Communication and Commitment with Constraints

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Abstract

I study the role of communication and commitment between an informed and an uninformed agent. The two agents contribute to a joint project such that *(i)* the agents' actions are substitutable, and *(ii)* the actions are constrained. In the absence of commitment and when decision-making is simultaneous, there is full information revelation as long as constraints are not binding. The presence of binding constraints results in only partial revelation of information in equilibrium. The most informative equilibrium is strictly pareto dominant. When decisions are sequential, information revealed is unchanged but the actions of the agents change, resulting in higher welfare. Finally, I characterize the ex ante optimal commitment mechanism for the uninformed agent. Providing greater commitment power strictly raises welfare of both agents and leads to greater overall efficiency.

Keywords: asymmetric information, cheap talk, commitment, strategic substitutes

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1 Introduction

A number of interactions in economics and political science involves joint decision-making by agents with information asymmetries. In international alliances, for example, members share private information and pool resources to achieve a common objective (e.g., unified defense, intelligence sharing or security cooperation). These joint tasks involve coordinated actions between the agents. When the actions are substitutable and there are binding constraints (e.g. fiscal, military) on the agents, it exacerbates the incentives of an informed agent to misrepresent private information in order to induce a higher action from the uninformed agent. The constraints therefore affects agents' capacity to contribute and coordinate efficiently.

Given the misaligment of incentives, any purely communication based decision-making protocol may not effectively aggregate private information. To overcome inefficiencies, alliances typically resort to ex-ante commitment mechanisms (e.g., through binding agreements). However, it is sometimes not possible to contract both agents' decisions. While the uninformed agent is able to commit to a contractible decision, it is likely that the informed agent retains decision-making authority, resulting in limited (imperfect) commitment.

This raises questions on how to organize decision-making when there is communication between agents, but no commitment. When commitment is feasible, what the optimal mechanism is when the informed agent's actions are non-contractible remains an open question. Finally, it is unclear whether constraining the autonomy of agents via binding commitment rules improves joint welfare of agents compared to decision-making under pure communication. To analyze these questions, this paper introduces a novel coordination game between two agents with action substitutability and constraints, and compares alternate decision making protocols that are communication and commitment based.

The theoretical work on communication with coordination motives has predominantly focused on strategic complementarities in actions (e.g. Alonso, Dessein, and Matouschek, 2008), while the problem of coordination with substitutable actions has not received sufficient attention. Further, the literature on commitment has focused on optimal delegation problems in which there is a single decision-maker with either perfect commitment (e.g. Alonso and Matouschek, 2008; Amador and Bagwell, 2013) or imperfect commitment (e.g. Bester and Strausz, 2001). In contrast, I introduce a novel form of imperfect commitment in this paper. Specifically, I focus on the case in which both agents are decision-makers, and there is limited commitment in that only one agent commits to a binding (contractible) decision while the other agent's action is non-contractible. The paper therefore combines the literature on communication with coordination motives, and contracting with imperfect commitment.

The coordination game between two agents with joint decision-making is such that *i)* private information is *soft*; *ii)* communication takes the form of cheap talk à la Crawford and Sobel (1982); *iii)* actions are imperfect substitutes; and *iv)* agents face constraints on their actions. The key feature of the model is that each agent has a coordination function that generates an output based on the individual actions. The concavity property of the underlying preferences implies that there is an unique value of the coordination function corresponding to each state of the world, and these values are different for the two agents. Crucially, the preferences exhibit a shared costs feature, instead of the free riding (marginal costs) property that is commonly observed in games with action substitutability (e.g., Dubey, Haimanko, and Zapechelnyuk (2006)). The way to interpret this feature is to think of a cost that each agent suffers from working together and taking joint actions. In alliances, for example, both partnering countries suffer losses when they contribute to a joint military operation. This could be a reputational cost incurred for partnering in a military operation with another country. These shared costs are exacerbated by the size of the overall joint operation and not restricted to only each countries' individual contribution to the effort.

In the baseline model, I analyze simultaneous decision-making with no commitment. To establish the existence and uniqueness of pure strategty equilibria in the decision-making stage, I use techniques of aggregative games developed by Jensen (2010). Once the uniqueness of actions is established for any set of beliefs between the agents, it is then possible to fully characterize the complete set of communication equilibria.

Three types of communication equilibria emerge in this setup, namely, i) *threshold*; ii) *partial pooling*; and iii) *hybrid equilibria*. In the threshold equilibria, the informed agent communicates truthfully only up to a certain threshold and pools all information beyond.[1] However, when the informed agent does not suffer from binding constraints on actions, all private information is communicated in equilibrium and there is full efficiency. It is only in the presence of binding constraints that full information revelation breaks down and there is some loss of information in equilibrium. The intuition is that in the absence of any constraints, both agents can take actions such that the joint coordination function for each agent corresponds with their first best levels, thereby precluding the need to misrepresent information.

In the partial pooling equilibria, the informed agent chooses multiple message pools up to a certain threshold and pools all the information above this threshold. The novel feature of these equilibria is that there is a continuum of partial pooling equilibria possible for each feasible threshold. Moreover, these equilibria are different in structure to the partitional equilibria of CS in that they are not monotonically increasing (or decreasing) in size. The size of the

[1] The threshold equilibria are similar to those derived by Kartik, Ottaviani, and Squintani (2007), and Kartik (2009). In both these papers, there is exaggerated communication in equilibrium which is in contrast to the truthful messaging equilibria characterized in this paper.

message pools depends on the equilibrium action of the marginal type in the interval, i.e., the indifference condition is pinned down by the action that achieves first best for the two adjacent messages. The hybrid equilibria combines the features of the above two equilibria. Specifically, the informed agent separates for some types within a threshold and pools some others. This gives rise to equilibria with pooling on either ends of the type space, and separation in between.[2]

Given the multiplicity of communication equilibria in the model, it is important to compare their welfare properties. Surprisingly, the communication equilibria exhibit an intuitive pareto ordering - the more informative threshold equilibrium is pareto dominant. This implies that welfare of agents is monotonically increasing in the amount of information revealed. When more information is revealed, both agents achieve their first best for a greater measure of types. Further, under the most informative equilibrium, the pooling message induces a greater action from the uninformed agent. The key intuition is that the informed agent has discretion in choosing her actions and making use of her private information on the pooling interval. This novel feature provides greater flexibility to the informed agent and allows for better coordination of agents' actions. It minimizes the inefficiency from under-allocation (over-allocation) for the informed (uninformed) agent, thereby improving welfare of both agents.

