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Inefficient Lock-in with Sophisticated and Myopic Players

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Abstract

Path-dependence in coordination games may lead to lock-in on inefficient outcomes, such as adoption of inferior technologies (Arthur, 1989) or inefficient economic institutions (North, 1990). We aim to find conditions under which lock-in is overcome by developing a solution concept that makes ex-ante predictions about the adaptation process following lock-in. We assume that some players are myopic, forming beliefs according to fictitious play, while others are sophisticated, anticipating the learning process of the myopic players. We propose a solution concept based on a Nash equilibrium of the strategies chosen by sophisticated players. Our model predicts that no players would switch from the efficient to the inefficient action, but deviations in the other direction are possible. Three types of equilibria may exist: in the first type lock-in is sustained, while in the other two types lock-in is overcome. We determine the existence conditions for each of these equilibria and show that the equilibria in which lock-in is overcome are more likely and the transition is faster when sophisticated players have a longer planning horizon, or when the history of inefficient coordination is shorter.

Keywords: game theory, learning, lock-in, farsightedness, coordination

JEL classification: C73, D83

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1 Introduction

One of the central problems of game theory is equilibrium selection in games with multiple Nash equilibrium. The problem is even more difficult in repeated games, where usual solution concepts permit a diverse set of sequences to be played on the equilibrium path. Any repetition of a stage game Nash equilibria could be supported by some subgame perfect Nash equilibrium, but even miscoordination can occur at the start of the game if the players are using strategies that implement efficient coordination only following such miscoordination. One reason for the multiplicity of equilibria is the lack of history dependence. As an example, consider figure 1 that represents two stages of a repeated game between players 1 and 2. Subgames starting at nodes $1b$ and $1c$ for player 1 are identical[1], therefore if there is an equilibrium that supports an action for player A in node $1b$, there will also be an equilibrium that supports this action in node $1c$. Nash equilibrium requires mutually consistent beliefs and actions but places no restrictions on how beliefs should depend on observed history. However, even though expecting the same action to be played is just as rational as expecting a different action (Goodman, 1983), there is robust experimental evidence that choices and beliefs do depend on past play, especially in games with multiple stable states (Van Huyck et al., 1990; Romero, 2015). We use this evidence to place additional restrictions on the belief formation process and develop a solution concept that depends on past play and refines the predictions of a subgame perfect Nash equilibrium.

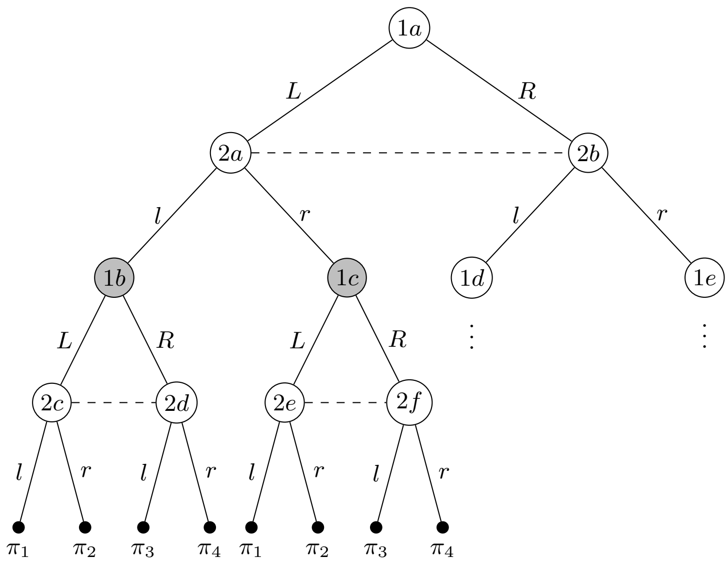

Figure 1: Two stages of a repeated two player game, where the first number indicates the player to whom the node belongs. End nodes display payoffs from the second stage.

[1]Except for the accumulated earnings that play no role under the standard assumptions of risk neutrality and selfishness.

Instead of using solution concepts, such as a subgame perfect Nash equilibrium, we could use learning models, which make predictions about the path of play based on outcomes in previous rounds. However, in learning models (see Fudenberg and Levine, 1998, Camerer, 2003) choices are determined only by observed history, ignoring the structure of upcoming rounds. In this paper the belief formation assumed in learning models is combined with an equilibrium concept to define a solution concept that takes into account both the observed history and the structure of future rounds.

Players in our model are assumed to be either "myopic" or "sophisticated". Myopic players behave as predicted by adaptive learning models: they form beliefs about the actions of other players, update beliefs based on observed history and choose a myopic best response. Sophisticated players have a certain planning horizon and compare payoffs of action plans that prescribe an action for each point in time within this planning horizon. We also assume that sophisticated players anticipate the learning process of myopic players and know about other sophisticated players, therefore our solution concept requires action plans of sophisticated players to be mutual best responses to each other.

One advantage of our solution concept is the ability to make predictions following a particular history of choices. Specifically, we are interested in convergence to an efficient state following previous coordination on an inefficient state. Standard solution concepts abstract from experience that players have prior to the game, although there is robust experimental evidence that behavioral spillovers occur if players experience the same game with different parameters (Romero, 2015, Kamijo et al., 2015) or if two different games are played consecutively (Devetag, 2005, Dolan and Galizzi, 2015). Likewise, in many real life situations decisions are made repeatedly and choices are sensitive to conventions that have been established in the past. It is important to have a theory that could explain how transitions to an efficient state depend on the history of play, but existing models are not able to do that. An adaptive learning model with a deterministic choice rule predicts that no player deviates from an inefficient state once it has been reached. In a subgame perfect Nash equilibrium the history of previous interactions plays no role. A model presented in this paper combines the two approaches and predicts that a transition from an inefficient to the efficient state can occur if certain conditions are satisfied, while transitions in the opposite direction never occur.

Our model also predicts that some players may deviate from an inefficient state, but none will deviate from the efficient one, therefore the efficient state is absorbing and there is a unique point in time when play transitions from the inefficient to the efficient state. For sophisticated players action paths that prescribe a switch from an efficient to an inefficient action are dominated, therefore sophisticated players will switch to the efficient action at most once. We calculate how such action plans of sophisticated players affect the switching period of myopic players, and how the latter affects sophisticated player payoffs. This mapping from sophisticated player action plans to payoffs is then used to determine the combinations of action plans that are mutual best responses to each other.

It is important to know whether inefficient lock-in (Arthur, 1989) can be overcome, and how the conditions can be changed to improve the chances of an efficiency-enhancing transition. We show that three types of equilibria may exist in the repeated game: in a "teaching equilibrium" sophisticated players switch to the efficient action at the start of the game, and myopic players switch later. In an "interior equilibrium" sophisticated players initially play the inefficient action, but switch to the efficient one and are subsequently followed by myopic players. In a "delay" equilibrium all sophisticated players choose the inefficient action for the entire duration of the game, and myopic players never switch. Inefficient lock-in is therefore overcome in the first two types of equilibria, but not in the third one. Point predictions cannot be made because of the multiplicy of equilibria, therefore instead we show how the speed of transition and the types of equilibria that exist respond to changes in game parameters. We find that as the planning horizon of sophisticated players increases, the teaching and the interior equilibria are more likely to exist, while the delay equilibrium is less likely. A longer history of inefficient coordination makes teaching equilibrium less likely and delays transitions. The effect of player composition is ambiguous: on one hand, a larger number of sophisticated players leads to a faster transition and higher profits in the teaching and interior equilibria, reducing incentives to completely stop teaching. On the other hand, as the number of sophisticated players grows, one player's actions have a smaller effect on myopic players, increasing incentives to delay teaching and leading to a potential breakdown of a teaching equilibrium.

Several other studies have extended the adaptive learning model with sophistication in different ways. Camerer et al. (2002a) and Camerer et al. (2002b) propose a sophisticated experience-weighted attraction (EWA) model in which some players are adaptive and learn using adaptive EWA, while others are sophisticated, anticipate how adaptive players learn and use strategic teaching. While conceptually this paper is similar to the model of sophisticated EWA, we develop a solution concept that can be used to make ex-ante predictions about the path of play in the game, while the parameters of sophisticated EWA can be estimated only ex-post. Ellison (1997) models a population of adaptive players, learning according to fictitious play, repeatedly matched in pairs to play a binary choice coordination game. Adding one rational player to the population of adaptive players can change the outcome from coordination on the inefficient equilibrium to coordination on the efficient one, as long as the number of players is fixed and the rational player is patient enough. Acemoglu and Jackson (2011) develop an overlapping generations model that shows how a social norm of low cooperation can be overturned by a single forward-looking player. Schipper (2011) uses an optimal control model with two players and shows how a strategic player can control an adaptive player in repeated games with strategic substitutes or strategic complements. Mengel (2014) studies adaptive players who are also forward-looking and finds that in two-player coordination games the efficient equilibrium may be stochastically stable, in contrast to the the case with only adaptive players.

2 Sophisticated Player Equilibrium

Consider n players, indexed by $i \in N \equiv \{1, 2, \ldots, n\}$, who play a repeated game in continuous time by choosing an action from a stage game action space $\{A, B\}$. We denote the time at which the game starts by 0, the duration of the remaining game by $\bar{T}$ and the duration of observed history by T', with $\bar{T}, T' \in (0, \infty)$. We implement the history of inefficient coordination by assuming that prior to time 0 only action B has been chosen.

We assume two types of players: m players are myopic and $n - m$ players are sophisticated. Throughout the paper we will index sophisticated players by $s \in S$ and myopic players by $i \in N \setminus S$. The two types of players follow different choice rules, respectively denoted by a_i and a_s, which prescribe an action for each moment in time. We will refer to a_i as a *choice function* and to a_s as an *action plan.* Denote the action of player i at time t by $a_i(t)$ and the action of player s by $a_s(t)$, where action A is coded as 1 and action B is coded as 0. Denote the combination of actions of all players except i by $a_{-i}(t) = \times_{j \in N \setminus \{i\}} a_j(t)$, with $a_{-i}(t) \in A_{-i}$, and denote the combination of actions of all sophisticated players except s by $a_{-s}(t) = \times_{j \in S \setminus \{s\}} a_j(t)$, with $a_{-s}(t) \in A_{-s}$. The payoff flow for player i at time t is $\pi_i(a_i(t), \times_{j \in N \setminus \{i\}} a_j(t))$. Similarly, denote the combination of choice functions of all myopic players except i by $a_{-i} = \times_{j \in \{N \setminus S\} \setminus \{i\}} a_j$ and the combination of action plans for all sophisticated players except s by $a_{-s} = \times_{j \in S \setminus \{s\}} a_j$.

The difference between a choice function for myopic players and an action plan for sophisticated players lies in how these functions are determined: choices of myopic players are determined by the history of play while the choices of sophisticated players must be optimal given the choices of all other players. Before specifying these two function we first have to define the beliefs and expected payoffs of myopic players.

Belief of a myopic player is a probability assigned to the event that a randomly chosen other group member chooses action A. Denote the belief of player i at time t by $x_i(t)$. Belief formation is assumed to follow a one parameter weighted fictitious play model,[2] proposed by Cheung and Friedman (1997). The original weighted fictitious play model is specified for two player games and we extend it to N-person games by assuming that a joint distribution

[2]Fictitious play corresponds to Bayesian updating of the probability that any group member will choose A, using a Dirichlet prior and assuming that the choice of each group member was independently drawn from the distribution about which players are learning.

of choices is used to form beliefs about the actions of group members, but players do not distinguish between the identities of others.[3] Beliefs are therefore homogeneous (Rapoport, 1985; Rapoport and Eshed-Levy, 1989): a single belief is formed about the probability that any other player will choose A. The fictitious play rule used to calculate myopic player beliefs is as follows:

$$x_i(t) = \frac{\int_{k=0}^{t} \gamma^k \sum_{j \in N \setminus \{i\}} \frac{a_j(t-k)}{n-1} \, \mathrm{d}k}{\int_{k=0}^{t+T'} \gamma^k \, \mathrm{d}k} \tag{1}$$

The integral in the numerator measures the weighted length of time in which action A has been observed, determined by the action plans of other group members. Observations prior to time 0 play no role because we assume that prior to time 0 only action B has been observed. The γ parameter measures the rate at which old observations are forgotten. We assume that $\gamma \in (0,1)$, where values close to 1 indicate that all past observations are given similar weights, while values close to 0 indicate that only the most recent experience is taken into account.

Expected payoffs of myopic players associated with each pure action are determined by beliefs $x_i(t)$, which are used to assign a probability to each action profile of other group members:

$$\begin{aligned} E\pi_i(a, x_i(t)) &= \sum_{a_{-i} \in A_{-i}} [Pr(a_{-i}(t) = a_{-i} | x_i(t)) \times \pi_i(a, a_{-i})] = \\ &= \sum_{a_{-i} \in A_{-i}} [x_i(t)^{(\sum a_{-i})} (1 - x_i(t))^{(n-1-\sum a_{-i})} \times \pi_i(a, a_{-i})], \quad \forall a \in \{1, 0\} \end{aligned} \tag{2}$$

Choice function $a_i(t, \times_{s \in S} \ a_s)$ prescribes an action for a myopic player i at any point in time $t \in [0, \bar{T}]$, conditional on the profile of action plans chosen by sophisticated players, $\times_{s \in S} \ a_s$. We assume that myopic players choose the action that maximizes immediate expected utility and ties are broken in favour of action A:

$$a_i(t, \times_{s \in S} a_s) = \begin{cases} 1 & \text{if } E\pi_i(1, x_i(t)) \geq E\pi_i(0, x_i(t)) \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

[3]There are several other ways how weighted fictitious play could be extended to N-person games. One way could be to assume that players form beliefs about the joint distribution of the actions of all others and update it using observed aggregate feedback: for example, Crawford (1995) assumes that players form beliefs and observe feedback about an order statistic of all the choices. Another way is to assume that separate beliefs are formed about every other player j based on the empirical distribution of j's choices (e.g. Monderer and Shapley, 1996). We combine the two approaches by assuming that players use the joint distribution of choices to form beliefs about the action of any opponent, but do not distinguish between their identities.

Action plans chosen by sophisticated players, $\times_{s \in S} a_s$, are explicitly included in the choice function to make it transparent that myopic player actions can be affected by sophisticated players. Note that the choice function depends only on the current round payoffs and beliefs, which are determined by observed history, therefore it is possible to anticipate myopic player choices at any history.

Sophisticated players anticipate the learning process of myopic players and are also far-sighted, thus at time 0 they choose an action plan for the interval $[0, T]$, where T is the length of the planning horizon of sophisticated players.

