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Liquidity Trap and Stability of Taylor Rules*

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Abstract

We study a productive economy with safe government bonds and fractional cash-in-advance constraint on consumption expenditures. Government issues bonds and levies taxes to finance public expenditures, while the Central Bank follows a feedback Taylor rules by pegging the nominal interest rate. We show that when the nominal interest rate is bound to be non-negative, under active policy rules a liquidity trap steady state does emerge besides the Leeper (1991) equilibrium. The stability of the two steady states depends, in turns, upon the amplitude of the liquidity constraint. When the share of consumption to be paid cash is set lower than one half, the liquidity trap equilibrium is unstable. The stability of Leeper equilibrium too depends dramatically upon the amplitude of the liquidity constraint. Policy and Taylor rules are thus theoretically rehabilitated since their targets, by contrast with a vast literature, may be now stable. We also show that a relaxation of the liquidity constraint is Pareto-improving and that the liquidity trap equilibrium Pareto-dominates the Leeper one, in view of the zero cost of money.

Keywords: Cash-in-Advance; Liquidity Trap; Monetary Policy; Multiple Equilibria.

JEL Classification: E31; E41; E43; E58

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1 Introduction

Policy makers since a long time, when conducting the monetary policy, follow active Taylor rules consisting in changing nominal interest rates more than one for one when inflation deviates from a given target. This policy aims at keeping inflation anchored to its long run target and thus to stabilize the monetary and financial sector of the economy. The traditional approach is based upon the analysis of the linearized versions of the truly non-linear models and focuses on a local analysis of the Leeper (1991) equilibrium: under active Taylor rules, such an equilibrium is locally unstable and thus unique. This approach, actually, may be misleading since it rules out the study of all the trajectories, as the liquidity trap, other than those consistent with the targets. Benhabib *et al.* (2001), indeed, show that the combination of active Taylor rules and a zero bound on the nominal interest rate creates a new steady state for the economy, a steady state that the authors call "unintended". Moreover, this steady state is stable and thus the economy converges toward it giving raise to a deflationary path with very low levels for the nominal interest rate. As Bullard and Russell (1999) argue, such a feature is consistent with the deflation and nominal interest rate regime observed in Japan in the recent years (see Krugman (1998); Bernanke (1999); Meltzer (1999)). Moreover, the issue of liquidity trap during the 1929 crises has been a source of debate (see among others, Damette and Parent (2015); Basile *et al.* (2010); Orphanides and Wieland (1998); Friedman and Schwartz (1963); James (2001); Hanes (2006) and Gandolfi (1974)). However, Benhabib *et al.* (2001) theoretical findings have not received great attention in the policy debate which appears to be deaft to the perils underlying the conduct of standard and celebrated Taylor rules.

Besides early contributions as that of Brunner and Meltzer (1968), other recent theoretical developments suggest new avenues for research. Orphanides and Wieland (1998) are among the first to have re-addressed the issue in the last twenty years. Schmitt-Grohé and Uribe (2009) call into question the zero interest rate policy as an appropriate strategy to escape the liquidity trap occurrence. They demonstrate that pursuing a zero interest rate policy is not a way to escape liquidity trap but, on the contrary, leads to maintain the economy in a stable liquidity trap equilibrium. Airaudo and Zanna (2012) show that Taylor rules generate, besides liquidity traps, also aggregate instability, endogenous cycles and chaotic dynamics in open economies. Svensson (2000) too studies the liquidity trap in an open economy. Kudoh and Nguyen (2010) analyze the effects of fiscal policy in an economy in which the Central Bank pursue Taylor rules. They find that they depend dramatically upon the conduct of monetary policy. Schmitt-Grohé and Uribe (2013) observe that the great contraction of 2008 pushed the US economy into a lasting liquidity trap characterized by zero nominal interest rates and inflation expectations below the target. At the same time, they find that output growth is recovered but nevertheless unemployment is increased. They refer to such a configuration as to "jobless recovery".

In our paper, following Benhabib *et al.* (2001), we confirm the utility of carrying out a global analysis rather that a pure local one, in order to unveil the existence of equilibrium outcomes other than the Leeper (1991) one, as liquidity traps, expectations-driven fluctuations and deterministic cycles. In our study, government issues bonds and levies taxes to finance public expenditures, while the Central Bank follows a feedback Taylor rules by pegging the nominal interest rate. The model is in infinite-horizon with endogenous labor supply and partial cash-in-advance constraint (CIA) on consumption expenditures as in Bosi *et al.* (2005). Within such a framework, we characterize the existence of stationary solutions and establish the conditions under which the different equilibria are or are not stable. All these things taken together lead to threshold phenomena in terms of the degree of liquidity of the economy such that, once one passes though them, some relevant change of stability occurs.

By focusing on the case where leisure and consumption are substitute goods, we find that under "passive" Taylor rules there is always an unique steady state, which can correspond either to the Taylor equilibrium or to the liquidity trap one. As is the case in Benhabib *et al.* (2001), under active Taylor rules, there may appear two stationary solutions simultaneously, one corresponding to the long-run Taylor target, the other sticking to a zero nominal interest rate. We first Pareto-rank all the stationary equilibrium candidates and determine the GDP associated to each of them. Actually, we find that the liquidity trap equilibrium Pareto-dominates the Taylor target, since the former entails a zero cost of money holding. However, in correspondence to the liquidity trap equilibrium, the Central Bank is not more able to further burst economic activity by an additional cut in the interest rate, since the latter is already sticked to its minimum level. We simultaneously show that as soon as the share of consumption to be paid cash decreases, the welfare associated to both the Taylor equilibrium and the liquidity one increases. This result is easily interpretable, once one keeps in mind that such a share represents the degree of financial market imperfection. By relaxing it, agents can thus buy more and more credit and thus avoid the transaction cost represented by the nominal interest rate. Of course, when the latter is zero, such a cost vanishes and Pareto optimality is restored, as it is case at the liquidity trap equilibrium.

The stability of each steady state arising in our economy depends dramatically upon the amplitude of the liquidity constraint and changes as the latter is made to vary continuously. When the share of the consumption good which must be bought cash is included between one half and one, under active Taylor rules, the liquidity trap equilibrium is stable and thus is reached for infinitely many initial conditions for the control variables. These results confirm Benhabib *et al.* (2001) finding, in which the liquidity trap equilibrium is entirely driven by agents' "state" of expectations. On the other hand, when the degree of financial imperfection is below one-half, the liquidity trap equilibrium becomes unstable and thus the conclusions of Benhabib *et al.* (2001) are reversed. It is now the Taylor target to be indeterminate and thus compatible with infinitely many agents' self-fulfilling beliefs. We also prove the existence of deterministic cycles around the Taylor target, meanwhile the dynamics characterizing the liquidity trap equilibrium is shown to be always monotonic.

Our results confirm the perils of analyzing uniquely the linearized versions of the truly non-linear models, since this rules out the study of all the trajectories, as the liquidity trap, other than those consistent with the targets. In addition, since the liquidity trap equilibrium is unstable for realistic low degrees of financial market imperfection, policy and Taylor rules are within our economy theoretically rehabilitated, not just as a tool for economic recovery but also as coordination devices as soon as the liquidity trap equilibrium seems to represent rather a "pathological" phenomenon, as claimed by the original Keynesian tradition (see Romer (1992) and Romer (2009)). In other words, as our paper shows, accounting for a partial CIA constraint allows to better appreciate the dynamic feature of the model and suggests that the Benhabib *et al.* (2001) results are not completely robust to some not negligible perturbation of the money demand.

The remainder of the paper is organized as follows. In Section 2 we present the economy; we describe the fiscal policy carried out by the government, the monetary policy pursued by the Central Bank, the households behavior and we derive the intertemporal equilibrium. Section 3 is devoted to the analysis of the stationary solutions and deals also with the welfare properties associated to them. The stability analysis represents the content of Section 4 meanwhile some concluding remarks are left to Section 5.

