A regime-switching Gegenbauer process to model unemployment in the G7 countries

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Unemployment Persistence: A Long-Standing Debate

- Since the early 1990s, the persistence of the unemployment rate has been widely studied.
- Economists linked this persistence to a long-term structural phenomenon or to a path-dependence phenomenon (hysteresis).
- Two key perspectives emerged:
    1. **Natural rate hypothesis**: Unemployment returns to an equilibrium level.
    2. **Hysteresis hypothesis**: Shocks can have permanent effects.

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2. **Hysteresis hypothesis**: Shocks can have permanent effects. $\Longrightarrow$ Changes in u_t implies changes in u_t^* (path-dependence)

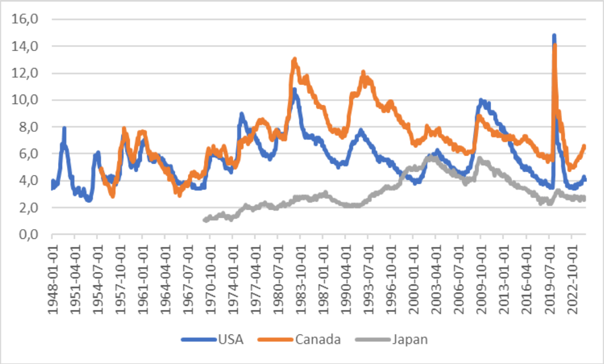


Figure 1: Monthly unemployment rates. United States. Canada and Japan S.A.

1. **Natural rate hypothesis**: Unemployment returns to an equilibrium level. $\Longrightarrow$ u_t fluctuates around u_t^*
2. **Hysteresis hypothesis**: Shocks can have permanent effects. $\Longrightarrow$ Changes in u_t implies changes in u_t^* (path-dependence)

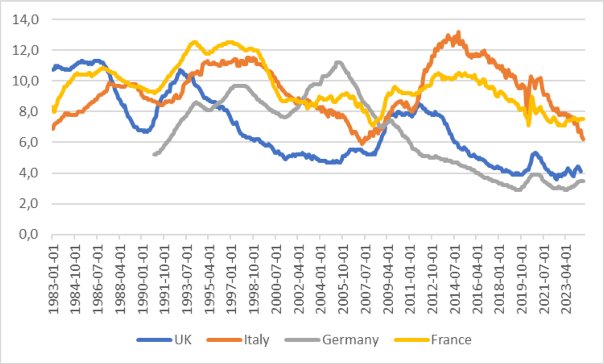


Figure 2: Monthly unemployment rates. UK, Italy, Germany and France S.A.

- Debate on hysteresis aligned with developments in time series analysis:
  1. Stochastic trends (unit roots, cointegration)
  2. Long-memory models (ARFIMA models).

- Theoretical literature : institutional rigidities originated from:.
  1. The labor market (insider-outsider effects, de-skilling duration mechanisms, institutional wage-setting norms)
  2. or the goods market (decline in capital accumulation and total factor productivity during economic recession phases, which affects the level of the long-term unemployment rate, or highly restrictive monetary policies).

- **Debate related to the short-term view**:
  1. Nonlinear deterministic models (nonlinear ARMA, regime switching models)
  2. Nonlinear stochastic models (Markov switching models).
- **Volatility due to economic shocks:**
  1. Unemployment reacts sharply to shocks.
  2. Policies struggle to stabilize fluctuations.
  3. "Yo-yo" behavior of unemployment dynamics.
- **Asymmetric regimes in unemployment cycles:**
  1. Easier to increase than to decrease.
  2. Downward transitions are often abrupt.

Aim of the paper

- We propose a model that combines both long memory and short-term nonlinear dynamics through smooth transition regime switching mechanism.
  1. **Gegenbauer model** : long-term persistence.
  2. **Regime-switching non-linear model** : short-term and medium term cyclical fluctuations with asymmetric dynamics.
- Contribution to the literature
  1. Long-memory part of the model generalizes the standard fractional integrated model to a two-factor Gegenbauer model
  2. we propose a new procedure to test the hypothesis of a simple linear model against a Gegenbauer regime-switching process
  3. Our modelling strategy: pre-treatment of the data through wavelet to avoid identification problems.
  4. We show that G7 countries'unemployment rate do follow a dynamic according to the model we propose

1. Introduction and Motivation
2. Theoretical Framework: Detailed Models
3. Modelling strategy: illustration in the case of Japan

Why Gegenbauer polynomials?

