Gender Wage Gaps across Skills and Trade Openness

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**Abstract**

Several empirical studies have shown that the effect of openness on the gender wage gap depends on the skill requirement of the workplace. This paper offers a theoretical explanation to understand that finding.

We integrate a statistical discrimination framework with the labour assignment approach to give general conditions under which the matching between firms and workers gives rise to a wider gender wage gap at the upper tail of the distribution, in accordance with empirical evidence. We further look at the effect of trade openness on the gender wage gap along the entire distribution. Workers’ characteristics vary in two dimensions, skills and job commitment. The inability to observe individual’s job commitment induces employers to base partly their decision on group average. Following the literature on labour and international trade, we assume that skills act as complements to technological upgrading. Exporting firms are more skill-intensive and pay higher wages; assuming further that worker’s job commitment is a complement to technological upgrading, we find that a reduction in trade costs increases wage inequality within-groups and has non-monotonic effects on between-group inequality. Trade openness reduces the gender wage gap among unskilled workers but increases the gender wage gap among high-skill workers.

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1 Introduction

Recent empirical studies have shown that the effect of openness on both the gender wage gap and the employment gap depends on the skill requirement of the job. Openness improves women’s access to paid employment and thus increases total female wage earnings in an economy, but those gains are concentrated in low-skill low-wage jobs within industries (see Ozler (2000), Ederington et al. (2009), Fafchamps (2009), Fafchamps et al. (2009), Juhn et al. (2012) which we discuss below). Despite a strong empirical interest, the channels of impact underlying this pattern have not been formalized. This paper seeks to offer an explanation for this phenomenon by relating skill-biased investments fostered by trade openness with discriminating behaviors that depend on jobs’ characteristics.

It is now well known that trade openness affects workers differently along the skill distribution; the skill premium is found to increase with trade exposure in both developed and developing countries\(^1\). At the same time, evidence on several countries shows that the adjusted gender wage gap is higher in the upper part of the wage distribution\(^2\). This pattern can be explained by three phenomenon. First, women are less often promoted to top jobs, the glass-ceiling effect. Second, within jobs, women are paid less than their male counterpart, especially in high-responsibility high-wage jobs. Moreover, wage dispersion across firms explains a large portion of the variation in individuals’ wages, in particular between men and women. There is evidence showing that the sorting of women into low-wage firms, a glass-door effect, accounts for part of the higher adjusted wage gaps at the top of distribution\(^3\). Given that gender discrimination is stronger at the top of the wage distribution and that trade openness tends to exacerbate wage inequality, one would expect a widening of the gender wage gap at the top of the wage distribution following

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\(^1\)Empirical assessments of the impact of trade openness on wage inequality are for instance, Bernard & Jensen (1997) on the US, Pavcnik et al. (2004) on Brazil, Brambilla et al. (2011) on several Latin-American countries. See also Goldberg & Pavcnik (2007) for a literature review. Theoretical models that explain how trade increases the skill-premium are Neary (2003), Yeaple (2005), Bustos (2011) and Sampson (2012) among others.

\(^2\)Recent examples in this literature include Nopo et al. (2010) for Latin American countries, de la Rica et al. (2008) for Spain and Albrecht et al. (2009) for the Netherlands.

\(^3\)Meyersson Milgrom et al. (2001) show that segregation into low-wage occupations and low-wage establishments explain part of the wage gap in Sweden while Amuedo-Dorantes & De la Rica (2006) come to similar conclusion for Spain. Javdani (2012) applies the methodology of Pendakur & Woodcock (2010) on Canadian data to decompose the gender wage gaps along the distribution into a within-firm glass-ceiling effect and a glass-door effect i.e. the under-representation of women in high-wage firms.
trade liberalization. This paper provides a description of the mechanism and the conditions under which trade openness leads to a reduction of the gender wage gap at the bottom of the distribution and an increase of the gap at the top.

Closely related to this paper, a few empirical studies have pointed out the heterogeneous effects of trade on the relative position of women depending on the skill requirement of the workplace. Joekes (1995) highlights that the expansion of the export manufacturing sectors in Morocco and Bangladesh created new sources of employment for women but in unskilled occupations, mainly in the textile and clothing industries. This result is confirmed by Fafchamps (2009) who finds that Moroccan exporters, concentrated in light industries such as textile and apparel where the workforce is mainly unskilled, employ significantly more women and pay them on average lower wages controlling for education. Ozler (2000) uses plant-level data from Turkey and shows that trade liberalization in the 1980s led to employment gains for women relative to men in the manufacturing sector. However, women continued to be employed in low-skill and low-pay jobs within plants. Furthermore, among plants with a high female share, as well as among large establishments, investments in machinery and equipment brought about a decline in the female share of employment. This finding supports the argument that employment gains for women following trade liberalization might be reversed as a consequence of technological upgrading. Ederington et al. (2009) use plant-level data for Columbia to study changes in employment within firms over the period 1985-1991. They show that plants that have the highest female share of employment are less intensive in capital and pay lower wages compare to the industry average. As for the role of openness, the share of exports in the plant’s total production is positively associated with the plant’s female share which implies that openness can be good for female employment opportunities. In the same direction, a reduction in tariffs has a positive effect on plants’ female share of employment. However, when they distinguish between the share of female among skilled workers and among unskilled workers, they uncover that those employment gains benefit mostly unskilled women while trade openness has been detrimental to skilled women. Indeed, an increase in the plant’s export share reduces the female share of skilled workers. As for a reduction in tariffs, it increases the demand for unskilled female labour but not the demand for skilled women among exporting plants, while it increases the demand for both types of female labour among non-exporting plants. Juhn et al. (2012) use plant-level data for Mexico to examine changes in gender wage and employment gaps. The results indicate that tariff reductions following the North American Free Trade Agreement increased the employment and wage bill share of women in unskilled
occupations but not among skilled workers. Implementing a two stage empirical strategy, they show that trade openness affects gender inequalities by increasing firms’ incentives to invest in new machinery equipment. Klein et al. (2010) evaluate the contribution of firms’ trade orientation to wage inequality across and within male and female groups of workers for the German manufacturing industries between 1993 and 2007. They find that women face a wage penalty compared to men and that this penalty is increasing with the skill level. The wage gap is smaller in exporting plants for low-skilled individuals but not for college educated individuals. They additionally find that the export-wage premium is increasing with the skill level of the individual within groups. To sum up, trade openness contributes to wage dispersion among men and among women, but the effect on the wage gap depends on the skill level of the workers. Finally, Oostendorp (2004) looks at the impact of sectoral trade shares on the gender wage gap within narrowly defined occupations for more than 80 countries. Exploiting the changes in trade intensity within a given occupation-sector-country cell over time, he finds that an increase in the sectoral trade share narrows the occupational wage gap for unskilled labour only and the occupational gender wage gap is lower in unskilled occupations compared to skilled occupations, the difference being bigger in developing countries.

