

# The Tree that Hides the Forest: A Note on Revealed Preference

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**Abstract** The common interpretation given to choice behavior that satisfies the traditional revealed preference axioms is that it results from the maximization of a single preference. We show that choice data alone does not enable one to rule out the possibility that the choice behavior that satisfies the revealed preference axioms is instead the result of the aggregation of a collection of distinct preferences. In particular, we show that any ordering is observationally equivalent to a majoritarian aggregation of a collection of distinct dichotomous orderings. We also show that any ordering is observationally equivalent to a Borda's aggregation of a collection of distinct linear orderings.

*Keywords:* Revealed preference theory; Rationalization; Dichotomous preferences; Aggregation rules; Choice data.

*JEL classification:* B4; D01; D71.

## 1 Introduction

Initiated by Samuelson (1938) and developed by Houthakker (1950), Arrow (1959), Richter (1966), Afriat (1967), Sen (1971) among many others, the revealed preference theory has established an equivalence between observable properties of choice behavior - the traditional revealed preference axioms - and the possibility for this behavior to be rationalized by a single preference<sup>1</sup>. A very common interpretation of this equivalence is that any choice behavior that satisfies the revealed preference axioms results from the maximization of a single preference. We show that this interpretation may be misleading. Specifically, we show that it is impossible to distinguish - using choice data - if the behavior of an agent (be it an individual or an institution) *that satisfies the traditional revealed preference axioms* is the result of a (direct) maximization of a single preference *or* the result of the aggregation of a collection of distinct preferences.

We show this for two widely known and used aggregation rules. First, we show that *any* ordering is (observationally) equivalent to an aggregation of a collection of distinct dichotomous orderings by the majority rule. Second, we show that *any* ordering is an aggregation of a collection of distinct linear orderings by the Borda's rule. From a technical perspective, the original content of these results can be seen as generalizations of the McGarvey's (1953) theorem. McGarvey (1953) shows that it is possible to view any complete binary relation as the aggregation of a collection of linear orderings by the majority rule. Here, we show that if one imposes transitivity to the binary relation, then a similar result holds for (i) a collection of dichotomous orderings and for (ii) the Borda's rule.

We believe that these results question in an interesting fashion the usual interpretation of the revealed preferences axioms as unequivocal implications of the *mono-rationality* of choice behavior. For instance, suppose that one observes the choice behavior of an household that satisfies the revealed preference

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<sup>1</sup>Examples of these axioms include the Weak and Strong Axioms of Revealed Preference and the Weak and Strong Congruence Axioms. As shown by Sen (1971), these axioms are all equivalent when applied to a choice function defined over all subsets of a finite set.

axioms. Our results imply that one may need additional evidence in order to disentangle if this behavior is coming from the decision of a “benevolent dictator” (one parent) or if it is instead the result of a collective decision (an aggregation of the preferences of the family members). Alternatively, suppose that one observes the choices of an individual decision maker. Our results indicate that if his or her choices satisfy the revealed preference axioms, then there exists a *sensible* internal process by which a collection of individual “selves” are aggregated into a single ordinal index. Then, it may be the case that the individual is not revealing the maximization of a “better off” relation but instead the aggregation of several preferences for which the ranking of alternatives may be different from one preference to the other.

The remainder of the paper is organized as follows. Section 2 is devoted to notation and preliminaries. In Section 3 we present our results. In Section 4 we discuss the implications of these results. We summarize our contribution in Section 5.