Standard AR(m) process with a long-memory component:

$$\Phi_m(L)y_t = \epsilon_t \Leftrightarrow y_t = \sum_{i=1}^{m} \phi_i y_{t-i} + \epsilon_t, \quad \epsilon_t \sim iid(0, \sigma_\epsilon)$$

m large $\longrightarrow$ under-identification of parameters (parcimony problem) : too many parameters to estimate (thing about 100 coefficients!)

Solution: find a weighting function with a small number of parameters $\longrightarrow$ standard literature (Almon polynomials,...).

Here : we need a function that capture long-memory for time series that have cyclical dynamics (important: long-term does not necessarily means trend $\longrightarrow$ Gegenbauer polynomials

Why Gegenbauer polynomials?

$$\nabla(d,u) = (1 - 2uL + L^2)^d = \sum_{j=0}^{\infty} C_j^d(u) L^j,$$

can be expanded by using the fractional binomial expansion

$$(1+x)^d = \sum_{k=0}^{\infty} \binom{n}{k} x^k, \quad x = -2uL + L^2.$$

Simple calculus leads to the following recurrence

$$\begin{cases} C_0^d(u) = 1, \\ C_1^d(u) = -2\ d\ u, \\ C_j^d(u) = 2u\left(\dfrac{d-1}{j} + 1\right) C_{j-1}^d(u) - \left(2\dfrac{d-1}{j} + 1\right) C_{j-2}^d(u), \ \text{for} j \geq 2. \end{cases}$$

Applying this to a time series

$$\nabla(d,u)y_t = \epsilon_t, \longleftrightarrow y_t = -\sum_{j=1}^{\infty} C_j^d(u) L^j y_t + \epsilon_t$$

Definition of the parameters

- $u = cos(\nu)$: define a frequency ($0 \leq \omega \leq \pi \longrightarrow 0 \leq u \leq 1$).
- d: define long-memory parameter (fractional number)
  1. if $d > 0$: persistence = long-memory captured by a lower frequency of the time series (dominant cycle of long duration)
  2. if $d < 0$: anti-persistence = short-memory cyclical fluctuations captured at the highest frequency of the time series (dominant cycle of short durations)
- It can be shown that such a process has a spectral density that explodes near the zero frequency (near $\omega = 0$)

Process:

$$(1 - 2u_1 L + L^2)^{d_1}(1 - 2u_2 L + L^2)^{d_2} y_t = \epsilon_t$$

- $u_1 = cos(\omega_1), u_2 = cos(\omega_2)$: define two low frequencies near the zero frequencies in the spectral density.
- d_1, d_2: define two long-run parameters corresponding to those frequencies.
- Allows modeling cyclical dependence at more than one frequency.

Filtering Representation

Filtered Series:

$$y_t = \sum_{m=0}^{\infty} \lambda_m x_{t-m}$$

where:

$$\lambda_m = \sum_{k=0}^{m} C_k^{(d_1)}(u_1) C_{m-k}^{(d_2)}(u_2)$$

- λ_m: weight of lag m derived from convolution of two Gegenbauer expansions.
- Provides a richer long-range memory structure.

There are therefore five possible scenarios.

- Case 1: $d_1, d_2 < 0$.

λ_m decreases quickly and reflects the presence of a short-memory behavior in the unemployment rate series. However, anti-persistence implies a strong volatility in the short-term cycles with high increases of the series followed by high decreases.

- Case 2: $d_1, d_2 > 0$ and $d_1 + d_2 < 1$.

The series also have long-term memory persistent behavior but is mean-reverting.

- Case 3: $d_1, d_2 > 0$ and $\mathrm{d}_1 + d_2 > 1$.

The series under investigation have a dynamic which is explosive and highly unstable. Shocks to the unemployment never dissipate.

- Case 4: $d_1 < 0$ and $d_2 > 0$.This corresponds to mixed memory of type I.

One of the components causes persistence, while the second is a source of anti-persistence (i.e. presence of significant short-term cyclical components).

In this case, the final effect depends on u_1 and u_2. It may happen, for example, that the initial response to a shock is strong at first, before its effects gradually dissipate.

- Case 5: $d_1 > 0$ and $d_2 < 0$. This corresponds to mixed memory of type II.

The situation is symmetrical to the previous case. It is possible that the effects of a shock seem to fade and disappear at first, before resurfacing after some periods ("bounce-back effects").

