

Consumer Durables in T(H)ANK Economies*

Emil Holst Partsch[†]

University of Copenhagen

Ivan Petrella[‡]

University of Warwick & CEPR

Emiliano Santoro[§]

University of Copenhagen

December 3, 2022

Abstract

We devise tractable heterogeneous-agent New Keynesian economies where households may infrequently participate in financial markets. Both nondurable and durable goods are available for consumption. To the extent that durables feature slow depreciation, a risk-sharing condition emerges, involving all households who can buy both types of consumption goods, irrespective of their financial status. In light of this, factors typically key in shaping monetary transmission in benchmark one-sector economies—such as fiscal transfers from financially unconstrained to constrained households—only affect household-specific durable expenditure, while being otherwise neutral to either type of sectoral demand. When introducing hand-to-mouth consumers with no access to both durable purchases and financial assets, fiscal transfers are no longer purely redistributive—neither at the household nor at the sectoral level—and tend to amplify the response of GDP to monetary shocks, unlike what found in one-sector economies featuring nondurables only.

Keywords: Heterogeneous agents, durable goods, monetary policy, fiscal transfers.

JEL codes: E21, E31, E40, E44, E52.

*We wish to thank Davide Debortoli, Erik Öberg, Søren Hove Ravn, and Petr Sedlacek for useful comments.

[†]Department of Economics, University of Copenhagen, Øster Farimagsgade 5, Copenhagen K, DK-1353, Denmark. Email: ehp@econ.ku.dk.

[‡]Warwick Business School, University of Warwick, Scarman Building, Gibbet Hill Road, Coventry, CV4 7AL, UK. Email: ivan.petrella@wbs.ac.uk.

[§]Department of Economics, University of Copenhagen, Øster Farimagsgade 5, Copenhagen K, DK-1353, Denmark. Email: emiliano.santoro@econ.ku.dk.

1 Introduction

In recent years, the macroeconomic literature has established various lines of enquiry on the connection between incomplete markets and household heterogeneity, with the aim of understanding both the aggregate and the distributive outcomes of shocks to the economy (Coibion et al., 2017; Kaplan et al., 2018; Kogan et al., 2020, among others). Concurrently, a certain interest has emerged in developing analytically tractable models that can capture the salient features of heterogeneous-agent (HA) economies (see, e.g., Bilbiie, 2018, 2020; Ravn and Sterk, 2021). That said, much of our understanding of the transmission of monetary policy—and most of the analytical literature employing HA New Keynesian (HANK) models is no exception—comes from one-sector economies where only nondurable goods are available for consumption. Yet, it is well known that large part of consumption fluctuations reflect movements in the durable component (both at the household and at the aggregate level; see Attanasio, 1999; Stock and Watson, 1999). Moreover, consumer spending on durables is far more sensitive to changes in the interest rate than is expenditure on nondurables and services (Mankiw, 1985).

This paper examines the role of consumer durables for monetary transmission. To this end, we devise different tractable HA economies with sticky prices where households may infrequently participate in financial markets. The common trait of these economies is that, conditional on their holdings of liquid financial assets, households may feature durables, along with nondurables, in their consumption baskets. A well-established property of representative-agent New Keynesian (RANK) economies involving consumption goods subject to slow depreciation is to preserve a quasi-constant shadow value (along with their stock), in the face of temporary shocks (Barsky et al., 2007). In light of this, the shadow value of income for an agent that buys durables mirrors changes in the price of the latter relative to that of nondurables. When transposing this logic to HA settings, an endogenous *risk-sharing* condition obtains among all consumers who may buy durables. We show this to be the case both in a two-agent New Keynesian (TANK) setting where limited participation to the financial market applies deterministically—so that households are invariably sorted into savers and hand-to-mouth (HtM) consumers—and in a setting characterized by idiosyncratic uncertainty, where consumers may switch between the two financial states: we refer to the latter as the 2-state HANK model. Even if HtM households do not access a saving technology—at least from time to time—they

can still smooth their nondurable consumption profile through durable purchases. In light of this, following a monetary shock both savers' and HtM households' nondurable consumption levels remain at the (symmetric) steady state—when the relative price of durables does not vary—or display analogous deviations from the steady state—net of a factor that depends on agent-specific curvature of nondurable utility—when sectors exhibit asymmetric price stickiness (and, thus, the relative price changes). In this second scenario, it is possible to derive an Euler equation for aggregate nondurable consumption where the elasticity of (expected) aggregate consumption growth to the real rate of interest depends on the share of HtM households, as well as on both households' degree of relative risk aversion. This factor loading indexes the so-called *HtM* channel in our 2-agent/state economies with asymmetric price stickiness across sectors.

A key theme of Bilbiie (2018) is that monetary policy transmission is shaped by fiscal redistribution of monopoly profits from savers to HtM consumers. A major departure of our TANK economy from this one-sector benchmark is that fiscal redistribution is neutral to both household-specific and sectoral nondurable consumption, regardless of how sectoral price stickiness is calibrated.¹ In fact, only preference heterogeneity may activate the HtM channel, in a setting where purchases of long-lived durables are available to any agent. This is because durables insulate HtM households' nondurable consumption from changes in sectoral profits that occur when demand (and, thus, real wages) vary, for whatever reason, and in either sector. Relatedly, due to household-specific durable expenditure adjusting in fulfillment of the risk-sharing condition, transfers are also neutral to the sectoral demand of durables, while being purely redistributive at the household level.

These properties necessarily survive when we move from the TANK to the 2-state HANK model, thus complementing the *HtM* channel with a *self-insurance* channel emerging from the interaction between aggregate and idiosyncratic uncertainty, in the vein of Bilbiie (2018, 2020). A notable feature of the aggregate nondurable Euler in the 2-state HANK model, as compared with its TANK counterpart, is that discounting (compounding) of news about future expenditure may arise when sectors exhibit asymmetric price stickiness; but, again, only to the extent that HtM households are more (less) risk averse than savers. In fact, even if they acknowledge that in some states of the world they might

¹In Bilbiie (2018), instead, fiscal transfers invariantly reduce constrained agents' income elasticity to aggregate income, dampening the effects of shocks and policies.

find themselves financially constrained, households are still able to access durable goods as a saving device, so that only preference heterogeneity modulates self insurance. This finding challenges the conventional emphasis on the interplay between idiosyncratic uncertainty and HtM behavior as a key driver of aggregate *nondurable* consumption (Bilbiie, 2008), especially in connection with the self-insurance channel, which is regarded as a powerful intertemporal propagator of the HtM channel. Seen in this perspective, the ability to buy durable goods, along with their role as a store of value, brings the 2-state HANK model closer to a setting with complete markets (to the extent that preference heterogeneity is considered of second-order importance).

In light of these properties, we introduce a third class of households that are limited in the access to both financial markets and durables—a category that we label *pure HtM*—assuming they may switch to/from a third state embodying these restrictions. Within this setting, we retrieve two core properties: *i*) first, fiscal redistribution becomes non-neutral with respect to both durable and nondurable sectoral production; *ii*) second, based on *i*) we observe that durables flip the behavior of the conditional volatility of GDP with respect to fiscal redistribution, relative to a comparable one-sector economy where GDP only accounts for the production of nondurables.² In our setting, regardless of the relative degree of sectoral price stickiness, fiscal transfers *amplifies* the conditional volatility of GDP, while the opposite holds true in a one-sector model à la Bilbiie (2018). Thus, not only do durables prove to be crucial in that they induce higher aggregate volatility—even if produced by a relatively small sector in the economy—but also the way they affect monetary transmission, both in the aggregate and at the household level, may bear very important implications about the interaction with fiscal policy.

Related literature This work relates to a broad literature employing saver-spender models to investigate the transmission of monetary policy (see Campbell and Mankiw, 1989; Mankiw and Zeldes, 1991) and fiscal policy (see Galí et al., 2007). Inspired by this tradition, Bilbiie (2008) devises a one-sector TANK model where profits and their redistribution through fiscal policy take center stage. While building up on this, our settings represent non-trivial two-sector extensions, where the *propagation* of monetary policy may change profoundly.

²All the model variations we consider feature sticky prices. However, contemplating nominal wage stickiness—and, thus, opening up to the possibility of modulating the cyclical of firm profits—does not affect this baseline principle.

In this respect, we relate to Barsky et al. (2007) and other contributions employing RANK models with durables to investigate the transmission of monetary policy (e.g. Erceg and Levin, 2006; Monacelli, 2009; Sudo, 2012; Tsai, 2016; Petrella et al., 2019) in that we report how profit redistribution and other structural characteristics interact with sectoral price stickiness, and may ultimately affect monetary transmission as observed in RA economies.

On the HANK front, Bilbiie (2018, 2020) surveys both the analytical and the quantitative literature. As for the first strand—which, to the best of our knowledge, has not examined the role of consumer durables for monetary transmission—he traces out the main differences between his framework, which emphasizes the role of cyclical inequality in HANK, and other contributions featuring cyclical income risk.³ Thus, we refer to his survey for a detailed mapping of the available contributions. As for the second strand of the literature, our paper closely relates to McKay and Wieland (2022), who show how embedding durables into an otherwise standard HANK economy is key to attenuating the forward guidance puzzle due to higher interest rate sensitivity of the demand for durables. Furthermore, in a companion paper we devise a calibrated two-sector setting featuring durable and nondurable production, showing how indirect (general-equilibrium) effects represent the bulk of monetary propagation to both types of consumption goods, as compared with intertemporal substitutability (see Holst Partsch et al., 2022).

Finally, we relate to some contributions examining households' adjustment of the durable-nondurable consumption mix in the face of transitory income shocks. In this respect, Parker (1999) suggests that constrained households cut back more on goods that exhibit high intertemporal substitution, because the utility cost of fluctuations in these is lower than goods that are less substitutable over time. Browning and Crossley (2000) formally shows this effect is equivalent to that characterizing the adjustment of luxury-goods expenditure in Hamermesh (1982).⁴ While our main focus is on the transmission of monetary policy shocks, a main point of tangency with these studies is that durables act as an "inefficient" saving technology that bears the burden of the adjustment, for they display a quasi-constant shadow value and, thus, close-to-infinite intertemporal substitutability. To some extent, Attanasio et al. (2020) retrieve this tendency in car expenditure during the Great Recession. However, they highlight that adjustment along the extensive

³See, e.g., Werning (2015), Acharya and Dogra (2020), Challe (2020), Ravn and Sterk (2021).

⁴Browning and Crossley (2009) complements this accelerator effect with irreversibility in durable purchases.

margin in their S-s model mostly reflects the emergence of adverse conditions during recessions, while procyclical variation along the intensive margin has mostly to do with households who are less severely affected by the contraction.

Structure The rest of the paper is organized as follows. In Section 2 we outline the baseline structure of our modular economies. Section 3 discusses the role of durables, thus reporting the behavior of both household-specific and sectoral consumption in the benchmark TANK economy. Section 4 introduces a 2-state HANK economy where consumers can switch between different states (financially constrained vs. unconstrained), while still holding a stock of durables. Section 5 focuses on aggregate amplification, extending the 2-state HANK economy to a 3-state economy, where the additional state contemplates the presence of agents with access to neither a saving technology nor durable purchases. Section 6 concludes.

2 Modeling strategy

We will proceed modularly, starting from a TANK model with savers and HtM households that buy both nondurables and durables. After discussing the main implications of this in connection with Bilbiie (2008, 2018, 2020), we will extend the model into a HANK economy with two states (i.e., savers vs. HtM households). The final step consists of elaborating an analytical HANK model featuring a 3-state structure, with potential transition among savers, *wealthy HtM* (who can smooth through durables but no financial markets), and *pure HtM* households (who hold neither durables nor financial assets).

The core of the model is a standard cashless dynamic general equilibrium economy augmented for limited asset market participation (LAMP). In line with Bilbiie (2008, 2018, 2020), we assume that a fraction of the households are excluded from asset markets, while others trade in complete markets for state-contingent securities (including a market for shares in firms). The main point of departure from conventional LAMP economies lies in differentiating consumption goods into nondurables and durables.

2.1 TANK economy

There is a continuum of households and two sectors of production, each of them populated by a single perfectly competitive final-good producer and a continuum of monopolistically competitive intermediate-goods producers setting prices on a staggered basis. There is also a government pursuing a redistributive fiscal policy and a nominal interest-rate monetary policy. A continuum of households is envisaged over the support $[0, 1]$, all having a similar utility function. A λ_S share is represented by households who can trade in all markets for state-contingent securities. We will interchangeably refer to these as assetholders or savers.

2.1.1 Households

Each assetholder chooses consumption, asset holdings, and leisure, solving a standard intertemporal problem featuring an additively separable CRRA time utility:

$$\max_{C_{S,t}, B_{S,t}, X_{S,t}, N_{S,t}} E_t \left\{ \sum_{i=0}^{\infty} \beta^i \left(\frac{C_{S,t}^{1-\sigma_S}}{1-\sigma_S} + \eta_S \frac{X_{S,t}^{1-\chi_S}}{1-\chi_S} - \varpi_S \frac{N_{S,t}^{1+\phi_S}}{1+\phi_S} \right) \right\}$$

s.t.

$$B_{S,t} + \Omega_{S,t} V_t \leq (1 + r_{t-1}) B_{S,t-1} + \Omega_{S,t-1} (V_t + P_{C,t} D_t) + W_t N_{S,t} - P_{C,t} C_{S,t} - P_{X,t} [X_{S,t} - (1 - \delta) X_{S,t-1}],$$

where $\beta \in (0, 1)$ is the discount factor, $\eta_S > 0$ and $\varpi_S > 0$ indicate how durable consumption and leisure are valued relative to nondurable consumption, $\phi_S > 0$ is the inverse of the labor supply elasticity, while $\sigma_S \geq 1$ and $\chi_S \geq 1$ index the curvature of the utility in nondurables and durables, respectively. $C_{S,t}$, $X_{S,t}$, $N_{S,t}$ are nondurable consumption, the stock of durables and hours worked by saver (the time endowment is normalized to unity). $P_{C,t}$ (taken as the numeraire) and $P_{X,t}$ are the nominal prices of nondurable and durable goods, respectively. There are two financial assets: a government-issued riskless bond paying a nominal return $r_t (> 0)$, denoted by $B_{S,t}$, and shares in monopolistically competitive firms, denoted by $\Omega_{S,t}$. V_t is the average market value at time t of the shares in the intermediate-good firms, while $D_t = D_{C,t} + D_{X,t}^C$ are total dividend payoffs aggre-

gated over the two sectors in terms of nondurable prices, with $D_{C,t}$ denoting profits from the nondurable goods sector and $D_{X,t}^C$ indicating profits from the durable goods sector (deflated by $P_{C,t}$).