## 2 Notation and Preliminaries

Let  $X$  be a finite set of alternatives denoted by  $x, y$ , etc., and assume that  $\#X = n \geq 3$ . Let  $A$  be any subset of  $X$  and  $\mathcal{P}(X)$  denote the set of all non-empty subsets of  $X$ . A choice function is a mapping  $C : \mathcal{P}(X) \rightarrow X$  that satisfies  $C(A) \in A$  for every  $A \in \mathcal{P}(X)$ . The standard interpretation is that  $C(A)$  is the chosen element from the set  $A$ . A binary relation  $R$  on  $X$  is a subset of  $X \times X$ . Following the convention in economics, we write  $xRy$  instead of  $(x, y) \in R$ . A binary relation  $R$  on  $X$  is (i) *reflexive* if  $xRx$  for all  $x \in X$ , is (ii) *complete* if either  $xRy$  or  $yRx$  for all *distinct*  $x, y \in X$ , is (iii) *transitive* if  $xRy$  and  $yRz$  imply  $xRz$  for any  $x, y, z \in X$ , and is (iv) *antisymmetric* if  $xRy$  and  $yRx$  imply that  $x = y$  for any  $x, y \in X$ . Denote by  $P$  and  $I$  the *asymmetric* and *symmetric* components of  $R$  respectively, i.e.,  $xPy \Leftrightarrow xRy \wedge \neg(yRx)$  and  $xIy \Leftrightarrow xRy \wedge yRx$ . Let  $I^1, \dots, I^{m(R)}$  denote the  $m(R)$  *indifference classes* of  $R$  from top to bottom such that for  $1 \leq j < l \leq m(R)$  if  $x, y \in I^j$  then  $xIy$  and if  $x \in I^j$  and  $y \in I^l$  then  $xPy$ . An *ordering* on  $X$  is a reflexive, complete, and transitive binary relation on  $X$ , and a *linear ordering* on  $X$  is an antisymmetric ordering on  $X$ . An ordering  $R$  is *dichotomous* if  $m(R) \leq 2$ .

For any ordering  $R$ , a choice function  $C$  is said to be *rationalized* by  $R$  if  $C(A) = \max_A R := \{x \in A : \forall y \in A \ xRy\}$  for all  $A \in \mathcal{P}(X)$ . One of the traditional revealed preference axioms, the Weak Axiom of Revealed Preference (WARP), states that if an alternative  $x$  is “revealed preferred” to  $y$  (i.e.,  $x$  is once chosen when  $y$  is available *and* rejected), then  $y$  is *not* revealed to be “at least as good as”  $x$  (i.e.,  $y$  is never chosen when  $x$  is available). Formally, WARP states that for all  $x, y \in X$  and  $A, B \in \mathcal{P}(X)$  if  $x \in C(A)$  and  $y \in A \setminus \{C(A)\}$  then  $\neg(y \in C(B) \text{ and } x \in B)$ . A choice function that satisfies this condition can be viewed as resulting from the maximization of a single preference, i.e.:

**Theorem 1** (*Arrow, 1959*) *A choice function  $C$  satisfies WARP if and only if it is rationalized by an ordering  $R$ .*

For any collection  $(R_1, \dots, R_k)$  of  $k$  orderings on  $X$ , we define the *majoritarian* and *Borda’s* aggregation rules, denoted respectively by  $R^{maj}(R_1, \dots, R_k)$  and  $R^{bor}(R_1, \dots, R_k)$ , by:

$$\begin{aligned} x R^{maj}(R_1, \dots, R_k) y &\iff \#\{i \leq k : x R_i y\} \geq \#\{i \leq k : y R_i x\} \\ x R^{bor}(R_1, \dots, R_k) y &\iff \sum_{i=1}^k \#\{z \in X : x P_i z\} \geq \sum_{i=1}^k \#\{z \in X : y P_i z\} \end{aligned}$$

The Borda’s aggregation rule applied to a collection of orderings always generates an ordering. The majoritarian aggregation rule generates a binary relation that is reflexive and complete but not necessarily transitive. However, the majority relation always generates a transitive binary relation whenever it aggregates a collection of dichotomous orderings, i.e.:

**Theorem 2** (*Inada 1964*) *Let  $(R_1, \dots, R_k)$  be a collection of  $k$  dichotomous orderings on  $X$ . Then the majoritarian aggregation  $R^{maj}(R_1, \dots, R_k)$  is an ordering on  $X$ .*

### 3 Results

We first establish that any ordering is equivalent to the majoritarian aggregation of a collection of *distinct* dichotomous orderings<sup>2</sup>:

**Theorem 3** *R is an ordering on X if and only if there exists a collection  $(R_1, \dots, R_k)$  of k distinct dichotomous orderings on X such that  $R^{maj}(R_1, \dots, R_k) = R$ .*