Rich dynamics of long-memory cyclical behavior

We therefore notice the advantage of having two long memory parameters here (instead of just one, as is usually the case).

- Brings out both the persistence inherent to the potential behavior of the unemployment rates.
- Also provides information about the short-term sensitivity of the unemployment rate to shocks (captured by anti-persistence).
- The dynamics can thus be more complex than that of well-behaved bell curves with transitory effects that dissipate over the long-term.

Standard Smooth Transition Autoregressive model (STAR):

$$X_t = \left[1 - \Phi\left(\mathrm{s}_t, \gamma, c\right)\right] \sum_{i=1}^{p} \alpha_1^1 \mathrm{X}_{t-i} + \left[\Phi\left(\mathrm{s}_t, \gamma, c\right)\right] \sum_{i=1}^{p} \alpha_1^2 \mathrm{X}_{t-i} + \epsilon_t,$$

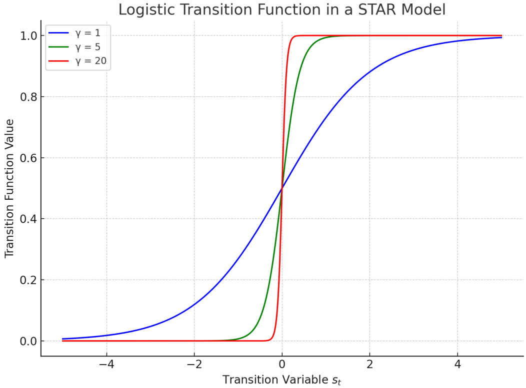
$\Phi\left(\mathrm{s}_t, \gamma, c\right)$: transition function that model the regime switching dynamics.

Two different parametrizations of this function, widely used in the literature:

Logistic : $\Phi\left(\mathrm{s}_t, \gamma, c\right) = \left[1 + exp\left(-\gamma(s_t - c)\right]^{-1}, \ \gamma > 0,$

Exponential : $\Phi\left(\mathrm{s}_t, \ \gamma, c\right) = \left[1 - exp(-\gamma(s_t - c)^2)\right], \ \gamma > 0.$

- X_t variable of interest (unemployment rate)
- s_t : transition variable that governs the switches between regimes (for instance, an indicator of the business cycle that impact labour markets).
- γ: parameter showing the strength of the transition and c is a threshold value.



Combination of the two previous models. we call the new model a regime-switching k-factor Gegenbauer STAR process (k=2)

$$\begin{cases} (1 - 2u_1 L + L^2)^{d_1} (1 - 2u_2 L + L^2)^{d_2} y_t = x_t, \\ x_t = \phi_1' w_t \Big[1 - G(s_t, \gamma, c) \Big] + \phi_2' w_t \Big[G(s_t \gamma, c) \Big] + \epsilon_t, \end{cases} \tag{1}$$

$$d_i \in [-0.5, 0.5) \qquad |u_i| \leq 1 \qquad \epsilon_t \sim iid(0, \sigma_\epsilon^2),$$
$$\phi_i' = (\phi_{i,0}, \phi_{i,1}, \phi_{i,2}, \ldots, \phi_{i,p}), \qquad i = 1, 2,$$
$$w_t = (1, x_{t-1}, \ldots, x_{t-p})', \qquad p \geq 1,$$
$$G(s_t, \gamma, c) = \Phi(s_t, \gamma, c).$$

Our model

Our model can be written as the linear combination of two 2-factor Gegenbauer processes, with two extreme regimes for the ARMA coefficients $\iff$ linear combination of two particular infinite autoregressive processes:

$$y_t = \left(\phi_{1,0} + \eta_1' \Omega_t\right)[1 - G(s_t, \gamma, c)] + \left(\phi_{2,0} + \eta_2' \Omega_t\right) G(s_t \gamma, c) + \epsilon_t,$$

where

$$\begin{aligned}
&\Omega_t = (y_{t-1}, \ldots, y_{t-\infty})', \\
&\eta_i' = \left(\eta_{i,1}, \eta_{i,2}, \ldots, \eta_{i,\infty}\right), \\
&\eta_{i,k} = \phi_{i,k}\left(1 - 2u_1 L + L^2\right)^{d_1}\left(1 - 2u_2 L + L^2\right)^{d_2}, \\
&i = 1, 2, \;\; k = 1, 2, \ldots, \infty.
\end{aligned}$$

Our model: Generalized impulse response function (GIRF)

The interpretation of the dynamics produced by the GIRF of a RSG-STAR model can therefore be tricky because the combination of the Gegenbauer and STAR components induces complex intricated behaviors.

