance based on Murphy-Topel standard errors.

Sigma-v and Sigma-u: are respectively the standard deviations of the zero-mean normal noise, and the pre-truncated inefficiency component's distribution.

Rho: is the correlation coefficient between the noise component and the selection process.

**Table 4** Race-specific stochastic earnings frontier (selection correction)

Panel A: Blacks	Women			Men	
	Blacks (no. observation = 4,240)			Blacks (no. observation = 3,503)	
	Coef.	St. Errors	M.Topel	Coef.	St. Errors
Years of educ	0.1659	(0.0038)***	[0.0035]***	0.0810	(0.0046)***
Experience	0.0235	(0.0034)***	[0.0035]***	0.0211	(0.0042)***
Experience2	-0.0004	(0.0001)***	[0.0001]***	-0.0003	(0.0001)***
Constant	0.2508	(0.0646)***	[0.0605]***	1.6428	(0.1372)***
Sigma-u	0.3964	(0.0179)***	***	0.0110	(0.1345)
Sigma-v	0.5123	(0.0102)***	***	0.4919	(0.0108)***
Rho	0.9741	(0.0172)***	***	-0.5483	(0.0586)***
LnL		-4,376.30			-3,472.07
Mean Eff (%)		77.20			99.13

**Table 4 (continued)**

<b>Panel B: Whites</b>	Women			Men		
	Whites (no. observation = 25,624)			Whites (no. observation = 34,118)		
	Coef.	St. Errors	M.Topel	Coef.	St. Errors	M.Topel
Years of educ	0.1492	(0.0021)***	[0.0021]***	0.1009	(0.0014)***	[0.0020]***
Experience	0.0245	(0.0015)***	[0.0015]***	0.0314	(0.0014)***	[0.0023]***
Experience2	-0.0004	(0.0000)***	[0.0000]***	-0.0005	(0.0000)***	[0.0000]***
Constant	0.6263	(0.0354)***	[0.0360]***	1.6536	(0.0260)***	[0.0359]***
Sigma-u	0.3467	(0.0189)***	***	0.3177	(0.0118)***	***
Sigma-v	0.4400	(0.0056)***	***	0.4463	(0.0076)***	***
Rho	0.6091	(0.0492)***	***	-0.5943	(0.0225)***	***
LnL		-26,952.38			-27,649.50	
Mean Eff (%)		78.51			83.35	

Notes: author's calculation from CPS ORG 2006.

\*\*\*, \*\*, \* represent, respectively, significance at the level of 1, 5, and 10%

The estimated Probit model for selection of being employed includes as explanatory variables: education, potential experience (and its square), U.S citizenship, race, living in a central city, being responsible of dependent child under 5, and being married.

M.Topel: Significance based on Murphy-Topel standard errors.

There is no inefficiency for black male workers: the frontier's coefficients and their standard errors are similar to those estimated with Heckman sample selection correction.

Sigma-v and Sigma-u: are respectively the standard deviations of the zero-mean normal noise, and the pre-truncated inefficiency component's distribution.

Rho: is the correlation coefficient between the noise component and the selection process.

**Table 5** Stochastic earnings metafrontier estimation

	Metafrontier on all sample	White males' frontier
Years of education	0.1009***	0.1009***
Experience	0.0314***	0.0314***
Experience2	-0.0005***	-0.0005***
Constant	1.6536***	1.6536***
Sigma-u	0.3408***	0.3177***
Sigma-v	3.00E-11	0.4463***
Log Likelihood	23,668.11	-27,649.50

Notes: author's calculation from CPS ORG 2006.

\*\*\*, \*\*, \* represent respectively, significance at the level of 1, 5, and 10%

Sigma-v and Sigma-u: are respectively the standard deviations of the zero-mean normal noise, and the pre-truncated inefficiency component's distribution.

