amse
école d'économie d'aix-marseille
aix-marseille school of economics

Working Papers / Documents de travail

Macroeconomic Shocks in the Fog: The Role of Endogenous Uncertainty

Anastasiia Antonova
Mykhailo Matvieiev
Céline Poilly

WP 2025 - Nr 38

Faculté d'économie et de gestion Aix Marseille Université
cnrs
amidex Aix Marseille Université
L'ÉCOLE DES HAUTES ÉTUDES EN SCIENCES SOCIALES
Centrale Méditerranée
sciencespo.aix

Macroeconomic Shocks in the Fog: The Role of Endogenous Uncertainty*

Anastasiia Antonova† Mykhailo Matvieiev‡ Céline Poilly§

December 18, 2025

Abstract

Recessions are often accompanied by heightened uncertainty. We look at the effect of endogenous uncertainty on aggregate demand and its implications for monetary policy. We enrich a non-linear New-Keynesian model with imperfect noisy information, where the precision of signals is pro-cyclical. The endogenous uncertainty channel amplifies aggregate demand effects through precautionary saving. Ultimately, it can even reverse sign of the output-gap response to a supply shock. Monetary policy can eliminate both pricing and information-induced inefficiencies by closing the output gap. Based on U.S. household income forecast errors data, we estimate a sizable and significant degree of pro-cyclicality in the precision of signals.

JEL classification: D81, D83, E21, E32, E40.

Keywords: Endogenous uncertainty, precautionary saving, aggregate demand, imperfect information.

---

*We thank participants of seminar series at the Banque de France, the ECB, the Universitad de Barcelona and the University of Konstanz, as well as Isaac Baley for useful comments. We acknowledge financial support from the French government under the "France 2030" investment plan managed by the French National Research Agency Grant ANR-17-EURE-0020, and by the Excellence Initiative of Aix-Marseille University - A*MIDEX. It was also supported by French National Research Agency Grant ANR-20-CE26-DEMUR. Céline Poilly thanks the Institut Universitaire de France for its financial support.

†Bank of Canada. Email: aantonova@bank-banque-canada.ca

‡Bank of Canada. Email: mykhailo.matvieiev@univ-amu.fr

§Aix Marseille Univ, CNRS, AMSE, Marseille, France. Email: celine.poilly@univ-amu.fr

1 Introduction

Bad times are uncertain times: recessions are often characterized not just by low output but also by heightened uncertainty. Empirical evidence suggests that this uncertainty is, at least in part, an endogenous response to downturns Ludvigson et al. (2021). One explanation for the endogeneity of uncertainty is the pro-cyclicality of information precision, as offered by Fajgelbaum et al. (2017): economic activity generates information, improving forecast accuracy and reducing uncertainty. While Fajgelbaum et al. (2017) study the implications of this mechanism in a Real Business Cycle model with flexible prices, its role in monetary models — which operate primarily through aggregate demand — remains largely unexplored. In this paper, we ask: how does pro-cyclical information precision shape aggregate demand, and what are its implications for monetary policy?

To address this question, we enrich the New-Keynesian noisy-information framework *à la* Lorenzoni (2009) with two dimensions: pro-cyclical precision of information, and precautionary saving motive. Precisely, we build a typical non-linear New-Keynesian model where the representative household has imperfect information about aggregate productivity and forms its beliefs based on the noisy signals. Endogenous uncertainty is modeled by assuming that the precision of these signals is positively related to the economy's output, capturing the learning-by-doing notion that economic activity generates information.[1] Agents learn by using the information contained in signals to update their beliefs in a Bayesian manner. Based on these beliefs, they make consumption and saving choices.

The pro-cyclical precision of signals in the non-linear model translates into counter-cyclical uncertainty that shapes the precautionary-saving motive: uncertainty depends on economic activity through the notion that activity generates information, while economic activity depends on uncertainty through consumption demand shaped by the precautionary-saving motive. This feedback loop constitutes a novel mechanism of shock propagation in the New-Keynesian model – the endogenous-uncertainty channel. The feedback loop takes place in the law of motion of beliefs, illustrating the fact that the levels of uncertainty and economic activity are jointly determined in equilibrium. Endogenous uncertainty affects households' consumption demand through a precautionary saving motive, which ultimately affects the output level in our New Keynesian environment with nominal rigidities, where output is partially determined by demand.[2]

[1]The implications of pro-cyclical signal precision have previously been studied in the context of the Real Business Cycle model by Van Nieuwerburgh and Veldkamp (2006), with a focus on the cyclical speed of learning, and by Fajgelbaum et al. (2017) and Ilut and Saijo (2021), who examined the implications of endogenous consumer confidence for firm investment decisions and economic activity. In contrast, we focus on the New-Keynesian aggregate demand amplification arising from the precautionary-saving channel.

[2]Ravn and Sterk (2017, 2021) and Challe (2020) analyze the aggregate-demand amplification through the precautionary-saving channel. However, they focus on counter-cyclical unemployment probability as a source of uninsured idiosyncratic risk for households in a HANK-SAM model. In contrast, we concentrate on uncertainty that arises from imperfect aggregate information within a representative agent New-Keynesian model.

To understand how does our endogenous-uncertainty mechanism alter the transmission of macroeconomic shocks, we first develop an analytical version of our model. To do so, we assume that prices are rigid only in the first period, which allows beliefs to be expressed solely in terms of exogenous variables. We then employ a risk-adjusted log-linearization that incorporates higher-order terms in the household's optimality condition. This is because our mechanism centers on precautionary saving, which requires accounting for risk-related terms in the approximation.

We analytically decompose the effect of productivity shocks (supply shock) on the aggregate demand (output gap) into two channels: the standard New-Keynesian channel, and the *endogenous-uncertainty channel*. Due to a standard New-Keynesian channel, the negative productivity shock leads to a positive output gap, since potential output drops below actual output due to the inability of prices to adjust upwards – a well-known result in the literature. In contrast, due to the endogenous-uncertainty channel the same negative productivity shock leads to a negative output gap. This is because a decrease in economic activity translates into lower information quality and higher uncertainty, depressing the aggregate demand through the precautionary saving motive. The ultimate effect of a productivity shock on the output gap depends on the strength of the endogenous-uncertainty channel. When the endogenous-uncertainty channel dominates, the negative supply shock becomes a "Keynesian-supply shock" (Guerrieri et al. (2022), that is, a supply shock that depresses the demand.

We also analyze the propagation of a government spending shock and establish that the endogenous-uncertainty channel mitigates the adverse effect of higher government spending on private consumption, that is, the crowding-out effect. When government spending increases, output goes up, driving down the uncertainty as more quality information becomes available. This lowers precautionary saving and thereby stimulates private consumption. Under certain conditions—specifically, when the shock is not very persistent and the monetary policy response is weak—the endogenous-uncertainty channel can even generate crowding-in, meaning that private consumption may rise in response to government spending as overall confidence in the economy improves.

Therefore, analytical results show that endogenous uncertainty provides an amplification mechanism for economic fluctuations, operating solely through aggregate demand. Further, we show that the adverse effects of the endogenous-uncertainty channel can be fully mitigated by monetary policy, as long as it fully stabilizes the output gap. The reason is that the flexible-price output is not affected by endogenous uncertainty: under flexible prices, output is fully determined by supply, and demand-side frictions are compensated by the price adjustment. Therefore, by closing the output gap, monetary policy is able to restore flexible prices and full-information allocation.

In the quantitative part of the paper, we quantify the amplification effect the endogenous-uncertainty channel engenders. For this purpose, we employ a full-fledged non-linear version of our New-Keynesian model, parametrized for the U.S. In particular, we estimate the parameters governing the information precision and its cyclicality by matching the time-series percentiles of the income-growth expectation error in the model and in the data. Specifically, we construct a series of aggregate income growth expectation errors from the Michigan Survey of Consumers; we first construct the individual income growth errors and then aggregate them across individuals, eliciting the error related to the aggregate income component. Our estimation procedure yields a significant degree of pro-cyclicality in the precision of signals, while the estimated signal-to-noise ratio is in the same range than Melosi (2014). Considering a one standard-deviation negative TFP shock, the endogenous uncertainty doubles the consumption fall, in deviation from its steady state, relative to the model with constant uncertainty.

Finally, we simulate the quantitative model response to a productivity shock and the government spending shock in the model with pro-cyclical precision of signal, and in the model where this mechanism is shut down. We find that the empirically relevant degree of cyclicality strongly amplifies the negative response of output to a negative productivity shock, compared to a case where the precision of information is constant. Moreover, a strict-inflation targeting rule removes the information-related distorsion, implying that the so-called "divine coincidence" (Blanchard and Gali (2007)) holds in our model.

Related Literature. First, we relate to the literature on imperfect information and learning within the New-Keynesian model Woodford (2001); Lorenzoni (2009) who study the role of noisy information of constant precision; our analysis contributes by focusing on the implications of pro-cyclical information precision.

Second, we relate to broader literature on implications of endogenous uncertainty arising from pro-cyclical signal precision in business cycle models. Existing studies consider various dimensions of uncertainty propagation, with a focus on investment (Saijo, 2017; Fajgelbaum et al., 2017; Schaal and Taschereau-Dumouchel, 2023), Knightian uncertainty (Ilut and Saijo, 2021), financial frictions (Benhabib et al., 2019; Straub and Ulbricht, 2024) or labor market frictions (Bernstein et al., 2024). In contrast, we investigate the implications of endogenous *consumer* uncertainty propagation through the precautionary saving channel, which operates via its effects on aggregate demand. Our analysis also relates to the empirical literature on the countercyclicality of aggregate uncertainty Bloom (2009); Jurado et al. (2015), and in particular Ludvigson et al. (2021), who provides evidence of the endogeneity of uncertainty to the business cycle.

Third, the demand-side amplification through the precautionary saving channel, driven by time-varying, endogenous uncertainty, makes our paper conceptually similar to the amplification mechanism in the HANK&SAM models of Ravn and Sterk (2017, 2021) and Challe (2020). However, the difference is that in our model, endogenous uncertainty arises from informational friction rather than labor market imperfection. As a result, uncertainty in our model is aggregate rather than idiosyncratic, and it triggers precautionary saving even under full risk sharing across households, making redistributive policies, such as insurance or transfers, ineffective. In this scene, we also speak to the literature examining the effect of aggregate uncertainty through precautionary saving Fernández-Villaverde et al. (2011); Leduc and Liu (2016); Basu and Bundick (2017); Fernández-Villaverde and Guerrón-Quintana (2020); we contribute by considering endogenous uncertainty.

Fourth, we relate to the literature on Keynesian supply shocks — supply-side disturbances that endogenously generate demand effects. Existing research identifies several mechanisms underlying this effect. For example, Fornaro and Wolf (2023) show that productivity-enhancing investment within the New-Keynesian model can generate the Keynesian supply effect. Similarly, Cesa-Bianchi and Ferrero (2021) and Guerrieri et al. (2022) demonstrate that sectoral productivity shocks can trigger strong aggregate demand effects in a multi-sectoral economy, while Bilbiie and Melitz (2023) explore amplification through the firm entry–exit multiplier. L'Huillier et al. (2024) find that Keynesian supply shocks can also emerge when incorporating non-rational (diagnostic) expectations. We contribute to this literature by analyzing an alternative mechanism for aggregate demand amplification, based on the feedback loop between uncertainty and economic activity through the precautionary saving channel. Unlike the mechanisms above, ours does not rely on production-side features but on endogenous fluctuations in consumer uncertainty, which influence consumption demand and ultimately the level of economic activity.

Fifth, we relate to a broader literature on pro-cyclical learning in noisy-information environments (Veldkamp, 2005; Van Nieuwerburgh and Veldkamp, 2006; Ordonez, 2009; Mäkinen and Ohl, 2015) and the imperfect information in general. Existing empirical works testing the Full Information Rational Expectations (FIRE) hypothesis (Mankiw et al., 2003; Andrade and Le Bihan, 2013; Coibion and Gorodnichenko, 2012, 2015) provide strong evidence against perfect information. We contribute by quantifying the cyclicality of consumers' information imperfection in the U.S. within a structural noisy-information model.

Layout The rest of the paper is organized as follows. Section 2 describes the theoretical model. Section 3 provides analytical results about the endogenous uncertainty channel. Section 4 provides an empirical quantification of our mechanism in the fully-fledged model and it discusses the transmission channels of macroeconomic shocks. Section 5 concludes.

2 A New-Keynesian Model with Endogenous Uncertainty

We base our model on the noisy information New-Keynesian framework, in the spirit of Woodford (2001) and Lorenzoni (2009), which we extend in two dimensions. First, drawing on the premise that economic activity generates information, we introduce pro-cyclical signal precision, leading to endogenous, time-varying uncertainty. Second, we account for household precautionary-saving behavior by departing from the linear framework. These two extensions give rise to a novel endogenous-uncertainty channel for aggregate shock propagation, driven by the feedback loop between consumer uncertainty and economic activity – a mechanism absent in the standard noisy information New-Keynesian model. We present our model in three steps. First, we lay out the model environment featuring the elements that are standard to a New-Keynesian model. Second, we describe the information structure and belief updating. Third, we define an equilibrium and characterize the joint determination of economic activity and agents' beliefs.

2.1 Environment

The economy features a representative household that consumes, saves, and supplies labor, and monopolistically competitive firms that use labor to produce differentiated goods and set prices subject to Rotemberg price adjustment costs. These differentiated goods are aggregated into a final consumption good purchased by the household and government. Policy is set by a central bank following a Taylor rule, with government spending financed by lump-sum taxes. Aggregate productivity shocks drive economic fluctuations. Note that all expectation operators, $\mathbb{E}(.)$, represent expectations conditional on the (imperfect) information available to agents at the time they make their decisions.

2.1.1 Household

A representative household chooses paths of consumption, saving portfolio, and labor to maximize its expected lifetime utility

$$\max_{C_t, L_t} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\eta}}{1-\eta} - \xi \frac{L_t^{1+\omega}}{1+\omega} \right), \tag{1}$$

where C_t denotes the household's consumption, and L_t is the labor supply (hours worked). Parameter η measures the household's relative aversion to risk, ω is the inverse Frisch elasticity of labor supply, ξ is a scale parameter and β is the discount factor. The household lifetime utility in Eq. (1) is subject to the following sequence of budget constraints

$$P_t C_t + Q_t B_t = B_{t-1} + P_t W_t L_t + D_t, \tag{2}$$

where P_t is the price of the final consumption good, W_t is the real wage, B_t are holdings of riskless bonds, Q_t is the bond price, D_t are lump-sum transfers from firm ownership and government. The household's optimization yields the Euler equation and the labor supply equation, given respectively by

$$Q_t = \beta \cdot \mathbb{E}_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\eta} \cdot \frac{1}{\pi_{t+1}} \right\}, \tag{3}$$

$$\xi L_t^{\omega} = C_t^{-\eta} W_t, \tag{4}$$

where $\pi_t \equiv \frac{P_t}{P_{t-1}}$ denotes the gross inflation rate.