- Firstly, the long-memory effects due to the Gegenbauer model produce long-lasting effects.
- Secondly, the potential presence of anti-persistence effects in the Gegenbauer process and some explosive roots of the STAR component both produce amplified responses of unemployment to shocks.
- Thirdly, the regime switching transition function can produce strong instabilities in the dynamics of the non-linear regime changes of the STAR component.

Our model: Generalized impulse response function (GIRF)

The response at a time horizon τ of the unemployment rate to a shock at time t of magnitude ν on the residual component ϵ_t is given by the following generalized impulse response function:

$$GIRF_\tau = \mathbb{E}\big[y_{t+\tau}|\epsilon_t = \nu, \Omega_{t-1}\big] - \mathbb{E}\big[y_{t+\tau}|\epsilon_t = 0, \Omega_{t-1}\big],$$

Ω_{t-1}: information set available up to time $t-1$.

GIRF = distance between two forecasts, with one for which the value of ϵ_t is non-zero.

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GIRF $=$ distance between two forecasts, with one for which the value of ϵ_t is non-zero.

We show that

$$GIRG_\tau = \nu \lambda_\tau,$$

with

$$\lambda_\tau = \sum_{j=0}^{\tau} \alpha_j^{(1)} \alpha_{\tau-j}^{(2)} \prod_{k=0}^{j-1} \Big[\quad \left(1 - G(s_{t+k}, \gamma, c)\right) \sum_{n=1}^{p} \left(\phi_{1n} L^{n-1}\right) + \quad G(s_{t+k}) \sum_{n=1}^{p} \left(\phi_{2n} L^{n-1}\right) \Big].$$

Remember:

$$\prod_{i=1}^{2} \nabla(d_i, u_i) = \left(\sum_{j=0}^{\infty} \alpha_j^{(1)} L^j \right) \left(\sum_{k=0}^{\infty} \alpha_k^{(2)} L^{j+k} \right)$$

where $\alpha_k^{(1)}$ corresponds to $\nabla(d_1, u_1)$ and $\alpha_k^{(2)}$ corresponds to $\nabla(d_2, u_2)$.

The following elements influence the response of y at the τ horizon:

(a) $\alpha_j^{(1)}$ and $\alpha_j^{(2)}$ capture the effects of long-term memory.

(b) The term after the product $\prod_{k=0}^{j-1}[\ldots]$ captures the effects of regime changes.

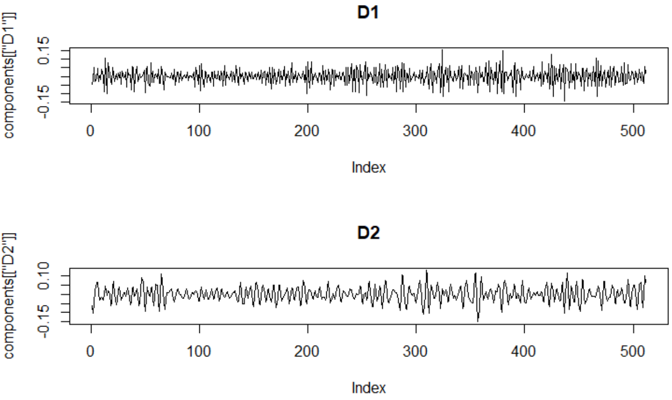
(c) The transition function $G(s_t, \gamma, c)$ introduces an asymmetry in the response.

- The short-term response of y_t (for example for $\tau = 1, 2$) depends
  - on the persistence induced by the long Gegenbauer memory
  - and the G function that determines the regime in which the response takes place.
- The medium-term response (for example for $\tau = 3, 5, 10$) depends on the influence of the Gegenbauer components. These make it possible to know whether the shocks dissipate quickly or whether they exhibit persistent memory dynamics.
  - If $d_1, d_2 > 0$, the shocks are persistent.
  - If $d_1, d_2 < 0$, the shocks dissipate quickly.
- The long-term response ($\tau \to \infty$) depends on two factors:
  -if the process is fractionally integrated ($d_1, d_2 > 0$), there is hysteresis and the effect of the initial shock never disappears,
  - if $d_1, d_2 < 0$, the GIRF converges quickly towards 0 and the unemployment rate is dominated by cycles with short durations.