**Table 6** Statistics of metafrontier efficiency measures

<b>Women</b>								
Efficiency measures	Blacks				Whites			
	Mean	St.d	Min	Max	Mean	St.d	Min	Max
Group Eff.	<b>0.772</b>	0.059	0.187	0.887	<b>0.785</b>	0.053	0.318	0.915
TGR	<b>0.527</b>	0.081	0.200	0.902	<b>0.646</b>	0.072	0.315	0.941
ME	<b>0.409</b>	0.073	0.041	0.709	<b>0.507</b>	0.067	0.199	0.812
<b>Men</b>								
Measures	Blacks				Whites			
	Mean	St.d	Min	Max	Mean	St.d	Min	Max
Group Eff.	<b>0.991</b>	0.000	0.991	0.992	<b>0.834</b>	0.039	0.410	0.920
TGR	<b>0.662</b>	0.031	0.570	0.869	<b>1.000</b>	0.000	1.000	1.000
ME	<b>0.657</b>	0.031	0.565	0.861	<b>0.834</b>	0.039	0.410	0.920

Notes: author's calculation from CPS ORG 2006.

Group Eff (group-specific earnings efficiency); TGR (technology gap ratio); ME (metafrontier earnings efficiency).

**Table 7 Paired Student tests**

Variables	t statistics			
	Black men	White women	Black women	White women
	versus	versus	versus	versus
	White men	Black men	Black men	Black women
TGR	-668.86***	-16.87***	-97.00***	-89.86***
ME	-330.80***	-220.45***	-198.06***	-82.66***

Notes: author's calculation from CPS ORG 2006.

\*\*\*, \*\*, \* represent, respectively, the difference is significant at the level of 1, 5, and 10%.

TGR (technology gap ratio); ME (metafrontier earnings efficiency).

**Table 8** Racial (among men) and gender (among whites) wage gaps at different percentiles (standard errors in parentheses)

<b>Panel A: Whites vs. Blacks</b> (Counterfactuals: Whites' characteristics and Blacks' coefficients)							
	5th	10th	25th	50th	75th	90th	95th
Raw wage gap	0.217	0.249	0.293	0.300	0.280	0.273	0.301
Counterfactual gender gap using general HC variables	0.187 (0.017)	0.201 (0.016)	0.227 (0.012)	0.234 (0.011)	0.211 (0.013)	0.221 (0.018)	0.240 (0.022)
Counterfactual gender gap using general HC variables and additional controls	0.154 (0.017)	0.167 (0.016)	0.205 (0.011)	0.207 (0.011)	0.189 (0.014)	0.202 (0.018)	0.201 (0.021)

**Table 8** (continued)

<b>Panel B: Men vs. Women</b> (Counterfactuals: Men's characteristics and Women's coefficients)							
	5th	10th	25th	50th	75th	90th	95th
Raw wage gap	0.149	0.200	0.246	0.236	0.231	0.225	0.250
Counterfactual gender gap using general HC variables	0.210 (0.018)	0.223 (0.013)	0.255 (0.012)	0.269 (0.013)	0.261 (0.015)	0.240 (0.019)	0.230 (0.026)
Counterfactual gender gap using general HC variables and additional controls	0.170 (0.018)	0.188 (0.013)	0.227 (0.012)	0.253 (0.012)	0.251 (0.014)	0.234 (0.018)	0.223 (0.023)

Notes: author's calculation from CPS ORG 2006.

**Additional controls:** U.S citizenship, race, living in a central city, being responsible of dependent child under 5, and being married.

**Counterfactual wage gaps** are obtained following Albrecht et al. (2003). It is a version of the Machado and Mata (2005) approach. We use Stata command *mmset*.

