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Energy Markets and CO_2 emissions: analysis by stochastic copula autoregressive model

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Abstract

We examine the dependence between the volatility of the prices of the carbon dioxide "CO_2" emissions with the volatility of one of their fundamental components, the energy prices. The dependence between the returns will be approached by a particular class of copula, the Stochastic Autoregressive Copulas (SCAR), which is a time varying copula that was first introduced by Hafner and Manner (2012)[1] in which the parameter driving the dynamic of the copula follows a stochastic autoregressive process. The standard likelihood method will be used together with Efficient Importance Sampling (EIS) method, to evaluate the integral with a large dimension in the expression of the likelihood function. The main result suggests that the dynamics of the dependence between the volatility of the CO_2 emission prices and the volatility of energy returns, coal, natural gas and Brent oil prices, do vary over time, although not much in stable periods but rise noticeably during the period of crisis and turmoils.

Key words: CO_2 emissions, dependence, SCAR copula, Efficient Importance Sampling, GAS model

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1. Introduction

Under the Kyoto protocol, OECD countries must reduce their emissions of greenhouse gas by a minimum of 5% from 1990 levels during the period $2008-2012$. In this framework, the European Union has decided to reduce CO_2 emissions by 8%. To do that, the EU has proposed a framework scheme known as the European Union Emission Trading Scheme (EU-ETS) to determine the price of CO_2 (carbon dioxide) emissions. In this context, european plants with large CO_2 emissions obtain from their governments allowances to emit metric tonnes of CO_2 equivalent. Permits can be traded in spot, future and option markets. In the european context, the first phase of trading was in the years $2005-2007$ and the second one was in the years $2008-2012$ coinciding with the introduction of the Kyoto protocol. The third trading phase started in 2013 and will last until December 2020. In the European Union, the higher production of carbon dioxide emissions is concentrated on the power generation sector and on the small number of plants. The power sector and the heat generation industry drive approximately 55% of the total allowance in the first phase and thus are the key players in the EU-ETS and their behavior greatly influences the carbon price dynamics. The purpose of the EU-ETS trading scheme is to encourage firms to reduce their emissions. For Paolella and Taschini (2008) [2] the scarcity of allowances will drive-up the trend in prices. However, the short life of the prices of $CO2$ emissions is associated with a large level of uncertainty. The price of carbon is usually determined by the market structure and institutional policies. The level of emissions depends on unexpected movements in energy demand, the prices of oil, gas, coal,. . . and weather conditions (temperatures, rainfall,. . .). Bredin and Muckley (2011) [3] show that this market is driven by its fundamental variables, and can be affected by economic growth and/or financial markets. So what are the factors that determine the price of CO_2? In a survey, Springer (2003) [4], shows that among the cofactors that determine the CO_2 emission allowance prices, energy prices and climatic conditions are fundamentals. The main drivers of the price of carbon can be categorized as factor driver by demand and supply forces. Thus, the key supply factors are the number of emission allowances, allocated to individual installations in the National Allocation Plans by the EU, as well as other regulatory uncertainties. The demand factor, however, is more dynamic and the allowance demand is strongly influenced by the demand for electricity. As a result, factors that influence the demand for electricity , such as (extreme) temperature, seasonality and general economic activity are also thought to drive the demand for carbon emission allowances. In the recent literature about the empirical relationship between European Union Allowances prices and its fundamentals, a large theoretical review of the determinants was made by Springer (2003)[4]. Christiansen et al (2005)[5] identifies economic growth, energy prices and weather conditions as key drivers of EUA prices. Chevallier et al (2008) [6] found that the industrial production impact positively (negatively) the carbon market during periods of economic expansion (recession), confirming the relationship between macroeconomics and the price of carbon. (Burniaux (2000)[7], Ciorba et al (2001)[8], Sjim (2005)[9] and van der Mensbrugghe (1998)) in the same way showed that energy prices influence CO_2 prices. Redmond and Convery (2007)[10], Battaler et al (2013)[11], Alberola et al (2008)[6] and all studies including energy variables, assumed geometrical brownian motion process for modeling energy prices. To model electricity, natural gas spot prices, commodity prices, or to describe energy commodities, we use a geometric brownian motion with mean reversion in a long term value θ in the drift term. Concerning the stochastic volatility model, Eydeland and Geman (2005)[12] extend the Heston model (1993)[13] to gas and/or electricity prices. The movements in price are, however, not independent. If they were, then it would be possible to form a portfolio with negligible volatility. To understand the relative magnitude of all these correlations and why they change, it is important to look at the economic factors behind the movements in asset prices. Changes in asset prices reflect changing forecasts of future payments. The information that changes the forecasts is often called "news". Every element of news affects all asset prices; this is one of the most important reasons why correlations change over time. The second important reason is the characteristics of the news change. Time variations arise only from substituting volatility in the innovation for dividends. If there is no predictability in expected returns, then this is also the conditional variances of returns. The longer the memory of the dividend process, the more important is this effect and the greater is the volatility. For these energy commodities, the price is strongly time dependent, and consequently, the covariance and the unconditional correlation are time dependent as well.

We examine, in this paper, the dependence between the conditional volatility of the prices of CO_2 emissions with the conditional volatility of their fundamentals (energy prices): coal, natural gas and Brent Oil as well as the SP-GSCI energy price. We focus on energy because it is used in industrial production and activities with high fossil fuel consumption, and consequently have large CO_2 emissions and as energy prices are one of the main factors determining and driving the carbon prices as stated above. The dependence between the returns will be approached by a particular class of dynamic copula, the Stochastic Autoregressive Copulas (SCAR), a time varying copula that was first introduced by Hafner and Manner (2008) [1] in which the parameter driving the dynamics of the copula follows a stochastic autoregressive process and takes into account the non linearity of the data. In this copula the parameters of volatilities and dependence are estimated by standard maximum likelihood together with Efficient Importance Sampling.

Our article contributes to the literature in several important aspects. We use the dynamic SCAR copula approach to examine the relationship over time between the variables in pairs. In other words, we examine the dynamics of the correlation or dependencies in term of conditional volatility pair by pair between the carbon dioxide emission prices and the other energy prices: (CO_2/Brent oil, CO_2/Natural Gas,CO_2/ SP energy index and the CO_2/coal). We used the dynamic SCAR copula approach, the choice of the best fitted copula model, presented as follows, for each pair mentioned above, respectively, is based on the log-likelihood criteria: the rotated Gumble copula, the Gumbel, the Normal copula and the Frank copula. We have observed for the last pair a strong correlation and common movement, after mid-2011, in the level of trends (obtained by a decomposition analysis in state space framework (see fig 4 in Annex). In addition it is important to mention the fact that since the CO_2 emission prices are traded essentially in European countries, our variables concern also the European markets, except the Natural Gas that is traded in the American market. However we have kept it since it does not differ from the evolution of Natural Gas prices in European countries. Copulas are a flexible, non-standard tools that help decomposing any multivariate distribution into marginal distributions, that describe individual behavior, and fully capture the dependence between the variables. The fact that dependencies can be modeled independently of the marginal distributions, contributed to the expansion of this approach especially since it can be applied over the various type of data and not only financial ones. By focusing on the particular case of the dynamic type of copula, we have improved our model further, since investigating the dependence structure between the commodities and CO_2 emission prices through time is much more realistic and efficient than doing it in a static way. In this context, we find some papers that used the copula approach either in its static version or its dynamic one introduced by Patton, with different commodities. However, in our knowledge, this is the first paper to deal with the implication of energy price commodities on CO_2 emission prices by means of the dynamic SCAR copula. To the best of our knowledge, copulas have been used in commodities markets by Zohrabyan (2014)[14], Kharoubi and German(2008)[15], Reboredo (2011)[16], Nguyen and Bhatti (2012)[17], Hammoudeh et al (2013)[18] and Syed et al(2014)[19]. In addition, the returns of the series are modeled by the Generalized Autoregressive Score (GAS) model that can deal with the jumps, and occasional and temporary changes in the returns better than the GARCH-type model and thus lessens the impact of occasional extreme observations in the series. With the GAS model, the time-varying parameter which characterizes the conditional distribution can be updated using the scaled score of the likelihood function. In section two we present the model and the estimation method used; in section three, the empirical results will be presented and discussed before concluding.