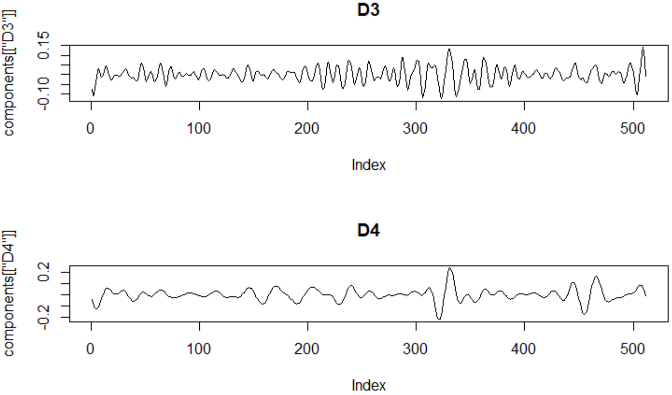
Wavelet decomposition of the time series (6-level Daubechies wavelet): D1,....D6 + S6 (trend component)

- First level D1 : high-frequency fluctuations (fine details of y_t: very noisy $\longrightarrow$ we filter the original series from this component (denoising)
- D2 to D5 levels : moderate frequency changes (fluctuations reflecting seasonal changes and the business cycle (short- to medium-duration cycles)
- D6 level : large scale components (long-duration cycles)
- S6 level (father wavelet) : trend components and structural shifts $\longrightarrow$ We de-trend the original tiem series

Wavelet decomposition of the original time series



Wavelet decomposition of the original time series



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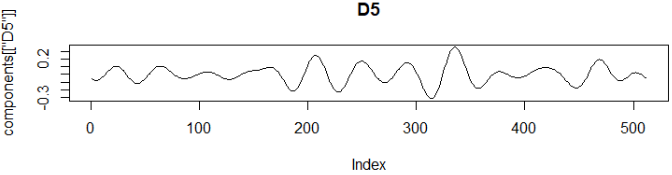

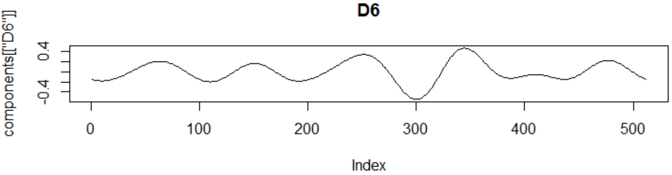

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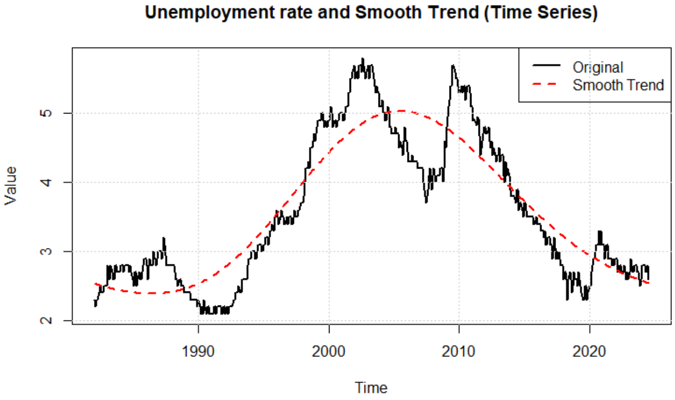

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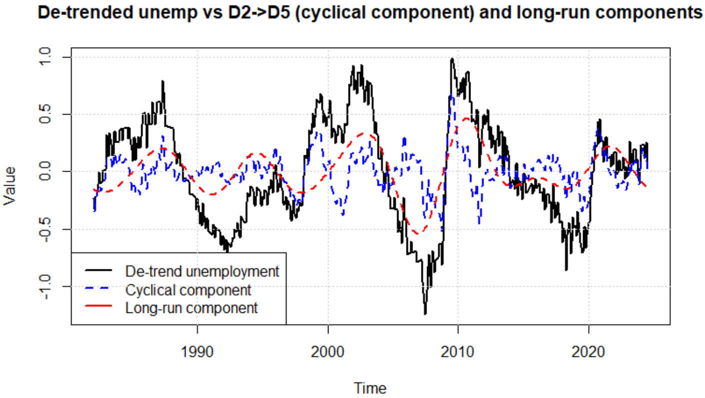
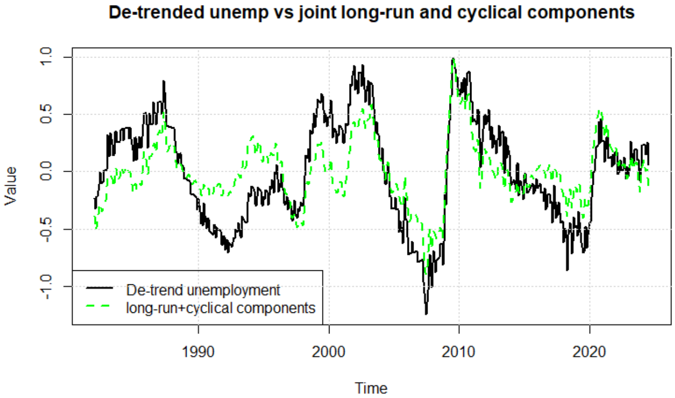