**Table 9** Statistics of efficiency measures (with additional controls)

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<b>Groups : Mean (median)</b>	<b>Technology Gap Ratio - TGR</b>			<b>Group-specific Earnings Efficiency</b>		
	<b>Model (1)</b>	<b>Model (2)</b>	<b>Model (3)</b>	<b>Model (1)</b>	<b>Model (2)</b>	<b>Model (3)</b>
<b>Females, Whites</b>	<b>0.65 (0.64)</b>	0.73 (0.72)	0.80 (0.79)	<b>0.79 (0.79)</b>	0.80 (0.81)	0.83 (0.83)
<b>Males, Blacks</b>	<b>0.66 (0.66)</b>	0.70 (0.69)	0.76 (0.76)	<b>0.99 (0.99)</b>	1.00 (1.00)	0.98 (0.99)
<b>Males, Whites</b>	<b>1.00 (1.00)</b>	1.00 (1.00)	1.00 (1.00)	<b>0.83 (0.84)</b>	0.85 (0.85)	0.76 (0.79)

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Notes: author's calculation from CPS ORG 2006.

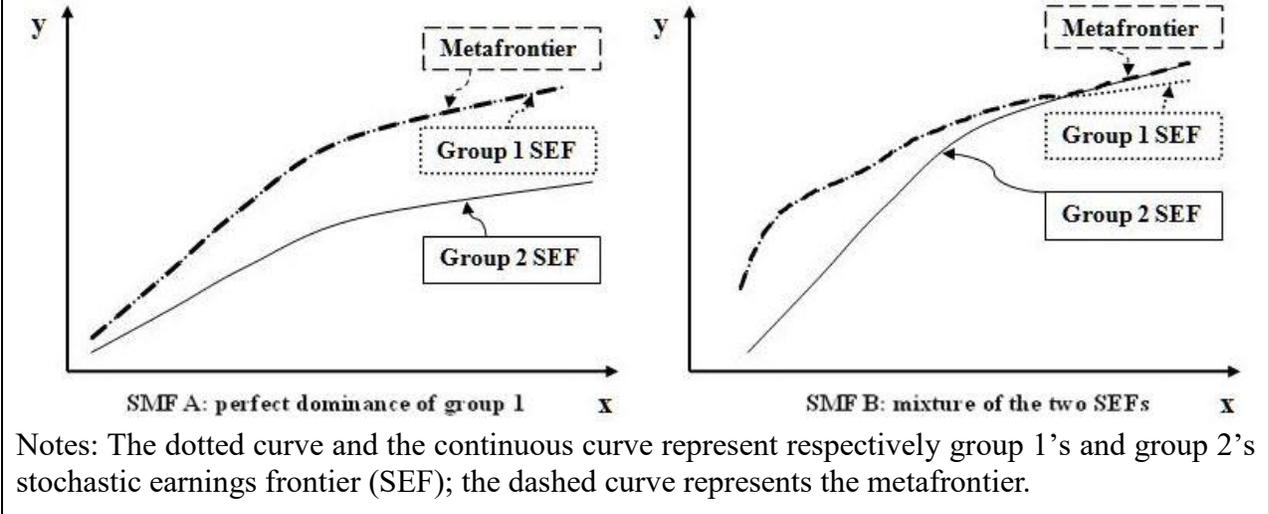
Model (1): baseline, only general human capital (HC) variables are included as determinants of the frontier.

Model (2): HC, job occupation and industry sectors (JO-IS) are included as determinants of the frontier.