2. The model

2.1. The SCAR model

We introduce the Stochastic Copula Autoregressive (SCAR) model proposed by Hafner and Manner (2012)[1], that can be seen as a multivariate stochastic volatility model. We consider the bivariate time series $(u_{1,t}, u_{2,t})$ for $t = 1 \dots T$ distributed using a time varying copula C with a dynamic parameter θ:

$$(u_{1,t}, u_{2,t}) \propto C(u_1, u_2|\theta_t) \tag{1}$$

where $\theta_t \in \Theta \subset \mathbb{R}$. We suppose that θ_t is driven by a latent stochastic process where $\theta_t = \Psi(\lambda_t)$ and $\Psi : \mathbb{R} \to \Theta$ is a predefined function to assure that the copula parameter is defined in its own domain, depending on the chosen copula.[1] λ_t is an unobservable underlying process that follows a first order autoregressive process:

$$\lambda_t = \alpha + \beta\lambda_{t-1} + \kappa\varepsilon_t, \ |\beta| < 1, \ \kappa > 0 \tag{2}$$

with ε_t is a Gaussian innovation process. The observed variables are transformed into uniform distribution. In the SCAR copula the dynamics are not generated by the data/observations as in the dynamic conditional correlation model or the copula based on the Patton model (Patton, 2006 [20]), but by an independent stochastic process. This model is non linear and can be written in its state-space form. The state equation given by

$$(u_t, v_t)|\lambda_t \sim C(u, v|\Psi(\lambda_t)) \tag{3}$$

and the transition equation:

$$\lambda_t = \alpha + \beta\lambda_{t-1} + \kappa\varepsilon_t \tag{4}$$

[1] For Frank copula the transformation Ψ is $\Psi(x) = x$, for the Clayton copula, it is $\Psi(x) = \exp(x)$, for the Gumbel copula $\Psi(x) = exp(x) + 1$ and for the Gaussian, the Ψ is the inverse Fisher transform, $\Psi(x) = \frac{\exp(2x)-1}{\exp(2x)+1}$.

2.2. Estimation

Before focusing on the dependence structure that can be defined using the copula function, we first need to obtain uniform inputs from the marginal distributions. In the first step, we fit a Generalized Autoregressive Score (GAS) model and then we extract standardized residuals. In the second step, by applying the empirical cumulative distribution function (ECDF) to the standardized residuals, we obtain the uniform inputs $u_{i,t} = F_i(\varepsilon_{i,t})$ where F_i is the *c.d.f.* of variable i. Other transformations are possible and relevant. Decoupling marginal and dependence parameters facilitates estimation. The joint log-likelihood function of the model can be split into two parts: the marginal likelihood(L_{X_1} and L_{X_2}) and the copula likelihood(L_C). The joint likelihood is written as follows:

$$L(\delta_1, \delta_2, \theta, X_1, X_2) = L_{X_1}(\delta_1; X_1) + L_{X_2}(\delta_2; X_2) + L_C(\theta; F(X_1, \delta_1), G(X_2, \delta_2)) \tag{5}$$

F and G are the respective marginal distributions, of the processes X_1 and X_2, δ_1 and δ_2 are the parameters of each distribution. The inference function for margins (IFM) will be used to estimate the parameters of the previous equation. In the first stage only the marginal distributions parameters are estimated:

$$\hat{\delta} = argmax L_{X_i}(\delta_i), \ i = 1, 2 \tag{6}$$

In the second stage, the dependence parameter or the copula parameter is estimated from the copula likelihood L_C w.r.t the estimated parameters of the marginal distributions.

$$\hat{\theta} = argmax L_C(\theta, \hat{\delta}) \tag{7}$$

The likelihood function of stochastic copula models is complex and cannot be evaluated numerically using only the ML. Instead we use a Maximum Likelihood technique by introducing the Efficient Importance Sampling (EIS) procedure introduced by Liesenfeld and Richard (2003)[21] and Richard and Zhang (2007)[22]. As the model has nonlinear state space form, the estimation procedure can also be considered in a nonlinear filtering algorithm and consequently can be related to the nonlinear filtering technique (Doucet et al , 2001[23]). A particular case of the latter is the method of importance sampling, discussed by Durbin and Koopman (2000, 2001)[24] and used to estimate time series models with a state space representation. The EIS procedure, a particular importance sampling technique, is a Monte Carlo (MC) technique aiming to evaluate integrals with high dimensions. Using this technique, we will evaluate efficiently and flexibly the likelihood function with high-dimensional variables in its expression. When we combine both the ML and the EIS, we call it the Maximum Likelihood-Efficient Importance Sampling (ML-EIS) method, and we obtain asymptotically efficient estimators. In this section, we describe and adapt the EIS method to estimate the parameters of our model.

The aim is to estimate the vector of parameters $\omega = (\alpha, \beta, \kappa)$ of the dependence structure. We consider the bivariate process $(U_1; U_2)$ with $U_1 = \{u_{1,t}\}_{t=1}^{T}$ and $U_2 = \{u_{2,t}\}_{t=1}^{T}$. The function $f(U_1, U_2, \Lambda; \omega)$ represents the joint density of the two variables $(U_1; U_2)$ and the latent process Λ defined by $\Lambda = \{\lambda_t\}_{t=1}^{T}$. The likelihood associated to (U_1, U_2) with the parameter ω is the following:

$$L(\omega; U_1, U_2) = \int f(U_1, U_2, \Lambda, \omega) d\Lambda \tag{8}$$

which can be factorized into a product of conditional densities:

$$L(\omega; U_1, U_2) = \int \prod_{t=1}^{T} f(u_{1,t}, u_{2,t}, \lambda_t | U_{1,t-1}, U_{2,t-1}, \Lambda_{t-1}, \omega) d\Lambda \tag{9}$$

Here $U_{1,t-1} = \{u_{1,1} ... u_{1,t-1}\}$, the same applies for $U_{2,t-1}$ and Λ_{t-1}.

The joint density is:

$$f(u_{1,t}, u_{2,t}, \lambda_t | U_{1,t-1}, U_{2,t-1}, \Lambda_{t-1}, \omega) \tag{10}$$

which can be decomposed into the copula density $c(u_{1,t}, u_{2,t} | \lambda_t, U_{1,t-1}, U_{2,t-1}, \omega)$ multiplied by the conditional density of λ_t w.r.t $(U_{1,t-1}, U_{2,t-1}, \Lambda_{t-1})$, $p(\lambda_t | U_{1,t-1}, U_{2,t-1}, \Lambda_{t-1}, \omega)$. Since the conditional density p is independent of past observations of $(U_{1,t-1}, U_{2,t-1})$, then we can get rid of it and the new expression of the likelihood function can be written as:

$$L(\omega; U_1, U_2) = \int \prod_{t=1}^{T} c(u_{1,t}, u_{2,t} | \lambda_t, U_{1,t-1}, U_{2,t-1}, \omega) \times p(\lambda_t | \Lambda_{t-1}, \omega) d\Lambda \tag{11}$$

A natural estimate of the likelihood is based upon drawing sample paths from the sequence of densities p also called the natural sampler, which are directly obtained from the statistical specification of the model. In other words, it would be better to simulate the T-integral likelihood instead of evaluating it since it cannot be determined by analytical or numerical methods because of its high dimensionality. Then, we simulate a large number N of trajectories $\left\{\tilde{\lambda}_t^{(i)}(w)\right\}_{t=1}^{T}$ from the natural sampler p. In that case, the likelihood function is written as :

$$\hat{L}_N(w, U_1, U_2) = \frac{1}{N}\sum_{i=1}^{N}\left[\prod_{t=1}^{T} c(u_{1t}, u_{2t}|\tilde{\lambda}_t^{(i)}(w), U_{1,t-1}, U_{2,t-1}, w)\right] \tag{12}$$