De-trended unemp vs joint long-run and cyclical components
Value
1.0
0.5
0.0
-0.5
-1.0
1990
2000
2010
2020
Time
De-trend unemployment
long-run+cyclical components

Re-pararametrization using the stirling approximation We show that the Gegenbauer component of our process can be approximated, for a sufficiently large truncation point, by :

$$z_t \approx \sum_{j=1}^{M} \psi_j y_{t-j}, \ \psi_j = \sum_{k=0}^{j} \frac{cos(ku_1)}{k^{1-d_1}} \frac{cos(ku_2)}{k^{1-d_2}}$$

M: truncation point
Define

$$G_k(u_k, d_k, L) = \left(1 - 2cos(\lambda)L + L^2\right)^d, \ k = 1, 2$$

Combination of the two previous models. we call the new model a regime-switching k-factor Gegenbauer STAR process (k=2)

$$\begin{cases} (1 - 2u_1 L + L^2)^{d_1} (1 - 2u_2 L + L^2)^{d_2} y_t = x_t + \epsilon_t, \\ x_t = \phi_1' w_t \Big[1 - G(s_t, \gamma, c) \Big] + \phi_2' w_t \Big[G(s_t \gamma, c) \Big], \end{cases} \quad (2)$$

The test is :

$H0$: y_t follows is a process (AR)

against

$H1$: y_t follows a RSG-LSTAR process.

We show that the test can be done using the following Lagrange multiplier test, after Taking a third-order Taylor expansion around $\gamma = 0$.

1. Step 1 : Estimate a Gegenbauer process on the original data (de-trend and de-noised) time series y_t . From the fitted values of this regression, take the estimated residuals $\hat{\nu}_t$.
2. Step 2. Estimate the following auxiliary regression

$$\hat{\nu}_t = a_0 + \sum_{j=1}^{P} a_j \hat{\nu}_{t-j} + \sum_{i=1}^{3} \sum_{j=1}^{P} \delta_{i,j} \hat{\nu}_{t-j} +$$

$$\lambda \sum_{j=1}^{t-1} \hat{\nu}_{t-j} \frac{cos(ju_1)}{j^{1-d_1}} \frac{cos(ju_2)}{j^{1-d_2}} + w_t,$$

$w_t \sim iid(0, \sigma_w)$, with $u_1, u_2, d-1, d_2$ obtained at Step 1.

- Step 3 : Apply the following test:

  $H0 : \delta_{i,j} = \lambda = 0$ (constrained regression)
  against
  $H1 : \delta_{i,j} \neq 0$ **and** $\lambda \neq 0$ (unconstrained regression)

  Denote SS_0 and SS_1, the sum od squared residuals of the regression under H_0 and under H_1.

- Step 4 : Compute the following Fisher statistic:

$$LM = \frac{(SS_0 - SS_1)/(k(p+1))}{SS_1/(T-(k+1)()p+1)} \sim F\big(k(p+1), T-(k+1)(p+1)\big)$$

  where k is the highest power in the Taylor expansion of the logistic function.

  Conclude towards rejection or no-rejection of H_0.