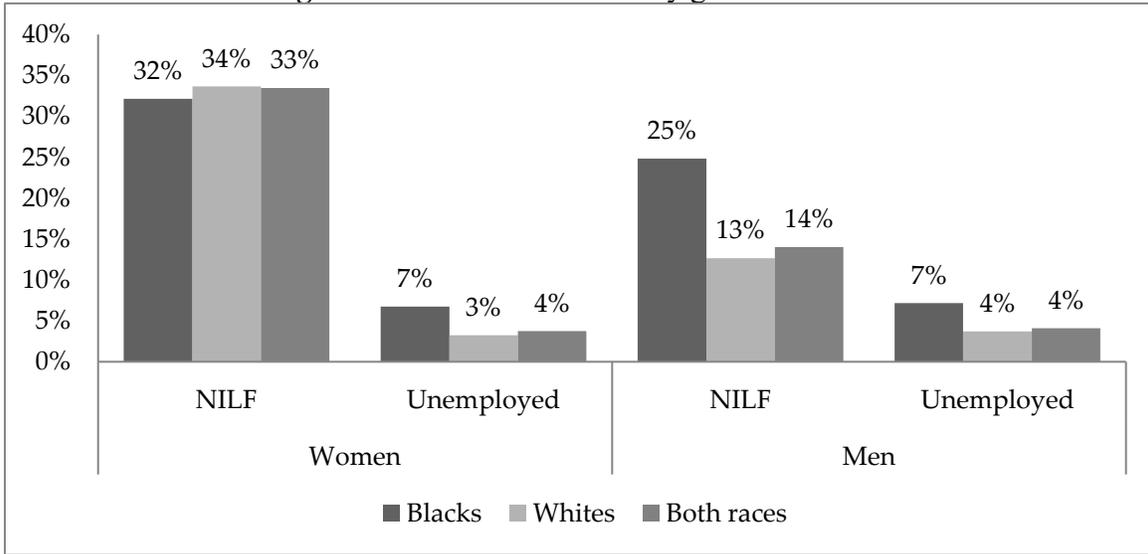
Model (3): Heteroscedastic inefficiency and noise components: years of education, experience, and individual characteristics are included as determinants. HC and JO-IS are included as determinants of the frontier.

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Figure 1 Stochastic metafrontier