However, this ignores the fact that the observable variables contain information on the latent process as quoted in Danielsson and Richard (1993)[25] and Liesenfeld and Richard (2003)[21] since the $\left\{\tilde{\lambda}_t^{(i)}(w)\right\}_{t=1}^{T}$ are simulated independently of the observed variables U_1 and U_2 and thus the information given by the data is not exploited. As a result, such estimates exhibit very large variance and require a large number of draws. To overcome this issue, the Efficient Importance Sampling method (EIS) aims to find or to construct new samplers such that the information on Λ contained in the data (U_1, U_2) is exploited. Let $\{m(\lambda_t|\Lambda_{t-1}, a_t)\}_{t=1}^{T}$ be a sequence of auxiliary samplers indexed by the parameters a_t to be estimated. The likelihood function can have the following expression:

$$L(\omega; U_1, U_2) = \int \prod_{t=1}^{T}\left[\frac{f(u_{1,t}, u_{2,t}, \lambda_t|U_{1,t-1}, U_{2,t-1}, \Lambda_{t-1}, \omega)}{m(\lambda_t|\Lambda_{t-1}, a_t)}\right]\prod_{t=1}^{T} m(\lambda_t|\Lambda_{t-1}, a_t)d\Lambda \tag{13}$$

which can be calculated by using N trajectories $\left\{\hat{\lambda}_t^{(i)}(a_t)\right\}_{t=1}^{T}$ drawn from the importance sampler m by

$$\tilde{L}_N(\omega, U_1, U_2) = \frac{1}{N}\sum_{i=1}^{N}\left(\prod_{t=1}^{T}\left[\frac{f(u_{1,t}, u_{2,t}, \tilde{\lambda}_t^{(i)}|U_{1,t-1}, U_{2,t-1}, \tilde{\Lambda}_{t-1}^{(i)}(a_{t-1}), \omega)}{m(\tilde{\lambda}_t^{(i)}(a_t)|\tilde{\Lambda}_{t-1}^{(i)}(a_{t-1}), a_t)}\right]\right) \tag{14}$$

The main idea behind the EIS method is to find a sequence of auxiliary constants $\{a_t\}_{t=1}^{T}$ and a function m providing a good match between the numerator and the denominator in the equation (14) so that the variance of $\tilde{L}$ is minimized. The problem is the high dimensionality and therefore we decompose the task into a sequence of manageable low-dimensional optimization sub-problems. The function f will be approximated by a function $k(\Lambda_t, a_t)$ such that:

$$m(\lambda_t|\Lambda_{t-1}, a_t) = \frac{k(\Lambda_t, a_t)}{\chi(\Lambda_{t-1}, a_t)} \tag{15}$$

and

$$\chi(\Lambda_{t-1}, a_t) = \int k(\Lambda_t, a_t) d\Lambda_t \tag{16}$$

The right match between f and m can be seen as a match between $f(\Lambda_t, U_{1,t}, U_{2,t}| \Lambda_{t-1}, U_{1,t-1}, U_{2,t-1}, \omega)$ and $k(\Lambda_t, a_t)$ its functional approximation, and also the density kernel of m.

Jointly, this gives us a method to obtain the auxiliary constants $\{a\}_{t=1}^{T}$. We need to solve the following least squares problems for each period t with low dimensions:

$$\hat{a}_t = \underset{a_t}{\operatorname{argmin}} \sum_{i=1}^{N} \left(\log \left[\tilde{F}_t \right] - c_t - \ln k(\tilde{\Lambda}_t^{(i)}(w); a_t \right)^2 \tag{17}$$

with

$$\tilde{F}_t = f\left(u_{1,t}, u_{2,t}, \tilde{\lambda}_t^{(i)}(w) | U_{1,t-1}, U_{2,t-1}, \tilde{\Lambda}_{t-1}^{(i)}(w) . \chi(\tilde{\Lambda}_{t-1}^{(i)}(w); \hat{a}_{t+1}) \right) \tag{18}$$

for $t = T..1$ and $\chi(\Lambda_T; a_{t+1}) \equiv 1$. The EIS estimate of the likelihood function is obtained by substituting the estimated sequence $\{\hat{a}_t\}_{t=1}^{T}$ and N draws from the importance sampler m into the likelihood function in the equation (14). In addition, if $k(\Lambda_t; a_t)$ as well as ξ belong to the exponential family, the EIS least-squares problem in (17) become linear in a_t. The expression of k proposed by Liesenfeld and Richard (2003)[21] is:

$$k(\Lambda_t; a_t) = p(\lambda|\lambda_{t-1}, \omega) \xi(\lambda_t, a_t) \tag{19}$$

with

$$\xi(\lambda_t, a_t) = \exp(a_{1,t}\lambda_t + a_{2,t}\lambda_t^2) \tag{20}$$

to simplify the least square system above and make it linear. In this case, the conditional mean and variance of m are the following:

$$\mu_t = \sigma_t^2 \times (\frac{\alpha + \beta\lambda_{t-1}}{\nu^2} + a_{1,t}), \tag{21}$$

and

$$\sigma_t^2 = \frac{\nu^2}{1 - 2\nu^2 a_{2,t}} \tag{22}$$

The explicit expressions of p, k and ξ are given by Liesenfeld and Richard (2003)[21]. The steps required to compute and implement the EIS estimation of the likelihood function are the following:

- Step(1): draw N trajectories called $\left\{\tilde{\lambda}_t^{(i)}(\omega)\right\}_{t=1}^{T}$ by using the natural sampler p.

- Step(2): For $T \to 1$, use these random draws to solve the back-recursive least-square regression problem defined above characterized by the following linear regression:

$$\log c(u_t, v_t|\theta_t(\omega)) + \log \xi(\tilde{\lambda}_t^{(i)}(\omega); \hat{a}_{t+1}) = cst + a_{1,t}\tilde{\lambda}_t^{(i)} + a_{2,t}\left[\tilde{\lambda}_t^{(i)}\right]^2 + error \tag{23}$$

- Step(3): Extract N trajectories $\left\{\tilde{\lambda}_t^{(i)}(\hat{a}_t)\right\}_{t=1}^{T}$ from m, the importance sampler, and solve the least squares problem in step 2 again. Iteration of both steps 1 and step 2 until the $\{\hat{a}_t\}_{t=1}^{T}$ converge.

- Step(4): Draw N trajectories from $\left\{\tilde{\lambda}_t^{(i)}(\hat{a}_t)\right\}_{t=1}^{T}$ from m, the importance sampler, from which the EIS estimate of the likelihood function is evaluated according to the equation (14).

Estimating the Underlying Process

Obtaining the set of the estimated parameters (α, β, κ) of the underlying process is crucial, however, not enough to determine exactly the path of the dependence pattern of observations. In other words, we need to get an estimate of the sequence of the process $\{\lambda_t\}$ as well as the function $\Psi(\lambda_t)$. For this matter we make use of the EIS method. The EIS exploits all the information provided by the set of variables and thus generates efficient samples of the underlying process $\{\lambda_t\}_{t=1}^{T}$, to which we apply the transformation Ψ. Finally, the dependence path or the smoothed estimate of $\Psi(\lambda_t)$ is obtained by this expression:

$$\Psi(\hat{\lambda}_{t/T}) = \frac{1}{N}\sum_{i=1}^{N}\Psi\left(\tilde{\lambda}_t^{(i)}(\hat{a}_t)\right) \tag{24}$$