Having identified a pareto dominant equilibrium, I analyze a sequential protocol in which the uninformed agent moves first (Stackelberg leader) after the communication round. When actions are sequential, the most informative threshold equilibrium of the coordination game is the same as in the simultaneous protocol.[3] However, the sequential protocol provides both agents a higher ex-ante welfare. This stems from the fact that once the uninformed agent's action is sunk, the informed agent observes this and correspondingly adjusts (moderate) her own action. The uninformed agent benefits from this *moderating effect*, and therefore takes a higher action in equilibrium when the information is pooled. This further translates into greater ex-ante welfare for the informed agent as well, resulting in higher overall efficiency.

Though sequential decision-making improves joint welfare of the agents, there is still inefficiency on the pooling interval. This arises because the uninformed agent takes an expected action while the informed agent exploits her private information. Typically, agents can rely on binding commitments to mitigate this inefficiency. To capture this, I analyze a *commitment protocol* under which the uninformed agent commits to a communication dependent incentive compatible *action rule*. Following this, the informed agent decides on the information to com-

[2]The hybrid equilibria bears some semblance in structure to the *central pooling equilibria* in the work of Bernheim and Severinov (2003). They characterize signaling equilibria in which there is pooling towards the middle of the spectrum of types and separation on either ends of the type space. Instead of central pooling, however, the hybrid equilibria exhibits pooling on both extremes, and induces separation in the middle of the type space.

[3]I will consider the most informative equilibrium for comparison purposes since it is ex ante efficient.

municate. Crucially, the informed agent moves after the contractible action (of the uninformed agent) is realized. Notice that this is similar to the optimal delegation problem except that there is an additional informed decision-maker whose action is non-contractible.

Since there is limited commitment, the informed agent, depending on the contracted commitment rule, can decide what to reveal and then subsequently what action to take. This adds a layer of complexity to the commitment problem faced by the uninformed agent. However, due to the result of Bester and Strausz (2001)[4], there exists an incentive feasible direct mechanism (Revelation Principle) in which the set of messages used by the informed agent corresponds with the type space, and uninformed agent commits to actions contingent on the type revealed. Further, the informed agent observes the commitment rule and best responds to this action. Since the revelation principle is applicable, the uninformed agent's problem is twofold: *i)* to choose an action rule that satisfies the informed agent's incentive compatibility constraints; and *ii)* to minimize the inefficiencies from coordination, conditional on satisfying the incentive compatibility conditions.

The optimal commitment rule exhibits three key features. The informed agent mimics the actions of the simultaneous protocol up to the most informative threshold. Beyond this, the informed agent minimizes coordination losses by committing to actions such that the informed agent always takes the highest action in the non-contractible stage. Finally, the uninformed agent *caps actions* beyond a (higher) threshold of information.[5] By committing to an ex ante decision rule, the uninformed agent incentivizes the informed agent to reveal more information in a way that benefits both agents.

Related Literature

This paper extends and contributes to the vast theoretical and applied literature of that studies communication in interdependent environments. The role of communication with strategic complementarities in actions have been widely studied and applied to varied settings (e.g. Alonso, Dessein, and Matouschek, 2008; Baliga and Morris, 2002; Dessein and Santos, 2006; Hagenbach and Koessler, 2010; Rantakari, 2008). Barring Alonso (2007), who considers a principal-agent setting in which an uninformed principal controls the decision rights and actions of the two players are either strategic complements or substitutes, none of the other papers have looked at incentive problems when the nature of coordination is such that both players' actions are substitutable.

[4]See Proposition 1.

[5]Without this capping, the uninformed agent would end up providing first best levels to the informed for all possible states, which would be equivalent to full delegation as in Dessein (2002). This form of full delegation is never optimal for the uninformed agent in this paper.

The literature on communication and commitment (Holmstrom, 1978) has delved into the question of optimal delegation by an uninformed Principal. Alonso and Matouschek (2008) characterize the necessary and sufficient conditions for interval delegation to be optimal under quadratic loss utility functions. Amador and Bagwell (2013) generalize this result for a broader class of welfare functions and also allow for money burning feature. In both these papers, there is only a single decision-maker (uninformed Principal) and there is perfect commitment. The paper by Ambrus and Egorov (2017) study contracting with both money burning and also allow for monetary transfers. On the other hand Krishna and Morgan (2008) look at contracting with imperfect commitment and monetary transfers. Contrastingly, my paper studies a case in which both agents are decision-makers and there is only limited commitment. Surprisingly, the optimal commitment mechanism in my paper resembles the interval delegation result in that the uninformed agent provides a cap on actions.

Another relevant paper is the work by Melumad and Shibano (1991), who characterize a deterministic commitment rule for the uninformed receiver in a standard cheap talk game. They find that commitment power to the uninformed receiver improves her welfare compared to pure cheap talk. However, the opposite is true for the informed sender. The optimal commitment rule in my work is also deterministic but on the other hand improves the welfare of both the informed and uninformed agents, which is contrary to their findings.

This paper is also related to the work on information sharing with substitutable actions. Gal-Or (1985) and Li (1985) study cournot competition with uncertainty about the market demand parameter. They find that communication breaks down in equilibrium and no information is credibly revealed by the firms. On the contrary, when the uncertainty is regarding cost parameters (Gal-Or, 1986; Goltsman and Pavlov, 2014) there exists informative communication equilibria with cheap talk. My paper on the other hand is concerned with scenarios where there are coordination incentives and shared costs that is a function of both agents' actions.

The rest of the paper proceeds as follows. In Section 2, I present a simple example to show the main intuition driving my results. Section 3 outlines the basic model and Section 4 presents conditions for full information revelation equilibrium. Section 5 contains the results pertaining to partial revelation equilibria. In Section 6, I present efficiency properties of equilibria and analysis of the sequential protocol follows in Section 7. In Section 8, I characterize the optimal commitment mechanism for the uninformed agent. Finally, Section 9 contains concluding remarks.