This paper offers a theoretical explanation for the finding that trade affects the gender wage gap differently across the skill distribution. The model features two groups of workers, men and women, whose characteristics vary in two dimensions, skills and job commitment which corresponds to workers’ availability and willingness to maintain a long and continuous working life. We assume that men and women have the same skill distribution that is perfectly observable. However, commitment is unobservable by the employer which generates statistical discrimination. In particular, employers discriminate against women because they have on average a lower labour market commitment4. As employers pay worker-specific wages, a woman is hired at a lower wage compared to a man with identical skills to compensate for the potential loss in case of lower commitment. This setting has been inspired by Lazear & Rosen (1990) dynamic model of statistical discrimination where women face a lower promotion probability along the job ladder of a given firm because of learning in top jobs and a lower propensity for women to remain on the job. In our

4Gender differences in labour market commitment stems from work interruptions typically due to maternity leave and child rearing. It also includes the impossibility to work overtime as well as lower energy on the job due to greater time spent on housework and childcare
model, workers are sorted across firms rather than types of jobs; moreover, the matching is not determined by learning but by technology differences that result from firms' endogenous investment decisions. Firms make a simultaneous decision on technology investment and hiring as in Yeaple (2005). They calculate the expected workers productivity conditional on the technology, their observation of worker's skills and their expectation about the worker's degree of commitment. We assume that high-skill workers are more productive than low-skill workers (absolute advantage) but they are more so in high-technology firms (comparative advantage). Similarly, strongly committed workers are always more productive than less-committed workers (absolute advantage) but they are more so in high-technology firms (comparative advantage).

The model gives the sorting of men and women across firms and the wage gap distribution in a closed economy setting. The most skilled workers are employed in the high-technology firms where the skill reward and the expected commitment reward are higher. Yet, to enter a high-technology firm, women need to have a higher skill level than men to compensate for the uncertainty on their level of commitment. This leads to higher gender wage gaps in the upper part of the distributions which fits within-country evidence.

We then shed light on the implications of international trade for the gender wage gap in the presence of such heterogeneity. We consider a monopolistic competition framework (Krugman (1980)) where two identical countries trade different varieties of a differentiated good. Trade is costly, generating both fixed and variable costs, so that only the most productive firms that use the high-technology engage in exporting along the lines of Melitz (2003). A reduction in trade costs spurs firms to adopt the high-technology to benefit from export revenues which increases the demand for skilled workers and strongly committed workers. We find that trade liberalization increases the gender wage gap in the upper tail of the distribution as it induces more firms to adopt skill-intensive production technologies where job commitment acts as a strategic complement. However, the effect on the mean gender wage gap is ambiguous.

This paper is related to two strands of literature. First, it contributes to a large body of work dealing with how trade openness, associated with firm heterogeneity, influences wage inequalities. In much of this literature, exporters differ from non-exporters as they are bigger, more productive, more skill-intensive and pay higher wages. See the influential paper by Bernard & Jensen (1997) for evidence on the exporter wage premium and skill-biased shift in labour demand in U.S. manufacturing during the 1980s. Bustos (2011) gives evidence...
related to recent trade models using labour assignment to provide insights about the impact of globalization on the labour markets (see for example Yeaple (2005), Ohnsorge & Trefler (2007), Costinot & Vogel (2010)). The novelty of the present paper is to introduce another dimension in worker heterogeneity that is unobservable to the firm and leads employer to discriminate on the basis of the average characteristic of the group the worker belongs to. We show that the wage dispersion induced by trade occurs both within gender groups, along the skill level, and between gender groups, generating changes in the gender wage gap adjusted for observable skills.

In this way, the paper also contributes to the literature that deals with the effect of the overall wage structure on the wage gaps across groups such as men and women by taking into account the effect of trade openness.\footnote{The wage structure is the values the labour market attaches to skills and other productive characteristics. For studies of the US labour market, see Blau and Kahn (1992, 1994, 2003)}

This work is also related to the literature on gender discrimination in the labour market and in particular to the papers studying the effect of trade openness on the adjusted gender wage gap. Most of those studies have focused on the competition effect of international trade in a setting with taste-based discrimination (see ?, Black & Brainerd (2004), Menon & van der Meulen Rodgers (2009) among others for empirical appraisals of the foreign competition effect, as well as Ederington et al. (2009) and Ben Yahmed (2012) for both empirical and theoretical analysis). The present study focuses on a channel different from the competition channel coupled with taste-based discrimination. It focuses on the channel of biased technological change induced by trade openness in a setting with statistical discrimination. One related paper is the one by Juhn et al. (2012) who provide an explanation for narrowing gender wage gaps among blue-collars but not among white-collars, a pattern they observe in Mexico in the aftermath of the NAFTA. They assume that the new technology reduces the need for physical strength in blue collar occupations so that the relative demand for female labour increases in those occupations. We depart from their setting by proposing a model with worker heterogeneity in two dimensions, an observable characteristic distributed equally among men and women, and an unobservable characteristic unequally distributed. Differences in expected productivity arise endogenously from the matching of firms and workers. This paper gives general conditions on the production technology under which we can generate non-monotonic effects of trade on gender inequal-
The rest of the paper is organized as follows. Next section describes the setup of the model. Section 3 provides the equilibrium in a closed economy where the distribution the gender wage gap across fits within-country evidence. In section 4, we characterize the open economy equilibrium and derive the implications of international trade and further reductions in trading costs for the distribution of the gender wage gap. The final section concludes.

2 Setup of the model

2.1 Demand

Preferences are identical across all consumers who choose a quantity of a homogeneous good and a quantity of varieties of a differentiated good. The utility function is Cobb Douglas between the differentiated good $X$ and the homogeneous good $Y$ and presents a CES sub-utility over the varieties $i$ of $X$. This function expresses a love of variety of consumers. Then

\[
U = Y^{1-\beta}X^\beta
\]

\[
X = \left( \sum_i x_i^\alpha \right)^{\frac{1}{\alpha}}
\]

where the elasticity of substitution across varieties of $X$ is given by $\sigma = \frac{1}{1-\alpha}$. The price index of the differentiated good $X$ is:

\[
P_X = \left( \sum_i p_i^{1-\sigma} \right)^{\frac{1}{1-\sigma}}.
\]

If all prices are equal, the price index is $P_X = pN^{\frac{1}{1-\sigma}}$. It decreases with $N$ the number of varieties produced and the elasticity of substitution $\sigma$. Consumers choose the share of their income $M$ they will devote to the differentiated good by maximizing their utility subject to their revenue constraint. The price of the homogeneous good is normalized to one.

\[
X = (\beta M)/P_X
\]

\[
Y = (1 - \beta)M
\]

Let us note $E = \beta M$ is the portion of income spent on the differentiated good. Consumers decide also how much of each variety they consume. As they value diversity, they
consume a positive amount of each symmetric variety and spend the same amount on each variety:

\[ x_i = \frac{E}{P_X} \left( \frac{p_i}{P_X} \right)^{-\sigma} \]  

(2)

The demand for variety \( i \) takes into account the average price of good \( X \). The term \( \frac{E}{P_X} \) corresponds to the aggregate demand for \( X \) while the price differential \( \frac{p_i}{P_X} \) models the competition effect between variety \( i \) and the other varieties.