2.3. The Model for the Marginal Distributions

It is crucial to capture the dynamic behavior of the time series data whether in their one-dimensional or multidimensional form. In this context, we can categorize time series with time-varying or dynamic parameters to two types of models: parameter driven models and observation driven models. A time series process is an observation driven model when the dynamics of its parameters are introduced by allowing the parameters to be functions of past observations, exogenous variables and lagged dependent variables. We consider the Generalized Autoregressive Score (GAS) model introduced by Harvey (2013)[26], which belongs to the latter class, and presents a set of advantages. The parameters can be predicted given past information of the data, and so the likelihood evaluation becomes more simple and straightforward; also there is the possibility of extensions to asymmetric, long memory, and other dynamics. Many examples of models are also observation driven, such as the generalized autoregressive conditional heteroskedasticity (GARCH) model (Engle (1982), Bollerslev (1986) and Engle and Bollerslev (1986))[27][28][29], the autoregressive conditional duration (ACD) model and the autoregressive conditional intensity (ACI) model by Engle and Russell (1998)[30], the dynamic conditional correlation (DCC) model (Engle, 2002)[31], the Poisson count model by Davis, Dunsmuir, and Streett (2003)[32]. We refer to the observation driven model based on the score function as Generalized auto-regression score model. The mechanism that drives the dynamics of the time-varying parameters is introduced by the score function. The GAS models are more general and encompass all the listed models above. It also overcomes some limitations in the GARCH one. First, it is robust to jumps and outliers and can model better heavy-tailed financial returns and distributions compared to other GARCH models (Harvey and Sucarrat (2014))[33]. Second, GARCH models are unable to capture the leverage effect since in the conditional variance they ignore the signs of the lagged residuals, unlike the GAS model that accomodate very well the most important characteristics of time-varying financial volatility: leverage, conditional fat-tailedness, conditional skewness and the decomposition of volatility into a short-term and a long-term component. Third, the asymptotic properties are much easier to obtain than with GARCH models. Finally, since the conditional score drives the dynamics of the model, GAS models acquire some attractive theoretical properties. In particular, a simple transformation of the score would allow for another type of model. If we consider the following model:

$$y_t = \sigma_t \varepsilon_t \tag{25}$$

where ε_t is a Gaussian disturbance with a unit variance and zero mean and σ_t is the standard deviation. In this case the GAS(1,1) is equivalent to the GARCH(1,1) and the variance can be written as the following

$$\sigma_{t+1}^2 = \alpha_0 + \alpha_1 y_t^2 + \beta_1 \sigma_t^2 \tag{26}$$

For the GAS(1,1) model combined with Student-t or a Skewed-Student distribution as suggested by Harvey and Chakravarty (2008), the conditional variance is given by:

$$\sigma_t^2 = \omega_1 + \alpha_1 u_{t-1} \sigma_{t-1}^2 + \phi_1 \sigma_{t-1}^2 \tag{27}$$

with

$$u_t = \frac{(\nu_1 + 1)\varepsilon_t^2}{\nu_1 - 2 + \varepsilon_t^2} - 1, \text{ if } \varepsilon_t \sim St(0, 1, \nu_1) \tag{28}$$

and

$$u_t = \frac{(\nu_1 + 1)\varepsilon_t \varepsilon_t^*}{(\nu_1 - 2) g_t \zeta_t^I} - 1, \text{ if } \varepsilon_t \sim SKSt(0, 1, \zeta, \nu_1) \tag{29}$$

3. Empirical results

3.1. Data description

The data set is composed of the CO_2 emission spot prices[2], and by different other energy commodities, and we want to examine their relationship or dependence in their conditional volatility over time. Let's note that CO_2 emission prices are not only driven by energy prices but also by seasonalities, weather related factors as well as industrial production. In this paper we do not study the relationship between the latter (industrial production) and CO_2 allowances, because many papers and work have confirmed the link between the two. Chevallier et al (2008)[6], among others, studied the effect of industrial production on carbon dioxide emission prices between 2005 and 2007. They only considered sectors covered by the EU ETS.

For the energy commodities considered, we take crude Brent oil[3] , natural gas (Natural Gas-Henry Hub /MMBTU)[4] and Coal (Coal ICE API2 CIF ARA).[5] Then we consider the variable SP Goldman Sachs Energy Total returns index

[2]Carbon emission prices correspond to the price of these permits emissions. It is traded in the The European Union Emissions Trading System (EU ETS) which was the first large greenhouse gas emissions trading scheme in the world, and remains the biggest. It Operates in the 28 EU countries and the three EEA-EFTA states (Iceland, Liechtenstein and Norway) and covers around 45% of the EU's greenhouse gas emissions

[3]it is one of the major classifications of oil and can serve as a major benchmark price for purchases of oil worldwide and typically refined oil in Northwest Europe. In fact, most Brent oil is destined for European markets and large parts of Europe now receive their oil from the brent oil. Petroleum production from Europe tends to be priced relatively to this oil.

[4] The Henry Hub pipeline is the pricing point for natural gas price on the New York Mercantile Exchange. This index is considered as a benchmark for the entire North American natural gas market and denominated in \$/mmbtu (millions of British thermal units)

[5]Here we consider the coal prices from the Intercontinental Exchange ICE, a market based in Atlanta, Georgia, that facilitates the electronic exchange of energy commodities. The API 2 index is the benchmark price reference for coal imported into Northwest Europe. The major coal importing ports in this region are Antwerp/Rotterdam/Amsterdam (ARA) and the term CIF (Cost,Insurance and Freight) indicates that insurance and all other charges up to the named port of destination are already included in the price quoted by the seller.

(SP-energy) [6] All price series are quoted in US dollars and were extracted from Datastream International in daily frequency, motivated by the fact that these indexes are products in financial markets and are traded each day except on non-working days (5 days a week). The sample begins in 16/01/2009 to 23/01/2014 giving us 1309 observations. It corresponds to the second phase of trading with carbon allowances parallel to the Kyoto protocol.

By referring to figure 2 in the appendix, the series in levels are non stationary and are integrated of order one. Then we consider $r_t = \ln(P_t) - \ln(P_{t-1})$, the returns [7] of each variable with P_t being the price at time t (Figure 3 in Annex). Referring to the plots of the different returns, there is evidence of time varying and clustering volatility. Meaning that high (low) values of volatility tend to be followed by high (low) values.

Table 1 presents a summary statistics of the variables. All returns have a mean close to zero. Neither of the series had a significant trend over the sample, since means are very small relative to the standard deviations. In order to detect a unit root in any of them, Augmented Dickey Fuller test was performed. We do reject the null-hypothesis of a unit root for all returns, all of them are stationary since the p-value is less than the critical level of 5% and integrated of order 0.

[6] The $S\&P$ GSCI (the Goldman Sachs Commodity Index) is the benchmark for investment in commodity markets and a measure of commodity performance over time. It is a tradable index that is readily available to market participants of the Chicago Mercantile Exchange. The index comprises 24 commodities from all commodity sectors and is a measure of commodity returns comparable to the $S\&P500$. In our case, we only focus on the energy component of the $S\&P$ GSCI that contains: Crude Oil, Brent Crude Oil, RBOB Gas, Heating Oil, GasOil and Natural Gas.

[7] Using returns or the first difference of log-asset price process, removes mean non-stationarity and facilitates measuring all variables in a comparable metric, thus enabling evaluation of analytic relationships amongst two or more variables despite originating from price series of unequal values. Third, exhibits the main statistical features and characteristics of financial data like stationarity, conditional volatility, autocorrelation of returns close to white noise and strong persistence in the squared returns, fat tailed distributions (see table1 for further details).