2 An Example

Consider a joint task to be executed by an uninformed agent A_1 and an informed agent A_2 (without loss of generality). A_2 perfectly observes signal about the state of the world θ, drawn from an uniform distribution $[0,1]$. The information is soft and A_2 communicates its private information by sending a cheap talk message $m(\theta)$ to A_1. Upon communication, both A_1 and A_2 take actions that affects both their payoffs. Let the utility functions be the following:

$$U^1 = -\left[\left(\frac{x_1 + \eta x_2}{1+\eta}\right) - \theta\right]^2$$

$$U^2 = -\left[\left(\frac{x_2 + \eta x_1}{1+\eta}\right) - \theta - b\right]^2$$

Observe that both players take actions that contribute to the project and these actions are substitutable in that $\frac{\partial^2 U^i}{\partial x_1 \partial x_2} < 0$, where $\eta \in (0,1)$ captures the degree of substitutability. Finally, the two players face constraints in that x_i has a domain $[-a, a]$. When A_2 truthfully reveals the true state of the world, i.e. $m(\theta) = \theta$, the two players solve the following best responses:

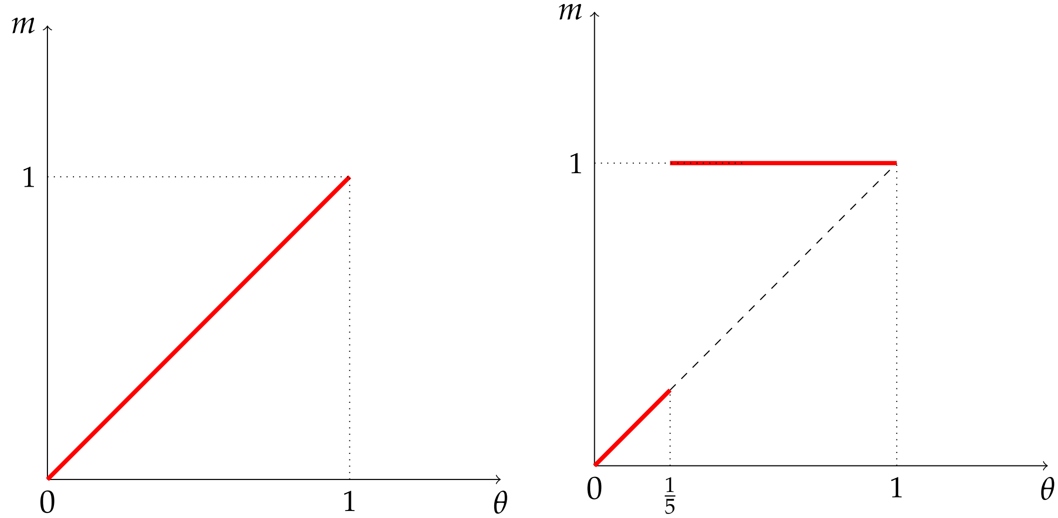
$$A_1 : x_1 = (1+\eta)\theta - \eta x_2$$

$$A_2 : x_2 = (1+\eta)(\theta + b) - \eta x_1$$

To simplify the exposition, let $b = \frac{2}{5}$ and $\eta = \frac{1}{2}$. The actions after truthful messaging are given by: $x_1^* = \theta - \frac{2}{5}$, $x_2^* = \theta + \frac{4}{5}$. Notice immediately that full information revelation is possible if $a \geqslant \frac{9}{5}$. This is because A_2 is able to reveal truthfully the highest type $\theta = 1$, and $x_2^*(1) = \frac{9}{5}$. This way, A_2 achieves first best. On the other hand, when $a < \frac{4}{5}$, no information can be credibly revealed by A_2.[6]

Finally, when $\frac{4}{5} < a < \frac{9}{5}$, A_2 has an incentive to reveal some information. To see this, let $a = 1$. Then, for any $\theta \in [0, \frac{1}{5}]$, A_2 reveals the state truthfully since her optimal action is within the domain of available actions (in this case $x_2^*(\frac{1}{5}) = 1$). But, for any $\theta > \frac{1}{5}$, A_2 cannot sustain a truthful messaging strategy since the constraints are binding for A_2 (i.e. $x_2 = 1$). Then the optimal action for A_1 is according to its best response function, which is $x_1 = \frac{3}{2}\theta - \frac{1}{2}$. This

[6]Suppose say $a = \frac{2}{5}$. Then the constraint is binding for all types. A_2 can inflate her signal in order to induce A_1 to allocate more. To see this, instead of $m(0) = 0$, say inflated message is $m(0) = \frac{2}{5}$. Then, A_1 best responds by allocating $x_1^* = \frac{2}{5}$. A_2 then contributes $x_2^* = \frac{3}{5} - \frac{1}{5} = \frac{2}{5}$. That is, by inflating her information the informed agent induces a higher action from A_1 whilst achieving first best. However this incentive to misrepresent means that messages do not carry credibility in equilibrium. A_2 can never credibly reveal any information to A_1 and therefore communication is rendered ineffective.



(a) $a \geq \frac{9}{5}$ (b) $a = 1$

Figure 1: When $a \geq \frac{9}{5}$, the action constraints are not binding for A_2, resulting in full information revelation. On the other hand, when $a \in (\frac{4}{5}, \frac{9}{5})$ there is only partial revelation of information. Specifically, for all states above $\frac{1}{5}$, the agent A_2 pools and sends a message $m = 1$.

cannot be an equilibrium since A_2 gets a payoff of $U_2 = -\left(\frac{1+\frac{1}{2}(\frac{3}{2}.\theta-\frac{1}{2})}{\frac{3}{2}} - \theta - \frac{2}{5}\right)^2 \neq 0$ where $\frac{1+\frac{1}{2}(\frac{3}{2}.\theta-\frac{1}{2})}{\frac{3}{2}} < \theta + \frac{2}{5}$ for $m = \theta > \frac{1}{5}$. This implies there is under-allocation from A_2's perspective if it reveals the truth. Therefore, A_2 has an incentive to exaggerate its information in order to induce the other agent to play a higher action. This precludes separation beyond $\theta = \frac{1}{5}$.

In fact, all types above this cutoff must pool and send the highest message, $m = 1$. This is primarily because the signals are (imperfectly) invertible in this environment. For instance, when $\theta = \frac{2}{5}$, A_2 would want to exaggerate and send a message of at least $m \geq \frac{3}{5}$, since this would ensure that her action constraints are not binding. Now suppose say A_2 sends $m = \frac{3}{5}$. This message could possibly come from any of the types $\theta \in (\frac{1}{5}, \frac{2}{5}]$, each of whom have incentives to deviate and send $m = \frac{3}{5}$. Hence, A_1 can invert the message and form beliefs accordingly.[7] But if this were the case, every type in the interval $(\frac{1}{5}, 1]$ would find it profitable to exaggerate even further. Therefore, there is at most a partially revealing equilibrium in which A_2 is truthful (separates) in the range $\theta \in [0, \frac{1}{5}]$ and pools for $\theta \in (\frac{1}{5}, 1]$ by sending the highest possible inflated message, $m = 1$.