Maximizing utility subject to this constraint gives the first-order necessary and sufficient conditions at each date and in each state:

$$1 = \beta E_t \left\{ \frac{C_{S,t+1}^{-\sigma_S} (1 + r_t)}{C_{S,t}^{-\sigma_S} (1 + \pi_{C,t+1})} \right\}, \quad (1)$$

$$\frac{V_t}{P_{C,t}} = \beta E_t \left\{ \frac{C_{S,t+1}^{-\sigma_S}}{C_{S,t}^{-\sigma_S}} \left(\frac{V_{t+1}}{P_{C,t+1}} + D_{t+1} \right) \right\}, \quad (2)$$

$$Q_t C_{S,t}^{-\sigma_S} = \eta_S X_{S,t}^{-\chi_S} + \beta(1 - \delta) E_t \{ Q_{t+1} C_{S,t+1}^{-\sigma_S} \}, \quad (3)$$

$$\varpi_S N_{S,t}^{\phi_S} = C_{S,t}^{-\sigma_S} \frac{W_t}{P_{C,t}}, \quad (4)$$

where $Q_t \equiv P_{X,t}/P_{C,t}$ and $(1 + \pi_{C,t+1}) \equiv \frac{P_{C,t+1}}{P_{C,t}}$.

The rest of the households (labeled non-assetholders or HtM households, and indexed by H) have no financial assets and solve

$$\max_{C_{H,t}, X_{H,t}, N_{H,t}} E_t \left\{ \sum_{i=0}^{\infty} \beta^i \left(\frac{C_{H,t}^{1-\sigma_H}}{1 - \sigma_H} + \eta_H \frac{X_{H,t}^{1-\chi_H}}{1 - \chi_H} - \varpi_H \frac{N_{H,t}^{1+\phi_H}}{1 + \phi_H} \right) \right\}$$

s.t.

$$C_{H,t} + Q_t [X_{H,t} - (1 - \delta) X_{H,t-1}] = \frac{W_t}{P_{C,t}} N_{H,t} + T_t^H,$$

where T_t^H denotes fiscal transfers. The first order conditions are

$$\varpi_H N_{H,t}^{\phi_H} = C_{H,t}^{-\sigma_H} \frac{W_t}{P_{C,t}}, \quad (5)$$

$$Q_t C_{H,t}^{-\sigma_H} = \eta_H X_{H,t}^{-\chi_H} + \beta(1 - \delta) E_t \{ Q_{t+1} C_{H,t+1}^{-\sigma_H} \}. \quad (6)$$

2.1.2 Firms

In each sector $j = \{C, X\}$, the final good is produced by a representative firm using a CES production function (with elasticity of substitution ε^j) to aggregate a continuum of intermediate goods indexed by i : $Y_{j,t} = \left(\int_0^1 Y_{j,t}(i)^{(\varepsilon_j-1)/\varepsilon_j} di \right)^{\varepsilon_j/(\varepsilon_j-1)}$. Final-good producers behave competitively, maximizing profits $P_{j,t}Y_{j,t} - \int_0^1 P_{j,t}(i)Y_{j,t}(i)di$ each period: for the j^{th} sector, $P_{j,t}$ is the overall price index of the final good and $P_{j,t}(i)$ is the price of intermediate good i . For $j = \{C, X\}$, the demand for each intermediate input is $Y_{j,t}(i) = (P_{j,t}(i)/P_{j,t})^{-\varepsilon_j} Y_{j,t}$ and the price index is $P_{j,t}^{1-\varepsilon_j} = \int_0^1 P_{j,t}(i)^{1-\varepsilon_j} di$. Each intermediate good is produced by a monopolistically competitive firm indexed by i , using a linear technology, $Y_{j,t}(i) = N_{j,t}(i)$, while bearing a nominal marginal cost W_t . The profit function in real terms is thus given by: $D_{j,t}(i) = (1 + \tau_j^S) [P_{j,t}(i)/P_{j,t}] Y_{j,t}(i) - (W_t/P_{j,t}) N_{j,t}(i) - T_{j,t}^F$, where $1 + \tau_j^S$ is a production subsidy, while $T_{j,t}^F$ stands for a lump-sum profit tax. We assume the subsidy to be set to eliminate the markup distortion in the steady state: the pricing condition under flexible prices, $P_{j,t}^*(i)/P_{j,t} = 1 = \varepsilon_j (W_{j,t}^*/P_{j,t}) [(1 + \tau_j^S) (\varepsilon_j - 1)]^{-1}$, allows us to pin down this value at $\tau_j^S = (\varepsilon_j - 1)^{-1}$. Financing the total cost of this subsidy by the profit tax ($T_{j,t}^F = \tau_j^S Y_{j,t}$) leads to aggregate sectoral profits $D_{j,t} = Y_{j,t} - (W_t/P_{j,t}) N_{j,t}$, which are zero in the steady state, thus allowing for full insurance in both nondurable and durable consumption—i.e. $C_S = C_H = C$ and $X_S = X_H = X$ —and implying $Q = 1$. Our core analysis will be conducted in economies that are log-linearized around this undistorted steady state. Log-linear variables will generally be denoted by the lower-case counterparts of level variables. However, in line with Bilbiie (2018) we define $d_{j,t} \equiv \ln(D_{j,t}/Y_j)$, which implies $d_{j,t} = -(w_t - p_{j,t})$.⁵ Moreover, in the remainder of the analysis ω_t will denote the real wage expressed in units of nondurables, i.e. $\omega_t \equiv w_t - p_{C,t}$.

Next, we allow for price setting in the vein of Calvo (1983) and Yun (1996). Intermediate-good firms in each sector $j = \{C, X\}$ adjust their prices infrequently, with θ_j being both the history-independent probability of keeping the price constant and the fraction of firms that keep their prices unchanged. Assetholders (who, in equilibrium, will hold all the shares) maximize the value of the firm, i.e. the discounted sum of future nominal profits, choosing the price $P_{j,t}(i)$ and using $\Lambda_{t,t+i}$, the relevant stochastic discount factor (pricing kernel) for nominal payoffs:

$$\max E_t \sum_{s=0}^{\infty} (\theta^s \Lambda_{t,t+s} [(1 + \tau_j^S) P_{j,t}(i) Y_{j,t,t+s}(i) - MC_{t+i} Y_{j,t,t+s}(i) - T_{j,t+s}^F]), \text{ subject to the}$$

⁵Notice that, due to the subsidy leading to an undistorted steady state, $d_{X,t} = d_{X,t}^C$.

demand equation, and where $\Lambda_{t,t+1}$ is S 's the marginal rate of intertemporal substitution between time t and $t + 1$. In equilibrium, each producer that chooses a new price $P_{j,t}(i)$ in period t will choose the same price and the same level output, so that the sectoral price index is $P_{j,t}^{1-\varepsilon_j} = (1 - \theta_j) (P_{j,t}^*)^{1-\varepsilon_j} + \theta_j P_{j,t-1}^{1-\varepsilon_j}$.

2.1.3 Government

The government conducts fiscal and monetary policy. Along with the tax and the subsidy applied to sectoral production, the former consists of a redistribution scheme that taxes S 's dividends at τ^D and rebates the proceedings to H , so that $T_t^H = \frac{\tau^D}{\lambda_H} D_t$. As stressed by Bilbiie (2020), this is key to determining the extent of indirect monetary policy transmission in his HA economies.

Monetary policy is conducted by means of a standard interest-rate rule that sets the nominal rate of interest in reaction to aggregate inflation and featuring a non-systematic component:

$$\frac{R_t}{R} = (1 + \pi_t)^{\phi_\pi} \exp(\nu_t), \quad (7)$$

where R is the steady-state (gross) nominal interest rate, ϕ_π denotes the degree to which the nominal interest rate responds to aggregate inflation, $\pi_t = \alpha \pi_{C,t} + (1 - \alpha) \pi_{X,t}$ (with $\alpha \in [0, 1]$), whose steady state has been implicitly assumed to be zero, as that for the sectoral inflation rates, and where $\nu_t = \rho_\nu \nu_{t-1} + \varepsilon_t^\nu$, ε_t^ν being i.i.d., zero-meaned and with variance σ_ν^2 .

2.1.4 Equilibrium and market clearing

A rational expectations equilibrium is a sequence of processes for all prices and quantities introduced above, such that the optimality conditions hold for all agents and all markets clear at any given time t . Specifically, labor market clearing requires that labor demand and total labor supply to be equal, $N_t = \lambda_H N_{H,t} + \lambda_S N_{S,t} = \sum_{j=\{C,X\}} N_{j,t}$. With uniform steady-state hours, this implies the log-linear relationship $n_t = \lambda_H n_{H,t} + \lambda_S n_{S,t}$.

State-contingent assets are in zero net supply (markets are complete and agents trad-

ing in them are identical), whereas equity market clearing implies that shareholdings of each assetholder are

$$\Omega_{S,t+1} = \Omega_{S,t} = \Omega = \frac{1}{\lambda_S}. \quad (8)$$

Finally, by Walras' Law, the goods markets also clear, so that $C_t \equiv \lambda_H C_{H,t} + \lambda_S C_{S,t}$ and $X_t \equiv \lambda_H X_{H,t} + \lambda_S X_{S,t}$: in light of full consumption risk-sharing in the steady state, once log-linearized these respectively translate into $c_t = \lambda_H c_{H,t} + \lambda_S c_{S,t}$ and $x_t = \lambda_H x_{H,t} + \lambda_S x_{S,t}$.

3 The role of durability

Consider households' Euler equations for durables. These may be solved forward to yield an expression for the households-specific shadow value of durables:

$$Q_t C_{z,t}^{-\sigma_z} = \eta_z E_t \left\{ \sum_{i=0}^{\infty} \beta^i (1 - \delta)^i X_{z,t+i}^{-\chi_z} \right\} \equiv \Lambda_{z,t}, \quad z = \{S, H\}. \quad (9)$$

As noted by Barsky et al. (2007), $\Lambda_{z,t}$ will be largely time-invariant to shocks with short-lived effects, when durables are long-lived enough. This means that the intertemporal elasticity of substitution in durables demand is close to infinite. Thus, for $\beta(1 - \delta)$ close to one, short-term movements in $X_{z,t}$ —as those generated by a temporary shock to fiscal spending or the nominal rate of interest—will affect the right side of the equation above relatively little, so that $Q_t C_{z,t}^{-\sigma_z} \approx \Lambda_z$. According to this, movements in the relative price of durables are forced to mirror those in either household's shadow value of income, thus reflecting the emergence of an endogenous risk-sharing condition, as enunciated in Proposition 1.

Proposition 1 *Assuming that durables feature low depreciation rates, so that their shadow value is quasi-constant, implies the emergence of a risk-sharing condition involving all households that can purchase durables in the TANK economy. Therefore, $\Lambda_S C_{S,t}^{\sigma_S} \approx \Lambda_H C_{H,t}^{\sigma_H}$.*

Thus, extending households' consumption possibilities to (long-lived) durables implies positive comovement between their consumption of nondurables, the extent of such comovement primarily depending on the relative curvature of the utility functions over

nondurables. A note of caution is warranted, at this stage, about the generality of this result. Allowing for non-separability between durables and nondurables in households' utility would not fundamentally alter the quasi-constancy property, given that the stock-flow ratio is high for durables that depreciate slowly, so that $X_{z,t}$ moves slowly enough to be regarded as nearly constant, in the face of temporary shocks (Barsky et al., 2007).⁶ In this respect, we should stress that Barsky et al. (2007) dig into the nature of a long-lived durable, conducting a robustness exercise on the interaction between sectoral price rigidity and the speed of depreciation, and checking how good an approximation is to regard the shadow value of durables as quasi-constant.⁷

In a log-linear setting, the result reported in Proposition 1 translates into

$$\sigma_H c_{H,t} = q_t = \sigma_S c_{S,t}. \quad (10)$$

Combining this with $c_t = \lambda_H c_{H,t} + \lambda_S c_{S,t}$ returns⁸

$$c_t = [1 - \lambda_H (1 - \gamma)] c_{S,t}, \text{ where } \gamma \equiv \frac{\sigma_S}{\sigma_H}, \quad (11)$$

so that we can combine the latter with savers' Euler for nondurables, as well as nondurables' market clearing (i.e., $y_{C,t} = c_t$), obtaining:

$$y_{C,t} = E_t y_{C,t+1} - \chi^{-1} (r_t - E_t \pi_{C,t+1}), \quad (12)$$

where $\chi \equiv \frac{\sigma_S}{1 - \lambda_H (1 - \gamma)}$. Therefore, provided that sectors exhibit asymmetric price stickiness—so that the relative price of durables deviates from the steady state in the face of monetary shocks—in this TANK setting intertemporal substitution over nondurable consumption depends on household heterogeneity in the curvature of nondurable utility and, con-

⁶Analogously, introducing an adjustment cost of durables to capture illiquidity would further inhibit changes in the stock, thus exerting little effect on the shadow value of durables (from the perspective of either type of household). We verify this property using a 2-sector HANK model with cyclical income risk and quadratic cost of adjustment for the stock of durable (see Holst Partsch et al., 2022). Within this setting, we also verify that the interest-rate sensitivity of durable holders is comparable, regardless of the amount of liquid financial assets held, in line with the key prediction of the present framework). These additional results are available, upon request, from the authors.

⁷Intuitively, a trade-off emerges between the rate of depreciation and price stickiness: if durable prices adjust quickly, the overall grip of price rigidity is diminished, so that changes in complements (or substitutes) to the durable are extremely short-lived, and the change in the stock of the durable itself is negligible. Barsky et al. (2007) conclude that, for a rather large parameter space, their durable is an "idealized" one.

⁸Note also that, after combining the two labor supply schedules with $c_{H,t} = \gamma c_{S,t}$, the following restriction applies: $\phi_S n_{S,t} = \phi_H n_{H,t}$.

ditional on such heterogeneity, on the share of constrained agents with respect to the total population. Specifically, increasing a positive wedge between the curvature of S 's nondurable consumption utility and that of H amplifies the impact of $r_t - E_t\pi_{C,t+1}$ on $\Delta E_t y_{C,t+1}$. A key property should also be highlighted, in light of the contribution of Bilbiie (2008):

Remark 1 *According to (12), there is no room for an inversion of the slope of the aggregate nondurable Euler equation in the TANK economy, as the elasticity of total nondurable demand to the real interest rate is always lower than zero.*

3.1 Monetary transmission in the TANK economy

We can now examine equilibrium behavior in both sectors, as well as household-specific expenditure in either type of good, conditional on monetary shocks. To elicit the distinctive role of durability in monetary transmission, we take an economy with symmetric price stickiness as the most straightforward extension of the one-sector framework. Thus, in line with Barsky et al. (2007), we alternatively consider the case of purely flexible prices of durables and nondurables. Based on this plan, we detail the behavior of *sectoral* purchases, as well as the determinants of *household-specific* consumption of durables and nondurables, with a specific focus on the role of fiscal redistribution. The complete log-linear TANK economy, as well as the analytics for each scenario, are reported in Appendix A.