	CO_2 emissions	Gas	Coal	Oil	SP-energy
Mean	-0.0006	-2.7525e-05	4.2118e-06	0.0006	0.00022
SD	0.0341	0.0340	0.011946	0.0178	0.0168
Min	-0.4465	-0.2408	-0.12895	-0.1197	-0.088
Max	0.2106	0.2619	0.083766	0.0946	0.0773
Kurtosis	28.6329	12.5663	20.237	6.3991	6.3462
Skewness	-1.2725	0.5763	-0.11873	-0.0674	-0.2886
ADF stat	-10.5679	-10.8643	-17.79	-11.6788	-11.4241
ADF p-val	0.001	0.001	0.01	00.01	0.001
Jarque Bera stat	36162.25	5059.982	12250	630.6941	628.431
Jarque Bera p-val	2.2e-16	2.2e-16	0	2.2e-16	2.2e-16
Q(20) stat	75.2516	103.9918	56.2686	24.0575	20.326
Q(20) p-val	2.473e-08	2.415e-13	0.0000265	0.2399	0.4377
$Q(20)^2$ stat	115.0913	544.7371	86.5206	433.0139	675.891
$Q(20)^2$ p-val	2.331e-15	2.2e-16	0	2.2e-16	2.2e-16
Arch LM stat	79.8655	263.5155	3.6239	180.6845	246.5221
Arch LM p-val	4.138e-09	2.2e-16	0.0001	2.2e-16	2.2e-16

Table 1: Descriptive statistics of the return series

For all series except natural gas, the returns distributions display negative skewness, which is evidence of non symmetric distribution, meaning, the probability of loss is superior to that of gain. Moreover, the data indicates an excess kurtosis, hence the tails of the distribution contain more observations than a Gaussian distribution. The Ljung Box statistic for autocorrelation indicates that the null hypothesis of no autocorrelation for the returns, and their squared returns can be rejected. There are exceptions concerning the SP-GSCI energy and Brent oil index where only their squared returns are auto-correlated. We also apply tests for ARCH effects by using the ARCH LM test of Engle (1982)[27]. All series exhibit strong evidence of serial correlation and indicate the presence of ARCH effects in their returns. The examination of the autocorrelation functions of the

series, the ACF of squares' returns, shows a similitude with a random walk, which means that prices cannot be predictable, strictly speaking. Their prices behave rather randomly and do not follow a certain trend or pattern, thus they verify the hypothesis of the efficient market (EHM) (see fig 3 in the Annex).
As mentioned in the introductory section, the processes are generated by Geometric Brownian motion:

$$dp(t) = \mu(t)dt + \sigma(t)dW(t),\ t \geq 0 \tag{30}$$

where $W(t)$ is a standard Wiener process. The main limitation of the generalized Wiener process is that both drift and volatility parameters are constant over time while it is well known that daily returns for instance are heteroscedastic. Ito's process allowed drift and volatility parameters to be time varying. Empirical studies have shown that a continuous diffusion model fails to explain some characteristics of returns. To overcome this inadequacy, we consider the jump diffusion process

$$dp(t) = \mu(t)dt + \sigma(t)dW(t) + \kappa(t)dq(t),\ t \geq 0 \tag{31}$$

where $dq(t)$ is a counting process equal to one when a jump happens in time t, otherwise 0. We must make a distinction between a shift and a random walk type variability. A random walk model involves a gradual random change over time, there is no mean average state, and in the absence of negative feedback, variance increases with time. A jump suggests an abrupt change in relation to the duration of a regime. And that's why we use the GAS model instead of a GARCH model to characterize the marginals, since it prevents and reduces the effect of jumps and temporary changes better than the latter.

3.2. The estimation

As stated above, our aim is to analyze the dynamics of the dependence linking conditional volatility over time, between CO_2 emission spot prices and a set of commodities prices: the energy commodities based on their volatilities. However, we do not consider the relationship between CO_2 emissions and the industrial production which is found to be strong, as stated by (Chevallier et al (2008)[6], Choi et al(2010)...). For that, we consider four pairs of bivariate time series, the CO_2 allowances price being a variable in each of these pair. Those pairs are[8]: CO_2 allowance/Natural gas, CO_2 allowances /Brent oil, CO_2allowances/coal, CO_2 allowances/SP energy. First, we specify the marginal distributions, second the one of the copula. We estimate separately for each variable the GAS model, introduced by Creal, Koopman, and Lucas (2013)[34], best fitted for heavy tailed and skewed distributions with Student's t or Skewed Student innovations to account for marginal time dependence of each variable. A summary of the marginal estimates is given in the table 2. The parameter estimates are positive, significantly different from zero with the level of error equal to 1%, except the constant in the mean and the variance equations, and a small value of standard error. All coefficients are significant. We find that the $GAS(1,1)$ with a student distribution for the innovations is the best fitted for for out of five variables. The coal is the only variable for which we choose the GAS model with skewed student distribution. Following Harvey and Chakravarty (2008)[35], we call the $GAS(1,1)$ model with a t distribution 'Beta-t-GARCH'. Since $|\phi_1| < 1$, the processes are stationary and the results suggest that the student distribution of the innovations do take into account the fat tailed character of all the returns (ν is between 2 and 10).

Having estimated the marginal distributions and transformed the standardized residuals into $U(0,1)$ random variables by the probability integral transform (PIT), the last step is to model the dependence structures between the CO_2 emission price returns and the other returns. Naturally, the estimations are performed by pairs. The SCAR or the time varying stochastic copula is then estimated using the method of EIS.[9] We present the copulas estimates in table 3, and we are interested in investigating the dynamics of the dependence struc-

[8]All series are taken in returns

[9]To reduce the Monte Carlo variation, N, the number of simulated trajectories in the EIS sampler, can take the value of 200.

	CO2	Brent	Natural Gas	SP-energy	Coal
cst	0.000048	0.000533	-0.000577	0.000471	-0.000311
	0.9261	0.1834	0.3946	0.1878	0.2495
ω_1	0.091893	0.018135	0.109353	0.015596	0.154355
	0.0457	0.1188	0.1109	0.1769	0.3696
α_1	0.154819	0.058149	0.098001	0.066217	0.193787
	0.0000	0.0003	0.0000	0.0008	0.0001
ϕ_1	0.995467	0.992936	0.990266	0.993506	0.978019
	0.0000	0.0000	0.0000	0.0000	0.0000
ν_1	5.619055	9.973243	5.401241	6.683429	2.064805
	0.0000	0.0005	0.0000	0.0000	0.0000
ζ	-	-	-	-	-0.028756
	-	-	-	-	0.4057
Log lik	2941.507	3547.334	2833.397	3653.464	4571.52

Table 2: Parameters estimates of the marginal distributions

tures. One of the questions commonly asked is ”which copula should we use?” We use an information criteria given by the maximization of the log-likelihood value. In table 3, we present only the best-fitted copula model. Further estimation concerning all the copulas considered in each pair are presented in the annex (tables 6-9 in Annex). Before evaluating and analyzing the estimates of the SCAR copula obtained by the ML-EIS technique we remind that the latent process λ_t describing the dependence is the following:

$$\lambda_t = \alpha + \beta\lambda_{t-1} + \kappa\varepsilon_t$$

to which we apply a function Ψ depending on the choice of the copula. We notice that β varies according to the variable and the copula family. It is also important to mention that the dynamic aspect of the dependence between the variables in each pair is confirmed since κ is different than 0 as stated by Hafner and Manner[1].

Copula (Log Likelihood)	α	β	κ
SP energy-CO_2			
Normal	0.0022	0.9884	0.0190
(26.1137)	-	-	-
Coal-CO_2			
Frank	0.0223	0.5661	0.9984
(0.7127)	0.0893	0.05675	0.09999
Gas-CO_2			
Gumbel	-0.0266	0.9931	0.1304
(1.3082)	-	-	-
Oil-CO_2			
Rotated Gumbel	-0.0078	0.9962	0.0682
(28.274)	0.0068	0.0029	0.0315

Table 3: Parameters estimates of the best fitted SCAR copula

Having estimated the copulas parameters, we could then obtain the dependence path over time of each pair of variables. However, before discussing the outcomes of the SCAR model, a brief examination of the correlation matrix of the CO_2 emissions and the commodities prices (tables 4 and 5) show us that there is a significant difference between correlations before and after mid-2011. On the whole sample, there are positive but low correlations between coal prices and carbon emission prices (0.19) and coal prices and natural gas prices (0.05). Also, there is a strong negative correlation between Brent oil price and carbon emission price. In this case, the CO_2 emissions are mainly explained by the behavior of the Brent oil and the natural gas. Now if we only consider the second part of the sample after the turning point (mid-2011), the landscape is totally different. The CO_2 emissions prices are mainly correlated to the coal and the Brent oil prices. The correlation with the natural gas is close to zero. The correlation between the coal and the CO_2 emissions has almost quadrupled

(0.77), and the correlation of Brent oil with CO_2 emissions is now positive and decreased from 0.58 to 0.39. The most important impact on the correlation for CO_2 emissions comes from the behavior of the prices of coal. The strong decrease of the coal prices has an impact on the size of the demand. We can see this aspect in figure 4 in Annex, where the level of the trend of the coal price and the carbon emissions price are represented. The trend components are obtained by a classical decomposition of the process in a state space representation.