Estimation of GG model on the long-term component of unemployment rate series (D6 level)

Table 1: Estimated Coefficients and Statistics

	u1	**fd1**	**u2**	**fd2**
Estimate	-0.605511	0.24189	0.9979276	0.9189
Std. Error	0.007889	0.02782	0.0001635	0.0176

Table 2: Gegenbauer Parameters

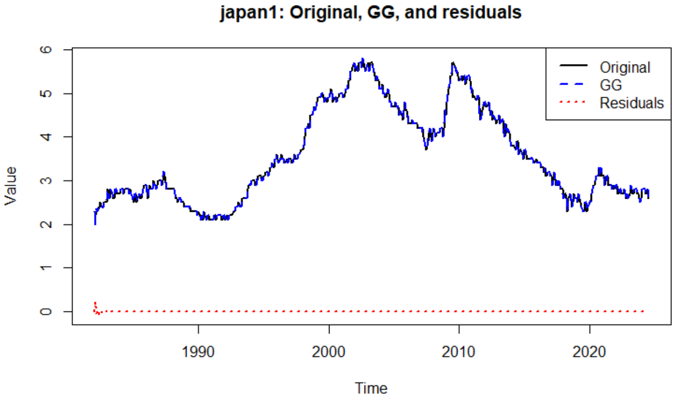
	Factor1	**Factor2**
Gegenbauer Frequency	0.3535	0.0102
Gegenbauer Period	2.8287	97.5785
Gegenbauer Exponent	0.2419	0.9189

Table 3: Estimated Coefficients and Statistics

	u1	fd1	u2	fd2
Estimate	-0.605511	0.24189	0.9979276	0.9189
Std. Error	0.007889	0.02782	0.0001635	0.0176

- u_2 close to 1 $\longrightarrow$ the spectral density has a pole near zero at frequency $\omega = arcos(0.92) = 0.01$ (Gegenbauer frequency).
- $0.5 < d_2 < 1 \longrightarrow$ non-stationary component but mean reverting: shocks are very persistent in the long-term, but not pure hysteresis. Long-term unemployment rate is non-stationary : possible multiple equilibria for u_t^*.
- $d_1 = 0.24 \longrightarrow$ another long-memory component at frequency $\omega = arcos(-0.605) = 0.35$.

Illustration for Japan



Linear Test Results for Unemployment Rate vs. RSG-LSTAR Model

Table 4: Linear Test Results

P	F-statistic	p-value
1	24.947	0.000
2	115.087	0.000
3	178.917	0.000
4	77.022	0.000
5	36.193	0.000
6	6.040	0.000

Transition variable: $\Delta_6 u_t =$ cumulated changes of unemployment over 6 months.

- H_0 always rejected for maxlag of the AR lags. $\longrightarrow$ suggests a RSG-LSTAR dynamics in the cyclical component of unemployment rate.

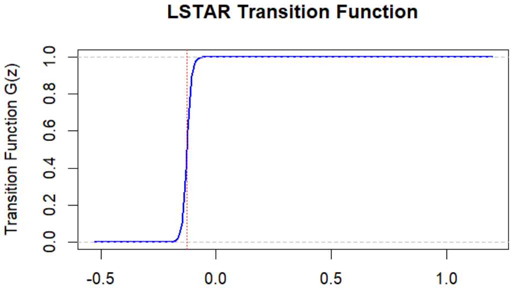
Two-Regime LSTAR Model Estimates

Parameter	Estimate	Std. Error	t-value	$\Pr(>\|z\|)$
Intercept (Low Regime)	0.002	0.003	0.864	0.388
$\phi_{L,1}$	2.100	0.071	29.446	$<$0.001 ***
$\phi_{L,2}$	-2.330	0.137	-16.983	$<$0.001 ***
$\phi_{L,3}$	1.642	0.136	12.058	$<$0.001 ***
$\phi_{L,4}$	-0.496	0.070	-7.075	$<$0.001 ***
Intercept (High Regime)	-0.003	0.003	-1.004	0.315
$\phi_{H,1}$	0.218	0.083	2.636	0.008 **
$\phi_{H,2}$	-0.237	0.163	-1.451	0.147
$\phi_{H,3}$	0.212	0.162	1.306	0.192
$\phi_{H,4}$	-0.174	0.082	-2.134	0.033 *
γ	100.000	-	-	-
θ	-0.126	0.025	-5.142	$<$0.001 ***

Fit: Residual variance = 0.000821, AIC = -3614, MAPE = 108.9%
Significance codes: *** $p < 0.001$, ** $p < 0.01$, $*$ $p < 0.05$

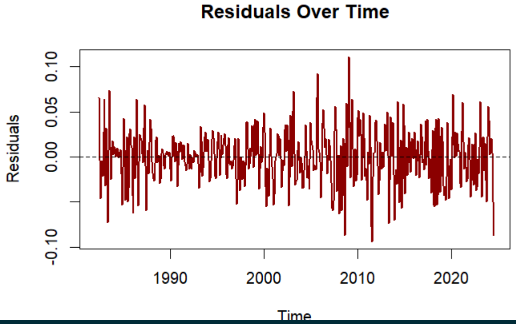
LSTAR Skeleton: Root Properties (Low and High Regimes)