2.1.2 Firms

A continuum of monopolistically competitive firms indexed by $i \in [0,1]$ produces differentiated goods $Y_t(i)$ which are then aggregated into the final output via the CES aggregator $Y_t = \left(\int_0^1 Y_t(i)^{\frac{\epsilon-1}{\epsilon}} \mathrm{d}i \right)^{\frac{\epsilon}{\epsilon-1}}$, where ϵ is the elasticity of substitution between differentiated goods. The demand for type-i good is

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon} Y_t, \tag{5}$$

where $P_t(i)$ is the price of good i, and the aggregate price index is $P_t = \left(\int_0^1 P_t(i)^{1-\epsilon} \mathrm{d}i \right)^{\frac{1}{1-\epsilon}}$.

Firm i produces its output using the following production technology

$$Y_t(i) = \tilde{A}_t L_t(i)^{1-\alpha}, \tag{6}$$

where $\tilde{A}_t$ is the aggregate productivity, which will be described in detail below, and $(1-\alpha)$ denotes the returns to scale.

Each firm i acts as a price-setter by choosing the price of its good, $P_t(i)$, while facing nominal quadratic costs of price adjustment a la Rotemberg (1982). Each firm maximizes its expected discounted stream of future profits

$$\max_{P_t(i)} \mathbb{E}_0 \sum_{t=0}^{\infty} Q_{0,t} \left\{ P_t(i) Y_t(i) - (1-\bar{\tau}) P_t W_t L_t(i) - \frac{\Phi}{2} \left(\frac{P_t(i)}{P_{t-1}(i)} - 1 \right)^2 P_t Y_t \right\}, \tag{7}$$

subject to the sequence of firm-specific demand given by Eq. (5). Here $\bar{\tau} = \epsilon^{-1}$ is a standard labor subsidy ensuring the efficiency of the flexible-price equilibrium. Let Φ denote the strength of price adjustment cost that drives the degree of price rigidity. Finally, $Q_{t,t+s} \equiv \beta^s \, \mathbb{E}_t \left[(C_{t+s}/C_t)^{-\eta} / \prod_{j=1}^{s} \pi_{t+j} \right]$ denotes the stochastic discount factor.

The first-order condition for profit maximization, evaluated in a symmetric equilibrium, yields the conventional New-Keynesian Phillips curve

$$\epsilon \left(1 - [1-\bar{\tau}] MC_t\right) = 1 - \Phi \left(\pi_t - 1\right) \pi_t + \Phi \mathrm{E}_t \left\{ Q_{t,t+1} \left(\pi_{t+1} - 1\right) \pi_{t+1} \frac{Y_{t+1}}{Y_t} \right\}, \tag{8}$$

where MC_t denotes the real marginal cost. The labor demand equation, derived from the firms' cost-minimization problem and evaluated at the symmetric equilibrium, is given by:

$$W_t = (1-\alpha)\, \tilde{A}_t MC_t L_t^{-\alpha}. \tag{9}$$

2.1.3 Policy and resource constraint

The central bank sets the nominal interest rate R_t according to the Taylor (1993) rule

$$\frac{R_t}{\bar{R}} = \left(\frac{\pi_t}{\bar{\pi}}\right)^{\phi_\pi}, \tag{10}$$

where $\bar{R} = 1/\beta$ is the steady-state nominal interest rate (with $\bar{\pi} = 1$). Parameter ϕ_π determines the sensitivity of the interest rate to inflation. The nominal rate and the bond prices are linked as $R_t = 1/Q_t$. We also assume that there is an exogenous path of government spending G_t financed through a lump-sum tax.

Aggregate output is either consumed by households and the government or spent as a price adjustment cost. Hence, the aggregate resource constraint is given by

$$Y_t = C_t + G_t + \frac{\Phi}{2}(\pi_t - 1)^2 Y_t. \tag{11}$$

2.1.4 Productivity

Log-productivity, $\tilde{a}_t \equiv \log(\tilde{A}_t)$, consists of two components: a persistent component a_t and a transitory component f_t, such that

$$\tilde{a}_t = a_t + f_t, \tag{12}$$

where f_t is an uncorrelated process $f_t \sim \mathcal{N}(0, \sigma_f^2)$ and a_t is an AR(1) process

$$a_t = (1-\rho_a)\bar{a} + \rho_a a_{t-1} + \epsilon_t^a. \tag{13}$$

Here ρ_a is the persistence of a_t, and the innovation is normally distributed $\epsilon_t^a \sim \mathcal{N}(0, \sigma_a^2)$. The two shocks, f_t and ϵ_t^a are mutually independent. We normalize steady-state productivity, such that $\bar{a} = 0$.

2.2 Information structure

We depart from the typical full-information model by assuming that agents observe the current level of productivity, $\tilde{a}_t$, but they *cannot* disentangle between the transitory component f_t and the persistent component a_t, which is a common assumption in the noisy information New-Keynesian literature (Lorenzoni, 2009). The persistent component of productivity constitutes the true state of the economy. While this true state is unknown to agents, they form beliefs about it based on the available noisy information.

At the beginning of period t, agents hold a prior belief regarding a_t. As period t progresses, agents update their beliefs using noisy signals obtained from two sources: (*i*) the observed realized productivity $\tilde{a}_t$, and (*ii*) an additional "learning-by-doing" signal of time-varying precision. As explained below, the precision of the learning-by-doing signal positively depends on the level of output, hence information quality is pro-cyclical.

2.2.1 Priors

Let Ω_t denote the information set available to households at the beginning of period t, before they receive any signals pertaining to the *current* period. Given this information, we denote the prior belief about the persistent productivity component as

$$a_t|\Omega_t \sim \mathcal{N}(\theta_t, \gamma_t^{-1}), \tag{14}$$

where θ_t is the perceived mean, and γ_t^{-1} is the perceived variance (γ_t is the precision of the information in Ω_t).

2.2.2 Signals

During period t, agents receive two noisy signals. The first signal, z_t, amounts to observing the productivity level $\tilde{a}_t$. The second signal, s_t, is the learning-by-doing signal.

Formally, signal z_t is simply $z_t = \tilde{a}_t$. Given this signal, the belief about the persistent productivity component is $a_t|z_t \sim \mathcal{N}(\tilde{a}_t, [\gamma^z]^{-1})$, where $\gamma^z = \sigma_f^{-2}$ is the signal precision. From Eq. (12), we can express the noisy signal z_t as

$$z_t = a_t + \epsilon_t^z \tag{15}$$

where $f_t = \epsilon_t^z \sim \mathcal{N}(0, [\gamma^z]^{-1})$ corresponds to a noise shock.

The second signal that agents receive regarding the persistent component a_t is a noisy learning-by-doing signal, denoted by s_t, with pro-cyclical precision. It is defined as

$$s_t = a_t + \epsilon_t^s, \tag{16}$$

where $\epsilon_t^s \sim \mathcal{N}(0, [\gamma_t^s]^{-1})$ is a noise shock. The precision of this signal is time-varying and increases with the level of economic activity, such that $\gamma_t^s = \gamma(Y_t/\bar{Y})$ where γ characterizes the strength of signal precision pro-cyclicality and $\bar{Y}$ is the steady-state level of output. The pro-cyclical precision of information is a novel feature of our model relative to the New-Keynesian imperfect-information framework of Lorenzoni (2009). It reflects the phenomenon that higher economic activity generates information, allowing agents to form more precise beliefs, in the spirit of Van Nieuwerburgh and Veldkamp (2006); Fajgelbaum et al. (2017); Ilut and Saijo (2021), among others.[3] More specifically, higher economic activity exposes agents to more interaction with the production technology, allowing them to learn more accurately about the underlying state of the economy. Here, we assume that the positive dependence of signal precision on output reflects the "learning-by-doing" nature of this information source, meaning that each unit of production generates information. Alternatively, one could interpret the pro-cyclicality of signal's precision through a social learning process, i.e. by assuming that workers trade information about productivity on the labor market. In Appendix A, we describe the microfoundations of these two learning protocols that both result in the same pro-cyclical information precision behavior within the context of our model.

2.2.3 Beliefs formation

Solving the signal extraction problem using Bayesian updating yields the agents' expectation of a_t based on the prior information Ω_t and two signals z_t and s_t

$$\mathrm{E}(a_t|\Omega_t, s_t, z_t) = \frac{\gamma_t\theta_t + \gamma^z z_t + \gamma_t^s s_t}{\gamma_t + \gamma^z + \gamma_t^s}, \tag{17}$$

$$\mathrm{Var}(a_t|\Omega_t, s_t, z_t) = \frac{1}{\gamma_t + \gamma^z + \gamma_t^s}, \tag{18}$$

where $\Omega_{t+1} = \{\Omega_t, z_t, s_t\}$ is the set of information available at the end of the period t (beginning of the period $t+1$). By combining the AR(1) productivity process in Eq. (13) with the mean belief in Eq. (17), we can derive the beliefs about a_{t+1} at the beginning of period $t+1$, denoted as $\theta_{t+1} \equiv \mathrm{E}(a_{t+1}|\Omega_{t+1})$

$$\begin{aligned}\theta_{t+1} &= (1-\rho_a)\bar{a} + \rho_a \mathrm{E}(a_t \mid \Omega_t, s_t, z_t) + \mathrm{E}(\epsilon^a_{t+1} \mid \Omega_{t+1}), \\ &= (1-\rho_a)\bar{a} + \rho_a \frac{\gamma_t\theta_t + \gamma^z z_t + \gamma_t^s s_t}{\gamma_t + \gamma^z + \gamma_t^s}. \end{aligned} \tag{19}$$

[3] Fajgelbaum et al. (2017) and Ilut and Saijo (2021) justify their modeling choices through the production-based information received by firms at a disaggregated level. Instead, in the spirit of Van Nieuwerburgh and Veldkamp (2006), we assume that the *aggregate* economic activity is the only source of information contained in the signal received by all agents

The precision of these beliefs, denoted as $\gamma_{t+1} \equiv [\text{Var}(a_{t+1}|\Omega_{t+1})]^{-1}$, is obtained by combining Eq. (13) with Eq. (18)

$$\gamma_{t+1} = \left[\rho_a^2 \, \text{Var}(a_t \mid \Omega_t, s_t, z_t) + (1-\rho_a)^2 \text{Var}(\bar{a}) + \text{Var}(\epsilon_t^a)\right]^{-1},$$
$$= \left[\frac{\rho_a^2}{\gamma_t + \gamma^z + \gamma_t^s} + \sigma_a^2\right]^{-1}. \tag{20}$$

Eq. (19) and (20) jointly establish the recursive law of motion for beliefs regarding the productivity component, a_t. They capture the feedback loop between signal's precision – and thus macroeconomic uncertainty – and economic activity.

2.3 Equilibrium

All equations of the model are summarized in Appendix B and the model's equilibrium is defined as follow

Definition. *A dynamic symmetric equilibrium in this economy is a sequence of quantity variables, $\{C_t, L_t, Y_t, G_t, D_t\}_{t=0}^{\infty}$, price variables, $\{P_t, W_t, Q_t, R_t\}_{t=0}^{\infty}$, and beliefs, $\{\theta_t, \gamma_t, z_t, s_t, \gamma_t^s\}_{t=0}^{\infty}$, such that: (i) households' and firms' optimality conditions are satisfied; (ii) all firms set their prices to P_t; (iii) beliefs follow their recursive law of motion; (iv) paths of quantities and prices are consistent with beliefs; (v) signal precision is consistent with output path; (vi) the interest rate is consistent with the monetary rule; and (vii) all markets clear.*

Note that the precision of information depends on output, while equilibrium output in turn depends on the precision of information, as stated in (*iv*)–(*v*) of the Equilibrium definition. This two-way feedback lies at the heart of our endogenous-uncertainty amplification mechanism: with pro-cyclical signal precision, uncertainty and economic activity are jointly determined in equilibrium.

3 The Uncertainty Channel: a Stylized Approach

In this section, we analyze the role of pro-cyclical information quality in the transmission of shocks within a tractable version of the model. Specifically, we show that the combination of endogenous uncertainty – stemming from pro-cyclical information quality – and the precautionary-saving motive gives rise to a novel transmission mechanism absent in the standard New-Keynesian model: the endogenous-uncertainty channel. Capturing the precautionary motive requires going beyond a standard linear approximation to include the higher-order terms through which uncertainty shapes agents' decisions; we achieve this using a risk-adjusted log-linear approximation, which allows uncertainty to matter while keeping the model tractable.

Specifically, in the spirit of Skinner (1988), we augment an otherwise linear Euler equation by an additional second-order term that captures precautionary-saving behavior.

In terms of notation, we use lowercase letters to denote the logarithms of their corresponding uppercase variables, such that $x_t = log(X_t)$ unless specified otherwise. Additionally, we denote deviations from the steady state with $\hat{x}_t$. All derivations are provided in Appendix C.

3.1 Tractability assumptions

We consider here a fully tractable version of our baseline setup, obtained by making several simplifying assumptions. First, we assume that nominal price rigidities apply only in the current period t. This implies that the inefficient wedge between the wage and the marginal product of labor exists only in the current period and not in future periods. Second, we assume that households save in real bonds to abstract from the inflation-risk premium created by inflation expectations. Third, we assume a linear production function ($\alpha = 0$), and a zero steady-state government spending ($\bar{G} = 0$).[4] Fourth, we assume that there is no prior information about productivity, meaning zero precision $\gamma_t = 0$.[5]

3.2 Inefficiency wedge and output path

Given our assumption, log-linearizing the labor demand Eq. (9) yields the link between wage and productivity in every period

$$w_{t+j} = mc_{t+j} + \tilde{a}_{t+j} \quad \text{with} \begin{cases} mc_{t+j} \neq 0 & \text{if} \quad j = 0 \\ mc_{t+j} = 0 & \text{if} \quad j > 0 \end{cases} . \tag{21}$$

Here, mc_{t+j} denotes the logarithm of real marginal cost, capturing the inefficient wedge between wages and productivity. In period t, real marginal cost is nonzero because price rigidity generates inefficiency. In later periods ($t + j$, $j > 0$), real marginal cost is zero, as the economy becomes efficient and nominal marginal cost equals the price.

By combining labor supply (4), labor demand (9), production function (6), and resource constraint (11), we obtain the following expressions for output in period t and subsequent periods

$$y_t = \left(\frac{1}{\omega + \eta}\right) mc_t + \left(\frac{1 + \omega}{\omega + \eta}\right) \tilde{a}_t + \left(\frac{\eta}{\omega + \eta}\right) g_t, \tag{22}$$

$$y_{t+j} = \left(\frac{1 + \omega}{\omega + \eta}\right) \tilde{a}_{t+j} + \left(\frac{\eta}{\omega + \eta}\right) g_{t+j}, \quad \text{for } j > 0. \tag{23}$$

[4] Assuming a linear production function and zero steady-state government spending simplifies the derivations, but is not required for tractability and does not affect our main theoretical results.

[5] This assumption simplifies exposition without affecting our main result. For a more general case allowing for an arbitrary value of γ_t, see Appendix C

where $g_t = \frac{G_t}{\bar{Y}}$ is the ratio of government spending to steady-state output.

Eq. (22) shows that period t output depends on the endogenous inefficient wedge mc_t. In contrast, for subsequent periods ($j > 0$), Eq. (23) shows that output is fully determined by exogenous factors—technology and government spending.