	Lcoal	LCO_2	Lbrent	Lnatgas	LSP_E
Lcoal	1	0.20	0.58	0.03	0.48
LCO_2	0.20	1	-0.45	0.31	-0.37
Lbrent	0.58	-0.45	1	-0.32	0.91
Lnatgas	0.042	0.31	-0.32	1	-0.25
$LSP-E$	0.48	-0.37	0.91	-0.25	1

Table 4: correlations between the returns for the whole sample

	Lcoal	LCO_2	Lbrent	Lnatgas	LSP-E
Lcoal	1	0.77	0.39	-0.05	0.09
LCO_2	0.77	1	0.35	0.0002	0.015
Lbrent	0.39	0.35	1	-0.19	0.82
Lnatgas	-0.05	0.0002	-0.19	1	-0.087
LSP-E	0.09	0.015	0.82	-0.08	1

Table 5: correlations between the returns from May 2011 to the end of period

For the pair CO_2 emissions/SP-energy, the normal copula is the best one based on the Log-likelihood criteria (LogL) value. The value of β (0.9884), indicates a high persistence in the dependence process. For the pair CO_2 emissions/Brent we retain the Rotated Gumbel copula with a high persistence level and a small value for κ. For CO_2/Natural gas, we notice that the LogL's of Frank and Gumbel copulas are very close (1.5 and 1.3 respectively), thus we should not just discard the Gumbel copula since its given dependence pattern is more informative than the one given by the former copula. For the last pair of variables, that is CO_2 emission prices with coal prices, the Frank copula is the best-fitting model. Let's mention that all the pairs of variables have a highly persistent dependence process. In terms of the log-likelihood value, the Gaussian copula is the most appropriate for the pair: CO_2 emissions with the SP-energy prices. For the CO_2 emissions and Brent oil, the rotated Gumbel is the best fitting model, the Gumbel copula for CO_2 emissions with natural gas and the Frank for the coal and CO_2 emissions.

Figure 1: Time dependence path of the pairs of variables

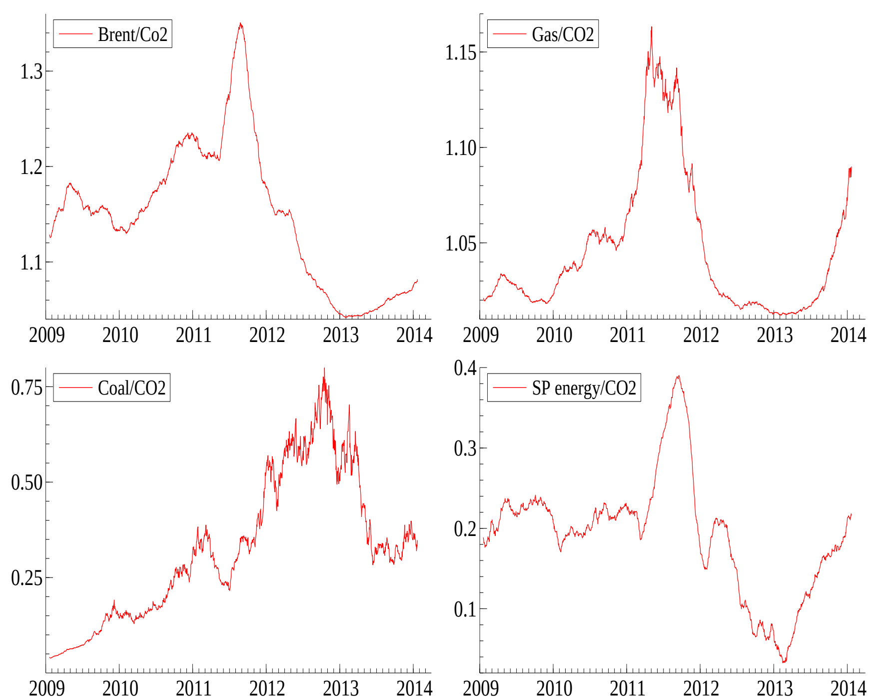

Figure 1 represents the dependence time dynamic captured by these copulas. The evolution of the relationship between the CO_2 and the Brent oil given by the top left of figure 1 is measured by the lower tail dependence based on the Rotated Gumbel copula. It confirms the hypothesis of greater dependence in the period of crisis which corresponds to mid 2011 until the beginning of 2012. The dependence path between the two variables is not linear and vary over time. In the beginning of the study, it is positive and is increasing, although not very strong, between 1.15 and 1.12, giving us a coefficient of correlation between 0.11 and 0.14. Nevertheless this shows the positive impact of oil prices on the CO_2 allowances both being in high demand, especially the former one. Also, it is important to mention that when considering this period of time from 2009 until the beginning 2011, the European economy still suffered from the harmful effects of the financial crisis and was still marked by a slowdown in the beginning of the period. We find a positive relation between the two mentioned variables in this stressful period of time, since the prices of fuel consumption in particular the Brent oil started to recover from their lowest level since 2009 contrarily to financial stock market returns and so did the CO_2 emission prices, a fact confirmed by Duc Khuong and *al* (2013)[18] among others. The strongest rise in dependence in the period between mid 2011 and the beginning of 2012 shows the positive impact of the Brent oil prices on the CO_2 allowances and emission prices that reached its highest level. In fact, both prices reached their maximum value in this period of time. However, the cause may differ from the previous period since both the European economy and world wide economy in general suffered from some crisis and instability caused by the wars and revolutions in the Middle East and North Africa especially Libya and Bahrein on the one hand, and the intensified debt crisis in the euro zone on the other hand. In fact, this finding confirms the increase of the correlation between the assets in an unstable and unsteady period. In the remaining period, the dependence decreased considerably. Oil prices remained high although they did not increase and consequently the CO_2 allowances spot prices decreased significantly. This can be attributed to the decrease in oil demand as well as some regulations and policies established by European countries to encourage and expand the use of renewable and clean energy. The dependence structure of CO_2 allowance prices, and the SP-GSCI Energy prices is given in the bottom right of figure 1. We can determine different phases of the time dependence between the pair of variables: prior to February 2011, this relationship is positive, dynamic, although not much, with values between 0.17 and 0.25. And then we have an expansion of the relation of the two variables in 2011, caused by the same factors stated above, instability and crisis, where the correlation attained a max of 0.4 in this period, followed naturally by a decrease until the beginning of 2013 but remaining positive nevertheless, and reaching almost 0.03 as a minimum correlation, to continue rising over the rest of the period of study. By studying the relationship between these two variables, we notice that the main driver of the correlation in the SP-Energy index is indeed the Brent oil (correlation equal to 0.2 compared to 0.04 for natural gas), a fact pointed out by Chevallier[6]. The dynamics of the dependence between natural gas prices and CO_2 emission prices vary over time and is positive, however little, compared to the relation between the latter one and the Brent oil. We notice the rise of the dependence over the same period of turmoil mentioned above. The dependence path of the Coal/CO_2 emissions presented in the figure 1 emphasizes the dynamic nature of the correlation between these variables. From the beginning of the period of study to mid-2011 the correlation was relatively low (it didn't exceed 0.3) compared to the after mid-2011 period, where it increased considerably and reached the value of 0.75. These results coincide with the results in table 4 and table 5 where we studied the correlation of the coal/CO_2 emissions on the whole sample, then by focusing only on the period from May 2011 to January 2014. Since people substituted oil and natural gas by coal, the demand and use for coal increased and so did the prices and the contracts to emit the CO_2 . This is why the dependence between the Coal and CO_2 emissions is the highest in the second period. So Brent oil, natural gas and coal contribute to the evolution of the CO_2 emission prices, although Brent oil and coal has the most part of it.