The example suggests a *novel trade-off* for information transmission with substitutability and

[7] Partition equilibria of the kind developed by CS are also ruled out on the interval $(\frac{1}{5}, 1]$. The incentive to exaggerate ensures that if there are two partitions, say, the high types in the lower partition would find it profitable to deviate to the higher partition, precluding the existence of an indifference type in the first place.

action constraints. The ability to truthfully reveal information depends on the actions available to the informed player. The informed agent A_2 is able to provide more information as the action set available to her is bigger. For the same reasons, when constraints are binding, there is an incentive to inflate private information and extract more actions from the uninformed agent A_1.

3 The Model

Consider a joint project between two agents, an uninformed agent A_1 and an informed agent A_2. The payoff from the project is dependent on state $\theta \in [0,1]$ and the actions taken by both agents. The state θ is distributed according to a cdf $F(.)$ and a corresponding density $f(.)$ with full support. Agent A_2 receives a perfectly observable private signal about the state θ. The set of possible actions available to the agents is constrained and given by $x_i \in V \subseteq \mathbb{R}$, where the set V is closed and compact.

Each agent's utility is given by $U(\phi^i(x_i, x_{-i}), \theta, b_i)$, where $\phi^i(.)$ is the agent-specific (symmetric) *joint action function* (henceforth *coordination function*). The coordination function $\phi^i(.)$ depends on agent i's action x_i, as well as the action of the other agent, x_{-i}. The function is represented by a mapping $\phi^i : V \times V \to Z \subset \mathbb{R}$. The bias parameter b_i measures the conflict of interest between the two agents. This captures the extent to which the goals of the agents differ according to the needs of the project.

The standard assumptions on the utility function of players hold. Specifically, $U : V^2 \times [0,1] \times \mathbb{R} \to \mathbb{R}$ is twice continuously differentiable, $U_{11}(.) < 0$, $U_{12}(.) > 0$, and $U_{13}(.) > 0$ such that U has a unique maxima for any given pair (θ, b_i). The utility functions satisfy the condition $\frac{\partial^2 U}{\partial x_i x_j} < 0$, implying that actions of the two agents are substitutable. For sake of exposition, let the bias of the uninformed agent be normalized to $b_1 = 0$ and that of the informed to $b_2 = b > 0$. Let $\bar{\phi}^1_\theta \equiv \arg\max_{\phi^1} U\left(\phi^1, \theta\right)$ and $\bar{\phi}^2_\theta \equiv \arg\max_{\phi^2} U\left(\phi^2, \theta, b\right)$ be the first best levels of joint actions for the two agents respectively, for a given θ.

Finally, I make the following assumptions on the functional form of the coordination function of the agents:

Assumption 1 *Increasing marginal contribution:* $\frac{\partial \phi^i(.)}{\partial x_i} > 0$

Assumption 2 *Positive spillover:* $\forall i, j \neq i : \frac{\partial \phi^i(.)}{\partial x_j} > 0$

Assumption 3 *Imperfect substitutability:* $\forall i, j \neq i : \frac{\left(\frac{\partial \phi^i}{\partial x_i}\right)}{\left(\frac{\partial \phi^i}{\partial x_j}\right)} > 1$

Assumption 1 ensures that the function is strictly increasing in each agent's own action, while the second assumption ensures the same with respect to the other agent's action. Assumption 3 implies that the *marginal contribution effect* dominates the *spillover effect*. Further, it rules out perfectly substitutable actions.[8]

Timing - Simultaneous Decision-Making with no Commitment:

Following Kartik (2009), let $M = \bigcup_\theta M_\theta$ be a Borel space of messages available to A_2 such that $\forall \theta, \theta' \in [0,1] : M_\theta \bigcap M_{\theta'} = \emptyset$. The "*Simultaneous Protocol*" proceeds in two stages.

- In the first stage, A_2 observes the true state $\theta \in [0,1]$ and sends a message $m \in M$ to A_1. Let this messaging strategy be defined by a mapping $\mu : [0,1] \to M$ and the message $m = \mu(\theta)$.
- In the second stage, both agents simultaneously take actions $\alpha_1 : M \to V$ and $\alpha_2 : [0,1] \times M \to V$.

Equilibrium

An equilibrium of the *simultaneous protocol* game is a Perfect Bayesian Equilibrium in pure strategies that satisfies the following properties:

- A_1 and A_2 simultaneously choose actions $(x_1^*(m), x_2^*(\theta, m))$ that maximizes their expected utility according to the optimization problem:

$$x_1^*(m) \equiv \arg\max_{x_1 \in V} \mathbb{E}_{\theta|m} \left[U\left(\phi^1(x_1, x_2^*(\theta, m)), \theta\right)\right] \text{ subject to } x_1 \in V \tag{1}$$

$$x_2^*(\theta, m) \equiv \arg\max_{x_2 \in V} \left[U\left(\phi^2(x_2, x_1^*(m)), \theta, b\right)\right] \text{ subject to } x_2 \in V \tag{2}$$

- the coordination function maximizes each players' expected utility conditional on their information, ie, $\phi^{1^*}(x_1^*(m), x_2^*(\theta, m)) \equiv \arg\max_{\phi^1} U\left(\phi^1(x_1, x_2), \theta\right)$ and $\phi^{2^*}(x_2^*(\theta, m), x_1^*(m)) \equiv \arg\max_{\phi^2} U\left(\phi^2(x_2, x_1), \theta, b\right)$
- the posterior beliefs, given by a cdf $P(\theta \mid m)$, are updated using Bayes' rule whenever possible, given the messaging rule $\mu^*(\theta)$

[8] When actions are perfect substitutes, notice that there is no interior equilibrium in the action stage. Take the example presented in Section 2 and substitute $\eta = 1$. The best responses are such that there exists only corner solutions in which the informed agent takes the highest action and the uninformed one, the lowest one possible. For this reason, I focus on imperfect substitutability of actions.