3.1.1 Symmetric price stickiness

When goods produced by both sectors display symmetric price stickiness, $q_t = 0$, so that also household-specific and aggregate nondurable consumption remain at their steady-state values, in light of (10) and (11). Thus, combining S 's Euler for nondurables and the Taylor rule, together with households' labor supply:

$$y_{C,t} = 0,$$

$$y_{X,t} = \frac{Y}{Y_X} \frac{1}{\zeta \psi (\rho_\nu - \phi_\pi)} \varepsilon_t^\nu, \text{ where } \zeta \equiv \phi_S [1 - \lambda_H (1 - \vartheta)]^{-1} \text{ and } \vartheta \equiv \phi_S / \phi_H.$$

As in the one-sector RANK model discussed by Barsky et al. (2007), movements in aggregate production are accounted for entirely by durable production, with nondurable production remaining at the steady state. This is because HtM households can smooth nondurable purchases through durables and, due to the combination of equally sticky sectoral prices and slow depreciation, they end up not adjusting their nondurable consumption at all, in the face of monetary shocks. A further remarkable property of the sectoral equilibrium in the TANK economy is that household heterogeneity only matters to the extent it affects household preferences about nondurable consumption and labor supply, thus allowing us to formulate Proposition 2.

Proposition 2 *Due to consumption risk-sharing, sectoral equilibrium production in the TANK economy under homogeneous preferences is equivalent to that obtained in the RANK benchmark.*

As we will see in the next subsection, this result extends to the asymmetric economies. With this picture in mind, obtaining equilibrium household-specific durable expenditure is straightforward. For illustrative purposes, we report this as a function of sectoral durable production, while (temporarily) neutralizing preference heterogeneity in terms of labor supply (so that $\zeta = \phi$), without loss of generality:

$$e_{S,t} = \left(1 + \phi \frac{\tau^D - \lambda_H}{\lambda_S} \right) y_{X,t}, \quad (13)$$

$$e_{H,t} = \left(1 + \phi \frac{\lambda_H - \tau^D}{\lambda_H} \right) y_{X,t}, \quad (14)$$

implying that the magnitude of τ^D relative to λ_H amplifies/attenuates the response of savers relative to that of HtM households. The effect is analogous to that highlighted by Bilbiie (2020) in his one-sector economy with nondurables. In fact, in connection with durable expenditure, our TANK economy features an externality imposed by H on S through the income effect of real wages. To see this, initially assume fiscal transfers are null and that $y_{X,t}$ increases as the result of a monetary expansion. Concurrently, H 's income increases due to the real wage expanding along the aggregate labor supply schedule (as long as $\phi > 0$) and, proportionally, also their demand for durables increases (when $\tau^D = 0$ they incur no negative income effect from profits going down). This produces an extra expansion in $y_{X,t}$, labor demand, and the real wage. As S 's income drops in the face

of falling profits, they work more hours to accommodate extra demand (with H 's labor supply expanding in tandem, given the endogenous risk-sharing condition and, on top of that, the assumption of symmetric preferences). When setting $\tau^D > 0$, instead, fiscal redistribution induces H to internalize the contraction in firm profits (again, as long as we have a finite elasticity of labor supply).

At the sectoral level, though, no amplification/attenuation is induced by τ^D . In fact, fiscal transfers are purely redistributive in any model where durables insulate HtM households from the adverse effects of profits going down as demand (and, thus, the real wage) expands, for whatever reason, and in either sector. Such *neutrality* of fiscal transfers has nothing to do with the degree of sectoral price stickiness, while only hinging on the goods-demand structure of the economy, as we will see in Section 3.1.2. In fact, when both savers and HtM households can buy durables, household-specific durable spending adjusts as a reflection of the risk-sharing condition applying—so that both household-specific nondurable consumption and labor supply move in tandem—and transfers have no impact on $y_{X,t}$.⁹ Notably, this property is also invariant to allowing agents to switch between the (financially) constrained and the unconstrained state (as we will see in Section 4). Only contemplating pure HtM consumers—whose expenditure is exclusively directed towards nondurables—will break the neutrality of fiscal transfers, yielding some important insights about the interaction of fiscal redistribution and the amplification of monetary shocks in the aggregate (see Section 5).

3.1.2 Asymmetric price stickiness

We allow parameter heterogeneity back in the picture. As for sectoral price stickiness, instead, assume that one sector at the time exhibits pure price flexibility. Formally: $\theta_j = 0$ and $\theta_i > 0$, with $j, i = \{C, X\}$ and $j \neq i$. Table 1 summarizes the elasticity of household-specific nondurable and durable expenditure to their respective sector-specific production. In the analysis of Bilbiie (2020), such elasticity is key to examine the cyclical behavior of aggregate nondurable consumption. We find it useful to take a similar standpoint.

Even in this case, fiscal redistribution and labor market characteristics are irrelevant to

⁹As an alternative to deriving $y_{X,t}$ from the aggregate block of the economy, this property can be appreciated by consolidating the household-specific budget constraints in light of the risk sharing condition (10) and the household-specific labor supplies, so as to express $y_{X,t}$ as a function of ω_t .

Table 1: Elasticity to sectoral production

Sectoral price stickiness	$c_{S,t}$	$c_{H,t}$	$e_{S,t}$	$e_{H,t}$
Flexible p_X , sticky p_C	$\frac{1}{1-\lambda_H(1-\gamma)}$	$\frac{\gamma}{1-\lambda_H(1-\gamma)}$	$\left[1 - \left(\frac{Y}{Y_C} - \frac{1-\tau^D}{\lambda_S}\right) \sigma_S\right] \frac{1}{1-\lambda_H(1-\gamma)}$	$\frac{1}{\lambda_H} - \frac{\lambda_S}{\lambda_H} \left[1 - \left(\frac{Y}{Y_C} - \frac{1-\tau^D}{1-\lambda}\right) \sigma_S\right] \frac{1}{1-\lambda_H(1-\gamma)}$
Sticky p_X , flexible p_C	$\frac{1}{1-\lambda_H(1-\gamma)}$	$\frac{\gamma}{1-\lambda_H(1-\gamma)}$	$\left[1 - \sigma_S \left(\frac{1-\tau^D}{\lambda_S} \frac{Y_X}{Y_C} - \frac{1}{\phi_S} \frac{Y}{Y_C}\right)\right] \frac{Y_C}{Y_C \zeta + \chi Y} \frac{1}{1-\lambda_H(1-\gamma)}$	$\frac{1}{\lambda_H} - \frac{\lambda_S}{\lambda_H} \left[1 - \sigma_S \left(\frac{1-\tau^D}{\lambda_S} \frac{Y_X}{Y_C} - \frac{1}{\phi_S} \frac{Y}{Y_C}\right)\right] \frac{Y_C}{Y_C \zeta + \chi Y} \frac{1}{1-\lambda_H(1-\gamma)}$

Notes: The first column details two scenarios of asymmetric sectoral price stickiness, in which one sector at a time displays full price flexibility. For each of these, the following two columns report the elasticity of S 's and H 's nondurable expenditure to sectoral nondurable production, while the last two columns report the elasticity of S 's and H 's durable expenditure to sectoral durable production.

households' nondurable consumption response to monetary shocks, due to the presence of durables.¹⁰ Instead, both $c_{S,t}$'s and $c_{H,t}$'s elasticity with respect to aggregate nondurable expenditure only hinges on the magnitude of σ_S relative to σ_H and, conditional on these being different, on how households split between savers and HtM. Whenever the curvature of H 's nondurable utility exceeds that of S , i.e. $\gamma < 1$, $c_{S,t}$ ($c_{H,t}$) moves more (less) than one-for-one with $y_{C,t}$. In light of this, nondurable consumption inequality, as captured by $c_{S,t} - c_{H,t}$, is procyclical when $\gamma < 1$. As for the population shares, instead, increasing λ_H inflates (deflates) the elasticity of household-specific nondurable consumption to its sectoral aggregate, for $\gamma < 1$ (> 1), as is expected on *a priori* grounds.

On the other hand, fiscal redistribution and labor market characteristics do matter for the behavior of household-specific durable consumption (and, thus, for the behavior of cyclical inequality). In fact, both $e_{S,t}$'s and $e_{H,t}$'s degree of comovement with aggregate durable expenditure hinges on τ^D and ϕ .¹¹ Before seeing how such features combine, it is important to recall how sectoral production behaves in response to monetary shocks. In fact: *i*) $y_{C,t}$ increases in the face of a monetary expansion, when durables feature flexible prices, while *ii*) it contracts when it is up to nondurables to display no price stickiness (assuming that the shock is persistent enough). As for $y_{X,t}$, this necessarily comoves negatively with $y_{C,t}$ (this property may be relaxed once adding pure HtM households that consume no durables—as we do in Section 5—or in 2-agent/state frameworks where movements in the relative price are mitigated by assuming sectoral degrees of sectoral price that are not too different). With this picture in mind, fiscal policy is always redis-

¹⁰Appendix A confirms the neutrality of fiscal transfers with respect to both durable and nondurable sectoral demand, while consumer heterogeneity only plays a role through preferences, as indexed by χ and ζ , depending on how sectoral price stickiness is set.

¹¹While extending the analysis to a two-sector economy, we will mainly focus on these two determinants, taking as given other household-specific or sector-specific traits, such as the curvature of nondurable consumption utility and the relative size of each sector.

tributive towards H 's durable expenditure, conditional on the sign of $y_{X,t}$'s response to the monetary shock. Whenever, $y_{X,t}$ contracts (expands), increasing τ^D attenuates (amplifies) the response of $e_{H,t}$. At the same time, whenever labor hours vary—and this is not the case when durables feature flexible prices, in which case $n_{S,t} = n_{H,t} = 0$ —increasing the elasticity of labor supply works in the same direction as τ^D , as the slope of the labor supply schedule drops, and a given demand increase corresponds to a more muted contraction of sectoral profits.

Therefore, regardless of how price stickiness is set across sectors, even though H has no access to financial assets, risk-sharing in nondurable consumption emerges, *de facto* allowing constrained households to smooth their nondurable consumption profile through durable purchases. Long-lived durables insulate H 's nondurable consumption from the effects of profits going down as real wages expand. On the other hand, H 's (and, thus, S 's) durable expenditure rests on the cyclicalities of sectoral profits with respect to aggregate durable production. In this respect, take the case of flexible prices in the durable sector first: following a monetary expansion, the real wage in units of nondurables (i.e., ω_t) increases, while households' labor supply remains at the steady state—explaining why ϕ plays no role, in this context—and also the real wage in units of durables remains unaffected (so that $d_{X,t} = 0$ too). At the same time, $d_{C,t}$ contracts: thus, as τ^D increases, H (S) has less (more) resources to buy durables, for given $y_{X,t}$. In the case of flexible prices in the nondurable sector (and relatively inertial monetary shocks), instead, a monetary loosening expands the real wage in units of durables (i.e., $\omega_t - q_t$), so that $d_{X,t}$ shrinks, while leaving ω_t (and, thus, $d_{C,t}$) unaffected. Concurrently, households' labor supply increases. For given $y_{X,t}$, while the first effect restricts (increases) H 's (S 's) resources to buy durables, as τ^D increases, the second effect expands either household's durable purchase opportunities, though less so as ϕ_z increases, for $z = \{S, H\}$.

In the next section, we extend the TANK economy into a HANK model allowing agents to switch between different financial states, so as to study how compounding/discounting of news about future nondurable expenditure interacts with the *HtM channel* we have outlined for this type of economies.

4 A 2-state HANK economy

TANK economies miss a key channel in that financially unconstrained agents do not face the possibility of becoming constrained in the future, and *vice versa*. In this section, we extend the model from the previous section to allow for this possibility. In fact, Bilbiie (2018, 2020) discusses how extending the TANK setting in this direction is chiefly important to explain short-lived shocks and policies. As in these contributions, we can envisage the problem as featuring a unit mass of households that infrequently participate in financial markets: when they do, they can adjust their portfolio frictionlessly, and receive dividends from firms in either sector. When they do not participate, they only receive the return on their bond holdings from the previous period. Denote the two states as S and H , respectively. The exogenous change of state follows a Markov chain: the probability to stay type S is ϱ_{SS} , while households have a probability ϱ_{HH} to stay type H (with transition probabilities ϱ_{SH} and ϱ_{HS} , respectively). We focus on stationary equilibria whereby the mass of H is, by standard analysis, $\lambda_H = \frac{\varrho_{SH}}{\varrho_{SH} + \varrho_{HS}}$, with $\varrho_{SS} \geq \varrho_{SH}$, implying that the probability to stay a saver is larger than the probability to become one.

We follow Bilbiie et al. (2022) in that we make some assumptions that allow for analytical tractability. First, households are members of a family, whose intertemporal utility is maximized by the head, given limits to risk-sharing. Households are located on two *islands* depending on their financial-market participation status: one island is for savers, and one for HtM households. The family head can transfer resources within islands, but only some resources can be transferred between islands.

In light of these assumptions, there is full insurance within type, in the face of idiosyncratic risk, but limited insurance across types. At the beginning of the period, the family head pools resources within the island. The aggregate shocks are revealed, and the family head determines the consumption/saving choice for each household in each island. Then households learn their next-period status and have to move to the corresponding island accordingly, taking bonds and their stock of durables with them. Different financial assets thus have different liquidity: only one of the two financial assets (bonds) can be used to self-insure before idiosyncratic uncertainty is revealed—i.e., is liquid—while stocks are illiquid, and cannot be used to self-insure.