### 2.2 Worker heterogeneity in observable and unobservable characteristics

The workforce is heterogeneous in both skills and job commitment. There is a continuum of skills \( s \) distributed among the population according to a distribution function \( L \) over the support \([0; \bar{s}]\). \( L(s) \) is the inelastic supply of labour with skill no greater than \( s \). We assume that men and women have the same exogenous skill distribution \( L_f(s) = L_m(s) = L(s) \). The mass of workers per group is normalized to one. As for the differences in job commitment, let us assume that there are two types of individuals, the highly-committed that spend the maximum time and effort in the firm over the period \( e = \bar{e} \) and the low-committed ones for which \( e = \bar{e} \). We simplify the model by assuming that men always demonstrate a high level of commitment \( Pr_m(e = \bar{e}) = 1 \) while women have a probability to favour labour market activity over their domestic activities equal to \( Pr_f(e = \bar{e}) = \eta \) with \( 0 < \eta < 1 \). There is no correlation between \( s \) and \( e \) which means that the probability of being highly committed to one’s job is independent of one’s skill level.

The skill of a worker can be perfectly observed by the employers however the level of job commitment is unobservable: employers cannot anticipate the time and energy a worker is

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7 In sociology, the preference theory developed in Hakim (2000) argues that differences in women’s preferences for combination of domestic activities and paid employment explain differences in labour market attachment among women. She sorts women in three categories: home-centered, adaptative and work-centered. Only women belonging to the last two categories participate to the labour market. We model the difference between these two groups by an exogenous difference in job commitment.

8 This amounts to a normalization of male probability of commitment. We actually just need that men are more likely to prefer work-centered lifestyles and thus have a higher probability to be highly committed.

9 Skill investment is exogenous in this model. Our results will hold if we allow for a correlation between skills and commitment sufficiently low compare to the degree of complementarity in the production function as we will see below.
going to put in the job. Even though employers expect some women to be highly committed, they are unable to know which ones prior hiring. As a consequence, employers use the average female labour market attachment to make expectations about female productivity. Labour productivity is increasing with both $s$ and $e$ and depends also on the technology $j$ in use, as we will see in the next sub-section.

### 2.3 Production

The productivity of a worker endowed with skills of level $s$ and a level of commitment $e$ when working with technology $j$ is noted $\varphi_j(s,e)$. Because employers cannot observe $e$, they form expectations based on the observable, the skill and the sex of the worker. We note $\tilde{\varphi}_{jg}(s)$ the expected productivity of a worker with skill $s$ from group $g$ as viewed by the employer prior hiring when technology $j$ is used. As men’s productivity is perfectly observable, $\tilde{\varphi}_{jm}(s) = \varphi_j(s,\bar{e})$. As for women’s productivity, employers form identical expectations given by:

$$\tilde{\varphi}_{jf}(s) = E(\varphi_j(s,e)|\eta) = \eta \varphi_j(s,\bar{e}) + (1 - \eta) \varphi_j(s,e).$$

Employers anticipate different productivities for a man and a woman endowed with the same skill level and working with the same technology:

$$\tilde{\varphi}_{jm}(s) > \tilde{\varphi}_{jf}(s) \quad \forall j \in \{l,h\} \text{ and } \forall s \in [0;\bar{s}].$$

In sector Y, the homogeneous good, firms produce under constant returns to scale and perfect competition using labour only. We assume that labour productivity does not depend on either workers’ skills or effort in this sector and we set $\varphi_Y = 1$. We note $c_Y$ the unit cost of production equal to the wage per efficient unit of labour: $c_Y = \frac{w_Y}{\varphi_Y}$. Under perfect competition in both product and labour markets, firms set prices equal to their unit cost of production $p_Y = \frac{w_Y}{\varphi_Y}$. We choose sector Y as the numeraire $p_Y = 1$, consequently we have $c_Y = w_Y = \varphi_Y = 1$.

In sector X, the differentiated good sector, firms operate under imperfect competition and increasing returns to scale. We assume that the sector is characterized by horizontal product differentiation and monopolistic competition where $N$ firms produce each a variety of the differentiated product. Firms have to pay a fixed investment cost to produce one variety. This innovation cost $F$ acts as an entry barrier which ensures that each variety is produced by only one firm. As varieties are not perfect substitutes, firms enjoy some market power that enable them to make positive operating profits and pay the fixed cost. After choosing its technology, the firm can produce a variety of good X hiring labour; we
differentiate male labour \( m \) and female labour \( f \). The following assumptions characterize the technology and the productivity function.

**Assumption A1. Fixed and variable costs**
Firms can invest in two different technologies indexed by \( j = \{l, h\} \). To acquire the the high-technology firms bear a higher fixed cost \( F_h > F_l \) but benefit from a higher productivity of labour for a given skill level and commitment, \( \varphi_h(s, e) > \varphi_l(s, e) \). If a worker has no skill, \( s = 0 \), his/her productivity is the same in sector \( X \) and \( Y \) : \( \varphi_h(0, e) = \varphi_l(0, e) = 1 \).

Firms choose the type of investment they make considering both its cost and the resulting gain in productivity. This specification is consistent with R&D being positively correlated with firm productivity (Klette & Kortum (2004) for example).

**Assumption A2. Log-supermodularity in skills and technology**
Skills acts as a strategic complement to technology upgrading:

\[
\frac{\varphi_h(s', e)}{\varphi_h(s, e)} > \frac{\varphi_l(s', e)}{\varphi_l(s, e)} \quad \text{for any } s' > s \quad \forall e
\]

The productivity gain derived from hiring a worker with a higher skill is greater under technology \( h \). This means that workers with higher skill levels have a comparative advantage in the sophisticated technology.

**Assumption A3. Log-supermodularity in commitment and technology**
Job commitment and technology upgrading are complementary:

\[
\frac{\varphi_h(s, \bar{e})}{\varphi_h(s, e)} > \frac{\varphi_l(s, \bar{e})}{\varphi_l(s, e)} \quad \text{for any } \bar{e} > e \quad \forall s
\]

This means that strongly committed workers have a comparative advantage in the high technology.

**Assumption A4. Log-supermodularity in skills and commitment**
Job commitment and skills are complementary:

\[
\frac{\varphi_j(s', \bar{e})}{\varphi_j(s, \bar{e})} > \frac{\varphi_j(s', e)}{\varphi_j(s, e)} \quad \text{for any } \bar{e} > e \quad \text{and } s' > s \quad \forall j = \{l, h\}
\]
A high skill level is more valuable when the worker’s job commitment is high.

Assumptions A2 to A4 requires the productivity function to be non separable in \( s, e \) and \( j \).

We further assume that labour markets are perfectly competitive and that employers are risk-neutral. Workers bear no search cost and wages are flexible. Firms take the wage rate per efficiency unit of labour as given. Writing \( \tilde{c}_j \) the expected unit cost under technology \( j \), firms set worker-specific wages in accordance with the expected productivity of each worker \( w_g(s) = \tilde{c}_j \tilde{\varphi}_{jg}(s) \).