- given the beliefs and second stage actions $x_1(m)$ and $x_2(\theta, m)$, A_2 chooses a messaging strategy that maximizes expected payoff in the first stage,

$$\mu^*(\theta) \in \arg\max_{m \in M} \mathbb{E}_{P(.|m)} \left[U\left(\phi^2(x_2(\theta, m), x_1(m)), \theta, b\right)\right]$$

A PBE always exists in games with cheap talk. This is a babbling equilibrium in which the agent A_2's message is ignored and A_1 takes an action based on the prior distribution of the state. In this paper, I try to identify conditions under which more informative equilibria emerge.

4 Full Information Revelation

When can the two agents share information efficiently? In other words, can all the private information held by A_2 be completely revealed to A_1, meaning $\mu(\theta) = \theta$ for all $\theta \in [0, 1]$. To see if a fully revealing equilibrium exists, it is important to understand the incentives of the informed agent A_2. For truthful messaging to be an equilibrium, A_2 must achieve first best for every possible state θ. Since A_2 is constrained, the bounds on her action set given by $\inf V = \underline{k}$ and $\sup V = \bar{k}$ directly affects A_2's ability to achieve first best. Therefore, the domain of available actions V acts as an *incentive compatibility constraint* for truth-telling.

Given this intuition, it is convenient to reformulate the second stage problem when A_2 has an *unrestricted* action domain to choose from. The following definition does precisely that.

Definition 1 ***Unconstrained Allocation****: Let $\bar{x}_2(\theta, m)$ be the optimal action of A_2 when i) $x_2 \in \mathbb{R}$; and ii) message m is believed by A_1 to be the true state.*

$$\bar{x}_2(\theta, m) \text{ solves } \max_{x_2 \in \mathbb{R}} U\left(\phi^2(x_2, \bar{x}_1(m)), \theta, b\right) \text{ subject to}$$
$$\bar{x}_1(m) \equiv \arg\max_{x_1 \in V} U\left(\phi^1(x_1, \bar{x}_2(\theta, m)), m\right)$$

Further, when communication is truthful ($m = \theta$), let the optimal action of players under the unconstrained optimization problem be $\bar{x}_1(\theta)$ and $\bar{x}_2(\theta) = \bar{x}_2(\theta, \theta)$.

Assumption 4 $\underline{k} \le \bar{x}_2(0) \le \bar{k}$

Definition 1 does not necessarily prescribe the action of A_2 in equilibrium. Instead, $\bar{x}_2(\theta, m)$ allows us to intuitively characterize the response of an informed agent when the message misrepresents the true state but is believed to be true by a naive A_1 (Kartik, Ottaviani, and Squintani, 2007; Ottaviani and Squintani, 2006). Assumption 4 ensures trivial outcomes are ruled out, i.e the case where no information is revealed credibly.[9] Finally, the following definition helps characterize the full information revelation equilibrium.

Definition 2 *Highest type incentive compatibility (**HTIC**)*[10] *:* $\bar{x}_2(1) \leq \bar{k}$

Definition 2 implies that the best response of A_2 after truthfully communicating the highest state $\theta = 1$ is within the domain of feasible actions. When HTIC is satisfied, it also implies that $\bar{x}_2(\theta) \leq \bar{k}$ for every $\theta < 1$.

Proposition 1 *A full information revelation equilibrium exists if and only if HTIC condition is satisfied.*

Proof. See Appendix A.1 ■

The *HTIC* condition ensures that the action constraints are never binding for A_2 under truthful revelation. This implies the informed agent can reveal her information and achieve full efficiency such that $x_2^*(\theta) = \bar{x}_2(\theta)$ for all $\theta \in [0,1]$. Despite the soft nature of information, there is full information transmission.

5 Partial Information Revelation

This section focuses on equilibria that emerge in the presence of binding constraints on the agents. The following assumption ensures an intuitive characterization of informative equilibria.

Assumption 5 $\underline{k} \leq \bar{x}_2(0,1) \leq \bar{k}$

Assumption 5 ensures any exaggeration by A_2[11] would induce an action that is within the bounds of the action constraints. This is useful is ruling out non-trivial cases and helps focus on the efficiency properties of informative communication equilibria.

The starting point of the analysis is to formulate the informed agent's incentive to misrepresent her information. This happens precisely when there exists states for which truthful communication can never be credible. Observe that when *HTIC* condition fails, then there must exist a cutoff $\bar{\theta}$ such that $\bar{x}_2(\bar{\theta}) = \bar{k}$. Let $G = \{\theta : \bar{x}_2(\theta) > \bar{k}\}$ be the set of states for which truthful revelation results in the constraint being binding on A_2. The cutoff state $\bar{\theta} = \sup\{[0,1] \backslash G\}$

[9] $\bar{x}_2(0) \leq \bar{k}$ provides an intuitive condition for any information transmission with action substitutability. When this fails, no information can be credibly revealed since A_1 always believes that A_2 is exaggerating its private information.

[10] HTIC is not related to the *No incentive to separate* (NITS) condition proposed by Chen, Kartik, and Sobel (2008).

[11] Note that this is a stronger version of assumption 4, which ensures feasibility of truthful communication for only the lowest type information.

therefore provides an upper bound on the extent of truthful communication. In other words, for any state $\theta > \bar{\theta}$, truthful reporting by A_2 ($m = \theta$) results in under-allocation (miscoordination). As a consequence, A_2 would find it profitable to misreport ($m > \theta$) and induce A_1 to allocate more in order to reduce this inefficiency from miscoordination. As a result, none of the messages beyond $\bar{\theta}$ are credible in any equilibria of the communication game.[12]

Lemma 1 *When HTIC is violated, all types in set G pool on the same message in every equilibrium of the communication game.*

Proof. See Appendix A.2 ■

The intuition behind Lemma 1 is the following. Suppose it was possible for A_2 to partition the set G into two - $G_1 = (\bar{\theta}, \bar{\theta}_g]$ and $G_2 = (\bar{\theta}_g, 1]$. Then, there are always types that are pooled in the first partition for whom A_2's optimal action is constrained by the bound $\bar{k}$. This implies that for some types in G_1, A_2 would have an incentive to exaggerate and pool with the higher types in G_2, precluding the possibility of such a partition in equilibrium. Therefore, in the presence of constraints, two things hold: *i)* at most, there is only partial revelation of information; and *ii)* no credible information is conveyed beyond $\bar{\theta}$. The next proposition characterizes the set of all partially revealing threshold equilibria.