Action plan a_s prescribes an action for a sophisticated player s at any point in time $t \in [0, T]$. Denote the set of all action plans by A_s. The action plan is assumed to be an open-loop strategy, which depends only on time and not on observed history. Sophisticated players face no strategic uncertainty about the actions of myopic players, but they do face uncertainty about the actions of other sophisticated players. Payoffs associated with an action plan a_s depend on the vector of action plans of other sophisticated players, a_{-s}, and on the choices of myopic players, whose choice function $a_i(t, a_s \times a_{-s})$ also depends on the action plans of all sophisticated players. The total payoff that a sophisticated player expects to earn over the period of length T is calculated as follows:

$$\Pi(a_s, a_{-s}, a_i(\cdot, a_s \times a_{-s})) = \int_0^T \pi[a_s(t), a_{-s}(t) \times a_i(t, a_s \times a_{-s})]dt \tag{4}$$

Since sophisticated players choose action plans and face no strategic uncertainty about the actions of myopic players, the game can be reduced to a static game between sophisticated players. Theoretical predictions in static games are typically made using a Nash equilibrium, so we follow the convention and require that sophisticated players choose action plans that are mutual best responses to each other.

Definition 1. *A combination of action plans $\times_{s \in S} a^*_s$ is a symmetric **sophisticated player equilibrium** if for each player $s \in S$, a^*_s satisfies*

$$\begin{aligned} \Pi(a^*_s, a^*_{-s}, a_i(\cdot, a^*_s, a^*_{-s})) \geq \Pi(a_s, a^*_{-s}, a_i(\cdot, a_s, a^*_{-s})), &\quad \forall a_s \in A_s \\ \text{and } a^*_s = a^*_j, &\quad \forall s, j \in S \end{aligned} \tag{5}$$

and $a_i(\cdot, a_s, a_{-s})$ is defined in (3).

If there were no myopic players, equation (5) would reduce to the standard Nash equilibrium. If all players were myopic, equation (5) would not apply, and the choices of all players would be calculated using the belief learning model. We will look at an intermediate case where both myopic and sophisticated players are present.

In the remainder of the paper we will characterize the symmetric sophisticated player equilibria for a repeated N-person critical mass coordination game.

3 Sophisticated Player Equilibrium in a Critical Mass Game

We are interested in determining conditions under which an inefficient convention could be replaced by an efficient one. One way how such a transition could take place is by strategic choice: sophisticated players could attempt to teach other players to play according to the efficient convention. To determine conditions under which such strategic teaching is possible we will characterize symmetric sophisticated player equilibria following lock-in to an inefficient state.

3.1 Critical Mass Game

Recall that we defined a sophisticated player equilibrium for a class of games with n players and an action space $\{A, B\}$. A special class of such games is a critical mass game, in which payoffs of each player depend on their action, $a_i(t)$ and on the total number of other group members who chose action A at time t, denoted by $r(a_{-i}, t) = \sum_{j \in N \setminus \{i\}} a_j(t)$, with $r(a_{-i}, t) \in \{0, 1, \dots, n-1\}$. The payoff flow for player i at time t is defined as follows:

$$\pi(a_i(t), a_{-i}(t)) = \begin{cases} H & \text{if } r(a_{-i}, t) \geq \theta \text{ and } a_i(t) = 1 \\ 0 & \text{if } r(a_{-i}, t) < \theta \text{ and } a_i(t) = 1 \\ M & \text{if } r(a_{-i}, t) \geq \theta \text{ and } a_i(t) = 0 \\ L & \text{if } r(a_{-i}, t) < \theta \text{ and } a_i(t) = 0 \end{cases} \tag{6}$$

To have a coordination game, we assume that $H > M$ and $L > 0$. The coordination requirement is determined by an exogenous threshold θ: action A generates a larger payoff than B if and only if at least θ other group members choose A. There are two stable states[4] in pure strategies if one point in time is considered in isolation: in the first stable state all players choose A and in the second one all players choose B. We assume that states are Pareto-ranked and define coordination on A as an efficient state by assuming that $H > L$. Finally, we assume that $M \geq L$, so that players who choose B also prefer a situation in which the threshold has been exceeded.

[4]We will use the term "state" rather than "equilibrium" when referring to a Nash equilibrium in a stage game to avoid confusion with the sophisticated player equilibrium.

Assumption 1: H > M $\geq$ L > 0.

We assume that there are at least 2 sophisticated players so that an equilibrium could be defined using equation 5. We also assume that the number of myopic players is sufficiently large to implement the efficient state, and the number of sophisticated players is small enough so that sophisticated players on their own could not implement the efficient state. If the latter condition was not satisfied, a sophisticated player equilibrium would reduce to the standard Nash equilibrium because sophisticated players would not need to take into account the learning process of myopic players.

Assumption 2: $2 \leq n - m < \theta \leq m$.

3.2 Choice Function of Myopic Players

Myopic players form beliefs about the actions of other players and choose an action that maximizes immediate payoffs. In this subsection we specify the choice function $a_i(t, a_s)$ that prescribes an action for player i at time t when sophisticated players are choosing action plans $\times_{s \in S} a_s$ (for brevity, we will omit the subscript under the product sign).

Proposition 1. *Suppose that in a game with payoffs defined by* (6) *at time t myopic player i holds beliefs $x_i(t)$. Then the choice function from* (3) *simplifies to:*

$$a_i(t, \times a_s) = \begin{cases} 1 & \textit{if } x_i(t) \geq I^{-1}_{\frac{L}{L+H-M}}(\theta, n-\theta) \\ 0 & \textit{otherwise} \end{cases} \tag{7}$$

where I^{-1} is the inverse of an incomplete regularized beta function.

Proof.

From (3), action A is chosen if the expected payoff of A at time t exceeds the expected payoff of B:

$$a_i(t) = 1 \Leftrightarrow E\pi(1, x_i(t)) \geq E\pi(0, x_i(t)) \tag{8}$$

In a critical mass game payoff depends only on the chosen action and on whether the number of other group members who chose A exceeds θ. Denote the subjective probability assigned to the latter event by $Pr[r(a_{-i}, t) \geq \theta | x_i(t)]$. Then expected payoffs in equation (2) can be defined as:

$$\begin{aligned} E\pi(1, x_i(t)) &= 0 \times (1 - Pr[r(a_{-i}, t) \geq \theta | x_i(t)]) + H \times Pr[r(a_{-i}, t) \geq \theta | x_i(t)] \\ E\pi(0, x_i(t)) &= L \times (1 - Pr[r(a_{-i}, t) \geq \theta | x_i(t)]) + M \times Pr[r(a_{-i}, t) \geq \theta | x_i(t)] \end{aligned} \tag{9}$$

The subjective probability that the threshold will be exceeded is calculated by adding the probabilities assigned to all action profiles of other players in which more than θ players choose A:

$$Pr[r(a_{-i},t) \geq \theta | x_i(t)] = \sum_{k=\theta}^{n-1} (x_i(t))^k (1-x_i(t))^{n-1-k} \binom{n-1}{k} \tag{10}$$

Use equations (9) and (10) to rewrite (8) the following way:

$$a_i(t) = 1 \Leftrightarrow \sum_{k=\theta}^{n-1} (x_i(t))^k (1-x_i(t))^{n-1-k} \binom{n-1}{k} \geq \frac{L}{L+H-M} \tag{11}$$

Notation in (11) is simplified using the definition of an incomplete regularized beta function:[5]

$$a_i(t) = 1 \Leftrightarrow I_{x(t)}(\theta, n-\theta) \geq \frac{L}{L+H-M} \tag{12}$$

Taking the inverse of (12) and substituting into (3) leads to the desired expression:

$$a_i(t, \times a_s) = \begin{cases} 1 & \text{if } x_i(t) \geq I^{-1}_{\frac{L}{L+H-M}}(\theta, n-\theta) \\ 0 & \text{otherwise} \end{cases}$$

■

Proposition 1 states that a myopic player chooses A instead of B if his probabilistic belief exceeds $I^{-1}_{\frac{L}{L+H-M}}(\theta, n-\theta)$, a threshold value that depends only on the game parameters. For brevity, we will refer to this threshold value by I^{-1}. We should note that the properties of inverse regularized beta functions imply that I^{-1} is increasing in L, M and θ, but decreasing in H and n.

Proposition 1 shows that myopic player actions can be determined by comparing their beliefs to a threshold value that is fixed in a given game. Once myopic player actions are know, Assumption 2 ensures that the efficient state is implemented if and only if all myopic players choose A. The next section will simplify the payoff calculation even further by showing that to know the payoff flow it is sufficient to know the first time when myopic player beliefs exceed the threshold value.

[5] An incomplete regularized beta function is defined as $I_c(a,b) = \sum_{k=a}^{a+b-1} c^k (1-c)^{a+b-1-k} \binom{a+b-1}{k}$. The function is well defined because $\frac{L}{L+H-M} \in (0,1)$, from Assumption 1.

3.3 Undominated Action Plans of Sophisticated Players

This section shows that although sophisticated players could use action plans that prescribe many switches from one action to the other, undominated action plans must prescribe at most one switch from action B to action A and no switches from action A to action B. The sophisticated player action space can therefore be restricted to a set of real numbers that denote a switching time from A to B.

Definition 2. *Denote by U_s (for "undominated") the set of action plan profiles in which no sophisticated player is choosing strictly dominated action plans:*

$$U_s = \{\times_{s \in S} a_s \in A_s | \nexists a_s' : \Pi[a_s', a_{-s}, a_i(\cdot, a_s' \times a_{-s})] > \Pi[a_s, a_{-s}, a_i(\cdot, a_s \times a_{-s})]\}$$

An action profile will be called *dominated* if it is not in set U_s, that is if in this action profile at least one sophisticated player is choosing a dominated action plan.

We will show that the set of undominated action plans cannot contain any strategies that prescribe a switch from A to B. The proof requires two additional lemmas.

Lemma 1. *If two action plans of the sophisticated player prescribe the same action, the payoff flow is higher for the action plan with which myopic player beliefs are higher:*

$$\begin{aligned} \pi[a_s'(t), a_{-s}(t) \times a_i(t, a_s' \times a_{-s})] &\geq \pi[a_s(t), a_{-s}(t) \times a_i(t, a_s \times a_{-s})] \\ &\quad \textit{if} \quad x(t)' \geq x(t) \quad \textit{and} \quad a_s'(t) = a_s(t) \end{aligned}$$

where $x(t)'$ is the belief held by myopic players if the sophisticated player uses action plan a_s' and $x(t)$ is the belief if the sophisticated player uses action plan a_s.

Proof: see Appendix A.2.

Lemma 1 shows that sophisticated players can only benefit from myopic players assigning a higher probability to others choosing A. The proof is based on an observation that the tendency for myopic players to choose A is increasing in their beliefs and sophisticated player payoffs are increasing in the number of players who choose action A.

Definition 3. *Denote by AB_M the set of action plan profiles for sophisticated players with which myopic players switch from A to B:*

$$\begin{aligned} AB_M = \{\times_{s \in S} a_s \in A_s | \exists t_1, t_2 \in [0, T] : \quad & t_1 < t_2 \\ & a_i(t_1, \times_{s \in S} a_s) = 1 \\ & a_i(t_2, \times_{s \in S} a_s) = 0\} \end{aligned}$$

Lemma 2. *All action plan profiles for sophisticated players with which myopic players switch from A to B are strictly dominated:*

$$AB_M \cap U_s = \emptyset$$

Proof: see Appendix A.2.

The intuition of Lemma 2 is straightforward: if myopic players ever switch to an efficient action A, the participation threshold will be exceeded as long as sophisticated players continue choosing action A. Consequently, sophisticated players who would choose B would lower their earnings. However, note that the proof rests on Assumption 2, which says that the number of myopic players exceeds the participation threshold. If this assumption did not hold, an argument about dominance could not be made because other sophisticated players may prevent efficient coordination by switching to B, which would make switching to B optimal.

Definition 4. *Denote by AB_S the set of action plan profiles for sophisticated players with which at least one sophisticated player switches from A to B:*

$$AB_S = \{\times_{s \in S} a_s \in A_s | \exists t_1, t_2 \in [0,T], s \in S: \begin{array}{l} t_1 < t_2 \\ a_s(t_1) = 1 \\ a_s(t_2) = 0\} \end{array}$$

Proposition 2. *Action plan profiles for sophisticated players that prescribe a switch from A to B for at least one sophisticated player are dominated:*

$$AB_S \cap U_s = \emptyset$$

Proof.

Take an action plan profile $\times_{s \in S} a_s \in AB_S$. We will show that in this action profile at least one sophisticated player must be choosing an action plan that is dominated.

If $\times_{s \in S} a_s \in AB_M$, at least one sophisticated player must be choosing a dominated action plan, from Lemma 2, and the proof would be completed. Alternatively, assume that $\times_{s \in S} a_s \in \{AB_S \setminus AB_M\}$. By the definition of AB_S, there must be a sophisticated player whose action plan prescribes a switch from A to B; denote the action plan of this player by $\widetilde{a}_s$ and denote the switching time prescribed by $\widetilde{a}_s$ by t'. Then there must be some small ϵ such that $\widetilde{a}_s(t) = 1$ if $t \in [t' - \epsilon, t')$ and $\widetilde{a}_s(t) = 0$ if $t \in [t', t' + \epsilon]$. Since $\times_{s \in S} a_s \notin AB_M$, myopic players switch from B to A at most once, thus their choices can be described by a number $\hat{t}(\widetilde{a}_s)$ that identifies this switching time: B is chosen in the interval $[0, \hat{t}(\widetilde{a}_s))$ and A is chosen in the interval $[\hat{t}(\widetilde{a}_s), T]$.

First, suppose that $t' \geq \hat{t}(\widetilde{a}_s)$, then myopic players would be choosing A at any time $t \geq t'$. Assumption 2 implies that the threshold will be exceeded at any such point in time, therefore an action plan $\widetilde{a}_s$ is dominated by an action plan that prescribes A at each point in time $t \geq \hat{t}(\widetilde{a}_s)$. Next, suppose that $t' < \hat{t}(\widetilde{a}_s)$ and $\hat{t}(\widetilde{a}_s) > T$. Then myopic players will choose B for the entire period that is taken into account by the sophisticated player, thus action plan $\widetilde{a}_s$ will be dominated by an action plan that prescribes B at all times.

Alternatively, suppose that $\hat{t}(\widetilde{a}_s) > t'$ and $\hat{t}(\widetilde{a}_s) \leq T$ (see an illustration in figure 2). Choose ϵ to be sufficiently small to satisfy $\hat{t}(\widetilde{a}_s) > t' + \epsilon$. Then for any $\widetilde{a}_s$ construct an action plan a'_s the following way:

$$a'_s(t) = \begin{cases} \widetilde{a}_s(t) & \text{if } t \in [0, t' - \epsilon) \cup (t' + \epsilon, T] \\ 0 & \text{if } t \in [t' - \epsilon, t'] \\ 1 & \text{if } t \in (t', t' + \epsilon] \end{cases}$$

In other words, a'_s is constructed by taking $\widetilde{a}_s$ and swapping choices prescribed in the interval $(t' - \epsilon, t')$ with choices prescribed in the interval $(t', t' + \epsilon)$. We will show that $\widetilde{a}_s$ is dominated by a'_s.