3.3 IS curve and law of motion of beliefs

To analyze the endogenous-uncertainty channel, we derive a risk-adjusted IS curve that incorporates the precautionary-saving motive by combining the Euler equation (3) with the resource constraint (11). The resulting expression is

$$y_t = (1 - \rho_g) g_t - \frac{1}{\eta}(r_t - \rho) + \mathrm{E_t}\{y_{t+1}\} - \frac{1}{2}(1 + \eta)\mathrm{Var_t}\{y_{t+1}\}, \tag{24}$$

where $\rho = -log(\beta)$, r_t is the real interest rate, ρ_g is the persistence of government spending, such that $g_{t+1} = \rho_g g_t$.

Eq. (24) shows the aggregate-demand determinants of output. The first three terms form a standard New-Keynesian IS curve (Galí, 2008): an increase in government spending g_t raises output, a higher real interest rate r_t reduces output by incentivizing households to save, and higher expected future output $E_t\{y_{t+1}\}$ positively influences current output. Importantly, our IS curve is augmented with a fourth term that reflects uncertainty about future output, expressed in terms of variance, $\mathrm{Var}_t\{y_{t+1}\}$. This term captures the negative link between current output and uncertainty about future output, as higher uncertainty forces households to reduce their current consumption to accumulate precautionary savings.

From Eq. (24), it is clear that current output depends on *beliefs* about future output, specifically $\mathrm{E_t}\{y_{t+1}\}$ and $\mathrm{Var_t}\{y_{t+1}\}$. Our tractable setup enables us to compute these beliefs analytically in terms of the exogenous variables $\tilde{a}_t$ and g_t using Eq. (23)

$$\mathrm{E_t}\{y_{t+1}\} = \left(\frac{1+\omega}{\omega+\eta}\right)\mathrm{E_t}\{\tilde{a}_{t+1}\} + \rho_g \left(\frac{\eta}{\omega+\eta}\right) g_t, \tag{25}$$

$$\mathrm{Var_t}\{y_{t+1}\} = \left(\frac{1+\omega}{\omega+\eta}\right)^2 \mathrm{Var_t}\{\tilde{a}_{t+1}\}. \tag{26}$$

Beliefs about output y_{t+1} can be constructed from productivity beliefs θ_{t+1} and γ_{t+1}. Specifically, $\mathrm{E_t}\{\tilde{a}_{t+1}\} = \mathrm{E_t}\{a_{t+1}\} = \theta_{t+1}$ and $\mathrm{Var_t}\{\tilde{a}_{t+1}\} = \mathrm{Var_t}\{a_{t+1}\} + [\gamma^z]^{-1} = \gamma_{t+1}^{-1} + [\gamma^z]^{-1}$. Using the law of motion for beliefs about the persistent component a_t, as given by Eq. (19) and (20), we derive the linear approximation of beliefs about future productivity, $\tilde{a}_{t+1}$, as follows

$$\mathrm{E_t}\{\tilde{a}_{t+1}\} = (1-\rho_a)\bar{a} + \rho_a[v s_t + (1-v)z_t], \tag{27}$$

$$\mathrm{Var_t}\{\tilde{a}_{t+1}\} = \sigma_a^2 + [\gamma^z]^{-1} + \frac{\rho_a^2}{\gamma}v(1+v\bar{y}) - \frac{\rho_a^2}{\gamma}v^2 \cdot y_t, \tag{28}$$

where $v \equiv \frac{\gamma}{\gamma+\gamma^z}$ is the steady-state weight given to signal s_t when forming beliefs.[6] Note that in Eq. (28), uncertainty about future productivity is decreasing in output, which is a consequence of procyclical precision of signal s_t.

We substitute the beliefs from Eq. (27) and (28) into Eq. (25) and (26), and then plug the result into the IS curve, Eq. (24) which yields output as a function of interest rate and exogenous variables. Expressing output in terms of deviations from the steady state gives

$$\hat{y}_t = f \cdot \left[-\frac{1}{\eta}\hat{r}_t + \frac{1+\omega}{\omega+\eta}\rho_a\big(v s_t + (1-v)z_t\big) + \Big(1 - \rho_g \cdot \frac{\omega}{\omega+\eta}\Big) g_t \right], \tag{29}$$

$$\text{where } f \equiv \left[1 - \frac{1}{2} \cdot \frac{\rho_a^2(1+\omega)^2(1+\eta)}{(\omega+\eta)^2} \cdot \frac{\gamma}{(\gamma^z+\gamma)^2} \right]^{-1}. \tag{30}$$

For economically reasonable parameter values— where positive signals s_t and z_t raise output—the composite parameter f is positive and satisfies $f \geq 1$. From Eq. (30), we see that parameter f depends on the quality of the pro-cyclical signal γ. Hence, f governs the intensity of amplification generated by endogenous uncertainty. When the pro-cyclical signal is absent, $\gamma = 0$, we have $f_{|\gamma=0} = 1$, that we label the constant-uncertainty case. When the pro-cyclical signal is present, $\gamma > 0$, we have amplification of the effect of shocks on output, that is $f_{|\gamma>0} > 1$, which corresponds to the endogenous-uncertainty case.

Note that f is non-monotonic in γ. When the signal s_t is highly imprecise ($\gamma \to 0$), we have $f \to 1$, and the endogenous-uncertainty channel is inactive because output has little effect on information precision. Conversely, when the signal is highly precise ($\gamma \to \infty$), representing an almost full-information case, we also have $f \to 1$; the channel is again inactive since additional economic activity provides negligible new information. In other words, the marginal value of information from output is irrelevant when signals are either very noisy or nearly perfect.

For intermediate values of signal pro-cyclicality, $\gamma \in (0, \infty)$, f first increases with γ, reaches a maximum at $\gamma = \gamma^z$, and then decreases. Hence, the endogenous-uncertainty channel is strongest for intermediate values of signal pro-cyclicality where output provides relevant information to improve the precision of the signal, i.e. parameter γ is neither too high nor too low. In Section 4, we exploit empirical data about consumers' forecast errors to quantify γ.

[6] Due to our assumption of no prior information being available ($\gamma_t = 0$), beliefs are constructed based on two signals, s_t and z_t, with weights v and $1-v$, respectively.

3.4 Aggregate demand effect of productivity shock

We now turn to the analysis of aggregate-demand amplification induced by the endogenous-uncertainty channel. To this end, we examine the economy's response to a productivity shock – an innovation in the level of the persistent productivity component, a_t. Following the New-Keynesian tradition, we use the output gap as our preferred measure of aggregate demand. Using Eq. (22), we first define the flexible-price output as

$$\hat{y}_t^f = \left(\frac{1+\omega}{\omega+\eta}\right)\tilde{a}_t + \left(\frac{\eta}{\omega+\eta}\right)g_t, \tag{31}$$

which is the output that would prevail in a flexible price economy. The output gap is then defined as the deviation of actual output from its flexible price counterpart, expressed as

$$\tilde{y}_t = \hat{y}_t - \hat{y}_t^f = \left(\frac{1}{\omega+\eta}\right)mc_t. \tag{32}$$

Note that in this economy, consumer price inflation also serves as a measure of aggregate demand, as it is fully determined by the output gap. To illustrate this, we log-linearize the Phillips curve (8), which, under our assumption of flexible prices in all periods except t, yields

$$\pi_t = \frac{\epsilon}{\Phi}\cdot mc_t = \frac{\epsilon(\omega+\eta)}{\Phi}\tilde{y}_t. \tag{33}$$

Let assume that the central bank controls the real interest rate and targets the output gap according to the rule $\hat{r}_t = \phi\tilde{y}_t$. This is equivalent to inflation targeting, as inflation and the output gap are linked through the Phillips curve, as given by Eq. (33). The corresponding monetary rule for inflation targeting is $\hat{r}_t = \phi'\pi_t$, where $\phi' = \phi\frac{\Phi}{\epsilon(\omega+\eta)}$.

Combining Eq. (29) and Eq. (31), we obtain that output gap satisfies

$$\left[1+\frac{\phi}{\eta}\cdot f\right]\cdot\tilde{y}_t = \frac{1+\omega}{\omega+\eta}\cdot\left[-\tilde{a}_t + \rho_a f(vs_t + (1-v)z_t)\right] + f\cdot\left(1-\frac{\omega\rho_g}{\omega+\eta}-\frac{\eta}{\omega+\eta}\right)g_t. \tag{34}$$

To construct the response of the output gap to productivity shock, we abstract from government spending by setting $g_t = 0$, and from noise shocks with $\epsilon_t^z = 0$ and $\epsilon_t^s = 0$. Under these assumptions, we have $z_t = s_t = \tilde{a}_t = a_t$. The link between productivity and the output gap is given by the following Proposition.

Proposition 1 (Aggregate demand effect of a productivity shock)**.** *The effect of a productivity shock a_t on the output gap $\tilde{y}_t$ is a sum of two effects, and is given by*

$$\tilde{y}_t = -\psi(1-\rho_a)\,a_t + \psi\rho_a(f-1)\,a_t, \tag{35}$$

where $\psi = \left[1 + \frac{\phi}{\eta} \cdot f\right]^{-1} \frac{1+\omega}{\omega+\eta}$. *The first term captures the standard New-Keynesian effect, while the second term represents the contribution of the endogenous-uncertainty channel.*

Proposition 1 characterizes the response of the output gap, $\tilde{y}_t$, to productivity shocks, a_t, decomposed into two effects. The first term in Eq. (35) captures the standard New-Keynesian mechanism: when $\rho_a < 1$, a *negative* productivity shock *raises* the output gap because actual output declines by less than it would under flexible prices. This happens because price rigidities prevent firms from increasing prices enough to replicate the flexible-price allocation, leaving prices inefficiently low. The resulting excess demand generates a positive output gap. The second term in Eq. (35) is active when $f > 1$ and captures the endogenous-uncertainty channel of shock transmission. Through this channel, a negative productivity shock leads to a *decrease* of the output gap. When the endogenous-uncertainty channel dominates the standard New-Keynesian channel–specifically, when $f > \rho_a^{-1}$ – a negative productivity shock generates a *negative* output gap. In other words, the endogenous-uncertainty channel can change the sign of the effect of productivity shocks on aggregate demand, relative to the textbook New-Keynesian model. In this case, a negative productivity shock behaves like a Keynesian-supply shock triggering a demand-driven recession.

Note that the effect of a productivity shock on flexible-price output, computed from Eq. (31), is $\hat{y}_t^f = \frac{1+\omega}{\omega+\eta} a_t$, and does not depend on the endogenous-uncertainty channel. The implications in terms of monetary policy are summarized in the following Corollary.

Corollary 1.1 (Aggregate-demand nature of the endogenous-uncertainty channel)**.** *The endogenous-uncertainty channel operates solely through the output gap. By closing the output gap, monetary policy can fully neutralize its effects.*

To understand the result in Corollary 1.1, note that flexible-price output given by Eq. (31) is not affected by the information imperfection: the market allocation under flexible prices is identical to that of a full-information model. This is because, under flexible prices, output is fully determined by supply, and demand-side frictions are compensated by the price adjustment. The demand-side information friction matters only as long as there is price rigidity, bringing about the aggregate-demand effects. Then, the stronger the information friction is, the larger the demand recession induced by a negative productivity shock. Hence, by closing the output gap, monetary policy is able to restore flexible prices and full-information allocation.

Finally, let examine how the endogenous-uncertainty channel affects the responses of output, inflation, and hours worked to a technology shock. Notice that the responses of actual output and inflation to the productivity shock are

$$y_t = f \cdot \psi \cdot \left[\frac{\phi}{\eta} + \rho_a\right] a_t, \tag{36}$$

$$\pi_t = \frac{\epsilon\rho_a(\omega+\eta)}{\Phi}\left[f - \frac{1}{\rho_a}\right] a_t. \tag{37}$$

Eq. (36) shows that the endogenous-uncertainty channel ($f > 1$) amplifies the output response to a productivity shock. The sign of inflation response in Eq. (37) tracks the sign of the output gap, due to the tight link between the two, see Phillips curve (33). In other words, inflation turns negative whenever the output gap is negative ($f > \rho_a^{-1}$).

The response of hours worked to productivity shock can be written as:

$$l_t = \tilde{l}_t + l_t^f = \tilde{y}_t - \frac{\eta-1}{\omega+\eta} a_t. \tag{38}$$

Here, $l_t^f = -\frac{\eta-1}{\omega+\eta} a_t$ denotes efficient hours worked in a flexible-price economy, and $\tilde{l}_t = \tilde{y}_t$ represents the inefficient hours gap. Efficient hours are negatively related to productivity as long as risk aversion satisfies $\eta \geq 1$, while the hours gap is positively related to the output gap. In the model without endogenous uncertainty, both efficient hours and the hours gap increase following a negative productivity shock, leading to higher total hours worked, consistent with the textbook New-Keynesian model (Galí, 2008). When the endogenous-uncertainty channel is present, however, the response of hours worked becomes ambiguous: efficient hours still rise, but the hours gap may fall in response to a negative productivity shock.

3.5 Crowding out/in effect of public spending shocks

We also examine the role of the endogenous-uncertainty channel in propagating the government spending shock. For this exercise, we assume that $z_t = s_t = \tilde{a}_t = 0$ and that $g_t > 0$, while monetary policy continues to respond to the output gap. The link between private consumption and government spending is given by the following Proposition.

Proposition 2 (Crowding out/in of private consumption)**.** *The link between private consumption and government spending is given by*

$$c_t = -f\tilde{\psi}\left[\rho_g + \frac{\phi}{\eta}\right] g_t + (f-1)\tilde{\psi}\left[1 + \frac{\eta}{\omega}\right] g_t \tag{39}$$

where $\tilde{\psi} = \frac{\omega}{1+\omega}\psi$*.*

Under constant uncertainty, i.e. $f = 1$, the first component only in the right-hand side of Eq. (39) survives. It captures the standard *crowding-out* effect in consumption: when government spending rises and is financed by a lump-sum tax, households' lifetime wealth falls, leading to a decline in private consumption. The strength of the crowding-out effect increases with government spending persistence, ρ_g, and with strength of monetary policy reaction, ϕ, as it was emphasized by Leeper et al. (2017). Two opposite forces emerge when we assume endogenous uncertainty, i.e. $f > 1$. On the one hand, endogenous uncertainty generates an additional term (last term) in Eq. (39) which captures the *crowding-in* effect in consumption of the endogenous-uncertainty channel: higher government spending boosts output, lowering uncertainty, which reduces household's precautionary savings and boots private consumption. On the other hand, endogenous uncertainty also amplifies the traditional crowding-out effect in consumption since the first term in Eq. (39) is multiplied by the intensity parameter f. The reason is that under endogenous uncertainty, consumers put more weight on expected output (see Equ. (30)). The final effect of a positive public spending shock on consumption depends on whether $(\rho_g + \phi/\eta)$ is lower or larger than $(1 + \eta/\omega)$. For instance, when the public spending shock is highly persistent (ρ_g large), the persistent negative wealth effect dominates the precautionary saving effect and thus the crowding-out of consumption is amplified.