2 The Economy

2.1 The Government and the Fiscal Policy

Let g_t denote the government public spending in real terms in period t and τ_t the tax revenue still in real terms. Let in addition $I_t \equiv 1 + i_t$ be the nominal interest factor in period t, i_t being the nominal interest rate relative to the same period, and B^g_{t+1} the nominal amount of safe government bonds issued in period t. Setting p_t the price of the (unique) consumption good produced in the economy in period t, the government budget constraint relative to period t is therefore given by

$$B^g_{t+1} = p_t g_t - p_t \tau_t + I_t B^g_t. \tag{1}$$

Let us assume in addition that the initial amount of nominal government bonds issued in period zero is $B_0^g > 0$. Let us observe that the real interest factor $R_t$satisfies

$$R_t = I_t \frac{p_{t-1}}{p_t}.$$

The government budget constraint (1) expressed in real terms gives

$$b_{t+1}^g = g_t - \tau_t + I_t b_t^g \frac{p_{t-1}}{p_t} \tag{2}$$

where $b_t^g \equiv B_t^g / p_{t-1}$ denotes the real amount of government bonds issued in period $t-1$. In the remainder of the paper we will focus on a fiscal policy consisting in a constant amount of real government expenditures $g_t = g$ for every $t \geq 1$ and in a flat tax $\tau_t \in [0, 1]$ on labor income w_t such that government income in period t is $\tau_t = \tau_w w_t l_t$. Finally, we assume a Ricardian framework, so that the government budget constraint (1) must be respected for all possible sequence of prices $\{p_t\}_{t=0}^{+\infty}$. In order to complete the description of the government behavior we must introduce the transversality condition ensuring that the present value of national debt, evaluated at the steady state, for t going to infinite is finite:

$$\lim_{T\to\infty} \frac{b_T}{(1+R)^T} < \infty \text{ where } b_T = R^T b_0 + \sum_{t=1}^{T} R^t (g - \tau)$$

which is obviously satisfied.

2.2 The Central Bank and the Monetary Policy

The Central Bank issues money against the purchase of government bonds through open market operations. Denoting B_{t+1}^{CB} the amount of nominal government bonds purchased by the Central Bank in period t and M_{t+1} the stock of nominal balances available in the economy at the outset of period t, the budget constraint of the Central Bank is

$$B_{t+1}^{CB} = I_t B_t^{CB} + M_{t+1} - M_t \tag{3}$$

which, setting $m_t \equiv M_t / p_t$ the real balances available at the beginning of period t, in real terms can be written as

$$b_{t+1}^{BC} = I_t \frac{p_{t-1}}{p_t} b_t^{BC} + m_{t+1} \frac{p_{t+1}}{p_t} - m_t. \tag{4}$$

Following Leeper (1991) and Kudoh and Nguyen (2010), we assume that the Central Bank follows a Taylor (1993) feedback rule:

$$I_t = I(\pi_t) = I^* \left(\frac{\pi_t}{\pi^*}\right)^{\gamma} \tag{5}$$

where $\pi_t \equiv \frac{p_t}{p_{t-1}}$ is the inflation factor between period $t-1$ and t, I^* and π^* are the implicit targets for, respectively, the nominal interest factor and for the inflation factor and $\gamma > 0$ is the elasticity of the nominal interest rate with respect to inflation satisfying

$$\frac{dI}{d\pi}\frac{\pi}{I} = \gamma. \tag{6}$$

When $\gamma < 1$ we will refer as to "passive" Taylor rules and when $\gamma > 1$ as to "active" Taylor rules. Notice that in the Taylor rules we do not include the output gap since in our model is by construction zero (see, e.g., Woodford (1993)) and, in addition, according to several empirical estimates (see, e.g., Clarida *et al.* (1998)), its coefficient falls within a range including very small values for many Central Banks. Since the Central Bank pegs the nominal interest rate, it must supply as much money as the household do demand in correspondence to the chosen interest rate.

2.3 Households

We consider an infinite horizon discrete time economy populated by a constant mass of agents whose size is normalized to one. The preferences of the representative agent are described by the following intertemporal utility function:

$$\sum_{t=0}^{+\infty} \beta^t \left[u(c_t) - v(l_t)\right] \tag{7}$$

where c_t is the unique consumption good, l_t the labor supply, p_t the price of consumption good and $\beta \in (0,1)$ the discount factor. The instantaneous utility function $u(c) - v(l)$ satisfies the following standard Assumption:

Assumption 1. *$u(c)$ is C^2 over $\mathbb{R}_+$, increasing and concave over $\mathbb{R}_+$. Moreover, $v(l)$ is C^2 over $\mathbb{R}_+$, strictly increasing and weakly convex.*

When maximizing (7) agents must respect the dynamic budget constraint:

$$p_t c_t + M_{t+1} + B_{t+1} = M_t + (1+i_t) B_t + (1-\tau_w) w_t l_t \tag{8}$$

where B_t denotes the safe nominal bonds issued by the government and held by the representative household, and w_t the nominal wage. Following analogous lines as in Hahn and Solow (1995), we assume in addition that agents must pay cash at least a share $q \in (0,1]$ of their consumption purchases:

$$q p_t c_t \leq M_t. \tag{9}$$

Denoting $b_t = B_t/p_t$ the real governments bonds held by the representative consumer at the outset of period $t-1$ and $\omega_t = w_t/p_t$ the real wage earned in period t, we get the intertemporal maximization problem of the representative agent:

$$\max_{(c_t,m_t,l_t,b_t)_{t=0}^{\infty}} \sum_{t=0}^{+\infty} \beta^t \left[u\left(c_t\right) - v\left(l_t\right)\right] \tag{10}$$

subject to the dynamic budget constraint

$$c_t + \frac{p_{t+1}}{p_t} m_{t+1} + \frac{p_{t+1}}{p_t} b_{t+1} = m_t + \left(1 + i_t\right) b_t + \left(1 - \tau_w\right) \omega_t l_t \tag{11}$$

and to the cash-in-advance constraint:

$$qc_t \leq m_t. \tag{12}$$

Let denote λ and μ the Lagrangian multiplier associated to the dynamic budget constraint and the cash-in-advance constraint, then the first order conditions are:[1]

$$\beta^t u'\left(c_t\right) = \lambda_t + q\mu_t, \beta^t v'\left(l_t\right) = \left(1 - \tau_w\right) \omega_t \lambda_t, \lambda_t - \tfrac{p_t}{p_{t-1}} \lambda_{t-1} + \mu_t = 0 \tag{13}$$

and the Fisher equation

$$-\tfrac{p_t}{p_{t-1}} \lambda_{t-1} + \lambda_t \left(1 + i_t\right) = 0. \tag{14}$$

2.4 Intertemporal Equilibrium

We assume a linear technology such that one unit of labor can be used to produce one unit of output y according to the linear production function

$$y_t = l_t \tag{15}$$

Equilibrium in the good market requires therefore $y_t = l_t$ in each period t meanwhile in the labor market firms, in view of the constant returns to scale technology, employs as much labor as it is supplied by the households. In the good market, total government expenditures g and total households consumption c must, in each period, equalize total production y :

$$g + c_t = y_t = l_t \text{ for each } t \geq 1. \tag{16}$$

[1] u', and v' denote respectively $\partial u(c)/\partial c$ and $\partial v(l)/\partial l$.

Notice that (16) is satisfied by the Walras' Law, once one takes into account the bonds market and the money market. Let us denote $\pi_t = p_t/p_{t-1}$. By manipulating appropriately (13)-(14), we obtain the following equations describing intertemporal equilibrium in terms of $(y_t, y_{t+1}, \pi_t, \pi_{t+1})$:

$$u'(y_t - g) = \frac{\beta u'(y_{t+1} - g)}{\pi_{t+1}} \left[\frac{qI(\pi_t) + 1 - q}{q + (1-q)I^{-1}(\pi_{t+1})} \right] \tag{17}$$

$$(1 - \tau_w)\beta u'(y_{t+1} - g) = (1 - q)\beta v'(y_{t+1}) + qv'(y_t)\pi_{t+1}, \tag{18}$$

where $I(\pi)$ is the Taylor rule defined in (5). Notice that y and π are variables which are not predetermined and therefore equilibrium is locally unique if and only if both roots of the Jacobian evaluated at the steady state under study lay outside the unit circle. In the opposite case where the steady state is a sink (both stable roots) or a saddle (one stable root), the system will be locally indeterminate and there will be infinitely many choices for the initial conditions ensuring the convergence toward the stationary solution. Once a particular trajectory $\{y_t, \pi_t\}_{t=0}^{\infty}$ has been selected, the price level prevailing in period zero is nevertheless indeterminate: once it has been chosen, on the other hand, all the other nominal variables are immediately derived.

3 Steady State Analysis

3.1 Existence and Multiplicity of Stationary Solutions

Our first task consists in studying the existence and the number of stationary solutions of the dynamic system defined by equations (17) and (18). For sake of precision, a steady state is a pair $(\pi, y) > (0, 0)$ satisfying the following planar system of equations:

$$\pi \left[q + (1-q)I^{-1}(\pi) \right] = \beta \left[qI(\pi) + 1 - q \right] \tag{19}$$

and

$$(1 - \tau_w)\beta u'(y - g) = (1 - q)\beta v'(y) + qv'(y)\pi \tag{20}$$

where the function $I(\pi)$ is defined in (5). The latter actually puts a lower bound on π, which we will refer as to $\pi_{\min}$. For $\pi > \pi_{\min}$, the nominal interest rate is positive and it is fixed on the basis of the Taylor rule, meanwhile, for $\pi \leq \pi_{\min}$, the gross nominal interest rate is kept equal to one. Since (19) includes uniquely the inflation rate, we derive the existence of a stationary π compatible with $I(\pi) \geq 1$ under the case of a passive Taylor rule, i.e. $\gamma < 1$, and under the case of an active Taylor rule, i.e. $\gamma > 1$. Once a stationary value for π has be found, from (20) one immediately derives the corresponding (and unique) stationary value for the output y. To find the stationary values for the inflation rate π, let us define the functions $G_0 \equiv \pi \left[q + (1-q)I^{-1}(\pi)\right]$ and $G_1 \equiv \beta \left[qI(\pi) + 1 - q\right]$.