In sum, the problem for the family head reads as:

$$\begin{aligned} & \max_{C_{S,t}, C_{H,t}, X_{S,t}, X_{H,t}, N_{S,t}, N_{H,t}, \Omega_{S,t}, Z_{S,t}, Z_{H,t}} E_t \left\{ \sum_{i=0}^{\infty} \beta^i \left[\lambda_S \left(\frac{C_{S,t+i}^{1-\sigma_S}}{1-\sigma_S} + \eta_S \frac{X_{S,t+i}^{1-\chi_S}}{1-\chi_S} - \varpi_S \frac{N_{S,t+i}^{1+\phi_S}}{1+\phi_S} \right) \right. \right. \\ & \quad \left. \left. + \lambda_H \left(\frac{C_{H,t+i}^{1-\sigma_H}}{1-\sigma_H} + \eta_H \frac{X_{H,t+i}^{1-\chi_H}}{1-\chi_H} - \varpi_H \frac{N_{H,t+i}^{1+\phi_H}}{1+\phi_H} \right) \right] \right\} \\ & \text{s.t.} \end{aligned}$$

$$\begin{aligned} C_{S,t} + Q_t \left[X_{S,t} - (1-\delta) \tilde{X}_{S,t-1} \right] + \Omega_{S,t} V_t + Z_{S,t} &= \frac{1+r_{t-1}}{1+\pi_{C,t}} B_{S,t-1} + \Omega_{S,t-1} (V_t + D_t) + \frac{W_t}{P_{C,t}} N_{S,t}, \\ C_{H,t} + Q_t \left[X_{H,t} - (1-\delta) \tilde{X}_{H,t-1} \right] + Z_{H,t} &= \frac{1+r_{t-1}}{1+\pi_{C,t}} B_{H,t-1} + \frac{W_t}{P_{C,t}} N_{H,t} + T_t^H, \\ \lambda_S \tilde{X}_{S,t} &= \lambda_S \varrho_{SS} X_{S,t} + \lambda_H \varrho_{HS} X_{H,t}, \\ \lambda_H \tilde{X}_{H,t} &= \lambda_S \varrho_{SH} X_{S,t} + \lambda_H \varrho_{HH} X_{H,t}, \\ \lambda_S B_{S,t} &= \lambda_S \varrho_{SS} Z_{S,t} + \lambda_H \varrho_{HS} Z_{H,t}, \\ \lambda_H B_{H,t} &= \lambda_S \varrho_{SH} Z_{S,t} + \lambda_H \varrho_{HH} Z_{H,t}, \end{aligned}$$

where, for $z \in \{H, S\}$, $\tilde{X}_{z,t}$ denotes the beginning-of-period- $t+1$ stock of durables, while $Z_{z,t}$ ($B_{z,t}$) denotes the end-of-period- t (beginning-of-period- $t+1$) stock of bonds. The zero-liquidity limit à la Krusell et al. (2011) implies that, even though S 's demand for bonds is well-defined, the supply is zero, so there are no bonds held in equilibrium. Under this assumption, the only equilibrium condition from this part of the model is the Euler equation for S 's bonds:

$$C_{S,t}^{-\sigma_S} = \beta E_t \left\{ \frac{1+r_t}{1+\pi_{C,t+1}} \left[\varrho_{SS} C_{S,t+1}^{-\sigma_S} + \varrho_{SH} C_{H,t+1}^{-\sigma_H} \right] \right\}. \quad (15)$$

As for H , being her constraint binding, the zero-liquidity assumption implies HtM behavior. Furthermore, the 2-state HANK model differs from its TANK counterpart with respect to the following durable Euler equations:

$$Q_t C_{S,t}^{-\sigma_S} = \eta_S X_{S,t}^{-\chi_S} + \beta(1-\delta) E_t \left\{ \varrho_{SS} Q_{t+1} C_{S,t+1}^{-\sigma_S} + \varrho_{SH} Q_{t+1} C_{H,t+1}^{-\sigma_H} \right\}, \quad (16)$$

$$Q_t C_{H,t}^{-\sigma_H} = \eta_H X_{H,t}^{-\chi_H} + \beta(1 - \delta) E_t \{ \varrho_{HH} Q_{t+1} C_{H,t+1}^{-\sigma_H} + \varrho_{HS} Q_{t+1} C_{S,t+1}^{-\sigma_S} \}. \quad (17)$$

Slow depreciation of durables, in conjunction with the states being persistent enough, implies that the right side of both equations is approximately constant, so that the relative price of durables is forced to mirror households' shadow value of income.¹²

It is then reasonable to treat the shadow value of as roughly constant in the face of a monetary disturbance, if durables are sufficiently long-lived; i.e., $\mathbf{Y}_t \approx (\mathbf{I} - \mathbf{A})^{-1} \mathbf{B}\mathbf{X}$. This implies a relationship analogous to (10) which, combined with market clearing, returns $c_t = [1 - \lambda_H (1 - \gamma)] c_{S,t}$, once more. Combining the latter with the log-linearized counterpart of the self-insurance equation (15),

$$c_{S,t} = \mu E_t c_{S,t+1} - \sigma_S^{-1} (r_t - E_t \pi_{t+1}), \text{ where } \mu \equiv \varrho_{SS} + \gamma \varrho_{SH}, \quad (18)$$

and with $c_t = y_{C,t}$, we obtain

$$y_{C,t} = \mu E_t y_{C,t+1} - \chi^{-1} (r_t - E_t \pi_{t+1}). \quad (19)$$

With idiosyncratic uncertainty (i.e., $\varrho_{SS} < 1$), the Euler for nondurables features discounting/compounding of news about future nondurable expenditure—as captured by the factor loading μ —depending on $\gamma \leq 1$.¹³ In either of the two cases, even if they acknowledge that in some state of the world they might find themselves financially constrained, households can still exploit durable goods as a store of value so that the marginal utility from nondurable consumption is equalized across states. As a consequence, different forms of household-specific consumption behave exactly in the same way they do in the TANK economy (cfr. Table 1), so that fiscal redistribution and labor market characteristics operate along the same direction across different scenarios, and do not interact with idiosyncratic risk. In light of this, also the 2-state HANK economy features a neutrality of

¹²To see that, it is convenient to express the two durable Euler equations as $\mathbf{Y}_t = \mathbf{A} E_t \mathbf{Y}_{t+1} + \mathbf{B}\mathbf{X}_t$, with $\mathbf{Y}_t = [Q_t C_{S,t}^{-\sigma_S}, Q_t C_{H,t}^{-\sigma_H}]'$ and $\mathbf{X}_t = [X_{S,t}^{-\chi_S}, X_{H,t}^{-\chi_H}]'$. The two eigenvalues of \mathbf{A} , $\beta(1 - \delta)$ and $\beta(1 - \delta)(\varrho_{HH} + \varrho_{SS} - 1)$, are both within the unit circle, so that $\mathbf{Y}_t = \sum_{i=0}^{\infty} \mathbf{A}^i \mathbf{B} E_t \mathbf{X}_{t+i}$ by forward iteration. As in the TANK economy, slow depreciation implies a high stock-flow ratio, so that even relatively large changes in the production of the durable over a moderate horizon have small effects on the stock. Moreover, Appendix B shows that, conditional on states being persistent enough, quasi constancy applies to the shadow value of durables in both states.

¹³Assuming homogeneous preferences, instead, implies that the Euler corresponds to that obtained in a RANK economy with no heterogeneity.

fiscal transfers with respect to both types of *sectoral* production.

Thus, we focus on the role of the compounding/discounting factor μ in affecting the elasticity of sectoral production to the monetary shock. In this respect, Proposition shows that μ amplifies the response of both $y_{C,t}$ and $y_{X,t}$, in either direction, when assuming asymmetric sectoral price rigidity. By contrast, it plays no role under symmetric sectoral price stickiness, in which case the model is isomorphic to the TANK economy and, thus—by virtue of Proposition 2—to the RANK benchmark, under homogeneous preferences.

Proposition 3 *In the 2-state HANK economy where $\theta_j = 0$ and $\theta_i > 0$, with $j, i = \{C, X\}$ and $j \neq i$, sectoral production is given by*

$$y_{C,t} = -\frac{1-\beta\rho_\nu}{(1-\beta\rho_\nu)(1-\rho_\nu\mu)+\psi_C(\phi_\pi-\rho_\nu)}\chi^{-1}\nu_t,$$

$$y_{X,t} = \frac{Y_C}{Y_X}\frac{1-\beta\rho_\nu}{(1-\beta\rho_\nu)(1-\rho_\nu\mu)+\psi_C(\phi_\pi-\rho_\nu)}\chi^{-1}\nu_t, \text{ when } \theta_X = 0 \text{ and } \theta_C > 0,$$

and

$$y_{C,t} = -\frac{1-\beta\rho_\nu}{(1-\beta\rho_\nu)(1-\rho_\nu\mu)-\phi_\pi\psi_X}\chi^{-1}\nu_t,$$

$$y_{X,t} = \frac{Y_C\zeta+\chi Y}{Y_X}\frac{1-\beta\rho_\nu}{(1-\beta\rho_\nu)(1-\rho_\nu\mu)-\phi_\pi\psi_X}\chi^{-1}\nu_t, \text{ when } \theta_X > 0 \text{ and } \theta_C = 0.$$

Notably, when $\gamma < 1$, the impact of monetary policy shocks on either form of sectoral consumption is attenuated, both with respect to the direct effect of the real rate of interest on $y_{C,t}$, and through discounting of future news about nondurable spending. This is a manifestation of the *self-insurance channel* in this economy, though the way this operates and interacts with the HtM channel is, again, different from Bilbiie (2020), due to the presence of durables. When good news about future aggregate nondurable production arrive, households recognize they will be constrained in the access to financial assets—in some state of the world—while displaying lower intertemporal substitution in nondurable consumption. In light of this, even being able to purchase durables and, through these smoothing nondurable purchases, households recognize they will not be able to make the most of the increase in $E_t y_{C,t+1}$.

As explained by Bilbiie (2020), the interaction between aggregate and idiosyncratic uncertainty represents the motive to self-insure, and more so as ϱ_{ss} drops, so that the HtM spell, as captured by λ_H , lasts longer. Unlike Bilbiie (2020), though, self-insurance operates with respect to nondurable consumption, *de facto*, only to the extent households display preference heterogeneity. The next section is partly aimed at addressing this short-

coming.

5 A 3-state HANK economy

We have seen how, in its 2-state version, the HANK model with long-lived durables impairs the propagation stemming from the interaction between idiosyncratic uncertainty and HtM behavior, which is typically regarded as a key driver of aggregate *nondurable* consumption. For this reason, we now devise a 3-state HANK economy where preference heterogeneity is switched off, and where household members can find themselves in any of the following three states: a state in which they can smooth consumption both over durables and nondurables, one with access to durable purchases but no financial assets—in which case they are considered *wealthy HtM*—and one without access to either durables or financial assets—in which case they are considered *pure HtM*.

We envisage a third island that we label K . Upon learning that they will move to this island, households drop their stock of durables, which are redistributed to S and H . The exogenous change of state follows a Markov chain: the probability to stay type S is p_{SS} , to stay type H is p_{HH} , and to stay type K is p_{KK} (with transition probabilities p_{fl} with $f, l = \{K, S, H\}$ and $f \neq l$). Even in this case, we focus on stationary equilibria.¹⁴ The head of family's optimization problem now reads as

$$\begin{aligned} \max_{C_{S,t}, C_{H,t}, C_{K,t}, X_{S,t}, X_{H,t}, N_{S,t}, N_{H,t}, N_{K,t}, \Omega_{S,t}, Z_{S,t}, Z_{H,t}} E_t \left\{ \sum_{i=0}^{\infty} \beta^i \left[\lambda_S \left(\frac{C_{S,t+i}^{1-\sigma_S}}{1-\sigma_S} + \eta_S \frac{X_{S,t+i}^{1-\chi_S}}{1-\chi_S} - \varpi_S \frac{N_{S,t+i}^{1+\phi_S}}{1+\phi_S} \right) \right. \right. \\ \left. \left. + \lambda_H \left(\frac{C_{H,t+i}^{1-\sigma_H}}{1-\sigma_H} + \eta_H \frac{X_{H,t+i}^{1-\chi_H}}{1-\chi_H} - \varpi_H \frac{N_{H,t+i}^{1+\phi_H}}{1+\phi_H} \right) \right. \right. \\ \left. \left. + \lambda_K \left(\frac{C_{K,t+i}^{1-\sigma_K}}{1-\sigma_K} - \varpi_K \frac{N_{K,t+i}^{1+\phi_K}}{1+\phi_K} \right) \right] \right\} \\ \text{s.t.} \end{aligned}$$

¹⁴The transition matrix is set so that the Markov chain is ergodic. The steady-state solution of the transition probabilities is reported in Appendix C.1.

$$\begin{aligned}
C_{S,t} + Q_t \left[X_{S,t} - (1 - \delta) \tilde{X}_{S,t-1} \right] + \Omega_{S,t} V_t + Z_{S,t} &= \frac{1 + r_{t-1}}{1 + \pi_{C,t}} B_{S,t-1} + \Omega_{S,t-1} (V_t + D_t) + \frac{W_t}{P_{C,t}} N_{S,t}, \\
C_{H,t} + Q_t \left[X_{H,t} - (1 - \delta) \tilde{X}_{H,t-1} \right] + Z_{H,t} &= \frac{1 + r_{t-1}}{1 + \pi_{C,t}} B_{H,t-1} + \frac{W_t}{P_{C,t}} N_{H,t} + T_t^H, \\
C_{K,t} + Z_{K,t} &= \frac{1 + r_{t-1}}{1 + \pi_{C,t}} B_{K,t-1} + \frac{W_t}{P_{C,t}} N_{K,t} + T_t^K \\
\lambda_S \tilde{X}_{S,t} &= (\varrho_{SS} \lambda_S + \varrho_{SK} \lambda_K) X_{S,t} + \varrho_{HS} \lambda_H X_{H,t}, \\
\lambda_H \tilde{X}_{H,t} &= \varrho_{SH} \lambda_S X_{S,t} + (\varrho_{HH} \lambda_H + \varrho_{HK} \lambda_K) X_{H,t}, \\
\lambda_S B_{S,t} &= \varrho_{SS} \lambda_S Z_{S,t} + \varrho_{HS} \lambda_H Z_{H,t} + \varrho_{KS} \lambda_K Z_{K,t}, \\
\lambda_H B_{H,t} &= \varrho_{SH} \lambda_S Z_{S,t} + \varrho_{HH} \lambda_H Z_{H,t} + \varrho_{KH} \lambda_K Z_{K,t}, \\
\lambda_K B_{K,t} &= \varrho_{SK} \lambda_S Z_{S,t} + \varrho_{HK} \lambda_H Z_{H,t} + \varrho_{KK} \lambda_K Z_{K,t}.
\end{aligned}$$

Under the assumptions above, the only equilibrium condition for the bond market is the Euler equation of S :

$$C_{S,t}^{-\sigma_S} = \beta E_t \left\{ \frac{1 + r_t}{1 + \pi_{C,t+1}} \left[\varrho_{SS} C_{S,t+1}^{-\sigma_S} + \varrho_{SH} C_{H,t+1}^{-\sigma_H} + \varrho_{SK} C_{K,t+1}^{-\sigma_K} \right] \right\}. \quad (20)$$

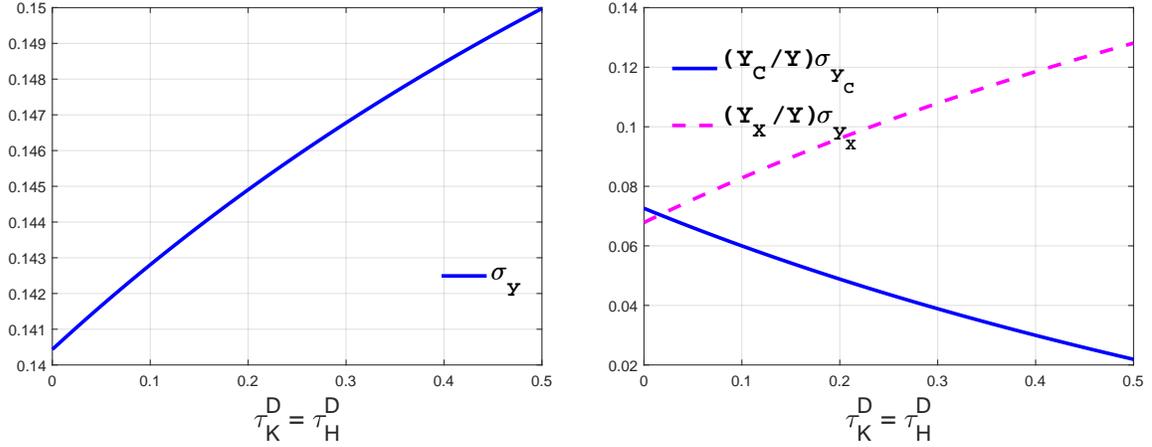
As for the demand of durables, this is determined by the combination of the following Euler equations for S and H :

$$Q_t C_{S,t}^{-\sigma_S} = \eta_S X_{S,t}^{-\chi_S} + \beta(1 - \delta) E_t \left\{ \frac{\varrho_{SS} \lambda_S + \varrho_{SK} \lambda_K}{\lambda_S} Q_{t+1} C_{S,t+1}^{-\sigma_S} + \varrho_{SH} Q_{t+1} C_{H,t+1}^{-\sigma_H} \right\}, \quad (21)$$

$$Q_t C_{H,t}^{-\sigma_H} = \eta_H X_{H,t}^{-\chi_H} + \beta(1 - \delta) E_t \left\{ \frac{\varrho_{HH} \lambda_H + \varrho_{HK} \lambda_K}{\lambda_H} Q_{t+1} C_{H,t+1}^{-\sigma_H} + \varrho_{HS} Q_{t+1} C_{S,t+1}^{-\sigma_S} \right\}. \quad (22)$$

Having examined the distinctive impact of durability in a two-agent economy with S and H households, we will now explore the role of K in determining sectoral and aggregate amplification of monetary shocks in the 3-state economy. In doing so, we abstract from preference heterogeneity, while retaining potential asymmetry in sectoral price stickiness and size. The complete log-linear economy and the analytical derivations are reported in Appendix C.2.

