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Procedural versus opportunity-wise equal treatment of alternatives: neutrality revisited*

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Abstract

We revisit the neutrality requirement in social choice theory. We propose a weakening of the standard neutrality condition, by allowing for different procedural treatment for different alternatives while entailing that alternatives enjoy same ex-ante possibility to be chosen. We compare these two conditions theoretically and computationally. Furthermore, we explore social choice problems in which this weakening resolves impossibilities that stem from a fundamental tension between neutrality and anonymity. Finally, we show that in certain social choice problems, this weakening provides an immediate refinement of anonymous, neutral, and Pareto optimal social choice rules towards retaining resoluteness.

Keywords: anonymity, neutrality, Pareto optimality, social choice functions

JEL Codes: D63, D71, D72, D74

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1 Introduction

Equal treatment between voters and between alternatives are among the core principles of democratic decision making. Equal treatment of voters is usually ensured by a condition called anonymity, which requires the social choice to be invariant under renaming voters. On the other hand, the typical condition to ensure equal treatment of alternatives is neutrality, which requires the social choice to change in compliance with renaming of alternatives.

The logical incompatibility between anonymity and neutrality while ensuring an untied outcome lies among the plethora of impossibilities in social choice theory. A social choice function (SCF) being a mapping from profiles of strict preferences into single alternatives, Moulin (1983, 1991) characterizes sizes of social choice problems which admit anonymous and neutral SCFs. More precisely, a social choice problem with n voters and m alternatives admits anonymous and neutral SCFs if and only if m cannot be written as the sum of some divisors of n that exceed 1 (Moulin, 1991, p. 253). When Pareto optimality is imposed on top of anonymity and neutrality, this requirement is strengthened to "n not having a prime factor less than m" (Moulin, 1983, p. 23). This last condition is equivalent to ask the greatest common divisor of $m!$ and n to be 1, as shown by Doğan and Giritligil (2015), who reconsider the problem through a group theoretic approach.[1]

How severe is this tension between anonymity and neutrality? Campbell and Kelly (2015) show the rarity of cases where anonymous and neutral SCFs exist: when the number of individuals is divisible by two or more distinct primes, only a small (finite) number of social choice problems as such admit anonymous and neutral SCFs. Also, when the number of alternatives exceeds the smallest prime dividing the number of individuals, an SCF is anonymous and neutral if it chooses alternatives in the bottom half of everyone's preferences.

Do all these results imply a lack of hope in guaranteeing equal treatment of

[1]Interestingly, as Bubboloni and Gori (2014) as well as Doğan and Giritligil (2015) show, $gcd(m!, n) = 1$ turns out to be necessary and sufficient for the existence of anonymous and neutral social welfare functions (*i.e.,* functions which assign to every preference profile a strict ranking of alternatives). Zwicker (2016) delivers an introduction to the theory of voting where major results regarding anonymity and neutrality are included.

voters and alternatives for untied collective choice? We reject this pessimism by identifying a weakening of neutrality which allows a vast range of possibilities while pandering to a very significant aspect of neutral treatment of alternatives. This new condition that we call equal opportunity for alternatives (EOA) requires that all alternatives are chosen at the same number of preference profiles. The moderated character EOA possesses puts forward an ex-ante fairness property that is more outcome-oriented compared to the classical approach that entails a more procedure-oriented equal treatment of alternatives.[2]

EOA is weaker than neutrality. As a result, one can think of using EOA instead of neutrality, when the latter turns out to be incompatible with anonymity. Of course, for this approach to make sense, there should be more permissive results regarding the compatibility between anonymity and EOA, which allow interesting SCFs that satisfy these two conditions while anonymity and neutrality contradict each other. Our paper addresses this matter.