3 The closed economy Equilibrium

3.1 Profit maximization under monopolistic competition

Firms operating with the same technology are symmetric, in particular they have the same expected productivity for a given worker of skill \( s \) and sex \( g \), that we note \( \tilde{\varphi}_{jg} \). Consequently the expected unit cost \( \tilde{c}_j \) is identical across firms of identical technologies. As a result, the technology \( j = \{h, l\} \) specifies all relevant firm’s variable. We can thus solve firms’ problem in sector X using two representative firms \( h \) and \( l \). Total production cost for a firm \( j \) can be written as :

\[
TC_j = \frac{1}{N_j} \sum_g \left( \int_{s \in S_{jg}} w_{jg}(s)l(s)ds \right)
\]

where \( S_{jg} \) is the set of skills of workers belonging to group \( g \) employed by a firm of type \( j \). \( N_j \) is the endogenous number of \( j \)-firms. Effective labour is used to produce \( q_j \) quantity of good X to be sold and to pay the fixed cost :

\[
q_j + F_j = \frac{1}{N_j} \sum_g \left( \int_{s \in S_j} \varphi(s)l(s)ds \right)
\]

We denote by \( \tilde{c}_j \) the expected cost per efficient unit of labour under technology \( j \), \( \tilde{c}_j = \frac{w_g(s)}{\tilde{\varphi}_{jg}(s)} \). The profit of a firm using technology \( j \) can written :

\[
\pi_j = p_j q_j - c_j(q_j + F_j)
\]
Firms maximize their expected profits with respect to quantities\textsuperscript{10}.

\[ \pi_j = \max_{q_j} \{ p_j q_j - \tilde{c}_j (q_j + F_j) \} \]

The first-order condition for equilibrium is:

\[ p_j = \frac{\sigma}{\sigma-1} \tilde{c}_j \]  \hspace{1cm} (3)

Under competitive labour markets and monopolistic competition, firms with technology \( j \) hire workers up to the point where the wage per efficiency unit of labour, \( \tilde{c}_j = w_j(s, \eta)/\varphi_j(s, \eta) \), equals the marginal revenue product \( p_j \frac{\sigma-1}{\sigma} \). Hence, employees working with the same technology are paid the same fraction of their respective expected productivity:

\[ w_{jq}(s) = p_j \frac{\sigma-1}{\sigma} \varphi_{jq}(s) \]

with \( 0 < \frac{\sigma-1}{\sigma} < 1 \).

3.2 The wage distribution for men and women

We follow Yeaple (2005) where workers with different skills sort across \( h \) and \( l \) type firms. In this paper, workers not only differ in their observable skill \( s \) but also in their unobservable degree of job commitment \( e \). Following the literature on job assignment, we assume that workers know the wage they can earn if they are matched to a given firm \( j \) and go to the firm that offers the highest wage.

**Proposition 1. Sorting of workers**

*If higher skill workers have a comparative advantage in \( h \)-type firms then \( h \)-type firms hire the most skilled workers of each group \( g \)*

We prove this result in the appendix by showing that if positive assortative matching is not followed, the value of the output and wages can increase by switching the assignment of workers to firms \textsuperscript{11}. This self-selection process implies that within each group there is a marginal worker who is indifferent between working in a low-tech firm or a high-tech firm.

\textsuperscript{10}Under monopolistic competition without any strategic interactions, competition on prices or on quantities lead to the same equilibrium result.

\textsuperscript{11}This sorting mechanism has been first suggested by Roy (1951) where workers self-select into the occupation that gives them the highest expected earnings.
The wage distribution for men and women is given by the function \( w_{jg}(s) = \tilde{c}_j \tilde{\varphi}_{jg}(s) \) where the wage of a worker is equal to the cost per unit of efficient labour times the expected productivity of the worker. We can give an expression for the wage that depends on firms’ technologies and the skill thresholds \( s_{jg} \) below which a worker from group \( g = \{ f, m \} \) is not hired by a firm \( j = \{ l, h \} \).

\[
\begin{align*}
    w_g(s) &= \begin{cases} 
        c_Y \varphi_Y = 1 & \text{if } s < s_{lg} \\
        \tilde{c}_l \tilde{\varphi}_{lg}(s) & \text{if } s_{lg} \leq s < s_{hg} \\
        \tilde{c}_h \tilde{\varphi}_{hg}(s) & \text{if } s_{hg} \leq s 
    \end{cases} 
\end{align*}
\] (4)

Among each group \( g = \{ m, f \} \), workers with a skill level equal to the threshold \( s_{lg} \) is indifferent between working in sector Y and working in a firm \( l \) in sector X. Similarly a worker with a skill level \( s_{hg} \) is indifferent between working in a firm using either technology \( h \) or \( l : \tilde{c}_l \tilde{\varphi}_{lg}(s_{hg}) = \tilde{c}_h \tilde{\varphi}_{hg}(s_{hg}) \). Consequently, we can rank the unit cost of production :

\[
\frac{\tilde{c}_l}{c_Y} = \frac{\varphi_Y(s_{lg})}{\tilde{\varphi}_{lg}(s_{lg})} = \frac{1}{\tilde{\varphi}_{lg}(s_{lg})} < 1 \quad \text{and} \quad \frac{\tilde{c}_h}{\tilde{c}_l} = \frac{\tilde{\varphi}_{lg}(s_{hg})}{\tilde{\varphi}_{hg}(s_{hg})} < 1
\] (5)

Firms in the diversified sector have lower unit cost of production than firms in sector Y. Within sector X, firms using the low technology have higher unit cost than firms using the high technology.

Using the indifference condition for both groups, we can rank the skill threshold required to men and women.

\[
\frac{\varphi_Y(s_{lf})}{\tilde{\varphi}_{lf}(s_{lf})} = \frac{\varphi_Y(s_{lm})}{\tilde{\varphi}_{lm}(s_{lm})} \Leftrightarrow \frac{1}{\tilde{\varphi}_{lf}(s_{lf})} = \frac{1}{\tilde{\varphi}_{lm}(s_{lm})}
\]

and

\[
\frac{\tilde{\varphi}_{lm}(s_{hm})}{\tilde{\varphi}_{hm}(s_{hm})} = \frac{\tilde{\varphi}_{lf}(s_{hf})}{\tilde{\varphi}_{hf}(s_{hf})} \Leftrightarrow \frac{\tilde{\varphi}_{lm}(s_{hm})}{\tilde{\varphi}_{hm}(s_{hm})} = \frac{\tilde{\varphi}_{lf}(s_{hf})}{\tilde{\varphi}_{hf}(s_{hf})}
\]

Using these two equations, we can state the following proposition :

**Proposition 2. Ranking of male and female skill requirements**

i) \( s_{jf} \) is a function of \( s_{jm} \) and \( \eta \)

ii) Given that \( Pr_f(e = \bar{e}) < Pr_m(e = \bar{e}) \) and \( \varphi_l \) is increasing in \( e \), employers using the technology \( l \) require from women a higher skill level \( s_{lf} > s_{lm} \)
iii) Under the assumptions A2 and A3, the skill threshold to work for a firm $h$ is higher for women $s_{hf} > s_{hm}$.

The proof is developed in the appendix.

Consequently, a woman working in sector Y can have a greater skill level than a man working in a firm $l$; this holds for workers with skills comprised between the male and female threshold for entering sector X, $s_{lm} \leq s \leq s_{lf}$. Similarly, a female worker employed in a firm $l$ can have a greater skill level than a men working in a firm $h$; this holds for workers with skills comprised between the male and female threshold for entering a firm $h$, $s_{hm} \leq s \leq s_{hf}$.