Regime	Root #	Real Part	Imag Part	Modulus	Angle (ř)	Stability	Cyclical
Low	1	0.2957	0.8807	0.9291	71.4406	Stable	Yes
Low	2	0.2957	-0.8807	0.9291	-71.4406	Stable	Yes
Low	3	0.7543	0.0753	0.7580	5.7045	Stable	Yes
Low	4	0.7543	-0.0753	0.7580	-5.7045	Stable	Yes
High	1	-0.3401	0.6197	0.7069	118.7567	Stable	Yes
High	2	-0.3401	-0.6197	0.7069	-118.7567	Stable	Yes
High	3	0.4491	0.3828	0.5901	40.4423	Stable	Yes
High	4	0.4491	-0.3828	0.5901	-40.4423	Stable	Yes



Residual Diagnostic Tests for LSTAR Model

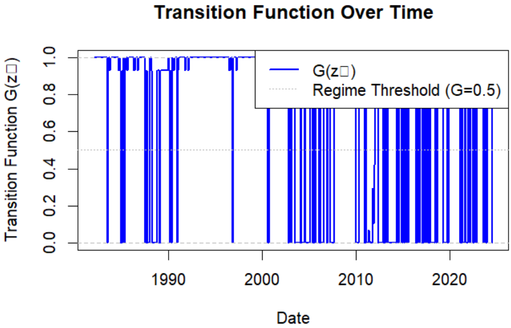
Test	Null Hypothesis	Stat Type	Statistic	p-value
Terasvirta's Neural Network	Linearity in mean	χ^2 (df = 2)	0.2744	0.8718
White Neural Network	Linearity in mean	χ^2 (df = 2)	0.1596	0.9233
Keenan's Test	AR process	F-stat	0.4480	0.5036
McLeod-Li Test	ARIMA process	Max p-value	—	0.0362
Tsay's Test	AR process	F-stat	1.2442	0.0734
Likelihood Ratio Test (TAR)	AR vs TAR model	χ^2	41.6508	0.2002



LSTAR Model: Root-Based Cycle Properties

Regime	Root #	Real Part	Imag Part	Modulus	Angle (ř)	Stability	Cyclical	Cycle Period
Low	1	0.2957	0.8807	0.9291	71.4406	Stable	Yes	5.04
Low	2	0.2957	-0.8807	0.9291	-71.4406	Stable	Yes	5.04
Low	3	0.7543	0.0753	0.7580	5.7045	Stable	Yes	63.11
Low	4	0.7543	-0.0753	0.7580	-5.7045	Stable	Yes	63.11
High	1	-0.3401	0.6197	0.7069	118.7567	Stable	Yes	3.03
High	2	-0.3401	-0.6197	0.7069	-118.7567	Stable	Yes	3.03
High	3	0.4491	0.3828	0.5901	40.4423	Stable	Yes	8.9
High	4	0.4491	-0.3828	0.5901	-40.4423	Stable	Yes	8.9

- The low regime has cycles around 5 and 63 months $\longrightarrow$ This regime has slower oscillations = smoother and slower-moving structure.
- The high regime shows a faster cycle of about 3 months $\longrightarrow$ This regime has faster fluctuations, possibly reflecting instability, volatility, and reactive dynamics.



- High regime: $G(s_t) = 1$

High Regime Years and Major Events in Japan

Year	Major Event in Japan
1981	Severe recession and oil crisis aftermath
1982	Japan becomes worlds largest creditor nation
1986	Yen appreciates rapidly post-Plaza Accord
1991	Asset bubble bursts; start of the Lost Decade
1992	GDP growth slows sharply; banking stress begins
1993	LDP loses power for first time since 1955
1995	Kobe earthquake; Tokyo subway sarin attack
1997	Asian Financial Crisis impacts Japan
1998	Bank bailouts amid deep recession
1999	Zero interest rate policy introduced
2001	Ehime Maru incident; Koizumi becomes PM
2008	Global Financial Crisis hits Japanese exports
2009	DPJ wins election, ending LDP dominance
2020	COVID-19 pandemic; Tokyo Olympics postponed

Table 5: Years in High Regime ($G \approx 1$) with Corresponding Major Events