3.6 Graphical illustration

A pro-cyclical signal precision implies a feedback loop since uncertainty (less precise signals) affects economic activity while economic activity affects uncertainty. In other words, prices, quantities, and uncertainty are jointly determined in equilibrium. To graphically illustrate the joint equilibrium adjustment of economic activity and uncertainty to a shock, we derive the aggregate demand (AD), aggregate supply (AS), and endogenous uncertainty (EU) curves. The AD curve is constructed by substituting the monetary rule $r_t = \bar{r} + \phi' \pi_t$ and output beliefs (25)-(26) into the IS curve, Eq. (24). The AS curve is obtained by substituting the output gap expression $\tilde{y}_t = y_t - y_t^f$ and flexible output Eq. (31) into the Phillips curve, Eq. (33). The EU curve is given by Eq. (28). The resulting set of curves is

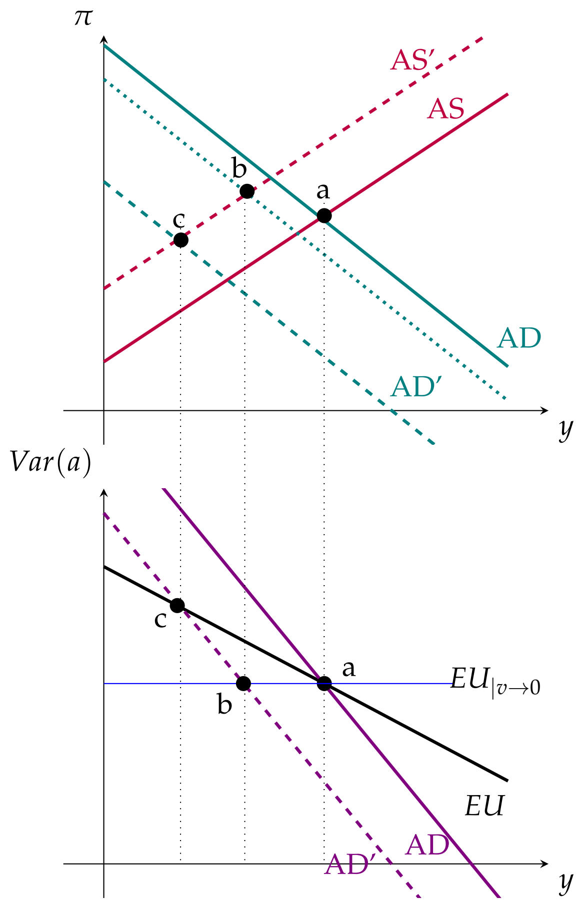
$$\textbf{AD:} \quad y_t = -\frac{1}{\eta}(\phi' \pi_t + \bar{r} - \rho) + \frac{1+\omega}{\omega+\eta} \mathrm{E_t}\{\tilde{a}_{t+1}\} - \frac{1}{2}(1+\eta)\left(\frac{1+\omega}{\omega+\eta}\right)^2 \mathrm{Var_t}\{\tilde{a}_{t+1}\},$$

$$\textbf{AS:} \quad \pi_t = \frac{\epsilon(\omega+\eta)}{\Phi}\left(y_t - \frac{1+\omega}{\omega+\eta}\tilde{a}_t\right),$$

$$\textbf{EU:} \quad \mathrm{Var_t}\{\tilde{a}_{t+1}\} = \sigma_a^2 + \frac{1}{\gamma^z} + \frac{\rho_a^2}{\gamma} v(1 + v\bar{y}) - \frac{\rho_a^2}{\gamma} v^2 y_t.$$

Figure 1 plots the AD and AS curves on the (y, π) plane, and AD and EU curves on the $(y, Var(a))$ plane. The EU curve is plotted for two cases: the down-sloping curve corresponds to an active endogenous-uncertainty channel, and the horizontal curve corresponds to the case without endogenous uncertainty (denoted as $EU_{|v\to 0}$). The general equilibrium of this economy is characterized by the point of intersection of AD with AS curves on the upper figure, and AD

Figure 1: Adjustments after negative productivity shock: illustration



Notes: The figure plots the equilibrium adjustment in economic activity (upper figure) and uncertainty (lower figure) to a negative productivity shock. Point (a) is the initial equilibrium, point (b) is the equilibrium without endogenous uncertainty, and point (c) is the equilibrium with endogenous uncertainty.

and EU curves on the lower figure.

Let the economy be initially at the equilibrium point (a). We consider the adjustment of the equilibrium to a negative productivity shock. The negative productivity shock shifts the AS and AD curves left – AS curves shifts as efficient output drops, and the AD curve shifts as expected future productivity declines. In the absence of endogenous-uncertainty channel, the new equilibrium is point (b) with lower output and higher inflation. However, if endogenous uncertainty is active, the equilibrium on the lower figure is given by the point of intersection of the new AD′ curve and the downward-sloping EU curve, that is, point (c). The point (c) features lower output and higher uncertainty. The fact that uncertainty increases in response to a shock translates into a further shift of the AD curve on the upper figure, to arrive at the equilibrium point (c) with a decrease in both output and inflation.

4 Quantification of Endogenous Uncertainty Channel

We now turn to the quantitative evaluation of the endogenous-uncertainty channel within the full, nonlinear New-Keynesian model laid out in Section 2. To this end, we parametrize the model with US data and solve it using a third-order perturbation method. We then examine the quantitative role of endogenous uncertainty in the transmission of productivity and public spending shocks, and analyze how this mechanism interacts with monetary policy. Further details on the solution method and simulation procedures are provided in Appendix E.

4.1 Parametrization

We parametrize the model at a quarterly frequency. The parameters fall into two categories. The first category contains standard parameters, which we calibrate to match conventional data moments or take directly from the literature. The second category consists of parameters specific to our endogenous-uncertainty mechanism. These include the signal-precision parameters γ and γ^z from the noisy-information block of the model, as well as σ_a; together, these parameters govern the law of motion for households' beliefs. We estimate this second set of parameters by matching moments of consumer income forecast errors constructed from the Michigan Survey of Consumers. Next, we describe (*i*) the calibration of the standard parameters, (*ii*) the construction of the survey-based forecast errors, and (*iii*) the estimation procedure for the information parameters.

4.1.1 Calibrated parameters

Table 1 reports the calibrated parameters, which are either chosen to match steady-state moments or taken from the literature. The time discount factor is set to $\beta = 0.99$, implying an annual interest rate of approximately 4% in the steady state. The coefficient of relative risk aversion is set to $\eta = 3$, and we assume a unitary Frisch elasticity of labor supply ($\omega = 1$), a standard value in the business-cycle literature. The scale parameter ξ is set to normalized steady-state labor to $\bar{L} = 0.3$. The elasticity of substitution across varieties is set to $\epsilon = 8$, which delivers a steady-state markup of roughly 14.5%, consistent with the estimates of Farhi and Gourio (2018). We set the production-function parameter $\alpha = 0.3$, implying a labor share of about 60%, in line with the empirical evidence in Autor et al. (2020). The price adjustment cost parameter is set to $\Phi = 100$, which maps – under our assumed elasticity of substitution – to a Calvo probability of price non-adjustment of approximately 0.75 (to a first-order approximation), implying an average price duration of about 3.5 quarters. This is in line with the findings of Nakamura and Steinsson (2013). The monetary policy parameter governing the response of the interest rate to inflation is set to the conventional value $\phi_\pi = 1.5$. Finally, the government spending-to-output ratio is set to

18%, matching its average level in US data.

Table 1: Calibrated parameters

Parameter	**Description**	**Value**	**Source/Target**
β	Discount factor	0.99	Annual interest rate, 4%
η	Degree of risk aversion	3	Standard Value
ω^{-1}	Frisch elasticity of labor supply	1	Standard Value
ϵ	Elast. of substitution btw goods	8	Price markup, 14.5%
α	labor-share complement	0.3	Labor Share, 60%
Φ	Price adjustment cost parameter	100	Frequency of price change, 25%
ϕ_π	Interest rate rule parameter	1.5	Standard Value
g/y	Government spending share of GDP	0.18	BEA, 18%

4.1.2 Forecast errors construction

To calibrate the imperfect-information block in our model (parameters γ and γ^z), we exploit variation in households' forecast errors over time. We construct the aggregate income growth forecast errors by aggregating individual forecast errors computed from the Michigan Survey of Consumers (MSC). The MSC is a rotating monthly panel survey in which respondents are eligible to be re-interviewed six months after the initial interview.[7] In each interview, respondents are asked to report their current household income (in dollars) as well as their expected income growth over the next 12 months.[8]

For each month, we consider only respondents who have been re-interviewed at least once. Let $\mathbb{E}_{t-12}[\Delta inc_{j,t}]$ denote the expected income growth reported by the j-th respondent during her first interview, referring to the income growth she expects to receive over the upcoming year. Using the data from two subsequent interviews, we also calculate the realized income growth of household j over the same year, $\Delta inc_{j,t}$. For each month, we compute the individual forecast errors as the difference between expected and realized income growth: $e^j_{t,t-12} = \mathbb{E}_{t-12}[\Delta inc_{j,t}] - \Delta inc_{j,t}$. We then aggregate errors across individuals to obtain a monthly series of the aggregate income growth forecast errors, $e^f_{t,t-12}$, calculated as $e^f_{t,t-12} = \Sigma^J_{j=1}\omega_{j,t}e^j_{t,t-12}$; here $\omega_{j,t}$ is the weight assigned to the j-th respondent's forecast error at time t ($\Sigma^J_{j=1}\omega_{j,t} = 1$) and provided by the MSC, J is the number of respondents. In Appendix D, we provide additional details on

[7] As explained by the Surveys of Consumers Technical Report (2024), "an independent cross-sectional sample is drawn each month, and those who completed interviews in a given month become eligible for re-interviews approximately six and twelve months later. Thus, each monthly sample is composed of a mix of interviews from the independent cross-sectional sample and the recontact sample."

[8] The current income corresponds to 'INCOME: total household income - current dollars' (the asked question is "*Now, thinking about your total income from all sources (including your job), how much did you receive in the previous year?*") and the expected income is 'FAMILY INCOME % u/d next year' (the asked question is "*By about what percent do you expect your income to (increase/decrease) during the next 12 months?*")

the aggregation procedure and Figure D.1 reports the distribution of the income forecast error over time. Intuitively, the approach exploits the fact that idiosyncratic income dispersion across households averages out, leaving only the aggregate component of income growth. The resulting series spans the period 1981m1–2019m12.

4.1.3 Estimation

Next, we outline our strategy for calibrating the remaining parameters γ, γ^z, σ_a, and ρ_a, which govern the evolution of households' uncertainty. To identify the parameters related to signal precision (γ^z and γ), we draw on the approach of Fajgelbaum et al. (2017), using selected moments of the forecast error distribution. In particular, we target the 5th and 95th percentiles of the aggregate forecast error distribution of income growth over time. The idea is that the 5th percentile corresponds to periods of recession, when the precision of the cyclical signal is low and uncertainty is largely driven by the time-invariant signal z, which helps identify γ^z. In contrast, the 95th percentile corresponds to periods of expansion, when output increases and the cyclical signal s_t receives greater weight in the posterior belief, which is informative for pinning down γ. The remaining two parameters that determine the evolution of uncertainty are the standard deviation and persistence of the fundamental shock, σ_a and ρ_a. We choose σ_a to match the empirical unconditional volatility of real private consumption in U.S. data over the period 1981Q1–2019Q4.[9] Parameter ρ_a is calibrated externally and set to 0.97.[10] The reason is that, in our setting, the persistence of the fundamental is inseparable from the informational content of the signal, as belief dynamics depend only on composite objects combining ρ_a with the signal-precision parameters γ and γ^z. Consumption persistence as an additional target does not help to overcome the issue, since this moment is likewise driven by both persistence and signal noise, leaving ρ_a difficult to isolate.

We parametrize the model by minimizing the squared distance between a vector of three empirical moments, $m^{\text{data}} \in \mathbb{R}^3$, and the corresponding model-simulated moments, $m^{\text{sim}}(\theta) \in \mathbb{R}^3$. Our simulated method of moments estimator is defined as

$$\hat{\theta} = \arg\min_{\theta} \ \big(m^{\text{data}} - m^{\text{sim}}(\theta)\big)' W \big(m^{\text{data}} - m^{\text{sim}}(\theta)\big),$$

where $[\gamma \ \gamma_z \ \sigma_a]'$ is the parameter vector and W is a symmetric, positive semi-definite 3×3 weighting matrix constructed from the covariance matrix of the empirical moments and estimated using stationary block bootstraps Politis and Romano (1994), which allows resampling without destroying the time dependence of the data. We use 1,000 bootstrap replications.

[9] The series is expressed in logs and one-sided HP-filtered before computing the standard deviation. The same transformation is applied to the model-generated data during the moment-matching procedure.

[10] This value is a typical estimate of the first-order TFP autocovariance obtained by regressing utilization-adjusted TFP on its lag; see, for example, Fajgelbaum et al. (2017); Antonova and Matvieiev (2025).

Table 2 presents the estimated parameter values along with the empirical and model moments.

Table 2: Model's parameters and matched moments

Panel A: Parameters Value		
Parameter	Description	Value
γ	Sensitivity of signal s_t precision wrt output	38.10 (3.63)
γ^z	Precision of signal z_t	5.27 (1.67)
σ_a	TFP level shock: s.d	0.018 (0.002)

Panel B: Matched Moments		
	Data, %	Model, %
Forecast error, 5$^{\text{th}}$ percentile	−6.91	−6.87
Forecast error, 95$^{\text{th}}$ percentile	6.11	6.14
Real consumption, s.d	1.04	1.04

Note: The values under brackets are the bootstrapped standard errors of the estimates.

We find that the model successfully reproduces the magnitude of the 5$^{\text{th}}$ and 95$^{\text{th}}$ percentiles of household forecast income errors as well as consumption volatility. The parameter governing the sensitivity of the cyclical signal to output, $\gamma = 38.1$, is positive and statistically significant, indicating that a pro-cyclical signal-to-noise ratio in our model aligns with the household expectations data. The estimated precision of the time-invariant signal, $\gamma^z = 5.27$, is also significant, suggesting that a sizable share of informational imperfections comes from factors unrelated to cyclical economic conditions. Finally, the parameter σ_a, which shapes dynamics through both the speed of information updating and the volatility of the fundamental, is estimated at 0.018, consistent with empirical consumption volatility. To gauge these estimation results, notice that the signal-to-noise ratio for the time-invariant signal equals $\sigma_a/(\gamma^z)^{-1} = 0.094$. By construction, the signal-to-noise ratio for signal s_t depends on fluctuations of output since it equals $\sigma_a/(\gamma \times Y_t/\bar{Y})^{-1}$. It means that a 1 percent deviation of output from a steady-state (normalized by unity) leads to a signal-to-noise ratio of 0.69. Interestingly, these values are in the range provided by Melosi (2014) who estimate the signal-to-noise ratios in a dispersed information model for the U.S.. Despite that our approach differs from him (in his model, firms are imperfectly informed and we use data on consumers' forecast for estimation), he finds a signal-to-noise ratio of 0.09 for monetary policy shocks and 0.6 for technology shocks.

4.2 IRFs Analysis

We now assess the role of the endogenous-uncertainty channel in the propagation of aggregate shocks. To this end, we solve the model from the perspective of a representative household with imperfect information about productivity, using a third-order perturbation method. We then construct conditional simulations in which the economy is hit by either a productivity shock or a government-spending shock. For each experiment, in addition to the baseline simulations, we also generate counterfactual impulse-response functions (IRFs) by shutting down the endogenous-uncertainty channel. In this counterfactual, information remains imperfect as in the baseline, but the level of uncertainty about productivity is held constant at its long-run value, taken from the baseline model. We next discuss the results of these two simulation exercises in turn.