We can now describe the wage distribution for both men and women. The slope of the wage profile becomes steeper at each group-specific skill threshold $s_{lg}$ and $s_{hg}$ because the technology $l$ enhances worker productivity compare to the technology used in sector Y and the technology $h$ features stronger skill complementarity than the technology $l$. Within group, the skill of any worker using the technology $l$ is lower than the skill of a worker using the technology $h$.

We can further give the distribution of the wage gap along the skill distribution where $WG(s) = \frac{w_m(s)}{w_f(s)}$ is the gap between a man and a woman of skill $s$.

$$
WG(s) = \begin{cases}
1 & \text{if } s \leq s_{lm} \\
\frac{\phi_{lm}(s)}{\phi_Y} & \text{if } s_{lm} \leq s \leq s_{lf} \\
\frac{\phi_{lm}(s)}{\phi_{lf}(s)} & \text{if } s_{lf} \leq s \leq s_{hm} \\
\frac{\phi_{hm}(s)}{\phi_{hf}(s)} & \text{if } s_{hm} \leq s \leq s_{hf} \\
\frac{\phi_{hm}(s)}{\phi_{hf}(s)} & \text{if } s_{hf} \leq s
\end{cases}
$$

Proposition 3. Under assumptions A1, A2 and A4, the gender wage gap is increasing in the skill level.

We give here an overview of the proof provided in the appendix. The assumption on the complementarity between skills and technology (A2) ensures that the gender wage gap is increasing in $s$ when men and women work with different technologies while the assumption on the complementarity between skills and commitment (A4) ensures that the gender wage gap is increasing in $s$ even for men and women hired by firms with identical technologies.
3.3 Free entry and market clearing

Investment in technology is unrestricted so that the number of firms adjusts until profits using either technology are zero. For each type of technology, the unit cost under which total revenues equal total (labour) costs is:

$$\hat{c}_j = \frac{\sigma}{\sigma - 1} P_{X}^{\sigma - 1} \left( \frac{E(\sigma - 1)}{F_j} \right) \left( \frac{1}{\sigma} \right)$$

(6)
The different fixed costs generate two productivity cutoffs. Producing with the technology \( h \) requires a higher productivity \( \tilde{c}_h < \tilde{c}_l \) to be able to make higher operating profits to pay for the higher fixed cost. Firms make their investment and human resources decisions jointly as the unit cost of producing with a given technology depends on the skill level of the workforce.

Relating the zero profit conditions for both types of firms, we obtain:

\[
\frac{\varphi_{hg}(s_{hg})}{\varphi_{lg}(s_{hg})} = \left( \frac{F_h}{F_l} \right)^{\frac{1}{2}}
\] (7)

This pins down the skill threshold to enter a firm \( h \) for both men and women as a function of the technologies’ parameters. An increase in the fixed cost to invest in the high technology increase the skill threshold required to workers.

Female and male total labour supply is assumed to be fixed and is divided across the tree types of firms.

The numbers of high-technology and low-technology firms in sector X are given by:

\[
N_h(q_h + F_h) = \int_{s \in S_{hf}} \varphi_h(s)l(s)ds + \int_{s \in S_{hm}} \varphi_m(s)l(s)ds
\]

\[
N_l(q_l + F_l) = \int_{s \in S_{lf}} \varphi_f(s)l(s)ds + \int_{s \in S_{lm}} \varphi_m(s)l(s)ds
\]

Using the free entry condition:

\[
N_h = \frac{1}{\sigma F_h} \left( \int_{s \in S_{hf}} \varphi_h(s)l(s)ds + \int_{s \in S_{hm}} \varphi_m(s)l(s)ds \right)
\] (8)

\[
N_l = \frac{1}{\sigma F_l} \left( \int_{s \in S_{lf}} \varphi_f(s)l(s)ds + \int_{s \in S_{lm}} \varphi_m(s)l(s)ds \right)
\] (9)

The number of firm \( j \) depends on the four skill thresholds \( s_{jg} \) with \( j = \{h, l\} \) and \( g = \{m, f\} \).

The threshold \( s_{hm} \) is pined down by the free entry condition in sector X while the sorting of workers across the two types of firms relates \( s_{hm} \) to \( s_{hf} \). The market clearing condition for good Y determines the skill threshold \( s_{lm} \).

To close the model, we finally use the market clearing condition in sector Y where the level of production is \( Y = \sum_g \int_{s} l(s)ds \).

The demand for good Y, given by the Cobb-Douglas preferences, must equal the production
of the good. Since $Y$ is the numeraire $p_Y = 1$, the market clearing condition is:

$$Y = (1 - \beta)M$$

where $M$ is total revenue which equals total wages (firms make no positive profits in equilibrium): $M = \sum g(\int_{s \in S_g} l(s)ds + \int_{s \in S_{hg}} \varphi_h(s)l(s)ds)$.

Consumption of good $Y$ is a function of the cost thresholds $s_{lg}$ and $s_{hg}$.

Using equations (13) and (5), replacing $M$ in the equation for the demand of good $Y$ and equalizing demand and production for good $Y$ we obtain:

$$\frac{\beta}{1-\beta} \hat{\varphi}_l(s_{lm}) \sum_g \int_{s_{lg}}^s l(s)ds = \sum_g \left( \int_{s_{lg}}^s \hat{\varphi}_l(s_{hm})l(s)ds + \frac{\hat{\varphi}_h(s_{hm})}{\hat{\varphi}_h(s_{hm})} \int_{s_{hg}}^s \hat{\varphi}_h(s)l(s)ds \right)$$

Equation (10) defines the skill threshold below which individuals are working in sector $Y$.

4 The open economy

4.1 Profit maximization and export patterns in the open economy

We assume that the domestic country trades with an identical foreign country so that we need to define the allocations and equilibrium in one country only. Markets are segmented because of a variable trade cost $\tau$ which includes fret and insurance costs along with tariffs. As a result, a firm may charge different prices on the domestic and foreign market. Besides, a firm incurs a fixed export cost $F^t$ to start exporting as in Melitz (2003). $F^t$ covers fixed market access costs such as setting up new distribution channels, shipping requirements as well as ensuring that the firm’s goods conforms to foreign standards and regulatory environment. The fixed cost generates a selection of firms into exporting as it is established by the empirical literature. Regardless of the export decision, a firm always incurs the investment cost $F^j$. Because this overhead production cost is already incurred, a firm would not export and not produce for its domestic market. Indeed domestic sells

\footnote{Assuming that the trading partner is identical allows to consider only one set of skill thresholds, $s_{lf}, s_{lm}, s_{hf}, s_{hm}$, which are common to each country. Country differences, for example in the skill distribution or in the technology, would give rise to different thresholds and equilibrium conditions that vary across countries.}
yield always strictly higher operating profits compare to sells to foreign markets because of the additional fixed and variable costs.

The demand for a variety $i$ of the differentiated good comes now from both domestic and foreign consumers who are assumed to have the same preferences:

$$x_i = p_i^{-\sigma} E P_X^{\sigma-1}$$
$$x_i^* = (p_i^*)^{-\sigma} E P_X^{\sigma-1}$$

where $p_i$ is the price of variety $i$ on the domestic market and $p_i^*$ is the price of variety $i$ when it is traded to a foreign market. $E$ is the share of the income spent on goods $X$. The price index is now:

$$P_X = \left( \sum_i p_i^{1-\sigma} + \sum_k p_k^{1-\sigma} \right)^{\frac{1}{1-\sigma}}$$

where $p_k^*$ is the price of variety $k$ traded by a foreign firm and sold on the domestic market. Firms are subjected to per-unit iceberg trade cost $\tau$. To address the foreign demand, a firm need to produce $q^t = \tau x_i^*$ as a share $\tau$ of the production is required for transportation.