4. Conclusion

The market dynamics of carbon dioxide emission prices have important policy implications. In this paper we study the relationship or the dependence between CO_2 prices and the energy prices through the dynamic copula case, the SCAR model. The returns are modeled by a GAS(1,1) model and can deal with the jumps and the temporary changes better than a GARCH type model. By doing so, we are able to take into account the non linearity of the data characterized by excess kurtosis, negative skewness and fat tails. We do have a significant impact of energy prices on the CO_2 emission prices, the dependence does vary over time and is not constant. It rises considerably when facing a period of turmoils, instability and wars, since the rise of energy prices encourage and push firms to substitute it by other type of energy and resources which imply a decrease in its demand and so the same goes for the CO_2 allowances, and that explains its high prices. Also we find that Brent oil as well as coal are the major factors driving the dependence between the aggregated energy prices and the CO_2 emission prices. Our results highlight that energy price volatility has a significant impact on CO_2 allowance prices. However, one should not disregard the fact that other factors should be taken into account in this matter. In fact, dioxide carbon prices volatility, can be affected by R & D in clean energy technologies and renewable energy sources. Also other measures could be introduced to reduce CO_2 emissions, like encouraging the development and use of alternative activities less intensive producers of the greenhouse.

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5. Appendix

5.1. *Basic theory on copulas*

For a complete and detailed review of the theory of copulas, their properties and types please refer to Nelsen(2006).

Examples of Copulas

Below we give some types of copulas functions along with the expressions of their density c and parameter θ. We also specify the exact and appropriate transformations Ψ we use for the SCAR specification depending on our choice of copulas.

- Frank Copula

  The density of Frank copula is given by

  $$c(u,v;\theta) = \frac{\exp((1+u+v)\theta)(\exp(\theta)-1)\theta}{\{\exp(\theta)+\exp((u+v)\theta)-\exp(\theta+u\theta)-\exp(\theta+v\theta)\}^2} \quad (32)$$

  Kendalls τ can be derived through the following expression depending on the parameter θ: $\tau = 1 - \frac{4(1-D_1)(\theta)}{\theta}$ where D is the Debye function $D_k(x) = \frac{k}{x^k}\int_0^x \frac{t^k}{e^t-1}dt$. Frank copula belongs to the family of Archimedean copulas and it does not exhibit tail dependence since it displays a symmetric dependence. Finally, this copula allows for both positive and negative dependence.

- Clayton Copula

  The density of the Clayton copula is:

  $$c(u,v;\theta) = u^{(-1-\theta)}v^{-1-\theta}(u^{-\theta}+v^{-theta}-1)^{-2-1/\theta}(1+\theta) \quad (33)$$

$\theta \in (o, +\infty)$. The functional form of Psi is given by $\Psi(x) = exp(x)$ meaning that the dependence or the copula parameter has a log-normal distribution. The expressions of the tau kendall, the upper tail dependence and the lower tail dependence are respectively:$\tau = \frac{\theta}{\theta+2}$, $\lambda_L = 2^{-1/\theta}$, and $\lambda_U = 0$ for all θ. The Clayton copula is also Archimedean, it displays asymmetry and it only allows for positive dependence.

- Gumbel Copula

  The density of the Gumbel copula is given by:

$$c(u,v;\theta) = \frac{\{log(u)log(v)\}^{\theta-1}\left\{\left[(-\log(u))^{\theta}+(-\log(v))^{\theta}\right]^{1/\theta}+\theta-1\right\}}{\left[(-\log(u))^{\theta}+(-\log(v))^{\theta}\right]^{2-1/\theta}uv} \times \exp\left\{\left[(\log(u))^{\theta}+(-\log(v))^{\theta}+(-\log(v))^{\theta}\right]^{1/\theta}\right\} \tag{34}$$

  with $\theta \in (1, +\infty)$ and the transformation $\Psi = exp(x) + 1$. The Gumbel copula belongs to the Archimedean family. τ, λ_U and $\lambda_L = 0$ can be expressed as the following: $\tau = 1 - \frac{1}{\theta}$, $\lambda_U = 2 - 2^{1/\theta}$ and $\lambda_L = 0$. And it only allows for positive dependence.

- Gaussian Copula

  Defining $x = \Phi^{-1}(u)$ and $y = \Phi^{-1}(v)$ where Φ is the CDF of a standard normal random variable, the density of the Gaussian copula is the following:

$$c(u,v;\theta) = \frac{1}{\sqrt{1-\theta^2}}exp(\frac{2\theta xy - x^2 - y^2}{2(1-\theta^2)} + \frac{x^2+y^2}{2}) \tag{35}$$

  with $\theta \in (-1, 1)$.We use the inverse Fisher transform to obtain the function Psi: $\Psi(x) = (\exp(2x)-1)/(\exp(2x)+1)$.The expression of the Kendall tau is $\tau = \frac{2}{\pi}arcsin(\theta)$. The Gaussian/normal copula has no tail dependence.

- Survival (Rotated) Copulas

  For a copula with a density c and a distribution C, the distribution of its survival copula $\bar{C}$ for a couple of variables(u, v) as well as its density $\bar{c}$ are given respectively by:$\bar{C}$=$C_\theta(1-u, 1-v)+u+v-1$ and $\bar{c}$=$c_\theta(1-u, 1-v)$.It actually represents the original copula c but with a 180° rotaion. The survival copula can be associated with the Gumbel and the Clayton family.