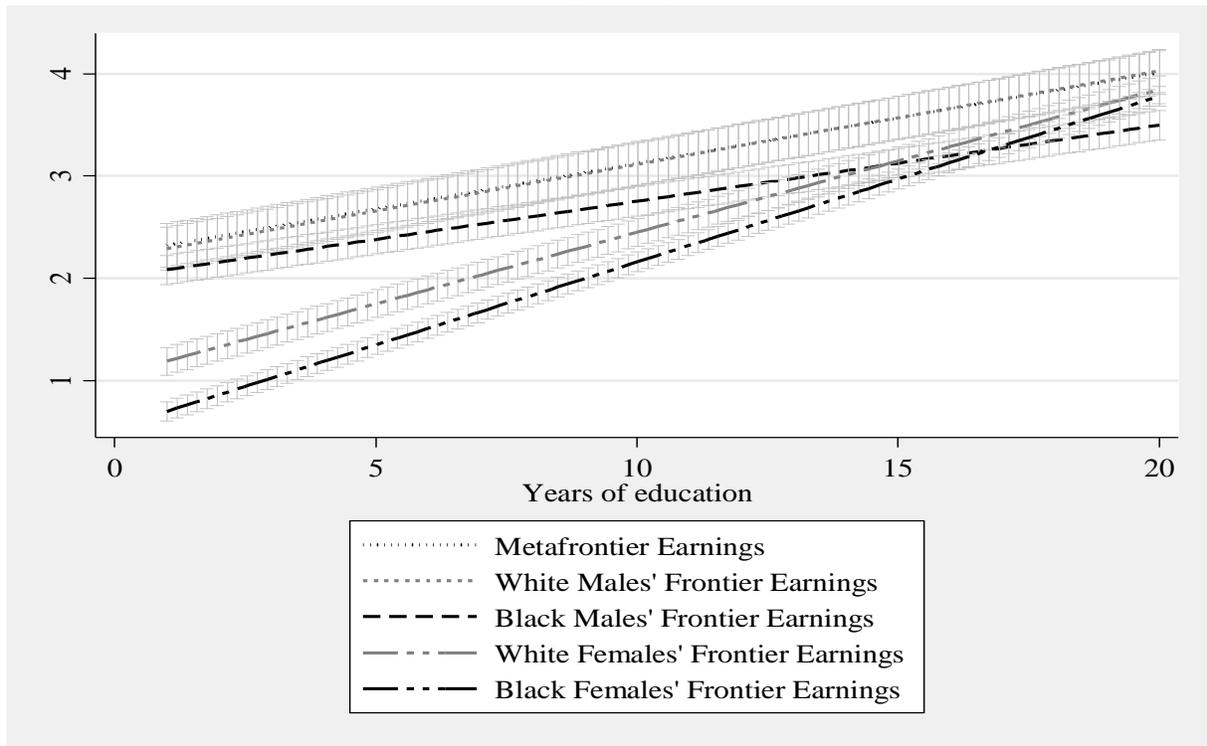


**Figure 2 Labor market status by gender and race**



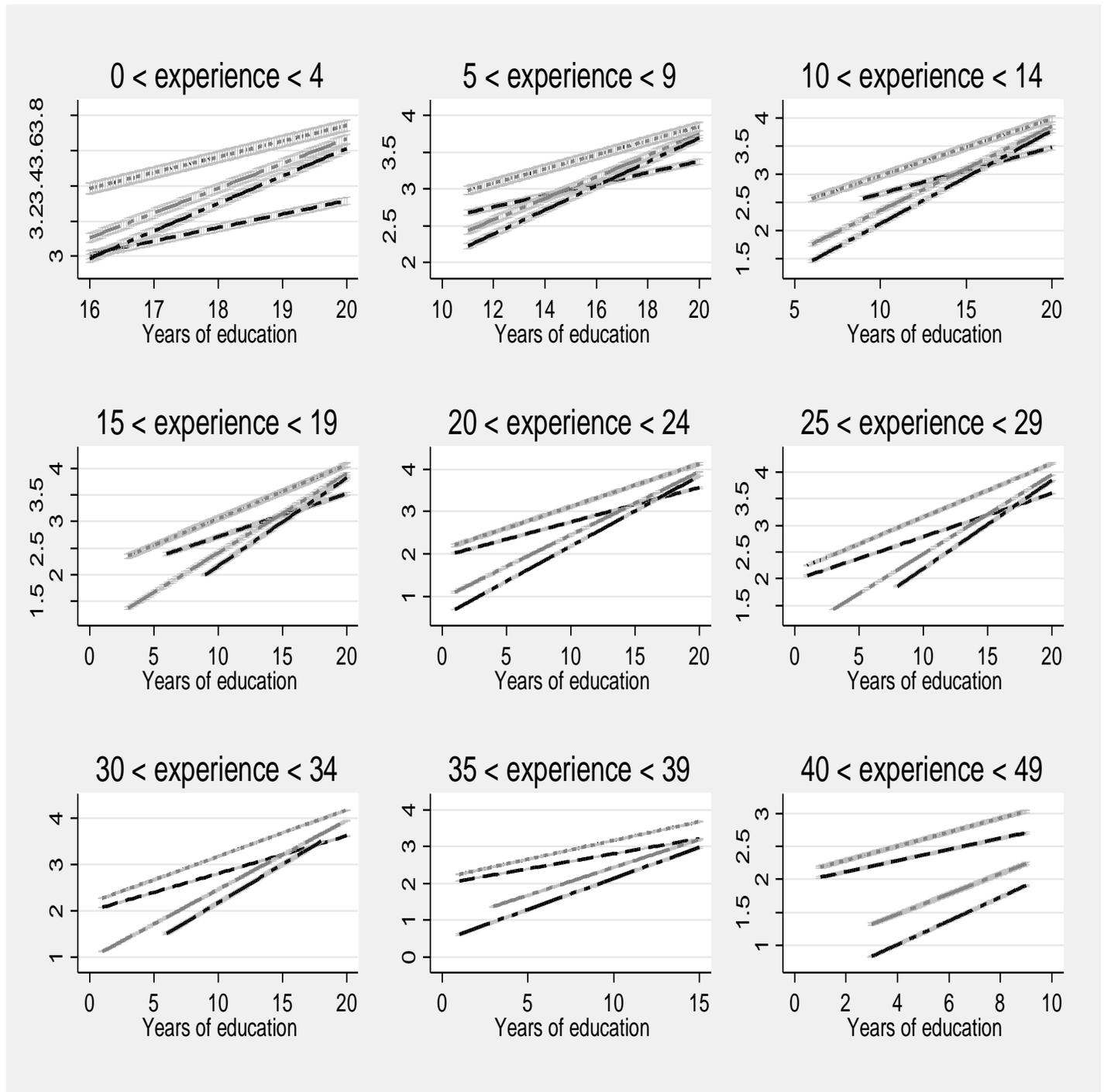
Notes: Author's calculation from CPS ORG 2006. Percentages of individuals not in the labor force (NILF) or unemployed, by gender and race.