We start by showing that not only EOA is weaker than neutrality, but also the class of EOA SCFs is considerably larger than those which are neutral. We do this by counting the number of neutral SCFs and the number of EOA SCFs for any given size of the social choice problem. An analytical comparison of these two numbers seems beyond reach, so we take a computational approach where we calculate the numbers of SCFs in each class for a small set of values for numbers of alternatives and individuals. These numerical exercises that we report on in the paper show strong tendencies in comparisons of the classes, especially in comparing neutral and EOA SCFs.

Being encouraged by these results, we revisit the tension between neutrality and anonymity and show the possibility of refining anonymous, neutral, and Pareto optimal social choice rules (which are forced to give tied outcomes) by replacing neutrality with EOA. Nevertheless, this positive result does not prevail over all conceivable social choice problems: we point to instances where EOA, anonymity, and Pareto optimality turn out to be incompatible. These are instances where anonymity contradicts neutrality even without Pareto optimality.

[2]Tyler (2000, 2006) counts neutrality as one of the four major characteristics of a fair procedure and discusses how procedural fairness is perceived differently than outcome-based fairness by the public in assessing legitimacy of social systems.

However, even in those cases, an EOA and anonymous SCF can be identified. This raises the question whether EOA and anonymity are always compatible, which we positively answer for a certain class of social choice problems, namely those where the number of alternatives exceeds the number of individuals..

The paper is organized as follows. Section 2 gives basic notation and notions. Section 3 introduces EOA and delivers results on its theoretical and computational comparisons to neutrality. Section 4 moves on to the analysis of compatibility of EOA with other standard axioms in social choice and provides positive results regarding cases where neutrality is previously shown to be incompatible. Section 5 concludes.

2 Basic notions and notation

Let A be a finite and nonempty set of $m \geq 2$ alternatives; N be a set of $n \geq 2$ individuals. Each pair (A, N) determines a *social choice problem* whose size is (m, n). For any i in N, let $P_i \in \mathcal{L}(A)$ denote the *preference* of i, where $\mathcal{L}(A)$ is the set of linear orders, *i.e.,* complete, antisymmetric, and transitive binary relations on A. Furthermore, $P_N \in \mathcal{L}(A)^N$ indicates a *profile*, an $n-$tuple of such individual preferences. A *social choice function* (SCF) is a mapping $f : \mathcal{L}(A)^N \to A$. An SCF f is *Pareto optimal* if, given any $P_N \in \mathcal{L}(A)^N$ and $y \neq f(P_N)$, there exists $i \in N$ such that $f(P_N)P_iy$.

For any non-empty finite set X, a *permutation* on X is a bijection $\sigma : X \leftrightarrow X$. Let Σ_X be the set of all permutations on X, hence $|\Sigma_A| = m!$. By abuse of notation, we also denote σ for permutations on $\mathcal{L}(A)$ and $\mathcal{L}(A)^N$. So, given a preference P_i, we have $xP_iy \iff \sigma(x)\sigma(P_i)\sigma(y)$, for all $x, y \in A$. Thus, $\sigma(P_N) = (\sigma(P_i))_{i \in N}$.

We say that P'_N is a *renaming* (of alternatives) of P_N iff there exists $\sigma \in \Sigma_A$ such that $P'_N = \sigma(P_N)$. We write $P_N \rho P'_N$ when P'_N is a renaming of P_N. Noting that $\rho \subseteq \mathcal{L}(A)^N \times \mathcal{L}(A)^N$ is an equivalence relation, we write $\mathcal{E}$ for the partition of $\mathcal{L}(A)^N$ provided by ρ. Note that, each profile P_N admits $m!$ renamings. Moreover, $|\mathcal{L}(A)^N| = m!^n$. Thus, $\mathcal{E}$ admits $m!^{n-1}$ equivalence classes, each of which contains $m!$ profiles. We write $\mathcal{E} = \{\mathcal{E}_i\}_{i \in \{1,\ldots,m!^{n-1}\}}$, with $|\mathcal{E}_i| = m!$, for all $i \in \{1, \ldots, m!^{n-1}\}$.