4.2.1 Productivity shock

Figure 2 reports the IRFs to a negative productivity shock in our baseline model, where the endogenous-uncertainty channel is active (solid blue lines), and in a counterfactual economy in which the endogenous-uncertainty channel is shut down (red lines with markers).

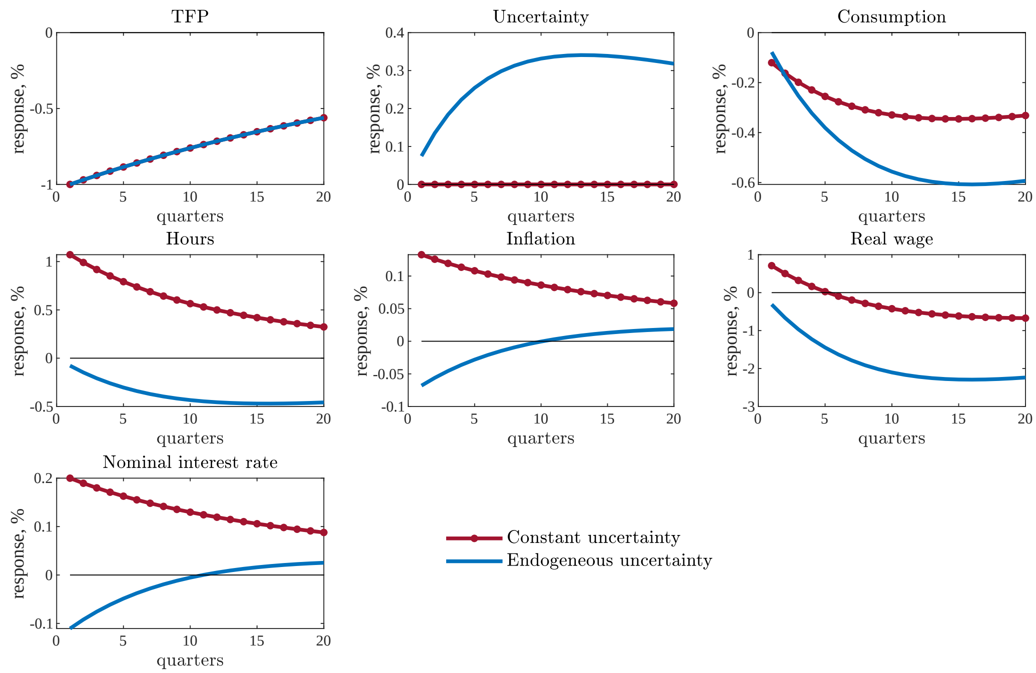
Let first consider the counterfactual where uncertainty is held constant. A decline in productivity reduces output, wages and hours worked while it generates inflation, which is in line with the traditional view that supply shocks cause output and prices to move in opposite directions.

In our baseline model instead, that features imperfect information and endogenous uncertainty, a negative TFP shock leads to a decline in productivity and a rise in uncertainty that reinforces the precautionary saving motive and thereby depresses aggregate demand and prices. The economy responds with deflation over the first several quarters and a sizable drop in consumption, reaching a peak decline of 0.6%, twice larger than in a model with constant uncertainty. Weak demand leads to lower nominal interest rates and a slack labor market; both employment and real wages decline. As a result, consumption, employment, and prices end up comoving, and our productivity shock acquires Keynesian features, namely a supply shock that is observationally similar to a demand shock. These results are in line with our theoretical prediction laid out in Section 3.

4.2.2 Public spending shock

We now turn to investigating the quantitative impact of the endogenous-uncertainty channel on the transmission of public spending shocks. We model this shock as a fully unexpected, zero-probability ("MIT") shock, meaning that households, when forming their beliefs, do not incorporate the possibility of a government spending innovation and thus take government spending to follow a fully deterministic path after the shock. We set the persistence of the public spending

Figure 2: IRFs to a negative productivity shock



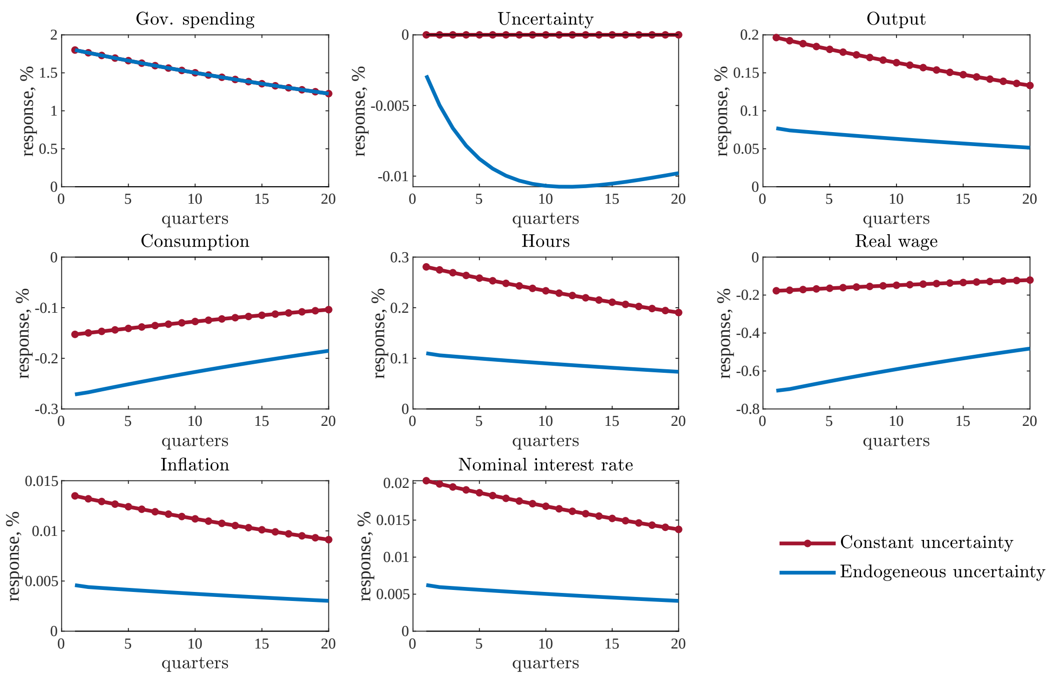
Note: The solid lines correspond to the response of the variables to a negative productivity shock in our baseline model with endogenous uncertainty. The lines with markers show the response in the counterfactual scenario where uncertainty is held constant. The response of uncertainty corresponds to the inverse of the response of γ_t. Consumption, hours and real wages are expressed in log. All IRFs are in deviation from their ergodic mean and multiplied by 100.

shock, ρ_g, to 0.98, and perturb the economy with a 1.8% increase in government spending.

Figure 3 shows the responses to a positive public spending shock in both the constant-uncertainty and endogenous-uncertainty settings. Let us first analyze the IRFs when uncertainty is held constant (red lines with markers). Consistent with the New-Keynesian literature, a positive public spending shock operates as a demand shock, pushing actual output above potential and thereby generating inflation. In response, the central bank raises the nominal interest rate. At the same time, the shock crowds out private consumption because higher public spending creates a negative wealth effect through increased taxes, leading households to consume less and work more.

In the presence of an endogenous response of uncertainty (solid blue lines), the expansionary impact of the positive public spending shock reduces the level of uncertainty. On the one hand, this mitigates households' precautionary saving behavior, which in turn limits the crowding-out effect on consumption. On the other hand, and as explained in Section 3, endogenous uncertainty reinforces the crowding-out of consumption, since consumers place a greater weight on their expectations, leading them to reduce consumption even further. Under our baseline calibration, the second effect dominates, and the stronger drop in consumption results in a more muted expansion in output. To illustrate the case where the first force dominates, Figure F.1 reports the IRFs to a public spending shock when $\rho_g = 0$. The low persistence dampens the negative wealth effect, as consumers expect only a very short-lived increase in future taxes. As a result, the precautionary-saving effect dominates, implying a reversal in the sign of the consumption response: a positive government spending shock actually crowds in private consumption.

Figure 3: IRFs to a positive public spending shock



Note: The solid lines correspond to the response of the variables to a positive public spending shock in our baseline model with endogenous uncertainty. The lines with markers show the response in the counterfactual scenario where uncertainty is held constant. The response of uncertainty corresponds to the inverse of the response of γ_t. Output, consumption, hours and real wages are expressed in log. All IRFs are in deviation from their ergodic mean and multiplied by 100.

4.3 Discussion: Exogenous versus endogenous uncertainty

Our previous results have shown that endogenous uncertainty – generated by pro-cyclical precision of signals – leads to strong amplification mechanisms in the transmission of both supply- and demand-driven (level) shocks. One might argue that a similar amplification effect could be obtained by combining first- and a second-order shocks since volatility shocks, which can be interpreted as exogenous uncertainty shocks also stimulate precautionary saving through second-order effects. We evaluate here to which extend a negative productivity shock with time-varying volatility generates the same effects than in our baseline model with imperfect information and endogenous uncertainty.

We express our model under full information ($\theta_t = a_t$; $\gamma_t \to \infty$) and we assume that technology follows a stochastic volatility (SV) process in the spirit of Born and Pfeifer (2014) and Basu and Bundick (2017) among many others

$$a_t = (1 - \rho_a)\bar{a} + \rho_a a_{t-1} + \sigma_{a,t}\epsilon_t^a, \tag{40}$$

$$\sigma_{a,t} = (1 - \rho_{\sigma_a})\,\bar{\sigma}_a + \rho_{\sigma_a}\sigma_{a,t-1} + \left(1 - \rho_{\sigma_a}^2\right)^{\frac{1}{2}} \eta_{\sigma_a}\epsilon_t^{\sigma_a}, \tag{41}$$

where $\epsilon_t^a \sim N(0,1)$ and $\epsilon_t^{\sigma_a} \sim N(0,1)$. Therefore, $\sigma_{a,t}$ measures technology uncertainty. Parameter ρ_{σ_a} drive the persistence associated with the volatility (second-order) shocks and η_{σ_a} governs the magnitude of the volatility shock.

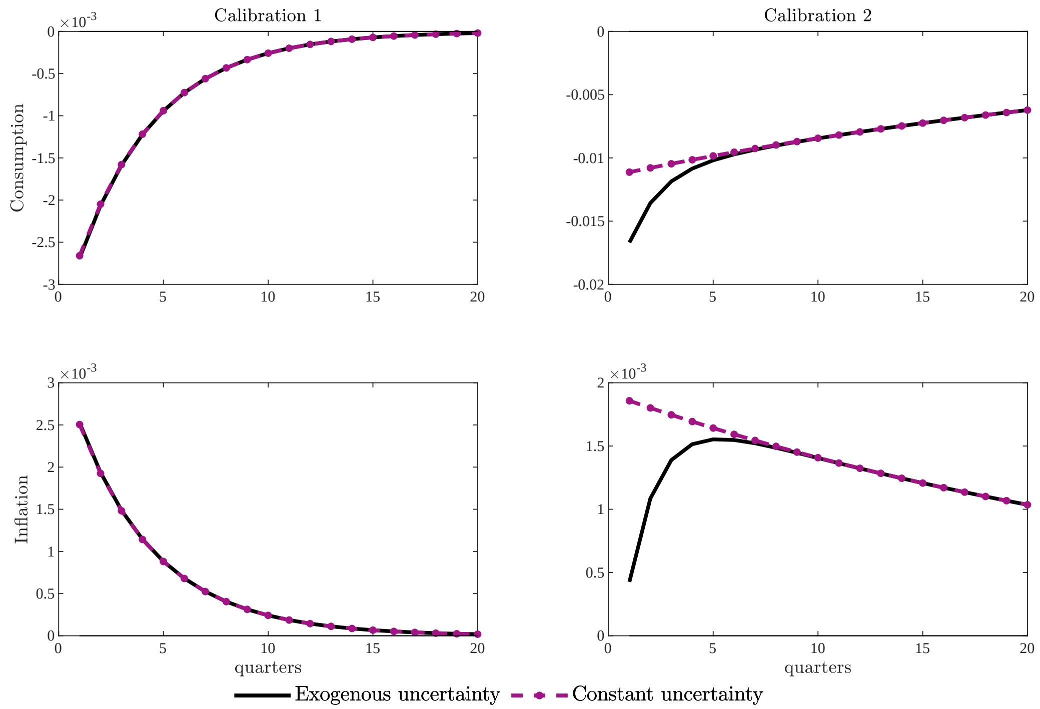
Figure 4 displays the IRFs of consumption and inflation to a negative productivity shock under two scenarios. In both cases, the dashed lines are the IRFs to a pure negative technology shock when the uncertainty channel is shut down ($\epsilon_t^{\sigma_a} = 0$) and the solid lines are the IRFs when the level productivity shock is combined with a negative volatility shock.[11] Compared to our model, uncertainty in this model is not generated through imperfect information but through stochastic volatility, which makes uncertainty fully exogenous and dependent on the calibration of the process (40)-(41). The two scenarios differ in the calibration of the shock process. In the left panel of Figure 4, we use the estimation results of Born and Pfeifer (2021) who estimate the SV process by Bayesian technics on U.S. data over the sample 1964q1-2015q4. This means that $\bar{\sigma}_a = 0.007$, $\rho_a = 0.773$, $\eta_{\sigma_a} = 0.002$ and $\rho_{\sigma_a} = 0.517$. Under this calibration, a positive one-standard-deviation innovation in volatility increases the average standard deviation of the innovation to the TFP shock from 1.80 to 1.8040. This roughly corresponds to the order of magnitude reported by Fernández-Villaverde et al. (2015) for fiscal shocks. The left panel of Figure 4 shows that under this calibration, the first-order effects of the negative productivity shock largely dominates the second-order effects such that the effects of uncertainty are negligible, despite that the difference between the two lines is not zero.

The right panel of Figure 4 reports the IRFs when we keep our baseline parametrization of σ_a and ρ_a for the AR(1) process while η_{σ_a} is calibrated so as to generate a extra drop in consumption similar in magnitude to the one obtained in Figure 2.[12] The persistence of the volatility process remains $\rho_{\sigma_a} = 0.517$. Two conclusions emerge. First, we calibrate $\eta_{\sigma_a} = 0.3$ which means that a positive one-standard-deviation innovation in volatility increases the average standard deviation of the innovation to the TFP shock from 1.80 to 2.52. This means that we need a huge increase in

[11]In practice, we add up the responses to negative level shocks with those to positive volatility shock, abstracting from the co-variates between the two shocks.

[12]The zero-uncertainty IRFs differ between the two columns since the calibration of $\bar{\sigma}_a$ and ρ_a are different.

Figure 4: IRFs to a negative technology shock with exogenous uncertainty



Note: The lines correspond to the response of the variables to a negative productivity shock driven by process (40)-(41) and under full information. The solid lines are when the stochastic volatility process is active ($\eta_{\sigma_a} > 0$). The dashed lines are when the stochastic volatility process is absent ($\eta_{\sigma_a} = 0$). The left panel corresponds to the responses when we calibrate the process following Born and Pfeifer (2021). The right panel corresponds to our own calibration. Consumption is expressed in log. All IRFs are in deviation from their ergodic mean and multiplied by 100.