*Proof* Given Theorem 2, only the second implication needs to be established. We distinguish three cases: (i)  $m(R) > 2$ , (ii)  $m(R) = 2$ , and (iii)  $m(R) = 1$ . For the first case, take a dichotomous ordering  $R_1$  such that the top indifference class is  $I^1$  and the bottom indifference class is  $\cup_{2 \leq j \leq m(R)} I^j$ . Then, take a second dichotomous ordering  $R_2$  such that the top indifference class is  $I^1 \cup I^2$  and the bottom indifference class is  $\cup_{3 \leq j \leq m(R)} I^j$ . Proceeding this way, we arrive at a collection  $(R_1, \dots, R_{m(R)-1})$  dichotomous orderings such that all  $x \in I^1$  are in the top indifference class of  $m(R) - 1$  dichotomous orderings, all  $x \in I^2$  are in the top indifference class of  $m(R) - 2$  dichotomous orderings, and so on. Then, straightforward verification shows that the binary relation induced by the majoritarian aggregation of  $(R_1, \dots, R_{m(R)-1})$  is the ordering  $R$ . For the second case, the previous construction generates a single dichotomous ordering  $R_1$ . Then, add to this dichotomous ordering a universally equivalent (dichotomous) ordering and denote it  $R_2$ . It follows that the binary relation induced by the majoritarian aggregation of  $(R_1, R_2)$  is the ordering  $R$ . For the third case, consider all pairs  $(x, y)$  of  $R$ . Then, for each of these pairs take two dichotomous orderings  $R_1$  and  $R_2$  such that  $xP_1yI_1a_1I_1a_2I_1\dots I_1a_{n-2}$  and  $yP_2xI_2a_1I_2a_2I_2\dots I_2a_{n-2}$  where  $a_1, \dots, a_{n-2}$  are the remaining alternatives of  $X$ . Proceeding this way, we arrive at a collection  $(R_1, \dots, R_k)$  of  $k = n(n - 1)$  dichotomous orderings such that these express opposing preferences and cancel each other with respect to any given pair of alternatives. Then, the binary relation induced by the majoritarian aggregation of  $(R_1, \dots, R_k)$  is the ordering  $R$ . □

The original content of Theorem 3 can be viewed as a generalization of McGarvey's (1953) theorem<sup>3</sup>. McGarvey (1953) shows that it is possible to view *any* complete binary relation  $R$  as the majoritarian aggregation of a collection of  $n(n - 1)$  *linear orderings*. Here, we show that by requiring the binary relation  $R$  to be transitive a similar result holds for a collection of *dichotomous orderings*<sup>4</sup>. Dichotomous orderings benefit from a very palatable interpretation, and is the only domain restriction *defined with respect to each individual binary relation* that is sufficient for the majoritarian aggregation rule to be always transitive when the number of binary relations is not necessarily odd<sup>5</sup>. This is relevant in terms of the implications of Theorem 3 that we discuss below.

It is worth noting that contrary to any ordering, it is *not* possible to view any complete binary relation as the majoritarian aggregation of a collection of dichotomous preferences. For instance, suppose that  $X = \{x, y, z\}$  and that  $xPy$ ,  $yPz$ , and  $zPx$ . It is easy to check that there is no collection of dichotomous orderings such that a majoritarian aggregation induces this complete binary relation. The interested reader may also notice that the number of binary relations constructed in Theorem 3 is considerably lower than that in McGarvey (1953)<sup>6</sup>. This points to the fact that the required number of *non-linear* orderings necessary for an ordering to be represented by the majority rule may be in general lower than the required number of *linear* orderings needed for an ordering (or a complete binary relation) to be represented by the same method. Now, we establish a similar result with respect to the Borda's aggregation rule:

**Theorem 4** *R is an ordering on X if and only if there exists a collection  $(R_1, \dots, R_k)$  of k distinct linear orderings on X such that  $R^{bor}(R_1, \dots, R_k) = R$ .*

<sup>2</sup>We require the collection of orderings to be distinct since otherwise any ordering is equivalent to any aggregation of the same single ordering or of a collection of identical ones to it.

<sup>3</sup>See e.g. Hollard and Breton (1996) and Gibson and Powers (2012) for other extensions of McGarvey (1953).

<sup>4</sup>Note that this result is not a corollary of McGarvey (1953). Although the McGarvey's (1953) theorem implies that any ordering can be viewed as a majoritarian aggregation of a collection of linear orderings, it does not entail that any ordering can be viewed as a majoritarian aggregation of a collection of dichotomous orderings.

<sup>5</sup>See Inada (1969) and Sen and Pattanaik (1969) for the remaining domain restrictions that are sufficient (and necessary) for the majoritarian aggregation rule to be always transitive when the number of binary relations is not necessarily odd. Contrary to dichotomous orderings, these restrictions are defined with respect to the admissible profile of binary relations.