Firms maximize their profits with respect to either price or quantity:

$$\pi_j = \max_{p_j} \{ p_j q_j + I^t (p_j^t q_j^t) - \tilde{c}_j (q_j + F_j + I^t (q_j^t + F^t)) \}$$

where $I^t$ equals 1 if the firm exports and $q_j^t = \tau x_i^*$. As marginal costs are constant, we can separate the profits they earn on each market. The pricing rule in the domestic market implies, exactly as in the autarky case, that the marginal cost of production equates the marginal revenue.

$$p_j \frac{\sigma - 1}{\sigma} = \tilde{c}_j$$

Firms who export will set higher prices in the foreign markets that reflect the increased marginal cost due to the transportation cost $\tau$ that is completely supported by the consumer (the standard mill pricing strategy):

$$p_j^t \frac{\sigma - 1}{\sigma} = \tau \tilde{c}_j \Leftrightarrow p_j^t = \tau p_j$$

The marginal cost of production is still given by $\frac{w_{ijg}(s)}{v_{jg}(s)}$ so that the sorting of workers across
firms stated in proposition 1 continues to hold, $h$-firms employ the workers with the highest skill level.

What are the firms that export?

Profits of a $j$-firm that serves only the domestic market are:

$$\pi_j = c_j^{1-\sigma} \frac{\sigma^{\alpha}}{(\sigma-1)^{\alpha}} EPX^{\sigma-1} - \tilde{c}_j F_j$$

Profits of a $j$-firm that serves both markets are:

$$\pi_j^t = c_j^{1-\sigma} \frac{\sigma^{\alpha}}{(\sigma-1)^{\alpha}} EPX^{\sigma-1}(1 + \tau^{1-\sigma}) - \tilde{c}_j (F_j + F^t)$$

A firm of type $j$ finds it profitable to export if $\pi_j \leq \pi_j^t$. Three cases arise:

1. $F^t \tau^{\sigma-1} \geq F_h$, no firm export
2. $F_l \leq F^t \tau^{\sigma-1} \leq F_h$, $h$ firms only export
3. $F^t \tau^{\sigma-1} \leq F_l$, both $l$ and $h$ firms export

We can see directly that if there are no fixed cost to export, $F^t = 0$, all firms are able to export and no level of variable cost $\tau > 1$ can generate the selection of the most productive firms into exporting. As the differences between exporters and non exporters -within sectors- are empirically pervasive, it is accepted that models with CES demand should assume a combination of fixed and variable trade costs to generate a sorting of firms according to their productivity.

From now on, we focus on the case 2 where only the high-technology firms are able to export. The free entry conditions determine which workers are employed by exporters. For $h$ firms, the zero-profit conditions implies:

$$\tilde{c}_h = (\sigma(F_h + F^t))^{1-\sigma} (-\frac{\sigma}{\sigma-1})^{\frac{1-\sigma}{\sigma}} EP^{\sigma-1} X (1 + \tau^{1-\sigma})^{\frac{1}{\sigma}}$$

We denote $\tilde{c}_a$ the marginal cost of $j$ firms under autarky. The expected cost per efficient unit of labour that firms $h$ can pay $\tilde{c}_h$ is larger under trade, $\tilde{c}_h > \tilde{c}_a$. This stems from the increase in market size that benefits to exporting firms.

The zero profit condition for $l$-firms is:

$$\tilde{c}_l = (\sigma F_l)^{1-\sigma} (-\frac{\sigma}{\sigma-1})^{\frac{1-\sigma}{\sigma}} (EPX^{\sigma-1})^{\frac{1}{\sigma}}$$
4.2 The wage distribution in the open economy

Trade openness has an impact on the skill-thresholds that define which type of men and women are hired by high-tech firms. As before, we can find the skill-threshold to enter a firm $h$ for by relating the two zero-profit conditions for $h$ and $l$ firms:

$$
\frac{\tilde{c}_h}{\tilde{c}_l} = \left( \frac{(F_h + F^t)}{F_l(1 + \tau^{1-\sigma})} \right)^{\frac{1}{\sigma}}
$$

From the above equation, we can see that the difference in marginal costs between the two types of firms is smaller under trade than under autarky, $\frac{\tilde{c}_h}{\tilde{c}_l} < \frac{c^a_h}{c^a_l}$.

Using the indifference conditions for the marginal workers of each group whose skill levels define the skill-threshold, $w_{lg}(s_{hg}) = w_{hg}(s_{hg}) \iff \frac{c_l}{c_l} = \frac{\varphi_{lg}(s_{hg})}{\varphi_{hg}(s_{hg})}$, we have:

**Proposition 4. When only $h$ firms export,**

i) the skill threshold to enter a firm $h$ is lower under trade compare to the autarky case for both groups, $s_{hg} < s^a_{hg}$ for $g = \{l, h\}$. More workers are matched with a high technology firms under trade.

ii) the skill threshold to enter a firm $h$ is still higher for women, $s_{hm} < s_{hf}$

iii) trade liberalization further reduces the skill requirement for both groups, 

$$
\frac{\partial \left( \frac{\tilde{c}_h}{\tilde{c}_l} \right)}{\partial \tau} > 0 \Rightarrow \frac{\partial s_{hg}}{\partial \tau} > 0 \ \forall g
$$

Although the expression for $\tilde{c}_l$ does not change, its value changes with openness. The decrease in the skill threshold $s_{hg}$ to enter $h$ firms raises wages for the most skill workers; this in turn raises total income which corresponds to a higher demand for the non-trade good. This effect will be explicit when the general equilibrium effect is highlighted. Sector $Y$ thus demands more labour. Consequently, we have a higher skill threshold to enter the manufacturing industry under trade $s_l > s^a_l$ and the marginal production cost of firms $l$ goes down $\tilde{c}_l < \tilde{c}^a_l$. Trade openness brings an increase in productivity in the manufacturing sector along with a higher demand for local services for instance, as a result some workers move from the manufacturing sector to the non-traded sectors; this is in line with general employment patterns.

**Proposition 5. When only $h$ firms export,**


i) the skill threshold to enter a firm $l$ is higher under trade compared to the autarky case for both groups, $s_{lg}^l > s_{lg}^a$

ii) the skill threshold to enter a firm $l$ is still higher for women, $s_{lm}^l < s_{lf}^l$

iii) the effect of trade liberalization is to further increase the skill threshold above which workers are employed in the traded sector, $\frac{\partial (\tilde{c}_l/c_Y)}{\partial \tau} > 0 \Rightarrow \frac{\partial \tilde{s}_{lg}}{\partial \tau} > 0 \ \forall g$

This is consistent with the stylized fact that the share of manufacturing employment among women is lower than among men and that trade openness does not reverse that trend.