Let us consider first the case $\gamma < 1$. Here the function G_0 coincides with the 45° line for $\pi \leq \pi_{\min}$ and then it describes a concave curve diverging to infinite with a limit slope equal to q. On the other hand, the function G_1 is equal to β for $\pi \leq \pi_{\min}$ and then it diverges to infinite with a limit slope equal to zero. It follows that, if $\pi_{\min} < \beta$, the two functions G_0 and G_1 intersect exactly once for some $\pi > \pi_{\min}$ in correspondence to which one has, as a consequence, $I(\pi) > 1$ (see Figure 1). On the other hand, if $\pi_{\min} > \beta$, the functions G_0 and G_1 intersect again only once but now for some $\pi < \pi_{\min}$. It follows that the economy will be located in a point corresponding to the liquidity trap, as it is depicted in Figure 2.

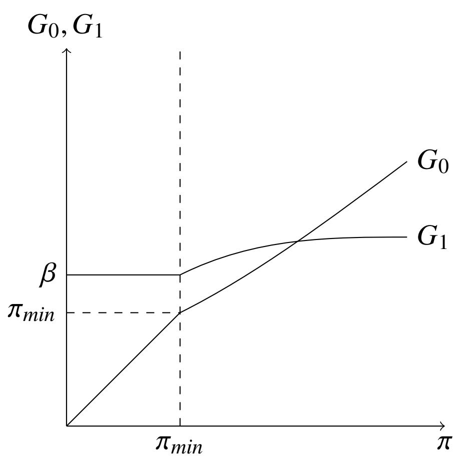

Figure 1: $\gamma < 1$ and $\pi_{min} < \beta$.

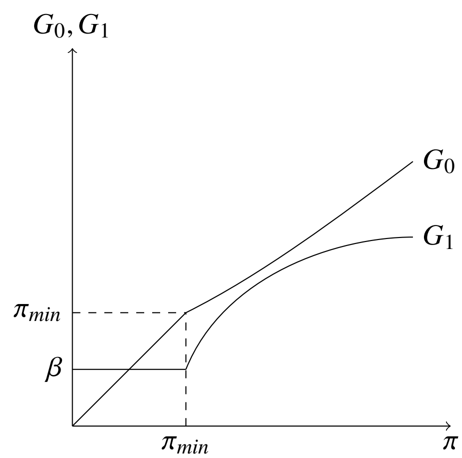

Figure 2: $\gamma < 1$ and $\pi_{min} > \beta$.

Let us now consider the case $\gamma > 1$. Again, the function G_0, for $\pi \leq \pi_{\min}$, coincides with the 45° line. Then it follows part of a parabola, before decreasing, then reaching a minimum for some $\hat{\pi}$ and eventually diverging to infinite with a limit slope equal to q. On the other hand, the function G_1 is, for $\pi \leq \pi_{\min}$, constant and equal to β, then it assumes the shape of a convex function which diverges to infinite with an infinite limit slope. It follows that two possible cases do arise. If $\beta < \pi_{\min}$, the curves G_0 and G_1 intersect twice, once in correspondence to a $\pi < \pi_{\min}$, giving thus raise to a liquidity trap equilibrium, and once for some $\pi > \pi_{\min}$ with an associate nominal gross interest rate larger than one (Leeper equilibrium, see Figure 3). If, conversely, $\beta > \pi_{\min}$, there is no stationary solution for the dynamic system defined by equations (17) and (18), as it is easily verifiable by looking at Figure 4.

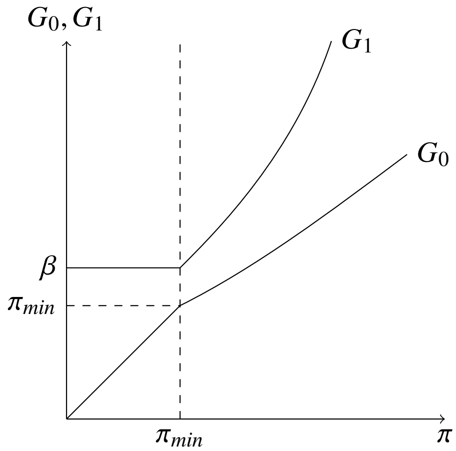

Figure 3: $\gamma > 1$ and $\pi_{min} < \beta$.

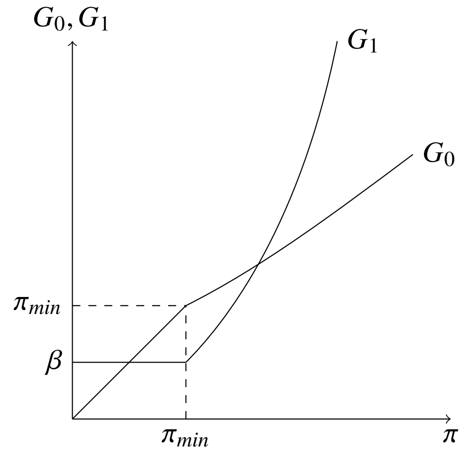

Figure 4: $\gamma > 1$ and $\pi_{min} > \beta$.

The following Proposition is thus proved.

Proposition 1. *Under Assumption 1, the following results prevail:*

i⌋ Let $\gamma < 1$ *and* $\pi_{\min} < \beta$*. Then the unique (Leeper) steady state is such that* $I(\pi) > 1$*;*

ii⌋ Let $\gamma < 1$ *and* $\pi_{\min} > \beta$*. Then the unique (liquidity trap) steady state is such that* $I(\pi) = 1$;

iii⌋ Let $\gamma > 1$ *and* $\pi_{\min} < \beta$*. Then there is no steady state;*

iv⌋ Let $\gamma > 1$ *and* $\pi_{\min} > \beta$*. Then there exists a (liquidity trap) steady state such that* $I(\pi) = 1$ *and a (Leeper) steady state such that* $I(\pi) > 1$.

■

3.2 Welfare Analysis

Once established the existence of two stationary solutions of the dynamic system described by equations (17)-(18), one may wonder at this point whether one of them Pareto-dominates the other one. To this end, let us totally differentiate (20) with respect to y and π in order to obtain

$$\frac{dy}{d\pi} = \frac{(1-\tau_w)\beta u''(y-g) - [(1-q)\beta + q\pi]\, v''(y)}{qv'(y)} < 0. \tag{21}$$

The other piece of information needed is provided by the differentiation of the stationary utility of the representative household, (7) which gives $u'(y-g) - v'(y)$ and by the stationary relationship

$$\frac{u'(y-g)}{v'(y)} = \frac{q\pi + (1-q)\beta}{(1-\tau_w)\beta} > 1. \tag{22}$$

Taking into account simultaneously (21) and (22) and by recalling to mind that the inflation rate corresponding to the liquidity trap equilibrium is lower than the one corresponding to the Leeper stationary solution, one has that households are better off in the former one, as stated in the following Proposition.

Proposition 2. *Under Assumption 1, the utility of the representative household, evaluated at the liquidity trap equilibrium, is larger than that corresponding to the Leeper equilibrium.*

■

Proposition 2 opens the door for some important considerations. The fact the liquidity trap dominates the Leeper equilibrium is indeed easily interpretable in the light of the fact that a lower inflation reduces the burden of the inflationary tax. This seems to suggest that a further decline in inflation below the liquidity trap equilibrium could entail a Pareto-improvement. However, the same definition of the liquidity trap puts a lower bound on the nominal interest rate (which cannot be negative) and thus, in view of the Taylor rule (5), on the inflation rate, which cannot be lower than $\pi_{\min}$. Were it not be the case, money would dominate government bonds in terms of return and the liquidity constraint would not more be binding, the households investing wealth exclusively in money balances. This confirms the policy implications of the liquidity trap: in correspondence to the latter, it is not more possible to stimulate economic activity by a further cut in the interest rate. It follows that monetary policy in this configuration is completely inefficacious.