The comparison of payoff flows generated by these two action plans is shown in figure 2. In the interval $[0, t + \epsilon)$ the sum of payoff flows is the same for both action plans $(\pi_1 + \pi_2 + \pi_3)$. Payoffs are equal because with both action plans myopic players choose B in this entire interval (both $\hat{t}(a'_s)$ and $\hat{t}(\widetilde{a}_s)$ exceed $t' + \epsilon$), therefore the participation threshold is never exceeded. Action plan $\widetilde{a}_s$ prescribes A for the same duration of time as a'_s, therefore the sum of payoffs in the interval $[0, t' + \epsilon)$ would be the same for both action plans.

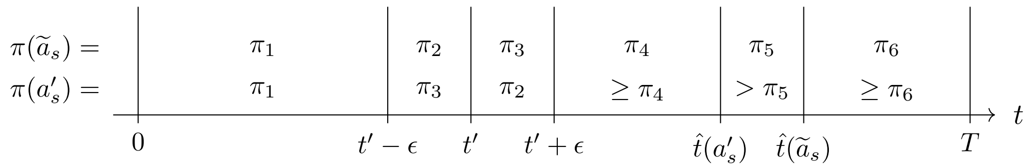

Figure 2: Payoff flows generated by action plans $\widetilde{a}_s$ and a'_s for the case $\hat{t}(\widetilde{a}_s) > t'$ and $\hat{t}(\widetilde{a}_s) \leq T$.

In the interval $[t' + \epsilon, T]$ the sum of payoffs generated by a'_s is strictly higher than that of $\widetilde{a}_s$. Since $\widetilde{a}_s(t) = a'_s(t)$, $\forall t \in (t' + \epsilon, T]$, any payoff difference between the two action plans in this interval must be due to the choices of myopic players. From equation 1, $x_i(t)$ would be the same under $\widetilde{a}_s(t)$ as under $a_s(t)'$ if γ was equal to 1. But as $\gamma \in (0, 1)$, older observations receive less weight and therefore myopic player beliefs would be strictly higher following a'_s than following $\widetilde{a}_s$ at any time $t \in (t' + \epsilon, T]$. Then Lemma 1 implies that the payoff flow is always weakly higher for a'_s at any time in the interval $[t' + \epsilon, T]$. To get strict dominance, note that $\hat{t}(a'_s) < \hat{t}(\widetilde{a}_s)$, for the following reasons. Since $\hat{t}(a'_s) \in (t' + \epsilon, T]$ and $x(t)$ is continuous, the switching period $\hat{t}(a'_s)$ must satisfy $x'_i(\hat{t}(a'_s)) = I^{-1}$. But since $\widetilde{x}_i(t) < x'_i(t)$, $\forall t \in (t' + \epsilon, T]$, it must also hold that $\widetilde{x}_i(\hat{t}(a'_s)) < x'_i(\hat{t}(a'_s)) = I^{-1}$. Consequently, the intersection of beliefs $\widetilde{x}_i(t)$ and belief threshold I^{-1} must occur strictly later, so that $\hat{t}(a'_s) < \hat{t}(\widetilde{a}_s)$. In the interval $(\hat{t}(a'_s), \hat{t}(\widetilde{a}_s))$ action plan $\widetilde{a}_s$ provides a flow of payoffs of at most L, while a'_s provides a payoff of H because more than θ players are choosing A.

The comparison of payoff flows associated with action plans $\widetilde{a}_s$ and a'_s is shown in figure 2. The sum of payoff flows generated by a'_s will be strictly higher than the sum of payoff flows generated by $\widetilde{a}_s$, therefore action plan $\widetilde{a}_s$ that prescribes switching from A to B is strictly dominated by another action plan a'_s.

■

The intuition of the proof is as follows: suppose that a sophisticated player switches from A to B. If myopic players switch from B to A at the same time or earlier, a sophisticated player would do better by always playing A instead. If myopic players never switch to A, there would be no incentive to play A in the first place. If myopic players switch at some time after the sophisticated player, the sophisticated player can strictly increase the earnings by teaching less at the start of the game and teaching more later.[6] Doing so would not reduce the payoffs prior to the switch, but would strictly decrease the switching time of the myopic players, because weighted fictitious play puts more weight on recent experience. Consequently, whenever sophisticated players are considering teaching for some period of time, they would be better off concentrating all the teaching just before the predicted switch of myopic players, thus a switch from A to B would never occur.

This section has shown that if sophisticated players do not choose dominated action plans, both myopic and sophisticated players will switch from B to A at most once, thus in the equilibrium the path of choices for either type can be described by a scalar indicating the switching time.

Each action path of myopic players that can be induced by undominated action paths of sophisticated players has the following structure:

$$a_i(t, \times a_s) = \begin{cases} 0 & \text{if } t \in [0, \hat{t}(\times a_s)) \\ 1 & \text{if } t \in [\hat{t}(\times a_s), T] \end{cases} \qquad \forall a_s \in U_s$$

Define $\hat{t} \in (0, \infty)$ as the **switching period** of myopic players. Note that $\hat{t} > 0$ because equation 1 implies that $x_i(0) = 0$, thus B is chosen at time 0.

[6] By "teaching" we mean choosing action A to induce myopic players to choose A in the future.

Each undominated action plan for sophisticated players has the following structure:

$$a_s(t) = \begin{cases} 0 & \text{if } t \in [0, y_s) \\ 1 & \text{if } t \in [y_s, T] \end{cases} \qquad \forall a_s \in U_s$$

Define $y_s \in [0, T]$ as a **strategy** for player s.

In the next section we will specify how the switching period of myopic players depends on the strategies of sophisticated players.

3.4 Optimal Switching Period for Myopic Players

The characterization of symmetric sophisticated player equilibria requires information about payoffs in an equilibrium and payoffs from potential deviations: in the first case all $(n-m)$ sophisticated players choose the same strategy, in the second case $(n-m-1)$ sophisticated players choose one strategy and one player chooses a different one. Denote the strategy of one sophisticated player by $y_s = y$ and the strategy of other $n-m-1$ sophisticated players by $y_j = \bar{y}$, for all $j \in \{S \setminus s\}$. Sophisticated player payoffs are determined by the switching period of myopic players, thus we first specify function $\hat{t}(y, \bar{y})$ that shows how the myopic player switching period depends on y and $\bar{y}$.

There are three cases to consider. In the first case, $\hat{t}(y, \bar{y}) > \max\{y, \bar{y}\}$, so that myopic players observe no other players choosing A from time 0 to time $\min\{y, \bar{y}\}$, a fraction of $\frac{n-m}{n-1}$ others choosing A from time $\max\{y, \bar{y}\}$ to $\hat{t}(y, \bar{y})$ and either a fraction of $\frac{1}{n-1}$ others choosing A from time $\bar{y}$ to time y (if $y > \bar{y}$) or a fraction of $\frac{n-m-1}{n-1}$ others choosing A from time y to $\bar{y}$ (if $\bar{y} > y$). Feedback observed by myopic players in this case is illustrated in figure 3.

In the second case, $y < \hat{t}(y, \bar{y}) < \bar{y}$. This will be true only if $\frac{1}{n-1} > I^{-1}$, that is if myopic players would switch to A after observing only one player choosing A. In this case each myopic player will observe no others choosing A from time 0 to y and a fraction of $\frac{1}{n-1}$ others choosing A from time y to $\hat{t}(y, \bar{y})$.

In the third case, $\bar{y} < \hat{t}(y, \bar{y}) < y$. Then each myopic player will observe no others choosing A from time 0 to $\bar{y}$ and a fraction of $\frac{n-m-1}{n-1}$ others choosing A from time $\bar{y}$ to $\hat{t}(y, \bar{y})$.

It is never possible that $\hat{t}(y, \bar{y}) < \min\{y, \bar{y}\}$ because at time $t \in [0, \min\{y, \bar{y}\})$ myopic players observe no others choosing A and therefore always choose B.

Proposition 3. *The switching period of myopic players is:*

$$\hat{t}(y, \bar{y}) = \begin{cases} \hat{t}_2(y) & \textit{if } y < \hat{t}_2(y) \le \bar{y} & \textit{and} & \frac{1}{n-1} > I^{-1} \\ \hat{t}_3(\bar{y}) & \textit{if } \bar{y} < \hat{t}_3(\bar{y}) \le y & \textit{and} & \frac{n-m-1}{n-1} > I^{-1} \\ \hat{t}_1(y, \bar{y}) & \textit{if } \max\{y, \bar{y}\} < \hat{t}_1(y, \bar{y}) & \textit{and} & \frac{n-m}{n-1} > I^{-1} \\ \infty & \textit{otherwise} \end{cases} \tag{13}$$

such that

$$\hat{t}_1(y,\bar{y}) = \frac{\log(\frac{n-m}{n-1} - I^{-1}) - \log(\gamma^{-\bar{y}}\frac{n-m-1}{n-1} + \gamma^{-y}\frac{1}{n-1} - \gamma^{T'}I^{-1})}{\log(\gamma)} \tag{14}$$

$$\hat{t}_2(y) = \frac{\log(\frac{1}{n-1} - I^{-1}) - \log(\gamma^{-y}\frac{1}{n-1} - \gamma^{T'}I^{-1})}{\log(\gamma)} \tag{15}$$

$$\hat{t}_3(\bar{y}) = \frac{\log(\frac{n-m-1}{n-1} - I^{-1}) - \log(\gamma^{-\bar{y}}\frac{n-m-1}{n-1} - \gamma^{T'}I^{-1})}{\log(\gamma)} \tag{16}$$

where y is the strategy of one sophisticated player and $\bar{y}$ is the strategy of other $(n-m-1)$ sophisticated players.

It is never possible that more than one condition of 13 is satisfied because $\hat{t}_1(y,\bar{y}) \leq \hat{t}_2(y)$ and $\hat{t}_1(y,\bar{y}) \leq \hat{t}_3(y)$ (see Lemma 10 in Appendix A).

Proof.

Case 1: $\hat{t}(y,\bar{y}) > \max\{y,\bar{y}\}$

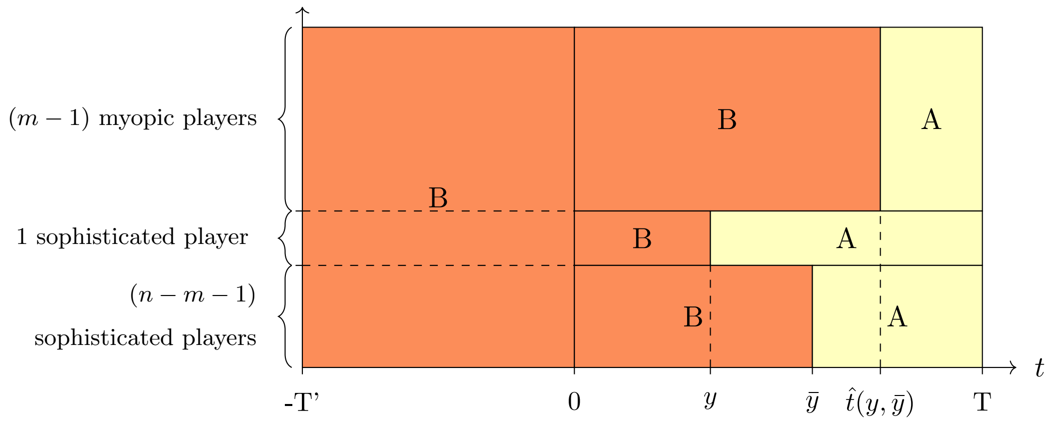

Figure 3: Illustration of the feedback observed by a single myopic player in the first case, where $\hat{t}(y,\bar{y}) > \max\{y,\bar{y}\}$. In this example $\bar{y} > y$. Vertical axis shows the fraction of other players choosing A or B, horizontal axis shows the passage of time. The first sophisticated player switches from B to A at time y, other $(n-m-1)$ sophisticated players switch at time $\bar{y}$ and myopic players switch at time $\hat{t}(y,\bar{y})$

Recall that beliefs of myopic players are calculated using weighted fictitious play from equation 1. If sophisticated players are using strategies y and $\bar{y}$, myopic player beliefs at any time $t \in (\max\{y,\bar{y}\}, \hat{t}(y,\bar{y})]$ will be calculated using the following rule:

$$
\begin{aligned}
x_i(t) =& \frac{\int_{k=0}^{t-\bar{y}} \gamma^k (\frac{n-m-1}{n-1}) \mathrm{d}k + \int_{k=0}^{t-y} \gamma^k (\frac{1}{n-1}) \mathrm{d}k}{\int_{k=0}^{t+T'} \gamma^k \mathrm{d}k} = \\
=& \frac{(\gamma^{t-\bar{y}} - 1)(\frac{n-m-1}{n-1}) + (\gamma^{t-y} - 1)(\frac{1}{n-1})}{\gamma^{t+T'} - 1}
\end{aligned}
$$

Expressions in the numerator correspond to the history observed by a myopic player at time $t \in (\max\{y, \bar{y}\}, \hat{t}(y, \bar{y})]$: $(n-m-1)$ sophisticated players are observed choosing A for a period of $t-\bar{y}$ and one sophisticated player is observed choosing A for a period of $t-y$. This feedback is illustrated in figure 3. The denominator measures the length of the entire history, including the T' rounds of inefficient coordination.

From Proposition 1, myopic players will choose A at time t if $x_i(t) \geq I^{-1}$:

$$
\begin{aligned}
a_i(t) = 1 \quad \Leftrightarrow \quad & \frac{(\gamma^{t-\bar{y}} - 1)(\frac{n-m-1}{n-1}) + (\gamma^{t-y} - 1)(\frac{1}{n-1})}{\gamma^{t+T'} - 1} \geq I^{-1} \quad \Leftrightarrow \\
\gamma^{t+T'}(\gamma^{-\bar{y}-T'} \frac{n-m-1}{n-1} & + \gamma^{-y-T'} \frac{1}{n-1} - I^{-1}) \leq \frac{n-m}{n-1} - I^{-1}
\end{aligned} \tag{17}
$$

If $\frac{n-m}{n-1} - I^{-1} \leq 0$, equation (17) is never satisfied because of the following relationship that contradicts (17):

$$
\gamma^{t+T'}(\gamma^{-\bar{y}-T'} \tfrac{n-m-1}{n-1} + \gamma^{-y-T'} \tfrac{1}{n-1} - I^{-1}) > \gamma^{t+T'}(\tfrac{n-m}{n-1} - I^{-1}) \geq \tfrac{n-m}{n-1} - I^{-1} \tag{18}
$$

The first inequality holds because $\gamma^{-\bar{y}-T'} > 1$ and $\gamma^{-y-T'} > 1$ and the second inequality holds because $\gamma^{t+T'} < 1$ and $\frac{n-m}{n-1} - I^{-1} \leq 0$. But (18) contradicts (17), therefore if $\frac{n-m}{n-1} - I^{-1} \leq 0$, equation (17) is never satisfied and myopic players would choose B at any time t.