The wage function has the same form than under autarky but the values of the skill thresholds $s_{lg}$ and $s_{hg}$ as well as the cost thresholds $\tilde{c}_l$ and $\tilde{c}_h$ has changed:

$$w_g(s) = \begin{cases} 
  c_Y \varphi_Y = 1 & \text{if } s \leq s_{lg} \\
  \tilde{c}_l \tilde{\varphi}_{lg}(s) & \text{if } s_{lg} \leq s \leq s_{hg} \\
  \tilde{c}_h \tilde{\varphi}_{hg}(s) & \text{if } s_{hg} \leq s 
\end{cases}$$ (13)

To measure the changes in the gender wage gap along the skill distribution, we compare the gap under autarky with the gaps under openness. The gender wage gap $\frac{w_m(s)}{w_f(s)}$ is now given by:

$$WG(s) = \begin{cases} 
  1 & \text{if } s \leq s_{lm} \\
  \frac{\tilde{\varphi}_{lm}(s)}{\tilde{c}_l} & \text{if } s_{lm} \leq s \leq s_{lf} \\
  \frac{\tilde{\varphi}_{lm}(s)}{\tilde{c}_l \tilde{\varphi}_{lf}(s)} & \text{if } s_{lf} \leq s \leq s_{hm} \\
  \frac{\tilde{\varphi}_{hm}(s)}{\tilde{c}_l \tilde{\varphi}_{hf}(s)} & \text{if } s_{hf} \leq s 
\end{cases}$$

Appendix D makes this result explicit.

**Proposition 6.** Following trade liberalization,

i) the gender wage gap is reduced at the bottom of the skill distribution as $s_{lm} > s_{lm}^a$

ii) the gender wage gap widens at the top of the distribution given that $s_{hm} < s_{hm}^a$ and the wage profile is steeper under technology $h$.
Figure 2: Changes in the gender wage gap with trade openness

4.3 Free-entry and market clearing

In the case where only $h$ firms export and using the free entry condition, the number of firms is given by:

$$N_h = \frac{1}{\sigma(F_h + F_f)} \left( \int_{s \in S_{hf}} \bar{\varphi}_{hf}(s) l(s) ds + \int_{s \in S_{hm}} \bar{\varphi}_{hm}(s) l(s) ds \right)$$ (14)
\[ N_l = \frac{1}{\sigma F_l} \left( \int_{s \in S_l} \tilde{\varphi}_l f(s) l(s) ds + \int_{s \in S_{lm}} \tilde{\varphi}_{lm} s) l(s) ds \right) \] (15)

More workers are hire by \( h \) firms under trade as the skill threshold is lower \( s_{hg} > s_{hg}^a \) \( \forall g \).

Finally, the market clearing condition for good \( Y \) determines the new skill threshold \( s_{lm} \). Good \( Y \) is not traded. The market clearing condition is still given by \( Y = (1 - \beta)M \), where \( M \) equals total wages in the open economy. Skilled workers’ wages have increased following trade liberalization as more firms adopt the high-technology. \( M = \sum_g (\int_{s \in S_{Yg}} l(s) ds + \int_{s \in S_{hmg}} w_{hmg}(s) l(s) ds) \).

Using (13) with the new skill thresholds,

\[ M = \sum_g (\int_{0}^{s_{mg}} l(s) ds + \tilde{c}_l \int_{s_{mg}}^{s_{hm}} \tilde{\varphi}_{lmg} s) l(s) ds + \tilde{c}_h \int_{s_{hm}}^{s_{hg}} \tilde{\varphi}_{hmg} s) l(s) ds) \]

Using equation (5) and equalizing demand and production for good \( Y \), we have:

\[ \frac{\beta}{1 - \beta} \tilde{\varphi}_l(s_{lm}) \sum_g \int_{0}^{s_{mg}} l(s) ds = \sum_g \left( \int_{s_{mg}}^{s_{hm}} \tilde{\varphi}_{lmg} s) l(s) ds + \frac{\tilde{\varphi}_l(s_{hm})}{\tilde{\varphi}_{hmg} s) l(s) ds} \right) \] (16)

This equation determines the skill threshold below which individuals are now working in sector \( Y \) and closes the model.

### 5 Conclusion

This paper offers a theoretical explanation for varying gender wage gap along the skill distribution and heterogeneous impacts of trade openness on the wage gap depending on the position along the skill distribution. We need three supermodularity assumptions on the labour productivity function to give general conditions under which we find the pattern observed in empirical studies. More precisely we show that if skills and job commitment are complements to technological upgrading and if skills and job commitment are complements, statistical discrimination based on job commitment expectations generates a higher gender wage gap at the upper part of the distribution. The closed economy part of the model thus puts forward one reason for the glass ceiling effect as well as the increase in residual wage disparity within gender groups as documented by numerous empirical assessments.
The analysis also provides insights into the impact of trade openness in a setting with intra-industry trade and monopolistic competition. First, we show that when trade openness induces technological change biased towards observable and unobservable productive characteristics, it increases the wage gap at the top of the skill distribution. Second, due to general equilibrium effects, the gender wage gap is reduced at the lower part of the distribution.

The paper adds to the understanding of the interactions between the overall wage structure of an economy and the gender wage gap and can be used to interpret more general shocks that affect the demand for observable and unobservable characteristics of workers. It provides a rationale for looking at the contribution of what we call employers’ requirement for commitment in shaping gender inequalities.

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26
A Sorting of heterogeneous workers across firms

We can prove by contradiction, that a high-technology firm hires workers with higher skill level compared to a low-technology firm.

Consider two workers with skill $s_1 < s_2$. Let us assume that worker 1 is hired by a firm $h$ and worker 2 is hired by a firm $l$.

Firm $h$ pays worker 1 so that its profit is maximized:

$$\frac{\sigma - 1}{\sigma} p_h = \frac{w_h(s_1)}{\hat{p}_h(s_1)}$$

Firm $l$ pays worker 2 so that its profit is maximized:

$$\frac{\sigma - 1}{\sigma} p_l = \frac{w_l(s_2)}{\hat{p}_l(s_2)}$$

Firm $l$ would not increase its profit by hiring worker 1 at a wage just above the one paid by a firm $l$:

$$\frac{\sigma - 1}{\sigma} p_l \leq \frac{w_h(s_1)}{\hat{p}_l(s_1)}$$

Firm $h$ would not increase its profit by hiring worker 2 at a wage just above the one paid by a firm $h$:

$$\frac{\sigma - 1}{\sigma} p_h \leq \frac{w_l(s_2)}{\hat{p}_h(s_2)}$$

Equations 1 and 3 implies that $\frac{p_l}{p_h} \leq \frac{\hat{p}_h(s_1)}{\hat{p}_l(s_1)}$.

Equations 2 and 4 implies that $\frac{p_l}{p_h} \geq \frac{\hat{p}_h(s_2)}{\hat{p}_l(s_2)}$.