Another interesting question is concerned with the effect of an increase of q on the welfare, evaluated at the steady state under study, of the representative agent. Since the amplitude q of the liquidity constraint can be viewed as a measure of the degree of the capital market imperfection, one is tempted to guess that an increase in the latter is Pareto-worsening. This is actually true, regardless to the specific stationary solution analyzed. To prove this, let us first look at equation (19): as we have already seen, it is independent upon the output y and therefore it allows to find the stationary solutions in terms uniquely of π. At this point, we can totally differentiate (19) with respect to π and q in order to obtain:

$$\frac{d\pi}{dq} = -\frac{(I(\pi) - 1)\,(\pi - \beta I(\pi))}{I(\pi) q\,(1 - \beta I'(\pi)) + (1 - q)\,(1 - \gamma)}. \tag{23}$$

By a direct inspection of (19), one immediately verifies that at the Leeper equilibrium it is $\pi = \beta I(\pi)$ and at the stationary solution corresponding to the liquidity trap it is by definition $I(\pi) = 1$. It follows that under both cases of figure, $d\pi/dq = 0$ and therefore the stationary inflation rate is independent upon the amplitude q of the liquidity constraint. On the other hand, by differentiating (20) with respect to y and q, one easily derives:

$$\frac{dy}{dq} = -\frac{v'(y)(\beta - \pi)}{\beta u''(y-g) - [(1-q)\beta + q\pi]\, v''(y)} \tag{24}$$

which, under the assumption $\pi > \beta$, is strictly negative. This information, gathered together with (22), establishes that

$$\frac{d}{dq}\left(u\left(y\left(q\right)-g\right) - v\left(y\left(q\right)\right)\right) < 0 \tag{25}$$

and therefore proves that an increase in the amplitude q of the liquidity constraint is Pareto-worsening, as claims the following Proposition.

Proposition 3. *Under Assumption 1, the utility of the representative household, evaluated at each stationary solution, is decreasing in* $q \in [0, 1]$.

■

4 Stability Analysis

In the sequel we analyze the stability of the liquidity trap steady state and of the Leeper one. We will show that it depends dramatically upon the amplitude de liquidity constraint q.

4.1 Liquidity trap

As we have seen, when the economy is near the liquidity trap equilibrium, the dynamic system loses one dimension and boils down to a simple one-dimensional difference equation in terms of the input lagged once. It is immediate to verify that the Jacobian, evaluated at the steady state, is

$$\frac{dy_{t+1}}{dy_t} = \frac{1-q}{q} \tag{26}$$

Therefore the following Proposition is immediately verified.

Proposition 4. *Under Assumption 1, let* $\pi_{min} > \beta$*. Then the liquidity trap equilibrium is unstable (locally determinate) when* $q \in (0, 1/2)$ *and is stable (locally indeterminate) when* $q \in (1/2, 1]$.

■

As stated in Proposition 4, for q sufficiently low the liquidity trap steady state becomes unstable. This suggests that the findings of Benhabib *et al.* (2001) are not robust: the liquidity trap steady is indeed stable only for high enough amplitude q of the liquidity constraint. When the latter is relaxed, conversely, the steady state from stable becomes unstable.

4.2 The Leeper Equilibrium

In order to appraise the stability properties of the Leeper equilibrium, let us introduce the following expressions:

$$\varepsilon_{cl} = \frac{\varepsilon_{ll}}{\varepsilon_{cc}+\varepsilon_{ll}}$$

where ε_{cl} is the elasticity of the offer curve with

$$\varepsilon_{cc} = -\frac{u''(c)c}{u'(c)},\ \varepsilon_{ll} = -\frac{v''(l)l}{v'(l)}$$

respectively, the elasticity of the marginal utility and the elasticity of the disutility of labor. Notice that under the gross substituability in consumption and leisure assumption, that we will retain here, ε_{cl} belongs to $(0, +\infty)$.

Let us linearize the dynamical system (17)-(18) around the a steady state by exploiting the fact that $I\beta = \pi$ and equation (6). We have thus the following Proposition.

Proposition 5. *Under Assumption 1, the characteristic polynomial is defined by $\mathscr{P}(\lambda) = \lambda^2 - \lambda\mathscr{T} + \mathscr{D}$ where:*

$$\mathscr{D}(\varepsilon_{cl}) = -\frac{q^2\gamma\pi\varepsilon_{cl}}{\beta(1-q)\left[(1-\gamma)\left(q+(1-q)\frac{\beta}{\pi}\right)+q\gamma\varepsilon_{cl}\right]} \tag{27}$$

$$\mathscr{T}(\varepsilon_{cl}) = -\frac{q\pi\left[(1-\gamma)\left(q+(1-q)\frac{\beta}{\pi}-\varepsilon_{cl}\right)+q\varepsilon_{cl}\right]}{\beta(1-q)\left[(1-\gamma)\left(q+(1-q)\frac{\beta}{\pi}\right)+q\gamma\varepsilon_{cl}\right]} \tag{28}$$

■

In view of the complicated form of the above expressions, it may seem that the study of the local dynamics of system (17)-(18) requires long and tedious computations. However, by applying the geometrical method adopted in Grandmont *et al.* (1998) and Cazzavillan *et al.* (1998), it is possible to analyze qualitatively the (in)stability of the characteristic roots of the Jacobian evaluated at the steady state of system defined by (17)-(18) and their bifurcations (changes in stability) by locating the point $(\mathscr{T}, \mathscr{D})$ in the plane and studying how $(\mathscr{T}, \mathscr{D})$ varies when the value of some parameter changes continuously. If $\mathscr{T}$ and $\mathscr{D}$ lie in the interior of the triangle $\mathscr{ABC}$ depicted in Figure 5, the stationary solution is a sink. In the opposite case, it is either a saddle, when $|\ \mathscr{T}\ |>|\ 1+\mathscr{D}\ |$, or a source. If we fix all the parameters of the model with exception of ε_{cl} (which we let vary from zero to $+\infty$) we obtain a parametrized curve

$\{\mathscr{T}(\varepsilon_{cl}), \mathscr{D}(\varepsilon_{cl})\}$ that describes a half-line Δ starting from the point $(\mathscr{T}_0, \mathscr{D}_0)$ when ε_{cl} is close to zero. The linearity of such locus can be verified by direct inspection of the expressions for $\mathscr{T}$ and $\mathscr{D}$ and from the fact they share the same denominator. This geometrical method makes it possible also to characterize the different bifurcations that may arise when ε_{cl} moves from zero to $+\infty$. In particular, as shown in Figure 5, when the half-line Δ intersects the line $\mathscr{D} = \mathscr{T} - 1$ (at $\varepsilon_{cl} = \varepsilon_{cl}^{\mathcal{T}}$), one eigenvalue goes through unity and a saddle-node bifurcation generically occurs; accordingly, we should expect a change in the number and in the stability of the steady states. When Δ goes through the line $\mathscr{D} = -\mathscr{T} - 1$ (at $\varepsilon_{cl} = \varepsilon_{cl}^{\mathcal{F}}$), one eigenvalue is equal to -1 and we expect a flip bifurcation: it follows that there will arise nearby two-period cycles, stable or unstable, according to the direction of the bifurcation. Eventually, when Δ intersects the interior of the segment $\mathscr{BC}$ (at $\varepsilon_{cl} = \varepsilon_{cl}^{\mathcal{H}}$), the modulus of the complex conjugate eigenvalues is one and the system undergoes, generically, a Hopf bifurcation. Therefore, around the stationary solution, there will emerge a family of closed orbits, stable or unstable, depending on the nature of the bifurcation (supercritical or subcritical).

Following Grandmont *et al.* (1998) and Cazzavillan *et al.* (1998), this analysis is also powerful enough to characterize the occurrence of sunspot equilibria around an indeterminate stationary solution of system (17)-(18) as well as along flip and Hopf bifurcations[2]. Actually, system defined by (17)-(18) has at each period t two non-predetermined variable, the output and the inflation factor. In such a configuration, the existence of local indeterminacy requires that at least one of the two characteristic roots associated with the linearization of the dynamic system (17)-(18) around the normalized steady state has modulus less than one. The bifurcation parameter we will adopt through our analysis is the elasticity of intertemporal substitution in consumption ε_{cl}. Then the variation of the Trace $\mathscr{T}$ and of the Determinant $\mathscr{D}$ in the $(\mathscr{T}, \mathscr{D})$ plane will be studied as ε_{cc} is made to vary continuously within the $(0, +\infty)$ interval. The relationship between $\mathscr{T}$ and $\mathscr{D}$ is given by a half-line $\Delta(\mathscr{T})$ (Figure 5). $\Delta(\mathscr{T})$ is obtained from (27)-(28) and yields to the following linear relationship:

$$\mathscr{D} = \Delta(\mathscr{T}) = \mathscr{S}\mathscr{T} + \mathscr{Z} \tag{29}$$

where $\mathscr{Z}$ is a constant term. The slope of $\Delta(\mathscr{T})$ is given by:

$$\mathscr{S} = -\frac{q\gamma}{1 - \gamma + q\gamma} \tag{30}$$

[2]In the case of supercritical flip bifurcation and s supercritical Hopf bifurcation, sunspot remain in a compact set containing in its interior, respectively, the stable two-period cycle and the stable closed orbit. Unstable cycles and closed orbits emerge in the opposite case of subcritical bifurcations.