Figure 1: Volatility and fiscal redistribution



Notes: Symmetric transition probabilities, $\sigma = 1$, $\phi = 1$, $Y_C = \alpha = 0.75$, $Y_X = 1 - Y_C$, $\beta = 0.97$, $\theta_X = \theta_C = 0.6$, $\delta = 0.025$, $\phi_\pi = 1.5$.

5.1 Symmetric price stickiness

We primarily focus on our benchmark setting featuring symmetric price stickiness. Figure 1 reports the conditional volatility of GDP, along with that of nondurable and durable production (weighed by their relative sectoral size), as well as their correlation, all as functions of τ_K^D , which is the subsidy rate applied to K (assumed to be equal to that applied to H , τ_H^D).¹⁵ A first element to highlight is that fiscal transfers are no longer purely redistributive. In fact, aggregate volatility increases in τ_K^D , displaying a pattern that is broadly in line with the conditional volatility of durable production. This squares with the common view that durables dictate the aggregate behavior of two-sector RANK economies, regardless of the pricing and demand structure of nondurables (Barsky et al., 2007). This is still the case in our HANK environment. Let us see why.

A good starting point to provide some intuition to the driving forces behind this fact is to inspect the equilibrium sectoral levels of production, whose derivation is reported in Appendix C.3:

¹⁵The numerical experiments in this section do not rely on the approximation embodying quasi constancy of the shadow value of durables. Doing so, however, produces virtually unchanged results, thus confirming the soundness of our analytics.

$$y_{C,t} = \frac{Y\phi}{Y_C\phi + Y\sigma} \left(\frac{(1+\phi)\lambda_K}{\phi} - \tau_K^D \right) \omega_t, \quad (23)$$

$$y_{X,t} = \frac{Y}{(1-\lambda_K)Y_X} \left(\frac{\lambda_S + \lambda_H}{\phi} + \tau_K^D \right) \omega_t, \quad (24)$$

where $\omega_t = \frac{1}{\psi(\rho_\nu - \phi_\pi)} \varepsilon_t^\nu$. Looking at $y_{C,t}$, it is immediate to infer that its conditional volatility drops with τ_K^D over the support.¹⁶ On the other hand, the conditional volatility of $y_{X,t}$ invariantly increases in τ_K^D . Thus, aggregating the solutions for $y_{C,t}$ and $y_{X,t}$, and being $\frac{1}{1-\lambda_K} > \frac{Y_C\phi}{Y_C\phi + Y\sigma}$ always verified—by virtue of the element on the left (right) side of the inequality being greater (lower) than one—allows us to show that the conditional volatility of GDP is amplified by fiscal transfers, as stated by Proposition 4.

Proposition 4 *In the 3-state HANK economy with symmetric price stickiness, the response of y_t to monetary disturbances increases in fiscal redistribution, whenever $\lambda_K > 0$ and $\phi < \infty$.*

Thus, unlike one-sector models featuring nondurables only, fiscal redistribution amplifies the conditional volatility of GDP, the presence of long-lived durables being crucial to this property.¹⁷ To provide further intuition to this result, assume an unexpected monetary expansion inducing the real wage (which is common across sectors) to increase. The attenuation of volatility in nondurable production is to be ascribed to K (whom, in the present setting, is the only household adjusting nondurable purchases). With $\tau_K^D > 0$, these agents internalize the downward pressure of wages on firm dividends, which becomes increasingly intense as τ_K^D rises. The potential inversion in σ_{y_C} then stems from the fact that, as this wealth effect becomes conspicuous at relatively high degrees of fiscal redistribution—and this is more easily accomplished when labor supply is relatively inelastic—a given increase in the real wage compresses nondurable expenditure. As for

¹⁶In fact, conditional volatility should drop up until τ_K^D is lower than $\frac{(1+\phi)\lambda_K}{\phi}$; this cannot be appreciated in the graph, though, given the selected values for λ_K and ϕ .

¹⁷One might suggest that, while not altering the emergence of risk sharing among agents buying durables, envisaging nominal wage rigidity would affect the cyclicity of sectoral profits (for a HANK example, see Broer et al., 2019) and, thus, the specific way fiscal redistribution shapes the conditional volatility of sectoral and aggregate production. Even in this case, though, we should stress that introducing durables—no matter how large a share of the economy they represent and how “sticky” their prices are—profoundly changes the properties of an otherwise standard one-sector economy with nominal wage stickiness. Analogous considerations apply to economies with a different asset structure (see, e.g., Ravn and Sterk, 2021).

the conditional volatility of durables, instead, increasing τ_K^D allows S to translate on K the negative income effect from firm profits, thus exploiting more resources available for her own consumption (who accounts, together with H , for the entire demand in that sector; their nondurable consumption remaining at the steady state, instead).¹⁸ Thus, increasing fiscal transfers to K magnifies the passthrough of shocks to the real wage.

Being durable production the dominant source of GDP volatility, and being its own conditional volatility increasingly sensitive to fiscal redistribution, the overall impact of τ_K^D on σ_Y is necessarily expansionary. Notably, this tendency may disappear as $\lambda_K \rightarrow 0$ and $\phi \rightarrow \infty$: in this limit situation, pure HtM households—who represent a prerequisite for the non-neutrality result—vanish, and labor supply becomes increasingly inelastic, thus limiting the income externality on S and—by virtue of the risk-sharing condition linking to her—on H . Thus, a key issue in considering a one-sector economy producing only nondurables is that fiscal redistribution interacts with monetary policy so as to smooth its effects, while the reverse implication emerges when contemplating consumer durables, no matter how large their sector of production is. After examining amplification/attenuation in the economies with extremely asymmetric price stickiness—where one sector at the time features purely flexible prices—the next subsection shows how the same tendency characterizes economies with *any* degree of asymmetric sectoral price stickiness.

5.2 Asymmetric price stickiness

We now examine comparative statics in the economies where sectoral price stickiness is fully asymmetric. Much like the analysis of Bilbiie (2020), it is possible to characterize the elasticity of pure HtM households' nondurable consumption to aggregate nondurable consumption, whenever price stickiness is asymmetric between sectors:

$$c_{K,t} = \frac{\mu_K}{\lambda_K} y_{C,t}, \tag{25}$$

¹⁸Varying τ_H^D is neutral to the allocation of durables between H and S , instead, just as in the TANK or the 2-state HANK economies.

where the derivation of μ_K is performed in Appendix C.4. Thus, starting from S 's Euler for nondurables (which is the only one holding in equilibrium), we may characterize the behavior of aggregate nondurable consumption:¹⁹

$$y_{C,t} = \frac{\lambda_K (\varrho_{SS} + \varrho_{SH}) (1 - \mu_K) + \varrho_{SK} \mu_K (\lambda_H + \lambda_S)}{(1 - \mu_K) \lambda_K} E_t y_{C,t+1} - \frac{1 - \lambda_K}{\sigma (1 - \mu_K)} (r_t - E_t \pi_{C,t+1}). \quad (26)$$

Notably, $\mu_K < (>) \lambda_K$ ensures discounting (compounding) of news about the future while attenuating (amplifying) the elasticity of $y_{C,t}$ to the real interest rate.²⁰ Therefore, the (intra-temporal) HtM channel is *complemented* by the (inter-temporal) self-insurance channel: bad (good) news about future nondurable production reduce (boost) today's demand for nondurables, implying less (more) need for self-insurance against the K state. Thus, given that $y_{X,t}$ and $y_{C,t}$ display close-to-perfect negative correlation when either sector features purely flexible prices, the volatilities of the two sectoral productions are also characterized by the same determinants. In light of this, we can simply focus on the behavior of μ_K .

Figure 3 in Appendix C.4 reports μ_K as a function of τ_K^D and ϕ . Starting from the scenario featuring flexible prices of durable goods, $c_{K,t}$ reacts more than one-to-one to changes in nondurable production under relatively low fiscal redistribution (and large λ_K).²¹ Assume a monetary tightening, which causes a contraction in $y_{C,t}$ and ω_t , with S and H substituting nondurables for durables, given that the latter become relatively cheaper. Focusing on K , recall that, under $\theta_X = 0$ and $\theta_C > 0$, $d_{X,t} = 0$, so that her income equals $\frac{Y}{Y_C} (n_{K,t} + \omega_t) + \frac{\tau_K^D}{\lambda_K} d_{C,t}$. Notice that raising ϕ attenuates the increase in $n_{K,t}$, thus acting as a further drag on K 's labor income. At the same time, as $d_{C,t} = -\omega_t$, dividends accruing from the nondurables sector necessarily expand—attenuating the impact of the contractionary monetary stance on $c_{K,t}$ —and more so as τ_K^D increases and/or λ_K drops, all else equal, as in this case pure HtM consumers progressively internalize the positive effect from fiscal redistribution. This effect counteracts the negative impulse on aggregate

¹⁹Except, again, for the case in which $\theta_X = \theta_C$.

²⁰According to the same conditions, procyclical (countercyclical) nondurable consumption inequality emerges.

²¹In fact, in Appendix C.4 we show how to retrieve the multiplier in Bilbiie (2020), when abstracting from durability.

nondurable consumption.

Turning to the scenario with nondurable goods featuring flexible prices, $\mu_K > \lambda_K$ tends to hold more easily under relatively large fiscal redistribution and/or under a relatively small λ_K . As nondurable goods become relatively cheaper, a monetary tightening now induces S and H to substitute durables for nondurables. Recall also that K 's income equals $\frac{Y}{Y_C} n_{K,t} + \frac{\tau_K^D}{\lambda_K} \frac{Y_X}{Y_C} d_{X,t}$. As $d_{X,t} = -(w_t - p_{X,t}) = q_t$, dividends from the durable sector expand, thus supporting K 's purchase of nondurables. Thus, increasing τ_K^D and/or reducing λ_K enhances such expansion. As for ϕ , instead, raising it amounts to limit the drop in K 's labor supply, rendering it increasingly inelastic and attenuating the drag on $c_{K,t}$.

On the connection between fiscal transfers and aggregate volatility Figure 2 shows that aggregate conditional volatility increases in the fiscal transfer, *even* in an economy with mild asymmetry in sectoral price stickiness, and where durables have relatively more flexible prices (see Panel (a), where we impose $\theta_X = 0.4$ and $\theta_C = 0.6$).²² To provide some analytical intuition, equations (27) and (28) express $y_{C,t}$ and $y_{X,t}$ as functions of the real wage (in units of nondurables) and the relative price of durables. These equations respectively generalize (23) and (24), in that relative-price changes shape both $y_{C,t}$ and $y_{X,t}$:

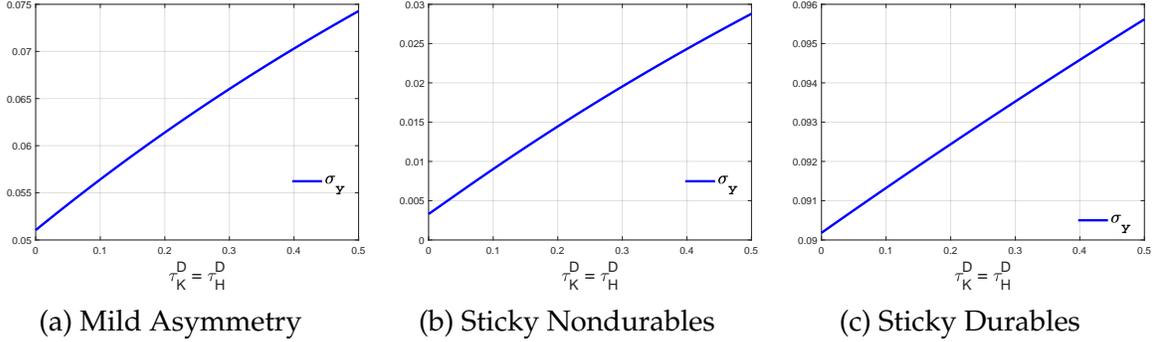
$$y_{C,t} = \frac{\phi Y}{\phi Y_C + \sigma Y} \left(\frac{\lambda_K (1 + \phi)}{\phi} - \tau_K^D \right) \omega_t + \left(\frac{\phi Y_X}{\phi Y_C + \sigma Y} \tau_K^D + \frac{\lambda_H + \lambda_S}{\sigma} \right) q_t, \quad (27)$$

$$y_{X,t} = \frac{Y}{Y_X (1 - \lambda_K)} \left(\frac{\lambda_S + \lambda_H}{\phi} + \tau_K^D \right) \omega_t - \left(\frac{\tau_K^D}{1 - \lambda_K} + \frac{(\phi Y_C + \sigma Y) (1 - \lambda_K) - \phi \sigma \lambda_K Y_X}{Y_X \phi \sigma (1 - \lambda_K)} \right) q_t. \quad (28)$$

As a result of a monetary expansion, both real wages and the relative price increase, inducing potentially contrasting effects on sectoral production. However, despite both ω_t and q_t are now endogenous to fiscal transfers (unlike the case of symmetric price sticki-

²²We take this setting as reflecting the main standpoint when it comes to calibrating multi-sector economies with nominal price rigidity and involving the durable-nondurable dichotomy. See, e.g., Bils and Klenow (2004) and Nakamura and Steinsson (2008). In fact, as we will show in the remainder of this section, our analysis retains a fair degree of generality, with respect to alternative calibrations of sectoral price rigidity.