**Figure 3** Comparison of group-specific frontier earnings relative to metafrontier earnings (in logarithm)



Notes: Author's calculation from CPS ORG 2006. The curves represent fitted values of group-specific earnings frontiers (with 95% confidence intervals). Those earnings frontiers are compared to the metafrontier, which is, in this graph, confounded with white men's earnings frontier.

**Figure 4** Comparison of group-specific frontier earnings relative to metafrontier earnings (in logarithm) at different levels of work experience



Notes: Author's calculation from CPS ORG 2006.

Black dotted curve, gray short-dashed curve, black dashed curve, gray long dashed-short dashed curve, and black long dashed-short dashed curve represent, respectively, fitted values of the metafrontier, white males' stochastic earnings frontier (SEF), black males' SEF, white females' SEF, and black females' SEF. The metafrontier is confounded with white males' frontier.

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## Appendix

**Table A.1** Probit estimates – (for sample selection correction)

<b>Dependent variable: being in employment in 2006</b>	<b>Coefficients</b>			
	<b>Black women</b>	<b>White women</b>	<b>Black men</b>	<b>White men</b>
Years of education	0.147***	0.107***	0.113***	0.104***
Experience	0.014	-0.014***	0.010	0.027***
Experience2	-0.001**	0.000	-0.001**	-0.001***
Being a foreign citizen	0.032	-0.300***	0.275***	-0.016
Living in a central city	-0.111***	-0.022	-0.121**	-0.067**
Dependent children under 5	-0.260***	-0.643***	0.036	0.102***
Being married	0.093**	-0.219***	0.445***	0.510***
Constant	-1.659***	-0.617***	-1.078***	-0.778***
Log L	-4375.438	-25258.504	-2972.704	-16632.229
Observations	6,938	40,613	5,152	40,792

Notes: author's calculation from CPS ORG 2006.  
 \*\*\*, \*\*, \* represent respectively, significance at the level of 1, 5, and 10%

**Table A.2** Group-specific earnings frontiers estimates (*without correction for sample selection*)

Dependent variable: logarithm of hourly wage	Coefficients			
	Black women	White women	Black men	White men
Years of education	0.130***	0.134***	0.100***	0.114***
Experience	0.019***	0.026***	0.026***	0.037***
Experience2	-0.000***	-0.000***	-0.000***	-0.001***
Constant	0.750***	0.928***	1.398***	1.426***
Sigma-u	0.004	0.284***	0.213**	0.397***
Sigma-v	0.430***	0.414***	0.438***	0.395***
Log L	-21,980	-2,221.861	-15,780	-2,439.871
Mean Efficiency	99.69	79.92	84.41	73.46
Observations	4,240	25,624	3,503	34,118

Notes: author's calculation from CPS ORG 2006.

\*\*\*, \*\*, \* represent respectively, significance at the level of 1, 5, and 10%

Sigma-v and Sigma-u: are respectively the standard deviations of the zero-mean normal noise, and the pre-truncated inefficiency component's distribution.