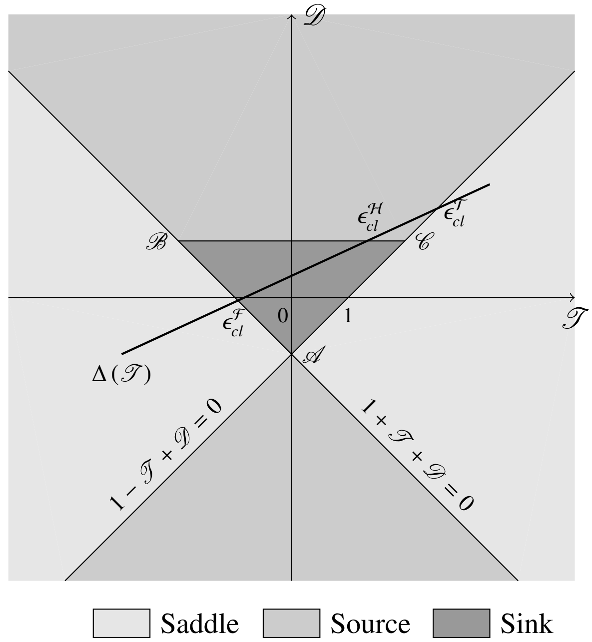

Figure 5: Stability triangle and $\Delta(\mathscr{T})$ segment.

When ε_{cl} is made to vary in the interval $(0, +\infty)$, $\mathscr{T}(\varepsilon_{cl})$ and $\mathscr{D}(\varepsilon_{cl})$ move linearly along the line $\Delta(\mathscr{T})$. As $\varepsilon_{cl} \in (0, +\infty)$, the properties of the line $\Delta(\mathscr{T})$ are derived from the consideration of its extremities. Actually, the starting point is the couple $(\lim_{\varepsilon_{cl}\to+\infty} \mathscr{T} \equiv \mathscr{T}_\infty, \lim_{\varepsilon_{cl}\to+\infty} \mathscr{D} \equiv \mathscr{D}_\infty)$. The coorresponding expressions are given by:

$$\mathscr{T}_\infty = \frac{\pi(1+\gamma-q)}{(1-q)\gamma\beta} \tag{31}$$

$$\mathscr{D}_\infty = -\frac{q\pi}{(1-q)\gamma\beta} \tag{32}$$

The coordinates of the origin are easily obtained and write:

$$\mathscr{T}_0 = -\frac{q\pi}{(1-q)\beta}, \mathscr{D}_0 = 0 \tag{33}$$

Finally, the half-line $\Delta(\mathscr{T})$ is pointing upward or downward depending on the sign of $\mathscr{D}'(\varepsilon_{cl})$:

$$\mathscr{D}'(\varepsilon_{cl}) = -\frac{q^2\gamma\pi(1-\gamma)\left(q+(1-q)\frac{\beta}{\pi}\right)}{\beta(1-q)\left[(1-\gamma)\left(q+(1-q)\frac{\beta}{\pi}\right)+q\gamma\varepsilon_{cl}\right]^2} \tag{34}$$

4.3 The benchmark case: $q = 1$

It useful to start our analysis by studying the benchmark case $q = 1$. This allows us to understand how the local dynamic features change as soon as q is relaxed from one to zero. As a matter of fact, we will show that the results do change dramatically with q.

Let us first analyse the stability of the liquidity trap steady state equilibrium. In view of the expression of the derivative $dy_{t+1}/dy_t = (1 - q)/q$, the liquidity trap steady state equilibrium, provided it exists, turns out to be stable (thus locally indeterminate). On the other hand, the Leeper steady state equilibrium, provided it exists, is a saddle for a passive monetary rules ($\gamma < 1$) and a source for active monetary rules ($\gamma > 1$). Recall to mind that, for $\gamma < 1$, the Leeper steady state equilibrium and the liquidity trap steady state equilibrium cannot exist simultaneously. As a matter of fact, for $\pi_{min} < \beta$, there exists only the Leeper steady state equilibrium meanwhile, for $\pi_{min} > \beta$, there exists only the liquidity trap steady state equilibrium. Moreover, under active policy rules, we have seen that both the Leeper steady state equilibrium and the liquidity trap steady state equilibrium exist when $\pi_{min} > \beta$ or no steady state exists at all ($\pi_{min} < \beta$). These results are in line with those of Benhabib *et al.* (2001) in a money-in-the-utility framework. Here, under active policy rules, the Leeper steady state equilibrium is unstable (locally determinate) meanwhile the liquidity trap steady state equilibrium is stable.

When $\gamma < 1$, the Leeper steady state equilibrium is stable (provided it exists) and thus there will be infinitely many initial conditions compatible with the transversality condition. On the other hand, when $\gamma > 1$, in correspondence to whatever initial condition located in the interval included between the liquidity trap equilibrium and the Leeper equilibrium, the system will converge toward the latter which therefore turns out to be quite robust. The unique way to attain the Leeper equilibrium will require agents to coordinate since the beginning on it (i.e. the system must jump immediately on it).

To prove all these results, let us simply observe that for $q = 1$, the trace and the determinant are $T = D = -\infty$ and lie on the half-line Δ with slope equals to $-\gamma$. Thus the following Proposition is immediately proved:

Proposition 6. *Under Assumption 1, let* $q = 1$ *in (27)-(28). Then the following results hold:*

i⌋ Let $\gamma < 1$ *and* $\pi_{min} < \beta$*. Then the Leeper steady state equilibrium is a saddle (locally indeterminate (and the liquidity trap steady state equilibrium does not exist;*

ii⌋ Let $\gamma < 1$ *and* $\pi_{min} > \beta$*. Then the liquidity trap steady state equilibrium is a saddle (locally indeterminate) and the Leeper steady state equilibrium does not exist;*

iii⌋ Let $\gamma > 1$ *and* $\pi_{min} < \beta$*. Then there does not exist any steady state equilibrium;*

iv] Let $\gamma > 1$ and $\pi_{min} > \beta$. Then the Leeper steady state equilibrium is a source (locally determinate) and the liquidity trap steady state equilibrium is a saddle (locally indeterminate).

■

Proposition 6 confirms Benhabib *et al.* (2001) results. Indeed, when the amplitude of the liquidity constraint is equal to one, under active Taylor rules, the Leeper equilibrium is locally determinate meanwhile the liquidity trap is locally indeterminate. In the following, we show that these features are not preserved when the amplitude of the liquidity constraint is relaxed. Actually, the two steady states can easily change the stability. In the next subsection, we will afford the study of the more general case where q is made to vary within the interval zero and one.

4.4 Passive Taylor rules: $\gamma < 1$

We now study the general case when q belongs to (0,1). We will analyze in sequel the different configurations that may arise.

Let $\gamma < 1$ and $\pi_{min} < \beta$ such that only the Leeper steady state equilibrium exists. It follows from equations (30)-(32) that the properties of the starting point $(\mathscr{T}_0, \mathscr{D}_0)$, of the slope $\mathscr{S}$ and of the end-point $(\mathscr{T}_\infty, \mathscr{D}_\infty)$ depend upon q. In addition, since $\gamma < 1$, in view of (34), we have $D'(\varepsilon_{cl}) < 0$. By a direct inspection of (33), we have $\mathscr{D}_0 = 0$ and $\mathscr{T}_0 < 0$ and thus the starting point is always on the abscissas' axis. Here we must introduce one critical value for q: q_2 such that $\mathscr{T}_0$ is equal to -1, with $q_2 = \beta/(\beta+\pi)$. Then, for $q \in (0, q_2)$, we have $\mathscr{T}_0 \in (-1, 0)$ and for $q \in (q_2, 1)$ $\mathscr{T}_0 < -1$. Since $\pi_{min} < \beta$, we have $q_2 > 1/2$. From (30) and $\gamma < 1$, we have $\mathscr{S} < 0$ and the slope belong to the interval $(-\gamma, 0)$. Finally, we consider the location of the end point $(\mathscr{T}_\infty, \mathscr{D}_\infty)$. In order to do this, we analyze $1 - \mathscr{T}_\infty + \mathscr{D}_\infty$ and $1 + \mathscr{T}_\infty + \mathscr{D}_\infty$. It follows from (31)-(32) that:

$$1 - \mathscr{T}_\infty + \mathscr{D}_\infty = 1 - \frac{(1+\gamma)\pi}{(1-q)\gamma\beta}, 1 + \mathscr{T}_\infty + \mathscr{D}_\infty = 1 + \frac{(1+\gamma-2q)\pi}{(1-q)\gamma\beta} \tag{35}$$