Figure 2: Volatility and fiscal redistribution: asymmetric stickiness



Notes. Panel (a): $\theta_X = 0.4$, $\theta_C = 0.6$; Panel (b): $\theta_X = 0$, $\theta_C = 0.6$; Panel (c): $\theta_X = 0.6$, $\theta_C = 0$. All other parameters: $\sigma = 1$, $\phi = 1$, $Y_C = \alpha = 0.75$, $Y_X = 1 - Y_C$, $\beta = 0.97$, $\delta = 0.025$, $\phi_\pi = 1.5$, symmetric transition probabilities.

ness), focusing on the factor loadings that involve τ_K^D helps us interpreting how fiscal redistribution may exert a first-order impact on sectoral volatility. Considering nondurable production first, we can see how increasing the fiscal transfer necessarily attenuates the impact of ω_t , while amplifying that of q_t : thus, a decline in σ_{y_C} must imply that the first effect dominates. This can also be inferred by reshuffling (27) as

$$y_{C,t} = \frac{Y\lambda_K(1+\phi)}{\phi Y_C + \sigma Y} \omega_t + \frac{\lambda_H + \lambda_S}{\sigma} q_t - \frac{\phi Y \tau_K^D}{\phi Y_C + \sigma Y} rmc_t, \quad (29)$$

from which it can be seen that increasing fiscal transfers attenuates the overall response through the negative factor loading attached to the average real marginal cost in the economy ($rmc_t = \frac{Y_C}{Y} \omega_t + \frac{Y_X}{Y} (w_t - p_{X,t})$). As for $y_{X,t}$, instead, increasing τ_K^D attenuates the expansionary impact of the relative price, while amplifying that of the real wage. For σ_{y_X} to decrease over most of the support of the transfer, it must be the case that the relative-price effect broadly dominates.

Turning our focus on aggregate volatility, we can express real GDP as the sum of two components, the second of which is represented by the average real marginal cost in the economy:

$$y_t = \mathfrak{F}(\omega_t, q_t) + \tau_K^D \left(\frac{1}{1 - \lambda_K} - \frac{\phi Y_C}{\phi Y_C + \sigma Y} \right) rmc_t, \quad (30)$$

where $\mathfrak{F}(\omega_t, q_t)$ is a function of the real wage in units of nondurables and the relative price,

and where factor loadings do not feature fiscal transfers. Focusing on the factor loading applying to the average real marginal cost, instead, allows us to infer that fiscal transfers amplify the overall monetary response. This is not just true regardless of how large the durable production sector is, but also of how “sticky” it is. In fact, as $\frac{1}{1-\lambda_K} > \frac{\phi_{Y_C}}{\phi_{Y_C} + \sigma_Y}$ is always verified (unless, again, limit situations are considered, in light of Proposition 4), fiscal redistribution amplifies a given change in the average real marginal cost.

6 Concluding remarks

Durables are key to the transmission of monetary policy. Not just because they are more interest-rate sensitive than nondurables—but also because they represent a store of value through which households may shape their nondurable consumption profile, even when they have no access to financial markets. We highlight this property within modular two-sector New Keynesian economies where part of the households are financially constrained, but might still be able to buy durables subject to slow depreciation, along with nondurables. The amplification/attenuation of household-specific and nondurable consumption in TANK and HANK economies where all households can buy durables only hinges on preference heterogeneity, whereas durable consumption at the household level also depends on other structural determinants, primarily the degree of fiscal redistribution from financially unconstrained to constrained households. Due to the specific goods-demand structure of the economy—with both households buying durables and nondurables—fiscal redistribution is neutral to either type of demand at the sectoral level. However, when contemplating the presence of households that are constrained in the access of both financial assets and durable purchases, such neutrality is broken, and we highlight that GDP (conditional) volatility increases in fiscal transfers, unlike one-sector TANK or HANK economies featuring nondurables only. Such prediction ultimately depends on how fiscal redistribution shapes the response of durables to monetary innovations. These results call for further research on monetary policy’s direct and indirect transmission in multi-sector settings with heterogeneous agents.

References

- Acharya, S. and Dogra, K. (2020). Understanding HANK: Insights From a PRANK. *Econometrica*, 88(3):1113–1158.
- Attanasio, O., Larkin, K. P., Ravn, M. O., and Padula, M. (2020). (S)Cars and the Great Recession. Nber working papers, National Bureau of Economic Research, Inc.
- Attanasio, O. P. (1999). Consumption. In Taylor, J. B. and Woodford, M., editors, *Handbook of Macroeconomics*, volume 1 of *Handbook of Macroeconomics*, chapter 11, pages 741–812. Elsevier.
- Barsky, R. B., House, C. L., and Kimball, M. S. (2007). Sticky-Price Models and Durable Goods. *American Economic Review*, 97(3):984–998.
- Bilbiie, F., Primiceri, G. E., and Tambalotti, A. (2022). Inequality and Business Cycles. Technical report, mimeo.
- Bilbiie, F. O. (2008). Limited asset markets participation, monetary policy and (inverted) aggregate demand logic. *Journal of Economic Theory*, 140(1):162–196.
- Bilbiie, F. O. (2018). Monetary policy and heterogeneity: An analytical framework. *CEPR Discussion Papers*, DP12601.
- Bilbiie, F. O. (2020). The new keynesian cross. *Journal of Monetary Economics*, 114:90–108.
- Bils, M. and Klenow, P. J. (2004). Some Evidence on the Importance of Sticky Prices. *Journal of Political Economy*, 112(5):947–985.
- Broer, T., Hansen, N.-J. H., Krussel, P., and Oberg, E. (2019). The new keynesian transmission mechanism: A heterogeneous-agent perspective. *Review of Economic Studies*, 0:1–25.
- Browning, M. and Crossley, T. F. (2000). Luxuries Are Easier to Postpone: A Proof. *Journal of Political Economy*, 108(5):1022–1026.
- Browning, M. and Crossley, T. F. (2009). Shocks, Stocks, and Socks: Smoothing Consumption Over a Temporary Income Loss. *Journal of the European Economic Association*, 7(6):1169–1192.

- Calvo, G. A. (1983). Staggered prices in a utility-maximizing framework. *Journal of Monetary Economics*, 12(3):383–398.
- Campbell, J. Y. and Mankiw, N. G. (1989). Consumption, income, and interest rates: Reinterpreting the time series evidence. *NBER Macroeconomics Annual*, 4:185–216.
- Challe, E. (2020). Uninsured Unemployment Risk and Optimal Monetary Policy in a Zero-Liquidity Economy. *American Economic Journal: Macroeconomics*, 12(2):241–283.
- Coibion, O., Gorodnichenko, Y., Kueng, L., and Silvia, J. (2017). Innocent bystanders? monetary policy and inequality. *Journal of Monetary Economics*, 88:70–89.
- Erceg, C. and Levin, A. (2006). Optimal monetary policy with durable consumption goods. *Journal of Monetary Economics*, 53(7):1341–1359.
- Galí, J., López-Salido, J. D., and Vallés, J. (2007). Understanding the effects of government spending on consumption. *Journal of the European Economic Association*, 5(1):227–270.
- Hamermesh, D. S. (1982). Social Insurance and Consumption: An Empirical Inquiry. *American Economic Review*, 72(1):101–113.
- Holst Partsch, E., Petrella, I., and Santoro, E. (2022). Durable Goods and Monetary Transmission in a HANK Economy. mimeo, University of Copenhagen.
- Kaplan, G., Moll, B., and Violante, G. L. (2018). Monetary policy according to HANK. *American Economic Review*, 108(3):697–743.
- Kogan, L., Papanikolaou, D., and Stoffman, N. (2020). Left behind: Creative destruction, inequality, and the stock market. *Journal of Political Economy*, 128(3):855–906.
- Krusell, P., Mukoyama, T., and Smith Jr., A. A. (2011). Asset prices in a Huggett economy. *Journal of Economic Theory*, 146(3):812–844.
- Mankiw, N. G. (1985). Consumer Durables and the Real Interest Rate. *The Review of Economics and Statistics*, 67(3):353–362.
- Mankiw, N. G. and Zeldes, S. P. (1991). The consumption of stockholders and nonstockholders. *Journal of Financial Economics*, 29(1):97–112.
- McKay, A. and Wieland, J. F. (2022). Forward guidance and durable goods demand. *American Economic Review: Insights*, 4(1):106–22.

- Monacelli, T. (2009). New Keynesian models, durable goods, and collateral constraints. *Journal of Monetary Economics*, 56(2):242–254.
- Nakamura, E. and Steinsson, J. (2008). Five Facts about Prices: A Reevaluation of Menu Cost Models. *The Quarterly Journal of Economics*, 123(4):1415–1464.
- Parker, J. A. (1999). The Reaction of Household Consumption to Predictable Changes in Social Security Taxes. *American Economic Review*, 89(4):959–973.
- Petrella, I., Rossi, R., and Santoro, E. (2019). Monetary Policy with Sectoral Trade-Offs. *Scandinavian Journal of Economics*, 121(1):55–88.
- Ravn, M. O. and Sterk, V. (2021). Macroeconomic Fluctuations with HANK & SAM: an Analytical Approach. *Journal of the European Economic Association*, 19(2):1162–1202.
- Stock, J. H. and Watson, M. W. (1999). Business cycle fluctuations in us macroeconomic time series. In Taylor, J. B. and Woodford, M., editors, *Handbook of Macroeconomics*, volume 1 of *Handbook of Macroeconomics*, chapter 1, pages 3–64. Elsevier.
- Sudo, N. (2012). Sectoral Comovement, Monetary Policy Shocks, and Input-Output Structure. *Journal of Money, Credit and Banking*, 44(6):1225–1244.
- Tsai, Y.-C. (2016). What Do Working Capital And Habit Tell Us About The Co-Movement Problem? *Macroeconomic Dynamics*, 20(1):342–361.
- Werning, I. (2015). Incomplete Markets and Aggregate Demand. NBER Working Papers 21448, National Bureau of Economic Research, Inc.
- Yun, T. (1996). Nominal price rigidity, money supply endogeneity, and business cycles. *Journal of Monetary Economics*, 37(2-3):345–370.

Appendix

A TANK economy

Log-linear economy

The TANK economy can be summarized by the following log-linear relationships:

Savers:

$$\begin{aligned}c_{S,t} &= E_t c_{S,t+1} - \frac{1}{\sigma_S} (r_t - E_t \pi_{t+1}) \\q_t - \sigma_S c_{S,t} &= -[1 - \beta(1 - \delta)] \chi_S x_{S,t} + \beta(1 - \delta) (E_t q_{t+1} - \sigma_S E_t c_{S,t+1}) \\ \phi_S n_{S,t} &= \omega_t - \sigma_S c_{S,t} \\c_{S,t} + \frac{Y_X}{Y_C} e_{S,t} &= \frac{Y}{Y_C} (\omega_t + n_{S,t}) + \frac{1-\tau^D}{\lambda_S} d_{C,t} + \frac{1-\tau^D}{\lambda_S} \frac{Y_X}{Y_C} d_{X,t} \\e_{S,t} &= q_t + \frac{1}{\delta} x_{S,t} - \frac{1-\delta}{\delta} x_{S,t-1}\end{aligned}$$

Hand-to-mouth:

$$\begin{aligned}q_t - \sigma_H c_{H,t} &= -[1 - \beta(1 - \delta)] \chi_H x_{H,t} + \beta(1 - \delta) (E_t q_{t+1} - \sigma_H E_t c_{H,t+1}) \\ \phi_H n_{H,t} &= \omega_t - \sigma_H c_{H,t} \\c_{H,t} + \frac{Y_X}{Y_C} e_{H,t} &= \frac{Y}{Y_C} (\omega_t + n_{H,t}) + \frac{\tau^D}{\lambda_H} d_{C,t} + \frac{\tau^D}{\lambda_H} \frac{Y_X}{Y_C} d_{X,t} \\e_{H,t} &= q_t + \frac{1}{\delta} x_{H,t} - \frac{1-\delta}{\delta} x_{H,t-1}\end{aligned}$$

Production and pricing:

$$\begin{aligned}y_{j,t} &= n_{j,t}, j = \{C, X\} \\d_{j,t} &= -(w_t - p_{j,t}), j = \{C, X\} \\\pi_{j,t} &= \beta E_t \pi_{j,t+1} + \psi_j r m c_{j,t}, \psi_j \equiv (1 - \theta_j)(1 - \beta \theta_j) / \theta_j, j = \{C, X\} \\r m c_{j,t} &= w_t - p_{j,t}, j = \{C, X\} \\q_t &= q_{t-1} + \pi_{X,t} - \pi_{C,t}\end{aligned}$$

Market clearing:

$$\begin{aligned}n_t &= \frac{Y_X}{Y} n_{X,t} + \frac{Y_C}{Y} n_{C,t} = \lambda_H n_{H,t} + \lambda_S n_{S,t} \\y_{C,t} = c_t &= \lambda_H c_{H,t} + \lambda_S c_{S,t} \\y_{X,t} &= \frac{1}{\delta} x_t - \frac{1-\delta}{\delta} x_{t-1} \\x_t &= \lambda_H x_{H,t} + \lambda_S x_{S,t}\end{aligned}$$

Monetary Policy:

$$\begin{aligned}r_t &= \phi_\pi \pi_t + \nu_t \\\pi_t &= \alpha \pi_{C,t} + (1 - \alpha) \pi_{X,t} \\\nu_t &= \rho_\nu \nu_{t-1} + \varepsilon_t^\nu\end{aligned}$$

where ω_t denotes the real wage expressed in units of nondurables, in percentage deviation from its steady state, i.e. $\omega_t \equiv w_t - p_{C,t}$.

Benchmark economy under symmetric sectoral price stickiness

In this case:

$$q_t = y_{C,t} = 0. \quad (31)$$

Thus, by combining the S 's Euler for nondurables and the Taylor rule we obtain

$$\pi_t = \frac{1}{\phi_\pi} E_t \pi_{t+1} - \frac{1}{\phi_\pi} \nu_t. \quad (32)$$

So that, assuming $\phi_\pi > 1$ is sufficient to iterate the equation forward and pin down the rate of inflation:

$$\pi_t = -\frac{1}{\phi_\pi} E_t \sum_{s=0}^{\infty} \left(\frac{1}{\phi_\pi} \right)^s \nu_{t+s} = \frac{1}{\rho_\nu - \phi_\pi} \varepsilon_t^\nu. \quad (33)$$

As $\sigma_{SCS,t} = 0$, labor supply implies

$$\phi_S n_{S,t} = \omega_t. \quad (34)$$

Since $\phi_S n_{S,t} = \zeta n_t$, aggregate inflation is dictated by

$$\pi_t = \beta E_t \pi_{t+1} + \zeta \psi n_t. \quad (35)$$

In light of $E_t \pi_{t+1} = 0$, $n_t = \frac{1}{\zeta \psi} \frac{1}{\rho_\nu - \phi_\pi} \varepsilon_t^\nu$ and

$$y_{X,t} = \frac{Y}{Y_X} y_t = \frac{Y}{Y_X} \frac{1}{\zeta \psi} \frac{1}{\rho_\nu - \phi_\pi} \varepsilon_t^\nu. \quad (36)$$

Thus, to obtain household-specific durable consumption, we plug household-specific labor supply and equilibrium profits into the budget constraints.