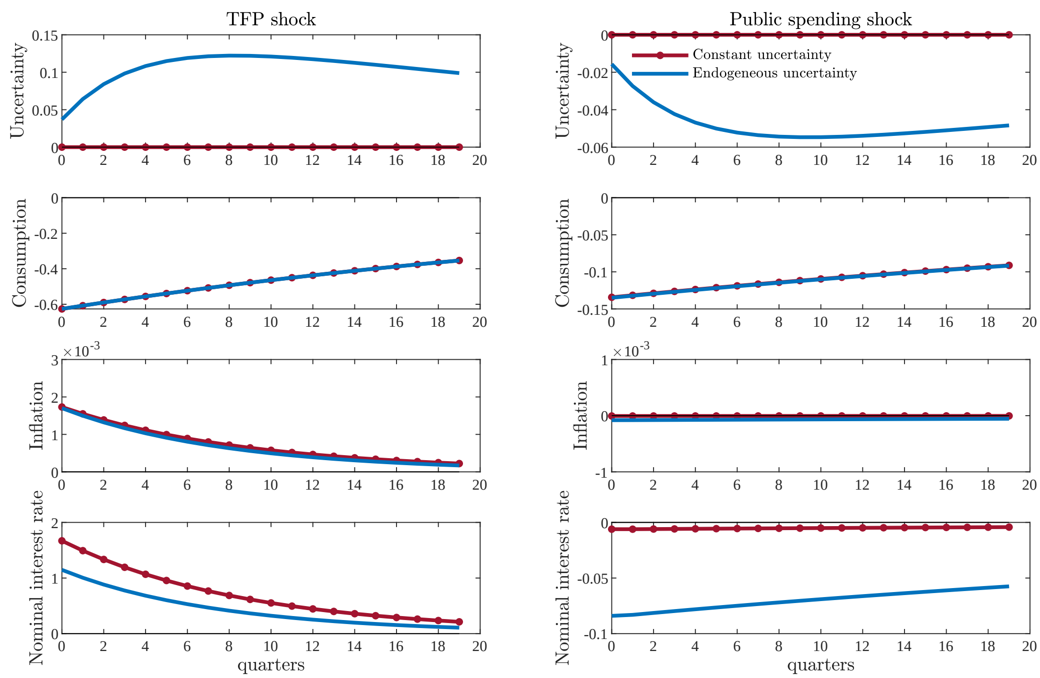
uncertainty to generate effects of the same magnitude than in our imperfect-information model. Under this calibration tough, we find that exogenous uncertainty is able to generate negative demand-driven effects since consumption drops by more after a negative productivity shock and inflation increases by less. However, the aggregate-demand effect driven by higher precautionary saving is not strong enough to reverse the sign of inflation, as we obtained in Figure 2. Second, comparing with Figure 2, we stress that our endogenous-uncertainty channel not only magnifies the demand-driven fluctuations, it also generates persistence through information frictions. While it is well accepted that imperfect information generates persistence in the transmission of shocks (see Melosi (2014) for instance), we find that endogenous uncertainty magnifies the long-last effect of shocks, which a model with exogenous uncertainty cannot do.[13]

[13]We can also generate an exogenous uncertainty shock in our baseline model by assuming that an exogenous shock on γ_t. We show in Figure F.1 that assuming a strong increase in uncertainty, by doubling it, generates a slightly smaller drop in consumption in response to a negative productivity shock, compared to the constant uncertainty case. This confirms that the endogenous-uncertainty channel acts as a strong amplification mechanism.

4.4 Implication for Monetary Policy

In Corollary 1.1, we showed that the endogenous-uncertainty channel has no impact on the flexible price output and henceforth, the monetary policy should be able to remove the associated aggregate demand fluctuations by eliminating the output gap. As standard in the canonical New-Keynesian model, this can be achieve by fully stabilizing prices, a well-known feature labeled as the "divine coincidence" (Blanchard and Gali (2007)).

Figure 5: IRFs to a productivity and public spending shock: the role of monetary policy



Note: The figure reports the IRFs under strict inflation targeting rule ($\phi_\pi = 50$). The left (right, resp.) panel corresponds the responses to a negative productivity (positive public spending, resp.) shock. The solid lines are the responses of the variables in our baseline model with time-varying uncertainty. The dashed lines are the responses when uncertainty is held constant. The response of uncertainty corresponds to the inverse of the response of γ_t. Consumption, hours and real wages are expressed in log. All IRFs are in deviation from their ergodic mean and multiplied by 100.

Figure 5 checks this assessment by reporting the IRFs to a negative technology (left panel) and a positive public spending (right panel) shock of a set of variables when we assume a strict inflation targeting rule, meaning that the nominal interest rate strongly reacts to any variation in inflation ($\rho_\pi = 1000$). The solid blue lines correspond to the responses in our baseline estimated model (solid blue lines) while the dashed green lines are when uncertainty is held constant. In line with our intuition, we find that strict inflation targeting removes demand-driven variations in inflation. Therefore, the responses of consumption and inflation under the endogenous-uncertainty case fully coincide with the one under constant uncertainty. This confirms that by closing the output gap, monetary policy is able to restore flexible prices and therefore the full-information allocation.

5 Conclusion

In this paper, we build a non-linear New-Keynesian model with pro-cyclical precision of signals. This allows us to introduces endogenous fluctuations in uncertainty while the non-linearity matters for consumption and saving decisions. The feedback loop between uncertainty and economic activity is the heart of the endogenous-uncertainty channel.

We show theoretically that endogenous uncertainty alters the effect of economic shocks on aggregate demand. Specifically, the negative supply shocks, which lead to an overheated demand in a standard New-Keynesian model, result in a depressed demand in the endogenous-uncertainty model. Similarly, the negative demand shocks, which depress demand in the standard New-Keynesian model, result in even more depressed demand in the endogenous-uncertainty model. In this way, endogenous uncertainty provides an amplification mechanism for economic fluctuations, operating solely through aggregate demand. Importantly, we find that the central bank can remove information-related distortion by closing the output gap since our endogenous-uncertainty mechanism has aggregate-demand effects only which are active as long as prices are rigid. As soon as the divine coincidence holds, strict inflation targeting allows to close the output gap and remove the amplification effects of uncertainty.

We also quantify the endogenous-uncertainty channel for the U.S. using a parametrized version of our model. In particular, we estimate the parameters governing information precision and its cyclicality by matching the model's forecast-error moments to the moments of aggregate income growth forecast errors constructed from the Michigan Survey of Consumers. In total, the feedback loop between information and economic activity generates strong and significant demand-driven effects that magnify the size and the persistence of responses of shocks.

References

Andrade, Philippe and Le Bihan, Hervé. Inattentive professional forecasters. *Journal of Monetary Economics*, 60(8):967–982, 2013.

Antonova, Anastasiia and Matvieiev, Mykhailo. News and firm entry: The role of the waiting option. *Journal of Economic Dynamics and Control*, 171:105034, 2025.

Autor, David, Dorn, David, Katz, Lawrence F, Patterson, Christina, and Van Reenen, John. The fall of the labor share and the rise of superstar firms. *The Quarterly journal of economics*, 135(2): 645–709, 2020.

Basu, Susanto and Bundick, Brent. Uncertainty shocks in a model of effective demand. *Econometrica*, 85(3):937–958, 2017.

Benhabib, Jess, Liu, Xuewen, and Wang, Pengfei. Financial markets, the real economy, and self-fulfilling uncertainties. *The Journal of Finance*, 74(3):1503–1557, 2019.

Bernstein, Joshua, Plante, Michael, Richter, Alexander W., and Throckmorton, Nathaniel A. A simple explanation of countercyclical uncertainty. *American Economic Journal: Macroeconomics*, 16(4):143–171, 2024.

Bilbiie, Florin O and Melitz, Marc J. Aggregate-demand amplification of supply disruptions: The entry-exit multiplier. Technical report, Cambridge Working Papers in Economics, 2023.

Blanchard, Olivier and Gali, Jordi. Real wage rigidities and the new keynesian model. *Journal of Money, Credit and Banking*, 39(s1):35–65, 2007.

Bloom, Nicholas. The Impact of Uncertainty Shocks. *Econometrica*, 77(3):623–685, 2009.

Born, Benjamin and Pfeifer, Johannes. Policy risk and the business cycle. *Journal of Monetary Economics*, 68:68–85, 2014.

Born, Benjamin and Pfeifer, Johannes. Uncertainty-driven business cycles: Assessing the markup channel. *Quantitative Economics*, 12(2):587–623, May 2021.

Cesa-Bianchi, Ambrogio and Ferrero, Andrea. The transmission of keynesian supply shocks. Technical report, CEPR Discussion Paper No. DP16430, 2021.

Challe, Edouard. Uninsured unemployment risk and optimal monetary policy in a zero-liquidity economy. *American Economic Journal: Macroeconomics*, 12(2):241–283, 2020.

Coibion, Olivier and Gorodnichenko, Yuriy. What can survey forecasts tell us about information rigidities? *Journal of Political Economy*, 120(1):116–159, 2012.

Coibion, Olivier and Gorodnichenko, Yuriy. Information rigidity and the expectations formation process: A simple framework and new facts. *American Economic Review*, 105(8):2644–2678, 2015.

Fajgelbaum, Pablo D., Schaal, Edouard, and Taschereau-Dumouchel, Mathieu. Uncertainty traps. *The Quarterly Journal of Economics*, 132(4):1641–1692, 2017.

Farhi, Emmanuel and Gourio, François. Accounting for macro-finance trends. *Brookings Papers on Economic Activity*, pages 147–223, 2018.

Fernández-Villaverde, Jesús and Guerrón-Quintana, Pablo A. Uncertainty shocks and business cycle research. *Review of Economic Dynamics*, 37:118–S146, 2020.

Fernández-Villaverde, Jesús, Guerrón-Quintana, Pablo, Rubio-Ramirez, Juan F, and Uribe, Martin. Risk matters: The real effects of volatility shocks. *American Economic Review*, 101(6):2530–61, 2011.

Fernández-Villaverde, Jesús, Guerrón-Quintana, Pablo, Kuester, Keith, and Rubio-Ramírez, Juan. Fiscal volatility shocks and economic activity. *American Economic Review*, 105(11):3352–3384, 2015.

Fornaro, Luca and Wolf, Martin. The scars of supply shocks: Implications for monetary policy. *Journal of Monetary Economics*, 140:S18–S36, 2023.

Foster, Andrew D and Rosenzweig, Mark R. Learning by doing and learning from others: Human capital and technical change in agriculture. *Journal of political Economy*, 103(6):1176–1209, 1995.

Galí, Jordi. Monetary policy, inflation, and the business cycle: An introduction to the new keynesian framework and its applications. Second Edition, Princeton Press, 2008.

Guerrieri, Veronica, Lorenzoni, Guido, Straub, Ludwig, and Werning, Iván. Macroeconomic implications of covid-19: Can negative supply shocks cause demand shortages? *American Economic Review*, 112(5):1437–1474, 2022.

Ilut, Cosmin and Saijo, Hikaru. Learning, confidence, and business cycles. *Journal of Monetary Economics*, 117:354–376, 2021.

Jurado, Kyle, Ludvigson, Sydney C, and Ng, Serena. Measuring uncertainty. *American Economic Review*, 105(3):1177–1216, 2015.

Leduc, Sylvain and Liu, Zheng. Uncertainty shocks are aggregate demand shocks. *Journal of Monetary Economics*, 82:20–35, 2016.

Leeper, Eric M, Traum, Nora, and Walker, Todd B. Clearing up the fiscal multiplier morass. *American Economic Review*, 107(8):2409–2454, 2017.

Lorenzoni, Guido. A theory of demand shocks. *American Economic Review*, 99:2050–84, 2009.

Ludvigson, Sydney C, Ma, Sai, and Ng, Serena. Uncertainty and business cycles: Exogenous impulse or endogenous response? *American Economic Journal: Macroeconomics*, 13(4):369–410, 2021.

L'Huillier, Jean-Paul, Singh, Sanjay R, and Yoo, Donghoon. Incorporating diagnostic expectations into the new keynesian framework. *Review of Economic Studies*, 91(5):3013–3046, 2024.

Mäkinen, Taneli and Ohl, Björn. Information acquisition and learning from prices over the business cycle. *Journal of Economic Theory*, 158:585–633, 2015.

Mankiw, N Gregory, Reis, Ricardo, and Wolfers, Justin. Disagreement about inflation expectations. *NBER macroeconomics annual*, 18:209–248, 2003.

Melosi, Leonardo. Estimating models with dispersed information. *American Economic Journal: Macroeconomics*, 6(1):1–31, 2014.

Nakamura, Emi and Steinsson, Jón. Price rigidity: Microeconomic evidence and macroeconomic implications. *Annu. Rev. Econ.*, 5(1):133–163, 2013.

Ordonez, Guillermo. Larger crises, slower recoveries: the asymmetric effects of financial frictions. Staff Report 429, Federal Reserve Bank of Minneapolis, 2009.

Politis, Dimitris N and Romano, Joseph P. The stationary bootstrap. *Journal of the American Statistical association*, 89(428):1303–1313, 1994.

Ravn, Morten O and Sterk, Vincent. Job uncertainty and deep recessions. *Journal of Monetary Economics*, 90:125–141, 2017.

Ravn, Morten O and Sterk, Vincent. Macroeconomic fluctuations with hank & sam: An analytical approach. *Journal of the European Economic Association*, 19(2):1162–1202, 2021.

Rotemberg, Julio J. Sticky prices in the United States. *Journal of political economy*, 90(6):1187–1211, 1982.

Saijo, Hikaru. The uncertainty multiplier and business cycles. *Journal of Economic Dynamics and Control*, 78:1–25, 2017.

Schaal, Edouard and Taschereau-Dumouchel, Mathieu. Herding through booms and busts. *Journal of Economic Theory*, 210:105669, 2023.

Skinner, Jonathan. Risky income, life cycle consumption, and precautionary savings. *Journal of monetary Economics*, 22(2):237–255, 1988.

Straub, Ludwig and Ulbricht, Robert. Endogenous uncertainty and credit ccrunches. *Review of Economic Studies*, 91(5):3085–3115, 2024.

Taylor, John B. Discretion versus policy rules in practice. In *Carnegie-Rochester conference series on public policy*, volume 39, pages 195–214. Elsevier, 1993.

Van Nieuwerburgh, Stijn and Veldkamp, Laura. Learning asymmetries in real business cycles. *Journal of monetary Economics*, 53(4):753–772, 2006.

Veldkamp, Laura L. Slow boom, sudden crash. *Journal of Economic theory*, 124(2):230–257, 2005.

Woodford, Michael. Imperfect common knowledge and the effects of monetary policy. NBER Working Papers 8673, National Bureau of Economic Research, Inc, 2001.

Macroeconomic Shocks in the Fog
Appendix

Anastasiia Antonova
Mykhailo Matvieiev
Céline Poilly

A Alternative learning frameworks

As a first possibility, consider a standard “learning-by-doing" assumption that each unit of production generates information. A noisy productivity signal for j-th unit of good produced

$$s_t(j) = a_t + \epsilon_t^s(j), \quad \epsilon_t^s(j) \sim \mathcal{N}(0,\ \gamma^{-1})$$

The total amount of goods produced is $\sum j = Y_t$. Then the average of these signals generated the overall noisy signal of precision equal the sum of precisions of the underlying signals:

$$s_t = \frac{1}{Y_t} \sum s_t(j) = a_t + \epsilon_t^s, \quad \epsilon_t^s \sim \mathcal{N}(0,\ [\gamma \cdot Y_t]^{-1}) \tag{A.1}$$

which yields pro-cyclical precision of information flow from signal s_t.