Alternatively, if $\frac{n-m}{n-1} - I^{-1} > 0$, condition (17) can be expressed the following way:

$$
\gamma^{-t} \geq \frac{\gamma^{-\bar{y}} \frac{n-m-1}{n-1} + \gamma^{-y} \frac{1}{n-1} - \gamma^{T'} I^{-1}}{\frac{n-m}{n-1} - I^{-1}} \tag{19}
$$

The left-hand side of (19) is strictly increasing in t and unbounded for any $\gamma \in (0,1)$, so (19) will be satisfied for some t, although not necessarily with $t \leq T$. Equation (19) is not satisfied for $t = 0$ because the RHS of (19) is always strictly larger than 1 (RHS is increasing in both y and $\bar{y}$, but RHS > 1 even if $y = \bar{y} = 0$ because $\frac{n-m}{n-1} - \gamma^{T'} I^{-1} > \frac{n-m}{n-1} - I^{-1}$) and $\gamma^{-t} < 1$. Consequently, (19) must be satisfied with equality at a unique value of t, which we denote by $\hat{t}_1(y, \bar{y})$, with $\hat{t}_1(y, \bar{y}) \in (0, \infty)$. This value is the first moment in time at which myopic players are indifferent between choosing A and B, thus it is exactly the switching period which we were looking for. To get an expression for $\hat{t}_1(y, \bar{y})$, require (19) to be satisfied with equality and rearrange the following way:

$$\hat{t}_1(y, \bar{y}) = \frac{\log(\frac{n-m}{n-1} - I^{-1}) - \log(\gamma^{-\bar{y}}\frac{n-m-1}{n-1} + \gamma^{-y}\frac{1}{n-1} - \gamma^{T'} I^{-1})}{\log(\gamma)} \tag{20}$$

Of course, $\hat{t}(y, \bar{y})$ can be calculated using (20) only if $\frac{n-m}{n-1} - I^{-1} > 0$, otherwise myopic players would always play B. The precise characterization of the switching period if case 1 is applicable is as follows:

$$\hat{t}(y, \bar{y}) = \begin{cases} \hat{t}_1(y, \bar{y}) & \text{if } \frac{n-m}{n-1} - I^{-1} > 0 \\ \infty & \text{otherwise} \end{cases} \tag{21}$$

Note that it is not required that $\hat{t}_1(y, \bar{y}) \leq T$, therefore it is possible that the planning horizon of a sophisticated player is too short to take re-coordination into account.

Case 2: $y < \hat{t} < \bar{y}$

Case 3: $\bar{y} < \hat{t} < y$

Proofs for Case 2 and Case 3 are in Appendix A.1.

■

Lemma 3. $\frac{\partial \hat{t}_2(y)}{\partial y} > 1$.

Proof: see Appendix A.2.

Lemma 3 implies that if $\hat{t}_2(0) < T$, it would be optimal for all sophisticated players to choose $y = 0$: increasing y by an amount of ε would increase the payoffs by εL, because of a longer delay, but would simultaneously decrease the payoffs by more than εH because of the longer switching period of myopic players.

3.5 Payoffs of Sophisticated Players

Proposition 3 shows how $\hat{t}(y, \bar{y})$, the switching period of myopic players, depends on sophisticated player strategies, if one player is using strategy y and all other players are using strategies $\bar{y}$. Proposition 4 will show how this specification can be used to calculate the sum of payoffs received by sophisticated players over the period that is taken into consideration.

Proposition 4. *If a sophisticated player s uses strategy $y_s = y$ and other sophisticated players use strategies $y_{-s} = \bar{y}$, total payoff received by player s over period $[0, T]$ is $\Pi(y, \bar{y})$ such that:*

$$
\Pi(y,\bar{y}) = \begin{cases} \Pi_1 = yL + (T - \hat{t}_1(y,\bar{y}))H & \textit{if } \hat{t}_1(y,\bar{y}) \leq T,\ \hat{t}_2(y) \geq \bar{y},\ \hat{t}_3(\bar{y}) \geq y & \text{(22a)} \\ \Pi_2 = yL + (T - \hat{t}_2(y))H & \textit{if } \hat{t}_2(y) < \bar{y} & \text{(22b)} \\ \Pi_3 = yL + (T - y)H & \textit{if } \hat{t}_3(\bar{y}) < y & \text{(22c)} \\ \Pi_4 = yL & \textit{if } \hat{t}_1(y,\bar{y}) > T & \text{(22d)} \end{cases}
$$

where $\hat{t}_1(y,\bar{y})$, $\hat{t}_2(y)$ *and* $\hat{t}_3(\bar{y})$ *are specified in Proposition 3.*

Proof.

The payoff function depends on the switching period of myopic players, which is determined by one of the four equations in condition (13). Each possibility is shown in figure 4. Consider panel (a), which illustrates a situation where all sophisticated players switch to A first[7], and myopic players follow later, therefore their switching time is calculated as $\hat{t}_1(y,\bar{y})$. The participation threshold is not exceeded at any time prior to $\hat{t}_1(y,\bar{y})$ and is exceeded afterwards, therefore the payoff flow of a sophisticated player is L prior to time y, 0 between time y and $\hat{t}_1(y,\bar{y})$ and H afterwards. The sum of payoffs in this case would be equal to $\Pi_1(y,\bar{y}) = yL + (T - \hat{t}_1(y,\bar{y}))H$. Panel (a), however, applies only if myopic players switch after all sophisticated ones, that is if $\hat{t}_2(y) \geq \bar{y}$ and $\hat{t}_3(\bar{y}) \geq y$, and if switching occurs prior to time T.

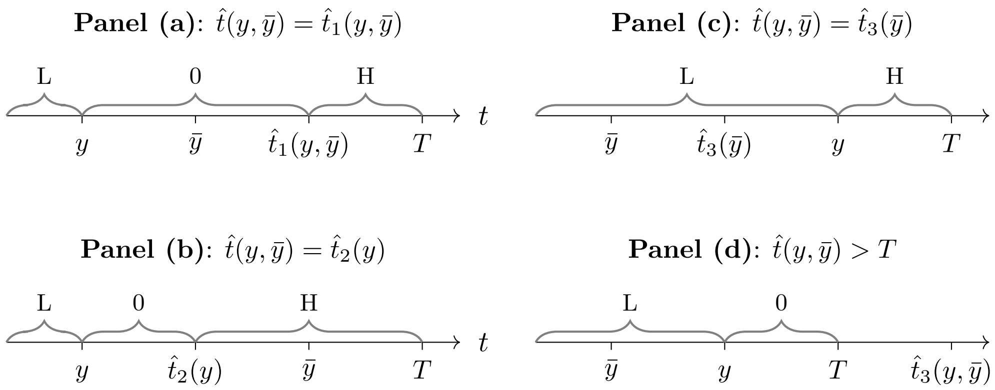

Figure 4: Stage game payoffs for every possible case. Panel numbering corresponds to equations in (22).

Another possibility is that myopic players switch after observing only one sophisticated player switching to A, a case illustrated in panel (b). Then the sophisticated player will receive a payoff flow equal to L at any time prior to y, a flow of 0 between time y and $\hat{t}_2(y)$ and a flow of H between $\hat{t}_2(y)$ and T. The sum of payoffs in this case would be equal to $\Pi_2(y,\bar{y}) = yL + (T - \hat{t}_2(y))H$. Panel (b) applies only if $\hat{t}_2(y) < \bar{y}$.

In a similar way, $(n-m-1)$ sophisticated players may switch first, followed by myopic players

[7] Panel (a) illustrates the situation with $y < \bar{y}$, but the payoff calculation for $y \geq \bar{y}$ would be equivalent.

and then by a single sophisticated player, illustrated in panel (c). Sophisticated player would receive L until time y, and would receive H afterwards. The sum of payoffs would therefore be equal to $\Pi_3(y, \bar{y}) = yL + (T - y)H$. Panel (c) applies only if $\hat{t}_3(\bar{y}) < y$.

Finally, myopic players may never switch to A, as illustrated in panel (d). In this case the sophisticated player would receive L until time y, and 0 afterwards, thus the total payoff would be $\Pi_4(y, \bar{y}) = yL$.

■

3.6 Characterisation of Symmetric Sophisticated Player Equilibria

Payoffs for each strategy of player s and the strategies of other sophisticated players are specified in (22). This specification transforms a repeated game into a static game played by sophisticated players, who are able to perfectly anticipate the choice path of myopic players. To make theoretical predictions, we can use the standard solution concept for static games – a Nash equilibrium – which requires mutual best responses for each player.

Proposition 2 shows that undominated action plans for sophisticated players can be identified by a strategy that identifies a switching time. We will therefore use the definition from (5) to call a combination of *strategies* (y^*, y^*) a symmetric sophisticated player equilibrium if it satisfies:

$$\Pi(y^*, \bar{y}^*) \geq \Pi(y, \bar{y}^*), \quad \forall y \in [0, T] \tag{23}$$
$$\text{and } y^* = \bar{y}^*$$

We will look at the existence of three types of equilibria: interior solutions with $y = \bar{y} \in (0, T)$, a corner solution with $y = \bar{y} = 0$ and a corner solution with $y = \bar{y} = T$. For each type we will determine the conditions under which an equilibrium exists, and the speed of transition to an efficient state.

3.6.1 Interior Sophisticated Player Equilibria

In this section we will derive the existence conditions for an interior equilibrium and show how the speed of transition to the efficient equilibrium depends on the game parameters.

Proposition 5. *A combination of strategies (y^*, y^*) with $y^* \in (0, T)$ is a sophisticated player equilibrium ("interior equilibrium") if and only if conditions* ***I1, I2, I3*** *and* ***I4*** *are satisfied:*

$$\hat{t}_1(y^*, y^*) < T, \qquad \textit{(I1)}$$

$$\frac{\log(\frac{n-m-H/L}{I^{-1}(n-1)})}{\log(\gamma)} - T' > 0, \qquad \textit{(I2)}$$

$$\hat{t}_1(y^*, y^*) - y^*L/H \leq \hat{t}_2(0), \qquad \textit{(I3)}$$

$$\hat{t}_1(y^*, y^*) - y^*L/H \leq T(1 - L/H), \qquad \textit{(I4)}$$

where equilibrium strategies are calculated by

$$y^* = \frac{\log(\frac{n-m-H/L}{I^{-1}(n-1)})}{\log(\gamma)} - T'$$

Proof.

The structure of the proof is shown in figure 5. First we need to specify the equilibrium payoffs. If condition **I1** holds, condition (22a) will hold as well, from Lemma 10, therefore $\Pi(y^*, y^*) = \Pi_1(y^*, y^*)$. If **I1** does not hold, $\Pi(y^*, y^*) = \Pi_4(y^*, y^*) = y^*L$, and an interior equilibrium will not exist because there is a profitable deviation to a strategy $y = T$ that provides a payoff of TL. Condition **I1** is therefore the first necessary condition for the existence of an interior equilibrium, and we will show that it is also jointly sufficient, together with conditions **I2**, **I3** and **I4**. These proofs are given in additional lemmas. Lemma 4 shows that equilibrium payoffs exceed deviation payoffs if and only if equilibrium payoffs exceed the payoffs of two endpoints, 0 and T, and the payoffs of 'neighboring' strategies, calculated by $\Pi_1(y, y^*)$. Lemma 5, 6 and 7 derive the conditions under which there are no profitable deviations for each case.

$$\begin{array}{l}
\Pi(y^*, y^*) \geq \Pi(y, y^*), \forall y \in [0, T] \xLeftrightarrow{\text{if not } \mathbf{I1}} \Pi_4(y^*, y^*) \geq \Pi_4(y, y^*), \forall y \in [0, T] \\
\qquad\qquad \Updownarrow \text{ if } \mathbf{I1} \\
\Pi_1(y^*, y^*) \geq \Pi(y, y^*), \forall y \in [0, T] \\
\qquad\qquad \Updownarrow \text{ Lemma 4} \\
\begin{cases}
\Pi_1(y^*, y^*) \geq \Pi_1(y, y^*), \forall y \in (y'', y') & \xLeftrightarrow{\text{Lemma 5}} \mathbf{I2} \\
\Pi_1(y^*, y^*) \geq \Pi_2(0, y^*) & \xLeftrightarrow{\text{Lemma 6}} \mathbf{I3} \\
\Pi_1(y^*, y^*) \geq \Pi_4(T, y^*) & \xLeftrightarrow{\text{Lemma 7}} \mathbf{I4}
\end{cases}
\end{array}$$

Figure 5: Structure of the proof for Proposition 5.

Lemma 4.

$$\Pi_1(y^*, y^*) \geq \Pi(y, y^*), \forall y \in [0, T] \quad \Leftrightarrow \quad \begin{cases} \Pi_1(y^*, y^*) \geq \Pi_2(0, y^*) \\ \Pi_1(y^*, y^*) \geq \Pi_1(y, y^*), \forall y \in [y'', y'] \\ \Pi_1(y^*, y^*) \geq \Pi_4(T, y^*) \end{cases}$$

If $\hat{t}_3(y^*) \leq T$*,* $y' = \hat{t}_3(y^*)$*, otherwise* y' *solves* $\hat{t}_1(y', y^*) = T$*. If* $\hat{t}_2(0) > y^*$*,* $y'' = 0$*, otherwise* y'' *solves* $\hat{t}_2(y'') = y^*$*.*

Proof: see Appendix A.2.

Lemma 5. $\Pi_1(y^*, y^*) \geq \Pi_1(y, y^*), \quad \forall y \in (y'', y')$*, if and only if condition* ***I2*** *is satisfied:*

$$\frac{\log(\frac{n-m-H/L}{I^{-1}(n-1)})}{\log(\gamma)} - T' > 0, \qquad (\textbf{\textit{I2}})$$

Proof: see Appendix A.2.

Lemma 5 specifies conditions under which there are no profitable deviations to strategies in the interval $[y'', y']$. In addition, equilibrium payoffs must be higher than the payoffs from choosing $y = 0$ and $y = T$. Conditions under which there are no incentives to deviate to such strategies are specified in Lemma 6 and Lemma 7.

Lemma 6. $\Pi_1(y^*, y^*) \geq \Pi_2(0, y^*)$ *if and only if condition* $I3$ *is satisfied:*

$$\hat{t}_1(y^*, y^*) - y^* L/H \ \leq \hat{t}_2(0), \qquad (\textbf{\textit{I3}})$$

Proof: see Appendix A.2.

Lemma 7. $\Pi_1(y^*, y^*) \geq \Pi_4(T, y^*)$ *if and only if condition* ***I4*** *is satisfied:*

$$\hat{t}_1(y^*, y^*) - y^* L/H \leq T(1 - L/H), \qquad (\textbf{\textit{I4}})$$

Proof: see Appendix A.2.

Taken together, Lemmas 5, 6 and 7 prove Proposition 5. Conditions **I1**, **I2**, **I3** and **I4** are jointly sufficient because if all of them are satisfied there are no incentives to deviate to any strategy in $[0, T]$. If one of these conditions is violated, there will be a strategy in some region that exceeds the equilibrium payoff.