Flexible prices of durables

From S 's labor supply:

$$\phi_S n_{S,t} = w_t - p_{X,t} + q_t - \sigma_{SCS,t}, \quad (37)$$

where $w_t - p_{X,t} = 0$ due to the assumption of flexible prices in the durables sector, and $q_t - \sigma_{SCS,t}$ is approximately null, due to durability. Analogous observations for H lead us to conclude that $n_{H,t} = n_{S,t} = n_t = y_t = 0$ and $y_{C,t} = -\frac{Y_X}{Y_C} y_{X,t}$, in line with Barsky et al. (2007). Therefore, the following autonomous system obtains under flexible prices in the durable sector:

$$y_{C,t} = E_t y_{C,t+1} - \chi^{-1} (r_t - E_t \pi_{C,t+1}), \quad (38)$$

$$\pi_{C,t} = \beta E_t \pi_{C,t+1} + \psi_C \chi y_{C,t}, \quad (39)$$

$$r_t = \phi_\pi \pi_{C,t} + \nu_t. \quad (40)$$

Conjecturing a solution of this type:

$$y_{C,t} = a_y \nu_t,$$

$$\pi_{C,t} = a_\pi \nu_t,$$

$$E_t y_{C,t+1} = a_y \rho_\nu \nu_t,$$

$$E_t \pi_{C,t+1} = a_\pi \rho_\nu \nu_t,$$

we obtain

$$a_y = -\frac{1 - \beta \rho_\nu}{\chi (1 - \beta \rho_\nu) (1 - \rho_\nu) + \psi_C \chi (\phi_\pi - \rho_\nu)},$$

$$a_\pi = -\frac{\psi_C}{(1 - \beta \rho_\nu) (1 - \rho_\nu) + \psi_C (\phi_\pi - \rho_\nu)},$$

so that

$$y_{C,t} = -\frac{1 - \beta\rho_\nu}{(1 - \beta\rho_\nu)(1 - \rho_\nu) + \psi_C(\phi_\pi - \rho_\nu)\chi} \frac{1}{\chi} \nu_t, \quad (41)$$

$$y_{X,t} = \frac{Y_C}{Y_X} \frac{1 - \beta\rho_\nu}{(1 - \beta\rho_\nu)(1 - \rho_\nu) + \psi_C(\phi_\pi - \rho_\nu)\chi} \frac{1}{\chi} \nu_t, \quad (42)$$

where $\sigma_H < \sigma_S$ implies higher reactivity of $y_{C,t}$ and $y_{X,t}$ in either direction (through the fact that χ is a negative function of $\sigma_S - \sigma_H$). As for agent-specific consumption, recall that $c_t = [1 - \lambda_H(1 - \gamma)]c_{S,t}$ and $c_t = \frac{1 - \lambda_H(1 - \gamma)}{\gamma}c_{H,t}$, implying:

$$c_{S,t} = -\frac{1 - \beta\rho_\nu}{(1 - \beta\rho_\nu)(1 - \rho_\nu) + \psi_C(\phi_\pi - \rho_\nu)\sigma_S} \frac{1}{\sigma_S} \nu_t, \quad (43)$$

$$c_{H,t} = -\frac{1 - \beta\rho_\nu}{(1 - \beta\rho_\nu)(1 - \rho_\nu) + \psi_C(\phi_\pi - \rho_\nu)\sigma_H} \frac{1}{\sigma_H} \nu_t, \quad (44)$$

so that the sign of the response follows from that of $y_{C,t}$. Finally, to obtain household-specific durable expenditure—which expressed in unit of nondurables is defined as $q_t + \frac{1}{\delta}x_{z,t} - \frac{1-\delta}{\delta}x_{z,t-1}$, with $z = \{S, H\}$ —we turn to the budget constraints. Thus, recall that $n_{H,t} = n_{S,t} = 0$, along with $d_{X,t} = -(w_t - p_{X,t}) = -\omega_t + q_t = 0$, and $\sigma_H c_{H,t} = q_t = \sigma_S c_{S,t}$, to obtain

$$e_{S,t} = \left[\left(\frac{Y}{Y_C} - \frac{1 - \tau^D}{1 - \lambda} \right) \sigma_S - 1 \right] \frac{Y_C}{Y_X} c_{S,t} \quad (45)$$

and, thus, $e_{H,t}$.

Flexible prices of nondurables

In this case, from S 's labor supply:

$$\phi_S n_{S,t} = \omega_t - \sigma_S c_{S,t}, \quad (46)$$

where $\omega_t = 0$ due to the assumption of flexible prices in the nondurables sector and $\sigma_S c_{S,t} = \chi y_{C,t}$. Thus, through $n_t = [1 - \lambda_H (1 - \vartheta)] n_{S,t}$ (where $\vartheta = \frac{\phi_S}{\phi_H}$) we obtain

$$n_t = y_t = -\frac{\chi}{\zeta} y_{C,t}, \quad (47)$$

where $\zeta = \phi_S [1 - \lambda_H (1 - \vartheta)]^{-1}$, so that

$$y_{C,t} = -\frac{Y_X}{Y_C \zeta + \chi Y} y_{X,t}. \quad (48)$$

Conjecturing

$$y_{C,t} = a_y \nu_t,$$

$$\pi_{X,t} = a_\pi \nu_t,$$

$$E_t y_{C,t+1} = a_y \rho_\nu \nu_t,$$

$$E_t \pi_{X,t+1} = a_\pi \rho_\nu \nu_t,$$

we obtain

$$a_y = -\chi^{-1} \frac{(1 - \beta \rho_\nu)}{(1 - \beta \rho_\nu)(1 - \rho_\nu) - \phi_\pi \psi_X},$$

$$a_\pi = \frac{\psi_X}{(1 - \beta \rho_\nu)(1 - \rho_\nu) - \phi_\pi \psi_X},$$

Thus

$$y_{C,t} = -\frac{(1 - \beta \rho_\nu)}{(1 - \beta \rho_\nu)(1 - \rho_\nu) - \phi_\pi \psi_X} \chi^{-1} \nu_t, \quad (49)$$

$$y_{X,t} = \frac{Y_C \zeta + \chi Y}{Y_X} \frac{(1 - \beta \rho_\nu)}{(1 - \beta \rho_\nu)(1 - \rho_\nu) - \phi_\pi \psi_X} \chi^{-1} \nu_t, \quad (50)$$

where the response of $y_{C,t}$ ($y_{X,t}$) to ν_t tends to be positive if the shock is persistent enough and where, again, $\sigma_H < \sigma_S$ implies higher reactivity of $y_{C,t}$ and $y_{X,t}$ in either direction (through the fact that χ is a negative function of $\sigma_S - \sigma_H$). As for agent specific consump-

tion:

$$c_{S,t} = -\frac{(1 - \beta\rho_\nu)}{(1 - \beta\rho_\nu)(1 - \rho_\nu) - \phi_\pi\psi_X\sigma_S} \frac{1}{\nu_t}, \quad (51)$$

$$c_{H,t} = -\frac{(1 - \beta\rho_\nu)}{(1 - \beta\rho_\nu)(1 - \rho_\nu) - \phi_\pi\psi_X\sigma_H} \frac{1}{\nu_t}, \quad (52)$$

so that the sign of the response follows from that of $y_{C,t}$. Finally, to obtain $e_{S,t}$ and $e_{H,t}$, we turn to the budget constraints, recalling that $d_{C,t} = -\omega_t = 0$, $n_{z,t} = -\frac{\sigma_z}{\phi_z}c_{z,t}$, $d_{X,t} = -(w_t - p_{X,t})$, and $\sigma_H c_{H,t} = q_t = \sigma_S c_{S,t}$:

$$e_{S,t} = \left[\sigma_S \left(\frac{1 - \tau^D}{1 - \lambda} \frac{Y_X}{Y_C} - \frac{1}{\phi_S} \frac{Y}{Y_C} \right) - 1 \right] \frac{Y_C}{Y_X} c_{S,t}, \quad (53)$$

$$e_{H,t} = \frac{1}{\lambda} y_{X,t} - \frac{1 - \lambda}{\lambda} e_{S,t}. \quad (54)$$

B Durables in the 2-state HANK economy

The 2-state HANK model differs from its TANK counterpart with respect to the following durable Euler equations:

$$Q_t C_{S,t}^{-\sigma_S} = \eta_S X_{S,t}^{-\chi_S} + \beta(1 - \delta) E_t \{ \varrho_{SS} Q_{t+1} C_{S,t+1}^{-\sigma_S} + \varrho_{SH} Q_{t+1} C_{H,t+1}^{-\sigma_H} \}, \quad (55)$$

$$Q_t C_{H,t}^{-\sigma_H} = \eta_H X_{H,t}^{-\chi_H} + \beta(1 - \delta) E_t \{ \varrho_{HH} Q_{t+1} C_{H,t+1}^{-\sigma_H} + \varrho_{HS} Q_{t+1} C_{S,t+1}^{-\sigma_S} \}. \quad (56)$$

We can take the two durable Euler equations and write them in compact form as

$$\mathbf{Y}_t = \mathbf{A} E_t \mathbf{Y}_{t+1} + \mathbf{B} \mathbf{X}_t, \quad (57)$$

with

$$\mathbf{Y}_t = \begin{bmatrix} Q_t C_{S,t}^{-\sigma_S} \\ Q_t C_{H,t}^{-\sigma_H} \end{bmatrix} \text{ and } \mathbf{X}_t = \begin{bmatrix} X_{S,t}^{-\chi_S} \\ X_{H,t}^{-\chi_H} \end{bmatrix}.$$

where

$$\mathbf{A} = \beta(1 - \delta) \begin{bmatrix} \varrho_{SS} & \varrho_{SH} \\ \varrho_{HS} & \varrho_{HH} \end{bmatrix},$$

$$\mathbf{B} = \begin{bmatrix} \eta_S & 0 \\ 0 & \eta_H \end{bmatrix}.$$

As the two eigenvalues of \mathbf{A} , $\beta(1 - \delta)$ and $\beta(1 - \delta)(\varrho_{HH} + \varrho_{SS} - 1)$, always lie within the unit circle, the system is stationary, and $\mathbf{Y}_t = \sum_{i=0}^{\infty} \mathbf{A}^i \mathbf{B} E_t \mathbf{X}_{t+i}$ by forward iteration. The associated eigenvectors are, instead:

$$\mathbf{E} = \begin{bmatrix} 1 & -\frac{\varrho_{SS}-1}{\varrho_{HH}-1} \\ 1 & 1 \end{bmatrix}.$$

Thus, we can rewrite \mathbf{A} as $\mathbf{E} \mathbf{V} \mathbf{E}^{-1}$, where $\mathbf{V} = \beta(1 - \delta) \text{diag}([1, (\varrho_{HH} + \varrho_{SS} - 1)])$. In light of this, $\mathbf{Y}_t = \sum_{i=0}^{\infty} \mathbf{A}^i \mathbf{B} E_t \mathbf{X}_{t+i}$ can be rewritten as a system of independent equations, $\tilde{\mathbf{Y}}_t = \sum_{i=0}^{\infty} \mathbf{V}^i E_t \tilde{\mathbf{X}}_{t+i}$, with

$$\begin{aligned} \tilde{\mathbf{Y}}_t &= \mathbf{E}^{-1} \mathbf{Y}_t \\ &= \frac{\varrho_{HH} - 1}{\varrho_{HH} + \varrho_{SS} - 2} \begin{bmatrix} 1 & \frac{\varrho_{SS}-1}{\varrho_{HH}-1} \\ -1 & 1 \end{bmatrix} \begin{bmatrix} Q_t C_{S,t}^{-\sigma_S} \\ Q_t C_{H,t}^{-\sigma_H} \end{bmatrix} \\ &= \frac{\varrho_{HH} - 1}{\varrho_{HH} + \varrho_{SS} - 2} Q_t \begin{bmatrix} C_{S,t}^{-\sigma_S} + \frac{\varrho_{SS}-1}{\varrho_{HH}-1} C_{H,t}^{-\sigma_H} \\ -C_{S,t}^{-\sigma_S} + C_{H,t}^{-\sigma_H} \end{bmatrix}, \end{aligned}$$

$$\begin{aligned} E_t \tilde{\mathbf{X}}_{t+i} &= \mathbf{E}^{-1} \mathbf{B} E_t \mathbf{X}_{t+i} \\ &= \frac{\varrho_{HH} - 1}{\varrho_{HH} + \varrho_{SS} - 2} \begin{bmatrix} 1 & \frac{\varrho_{SS}-1}{\varrho_{HH}-1} \\ -1 & 1 \end{bmatrix} \begin{bmatrix} \eta_S X_{S,t}^{-\chi_S} \\ \eta_H X_{H,t}^{-\chi_H} \end{bmatrix} \\ &= \frac{\varrho_{HH} - 1}{\varrho_{HH} + \varrho_{SS} - 2} \begin{bmatrix} \eta_S X_{S,t}^{-\chi_S} + \frac{\varrho_{SS}-1}{\varrho_{HH}-1} \eta_H X_{H,t}^{-\chi_H} \\ -\eta_S X_{S,t}^{-\chi_S} + \eta_H X_{H,t}^{-\chi_H} \end{bmatrix}. \end{aligned}$$

In extensive form, the two independent equations are:

$$Q_t \left(C_{S,t}^{-\sigma_S} + \frac{1 - \varrho_{SS}}{1 - \varrho_{HH}} C_{H,t}^{-\sigma_H} \right) = \sum_{i=0}^{\infty} [\beta(1 - \delta)]^i \left\{ \eta_S X_{S,t}^{-\chi_S} + \frac{1 - \varrho_{SS}}{1 - \varrho_{HH}} \eta_H X_{H,t}^{-\chi_H} \right\},$$

$$Q_t \left(-C_{S,t}^{-\sigma_S} + C_{H,t}^{-\sigma_H} \right) = \sum_{i=0}^{\infty} [\beta(1 - \delta) (\varrho_{HH} + \varrho_{SS} - 1)]^i \left\{ -\eta_S X_{S,t}^{-\chi_S} + \eta_H X_{H,t}^{-\chi_H} \right\}.$$

These two equations establish a direct connection between the average and the gap between the state-specific shadow values of durables and, respectively, the average and the gap between the state-specific discounted marginal utilities of the service flow of durables. To the extent that durables are long-lived enough (i.e., $\delta \rightarrow 0$), but also that states are persistent enough (i.e., $\varrho_{HH} \rightarrow 1$ and $\varrho_{SS} \rightarrow 1$), the argument of Barsky et al. (2007) still applies in the face of temporary shocks, as in this case the future terms affecting Y_t remain nearly constant, i.e. $Y_t \approx (\mathbf{I} - \mathbf{A})^{-1} \mathbf{B}\mathbf{X}$.