3 Neutrality *vs.* equal opportunity for alternatives

We now turn to our central discussion. First, we formally define neutrality of social choice functions.

Definition 1. *An SCF* $f : \mathcal{L}(A)^N \to A$ *is* neutral *iff for all* $\sigma \in \Sigma_A$*, we have* $f(\sigma(P_N)) = \sigma(f(P_N))$.

Next, let $W_f(x)$ denote the set of all profiles at which x is chosen under f, hence $W_f(x) = \{P_N \in \mathcal{L}(A)^N : f(P_N) = x\}$, with $|W_f(x)| = w_f(x)$. We are ready to define our new condition on SCFs.

Definition 2. *An SCF* $f : \mathcal{L}(A)^N \to A$ *satisfies* equal opportunity for alternatives *(henceforth "*satisfies EOA*") iff* $w_f(x) = w_f(y)$*, for all* $x, y \in A$.

We write $\mathcal{F}$ for the set of all SCFs from $\mathcal{L}(A)^N$ to A; $\mathcal{F}^{NEUTRAL}$ for the set of all SCFs that satisfy neutrality; $\mathcal{F}^{EOA}$ for the set of all SCFs that satisfy EOA.

Theorem 1. $\mathcal{F}^{NEUTRAL} \subsetneq \mathcal{F}^{EOA}$.

Proof. Let $A = \{x_1, \ldots, x_m\}$. Take any $f \in \mathcal{F}^{NEUTRAL}$, any $\mathcal{E}_t \in \sigma$, and any $P_N \in \mathcal{E}_t$. Let $f(P_N) = x_i$ for some $i \in \{1, \ldots, m\}$. For any $i, j \in \{1, \ldots, m\}$, let

$$\mathcal{E}_t^{ij} = \{P'_N \in \mathcal{E}_t : P'_N = \sigma(P_N) \text{ for some } \sigma \in \Sigma \text{ with } \sigma(x_i) = x_j\}.$$

Note that $\{\mathcal{E}_t^{ij}\}_{j \in \{1,\ldots,m\}}$ partitions $\mathcal{E}_t$. By neutrality, we have that $f(P'_N) = x_j$, for all $P'_N \in \mathcal{E}_t^{ij}$. As $|\mathcal{E}_t^{ij}| = |\mathcal{E}_t^{ik}|$, for all $j, k \in \{1, \ldots, m\}$, we have $w_f(x_j) = w_f(x_k)$, for all $j, k \in \{1, \ldots, m\}$, when f's domain is restricted to $\mathcal{E}_t$. As $t \in \{1, \ldots, m!^{n-1}\}$ is chosen arbitrarily, this is true for the whole domain.

To show the strictness of the inclusion, let $g : \mathcal{L}(A)^N \to A$ be the plurality rule which picks at any profile the alternatives which are ranked first by the highest number of individuals and possible ties are broken with respect to the preference of individual 1. Clearly g is neutral and satisfies EOA. Now fix some $x, y \in A$ and define the SCF $g' : \mathcal{L}(A)^N \to A$ such that

$$g'(P_N) = \begin{cases} x & \text{if } g(P_N) = y, \\ y & \text{if } g(P_N) = x, \\ g(P_N) & \text{otherwise.} \end{cases}$$

Note that g' is not neutral, but retains EOA. $\square$

The rule that is used to show the strictness of the inclusion in Theorem 1 is not Pareto optimal. Nevertheless, $\mathcal{F}^{EOA} \backslash \mathcal{F}^{NEUTRAL}$ admits Pareto optimal SCFs as well. For example, fixing some $x, y \in A$, consider the SCF that selects the best alternative for individual 1 whenever a majority of individuals prefer x to y, and chooses the best alternative for individual 2 otherwise. This SCF, which is Pareto optimal, satisfies EOA but fails neutrality.

These observations raise the following issue. How large is $\mathcal{F}^{EOA}$ compared to $\mathcal{F}^{NEUTRAL}$ and which interesting SCFs, if any, does it contain? We address the first question through a counting approach.