Proposition 2 *Under assumptions 1-5, when HTIC is violated, there are Partially Revealing Threshold Equilibria (PRTE) such that, $\forall \theta^* \in [0, \bar{\theta}]$: $m = \theta$ if $\theta \in [0, \theta^*]$ and $m = 1$ if $\theta \in (\theta^*, 1]$.*

Proof. See Appendix A.3 ■

Two things stand out from Proposition 2. First, it suggests that inflated messaging occurs only above a certain cutoff state, while every message within the cutoff is truthful.[13] Second, A_2 could possibly choose how much information to reveal in equilibrium. Though the PRTE $\theta^* = \bar{\theta}$ is the *most informative equilibrium*, it does not necessarily restrict them from providing less information to A_1. The $\bar{\theta}$ equilibrium provides a lower bound on the communication barrier. In contrast, the pure babbling equilibrium of the game (where $\theta^* = 0$) represents the one with the most communication barrier in which there is complete breakdown of information sharing between the agents.

[12]This resembles the credibility notion of *self-signaling*, identified by Aumann (1990), and Farrell and Rabin (1996). When the unconstrained action is above the bound, it implies that the action constraints are binding, and the equilibrium actions is $x_2^*(\theta) = \bar{k}$. Given imperfect substitutability, the informed agent's action has a *positive spillover* implying that $U_1(\phi^{2^*}(\bar{k}, x_1^*(\theta)), \theta, b) > 0$. This '*positive spillover effect*' implies that communication ceases to be credible, since A_2 (strictly) prefers to induce a higher action from A_1, by inflating her private information. See Baliga and Morris (2002) for more on this point.

[13]On a similar vein, Ottaviani and Squintani (2006) construct a cutoff equilibrium in which messages are revealing (albeit inflated) below the threshold, and for states above the cutoff, information transmission is partitional in nature. See also Kartik (2009) in which the exaggeration in communication is driven by lying costs.

In fact, the informed agent could choose to partition the information within the interval $[0,\bar{\theta})$, instead of revealing them truthfully. This is so because, under any PRTE, the constraints are satisfied with slack for any type in this interval. As a result, there is always a possibility to pool any type $\theta \in [0,\bar{\theta})$ with lower types within the interval such that the incentive compatibility conditions are satisfied. This gives rise to possibly multiple discontinuous partitions (Bernheim and Severinov (2003)).[14] The following proposition characterizes all such *hybrid equilibria*.

Proposition 3 *Hybrid equilibria: Fix a PRTE with threshold $\theta^* < \bar{\theta}$. For every such θ^* equilibrium, there exists partitions in the separating interval $[0,\theta^*]$ such that A_2 pools some types and separates on other types.*

Proof. Appendix A.4 ■

The intuition for the existence of hybrid equilibria is straightforward. The only incentive constraint that requires to be satisfied to sustain pooling of types within the interval is that the marginal type has no incentive to deviate. This is equivalent to requiring that the IC condition —action constraints not binding— is satisfied for the highest type within the pooling interval. However, for any type $\theta < \bar{\theta}$, it is true that $x_2(\theta) < \bar{k}$. From the continuity property of ϕ^2 and $U(.)$ there is always a $\delta > 0$ such that instead of revealing $m = \theta$, if A_2 sends a pooling message $m_{pool} = (\theta - \delta, \theta]$, the optimal action for agent A_2 is such that the constraints are not binding, i.e. $\bar{x}_2(\theta, m_{pool}) \leq \bar{k}$.

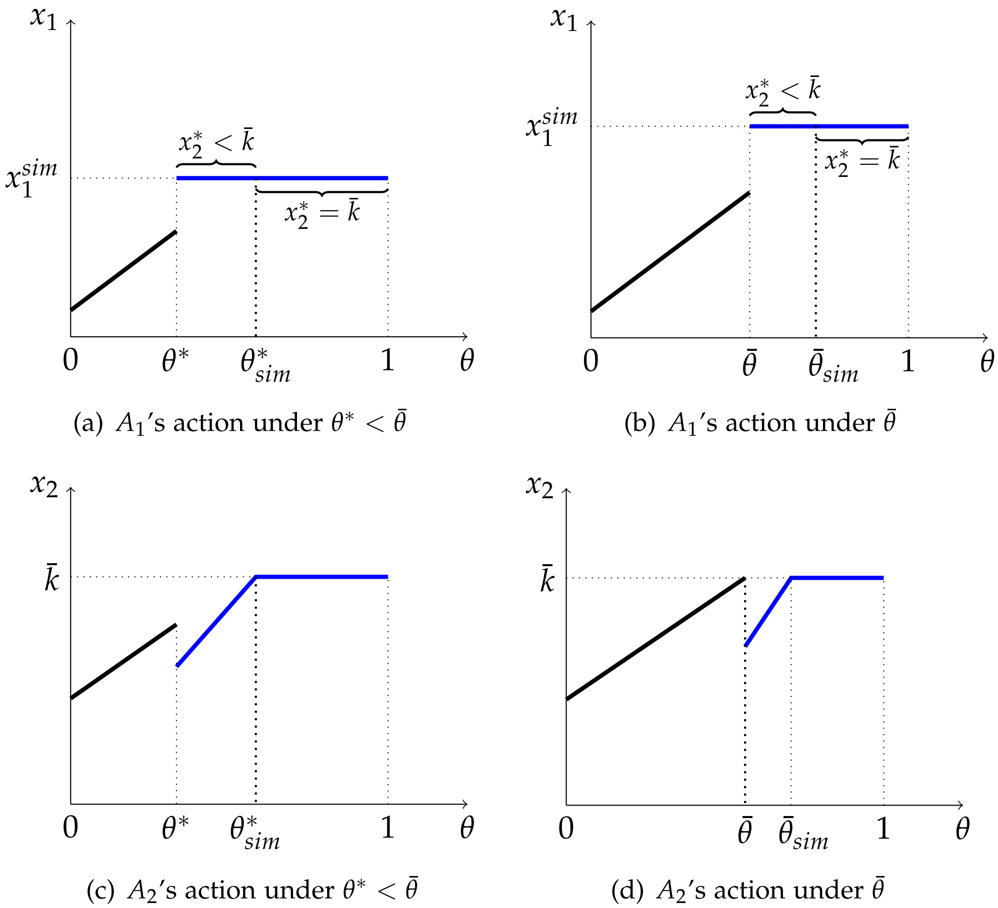
6 Efficiency

As is the case with cheap talk models, there is a multiplicity of equilibria in this setup. An important question that arises is the relationship between information and welfare of agents, or alternatively, the extent of communication barriers and efficiency. To understand the efficiency properties of equilibria, it is necessary to pin down the response of A_1 to a pooling message by A_2 under any threshold equilibrium θ^*. The following lemma characterizes this.