We get $1 - \mathscr{T}_\infty + \mathscr{D}_\infty > 0$ if $q < q_0$ and $1 + \mathscr{T}_\infty + \mathscr{D}_\infty > 0$ if $q < q_1$, with:

$$q_0 = 1 - \frac{(1+\gamma)\pi}{\gamma\beta}, q_1 = \frac{(1+\gamma)\pi + \gamma\beta}{2\pi + \gamma\beta} \tag{36}$$

Let us introduce one critical value for γ: γ_0 such that if $\gamma > \gamma_0$ then $q_2 < q_1$ and if $\gamma < \gamma_0$ then $q_2 > q_1$, with $\gamma_0 = (\beta - \pi)/(2\beta + \pi)$

Then we obtain the following implications: i) if $\gamma < \gamma_0$ then $q_0 < q_1 < q_2$; ii) if $\gamma > \gamma_0$ then $q_0 < q_2 < q_1$.

To complete the characterization of the local dynamics, let us first consider $\gamma < \gamma_0$. When $q \in (0, q_0)$, the location of the end-point is given by $1 - \mathscr{T}_\infty + \mathscr{D}_\infty > 0$ and $1 + \mathscr{T}_\infty + \mathscr{D}_\infty > 0$, and the the starting point $(\mathscr{T}_0, \mathscr{D}_0)$ lies in the $\mathscr{A}\mathscr{B}\mathscr{C}$ triangle; the steady state is thus a sink (locally indeterminate). This case is represented by the half-line Δ_0 in Figure 6.

When $q \in (q_0, q_1)$, the end-point satisfies $1 - \mathscr{T}_\infty + \mathscr{D}_\infty < 0$ and $1 + \mathscr{T}_\infty + \mathscr{D}_\infty > 0$, meanwhile the starting point $(\mathscr{T}_0, \mathscr{D}_0)$ lies in the $\mathscr{A}\mathscr{B}\mathscr{C}$ triangle. By relaxing continuously ε_{cl}, we obtain that the steady state is first a sink (locally indeterminate) and then, through a transcritical bifurcation, becomes a saddle (locally indeterminate). The corresponding case is represented by the half-line Δ_1 in Figure 6.

When $q \in (q_1, q_2)$, the end-point is such that $1 - \mathscr{T}_\infty + \mathscr{D}_\infty < 0$ and $1 + \mathscr{T}_\infty + \mathscr{D}_\infty < 0$, meanwhile the starting point $(\mathscr{T}_0, \mathscr{D}_0)$ lies in the $\mathscr{A}\mathscr{B}\mathscr{C}$ triangle. By relaxing continuously ε_{cl}, we obtain that the steady state is first a sink (locally indeterminate), then, through a transcritical bifurcation, it becomes a saddle (locally indeterminate), and fin,ally through a flip bifurcation, it becomes a source (locally determinate). This case case is summerized by the half-line Δ_2 in Figure 6.

Finally, when $q \in (q_2, 1)$, the location of the end-point satisfies $1 - \mathscr{T}_\infty + \mathscr{D}_\infty < 0$ and $1 + \mathscr{T}_\infty + \mathscr{D}_\infty < 0$. At the same time, the starting point $(\mathscr{T}_0, \mathscr{D}_0)$ lies outside the $\mathscr{A}\mathscr{B}\mathscr{C}$ triangle; by relaxing continuously ε_{cl}, we obtain that the steady state is first a sink (locally indeterminate) and then, through a transcritical bifurcation, it becomes a source (locally determinate). The half-line Δ_3 in Figure 6 describes the local dynamics and its changes in stability.

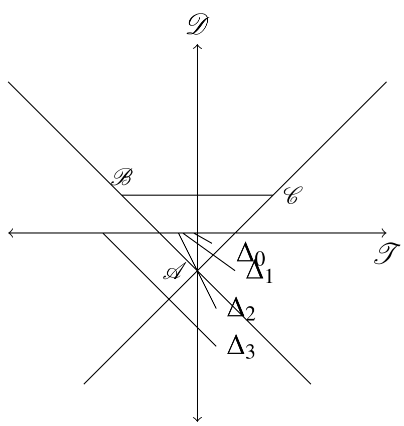

Figure 6: $\gamma \in (0, \gamma_0)$

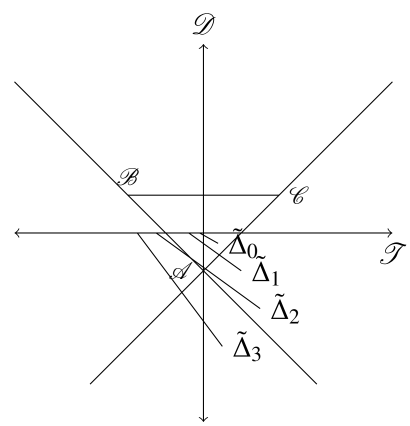

Figure 7: $\gamma \in (\gamma_0, 1)$

Let us consider now the case $\gamma > \gamma_0$. When $q \in (0, q_0)$, the end-point is such that $1 - \mathscr{T}_\infty + \mathscr{D}_\infty > 0$ and $1 + \mathscr{T}_\infty + \mathscr{D}_\infty > 0$, meanwhile the starting point $(\mathscr{T}_0, \mathscr{D}_0)$ lies in the $\mathscr{A}\mathscr{B}\mathscr{C}$ triangle. The steady state is then bound to be a sink (locally indeterminate). The half-line $\hat{\Delta}_0$ in Figure 7 summerizes such a picture.

When $q \in (q_0, q_2)$, we find that the end-point lies on the line satisfying $1 - \mathscr{T}_\infty + \mathscr{D}_\infty < 0$ and

$1 + \mathscr{T}_\infty + \mathscr{D}_\infty > 0$. The starting point $(\mathscr{T}_0, \mathscr{D}_0)$ conversely lies in the $\mathscr{ABC}$ triangle. By relaxing continuously ε_{cl}, we obtain that the steady state is first a sink (locally indeterminate) and then, through a transcritical bifurcation, it becomes a saddle (locally indeterminate). These features are described by the half-line $\hat{\Delta}_1$ depicted in Figure 7.

On the other hand, when $q \in (q_2, q_1)$, the end-point satisfies $1 - \mathscr{T}_\infty + \mathscr{D}_\infty < 0$ and $1 + \mathscr{T}_\infty + \mathscr{D}_\infty > 0$, meanwhile the starting point $(\mathscr{T}_0, \mathscr{D}_0)$ is now located outside the $\mathscr{ABC}$ triangle. By relaxing continuously ε_{cl}, we obtain that the steady state is thus first a sink (locally indeterminate) and then, through a transcritical bifurcation, it becomes a saddle (locally indeterminate). The half-line $\hat{\Delta}_2$ in Figure 7 refers to such a case.

Finally when $q \in (q_2, 1)$, the end-point is such that $1 - \mathscr{T}_\infty + \mathscr{D}_\infty < 0$ and $1 + \mathscr{T}_\infty + \mathscr{D}_\infty < 0$. On the other hand, the starting point $(\mathscr{T}_0, \mathscr{D}_0)$ lies outside the $\mathscr{ABC}$ triangle. Thus, by relaxing continuously ε_{cl}, one obtains that the steady state is first a sink (locally indeterminate) and then, through a transcritical bifurcation, it becomes a saddle (locally indeterminate). This case is summerized by the half-line $\hat{\Delta}_3$ depicted in Figure 7.