It is well-known that cardinality of $\mathcal{F}$ is $m^{m!^n}$. The following theorem gives the numbers of neutral and EOA SCFs, as functions of m and n.

Theorem 2. *The following equalities hold.*

(i) $|\mathcal{F}^{NEUTRAL}| = m^{m!^{n-1}}$.

(ii) $|\mathcal{F}^{EOA}| = \frac{(m!^n)!m!}{(m!^{n-1}(m-1)!)!^m}$.

Proof. *(i)* Take any $\mathcal{E}_t \in \sigma$ and pick any $P_N \in \mathcal{E}_t$. Let $f(P_N) = x_i$ for some $i \in \{1, \ldots, m\}$. Neutrality, together with the definition of $\mathcal{E}_t$, determines $f(P'_N)$ for any $P'_N \in \mathcal{E}_t$. Hence there are m neutral SCFs that can be defined on $\mathcal{E}_t$. As $t \in \{1, \ldots, m!^{n-1}\}$, there are $m^{m!^{n-1}}$ neutral SCFs altogether.

(ii) First observe that, given any two natural numbers p and q, there are $\frac{(pq)!}{q!^p}$ ways to partition a set of cardinality pq into p sets, each with cardinality q. Hence, there are $\frac{(m!^n)!}{(m!^{n-1}(m-1)!)!^m}$ ways to partition $\mathcal{L}(A)^N$ with cardinality $m!^n$ into m sets, each with cardinality $m!^n/m = m!^{n-1}(m-1)!$. For each of these ways $m!$ distinct functions can be defined. As a result, $\frac{(m!^n)!}{(m!^{n-1}(m-1)!)!^m} \times m!$ functions that satisfy EOA can be constructed.

$\square$

The following remark delivers some numerical observations, which rely on computations for small values of m and n that are provided in Appendix A.[3]

Remark 1. *We observe the following, as m and/or n increase.*

(i) $|\mathcal{F}^{NEUTRAL}|/|\mathcal{F}| \to 0$.

(ii) $|\mathcal{F}^{EOA}|/|\mathcal{F}| \to 0$.

(iii) $|\mathcal{F}^{\mathcal{NEUTRAL}}|/|\mathcal{F}^{EOA}| \to 0$.

The first and second observations in Remark 1 point to the fact that both neutrality and EOA are essentially difficult conditions to satisfy since as the size of the problem grows, the ratios of numbers of these functions to the number of all possible functions diminish. The last observation, on the other hand, shows that neutrality is essentially more difficult to satisfy, compared to EOA, as the number of neutral SCFs becomes negligible compared to EOA SCFs as the size of the social choice problem grows.

4 Anonymity and EOA

Anonymity is the other central axiom of social choice theory that we discuss in this paper. Given any P_N, let $\sigma(P_N) = (P_{\sigma(i)})_{i \in N}$ denote the profile obtained by a permutation $\sigma \in \Sigma_N$ of individuals.

Definition 3. *An SCF f is* anonymous *iff $f(P_N) = f(\sigma(P_N))$, $\forall \sigma \in \Sigma_N$.*

We know since Moulin (1983, 1991) the tension between anonymity and neutrality. We quote below his two theorems on how the existence of anonymous and neutral SCFs depends on the size of the social choice problem. First, we state two conditions for any two integers $n, m \geq 2$. For any $n \in \mathbb{N}$, let $\mathcal{D}(n)$ denote the set of all divisors of n that are greater than 1, and $\mathcal{D}^*(n) \subseteq \mathcal{D}(n)$ denote the set of prime factors of n.

$C_1(m, n)$: $x \in \mathcal{D}^*(n) \implies x > m$.

[3] We are providing computational results for only some small values of m and n because as m and n increase, these values grow dramatically. As diminution in the ratios are also fast, these values appear to be sufficient for our conclusions in Remark 1.