■

3.6.2 Corner Solution $y^* = 0$

In a second type of a symmetric sophisticated player equilibrium all sophisticated players switch to A at the start of the game, so that equilibrium strategies are $y^* = y^* = 0$.

Proposition 6. *A combination of strategies* $(0,0)$ *is a sophisticated player equilibrium ("teaching equilibrium") if and only if conditions* ***T1*** *and* ***T2*** *are satisfied:*

$$\begin{aligned} \frac{n-m-H/L}{n-1} &\leq \gamma^{T'} I^{-1}, \quad \textbf{\textit{(T1)}} \\ \hat{t}_1(0,0) &\leq T(1-L/H), \quad \textbf{\textit{(T2)}} \end{aligned}$$

Proof.

$$\begin{array}{lcl} \Pi(0,0) \geq \Pi(y,0), \forall y \in [0,T] & \overset{\text{if } \hat{t}_1(0,0)>T}{\Longleftrightarrow} & \Pi_4(0,y^*) \geq \Pi(y,y^*), \forall y \in [0,T] \\ \quad \Updownarrow \text{ if } \hat{t}_1(0,0)\leq T & & \\ \Pi_1(0,0) \geq \Pi(y,0), \forall y \in [0,T] & & \\ \quad \Updownarrow & & \\ \left\{ \begin{array}{l} \Pi_1(0,0) \geq \Pi_1(y,0), \forall y \in [0,y') \\ \Pi_1(0,0) \geq \Pi_4(T,0) \end{array} \right. & \begin{array}{c} \xLeftrightarrow{\text{Lemma 8}} \\ \xLeftrightarrow{\text{Lemma 9}} \end{array} & \begin{array}{l} \textbf{T1} \\ \textbf{T2} \end{array} \end{array}$$

Figure 6: Structure of the proof for Proposition 6.

Structure of the proof is shown in figure 6 and is similar to the proof of the interior equilibrium. The teaching equilibrium exists if (23) is satisfied for $y^* = 0$:

$$\Pi(0,0) \geq \Pi(y,0), \quad \forall y \in [0,T]$$

If $\hat{t}_1(0,0) > T$, condition (22d) is satisfied and equilibrium payoffs are determined by $\Pi(0,0) = \Pi_4(0,0) = 0$, while deviation payoffs are determined by $\Pi(y,0) = yL$. Then a teaching equilibrium would not exist because there is a profitable deviation to strategy $y = T$ that provides a payoff of TL. If $\hat{t}_1(0,0) \leq T$, equilibrium payoffs are calculated by $\Pi_1(0,0)$. Condition $\hat{t}_1(0,0) \leq T$ is therefore necessary for the existence of an interior equilibrium. We do not list this condition separately because it is implied by **T2**.

Deviation payoffs are determined in a similar way to the deviation payoffs for an interior equilibrium. Payoffs for a small $y \in [0,t']$ are calculated by $\Pi_1(y,0)$, where y' solves $\hat{t}_1(y',0) = T$. If the deviation is larger, that is $y \in [t',T]$, myopic player would never switch to A and deviation profits would be calculated by $\Pi_4(y,0) = yL$. All strategies in this interval would be dominated by strategy $y = T$ that provides a payoff of TL. Overall, there are two requirements that need to be satisfied for a teaching equilibrium to exist. First, equilibrium payoffs should be higher than the payoffs from any other $y \in [0,y')$, calculated by $\Pi_1(y,0)$. We will derive the conditions under which this requirement is satisfied in Lemma 8. Second, equilibrium payoffs should be higher than the payoff of strategy $y = T$; we will derive the conditions for this requirement in Lemma 9.

Lemma 8. $\Pi_1(0,0) \geq \Pi_1(y,0), \quad \forall y \in [0, y')$ *if and only if condition **T1** is satisfied:*

$$\frac{n - m - H/L}{n-1} \leq \gamma^{T'} I^{-1} \quad \textbf{\textit{(T1)}}$$

where y' solves $\hat{t}_1(y', 0) = T$.

Proof: see Appendix A.2.

Lemma 9. $\Pi_1(0,0) \geq \Pi_4(T,0)$ *if and only if condition **T2** is satisfied:*

$$\hat{t}_1(0,0) \leq T(1 - L/H) \quad \textbf{\textit{(T2)}}$$

Proof.

Deviation payoffs are calculated from (22d): $\Pi_4(T,0) = TL$. There are no incentives to deviate if

$$\Pi_1(0,0) \geq \Pi_4(T,0) \quad \Leftrightarrow \quad \hat{t}_1(0,0) \leq T(1 - L/H)$$

■

If both **T1** and **T2** hold, equilibrium payoffs are calculated by $\Pi_1(0,0)$ and there are no incentives to deviate neither to neighbouring strategies nor to strategy $y = T$. If one of these conditions is violated, there would be profitable deviation and a teaching equilibrium would not exist.

■

3.6.3 Corner Solution $y^* = T$

In the third type of a symmetric sophisticated player equilibrium all sophisticated players choose B for the entire duration of the game, that is $y^* = \bar{y}^* = T$.

Proposition 7. *A combination of strategies (T,T) is a sophisticated player equilibrium ("delay equilibrium") if and only if condition **D1** is satisfied:*

$$\hat{t}_2(0) \geq T(1 - L/H) \qquad \textbf{\textit{(D1)}}$$

$$
\begin{array}{lcl}
\Pi(T,T) \geq \Pi(y,T), \forall y \in [0,T] & & \\
\Updownarrow & & \\
\Pi_4(T,T) \geq \Pi(y,T), \forall y \in [0,T] & & \\
\Updownarrow & & \\
\left\{ \begin{array}{l} \Pi_4(T,T) \geq \Pi_2(0,T) \\ \Pi_4(T,T) \geq \Pi_4(y,T), \forall y \in [y',T] \end{array} \right. & \begin{array}{c} \Longleftrightarrow \\ \Longleftrightarrow \end{array} & \begin{array}{l} \textbf{D1} \\ \text{Always satisfied} \end{array}
\end{array}
$$

Figure 7: Structure of the proof for Proposition 7.

Proof.

If there is a symmetric equilibrium with $y^* = T$, it must hold that:

$$\Pi(T,T) \geq \Pi(y,T), \quad \forall y \in [0,T]$$

The structure of the proof is shown in figure 7. Condition (22d) is satisfied, therefore equilibrium payoffs are $\Pi(T,T) = \Pi_4(T,T) = TL$. Deviation payoffs $\Pi(y,T)$ are calculated either as $\Pi_4(y,T)$ if $y \in [y',T]$ or as $\Pi_2(y,T)$ if $y \in [0,y')$, where y' solves $\hat{t}_2(y') = T$. In the former case $\Pi_4(y,T) = yL$, which is less that the payoff of TL provided by strategy $y = T$, therefore the delay equilibrium would exist. In the latter case deviation payoffs are equal to:

$$\Pi(y,T) = \Pi_2(y,T) = yL + (T - \hat{t}_2(y))H$$

Lemma 3 implies that $\text{argmax}_y(\Pi_2(y,T)) = 0$, that is the most profitable deviation is to strategy $y = 0$. There will be no incentives to deviate to this strategy if the following holds:

$$
\begin{aligned}
\Pi_4(T,T) \geq \Pi_2(y,T) &\quad \Leftrightarrow \\
TL \geq (T - \hat{t}_2(0))H &\quad \Leftrightarrow \\
\hat{t}_2(0) \geq T(1 - L/H) &
\end{aligned}
$$

If this condition is satisfied, there will be no incentives to deviate to $y = 0$ and there would be no other profitable deviations, therefore a delay equilibrium would exist. If this condition is not satisfied, payoffs could be increased by choosing strategy $y = 0$.

■

Table 1: Summary of the types of symmetric Nash equilibria that may exist, speed of transition to the efficient state and the conditions that need to be satisfied for the particular type of equilibrium to exist.

Equilibrium	**Teaching**	**Interior**	**Delay**
Equilibrium strategy	$y^* = 0$	$y^* = \frac{\log(\frac{n-m-H/L}{I^{-1}(n-1)})}{\log(\gamma)} - T'$	$y^* = T$
Speed of transition	$\hat{t}(y^*, y^*) = \hat{t}_1(0,0)$	$\hat{t}(y^*, y^*) = \hat{t}_1(y^*, y^*)$	$\hat{t}(y^*, y^*) > T$
Equilibrium payoffs	$\Pi(0,0) = (T - \hat{t}_1(0,0))H$	$\Pi(y^*, y^*) = y^*L + (T - \hat{t}_1(y^*, y^*))H$	$\Pi(T,T) = TL$
		Existence conditions	
No deviation to neighbouring strategies	**T1:** $\frac{n-m-H/L}{n-1} \leq \gamma^{T'} I^{-1}$	**I2:** $y^* > 0$	–
No deviation to $y = 0$	–	**I3:** $\hat{t}_1(y^*, y^*) - y^*L/H \leq \hat{t}_2(0)$	**D1:** $T(1 - L/H) \leq \hat{t}_2(0)$
No deviation to $y = T$	**T2:** $\hat{t}_1(0,0) \leq T(1 - L/H)$	**I1:** $\hat{t}_1(y^*, y^*) < T$ **I4:** $\hat{t}_1(y^*, y^*) - y^*L/H \leq T(1 - L/H)$	–

4 Summary and Comparative Statics

Overall, three types of symmetric Nash equilibria can exist: in a "teaching" equilibrium all sophisticated players play A for the entire duration of the game and myopic players switch to A at some time $\hat{t}_1(0,0)$; in a "delay" equilibrium all sophisticated players choose B for the entire duration of the game, and myopic players never switch to A; in an interior equilibrium sophisticated players start by playing B and switch to A at time y^* while myopic players switch to A at time $\hat{t}_1(y^*,y^*)$. Table 1 summarizes all the conditions that need to be satisfied for each type of equilibrium to exist. Depending on the combination of parameters, it is possible that multiple equilibria will exist at the same time or that no symmetric equilibrium will exist.

We would like to make theoretical predictions about how the path of play depends on the game parameters, but precise predictions cannot be made due to the multiplicity of equilibria. Therefore we separately investigate how the factors of interest affect the existence conditions of each type of equilibria and the speed of transition to an efficient state. The factors that we consider are the length of planning horizon of sophisticated players (T), the number of myopic players (m) and the strength of initial lock-in (T').

4.1 Planning Horizon of the Sophisticated Players

The first parameter of interest is T, the length of the planning horizon for sophisticated players.

Proposition 8. *If sophisticated players have a longer planning horizon, then:*

1. *The speed of transition in any equilibrium is not affected.*
2. *Teaching equilibrium exists for a larger set of values of other parameters.*
3. *Interior equilibrium exists for a larger set of values of other parameters.*
4. *Delay equilibrium exists for a smaller set of values of other parameters.*

Proof.

Part 1 follows from the definition of the switching period, which depends only on myopic players who do not take future payoffs into account. For part 2, note that only condition **T2** depends on the planing horizon, and **T2** is satisfied for a larger set of parameters when T is higher. For part 3, note that conditions **I2** and **I4** depend on the length of the planning horizon, and both are satisfied for a larger set of parameters when T is larger. Part 4 holds because condition **D1** is satisfied for a smaller set of parameters when T is larger.

■

4.2 Player Composition

The second variable of interest is m, the number of myopic players, which reflects a different aspect of sophistication than the length of the planning horizon. Instead of making sophisticated players more sophisticated, we look at the effect of replacing some myopic players with sophisticated ones, while keeping the total number of players constant.

Proposition 9. *If there are more sophisticated players, then:*

1. *Transition is faster in the interior and in the teaching equilibria.*
2. *The effect on the existence of a teaching equilibrium or an interior equilibrium is ambiguous:*
   (a) *there are more incentives to deviate to neighboring action plans*
   (b) *there are less incentives to never choose A.*
3. *There is no change in the existence of the delay equilibrium.*

Proof.

This proof as well as other proofs on comparative statics rely on additional lemmas presented in Appendix A.3. For part 1, see Lemmas 11 and 12. To see part 2 for the teaching equilibrium, note that both condition **T1** and condition **T3** depend on player composition. A smaller number of myopic players leads to **T1** being satisfied for a smaller set of values of other parameters. On the other hand, a smaller number of myopic players makes condition **T2** satisfied for a larger set of parameters because $\hat{t}_1(0,0)$ is increasing in m (see Lemma 11). For the interior equilibrium, all four conditions depend on the number of sophisticated players. Incentives to deviate to neighbouring strategies are determined by condition **I2**, which is satisfied for a smaller set of parameters when there are more sophisticated players. To see it, notice that $\frac{\partial y^*}{\partial m} > 0$ (Lemma 12), therefore as m decreases so does y^*, therefore **I2** is less likely to be satisfied. Incentives to deviate to corner solutions are determined by conditions **I1**, **I3** and **I4**, all of which are satisfied for a larger set of parameters when there are more sophisticated players. Conditions **I3** and **I4** are satisfied for a larger set of parameters because $\frac{\partial \hat{t}_1(y^*,y^*)}{\partial m} > \frac{\partial y^*}{\partial m} > \frac{\partial y^*}{\partial m} L/H$ (see Lemma 14). Condition **I1** is also satisfied for a larger set of parameters because $\frac{\partial \hat{t}_1(y^*,y^*)}{\partial m} > 0$, from Lemma 12. Part 3 holds because outcomes in the delay equilibrium are not affected by the number of myopic players.

■

The finding that an increase in the number of sophisticated players can reduce the incentives to use strategic teaching may sound counterintuitive, but it is a result of decreased delay costs as the number of teaching players grows. When the number of sophisticated players is large and all of them are choosing A in the teaching equilibrium, the decision of a single sophisticated player to delay teaching has only a small negative effect on the transition period, making free-riding an attractive alternative that could lead to a break-down of a teaching equilibrium. But if a teaching equilibrium does exist, a larger number of sophisticated players would make transition faster.

4.3 Length of the History of Inefficient Coordination

The third factor that we look at is the strength of the initial lock-in to an inefficient state, measured by the length of history of inefficient coordination, T'.

Proposition 10. *If the history of inefficient coordination is longer, then:*

1. *Transition is slower in the teaching equilibrium but faster in an interior equilibrium.*
2. *Teaching equilibrium exists for a smaller set of parameter values*
3. *The effect on the existence of an interior equilibrium is ambiguous*
4. *Delay equilibrium exists for a smaller set of parameter values.*

Proof.