The second possibility is to consider a combination of learning by doing and social learning, see Foster and Rosenzweig (1995). Let us assume that each worker i (out of total employment L_t) gets a noisy signal about productivity for each unit j produced (learning by doing)

$$s_t(i,j) = a_t + \epsilon_t^s(i,j),\ \epsilon_t^s(i,j) \sim \mathcal{N}(0,\ \gamma^{-1}) \tag{A.2}$$

Each worker produces $y_t = \frac{Y_t}{L_t}$ goods, hence has an overall signal about productivity computed as the average of her learning by doing signals

$$s_t(i) = a_t + \epsilon_t^s(i),\ \epsilon_t^s(i) \sim \mathcal{N}(0,\ (\gamma \cdot y_t)^{-1}) \tag{A.3}$$

Finally, workers meet and exchange their information about productivity (social learning). The overall signal is the average of worker-specific signals and has precision $\gamma \cdot y_t \cdot L_t = \gamma_t \cdot Y_t$

$$s_t = a_t + \epsilon_t^s, \quad \epsilon_t^s \sim \mathcal{N}(0,\ [\gamma \cdot Y_t]^{-1}) \tag{A.4}$$

B Model equations

Household:

$$1 = \beta E_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\eta} \frac{R_t}{\pi_{t+1}} \right\}, \tag{B.1}$$

$$W_t = L_t^{\omega} C_t^{\eta}. \tag{B.2}$$

Firm:

$$MC_t = \frac{1}{1-\alpha} \frac{W_t}{\tilde{A}_t} L_t^{\alpha}, \tag{B.3}$$

$$Y_t = \tilde{A}_t L_t^{\alpha}. \tag{B.4}$$

Price-setting:

$$\epsilon \left(1 - MC_t\right) = 1 - \Phi \left(\pi_t - 1\right) \pi_t + \Phi \mathrm{E}_t \left\{ Q_{t,t+1} \left(\pi_{t+1} - 1\right) \pi_{t+1}^2 \frac{Y_{t+1}}{Y_t} \right\}, \tag{B.5}$$

where the discount factor is $Q_{t,t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\eta} \frac{1}{\pi_{t+1}}$.

Market clearing:

$$Y_t = C_t + G_t + \frac{\Phi}{2} \left(\pi_t - 1\right)^2 Y_t. \tag{B.6}$$

Monetary policy:

$$\frac{R_t}{\bar{R}} = \left(\frac{\pi_t}{\bar{\pi}} \right)^{\phi_\pi} \left(\frac{Y_t}{\bar{Y}} \right)^{\phi_y}. \tag{B.7}$$

Evolution of beliefs about a_t:

$$\theta_{t+1} = (1-\rho_a)\bar{a} + \rho_a \frac{\gamma_t \theta_t + \gamma^z z_t + \gamma_t^s s_t}{\gamma_t + \gamma^z + \gamma_t^s}, \tag{B.8}$$

$$\gamma_{t+1}^{-1} = \frac{\rho_a^2}{\gamma_t + \gamma^z + \gamma_t^s} + \sigma_a^2. \tag{B.9}$$

C Risk-adjusted linear model

Market clearing. Reduced form price rigidity only in the present period: $MC_t \neq P_t$ but $MC_{t+j} = P_{t+j}$ for all $j > 0$. Combining (B.2), (B.3), and (B.6) and log-linearizing, we obtain

$$y_t = \left(\frac{1}{\omega + \eta} \right) mc_t + \left(\frac{1+\omega}{\omega + \eta} \right) \tilde{a}_t + \left(\frac{\eta}{\omega + \eta} \right) g_t \tag{C.1}$$

$$y_{t+j} = \left(\frac{1+\omega}{\omega + \eta} \right) \tilde{a}_{t+j} + \left(\frac{\eta}{\omega + \eta} \right) g_{t+j}, \quad j > 0 \tag{C.2}$$

IS equation. Assume that the household saves only in real bonds. We perform a risk-adjusted log-linearization of a corresponding Euler equation B.1. There are two stages to this linearization: first, derive the risk premium (in the spirit of Skinner (1988)) and then do a standard log-linearization. Our target is to derive the Euler equation in the form of a certainty-equivalent Euler equation adjusted for risk premium. We define certainty-equivalent consumption as consumption that households would choose if future income was certain and equal to its expected value. First, consider a nonlinear Euler equation:

$$u'(C_t) = \beta(1+r_t)E_t u'(C_{t+1})$$

Consider the point $C^e_{t+1} = E_t C_{t+1}$. To derive the risk-premium arising due to the consumption uncertainty (a la Skinner) we take the 2nd order Taylor expansion for RHS around C^e_{t+1}

$$u'(C_t) = \beta(1+r_t)E_t\left\{u'(C^e_{t+1}) + u''(C^e_{t+1})\cdot(C_{t+1}-C^e_{t+1}) + \frac{1}{2}u'''(C^e_{t+1})\cdot(C_{t+1}-C^e_{t+1})^2\right\}$$

Taking expectations form the RHS:

$$u'(C_t) = \beta(1+r_t)\left(u'(C^e_{t+1}) + \frac{1}{2}u'''(C^e_{t+1})\cdot E_t(C_{t+1}-C^e_{t+1})^2\right)$$

We factor out the certainty-equivalent part:

$$u'(C_t) = \beta(1+r_t)u'(C^e_{t+1})\left(1 + \frac{1}{2}\frac{u'''(C^e_{t+1})}{u'(C^e_{t+1})}\cdot E_t(C_{t+1}-C^e_{t+1})^2\right)$$

Multiplying and dividing by $(C^e_{t+1})^2$ we obtain the expression in terms of the relative risk aversion:

$$u'(C_t) = \beta(1+r_t)u'(C^e_{t+1})\left(1 + \frac{1}{2}\frac{u'''(C^e_{t+1})}{u'(C^e_{t+1})}(C^e_{t+1})^2\cdot E_t\left(\frac{C_{t+1}-C^e_{t+1}}{C^e_{t+1}}\right)^2\right)$$

This equation can be rewritten as:

$$u'(C_t) = \beta(1+r_t)(1+\psi_t)u'(E_t C_{t+1}) \tag{C.3}$$

where $\psi_t = \frac{1}{2}\frac{u'''(C^e_{t+1})}{u'(C^e_{t+1})}(C^e_{t+1})^2\cdot E_t\left(\frac{C_{t+1}-C^e_{t+1}}{C^e_{t+1}}\right)^2$ is time-varying risk premium arising from uncertainty about future consumption.

We have CRRA utility $u(C) = \frac{C^{1-\eta}}{1-\eta}$. From now on, for the Euler equation and all equations that follow (including the low of motion of uncertainty), we work with *linear* approximations. Taking the logs from Equation C.3 we get

$$-\eta log(C_t) = log\beta + log(1+r_t) + log(1+\psi_t) - \eta log(E_t c_{t+1}) \tag{C.4}$$

Next we denote $c_t = log(C_t)$. Using the fact that $log(1 + x_t) \approx x_t$ and $logE_tX_{t+1} = E_tlog(X_t)$ to the first order, we get

$$c_t = -\frac{1}{\eta}\log(\beta) + E_tc_{t+1} - \frac{1}{\eta}r_t - \frac{1}{\eta}\psi_t$$

With CRRA utility we have

$$\psi_t = \frac{1}{2}\eta(1+\eta)\cdot E_t\left(\frac{C_{t+1} - C^e_{t+1}}{C^e_{t+1}}\right)^2$$

Noting that $\frac{C_{t+1}-C^e_{t+1}}{C^e_{t+1}} \approx c_{t+1} - Ec_{t+1}$ and $E_t(c_{t+1} - E_tc_{t+1})^2 = Var_tc_{t+1}$, and substituting for the log-linear resource constraint around the steady state with $\bar{G} = 0$ such that $y_t = c_t + g_t$ (where $g_t = \frac{G_t}{\bar{Y}}$) we obtain:

$$y_t - g_t = E_ty_{t+1} - \rho_g g_t - \frac{1}{\eta}(r_t - \rho) - \frac{1}{2}(1+\eta)Var_ty_{t+1} \tag{C.5}$$

where $\rho = -log(\beta)$.

Evolution of beliefs. We linearize beliefs (B.8), (B.9) around a stationary point $\gamma_t = \bar{\gamma}$, $\gamma^s_t = \bar{\gamma}^s$, $\theta_t = s_t = \tilde{a}_t = \bar{a}$. We obtain

$$\begin{aligned}\theta_{t+1} = (1-\rho_a)\bar{a} + \rho_a\bar{a} + \rho_a\frac{\bar{\gamma}}{\bar{\gamma}+\gamma^z+\bar{\gamma}^s}(\theta_t - \bar{a}) + \rho_a\bar{a} + \rho_a\frac{\gamma^z}{\bar{\gamma}+\gamma^z+\bar{\gamma}^s}(\tilde{a}_t - \bar{a}) + \rho_a\frac{\bar{\gamma}^s}{\bar{\gamma}+\gamma^z+\bar{\gamma}^s}(s_t - \bar{a}) + \\ + \left(\frac{\bar{\theta}}{\bar{\gamma}+\gamma^z+\bar{\gamma}^s} - \frac{\bar{\gamma}\bar{\theta}+\gamma^z\bar{\tilde{a}}+\bar{\gamma}^s\bar{s}}{(\bar{\gamma}+\gamma^z+\bar{\gamma}^s)^2}\right)(\gamma_t - \bar{\gamma}) + \left(\frac{\bar{s}}{\bar{\gamma}+\gamma^z+\bar{\gamma}^s} - \frac{\bar{\gamma}\bar{\theta}+\gamma^z\bar{\tilde{a}}+\bar{\gamma}^s\bar{s}}{(\bar{\gamma}+\gamma^z+\bar{\gamma}^s)^2}\right)(\gamma^s_t - \bar{\gamma}^s) = \\ = (1-\rho_a)\bar{a} + \rho_a\frac{\bar{\gamma}\theta_t + \gamma^z\tilde{a}_t + \bar{\gamma}^s s_t}{\bar{\gamma}+\gamma^z+\bar{\gamma}^s}\end{aligned}$$

Now, let us denote the prior information and productivity observation as a joint signal $z_t = \frac{\bar{\gamma}}{\bar{\gamma}+\gamma^z}\theta_t + \frac{\gamma^z}{\bar{\gamma}+\gamma^z}\tilde{a}_t$ of precision $\bar{\gamma} + \gamma^z$. Also, let us denote $v = \frac{\bar{\gamma}^s}{\bar{\gamma}+\gamma^z+\bar{\gamma}^s}$. Then, we can rewrite the next period belief as:

$$\theta_{t+1} = (1-\rho_a)\bar{a} + \rho_a[vs_t + (1-v)z_t] \tag{C.6}$$

Assuming that $\gamma_t = \bar{\gamma}$, using the fact that $\gamma^s_t = \gamma\frac{Y_t}{\bar{Y}}$, and the definition of v we obtain

$$\begin{aligned}\gamma^{-1}_{t+1} = \frac{\rho_a^2}{\bar{\gamma}+\gamma^z+\bar{\gamma}^s} + \sigma_a^2 - \frac{\rho_a^2}{(\bar{\gamma}+\gamma^z+\bar{\gamma}^s)^2}(\gamma_t - \bar{\gamma} + \gamma^s_t - \bar{\gamma}^s) = \\ = \rho_a^2\left(\frac{v}{\bar{\gamma}^s} - \frac{v^2}{\bar{\gamma}^s}\frac{\gamma^s_t - \bar{\gamma}^s}{\bar{\gamma}^s}\right) + \sigma_a^2 = \rho_a^2\left(\frac{v}{\bar{\gamma}^s} - \frac{v^2}{\bar{\gamma}^s}(y_t - \bar{y})\right) + \sigma_a^2\end{aligned}$$

which equals

$$\gamma^{-1}_{t+1} = \frac{\rho_a^2}{\gamma}v(1 - v(y_t - \bar{y})) + \sigma_a^2 \tag{C.7}$$

Finally, we are interested in $E_t\tilde{a}_{t+1}$ and $Var\tilde{a}_{t+1}$, which are obtained from the updated beliefs about a_{t+1} as

$$E_t(\tilde{a}_{t+1}) = E_t a_{t+1} = (1-\rho_a)\bar{a} + \rho_a[v s_t + (1-v)z_t] \tag{C.8}$$

$$Var(\tilde{a}_{t+1}) = \gamma_{t+1}^{-1} + \sigma_f^2 = \sigma_a^2 + \sigma_f^2 + \frac{\rho_a^2}{\gamma} v(1+v\bar{y}) - \frac{\rho_a^2}{\gamma} v^2 \cdot y_t \tag{C.9}$$

Output gap response to productivity shock. Combining IS equation (C.5) with (C.2) and beliefs (C.8), (C.9) we obtain

$$\left[1 - \frac{1}{2}\frac{(1+\omega)^2(1+\eta)}{(\omega+\eta)^2} \cdot \frac{\rho_a^2}{\gamma} \cdot v^2\right] \cdot y_t = -log(\beta) - \frac{1}{\eta} \cdot r_t + (1-\rho_g + \rho_g \frac{\eta}{\omega+\eta}) g_t +$$
$$+ \frac{1+\omega}{\omega+\eta} \cdot ((1-\rho_a)\bar{a} + \rho_a[v s_t + (1-v)z_t]) - \frac{1}{2}\frac{(1+\omega)^2(1+\eta)}{(\omega+\eta)^2}(\sigma_a^2 + \sigma_f^2 + \frac{\rho_a^2}{\gamma} v(1+v\bar{y}))$$

Taking the log deviations from the steady state, we get

$$\left[1 - \frac{1}{2}\frac{(1+\omega)^2(1+\eta)}{(\omega+\eta)^2} \cdot \frac{\rho_a^2}{\gamma} \cdot v^2\right] \cdot \hat{y}_t = -\frac{1}{\eta}\hat{r}_t + \frac{1+\omega}{\omega+\eta}\rho_a[v\hat{s}_t + (1-v)\hat{z}_t] + (1-\rho_g \cdot \frac{\omega}{\omega+\eta})\hat{g}_t$$

Let us denote $f = \left[1 - \frac{1}{2}\frac{(1+\omega)^2(1+\eta)}{(\omega+\eta)^2} \cdot \frac{\rho_a^2}{\gamma} \cdot v^2\right]^{-1}$, which gives

$$\hat{y}_t = -\frac{f}{\eta}\hat{r}_t + f\frac{1+\omega}{\omega+\eta}\rho_a[v\hat{s}_t + (1-v)\hat{z}_t] + f(1-\rho_g \cdot \frac{\omega}{\omega+\eta})\hat{g}_t \tag{C.10}$$

From C.1 we have the natural output (in log-deviation from the steady state) equal $\hat{y}_t^n = \frac{1+\omega}{\omega+\eta}\hat{\bar{a}}_t + \frac{\eta}{\omega+\eta}\hat{g}_t$. Let the output gap be denoted as $\tilde{y}_t = \hat{y}_t - \hat{y}_t^n$. Let the monetary policy response be such that $r_t = \phi \cdot \tilde{y}_t$. Then, from (C.10) we can write

$$\left[1 + \frac{\phi}{\eta} \cdot f\right] \cdot \tilde{y}_t = \frac{1+\omega}{\omega+\eta} \cdot [-\hat{\bar{a}}_t + \rho_a f(v\hat{s}_t + (1-v)\hat{z}_t)] + f(1-\rho_g \cdot \frac{\omega}{\omega+\eta} - \frac{\eta}{\omega+\eta})\hat{g}_t \tag{C.11}$$

Now, consider the change in productivity Δa_t. Change in productivity is reflected in the corresponding change in $\Delta\hat{\bar{a}}_t = \Delta a_t$, $\Delta a_t \hat{s}_t = \Delta a_t$, and $\Delta\hat{z}_t = \frac{1}{1+\bar{\gamma}\sigma_f^2}\Delta a_t$ (since z_t is the combination of prior belief and observation of the current productivity); to simplify further, we assume that $\bar{\gamma} = 0$, that is, no prior information is available about the productivity. Then the response of the output gap to productivity shock is computed from

$$\frac{\omega+\eta}{1+\omega}\left[1 + \frac{\phi}{\eta} \cdot f\right]\Delta\tilde{y}_t = [\rho_a f - 1] \cdot \Delta a_t = -(1-\rho_a) \cdot \Delta a_t + \rho_a(f-1) \cdot \Delta a_t \tag{C.12}$$

It is clear that $f \geq 1$ always, as is the effect of endogenous uncertainty channel. Endogenous uncertainty channel is absent when: 1) no learning from economic activity $\gamma = 0$, then we have $v = 0$ and $f = 1$, or 2) full information model $\gamma \to \infty$ (or $\sigma_f^2 = 0$), with $v = 1$ and $f = 1$.