Which implies that $\frac{\hat{p}_h(s_2)}{\hat{p}_l(s_2)} \leq \frac{\hat{p}_h(s_1)}{\hat{p}_l(s_1)}$. But this contradicts the assumption that more skilled workers have a comparative advantage in the high-technology.
B Ranking of male and female skill requirements

The indifference condition states that:

\[
\tilde{\varphi}_{hf}(s_{hf}) \frac{\varphi_{hm}(s_{hm})}{\varphi_l(s_{hm})} = \tilde{\varphi}_{lf}(s_{hf})
\]

\[
\iff \eta \varphi_h(s_{hf}, \bar{e}) + (1 - \eta) \varphi_h(s_{hf}, e) = \frac{\eta \varphi_l(s_{hf}, \bar{e}) + (1 - \eta) \varphi_l(s_{hf}, e)}{\varphi_l(s_{hm}, \bar{e})} = \frac{\eta \varphi_l(s_{hf}, \bar{e}) + (1 - \eta) \varphi_l(s_{hf}, e)}{\varphi_l(s_{hm}, \bar{e})}
\]

That we can rearrange

\[
\iff \frac{\eta + (1 - \eta) \varphi_l(s_{hf}, \bar{e})}{\varphi_l(s_{hf}, \bar{e})} = \frac{\varphi_l(s_{hm}, e) \varphi_h(s_{hf}, e)}{\varphi_l(s_{hm}, \bar{e}) \varphi_l(s_{hf}, \bar{e})}
\]

Let us prove by contradiction that \( s_{hf} > s_{hm} \).

Suppose that \( s_{hf} = s_{hm} = s_h \), the condition is now

\[
\frac{\eta + (1 - \eta) \varphi_l(s_{hf}, \bar{e})}{\varphi_l(s_{hf}, \bar{e})} = \frac{\varphi_l(s_{hm}, e) \varphi_h(s_{hf}, e)}{\varphi_l(s_{hm}, \bar{e}) \varphi_l(s_{hf}, \bar{e})} \iff \varphi_h(s_{hf}, e) = \frac{\varphi_l(s_{hm}, e) \varphi_h(s_{hf}, \bar{e})}{\varphi_l(s_{hf}, \bar{e})}
\]

Which contradicts the supermodularity assumption between technology and commitment. So that the male and female skill requirements cannot be equal.

Suppose now that \( s_{hf} < s_{hm} \). By supermodularity between technology upgrading and skills, we know that:

\[
\frac{\eta + (1 - \eta) \varphi_h(s_{hf}, \bar{e})}{\varphi_h(s_{hf}, \bar{e})} < 1
\]

By supermodularity between technology upgrading and skills and the fact that labour productivity is increasing in skills, we know that:

\[
\frac{\varphi_h(s_{hm}, \bar{e})}{\varphi_h(s_{hf}, \bar{e})} > \frac{\varphi_l(s_{hm}, \bar{e})}{\varphi_l(s_{hf}, \bar{e})} > 1
\]
Combining the two we have:

\[
\frac{\eta + (1 - \eta) \frac{\varphi_h(s_{hf}, e)}{\varphi_l(s_{hf}, e)} \varphi_l(s_{hm}, \bar{e})}{\eta + (1 - \eta) \frac{\varphi_l(s_{hf}, e)}{\varphi_l(s_{hf}, e)}} < \frac{\varphi_h(s_{hm}, e)}{\varphi_h(s_{hf}, \bar{e})}
\]

which contradicts the indifference condition. The female skill requirement to be hired by a high-tech firm cannot be lower than the male skill requirement.

**C  Proof of proposition 3: the gender wage gap is increasing in the level of skills**

We explain here under which assumptions the gender wage gap increases with \( s \). The expression for the wage gap is:

\[
WG(s) = \begin{cases} 
1 & \text{if } s \leq s_{lm} \\
\frac{\tilde{c}_l \tilde{\varphi}_{lm}(s)}{\tilde{\varphi}_Y(s)} & \text{if } s_{lm} \leq s \leq s_{lf} \\
\frac{\tilde{\varphi}_{lm}(s)}{\tilde{\varphi}_{lf}(s)} & \text{if } s_{lf} \leq s \leq s_{hm} \\
\frac{\tilde{c}_h \tilde{\varphi}_{hm}(s)}{\tilde{c}_l \tilde{\varphi}_{lf}(s)} & \text{if } s_{hm} \leq s \leq s_{hf} \\
\frac{\tilde{\varphi}_{hm}(s)}{\tilde{\varphi}_{hf}(s)} & \text{if } s_{hf} \leq s 
\end{cases}
\]

There is no wage gap between men and women who work in sector \( Y \) as labour productivity in \( Y \) does not depend on either skills or job commitment. This is relevant for jobs involving routine tasks. When the workers’ skill levels are comprised between \( s_{lm} \) and \( s_{lf} \), the wage gap equals \( \frac{\tilde{c}_l \tilde{\varphi}_{lm}(s)}{\tilde{\varphi}_Y(s)} \) which is greater than 1 and increasing in \( s \) from (A1). For that part to be increasing in \( s \), we actually need that technology \( l \) features stronger complementarities with skills compare to the technology used in sector \( Y \).

When \( \varphi \) is supermodular in skills and technology upgrading (A2), \( \frac{\tilde{c}_h \tilde{\varphi}_{hm}(s)}{\tilde{c}_l \tilde{\varphi}_{lf}(s)} \) is increasing in \( s \).

The supermodularity of \( \varphi \) in skills and commitment (A4) ensures that the ratios \( \frac{\tilde{\varphi}_{lm}(s)}{\tilde{\varphi}_{lf}(s)} \) and \( \frac{\tilde{\varphi}_{hm}(s)}{\tilde{\varphi}_{hf}(s)} \) are increasing in \( s \).
D Proof of proposition 5 on the changes in the gender wage gap with trade openness

How has the gender wage gap changed compared to the autarky case? Changes in the gender wage gap \( \frac{W(s)}{W^a(s)} \) are non-linear in \( s \).

For \( s \in [0; s_{lm}] \), there is no wage gap under either trade or autarky.

For \( s \in [s_{lm}; s_{lm}] \), \( \frac{W(s)}{W^a(s)} = \frac{1}{c_l \hat{\phi}_{lm}(s)} \), the wage gap is lower under trade as more men are employed in sector \( Y \) where there is no wage gap.

For \( s \in [s_{lm}; s_{lm}] \), \( \frac{W(s)}{W^a(s)} = \frac{\hat{\phi}_{lm}}{\hat{\phi}_{lm}(s)} \), the wage gap is lower under trade as the unit cost of \( l \)-firms has decreased.

For \( s \in [s_{lm}; s_{lm}] \), \( \frac{W(s)}{W^a(s)} = \frac{\hat{\phi}_{hm}(s)}{\hat{\phi}_{lm}(s)} \), the wage gap is higher under trade.

For \( s \in [s_{lm}; s_{lm}] \), \( \frac{W(s)}{W^a(s)} = \frac{\hat{\phi}_{hm}(s)}{\hat{\phi}_{lm}(s)} \) which is greater than 1 as sorting implies that \( \hat{\phi}_{hm}(s) > \hat{\phi}_{lm}(s) \) for \( s > s_{hm} \). The wage gap is higher under trade.

For \( s \in [s_{hm}; s_{hf}] \), \( \frac{W(s)}{W^a(s)} = \frac{\hat{\phi}_{hm}(s)}{\hat{\phi}_{lm}(s)} \) which is greater than 1. The wage gap has increased with trade.

For \( s \in [s_{hf}'; \bar{s}] \), \( \frac{W(s)}{W^a(s)} = 1 \)