Part 1 holds because the derivative of $\hat{t}_1(0,0)$ with respect to T' is positive while the derivative of $\hat{t}_1(y^*,y^*)$ is negative, as shown in Lemma 11 and Lemma 12. For part 2, parameter T' affects conditions **T1** and **T2**. An increase in T' leads to **T1** being satisfied for a smaller set of parameter values, because $\gamma^{T'}$ goes down. Condition **T2** is also less likely to be satisfied because of an increase in $\hat{t}_1(0,0)$. For part 3, notice that an increase in T' satisfies conditions **I1**, **I3** and **I4** for a larger set of parameter values, but satisfies condition **I2** for a smaller set of parameter values. Condition **I1** is satisfied for a larger set of parameter values because $\frac{\partial \hat{t}_1(y^*,y^*)}{\partial T'} < 0$. Conditions **I3** and **I4** are also satisfied for a larger set of parameter values because $\frac{\partial \hat{t}_1(y^*,y^*)}{\partial T'} = \frac{\partial y^*}{\partial T'} < \frac{\partial y^*}{\partial T'} L/H$ and $\frac{\hat{t}_2(0)}{\partial T'} > 0$, from Lemma 11, 12 and 13. Condition **I2** is satisfied for a smaller set of parameters because $\frac{\partial y^*}{\partial T'} < 0$, from Lemma 14.

■

Part 1 of Proposition 10 states that the history of inefficient coordination affects the transition speed in opposite ways in the teaching and in the interior equilibrium. The opposite sign of this effect is a result of changes of the equilibrium strategy in the interior equilibrium. If the equilibrium strategy in an interior equilibrium was held constant, a longer history of inefficient coordination would lead to a slower transition. However, to offset an increased history of inefficient coordination, in an equilibrium sophisticated players have to start teaching earlier. Lemma 12 shows that the latter effect is even stronger than the former.

5 Conclusion

In this paper we present a model that combines the notion of strategic and farsighted players, favored by game-theoretic solution concepts, with a notion of adaptive players, favored by learning models. We assume two types of players: myopic players make choices based on observed history of play while sophisticated players have correct beliefs about the actions of all other players, plan ahead and choose actions that maximize the sum of payoff flows. To make predictions in this modified game we propose a new solution concept based on a Nash equilibrium between sophisticated players who take the learning process of the myopic players into account.

This solution concept is applied to a critical mass coordination game in which play has converged to an inefficient state. The construction of a sophisticated player equilibrium involves several steps. Proposition 1 shows that myopic players will choose the efficient action if their beliefs exceed a certain threshold. Furthermore, in the sophisticated player equilibrium myopic players will switch from an inefficient to the efficient action at most once. The single switch and the assumption that there are sufficiently many myopic players means that the efficient state is absorbing, therefore the switching time is the only information needed for sophisticated players to calculate their payoffs. Proposition 3 shows exactly how the switching time of myopic players can be calculated if beliefs were formed using weighted fictitious play. The switching time depends on the strategies taken by sophisticated players, which could prescribe many switches from one action to the other. The task of finding the switching time of the myopic players is therefore greatly simplified by Proposition 2, which shows that only the sophisticated player strategies prescribing at most one switch from the inefficient to the efficient action survive the elimination of strictly dominated strategies, allowing a strategy to be identified by the switching time.

The ability to anticipate the speed of a transition allows sophisticated players to calculate how their payoffs depend on their own strategies and on the strategies chosen by other sophisticated players. The mapping from strategies to payoffs specified in Proposition 4 is used to identify strategy profiles in which all sophisticated players are best responding to each other. Three types of symmetric equilibria are possible: sophisticated players may play the efficient action right away, they may switch to the efficient action later or they may never switch. In the first two cases myopic players eventually start playing the efficient action, while in the third case all players choose the inefficient action. Which types of equilibria exist and how long the transition to an efficient state takes depends on the game parameters, as specified in Propositions 5, 6 and 7. Finally, Propositions 8, 9 and 10 show how these existence conditions depend on the history of inefficient coordination, length of the planning horizon of sophisticated players and the player composition. As the planing horizon of sophisticated players increases, teaching and interior equilibria exist for a larger set of parameters, while the delay equilibrium exists for a smaller set of parameters. A larger number of sophisticated players leads to faster transition to the efficient state in an interior or in a delay equilibrium, but the effect on the existence conditions is ambiguous: there are more incentives to deviate to neighboring strategies, but less incentives to deviate to corner solutions. Finally, we show that a longer history of observed inefficient coordination leads to a slower transition in a teaching equilibrium and to a smaller set of parameters under which a teaching equilibrium exists, while the set of parameters under which the delay equilibrium exists is larger. On the other hand, the transition to an efficient state in an interior equilibrium is faster, because a longer history of inefficient coordination forces sophisticated players to start teaching earlier.

The problem that motivated this paper was the lack of a suitable theoretical model that could be used to make predictions in a game in which inefficient conventions have already been established. A small change in the assumptions – instead of assuming all players to be farsighted we assume that some players are learning from history – leads to large differences in theoretical predictions. Not only can the new model be used to model inefficient conventions through the beliefs of myopic players, but it also reduces the set of predictions to only three types of equilibria, in contrast to almost limitless predictions made by standard solution concepts.

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A Appendix

A.1 Proof of Case 2 and Case 3 of Proposition 3

Case 2: $y < \hat{t} < \bar{y}$

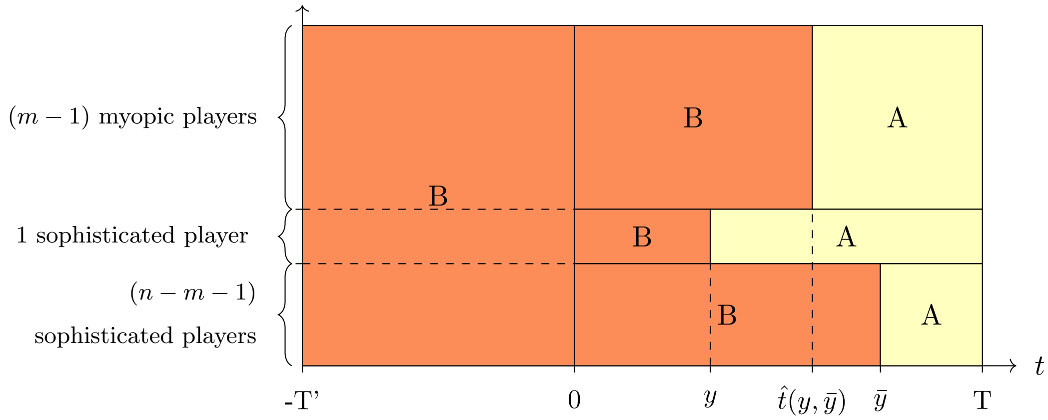

Figure 8: Illustration of the second case, where $y < \hat{t}(y, \bar{y}) \leq \bar{y}$. The height of the figure shows the fraction of players choosing action A or action B, the width shows the passage of time. The first sophisticated player switches from B to A in period y, other (n-m-1) sophisticated players switch in period $\bar{y}$ and the myopic players switch in period $\hat{t}$.

The second possibility is that $\hat{t}(y, \bar{y}) \leq \bar{y}$, that is myopic players switch to A earlier than $(n-m-1)$ sophisticated players. In this case the actual value of $\bar{y}$ will have no influence on the switching period of myopic players, as they will never observe any of the $(n-m-1)$ sophisticated players choosing A. Therefore the switching period will be a function only of the strategy chosen by a single sophisticated player. At time $t \in (y, \hat{t}]$ beliefs of a myopic player i are $x_i(t)$:

$$\begin{aligned} x_i(t) =& \frac{\int_{k=0}^{t-y} \gamma^k (\frac{1}{n-1}) \mathrm{d}k}{\int_{k=0}^{t+T'} \gamma^k \mathrm{d}k} = \\ =& \frac{(\gamma^{t-y} - 1)(\frac{1}{n-1})}{\gamma^{t+T'} - 1} \end{aligned}$$

Player i will choose A in t if:

$$\begin{aligned} x_i(t) \geq I^{-1} \quad &\Leftrightarrow \\ \gamma^{t+T'}(\gamma^{-y-T'} \frac{1}{n-1} - I^{-1}) \leq \frac{1}{n-1} - I^{-1} \end{aligned} \tag{24}$$

If $\frac{1}{n-1} - I^{-1} \leq 0$, equation (24) is never satisfied. To see this, notice the following relationship that contradicts (24):

$$\gamma^{t+T'}(\gamma^{-y-T'}\frac{1}{n-1}-I^{-1})>\gamma^{t+T'}(\frac{1}{n-1}-I^{-1})\geq\frac{1}{n-1}-I^{-1}$$

The latter equation holds because $\gamma^{-y-T'}>1$, $\gamma^{t+T'}<1$ and $\frac{1}{n-1}-I^{-1}\leq 0$.

Alternatively, if $\frac{1}{n-1}-I^{-1}>0$, (24) will be satisfied with equality at time $\hat{t}_2(y)\in(0,\infty)$ that satisfies:

$$\gamma^{-\hat{t}_2(y)}=\frac{\gamma^{-y}\frac{1}{n-1}-\gamma^{T'}I^{-1}}{\frac{1}{n-1}-I^{-1}}\quad\Leftrightarrow$$

$$\hat{t}_2(y)=\frac{\log(\frac{1}{n-1}-I^{-1})-\log(\gamma^{-y}\frac{1}{n-1}-\gamma^{T'}I^{-1})}{\log(\gamma)}\tag{25}$$

$\hat{t}(y,\bar{y})$ can be calculated using (25) only if $\frac{1}{n-1}-I^{-1}>0$, otherwise myopic players would never switch from A to B. The switching period if case 2 applies can be expressed as follows:

$$\hat{t}(y,\bar{y})=\begin{cases}\hat{t}_2(y) & \text{if } \frac{1}{n-1}-I^{-1}>0\\ \infty & \text{otherwise}\end{cases}\tag{26}$$

Case 3: $\bar{y}<\hat{t}<y$

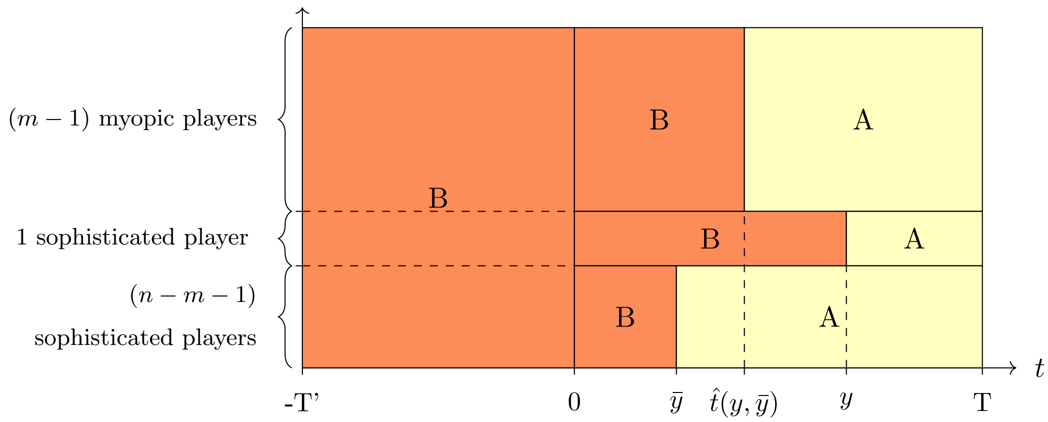

Figure 9: Illustration of the third case, where $\bar{y}<\hat{t}(y,\bar{y})\leq y$. Height of the figure shows a fraction of players choosing action A or action B, the width shows the passage of time. The first sophisticated player switches from B to A in period y, other $(n-m-1)$ sophisticated players switch in period $\bar{y}$ and myopic players switch at time $\hat{t}(y,\bar{y})$.

The third possibility is that $\bar{y}<\hat{t}(y,\bar{y})\leq y$, that is at first $(n-m-1)$ sophisticated players switch to A, then m myopic players switch and the last sophisticated player may switch some time after the myopic ones. In this case the switching time is a function only of $\bar{y}$. At time

$t \in (\bar{y}, \hat{t}(y, \bar{y})]$ beliefs of a myopic player i are $x_i(t)$:

$$\begin{aligned} x_i(t) =& \frac{\int_{k=0}^{t-\bar{y}} \gamma^k (\frac{n-m-1}{n-1}) \mathrm{d}k}{\int_{k=0}^{t+T'} \gamma^k \mathrm{d}k} = \\ =& \frac{(\gamma^{t-\bar{y}} - 1)(\frac{n-m-1}{n-1})}{\gamma^{t+T'} - 1} \end{aligned}$$

Player i will choose A in t if:

$$x_i(t) \geq I^{-1} \Leftrightarrow$$

$$\gamma^{t+T'} (\gamma^{-\bar{y}-T'} \frac{n-m-1}{n-1} - I^{-1}) \leq \frac{n-m-1}{n-1} - I^{-1} \tag{27}$$

If $\frac{n-m-1}{n-1} - I^{-1} \leq 0$, condition (27) is never satisfied. To see this, notice the following relationship that contradicts (27):

$$\gamma^{t+T'} (\gamma^{-\bar{y}-T'} \frac{n-m-1}{n-1} - I^{-1}) > \gamma^{t+T'} (\frac{n-m-1}{n-1} - I^{-1}) \geq \frac{n-m-1}{n-1} - I^{-1}$$

The latter conditions holds because $\gamma^{-\bar{y}-T'} > 1$, $\gamma^{t+T'} < 1$ and $\frac{n-m-1}{n-1} - I^{-1} \leq 0$. Therefore if $\frac{n-m-1}{n-1} - I^{-1} \leq 0$, equation (27) is never satisfied and myopic players would choose B at any time t.

Alternatively, if $\frac{n-m-1}{n-1} - I^{-1} > 0$, (27) will be satisfied with equality at time $\hat{t}_3(y) \in (0, \infty)$ that satisfies:

$$\gamma^{-\hat{t}_3(\bar{y})} = \frac{\gamma^{-\bar{y}} \frac{n-m-1}{n-1} - \gamma^{T'} I^{-1}}{\frac{n-m-1}{n-1} - I^{-1}} \Leftrightarrow \tag{28}$$

$$\hat{t}_3(\bar{y}) = \frac{\log(\frac{n-m-1}{n-1} - I^{-1}) - \log(\gamma^{-\bar{y}} \frac{n-m-1}{n-1} - \gamma^{T'} I^{-1})}{\log(\gamma)} \tag{29}$$

$\hat{t}(y, \bar{y})$ can be calculated using (29) only if $\frac{n-m-1}{n-1} - I^{-1} > 0$. Therefore, the switching period if case 3 applies can be expressed as follows:

$$\hat{t}(y, \bar{y}) = \begin{cases} \hat{t}_3(\bar{y}) & \text{if } \frac{n-m-1}{n-1} - I^{-1} > 0 \\ \infty & \text{otherwise} \end{cases} \tag{30}$$

A.2 Proof of Lemmas

Lemma 1: *If two action plans of the sophisticated player prescribe the same action, the payoff flow is higher for the action plan with which myopic player beliefs are higher:*

$$\pi[a'_s(t), a_{-s}(t) \times a_i(t, a'_s \times a_{-s})] \geq \pi[a_s(t), a_{-s}(t) \times a_i(t, a_s \times a_{-s})]$$
$$\text{if} \quad x(t)' \geq x(t) \quad \text{and} \quad a'_s(t) = a_s(t)$$

where $x(t)'$ is the belief held by myopic players if sophisticated player uses action plan a'_s and $x(t)$ is the belief if sophisticated player uses action plan a_s.