**Table A.3** Group-specific earnings frontiers and earnings metafrontier estimates – Robustness specification (Model (2)) (*corrected for sample selection*)

Dependent variable: logarithm of hourly wage	Coefficients			
	White women	Black men	White men	Metafrontier (on the three groups)
Years of education	0.103***	0.053***	0.076***	0.076***
Experience	0.025***	0.023***	0.031***	0.030***
Experience2	-0.000***	-0.000***	-0.000***	-0.000***
Management, business, financial, and related job occupations	0.296***	0.346***	0.244***	0.244***
Goods producing: mining, construction and manufacturing	Reference for industries and sectors			
Service producing: education and health	-0.136***	-0.165***	-0.209***	-0.209***
Service producing: wholesale and retail trade	-0.100***	-0.099***	-0.105***	-0.105***
Service producing: finance	-0.002	-0.060**	-0.006	-0.006***
Service producing: other	-0.121***	-0.062**	-0.109***	-0.109***
Constant	1.312***	1.990***	1.974***	1.985***
Lnsigma-u2	-2.436***	-11.242	-2.491***	-2.949***
Lnsigma-v2	-1.890***	-1.492***	-1.661***	-48.923
AtanRho	0.178**	-0.650***	-0.687***	-
Log L	-25560.741	-3318.0146	-26408.396	47359.4345

Observations	25,624	3,503	34,118	63,245
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Notes: author's calculation from CPS ORG 2006.

\*\*\*, \*\*, \* represent respectively, significance at the level of 1, 5, and 10%

**Lnsigma-v2** and **Lnsigma-u2**: are respectively the logarithms of the variances of the zero-mean normal noise, and the pre-truncated inefficiency component's distribution.

**AtanRho**: is the inverse hyperbolic tangent of the correlation coefficient between the noise component and the selection process.

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**Table A.4** Group-specific earnings frontiers and earnings metafrontier estimates - Robustness specification (Model (3)) (*corrected for sample selection*)

<b>Dependent variable: logarithm of hourly wage</b>	<b>Coefficients</b>			
	<b>White women</b>	<b>Black men</b>	<b>White men</b>	<b>Metafrontier (on the three groups)</b>
Years of education	0.104***	0.059***	0.129***	0.129***
Experience	0.025***	0.022***	0.037***	0.037***
Experience2	-0.000***	-0.000**	-0.001***	-0.001***
Management, business, financial, and related job occupations	0.291***	0.346***	0.228***	0.228***
Goods producing: mining, construction and manufacturing	Reference for industries and sectors			
Service producing: education and health	-0.104***	-0.094***	-0.110***	-0.111***
Service producing: wholesale and retail trade	-0.008	-0.057*	-0.013	-0.015***
Service producing: finance	-0.127***	-0.057**	-0.108***	-0.109***
Constant	1.317***	1.895***	1.082***	1.089***
<b>Determinants of <math>\ln\sigma_{-u2}</math></b>				
Years of education	0.020*	0.208**	0.028***	0.022***
Experience	0.124**	0.483***	0.216***	0.157***
Being a foreign citizen	0.393	3.126***	0.031	0.247***
Living in a central city	-0.157	-36.530	-0.197***	-0.011

Number of children	-0.331***	-0.114	-0.676***	-0.180***
Being married	-1.109***	0.335	-0.138***	0.158***
Constant	-4.596***	-18.376***	-5.001***	-6.295***
<b>Determinants of Lnsigma-v2</b>				
Years of education	0.010***	-0.000	0.002	0.121***
Experience	0.064***	0.053***	-0.036***	-1.490***
Being a foreign citizen	0.227**	-0.139	0.203**	0.363*
Living in a central city	0.058***	0.021	0.058***	0.208***
Number of children	-0.003	-0.040	0.117***	-0.454***
Being married	0.207***	0.024	0.124***	1.113***
Constant	-3.080***	-2.295***	-1.514***	2.279***
AtanRho	-0.194	-0.547***	0.855***	-
Log L	-25322.819	-3278.7019	-26109.666	68339.5987
Observations	25,624	3,503	34,118	63,245

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Notes: author's calculation from CPS ORG 2006.