$C_2(m,n)$: $\nexists X \subseteq \mathcal{D}(n)$ such that $m = \sum_{x \in X} x$.

Note that $C_1(m,n)$ implies $C_2(m,n)$, for all $n, m \geq 2$.

Theorem 3 (Moulin (1983))**.** *There exists an anonymous, neutral, and Pareto optimal SCF if and only if $C_1(m,n)$ holds.*[4]

Theorem 4 (Moulin (1991))**.** *There exists an anonymous and neutral SCF if and only if $C_2(m,n)$ holds.*

These two results indicate a difficulty in maintaining anonymity, neutrality, and Pareto optimality together. Hence, to satisfy these axioms in a given social choice problem, one must give up on resoluteness, the requirement that we choose only one alternative for each profile, and allow for set-valued choices. We define a *social choice rule* (SCR) as a correspondence $f : \mathcal{L}(A)^N \to 2^A \backslash \{\emptyset\}$.

What is the impact of replacing neutrality by EOA on the impossibilities of Moulin (1983, 1991)? Would this replacement pave the way to the identification of interesting SCFs which are anonymous and "almost neutral"? The result below delivers some hope on this.

Theorem 5. *There exists a social choice problem (A, N) which admits an anonymous, EOA, and Pareto optimal SCF while it admits no anonymous and neutral SCF.*

Proof. Take any A with $|A| = 4$ and let $N = \{1, 2\}$. Clearly, $C_2(m,n)$ fails to hold, hence by Theorem 4, (A, N) does not admit an anonymous and neutral SCF. For any $x, y \in A$, let $T_{xy} \subset \mathcal{L}(A)^N$ denote the set of profiles where agent 1 ranks x first and agent 2 ranks y first. Hence $\{T_{xy}\}_{x \neq y}$ partitions the set of profiles where there is no unanimously top ranked alternative. Given any $x, y \in A$, $|T_{xy}| = 3!^2 = 36$ and note that

$$T_{yx} = \{P'_N \in \mathcal{L}(\mathcal{A})^N : \sigma(P_N) = P'_N \text{ for some } P_N \in T_{xy} \text{ and } \sigma \in \Sigma_N\}.$$

Now, let $g : \mathcal{L}(A)^N \to A$ be an SCF that picks at any profile the alternative that is ranked first by both of the agents, if exists. Furthermore, let g be such

[4] $C_1(m,n)$ is equivalent to the condition that $gcd(n, m!) = 1$, as required by Theorem 3.3 in Doğan and Giritligil (2015) and Theorem 3.

that for all $P_N \in T_{xy}$, $g(P_N) = x$ if $zP_1t \iff zP_2t$ for $z, t \in A\backslash\{x, y\}$ and $g(P_N) = y$ otherwise. Hence we have that $|\{P_N \in T_{xy} : g(P_N) = x\}| = |\{P_N \in T_{xy} : g(P_N) = y\}| = 18$. Furthermore, $\forall P_N \in T_{xy}$, let $g(P_N) = g(\sigma(P_N))$, for all permutations σ on N, so that g retains anonymity. Finally, as g picks an alternative only if it is ranked first by an individual, it is also Pareto optimal. □

The SCF defined in the proof of Theorem 5 is a refinement of the plurality rule which also satisfies the well-known monotonicity condition of social choice theory.[5] This raises the question of possibility of refining interesting SCRs that are necessarily set-valued by Theorems 3 and 4 into anonymous and EOA SCFs. The following theorem advises some caution on this.

Theorem 6. *There exists a social choice problem which admits no anonymous, EOA, and Pareto optimal SCF.*

Proof. Let $A = \{x, y\}$ and $N = \{1, 2\}$. We have four possible profiles, P_N, P'_N, P''_N, P'''_N as shown below.