Lemma 2 *For any information threshold θ^*, agent A_1's equilibrium action on receiving the message $m_{pool}^{\theta^*} = (\theta^*, 1]$ is given by $x_1^{sim}(m_{pool}^{\theta^*})$ that solves,*

$$\arg\max_{x_1 \in V} \int_{\theta^*}^{\theta^*_{sim}} U(\phi^1(x_1, x_2^*(t, m_{pool}^{\theta^*})), t) dP(t \mid m_{pool}^{\theta^*}) + \int_{\theta^*_{sim}}^{1} U(\phi^1(x_1, \bar{k}), t) dP(t \mid m_{pool}^{\theta^*})$$

[14] Notice however that in all such equilibria, the types belonging to $G = (\bar{\theta}, 1]$ are always pooled together.



(a) A_1's action under $\theta^* < \bar{\theta}$

(b) A_1's action under $\bar{\theta}$

(c) A_2's action under $\theta^* < \bar{\theta}$

(d) A_2's action under $\bar{\theta}$

Figure 2: i) ▬ *interval of separation*: $m(\theta) = \theta$; ii) ▬ *interval of pooling*: $m_{pool} = 1$

Proof. See Appendix A.5 ■

The above lemma states that the best-response of A_1 to a pooling message entails an important trade off. Specifically, A_1's action is such A_2's action is always binding for some types within the pooling interval, i.e. there exists a measure of types $(\theta^*, \theta^*_{sim}]$ such that $\forall \theta \in (\theta^*, \theta^*_{sim}] : x_2^*(\theta, m_{pool}^{\theta^*}) \leq \bar{k}$ and for all other types $(\theta^*_{sim}, 1]$, $x_2^*(\theta, m_{pool}^{\theta^*}) = \bar{k}$.

Figure 2 illustrates this point. Notice that there is non-monotonicity in A_2's action at θ^* because of the discontinuous jump in A_1's response upon receiving the pooling message. Since A_1's action has a discontinuity at θ^*, the informed agent is able to readjust her actions to achieve first best. Further, since A_1's action is not high enough, there is always an interval of types —$(\theta^*_{sim}, 1]$ —for which the constraint is binding for A_2.

Lemma 2 clearly illustrates the benefit for the informed agent from revealing more information. First, it maximizes welfare on the interval $[0, \theta^*_{sim}]$. Second, as θ^* increases, A_1's action $x_1^{sim} = x_1^*(m_{pool}^{\theta^*})$ also increases on the pooling interval. This further implies that on the interval $(\theta^*_{sim}, 1]$, extracting a higher action from A_1 is welfare improving for the informed agent, given

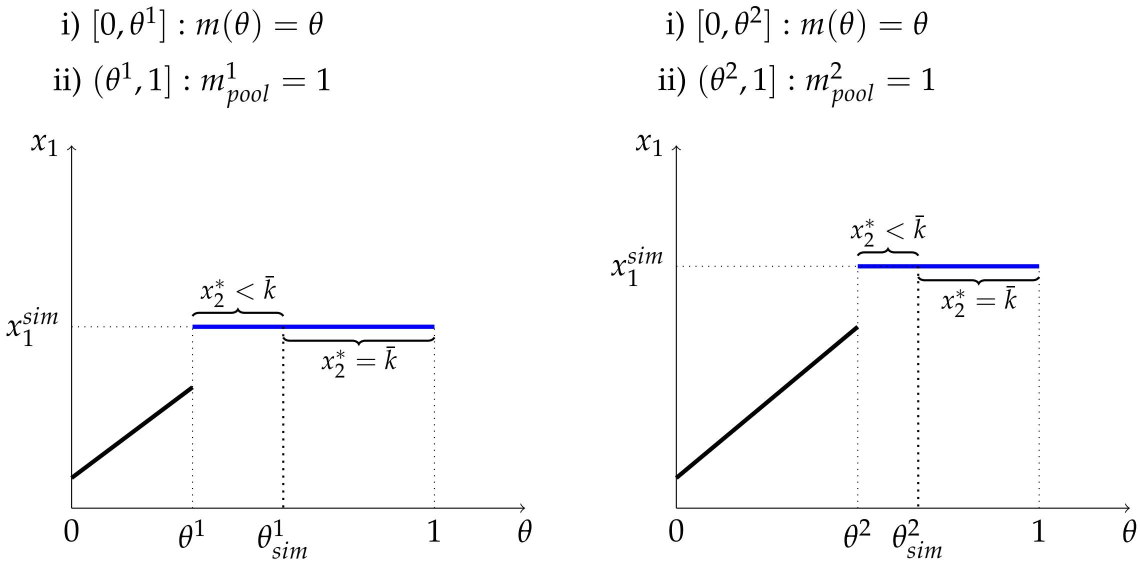


Figure 3: a) $\theta^1 < \theta^2$; b) $\theta^1_{sim} < \theta^2_{sim}$; c) $x_1^{sim}(m^1_{pool}) < x_1^{sim}(m^2_{pool})$

the positive spillover effect.

Proposition 4 *The most informative equilibrium, $\theta^* = \bar{\theta}$, is ex-ante efficient for both agents.*

Proof. See Appendix A.6 ■

Both agents benefit from more information sharing. In other words, the more severe communication barriers are, greater the welfare inefficiencies for the agents. A greater threshold of information means the constraints on the action set is not binding (increases efficiency) for a greater measure of types for A_2 and also entails a higher action from the A_1 on the pooling interval. Both of these effects provide A_2 with a greater ex-ante welfare. Figure 3 shows these trade offs. On the left, under a less informative threshold, A_1's action is lower on the pooling interval and this directly affects the informed agent's ability to achieve first best.

7 Sequential Protocol

Given the partial revelation of information in the presence of binding constraints, there is an efficiency loss for the two agents. In particular, the most informative equilibrium entails welfare losses for the agents. In this section, I study sequential decision-making under which the uninformed agent A_1 moves first and takes a decision after the communication round.