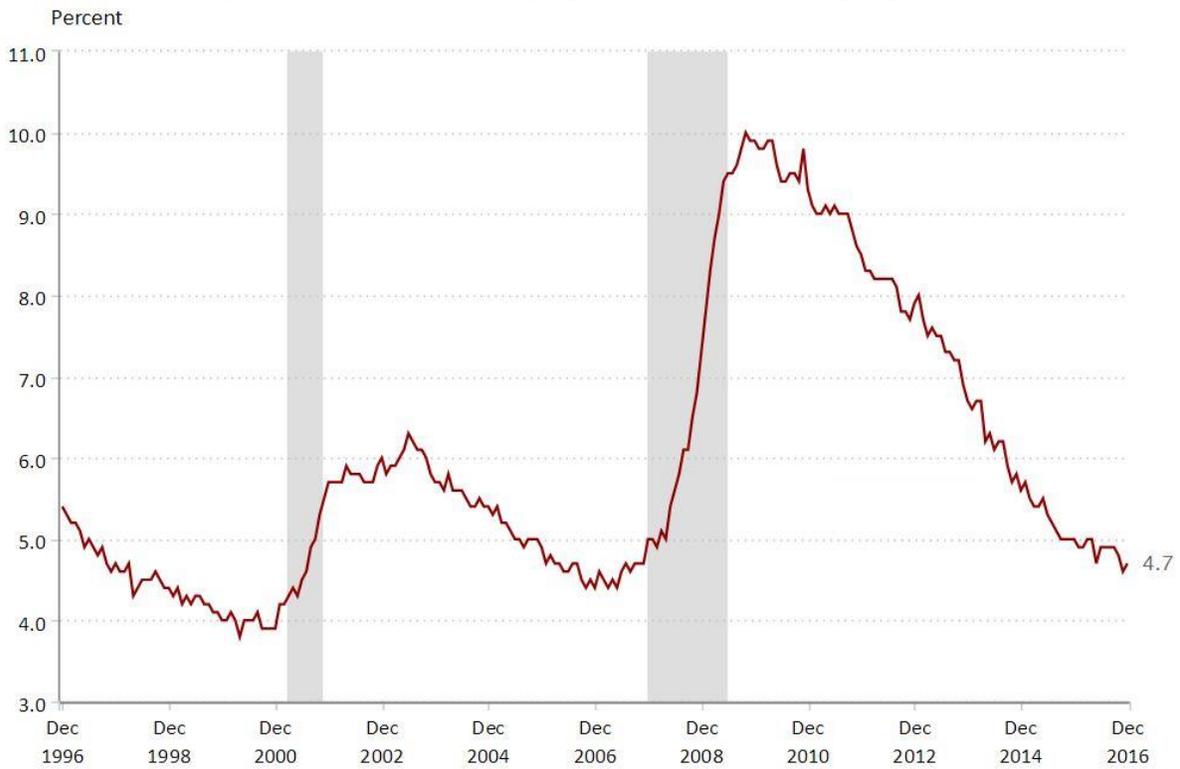
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**Lnsigma-v2** and **Lnsigma-u2**: are respectively the logarithms of the variances of the zero-mean normal noise, and the pre-truncated inefficiency component's distribution.

**AtanRho**: is the inverse hyperbolic tangent of the correlation coefficient between the noise component and the selection process.

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**Figure A.1** Civilian unemployment rate (seasonally adjusted)



Notes: From Bureau of Labor Statistics. Shaded areas are economic recessions (National Bureau of Economic Research). All races, genders, and ages are included.