The above results are summarized in the following Proposition which is immediately proved:

Proposition 7. *Under Assumption 1, let $\pi_{min} < \beta$. Then the following results hold:*

i⌋ Let $\gamma < \gamma_0$. When $q < q_0$, the steady state is a sink, i.e. locally indeterminate. When $q \in (q_0, q_1)$, the steady state is a sink, i.e. locally indeterminate, for $\varepsilon_{cl} < \varepsilon_{cl}^{\mathcal{T}}$, and a saddle, i.e. locally indeterminate, for $\varepsilon_{cl} > \varepsilon_{cl}^{\mathcal{T}}$. When $q \in (q_1, q_2)$, the steady state is a sink, i.e. locally indeterminate, for $\varepsilon_{cl} < \varepsilon_{cl}^{\mathcal{T}}$, and a saddle, i.e. locally indeterminate, for $\varepsilon_{cl} \in (\varepsilon_{cl}^{\mathcal{T}}, \varepsilon_{cl}^{\mathcal{F}})$, and a source, i.e. locally determinate, for $\varepsilon_{cl} > \varepsilon_{cl}^{\mathcal{F}}$. When $q > q_2$, the steady state is a saddle, i.e. locally indeterminate, for $\varepsilon_{cl} < \varepsilon_{cl}^{\mathcal{T}}$ and a source, i.e. locally determinate, for $\varepsilon_{cl} > \varepsilon_{cl}^{\mathcal{T}}$;

ii⌋ Let $\gamma > \gamma_0$. When $q < q_0$, the steady state is a sink, i.e. locally indeterminate. When $q \in (q_0, q_2)$, the steady state is a sink, i.e. locally indeterminate, for $\varepsilon_{cl} < \varepsilon_{cl}^{\mathcal{T}}$, and a saddle, i.e. locally indeterminate, for $\varepsilon_{cl} > \varepsilon_{cl}^{\mathcal{T}}$. When $q \in (q_2, q_1)$, the steady state is a saddle, i.e., locally indeterminate, for $\varepsilon_{cl} < \varepsilon_{cl}^{\mathcal{F}}$, a sink, i.e. locally indeterminate, for $\varepsilon_{cl} \in (\varepsilon_{cl}^{\mathcal{F}}, \varepsilon_{cl}^{\mathcal{T}})$, and a saddle, i.e. locally indeterminate, for $\varepsilon_{cl} > \varepsilon_{cl}^{\mathcal{T}}$. When $q > q_1$, the steady state is a saddle, i.e. locally indeterminate, for $\varepsilon_{cl} < \varepsilon_{cl}^{\mathcal{T}}$, and a source, i.e. locally determinate, for $\varepsilon_{cl} > \varepsilon_{cl}^{\mathcal{T}}$.

In addition, when ε_{cl} goes through $\varepsilon_{cl}^{\mathcal{F}}$ and $\varepsilon_{cl}^{\mathcal{T}}$, the steady state undergoes, respectively, a flip bifurcation and a transcritical one.

■

In the next subsection, we carry out the analysis of the case corresponding to active Taylor rules.

4.5 Active Taylor rules: $\gamma > 1$

We now consider the case $\gamma > 1$ and $\pi_{min} > \beta$ in order to ensure that both the Leeper steady state equilibrium and the liquidity trap one do exist.

Under these inequalities, in the light of the fact that $q_1 > 1$ and $q_0 < 0$, we have that the localisation of $(\mathscr{D}_\infty, \mathscr{T}_\infty)$ is given by $1 - \mathscr{T}_\infty + \mathscr{D}_\infty < 0$ and $1 + \mathscr{T}_\infty + \mathscr{D}_\infty > 0$. In addition, since $\gamma > 1$, in view of (34), we have $D'(\varepsilon_u) > 0$. Here the slope can be either positive or negative, depending upon the value of γ. Let us introduce the critical value for γ: γ_1 such that if $\gamma > \gamma_1$ then $\mathscr{S} > 0$ and if $\gamma < \gamma_1$ then $\mathscr{S} < 0$, with $\gamma_1 = 1/(1-q)$. Since $\pi_{min} > \beta$, we have $q_2 < 1/2$. The Leeper equilibrium and the liquidity trap equilibrium thus coexist; in addition, the liquidity trap steady state changes its stability.

Let us consider first the case $\gamma \in (1, \gamma_1)$. In the light of the above considerations, we obtain that for $q \in (0, q_2)$, the starting point $(\mathscr{T}_0, \mathscr{D}_0)$ lies in the $\mathscr{A}\mathscr{B}\mathscr{C}$ triangle. By relaxing continuously ε_{cl}, we find that the steady state is first a sink (locally indeterminate) and then, through a flip bifurcation, it becomes a saddle (locally indeterminate).

Finally, when $q \in (q_2, 1)$, with $q_2 < 1/2 < 1$, the starting point $(\mathscr{T}_0, \mathscr{D}_0)$ lies outside the $\mathscr{A}\mathscr{B}\mathscr{C}$ triangle. Here the steady state is first a saddle (locally indeterminate); then, through a flip bifurcation, it becomes a sink (locally indeterminate) and finally, through a transcritical bifurcation, it reaches a saddle configuration (locally indeterminate). These cases are described in Figure 8.

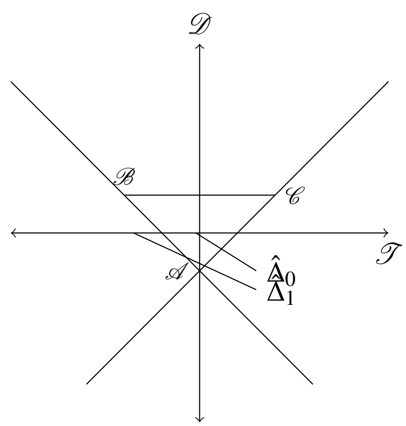

Figure 8: $\gamma \in (1, \gamma_1)$

Let us consider now the case $\gamma > \gamma_1$. Let us introduce the critical value for γ: γ_2 such that if $\gamma \gtrless \gamma_2$ then $\mathscr{S}$ is greater or less than one, with $\gamma_2 = 1/(1-2q)$. Notice that γ_2 does exist if and only if $q < 1/2$. Under the inequality $q < 1/2$, we get $\gamma_2 > \gamma_1$.

Let now consider $q \in (0, q_2)$ and let $\gamma \in (\gamma_1, \gamma_2)$ be such that $\mathscr{S} > 1$. The starting point $(\mathscr{T}_0, \mathscr{D}_0)$ lies inside the $\mathscr{A}\mathscr{B}\mathscr{C}$ triangle. By relaxing continuously ε_{cl}, we obtain that the steady state is first a sink (locally indeterminate), then, through a Hopf bifurcation, it becomes a source (locally determinate) and finally, through a flip bifurcation, it becomes a saddle (locally indeterminate). Let now be $\gamma > \gamma_2$ such that $\mathscr{S} < 1$. The starting point $(\mathscr{T}_0, \mathscr{D}_0)$ now lies inside the $\mathscr{A}\mathscr{B}\mathscr{C}$ triangle, and by relaxing continuously ε_{cl}, we obtain that the steady state is first a sink (locally indeterminate), then, through a flip bifurcation, it becomes a saddle (locally indeterminate). Afterwards, through a transcritical bifurcation, it becomes a source (locally determinate) and finally, through a flip bifurcation, it becomes a saddle (locally indeterminate).

Consider now the case $q \in (q_2, 1/2)$ and let $\gamma \in (\gamma_1, \gamma_2)$ be such that $\mathscr{S} > 1$. The starting point $(\mathscr{T}_0, \mathscr{D}_0)$ lies outside the $\mathscr{A}\mathscr{B}\mathscr{C}$ triangle. It follows that, by relaxing continuously ε_{cl}, the steady state is first a saddle (locally indeterminate), then, through a flip bifurcation, it becomes a sink (locally indetermiante), then, through a Hopf bifurcation, it becomes a source (locally determinate) and finally, through a transcritical bifurcation, it becomes a saddle (locally indeterminate).

Let now set $\gamma > \gamma_2$ such that $\mathscr{S} < 1$. The starting point $(\mathscr{T}_0, \mathscr{D}_0)$ is now located outside the $\mathscr{A}\mathscr{B}\mathscr{C}$ triangle. As soon as one relaxes continuously ε_{cl}, the steady state appears to be first a saddle (locally indeterminate), then, through a flip bifurcation, it becomes a sink (locally indetermiante), then, through a transcritical bifurcation, it becomes a source (locally determinate) and finally, through a flip bifurcation, it is a saddle (locally indeterminate).

Let us now assume $q \in (1/2, 1)$ and let $\gamma > \gamma_1$ since $q > 1/2$ and $\mathscr{S} > 1$. It is easily verifiable that the starting point $(\mathscr{T}_0, \mathscr{D}_0)$ now lies outside the $\mathscr{A}\mathscr{B}\mathscr{C}$ triangle and that, by relaxing continuously ε_{cl}, one has that the steady state is first a saddle (locally indeterminate), then, through a flip bifurcation, it becomes a sink (locally indeterminate); furthermore, through a Hopf bifurcation, the steady state becomes a source (locally determinate) and finally it becomes a saddle (locally indeterminate) by undergoing a transcritical bifurcation. These cases are depicted in Figures 9 and 10.

The above results are summarized in the following two Propositions. The first one refers to the case $q < 1/2$, meanwhile the second one to the case $q > 1/2$.