C 3-state HANK economy

C.1 Transition probabilities in the steady state

Let us consider the transition probabilities across three states/islands $[S, H, K]$ and assume those are governed by the following transition probabilities

$$\mathbf{P} = \begin{bmatrix} \varrho_{SS} & \varrho_{SH} & \varrho_{SK} \\ \varrho_{HS} & \varrho_{HH} & \varrho_{HK} \\ \varrho_{KS} & \varrho_{KH} & \varrho_{KK} \end{bmatrix}. \quad (58)$$

Denote with $\boldsymbol{\lambda} = [\lambda_S, \lambda_H, \lambda_K]$ the share of population within each of the states/islands. The stationary distribution is found by solving the system of equations $\boldsymbol{\lambda}\mathbf{P} = \boldsymbol{\lambda}$:

$$\boldsymbol{\lambda} = \begin{bmatrix} \frac{\varrho_{KH}\varrho_{HS} + \varrho_{KS}\varrho_{HS} + \varrho_{HK}\varrho_{KS}}{\varrho_{KH}\varrho_{HS} + \varrho_{KS}\varrho_{HS} + \varrho_{HS}\varrho_{SK} + \varrho_{SH}\varrho_{KH} + \varrho_{SK}\varrho_{KH} + \varrho_{SH}\varrho_{KS} + \varrho_{HK}\varrho_{KS} + \varrho_{HK}\varrho_{SH} + \varrho_{HK}\varrho_{SK}} \\ \frac{\varrho_{SH}\varrho_{KH} + \varrho_{SK}\varrho_{KH} + \varrho_{SH}\varrho_{KS}}{\varrho_{KH}\varrho_{HS} + \varrho_{KS}\varrho_{HS} + \varrho_{HS}\varrho_{SK} + \varrho_{SH}\varrho_{KH} + \varrho_{SK}\varrho_{KH} + \varrho_{SH}\varrho_{KS} + \varrho_{HK}\varrho_{KS} + \varrho_{HK}\varrho_{SH} + \varrho_{HK}\varrho_{SK}} \\ \frac{\varrho_{HS}\varrho_{SK} + \varrho_{HK}\varrho_{SH} + \varrho_{HK}\varrho_{SK}}{\varrho_{KH}\varrho_{HS} + \varrho_{KS}\varrho_{HS} + \varrho_{HS}\varrho_{SK} + \varrho_{SH}\varrho_{KH} + \varrho_{SK}\varrho_{KH} + \varrho_{SH}\varrho_{KS} + \varrho_{HK}\varrho_{KS} + \varrho_{HK}\varrho_{SH} + \varrho_{HK}\varrho_{SK}} \end{bmatrix} \quad (59)$$

C.2 Log-linear economy

Savers:

$$\begin{aligned}
c_{S,t} &= \varrho_{SS} E_t c_{S,t+1} + \varrho_{SH} E_t c_{H,t+1} + \varrho_{SK} E_t c_{K,t+1} - \frac{1}{\sigma} (r_t - E_t \pi_{C,t+1}) \\
q_t - \sigma c_{S,t} &= -[1 - \beta(1 - \delta)] \chi x_{S,t} \\
&+ \beta(1 - \delta) \left[\frac{\varrho_{SS} \lambda_S + \varrho_{SK} \lambda_K}{\lambda_S} (E_t q_{t+1} - \sigma E_t c_{S,t+1}) + \varrho_{SH} (E_t q_{t+1} - \sigma E_t c_{H,t+1}) \right] \\
\phi n_{S,t} &= \omega_t - \sigma c_{S,t} \\
c_{S,t} + \frac{Y_X}{Y_C} e_{S,t} &= \frac{Y}{Y_C} (\omega_t + n_{S,t}) + \frac{1 - \tau_K^D - \tau_H^D}{\lambda_S} d_{C,t} + \frac{1 - \tau_K^D - \tau_H^D}{\lambda_S} \frac{Y_X}{Y_C} d_{X,t} \\
e_{S,t} &= q_t + \frac{1}{\delta} x_{S,t} - \frac{1 - \delta}{\delta} (\varrho_{SS} \lambda_S x_{S,t-1} + \varrho_{HS} \lambda_H x_{H,t-1})
\end{aligned}$$

Wealthy hand-to-mouth:

$$\begin{aligned}
q_t - \sigma c_{H,t} &= -[1 - \beta(1 - \delta)] \chi x_{H,t} \\
&+ \beta(1 - \delta) \left[\varrho_{HS} (E_t q_{t+1} - \sigma E_t c_{S,t+1}) + \frac{\varrho_{HH} \lambda_H + \varrho_{HK} \lambda_K}{\lambda_H} (E_t q_{t+1} - \sigma E_t c_{H,t+1}) \right] \\
\phi n_{H,t} &= \omega_t - \sigma c_{H,t} \\
c_{H,t} + \frac{Y_X}{Y_C} e_{H,t} &= \frac{Y}{Y_C} (\omega_t + n_{H,t}) + \frac{\tau_H^D}{\lambda_H} d_{C,t} + \frac{\tau_H^D}{\lambda_H} \frac{Y_X}{Y_C} d_{X,t} \\
e_{H,t} &= q_t + \frac{1}{\delta} x_{H,t} - \frac{1 - \delta}{\delta} (\varrho_{HH} \lambda_H x_{H,t-1} + \varrho_{SH} \lambda_S x_{S,t-1})
\end{aligned}$$

Pure hand-to-mouth:

$$\begin{aligned}
\phi n_{K,t} &= \omega_t - \sigma c_{K,t} \\
c_{K,t} &= \frac{Y}{Y_C} (\omega_t + n_{K,t}) + \frac{\tau_K^D}{\lambda_K} d_{C,t} + \frac{\tau_K^D}{\lambda_K} \frac{Y_X}{Y_C} d_{X,t}
\end{aligned}$$

Production, pricing and profits:

$$\begin{aligned}
y_{j,t} &= n_{j,t}, \quad j = \{C, X\} \\
rmc_{j,t} &= w_t - p_{j,t}, \quad j = \{C, X\} \\
d_{j,t} &= -rmc_{j,t}, \quad j = \{C, X\} \\
\pi_{j,t} &= \beta E_t \pi_{j,t+1} + \psi_j rmc_{j,t}, \quad \psi_j \equiv (1 - \theta_j)(1 - \beta \theta_j) / \theta_j, \quad j = \{C, X\} \\
q_t &= q_{t-1} + \pi_{X,t} - \pi_{C,t}
\end{aligned}$$

Market clearing:

$$\begin{aligned}
n_t &= \frac{Y_X}{Y} n_{X,t} + \frac{Y_C}{Y} n_{C,t} = \lambda_H n_{H,t} + \lambda_K n_{K,t} + \lambda_S n_{S,t} \\
y_{C,t} &= c_t = \lambda_H c_{H,t} + \lambda_K c_{K,t} + \lambda_S c_{S,t} \\
y_{X,t} &= \frac{1}{\delta(1 - \lambda_K)} x_t - \frac{1 - \delta}{\delta(1 - \lambda_K)} x_{t-1}
\end{aligned}$$

Monetary policy:

$$\begin{aligned}
r_t &= \phi_\pi \pi_t + \nu_t \\
\pi_t &= \alpha \pi_{C,t} + (1 - \alpha) \pi_{X,t} \\
\nu_t &= \rho_\nu \nu_{t-1} + \varepsilon_t^\nu
\end{aligned}$$

C.3 Sectoral dynamics in the benchmark economy

The real wage can be determined as in the TANK economy. Combine the Euler for nondurables and the Taylor rule to obtain

$$\pi_t = \frac{1}{\phi_\pi} E_t \pi_{t+1} - \frac{1}{\phi_\pi} \nu_t, \quad (60)$$

So that, by assuming $\phi_\pi > 1$, is sufficient to iterate the equation forward and pin down the rate of inflation:

$$\pi_t = -\frac{1}{\phi_\pi} E_{t,s=0}^\infty \left(\frac{1}{\phi_\pi} \right)^s \nu_{t+s} = \frac{1}{\rho_\nu - \phi_\pi} \varepsilon_t^\nu, \quad (61)$$

Thus, from the NKPC:

$$\omega_t = \frac{1}{\psi(\rho_\nu - \phi_\pi)} \varepsilon_t^\nu. \quad (62)$$

Take now H 's and S 's budget constraints, and aggregate them, considering that *i*) $c_{H,t} = c_{S,t} = q_t = 0$ and, thus $\omega_t = w_t - p_{X,t}$, so that $d_{C,t} = d_{X,t} = -\omega_t$; *ii*) $n_{S,t} = n_{H,t} = \frac{1}{\phi} \omega_t$. Thus, multiply both sides of the constraint by $1/(1 - \lambda_K)$ to obtain

$$y_{X,t} = \frac{Y}{(1 - \lambda_K) Y_X} \left(\frac{\lambda_S + \lambda_H}{\phi} + \tau_K^D \right) \omega_t. \quad (63)$$

Take now K 's budget constraint, and combine it with $c_{K,t} = \frac{1}{\lambda_K} y_{C,t}$, $n_{K,t} = \frac{1}{\lambda_K} n_t - \frac{\lambda_S}{\lambda_K} n_{S,t} - \frac{\lambda_H}{\lambda_K} n_{H,t}$, and $n_{S,t} = n_{H,t} = \frac{1}{\phi} \omega_t$:

$$y_{C,t} = \frac{Y}{Y_C} \left(\lambda_K - \tau_K^D - \frac{\lambda_S + \lambda_H}{\phi} \right) \omega_t + \frac{Y}{Y_C} y_t. \quad (64)$$

Consider y_t from the definition of aggregate hours, and then combine this with the labor supply schedule, in each state (recall that $n_{K,t} = \frac{1}{\phi} \omega_t - \frac{\sigma}{\phi} c_{K,t}$):

$$y_t = \frac{1}{\phi} \omega_t - \frac{\sigma}{\phi} y_{C,t}.$$

Thus, combining the latter with (64):

$$y_{C,t} = \frac{Y\phi}{Y_C\phi + Y\sigma} \lambda_K \left(\frac{1 + \phi}{\phi} - \frac{\tau_K^D}{\lambda_K} \right) \omega_t.$$

C.4 Amplification under asymmetric price stickiness

We aggregate the labor supply schedules of households in each of the three states to obtain the aggregate wage schedule:

$$\phi n_t = \omega_t - \sigma c_t. \quad (65)$$

Let us now consider the case of *flexible prices for durables*. Combine the pure HtM households' labor supply with her budget constraint, using $d_{j,t} = -w_{j,t}$ and recalling that $\omega_{X,t} = 0$, to obtain

$$\omega_t = \frac{\left(\phi + \sigma \frac{Y}{Y_C} \right) \lambda_K}{\lambda_K + \phi (\lambda_K - \tau_K^D)} \frac{Y_C}{Y} c_{K,t}. \quad (66)$$

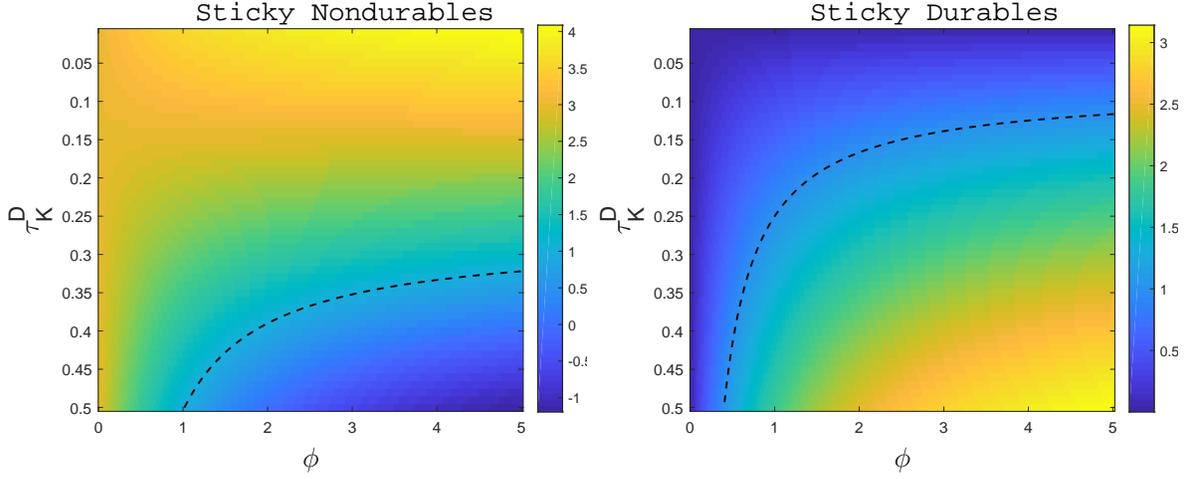
Plugging this into the the aggregate wage equation, and relying on $y_t = n_t$:

$$\phi y_t = \frac{\left(\phi + \sigma \frac{Y}{Y_C} \right) \lambda_K}{\lambda_K + \phi (\lambda_K - \tau_K^D)} \frac{Y_C}{Y} c_{K,t} - \sigma c_t. \quad (67)$$

This equation is the key to deriving K 's consumption as a function of total nondurable production. Furthermore, at this stage it is possible to prove the equivalence with the multiplier in Bilbiie (2020), by simply setting $Y_C = Y$. Recall again that $(w_t - p_{X,t}) = 0$. Thus, by appealing to K 's labor supply and $\sigma_H c_{H,t} = q_t = \sigma_S c_{S,t}$, we can show again that $n_{H,t} = n_{S,t} = 0$, so that $n_t = \lambda_K n_{K,t}$. In light of this:

$$c_{K,t} = \frac{[\lambda_K + \phi (\lambda_K - \tau_K^D)] [\lambda_K Y + \phi (\lambda_K Y - \tau_K^D Y_C)]}{\left(\phi + \sigma \frac{Y}{Y_C} \right) [\lambda_K Y + \phi (\lambda_K Y - \tau_K^D Y_C)] \frac{Y_C}{Y} - \phi [\lambda_K + \phi (\lambda_K - \tau_K^D)] [\lambda_K Y_C - \sigma (\lambda_K Y - \tau_K^D Y_C)]} \cdot \frac{\sigma}{\lambda_K} y_{C,t}$$

Figure 3: Amplification in nondurable production



Notes: The figure reports μ_K/λ_K as a function of ϕ and τ_K^D ; broken line denotes the unitary value. Calibration: left panel: $\theta_X = 0$ and $\theta_C = 0.6$; right panel: $\theta_X = 0.6$ and $\theta_C = 0$. Common parameters: symmetric transition probabilities, $\sigma = 1$, $Y_C = \alpha = 0.75$, $Y_X = 1 - Y_C$, $\beta = 0.97$, $\delta = 0.025$, $\phi_\pi = 1.5$.

(68)

As for the case of *flexible prices for nondurables*, recall that $\omega_t = d_{C,t} = 0$. Thus, K 's labor supply implies $n_{K,t} = -\frac{\sigma}{\phi}c_{K,t}$ and, her budget constraint:

$$\left(1 + \frac{\sigma Y}{\phi Y_C}\right) c_{K,t} = \frac{\tau_K^D Y_X}{\lambda_K Y_C} \sigma c_{S,t}, \quad (69)$$

In turn, using $c_{S,t} = \frac{1}{\lambda_H + \lambda_S} c_t - \frac{\lambda_K}{\lambda_H + \lambda_S} c_{K,t}$ and $c_t = y_{C,t}$, we can prove that

$$c_{K,t} = \frac{\tau_K^D \sigma \phi Y_X}{\phi Y_C (\lambda_H + \lambda_S) + \sigma Y (\lambda_H + \lambda_S) + \tau_K^D \sigma \phi Y_X} \frac{1}{\lambda_K} y_{C,t}. \quad (70)$$