<sup>6</sup>This is strictly true except when  $m(R) = 1$ , since in that case the number of binary relations constructed is the same. See e.g. Stearns (1959), Deb (1976), and Brams and Fishburn (2002) for results and discussions concerning the minimal number of binary relations necessary to express any complete binary relation over a set made of  $n$  alternatives.

*Proof* To prove the non-trivial implication, we distinguish two cases: (i)  $m(R) < n$  and (ii)  $m(R) = n$ . For the first case (non-linear ordering), take two linear orderings  $R_1$  and  $R_2$  such that (1) for all  $x \in I^j$  and all  $y \in I^l$  with  $1 \leq j < l \leq m(R)$  one has  $xP_1y$  and  $xP_2y$ , and (2) for all  $x_p \in I^j$  with  $p = 1, \dots, q$  one has  $x_1P_1x_2P_1\dots P_1x_q$  and  $x_qP_2x_{q-1}P_2\dots P_2x_1$ . Since  $m(R) < n$ , (2) guarantees that the two linear orderings are distinct. Then, straightforward verification shows that the binary relation induced by the Borda's aggregation of  $(R_1, R_2)$  is the ordering  $R$ . For the second case (linear ordering), consider all pairs  $(x, y)$  of  $R$ . Then, for each of these pairs such that  $xPy$  take two linear orderings  $P_1$  and  $P_2$  such that  $xP_1yP_1a_1P_1a_2P_1\dots P_1a_{n-2}$  and  $a_{n-2}P_2a_{n-3}P_2\dots P_2a_1P_2xP_2y$ ; For each of these pairs such that  $xIy$  take two linear orderings  $P_1$  and  $P_2$  such that  $xP_1yP_1a_1P_1a_2P_1\dots P_1a_{n-2}$  and  $a_{n-2}P_2a_{n-3}P_2\dots P_2a_1P_2yP_2x$  where  $a_1, \dots, a_{n-2}$  are the remaining alternatives of  $X$ . Proceeding this way, we arrive at a collection  $(R_1, \dots, R_k)$  of  $k = n(n - 1)$  linear orderings such that two of these express  $R$ 's strict preference or indifference between any given pair of alternatives while all other linear orderings cancel with respect to this pair. Then, the binary relation induced by the Borda's aggregation of  $(R_1, \dots, R_k)$  is the ordering  $R$ .  $\square$

This result shows that an ordering is formally equivalent to the Borda's aggregation of a collection of distinct preferences. The interested reader may notice that whenever the binary relation is not a linear ordering, then one needs only two preferences to induce it by the Borda's rule. In a recent paper, Kelly and Qi (2016) show that for a fixed  $k \geq 2$  any ordering is in the range of the Borda's rule except when  $k$  is odd and  $n$  is even. This subsumes our result with the exception of the case in which  $R$  is a linear ordering (and since we look at a collection of *distinct* preferences). So we do not wish to overemphasize the novelty of our contribution on this front and prefer instead to concentrate on the implications that this result entails.

In particular, these theorems show the *equivalence* between a single preference and two aggregations of a collection of *distinct* preferences: it is possible to generate any single preference from these aggregations and these aggregations always generate a single preference. Then, it follows that the maximization of a single preference is not observationally distinguishable from these aggregations. This implication is captured in the following corollaries to our theorems and Arrow (1959):

**Corollary of Theorems 1 and 3** *A choice function  $C$  satisfies WARP if and only if it is rationalized by the majoritarian aggregation  $R^{maj}(R_1, \dots, R_k)$  of a collection  $(R_1, \dots, R_k)$  of  $k$  distinct dichotomous orderings.*

**Corollary of Theorems 1 and 4** *A choice function  $C$  satisfies WARP if and only if it is rationalized by the Borda's aggregation  $R^{Bor}(R_1, \dots, R_k)$  of a collection  $(R_1, \dots, R_k)$  of  $k$  distinct linear orderings.*

## 4 Discussion

Whenever a choice function satisfies WARP (or any equivalent axiom), our results show that the choice behavior can not only be rationalized by the direct maximization of a single preference but also by two *sensible* aggregations of a larger collection of preferences. The novelty of this claim is not that other interpretations are possible, since this is inherent to rationalization results that only tell us that observable behavior can be emerging *as if* the agent is performing a specific decision process. But as Sen (1973), among many others, has forcefully argued, revealed preference theory is only meaningful if it is anchored to the revelation of a *sensible* decision making process. Interpreted in this light, the novelty is to show that the most common interpretation given to the rationalization of behavior that satisfies the traditional revealed preference axioms is *formally equivalent to other sensible interpretations*.