Proposition 8. *Under Assumption 1, let $\pi_{min} > \beta$. Then the following results prevail:*

i] Let $\gamma \in (1, \gamma_1)$. When $q < q_2$, the steady state is a sink, i.e. locally indeterminate, for $\varepsilon_{cl} < \varepsilon_{cl}^{\mathcal{T}}$, and a saddle, i.e. locally indeterminate, for $\varepsilon_{cl} > \varepsilon_{cl}^{\mathcal{T}}$. When $q \in (q_2, 1/2)$, the steady state is a saddle, i.e. locally indeterminate, for $\varepsilon_{cl} < \varepsilon_{cl}^{\mathcal{F}}$, a sink, i.e. locally indeterminate, for $\varepsilon_{cl} \in (\varepsilon_{cl}^{\mathcal{F}}, \varepsilon_{cl}^{\mathcal{T}})$, and a saddle, i.e. locally indeterminate, for $\varepsilon_{cl} > \varepsilon_{cl}^{\mathcal{T}}$;

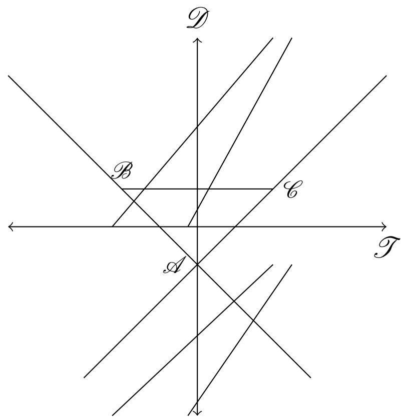

Figure 9 : $\gamma \in (\gamma_1, \gamma_2)$

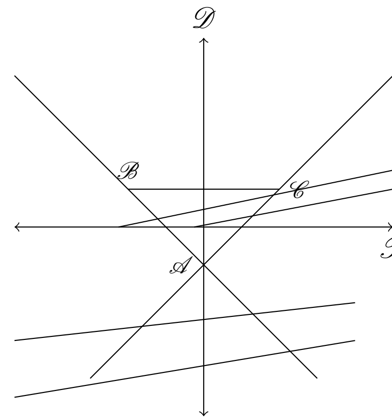

Figure 10: $\gamma > \gamma_2$

ii⌋ *Let $\gamma \in (\gamma_1, \gamma_2)$. When $q < q_2$, the steady state is a sink, i.e. locally indeterminate, for $\varepsilon_{cl} < \varepsilon_{cl}^{\mathcal{H}}$, a source, i.e., locally determinate, for $\varepsilon_{cl} \in (\varepsilon_{cl}^{\mathcal{H}}, \varepsilon_{cl}^{\mathcal{F}})$ and a saddle, i.e. locally indeterminate, when $\varepsilon_{cl} > \varepsilon_{cl}^{\mathcal{F}}$. When $q \in (q_2, 1/2)$, the steady state is a saddle, i.e. locally indeterminate, for $\varepsilon_{cl} < \varepsilon_{cl}^{\mathcal{F}}$, a sink, i.e. locally indeterminate, for $\varepsilon_{cl} \in (\varepsilon_{cl}^{\mathcal{F}}, \varepsilon_{cl}^{\mathcal{H}})$, a source, i.e. locally determinate, for $\varepsilon_{cl} \in (\varepsilon_{cl}^{\mathcal{H}}, \varepsilon_{cl}^{\mathcal{T}})$ and a saddle, i.e. locally indeterminate, for $\varepsilon_{cl} > \varepsilon_{cl}^{\mathcal{T}}$;*

iii⌋ *Let $\gamma > \gamma_2$. When $q < q_2$, the steady state is a sink, i.e., locally indeterminate, for $\varepsilon_{cl} < \varepsilon_{cl}^{\mathcal{T}}$, a saddle, i.e. locally indeterminate, for $\varepsilon_{cl} \in (\varepsilon_{cl}^{\mathcal{T}}, \varepsilon_{cl}^{\mathcal{T}})$, a source, i.e. locally determinate when $\varepsilon_{cl} in (\varepsilon_{cl}^{\mathcal{T}}, \varepsilon_{cl}^{\mathcal{F}})$ and a saddle, i.e. locally indeterminate, for $\varepsilon_{cl} > \varepsilon_{cl}^{\mathcal{F}}$. When $q \in (q_2, 1/2)$, the steady state is a saddle, i.e. locally indeterminate, for $\varepsilon_{cl} < \varepsilon_{cl}^{\mathcal{F}}$, a sink, i.e. locally indeterminate, for $\varepsilon_{cl} \in (\varepsilon_{cl}^{\mathcal{F}}, \varepsilon_{cl}^{\mathcal{T}})$, a saddle, i.e. locally indeterminate, for $\varepsilon_{cl} \in (\varepsilon_{cl}^{\mathcal{T}}, \varepsilon_{cl}^{\mathcal{T}})$, a source, i.e. locally determinate when $\varepsilon_{cl} in (\varepsilon_{cl}^{\mathcal{T}}, \varepsilon_{cl}^{\mathcal{F}})$ and eventually a saddle, i.e. locally indeterminate, for $\varepsilon_{cl} > \varepsilon_{cl}^{\mathcal{F}}$.*

In addition, when ε_{cl} goes through $\varepsilon_{cl}^{\mathcal{F}}$, $\varepsilon_{cl}^{\mathcal{T}}$, and $\varepsilon_{cl}^{\mathcal{H}}$ the steady state undergoes, respectively, a flip bifurcation, a transcritical bifurcation and a Hopf bifurcation.

■

The next Proposition completes our analysis.

Proposition 9. *Under Assumption 1, let $\pi_{min} > \beta$. Then the following results hold:*

i⌋ *Let $\gamma \in (1, \gamma_1)$. When $q > 1/2$, the steady state is a saddle, i.e. locally indeterminate, for $\varepsilon_{cl} < \varepsilon_{cl}^{\mathcal{F}}$, a sink, i.e. locally indeterminate, for $\varepsilon_{cl} \in (\varepsilon_{cl}^{\mathcal{F}}, \varepsilon_{cl}^{\mathcal{T}})$ and a saddle, i.e. locally indeterminate, for $\varepsilon_{cl} > \varepsilon_{cl}^{\mathcal{T}}$;*

ii⌋ *Let $\gamma > \gamma_1$ When $q > 1/2$, the steady state is a saddle, i.e. locally indeterminate, for $\varepsilon_{cl} < \varepsilon_{cl}^{\mathcal{F}}$, a sink, i.e. locally indeterminate, for $\varepsilon_{cl} \in (\varepsilon_{cl}^{\mathcal{F}}, \varepsilon_{cl}^{\mathcal{H}})$, a source, i.e. locally determinate, for $\varepsilon_{cl} \in (\varepsilon_{cl}^{\mathcal{H}}, \varepsilon_{cl}^{\mathcal{T}})$ and a saddle, i.e. locally indeterminate, for $\varepsilon_{cl} > \varepsilon_{cl}^{\mathcal{T}}$.*

In addition, when ε_{cl} goes through $\varepsilon_{cl}^{\mathcal{F}}$, $\varepsilon_{cl}^{\mathcal{T}}$, and $\varepsilon_{cl}^{\mathcal{H}}$ the steady state undergoes, respectively, a flip bifurcation, a transcritical bifurcation and a Hopf bifurcation.

■

5 Conclusion

In this paper we have provided a theoretical contribution to the debate running around the plausibility of the emergence of the liquidity trap as well as its stability features. In order to motivate a positive money demand, we have assumed that agents must pay cash a given share of the value of consumption expenditures. We show that the liquidity trap is not bound to be a stable equilibrium but that, instead, its stability depends dramatically upon the degree of liquidity of the economy, namely the degree of financial market imperfection. By showing that the liquidity trap is not necessarily the unique stable stationary solution of the economy, the original intuition of Keynes is henceforth consolidated on a theoretical point of view: in contrast with Benhabib *et al.* (2001), the liquidity trap represents again a limit case that can be avoided by means of an appropriate public policy aiming at coordinating agents toward the Taylor target. In contrast to a money-in-the utility approach, we are able, in fact, to capture the degree of market imperfection by simply letting the amplitude of the liquidity constraint on consumption expenditures to vary. However, we have proved that there is not always a rationale for escaping from the liquidity trap equilibrium, since it Pareto-dominates the Taylor target, in view of the zero interest rate associated to it and the consequent zero cost of money holding. In addition, the dynamic around the liquidity trap equilibrium is always monotonic, in contrast to the behavior characterizing the neighborhood of the Taylor equilibrium which can be easily cyclical.

A natural extension of the economy here treated, is to account for physical capital accumulation: such an asset will be indeed held by agents in order to carry over wealth from one period to another, beside government bonds and money. Also accounting for international trade would be an interesting issue to explore, in view of the close ties that would arise between the monetary policies implemented in each country involved.

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