Proof:

Consider two action plans a_s and a'_s that prescribe the same action at time t, but prescribe different actions prior to time t so that myopic players would hold higher beliefs following the history generated by a'_s.

From equation (7), $a_i(t, a_s \times a_{-s})$ is weakly increasing in beliefs $x_i(t)$, therefore:

$$a_i(t, a'_s \times a_{-s}) \geq a_i(t, a_s \times a_{-s})$$

Since we hold the action plans of other strategic players constant, a higher tendency to choose A by myopic players increases the total number of other players who choose A at time t. Because H > 0 and M ≥ 0, equation (6) implies that payoffs are weakly increasing in the number of other players choosing A, therefore the payoff generated by a'_s must be at least as high as the payoff generated by a_s:

$$\pi[a'_s(t), a_{-s}(t) \times a_i(t, a'_s \times a_{-s})] \geq \pi[a_s(t), a_{-s}(t) \times a_i(t, a_s \times a_{-s})]$$

■

Lemma 2: *All action plan profiles for sophisticated players with which myopic players switch from A to B are strictly dominated:*

$$AB_M \cap U_s = \emptyset$$

Proof.

Suppose that $\times_{s \in S} a_s \in AB_M$, then there are two points in time t_1 and t_2 with $t_1 < t_2$ such that myopic players choose A at time t_1 and B at time t_2 . Find the first switching period $t_s \in (t_1, t_2]$ such that A is chosen in the interval $[t_1, t_s)$, but B is chosen at time t_s. Since all myopic players share the same history, the value of t_s will be the same for each myopic player so no myopic player will choose A in the interval $[t_1, t_s)$. If a myopic player observed all other sophisticated players choosing A in the interval $[t_1, t_s)$, the fictitious play rule would imply that $x_i(t_s) \geq x_i(t_1)$ therefore if A was optimal at time t_1 it will also be optimal at time

t_s, contradicting the definition of t_s. Therefore if a myopic player chooses B at time t_s, at least one sophisticated player must be choosing B in the interval $[t_1, t_s)$, that is $a_s(t) = 0$ for some $s \in S$ and $t \in [t_1, t_s)$. Denote the action plan of this sophisticated player by $\widetilde{a}_s$. We will show that $\widetilde{a}_s$ is dominated by an action plan a'_s that prescribes A in the entire interval $[t_1, t_s)$ and is otherwise the same as $\widetilde{a}_s$. First, the sum of payoff flows generated by a'_s in the interval $[t_1, t_s)$ is strictly higher than that generated by $\widetilde{a}_s$ because all myopic players are choosing A in this interval, and therefore Assumption 2 implies that the threshold will be exceeded. Second, payoffs generated in the interval $(t_s, T]$ will be equal or higher than those of $\widetilde{a}_s$ because myopic players will hold higher beliefs if a'_s is chosen (due to more A choices being observed) and consequently Lemma 1 implies that higher beliefs will lead to weakly higher payoffs for the sophisticated player at any time $t > t_s$.

■

Lemma 3: $\frac{\partial \hat{t}_2(y)}{\partial y} > 1$.

Proof.

Use the definition of $\hat{t}_2(y)$ from equation (25):

$$\hat{t}_2(y) = \frac{\log(\frac{1}{n-1} - I^{-1}) - \log(\gamma^{-y}\frac{1}{n-1} - \gamma^{T'} I^{-1})}{\log(\gamma)}$$

The partial derivative is calculated as follows:

$$\begin{aligned}\frac{\partial \hat{t}_2(y)}{\partial y} &= \frac{1}{-\log(\gamma)} \times \frac{1}{\gamma^{-y}\frac{1}{n-1} - \gamma^{T'} I^{-1}} \times \gamma^{-y} \frac{-1}{n-1} \log(\gamma) = \\ &= \frac{\gamma^{-y}\frac{1}{n-1}}{\gamma^{-y}\frac{1}{n-1} - \gamma^{T'} I^{-1}} > 1\end{aligned}$$

■

Lemma 4:

$$\Pi_1(y^*, y^*) \geq \Pi(y, y^*), \forall y \in [0, T] \quad \Leftrightarrow \quad \begin{cases} \Pi_1(y^*, y^*) \geq \Pi_2(0, y^*) \\ \Pi_1(y^*, y^*) \geq \Pi_1(y, y^*), \forall y \in [y'', y'] \\ \Pi_1(y^*, y^*) \geq \Pi_4(T, y^*) \end{cases}$$

If $\hat{t}_3(y^*) \leq T$, $y' = \hat{t}_3(y^*)$, otherwise y' solves $\hat{t}_1(y', y^*) = T$. If $\hat{t}_2(0) > y^*$, $y'' = 0$, otherwise y'' solves $\hat{t}_2(y'') = y^*$.

Proof. To specify the deviation payoff, $\Pi(y, y^*)$, we will first look at deviations upwards ($y > y^*$) and then at deviations downwards ($y < y^*$). First, consider a deviation upwards to a strategy $y = y_D > y^*$. The calculation of payoff $\Pi(y_D, y^*)$ depends on the size of the deviation: if y_D is sufficiently small, the payoff is determined by $\Pi(y_D, y^*) = \Pi_1(y_D, y^*)$, but if y is large, myopic players may switch to A prior to y (see an illustration in figure 10, panel a), or myopic players may never switch to A (figure 10, panel b). The first option is possible only if the myopic players switch to A without ever observing player s choose A, that is if $\hat{t}_3(y^*) < T$. Then the deviation payoffs for an action plan $y_D \in (\hat{t}_3(y^*), T]$ are calculated by $\Pi_3(y_D, y^*)$. But $\Pi_3(y, y^*)$ is decreasing in y, thus any strategy in this interval would be strictly dominated by strategy $y = \hat{t}_3(y^*)$. In figure 10 we indicate dominance with an arrow pointing towards the dominant strategy. Checking for profitable deviations upwards therefore only requires checking for potential deviations in the interval $(y^*, \hat{t}_3(y^*)]$. Also note that $y_D \leq \hat{t}_3(\bar{y})$ together with condition **I1** imply that deviation payoffs for strategies $y_D \in (0, \hat{t}_3(\bar{y}))$ are equal to $\Pi_1(y_D, y^*)$.

The second possibility is that $\hat{t}_3(y^*) \geq T$, so that myopic players do not switch prior to T if they observe only $n - m - 1$ sophisticated players switching at y^* (see figure 10, panel b). Then because $\hat{t}_1(T, y^*) = \hat{t}_3(y^*) > T$, $\hat{t}_1(y^*, y^*) < T$ (from condition **I1**) and $\hat{t}_1(\cdot, y^*)$ is continuous, there must be a number $y' \in (y^*, T)$ such that $\hat{t}_1(y', y^*) = T$. If $y_D \in (y^*, y']$, (22a) is satisfied and $\Pi(y_D, y^*) = \Pi_1(y_D, y^*)$, because $\hat{t}_1(y, y^*) \leq T$, $\hat{t}_2(y_D) > y_D > y^*$ and $\hat{t}_3(y^*) > T > y^*$. The payoff from any $y_D > y'$ is determined by $\Pi_4(y, y^*) = yL$, and thus all strategies $y_D \in (y', T]$ are dominated by $y_D = T$. Overall, to check for the existence of an interior equilibrium it is sufficient to compare equilibrium payoffs to the payoffs from $y_D \in (y^*, y') \cup T$.

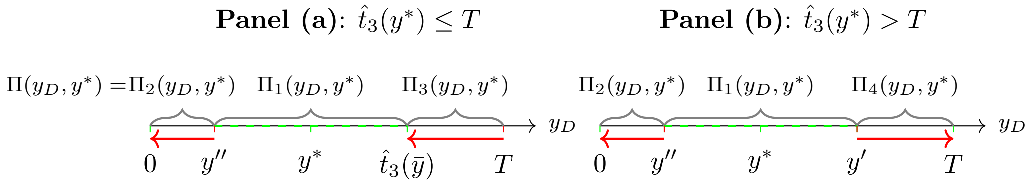

Figure 10: Calculation of deviation payoffs, $\Pi(y_D, y^*)$ for every possible value of y_D. Green dashed line and green ticks mark undominated strategies. Red arrows mark dominated strategies and the arrow points to the dominant strategy.

Now consider a possible deviation downwards to $y_D < y^*$. If y_D is only slightly below y^*, the switching period is $\hat{t}_1(y_D, y^*)$ and the deviation payoffs are $\Pi_1(y_D, y^*)$. But if y_D is low enough, myopic players may switch to A prior to y^*, at time $\hat{t}_2(y_D)$. If this does not happen, that is if $\hat{t}_2(0) > y^*$, payoffs from all deviations downwards are calculated by $\Pi_1(y_D, y^*)$. Otherwise, if $\hat{t}_2(0) \leq y^*$, there will be some value y'' that satisfies $\hat{t}_2(y'') = y^*$. For any y below this value, payoffs will be determined by $\Pi_2(y, y^*)$. From Lemma 3, any $y \in (0, y'')$ is dominated by $y = 0$, therefore to check if there are any profitable deviations downwards it is necessary to compare equilibrium payoffs to payoffs from strategies $y_D \in (y'', y^*) \cup 0$.

■

Lemma 5: $\Pi_1(y^*, y^*) \geq \Pi_1(y, y^*), \quad \forall y \in (y'', y')$, if and only if condition **I2** is satisfied:

$$\frac{\log(\frac{n-m-H/L}{I^{-1}(n-1)})}{\log(\gamma)} - T' > 0, \qquad (\mathbf{I2})$$

Proof.

We will calculate the first derivative of the profit function and determine under what conditions the derivative at the equilibrium point is equal to 0 and the second derivative is is non-positive, which ensures that the equilibrium is a maximum point and there are no incentives to deviate to strategies in the nearest neighbourhood. Instead of taking the derivative of the profit function, we will first transform it by applying a strictly increasing function $-\gamma^{(\cdot/H)}$, which preserves the sign of the derivative when $\gamma \in (0, 1)$. The transformed payoff function is calculated as follows:

$$\begin{aligned}
-\gamma^{\Pi(y,y^*)/H} &= -\gamma^{yL/H+T}\gamma^{-\hat{t}_1(y,y^*)} = \\
&= \frac{1}{I^{-1} - \frac{n-m}{n-1}}(\gamma^{y(L/H-1)+T}\frac{1}{n-1} + \gamma^{yL/H+T-y^*}\frac{n-m-1}{n-1} - \gamma^{yL/H+T+T'}I^{-1}) = \\
&= \frac{\gamma^{yL/H+T}}{I^{-1} - \frac{n-m}{n-1}}(\gamma^{-y}\frac{1}{n-1} + \gamma^{-y^*}\frac{n-m-1}{n-1} - \gamma^{T'}I^{-1}) \qquad (31)
\end{aligned}$$

where $\gamma^{\hat{t}_1(y,y_{-i})}$ has been substituted from (19). Differentiate the transformed profit function in (31) with respect to y to get

$$\begin{aligned}
\frac{\partial - \gamma^{\Pi(y,y^*)/H}}{\partial y} &= \frac{\log(\gamma)}{I^{-1} - \frac{n-m}{n-1}} \times (\gamma^{yL/H+T-y}\frac{1}{n-1}(L/H-1)+ \\
&+ \gamma^{yL/H+T-y^*}\frac{n-m-1}{n-1}L/H - \gamma^{yL/H+T+T'}I^{-1}L/H) = \\
&= \frac{\log(\gamma)\gamma^{yL/H+T}}{I^{-1} - \frac{n-m}{n-1}}\left(\gamma^{-y}\frac{1}{n-1}(L/H-1) + \gamma^{-y^*}\frac{n-m-1}{n-1}L/H - \gamma^{T'}I^{-1}L/H\right) \\
& \qquad (32)
\end{aligned}$$

The first derivative is non-negative if:

$$
\begin{aligned}
\frac{\partial - \gamma^{\Pi(y,y^*)/H}}{\partial y} &\geq 0 \quad \Leftrightarrow \\
\gamma^{-y}\frac{L/H-1}{n-1} + \gamma^{-y^*}\frac{n-m-1}{n-1}L/H - \gamma^{T'}I^{-1}L/H &\geq 0 \quad \Leftrightarrow \\
\gamma^{-y} &\leq \frac{\gamma^{-y^*}(\frac{n-m-1}{n-1}) - \gamma^{T'}I^{-1}}{\frac{H/L-1}{n-1}}
\end{aligned} \tag{33}
$$

The first derivative at point $y = y^*$ is non-negative if:

$$
\begin{aligned}
\left.\frac{\partial - \gamma^{\Pi(y,y^*)/H}}{\partial y}\right|_{y=y^*} &\geq 0 \quad \Leftrightarrow \\
\gamma^{-y^*}\frac{H/L-1}{n-1} \leq \gamma^{-y^*}(\frac{n-m-1}{n-1}) &- \gamma^{T'}I^{-1} \quad \Leftrightarrow \\
\gamma^{T'+y^*} &\leq \frac{n-m-H/L}{I^{-1}(n-1)} \quad \Leftrightarrow \\
y^* &\geq \frac{\log(\frac{n-m-H/L}{I^{-1}(n-1)})}{\log(\gamma)} - T'
\end{aligned} \tag{34}
$$

The derivative is equal to 0 only if y^* satisfies (34) with equality:

$$
y^* = \frac{\log(\frac{n-m-H/L}{I^{-1}(n-1)})}{\log(\gamma)} - T' \tag{35}
$$

There will be at most one y^* that satisfies (35) for any given set of parameters, therefore there can be at most one interior equilibrium in a given game, and the equilibrium strategy is determined by equation (35). A necessary condition for the existence of an interior equilibrium is $0 < y^* < T$. But note that condition **I1** from Proposition 5 implies that $y^* < T$ because $y^* < \hat{t}_1(y^*, y^*)$, therefore the only additional condition is that $y^* > 0$.

Condition I2:

$$
\frac{\log(\frac{n-m-H/L}{I^{-1}(n-1)})}{\log(\gamma)} - T' > 0
$$

The second derivative is obtained by differentiating (32) with respect to y:

$$
\begin{aligned}
\frac{\partial^2 - \gamma^{\Pi(y,y^*)/H}}{\partial y^2} = \frac{\log(\gamma)^2\gamma^{yL/H+T}}{I^{-1} - \frac{n-m}{n-1}} &\times (\gamma^{-y}\frac{1}{n-1}(L/H-1)^2 + \\
&+ \gamma^{-y^*}\frac{n-m-1}{n-1}(L/H)^2 - \gamma^{T'}I^{-1}(L/H)^2)
\end{aligned} \tag{36}
$$

The second order derivative is negative if:

$$\frac{\partial^2 - \gamma^{\Pi(y,y^*)/H}}{\partial y^2} < 0 \quad \Leftrightarrow$$

$$\gamma^{-y}\frac{1}{n-1}(L/H-1)^2 + \gamma^{-y^*}\frac{n-m-1}{n-1}(L/H)^2 - \gamma^{T'}I^{-1}(L/H)^2 > 0 \tag{37}$$

If condition **I2** is satisfied, the expression of y^* in (35) can be used to rewrite (37) as follows:

$$\gamma^{y^*-y} > \frac{L}{L-H} \tag{38}$$

Because $L < H$ and $\gamma \in (0,1)$, condition (38) is satisfied for all y. The first order condition is therefore both necessary and sufficient for $y = y^*$ to be a local maximum point. Moreover, equation (38) states that the second derivative is negative not only at $y = y^*$, but also for any other value of y. Since the first derivative is equal to 0 at point $y = y^*$, and it is decreasing at all y, the payoff function must be increasing at any point $y < y^*$ and decreasing at any point $y > y^*$. Continuity of the profit function therefore implies that $y = y^*$ is not only a local, but also a global maximum in the interval (y'', y) as long as condition **I2** is satisfied.