P_1	P_2	P'_1	P'_2	P''_1	P''_2	P'''_1	P'''_2
a	a	b	b	a	b	b	a
b	b	a	a	b	a	a	b

Pareto optimality implies choosing a at P_N and b at P'_N. Moreover, $f(P''_N) = f(P'''_N)$ by anonymity. Hence, $|\{P_N \in \mathcal{L}(A)^N : f(P_N) = a\}| \neq |\{P_N \in \mathcal{L}(A)^N : f(P_N) = b\}|$, a failure of EOA. □

Remark 2. *Although the social choice problem in Theorem 6 admits no anonymous, EOA, and Pareto optimal SCF, it does admit an anonymous and EOA SCF. To see this, consider $g : \mathcal{L}(A)^N \to A$ such that $g(P_N) = g(P'_N) = a$ and $g(P''_N) = g(P'''_N) = b$, which is both anonymous and EOA.*

[5] An SCF f is *monotonic* iff for any two profiles $P_N, P'_N \in \mathcal{L}(A)^N$ and any $x \in A$ such that $f(P'_N) = x$ with

- $xP'_iy \implies xP_iy, \forall y \in A$ and $\forall i \in N$, and
- $yP'_iz \iff yP_iz, \forall y, z \in A\backslash\{x\}$ and $\forall i \in N$,

we have $f(P_N) = x$. For discussion on monotonicity conditions in social choice theory, one may refer to Fishburn (1982) and Sanver and Zwicker (2012).

So we have more permissive results if we dispense with the idea of Pareto optimality altogether. This raises the question of how general is the compatibility between EOA and anonymity, an issue which we elaborate in the sequel.

We say that P'_N is a renaming (of individuals) of P_N iff there exists $\sigma \in \Sigma_N$ such that $P'_N = \sigma(P_N)$. We write $P_N \pi P'_N$ when P'_N is a renaming of P_N. Noting that $\pi \subseteq \mathcal{L}(A)^N \times \mathcal{L}(A)^N$ is an equivalence relation, we write Π for the partition of $\mathcal{L}(A)^N$ provided by π. An element of Π is called an *anonymous equivalence class.*

Theorem 7. *For all social choice problems such that $m > n$, there exists an anonymous and EOA social choice function.*

Proof. It can be shown that given a social choice problem, we have

$$|\Pi| = \binom{m! + n - 1}{n} = \sum_{k=1}^{n} \binom{m!}{k} \binom{n-1}{k-1}$$

where $\binom{m!}{k}\binom{n-1}{k-1}$ is the number of ways to have k orders appear in n individuals' preferences. To see this, note that h orders can be distributed to n agents while each order appears at least once in $\binom{n-1}{h-1}$ ways, and h orders can be chosen out of $m!$ in $\binom{m!}{h}$ ways (see (Feller, 1968, p. 38)).

Given $k \leq n$ orders, each vector $\mathbf{n}_k = (n_1, \ldots, n_k)$ such that

- $i < j \implies n_i \geq n_j$ for all $i, j \in \{1, \ldots, k\}$,
- $n_i \geq 1$ for all $i \in \{1, \ldots, k\}$, and
- $\sum_{i=1}^{k} n_i = n$

defines a way of distributing k orders to n individuals where each order appears at least once. Let ν_k be the set of all vectors as such and let $\delta(\mathbf{n}_k)$ denote the number of anonymous equivalence classes where k orders are distributed according to $\mathbf{n}_k$, and in each of which there are equal number of profiles. We have that

$$\binom{n-1}{k-1} = \sum_{\mathbf{n}_k \in \nu_k} \delta(\mathbf{n}_k).$$

For any $\mathbf{n}_k$, there are $\binom{m!}{k}\delta(\mathbf{n}_k)$ anonymous equivalence classes. As for all $k < m$, it is easy to see that $\binom{m!}{k}$ is divisible by m, the proof is completed. $\square$

5 Conclusion

We identify an equal opportunity condition for alternatives (called EOA), which requires each alternative to be chosen at the same number of profiles. Our condition ensures an opportunity-wise equal treatment of alternatives without imposing the procedural equal treatment requirement of the standard neutrality axiom in social choice theory. As a result, it is weaker than neutrality while preserving a spirit of outcome-oriented fairness.