5.2. Tables and figures

Figure 2: Time Series plots

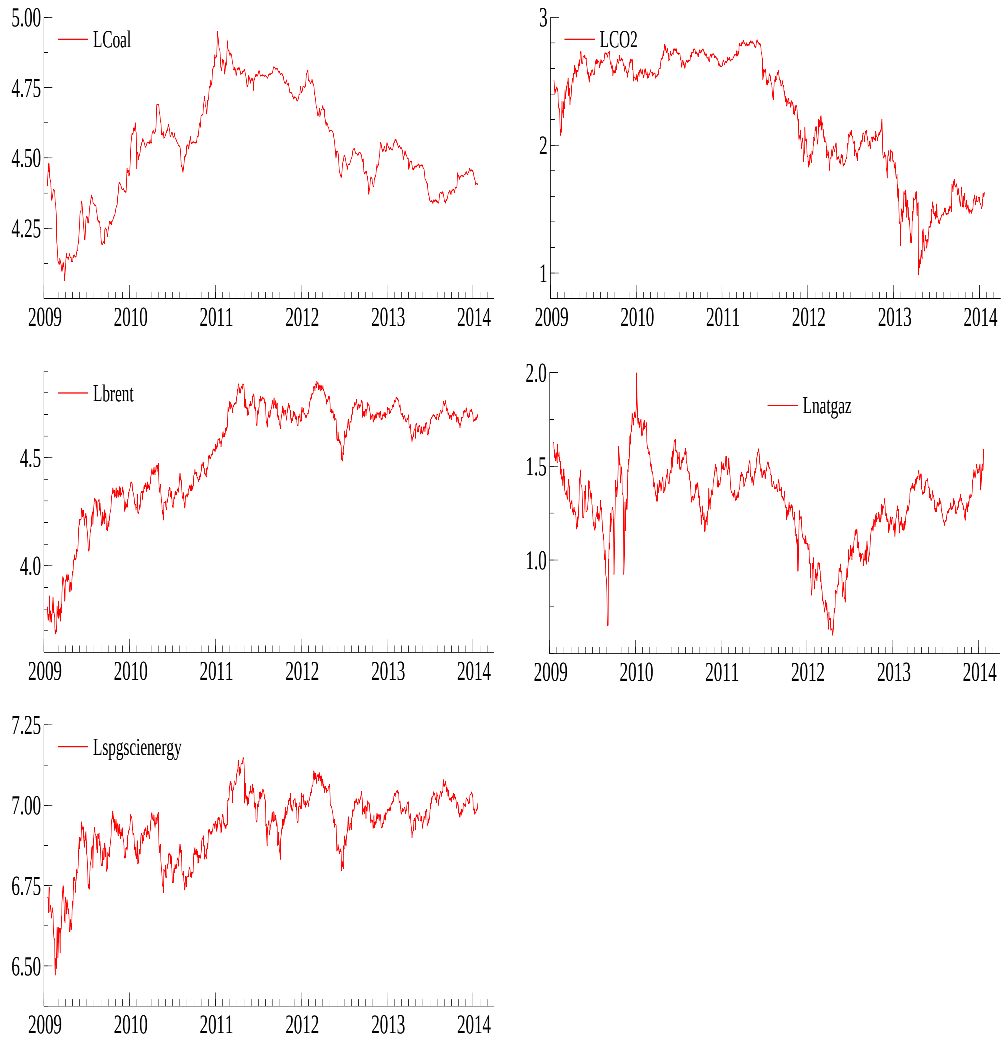

Figure 3: Plots of the Returns and some stylised facts

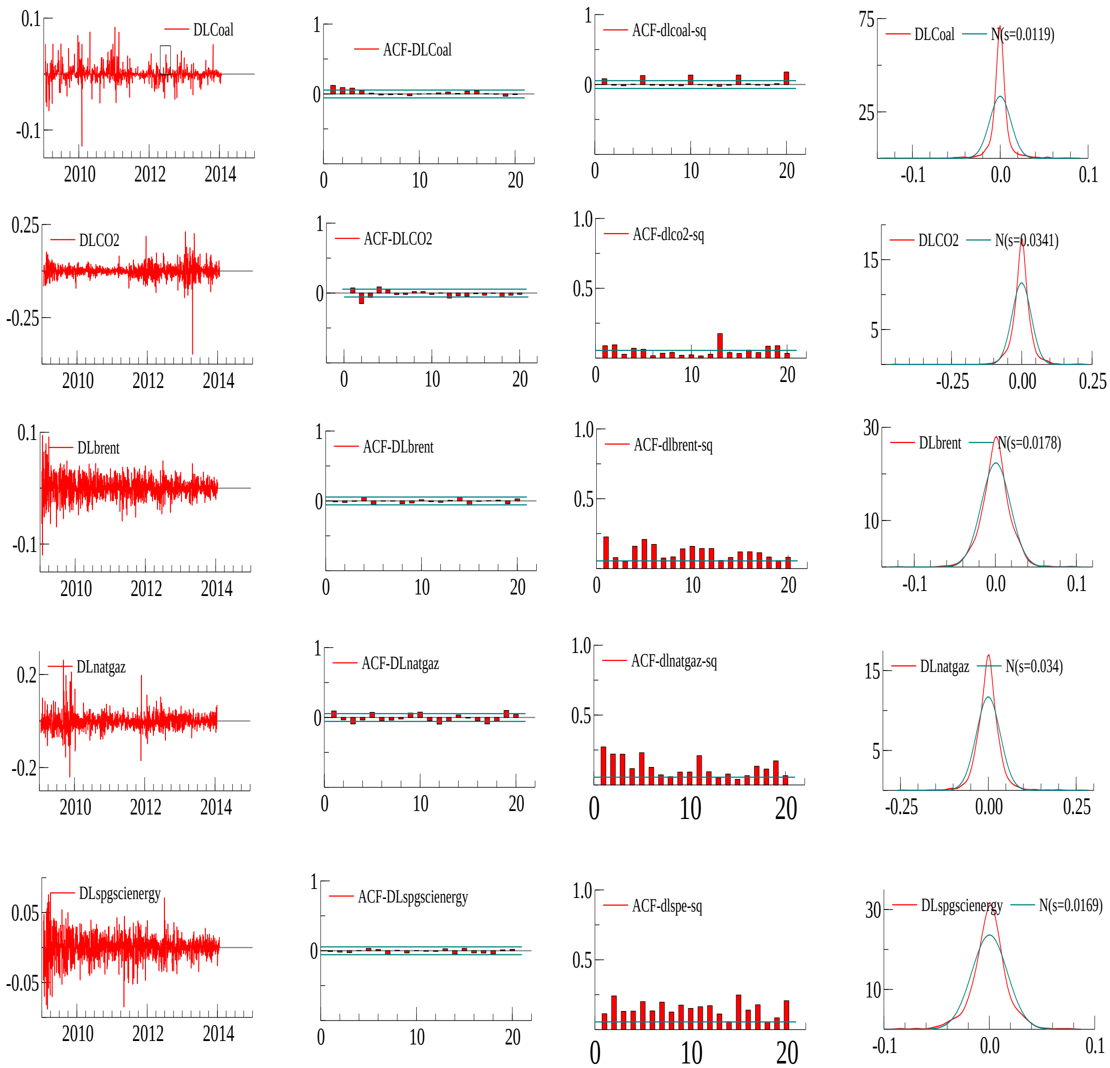

Figure 4: Common levels between the Coal and CO_2 emissions after mid-2011

Copula	α	β	κ
(Log Lkikelihood)	SE	SE	SE
Gumbel	-0.0100	0.9955	0.0625
(21.4517)	0.0084	0.0035	0.0398
Clayton	-0.0068	0.9956	0.0730
(26.4112)	0.0072	0.0042	0.0373
Normal	0.0787	0.6353	0.1529
(27.8781)	0.0659	0.3005	0.0899
Frank	0.6556	0.4988	0.9306
(26.130)	15.8773	11.3559	16.3074
Rotated Gumbel	-0.0078	0.9962	0.0682
(28.274)	0.0068	0.0029	0.0315
Rotated Clayton	0.0071	0.9797	0.2184
-13.3394	-	-	-

Table 6: Parameters estimates of the SCAR copula: CO_2 and the Brent oil

Copula	α	β	κ
(Log Lkikelihood)	SE	SE	SE
Gumbel	-0.0266	0.9931	0.1304
(1.3082)	-	-	-
Clayton	-0.0339	0.9899	0.1245
(1.0491)	0.0544	0.0154	0.1470
Normal	0.0066	0.8371	0.0010
(1.0557)	-	-	-
Frank	0.1201	0.6237	0.6045
(1.5428)	-	-	-
Rotated Clayton	0.0182	0.9718	0.2234
(-65.6433)	0.0060	0.0035	-

Table 7: Parameters estimates of the SCAR copula: CO_2 and the Natural Gas

Copula (Log Lkikelihood)	α SE	β SE	κ SE
Normal	-0.0024	0.6858	0.0691
(0.1430)	0.0099	0.04882	0.01052
Frank	0.0223	0.5661	0.9984
(0.7127)	0.0893	0.05675	0.09999
Rotated Gumbel	-0.1345	0.9909	0.1925
($-3.0339e-006$)	0.085339	0.012646	0.07565
Rotated Clayton	-0.1221	0.9879	0.4631
(-72.232)	0.2064	0.0099	0.6098

Table 8: Parameters estimates of the SCAR copula: CO_2 and the Coal

Copula (Log Lkikelihood)	α SE	β SE	κ SE
Gumbel	-0.0197	0.9920	0.1172
(22.3783)	0.0104	0.0043	0.0333
Clayton	-0.0167	0.9906	0.1121
(23.5969)	0.0116	0.0065	0.0403
Normal	0.0022	0.9884	0.0190
(26.1137)	-	-	-
Frank	0.0119	0.9894	0.1113
(25.8460)	0.0120	0.0104	0.0693
Rotated Gumbel	-0.0541	0.9761	0.2058
(24.0082)	-	-	-
Rotated Clayton	0.0053	0.9785	0.2215
(-12.6788)	0.0038	0.0020	-

Table 9: Parameters estimates of the SCAR copula: CO_2 and SP energy