■

Lemma 6: $\Pi_1(y^*, y^*) \geq \Pi_2(0, y^*)$ if and only if condition $I3$ is satisfied:

$$\hat{t}_1(y^*, y^*) - y^*L/H \;\leq \hat{t}_2(0), \qquad \textbf{(I3)}$$

Proof.

Use the profit specification in (13) to get the following expressions for the two profit functions:

$$\Pi_1(y^*, y^*) = yL + (T - \hat{t}_1(y^*, y^*))H$$

$$\Pi_2(0, y^*) = (T - \hat{t}_2(0))H$$

There are no incentives to deviate to $y = T$ if the former expression exceeds the latter:

$$\Pi_1(y^*, y^*) \geq \Pi_2(0, y^*) \quad \Leftrightarrow \quad \hat{t}_1(y^*, y^*) - y^*L/H \;\leq \hat{t}_2(0)$$

■

Lemma 7: $\Pi_1(y^*, y^*) \geq \Pi_4(T, y^*)$ if and only if condition **I4** is satisfied:

$$\hat{t}_1(y^*, y^*) - y^*L/H \leq T(1 - L/H), \qquad \textbf{(I4)}$$

Proof.

From (13), deviation payoffs are as follows:

$$\Pi_4(T, y^*) = TL$$

There are no incentives to deviate to $y = T$ if

$$\Pi_1(y^*, y^*) \geq \Pi_4(T, y^*) \quad \Leftrightarrow \quad \hat{t}_1(y^*, y^*) - y^* L/H \leq T(1 - L/H)$$

■

Lemma 8: $\Pi_1(0,0) \geq \Pi_1(y,0), \quad \forall y \in [0, y')$ if and only if condition **T1** is satisfied:

$$\frac{n - m - H/L}{n-1} \leq \gamma^{T'} I^{-1} \quad \textbf{(T1)}$$

where y' solves $\hat{t}_1(y', 0) = T$.

Proof.

Payoffs for any $y \in [0, y')$ are calculated the following way, from equation (22a):

$$\Pi_1(y, 0) = yL + (T - \hat{t}_1(y, 0))H \tag{39}$$

A necessary condition for the payoff to be maximized at $y = 0$ is the non-positive sign of the first derivative of (39) with respect to y at $y = 0$. We first apply a strictly increasing function $-\gamma^{(\cdot/H)}$ to the payoff function an then differentiate the transformed function with respect to y to obtain the following condition:

$$\frac{\partial - \gamma^{\Pi_1(y,0)/H}}{\partial y} \leq 0 \quad \Leftrightarrow$$

$$\frac{\log(\gamma)\gamma^T}{I^{-1} - \frac{n-m}{n-1}} \times \gamma^{-y}(\frac{1}{n-1}(L/H - 1) + \frac{n-m-1}{n-1} L/H - \gamma^{T'} I^{-1} L/H) \leq 0 \quad \Leftrightarrow$$

$$\gamma^{-y} \frac{1}{n-1}(L/H - 1) + \frac{n-m-1}{n-1} L/H - \gamma^{T'} I^{-1} L/H \leq 0 \tag{40}$$

Inequality (40) must hold for $y = 0$:

$$
\begin{aligned}
\left.\frac{\partial - \gamma^{\Pi_1(y,0)/H}}{\partial y}\right|_{y=0} &\leq 0 \quad \Leftrightarrow \\
\frac{1}{n-1}(L/H-1) + \frac{n-m-1}{n-1}L/H - \gamma^{T'} I^{-1} L/H &\leq 0 \quad \Leftrightarrow \\
L/H\frac{n-m}{n-1} - \frac{1}{n-1} &\leq \gamma^{T'} I^{-1} L/H \quad \Leftrightarrow \\
\frac{n-m-H/L}{n-1} &\leq \gamma^{T'} I^{-1} \qquad (41)
\end{aligned}
$$

To obtain the second derivative, differentiate the the left-hand side of (40) with respect to y and simplify to get:

$$
\frac{\partial^2 - \gamma^{\Pi_1(y,0)/H}}{\partial y^2} = \frac{1}{n-1}(L/H-1)(-1)\log\gamma
$$

Note that the second derivative is always negative because $\gamma \leq 1$ and $H > L$. If condition **T1** is satisfied, the first derivative will be non-positive at point $y = 0$, and it will non-positive for any $y \in (0, t')$. Payoffs would therefore be maximized by choosing $y = 0$. If **T1** does not hold, the first derivative is positive at point $y = 0$ and profits could be increased by choosing $y > 0$.

■

Lemma 10. *$\hat{t}_1(y, \bar{y}) \leq \hat{t}_2(y)$ and $\hat{t}_1(y, \bar{y}) \leq \hat{t}_3(\bar{y})$*

Proof.

Note that $\hat{t}_1(y, \bar{y})$ is increasing both in y and in $\bar{y}$, from equation (20). If y is held constant, at any given time t the maximum value of $\hat{t}_1(y, \bar{y})$ will be reached at $\bar{y} = t$. Substituting $\bar{y} = t$ into equation (17) reduces it to equation (24), thus $\max_{\bar{y}} \hat{t}_1(y, \bar{y}) = \hat{t}_2(y)$. Likewise, setting $y = t$ in equation (17) reduces it to equation (27), thus $\max_y \hat{t}_1(y, \bar{y}) = \hat{t}_3(\bar{y})$. Therefore $\hat{t}_1(y, \bar{y})$ can never exceed $\hat{t}_2(y)$ or $\hat{t}_3(\bar{y})$.

■

A.3 Comparative Statics

A.3.1 Speed of Transition in the Teaching Equilibrium

We will prove the effect of the parameter changes on the general function $\hat{t}_1(y, y)$, and all results will of course hold for the special case $y = 0$.

Lemma 11. *Speed of transition to the efficient state in a teaching equilibrium depends on the parameter values the following way:*

1. $\frac{\partial \hat{t}_1(y,y)}{\partial y} > 0$
2. $\frac{\partial \hat{t}_1(y,y)}{\partial m} > 0$
3. $\frac{\partial \hat{t}_1(y,y)}{\partial T'} > 0$

Proof.

Assume that a teaching equilibrium exists, so that $\hat{t}_1(0,0) < T$ and $\frac{n-m}{n-1} > I^{-1}$. We will show how the speed of transition in this type of equilibrium respond to changes in parameter values. The switching period $\hat{t}_1(y,y)$ is calculated using equation (20):

$$\hat{t}_1(y,y) = \frac{1}{-\log(\gamma)}\left[\log(\gamma^{-y}\frac{n-m}{n-1} - \gamma^{T'}I^{-1}) - \log(\frac{n-m}{n-1} - I^{-1})\right] \tag{42}$$

1. *Derivative with respect to y:*

$$\frac{\partial \hat{t}_1(y,y)}{\partial y} = \frac{\frac{n-m}{n-1}}{\frac{n-m}{n-1} - \gamma^{y+T'}I^{-1}} \tag{43}$$

$\frac{\partial \hat{t}_1(y,y)}{\partial y} > 0$ because $\frac{n-m}{n-1} > I^{-1}$ and $\gamma \in (0,1)$.

2. *Derivative with respect to m:*

$$\begin{aligned}\frac{\partial \hat{t}_1(y,y)}{\partial m} &= -\frac{1}{\log(\gamma)}\frac{1}{n-1}\left(\frac{1}{\frac{n-m}{n-1} - I^{-1}} - \frac{1}{\frac{n-m}{n-1} - \gamma^{T'+y}I^{-1}}\right) = \\ &= -\frac{1}{\log(\gamma)}\frac{1}{n-1}\frac{I^{-1}(1-\gamma^{T'+y})}{(\frac{n-m}{n-1} - I^{-1})(\frac{n-m}{n-1} - \gamma^{T'+y}I^{-1})}\end{aligned} \tag{44}$$

$\frac{\partial \hat{t}_1(y,y)}{\partial m} > 0$ because $\frac{n-m}{n-1} > I^{-1}$ and $\gamma \in (0,1)$.

3. *Derivative with respect to T':*

$$\frac{\partial \hat{t}_1(y,y)}{\partial T'} = -\frac{1}{\log(\gamma)}\frac{1}{\gamma^{-y}\frac{n-m}{n-1} - \gamma^{T'}I^{-1}} \times -I^{-1}\gamma^{T'}\log(\gamma) \tag{45}$$

$\frac{\partial \hat{t}_1(y,y)}{\partial T'} > 0$ because $\frac{n-m}{n-1} > I^{-1}$ and $\gamma \in (0,1)$.

■

A.3.2 Speed of Transition in the Interior Equilibrium

Lemma 12. *Speed of transition to the efficient state in an interior equilibrium depends on the parameter values the following way:*

1. $\dfrac{\partial \hat{t}_1(y^*, y^*)}{\partial m} > 0$
2. $\dfrac{\partial \hat{t}_1(y^*, y^*)}{\partial T'} = -1$

Proof.

Assume that an interior equilibrium exists, so that $\hat{t}_1(y^*, y^*) < T$ and $\frac{n-m}{n-1} > I^{-1}$. In an interior equilibrium changes in parameter values affect both the equilibrium strategies of sophisticated players and the switching period of myopic players, holding the strategies of sophisticated players constant. To measure the total effect we substitute the expression of y^* from equation (35) into (42) to obtain the following result:

$$\hat{t}_1(y^*, y^*) = \frac{1}{-\log(\gamma)} \left[\log \left(\frac{I^{-1}(n-1)\gamma^{T'}}{n-m-H/L} \frac{n-m}{n-1} - \gamma^{T'} I^{-1} \right) - \log \left(\frac{n-m}{n-1} - I^{-1} \right) \right] =$$
$$= \frac{1}{-\log(\gamma)} \left[\log(H/L) + \log(\gamma^{T'}) + \log(I^{-1}) - \log(n-m-H/L) - \log \left(\frac{n-m}{n-1} - I^{-1} \right) \right]$$

1. *Derivative with respect to m:*
$$\frac{\partial \hat{t}_1(y^*, y^*)}{\partial m} = -\frac{1}{\log(\gamma)} \left(\frac{1}{n-m-H/L} + \frac{n-1}{\frac{n-m}{n-1} - I^{-1}} \right) \tag{46}$$
$\frac{\partial \hat{t}_1(y^*, y^*)}{\partial m} > 0$ because $\frac{n-m}{n-1} > I^{-1}$, $\gamma \in (0,1)$ and $n - m - H/L > 0$ (if an interior equilibrium exists).

2. *Derivative with respect to T':*
$$\frac{\partial \hat{t}_1(y^*, y^*)}{\partial T'} = -1 \tag{47}$$

■

A.3.3 Speed of Transition if One Player is Teaching

Here we will calculate how the parameters of interest affect $\hat{t}_2(0)$, which measures the transition speed if a single sophisticated player always plays A while all others play B. This derivative is necessary for Proposition 9 and Proposition 10 because the existence of a delay equilibrium depends on $\hat{t}_2(0)$

Lemma 13. *Speed of transition to the efficient state if only one sophisticated player is choosing A depends on the parameter values the following way:*

1. $\dfrac{\partial \hat{t}_2(0)}{\partial m} = 0$
2. $\dfrac{\partial \hat{t}_2(0)}{\partial T'} > 0$

Proof.

Suppose that $\hat{t}_2(0) < T$, which holds only if $\frac{1}{n-1} > I^{-1}$. Then $\hat{t}_2(0)$ is calculated the following way, from expression 25:

$$\hat{t}_2(0) = \frac{\log(\frac{1}{n-1} - I^{-1}) - \log(\frac{1}{n-1} - \gamma^{T'} I^{-1})}{\log(\gamma)} \tag{48}$$

1. *Derivative with respect to m:*

$$\frac{\partial \hat{t}_2(0)}{\partial m} = 0 \tag{49}$$

2. *Derivative with respect to T':*

$$\frac{\partial \hat{t}_2(0)}{\partial T'} = \frac{\gamma^{T' I^{-1}}}{\frac{1}{n-1} - \gamma^{T'} I^{-1}} \tag{50}$$

$\frac{\partial \hat{t}_2(0)}{\partial T'} > 0$ because $\frac{1}{n-1} > I^{-1}$.

■

A.3.4 Equilibrium Strategies in the Interior Equilibrium

Another variable if interest is the strategy used by sophisticated players in an interior equilibrium, y^*, which has an effect on the existence conditions of the interior equilibrium.

Lemma 14. *The strategies used by sophisticated players in an interior equilibrium depend on parameter values the following way:*

1. $\dfrac{\partial y^*}{\partial m} > 0$

2. $\dfrac{\partial y^*}{\partial T'} = -1$

In addition:

3. $\dfrac{\partial y^*}{\partial m} < \dfrac{\partial \hat{t}_1(y^*, y^*)}{\partial m}$

Proof.

Equilibrium strategy is determined by equation (35):

$$y^* = \frac{\log(\frac{n-m-H/L}{I^{-1}(n-1)})}{\log(\gamma)} - T'$$

1. *Derivative with respect to m:*

$$\frac{\partial y^*}{\partial m} = -\frac{1}{\log(\gamma)}(\frac{1}{n-m-H/L}) \tag{51}$$

   $\frac{\partial y^*}{\partial m} > 0$ because $n - m - H/L > 0$ (because an interior equilibrium exists).

2. *Derivative with respect to T':*

$$\frac{\partial y^*}{\partial T'} = -1 \tag{52}$$

3. *Comparison to the derivative of $\hat{t}_1(y^*, y^*)$:*

   Recall the derivative of $\hat{t}_1(y^*, y^*)$ from equation (46):

$$\frac{\partial \hat{t}_1(y^*, y^*)}{\partial m} = -\frac{1}{\log(\gamma)}(\frac{1}{n-m-H/L} + \frac{n-1}{\frac{n-m}{n-1} - I^{-1}})$$

   The derivative of y^* calculated in (51) is strictly lower than the derivative of $\hat{t}_1(y^*, y^*)$ because $\frac{n-1}{\frac{n-m}{n-1} - I^{-1}} > 0$.

■