Crowding in private consumption. Now consider an increase in government spending Δg_t. The response of private consumption is $\Delta c_t = \Delta y_t - \Delta g_t$. The response of output to government spending shock is given from C.10 as $\Delta y_t = -\frac{1}{\eta} f \Delta r_t + f(1 - \rho_g \cdot \frac{\omega}{\omega+\eta})\Delta g_t$ and the response of interest rate is given from C.11 as $\Delta r_t = \phi \Delta \tilde{y}_t = \frac{\phi f (1-\rho_g \cdot \frac{\omega}{\omega+\eta} - \frac{\eta}{\omega+\eta})}{1+\frac{\phi}{\eta} f}$. Then, the consumption response to government spending shock is

$$\Delta c_t = \left[-1 + \frac{(1 - \rho_g \cdot \frac{\omega}{\omega+\eta} + \frac{\phi}{\omega+\eta}) f}{1 + \frac{\phi f}{\eta}} \right] \Delta g_t$$

The response of consumption depends on the persistence of government spending. Rearranging the terms we get

$$\left[1 + \frac{\phi f}{\eta} \right] \Delta c_t = (f-1)(1 - \rho_g \cdot \frac{\omega}{\omega+\eta} - \frac{\phi}{\eta} \frac{\omega}{\omega+\eta}) \Delta g_t - (\rho_g \cdot \frac{\omega}{\omega+\eta} + \frac{\phi}{\eta} \frac{\omega}{\omega+\eta}) \Delta g_t \qquad \text{(C.13)}$$

The first term is the crowding in effect from the endogenous uncertainty channel ($f > 1$). The second effect is the grounding out effect from the traditional government spending channel. When government spending is persistent (large ρ_g), the standard crowding out effect is strong and the endogenous uncertainty crowding in is weak.

D Aggregating individual forecast errors

We derive the recursive process for aggregate forecast error in the Bayesian learning framework with imperfect information and procyclical precision. Let aggregate income follow the AR1 process:

$$inc_t = \rho \cdot inc_{t-1} + (1-\rho) \cdot \bar{inc} + \varepsilon_t, \quad \varepsilon_t \sim \mathcal{N}(0, \sigma^2)$$

An individual household has income is:

$$inc_t^i = inc_t + v_t^i, \quad v_t^i \sim \mathcal{N}(0, (\gamma_t^v)^{-1})$$

v_t^i captures the individual component of income; γ_t^v is a precision parameter governing the dispersion of income distribution across individuals. The household does not observe inc_t directly and has a set of information Ω_t^i available at the beginning of period t. The corresponding prior belief about inc_t is

$$inc_t | \Omega_t^i \sim \mathcal{N}(\theta_t^i, (\gamma_t)^{-1})$$

where $E[inc_t|\Omega_t^i] = \theta_t^i$. During t household observes g_t^i and a noisy signal about inc_t

$$s_t^i = inc_t + u_t^i, \quad u_t^i \sim \mathcal{N}(0, (\gamma_t^u)^{-1})$$

Define expectation error as $e_{t-1}^i = \theta_t^i - inc_t^i$. Household updates information about inc_t so that: $E[inc_{t+1}|\Omega_{t+1}^i] = \rho E[inc_t|\Omega_{t+1}^i] + (1-\rho)\bar{inc}$ where the optimal combination of signals yields $E[inc_t|\Omega_{t+1}^i] = \frac{\gamma_t\theta_t^i + \gamma_t^v \cdot inc_t^i + \gamma_t^u s_t^i}{\gamma_t + \gamma_t^v + \gamma_t^u}$ and $\gamma_{t+1} = \left[\frac{\rho^2}{\gamma_t + \gamma_t^v + \gamma_t^u} + \sigma^2\right]^{-1}$. The individual expectation error is then:

$$\begin{aligned} e_t^i = \theta_{t+1}^i - inc_{t+1}^i = \rho\frac{\gamma_t\theta_t^i + \gamma_t^v \cdot inc_t^i + \gamma_t^u s_t^i}{\gamma_t + \gamma_t^v + \gamma_t^u} + E_t^i v_{t+1}^i - \rho \cdot inc_t - v_{t+1}^i - \varepsilon_{t+1} = \\ = \rho\frac{\gamma_t e_t^i + (\gamma_t + \gamma_t^v) v_t^i + \gamma_t^u u_t^i}{\gamma_t + \gamma_t^v + \gamma_t^u} + E_t^i v_{t+1}^i - v_{t+1}^i - \varepsilon_{t+1} \end{aligned}$$

Taking cross-sectional expectations (aggregating across individuals) and using that $E[v_t^i] = E[u_t^i] = E[E_{t-1}^i v_t^i] = 0$ because noise is idiosyncratic we obtain the average expectation error $e_t = E[e_t^i]$ as:

$$e_{t+1} = \rho\frac{\gamma_t}{\gamma_t + \tilde{\gamma}_t} \cdot e_t - \varepsilon_{t+1}$$

where $\tilde{\gamma}_t = \gamma_t^v + \gamma_t^u$ is the joint precision of two noisy signals. For a given prior precision γ_t, the persistence of forecast error is decreasing in signal precision $\tilde{\gamma}_t$. Finally, note that this aggregation relies on the fact that the precision of belief is the same across individuals, even though the mean belief is idiosyncratic.

E Numerical solution and simulation details

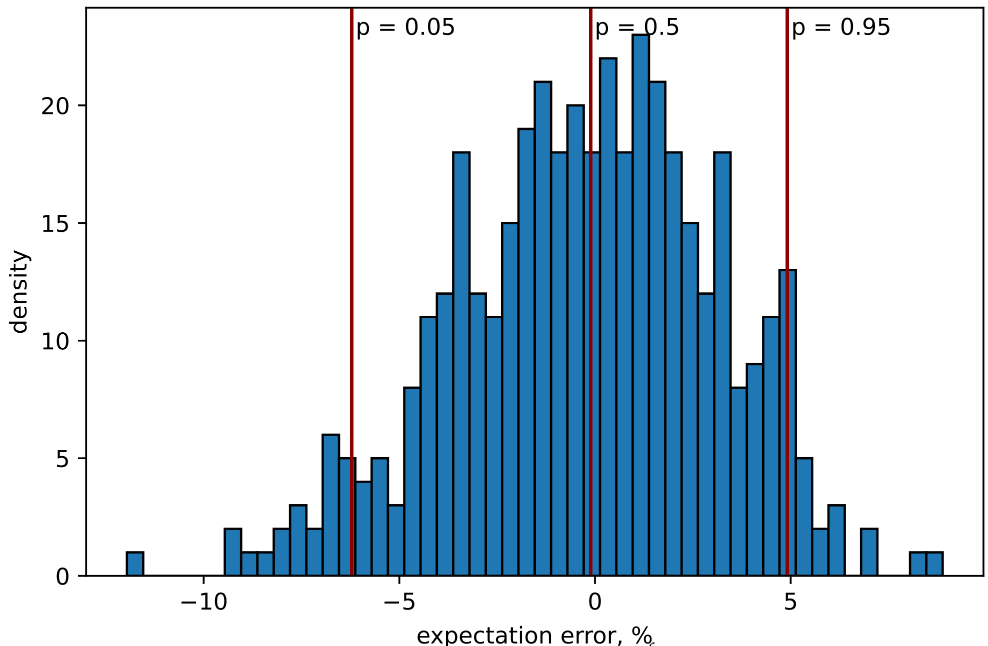
We solve the rational expectation model from the point of view of a representative household having imperfect information about productivity. That is, beliefs θ_t and γ_t are the observable state variables but not the persistent productivity component a_t. As households compute their expectations based on these beliefs, they perceive deviation of signals from these beliefs as realizations of expectation errors. Given the prior information Ω_t, observing signals z_t and s_t consists of the expected part θ_t and the innovation part. Signal z_t is

$$z_t = \theta_t + \tilde{e}_t^z,$$

where $\tilde{e}_t^z = (a_t - \theta_t) + \epsilon_t^z$ is innovation, consisting of expectation error $a_t - \theta_t$ and realization of temporary productivity component $f_t = \epsilon_t^z$. Similarly, the public signal s_t is

$$s_t = \theta_t + \tilde{e}_t^s$$

Figure D.1: Distribution over time of the average income forecast errors



Note: The figure displays the distribution of the average Forecast error, $e^f_{t,t12}$, computed from the Michigan Survey of Consumers.

where $\tilde{e}^s_t = (a_t - \theta_t) + \epsilon^s_t$ is a sum of expectation error $(a_t - \theta_t)$ and a realization if signal noise ϵ^s_t. Hence, treating errors $\tilde{e}^z_t$ and $\tilde{e}^s_t$ as exogenous disturbances (which they are from the point of view of a household) account for the imperfect information of the household in the dynamic RE model.

Let e^a_t denote the expectation error between realized productivity component, a_t, and the expected one, $\mathrm{E}(a_t|\Omega_t)$, such that $e^a_t \equiv a_t - \theta_t$. Notice that expectation error is drawn from the distribution $e^a_t \sim \mathcal{N}(0, \gamma_t^{-1})$. We can therefore rewrite the observed signals in terms of the expected element θ_t and standard normal innovations scaled by the corresponding (time-varying) standard deviations

$$z_t = \theta_t + e^a_t + \epsilon^z_t = \theta_t + [\gamma_t]^{-1/2}\epsilon^e_t + [\gamma^z]^{-1/2}\epsilon^z_t \tag{E.1}$$

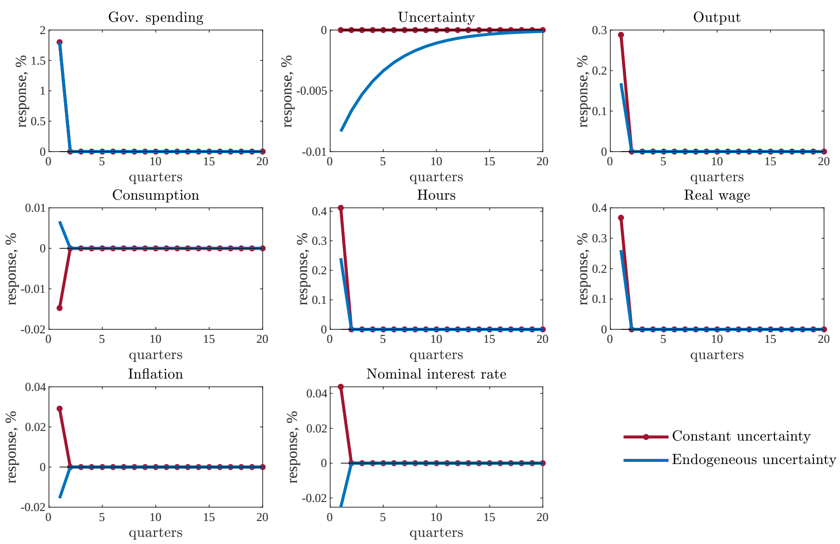
$$s_t = \theta_t + e^a_t + \epsilon^s_t = \theta_t + \gamma_t^{-1/2}\epsilon^e_t + [\gamma^s_t]^{-1/2}\epsilon^s_t \tag{E.2}$$

where ϵ^e_t, ϵ^z_t and ϵ^s_t are i.i.d. drawn from a standard normal distribution. We solve the model consisting of equations (B.1) - (B.9) and (E.1) - (E.2) using third-order perturbation method, allowing us to capture the precautionary saving channel. Simulating the model response to shocks amounts to recursive construction of expectation error e^a_t and computing the model path conditional on its realization.

F Robustness Analysis

F.1 Public spending shock: zero-persistence

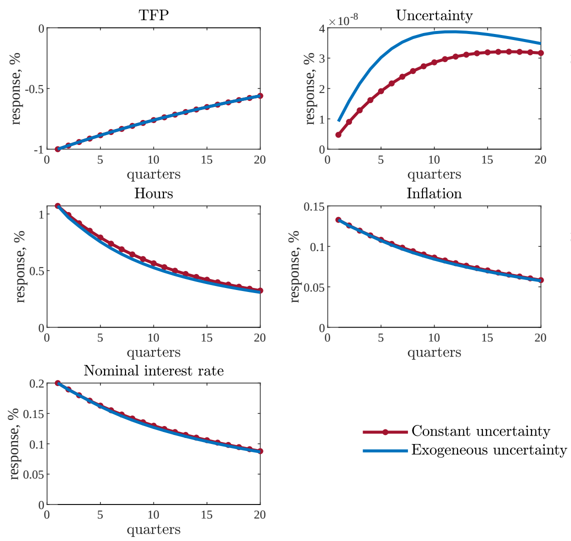
Figure F.1: IRFs to a positive public spending shock: $\rho_g = 0$



Note: The solid lines correspond to the response of the variables to a positive public spending shock in a model with time-varying uncertainty. The dotted lines show the response in the counterfactual scenario where uncertainty is held constant. In both case, the process driven public spending has no persistence ($\rho_g = 0$). The response of uncertainty corresponds to the inverse of the response of γ_t. Output, consumption, hours and real wages are expressed in log. All IRFs are in deviation from their ergodic mean and multiplied by 100.

F.2 Productivity shock and time-varying volatility

Figure F.1: IRFs to a negative TFP shock with exogenous uncertainty



Note: The solid lines correspond to the response of the variables to a negative productivity shock in our baseline model when uncertainty doubles exogenously. The lines with markers show the response in the counterfactual scenario where uncertainty is held constant (as in Figure 2). The response of uncertainty corresponds to the inverse of the response of γ_t. Consumption, hours and real wages are expressed in log. All IRFs are in deviation from their ergodic mean and multiplied by 100.