This weakening is considerable indeed. As a function of the size of the social choice problem, we count the neutral SCFs (first time in the literature, as to the best of our knowledge), we count the EOA SCFs, and observe that replacing neutrality with EOA opens up a significantly large possibility of defining SCFs. This is of particular importance as we know that neutrality is too strong to be maintained alongside with anonymity, which is another basic equal treatment condition of social choice theory.

In fact, we are able to show instances where anonymity and neutrality are incompatible while it is possible to define SCFs that are anonymous, EOA, and furthermore Pareto optimal and monotonic. We show this through a refinement of plurality rule, hence the prospects are valuable also in that we might retain anonymous and neutral SCFs to the extent it is possible and complement them by replacing neutrality with EOA in instances where ties are inevitable. On the other hand, we also show instances where EOA, anonymity and Pareto optimality are incompatible, but EOA and anonymity are compatible.

These findings bring us to the more general question of replacing neutrality with EOA in Theorems 3 and 4 and determining the corresponding characterization. We conjecture that anonymity and EOA are always compatible, independent of the size of the social choice problem. Our Theorem 7 brings a partial answer to this by affirming the conjecture for cases where the number of alternatives exceeds the number of individuals. However, the complicated combinatorial nature of the question disables us to affirm the conjecture for the remaining cases, or to present general characterization results when Pareto optimality enters into the picture.

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A Observations on the numbers of neutral and EOA SCFs

Tables (1-3) below show different ratios the observations made in Section 3 are based on. In all tables $\underline{\mathbf{0}}$ represents numbers smaller than 10^{-10mn}.

$\lvert\mathcal{F}^{NEUTRAL}\rvert/\lvert\mathcal{F}^{EOA}\rvert$	$n=2$	$n=3$	$n=4$
$m=2$	0.333333	0.114286	0.00994561
$m=3$	3.58965×10^{-14}	5.73212×10^{-85}	$\underline{\mathbf{0}}$
$m=4$	$\underline{\mathbf{0}}$	$\underline{\mathbf{0}}$	$\underline{\mathbf{0}}$

Table 1: The ratio of $\lvert\mathcal{F}^{NEUTRAL}\rvert/\lvert\mathcal{F}^{EOA}\rvert$ for different pairs (m, n).

$\lvert\mathcal{F}^{NEUTRAL}\rvert/\lvert\mathcal{F}\rvert$	$n=2$	$n=3$	$n=4$
$m=2$	0.25	0.0625	0.00390625
$m=3$	4.85694×10^{-15}	1.31273×10^{-86}	$\underline{\mathbf{0}}$
$m=4$	$\underline{\mathbf{0}}$	$\underline{\mathbf{0}}$	$\underline{\mathbf{0}}$

Table 2: The ratio of $\lvert\mathcal{F}^{NEUTRAL}\rvert/\lvert\mathcal{F}\rvert$ for different pairs (m, n).

$\lvert\mathcal{F}^{EOA}\rvert/\lvert\mathcal{F}\rvert$	$n=2$	$n=3$	$n=4$	$n=5$
$m=2$	0.75	0.546875	0.392761	0.2799
$m=3$	0.135304	0.0229012	0.0038267	0.000638057
$m=4$	0.00175989	0.0000149993	1.27583×10^{-7}	1.08512×10^{-9}
$m=5$	8.19334×10^{-7}	5.69061×10^{-11}	3.95181×10^{-15}	$\underline{\mathbf{0}}$
$m=6$	8.12216×10^{-12}	$\underline{\mathbf{0}}$	$\underline{\mathbf{0}}$	$\underline{\mathbf{0}}$

Table 3: The ratio of $\lvert\mathcal{F}^{EOA}\rvert/\lvert\mathcal{F}\rvert$ for different pairs (m, n).