This relates our results to the analysis that use the revealed preference methodology to characterize collective or nonstandard decision models. There is by now an established literature on the rationalization of decision models with multiple preferences (e.g. Manzini and Mariotti 2007; Masatlioglu et al. 2012). These contributions can be seen as a first step to find an axiomatic counterpart to the psychological and empirical literature on multiple identities/selves (e.g. Turner 1985; Hoff and Pandey 2006; LeBoeuf et al. 2010; Benjamin et al. 2010). In this respect, our results point towards sensible rationalizations of the standard rational choice behavior by two decision models with multiple preferences.

Our results also relate with the emerging field of empirical revealed preference theory. Following Afriat (1967) and Varian (1982a, 1982b), empirical revealed preference theory has become more feasible and applicable to different fields of economics<sup>7</sup>. One of its major applications has been to the domain of

<sup>7</sup>Authors in this field have focused on consumption data, albeit many economic decisions are discrete in nature. A choice function is relevant as it can be regarded as a generic tool to analyze any type of choice (see Sen 1973).

household economics (e.g. Browning and Chiappori 1998; Cherchye et al. 2010). This is a good example, as noted in the introduction, to illustrate the implications of our results in terms of the (im)possibility to distinguish between individual and collective decisions. Our results indicate that when it comes to discrete decisions, it may be the case that the observable properties of the choice behavior of an household are not sufficient to ascertain if the household is a unitary decision entity or if it is instead following a collective decision model. Equivalent implications hold for the choice behavior of committees, teams, or any other organization as long as the decision process is unknown to the observer.

Finally, our results may shed some light to the conceptual and methodological apparatus that is often used in economics. In a recent contribution to a book on the *Foundations of Positive and Normative Economics*, Gul and Pesendorfer (2008) have endorsed a view of “mindless economics” with the following premises:

“In the standard approach, the terms “utility maximization” and “choice” are synonymous. A utility function is always an ordinal index that describes how the individual ranks various outcomes and how he behaves (chooses) given his constraints (available options). The relevant data are revealed preference data, that is, consumption choices given the individuals constraints.” (Gul and Pesendorfer 2008, 7)

We can make two comments in the light of our analysis. First, our results indicate that “utility maximization” and “choice” are not necessarily synonymous. Using Samuelson’s own lexicon (in Archibald et al. 1963), satisfying WARP may be perfectly “realistic” at the same time that “maximizing ordinal utility” *is not*. This is so, since, as argued before, it is not possible to exclude that the behavior that satisfies WARP results instead from one of two aggregations of a collection of distinct preferences<sup>8</sup>. Second, our results illustrate one of the difficulties of relying exclusively on “revealed preference data”. Specifically, they indicate that different models are not necessarily distinguishable using choice data alone. This brings an additional argument in favor of two stances: 1) that the revealed preference principle cannot be *always* used as a criterion for selecting between two modeling approaches (see e.g. Spiegler 2008), and 2) that we may need to combine choice observations with other types of data in order to understand the underlying reasons of behavior (see e.g. Schotter 2008). If our results only provide a mild argument in favor of these views, they highlight the internal tension that may emerge from mindless economics.

## 5 Conclusion

The message of this note can be summarized in one sentence: a single rationalization may hide multiple rationalizations. In effect, when one observes a choice function that can be rationalized by a single ordering, one can not exclude that this rationalization results in fact from a (majoritarian or Borda) aggregation of a larger collection of individual orderings/linear orderings. Then, given the theoretical prominence to favor choice data in economics (e.g. Gul and Pesendorfer 2008; Binmore 2009) and the increasingly application of empirical revealed preference theory in fields such as household economics, these remarks highlight the relevance of circumspection on the interpretation of this type of data: won’t it be *the tree that hides the forest*.

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<sup>8</sup>Otherwise, our results do not question the possibility to falsify the hypothesis that some observable behavior results from utility maximization. Indeed, whenever a choice function violates WARP it is possible to exclude that this behavior results from utility maximization (at least in principle; see e.g. Hausman 2000). In this respect, our results only tell us that it is also possible to exclude that this behavior results from two aggregations of a collection of distinct preferences.

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