The sequential protocol proceeds as follows:

- A_2 observes the true state $\theta \in [0,1]$, sends a message $m \in M$ to A_1 such that $\mu : [0,1] \to M$

- A_1 observes the message m and commits to an action, which is a decision mapping from the message set to the action set, $\alpha_1 : M \to V$.
- Finally, A_2 observes the actions of A_1 and decides on an action $\alpha_2 : [0,1] \times V \to V$

Notice the critical difference in this protocol. By moving first, the uninformed agent provides an additional layer of information to A_2. The presence of constraints implies that despite such sequential decisions, A_2 is unable to credibly convey the true state beyond the $\bar{\theta}$ threshold. This is driven by the earlier observations under simultaneous protocol. The following proposition lays out this result.

Proposition 5 *Every PRTE under simultaneous protocol is also an equilibrium under sequential protocol.*

Proof. See Appendix A.7 ■

Given a set of actions, under the most informative threshold equilibrium $\bar{\theta}$,[15] the sequential protocol provides the same (ex-ante) welfare to the agents on the interval $[0, \bar{\theta})$ compared to the case of simultaneous actions. The crucial difference between the two protocols arises on the uninformative domain of the state space, when the communication barrier is reached. Since A_1 takes an action before A_2 after the message $m_{pool} = (\bar{\theta}, 1]$, the equilibrium action under simply solves the following:

$$x_1^{pa} = x_1^{pa}(m_{pool}) \equiv \arg\max_{x_1 \in V} \int_{\bar{\theta}}^{\bar{\theta}_{pa}} U\left(\phi^1\left(x_1, x_2^{pa}(t, x_1)\right), t\right) dF + \int_{\bar{\theta}_{pa}}^{1} U\left(\phi^1\left(x_1, \bar{k}\right), t\right) dF \qquad (3)$$

When A_2 observes A_1's action, there is an additional *undoing effect* ($\frac{dx_1}{dx_2} < 0$) in that A_2 can adjust its action depending on the actions of A_1. This undoing effect implies that A_1 takes a higher action on the pooling interval compared to the .

Lemma 3 $x_1^{pa} > x_1^{sim}$

For agent A_2, the welfare improvement under sequential decisions directly follows from Lemma 3. Specifically, A_2's constraint is now binding for a smaller interval of types $[\bar{\theta}_{pa}, 1]$ (see Figure 4). However, over this interval since $x_1^{pa} > x_1^{sim}$ and $U_1 > 0$, the expected utility for

[15] Given the efficiency properties of equilibria, I will henceforth compare the welfare outcomes under only the most informative equilibrium across protocols.

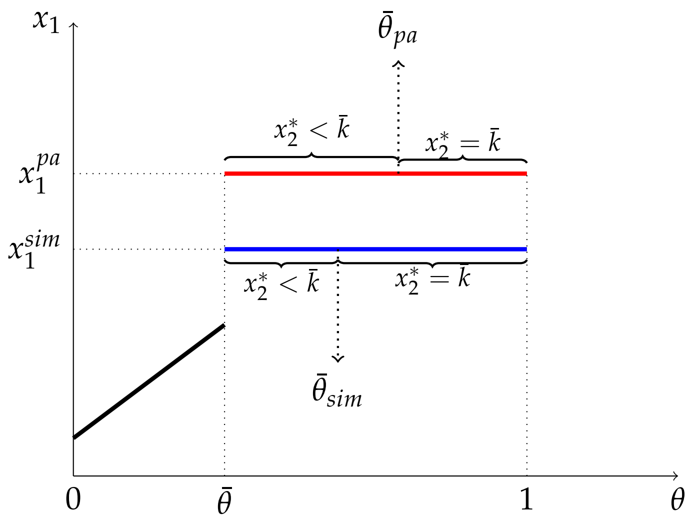


Figure 4: i) $[0, \bar{\theta}]$: *interval of separation*; ii) $(\bar{\theta}, 1]$: *interval of pooling*

A_2 is greater under ex-post commitment. For the uninformed agent A_1, the reason is intuitive. Suppose under sequential protocol A_1 contributes $x_1^{sim} + \epsilon$ on the pooling interval. A_2 observes this action and readjusts her own action downwards. This readjustment is akin to the undoing effect, and it mitigates the extent of miscoordination from over-allocation for A_1, resulting in increased welfare.

Proposition 6 *Sequential Protocol provides a higher ex-ante welfare to both agents compared to Simultaneous Protocol.*

Proof. See Appendix A.8 ■

8 Optimal Commitment

Making the uninformed agent the Stackelberg leader improved overall joint welfare for the agents. However, there is still a source of inefficiency under sequential protocol on the pooling interval. Specifically, there is a discontinuous jump in A_1's action at $\bar{\theta}$ (see Figure 2(b)). Correspondingly, A_2's action is also non-monotonic (see Figure 2(d)). A_1 fails to extract maximum possible action $\bar{k}$ from A_2 on the pooling interval. As a result, A_1 is over-allocating in order to satisfy agent A_2's first best $\bar{\varphi}_\theta^2$ on the interval $(\bar{\theta}, \bar{\theta}_{pa}]$.[16]

By instead committing to an ex-ante action rule, A_1 can mitigate some of this inefficiency. The *commitment rule* is equivalent to A_1 choosing an ex-ante action that is contingent on the

[16]For similar arguments, the same miscoordination concerns are valid under the simultaneous protocol.

information communicated by A_2. The optimal commitment rule problem for A_1 is given by the following:

$$\underset{x_R^{fa}(\theta)\in V}{\text{argmax}} \int_0^1 U\left(\phi^1\left(x_1^{fa}(\theta), x_2^{fa}(\theta, x_1^{fa}(\theta))\right), \theta\right) dF \text{ such that } \forall \theta', \theta'' \in [0,1]:$$

$$U\left(\phi^2\left(x_2^{fa}(\theta', x_1^{fa}(\theta')), x_1^{fa}(\theta')\right), \theta', b\right) \geq U\left(\phi^2\left(x_2^{fa}(\theta', x_1^{fa}(\theta'')), x_1^{fa}(\theta'')\right), \theta', b\right)$$

$$x_2^{fa}(\theta, x_1^{fa}(\theta)) \equiv \underset{x_2\in V}{\text{argmax}}\, U\left(\phi^2(x_2, x_1^{fa}(\theta)), \theta, b\right)$$

The problem for agent A_1 boils down to choosing a sequence of actions for every state $\theta \in [0,1]$ such that it maximizes the agent's overall expected utility conditional on the IC constraint that ensures truthful revelation for all types of A_2's private information. From the Revelation Principle, if A_1 mimics the actions under sequential protocol, it can guarantee at least as much welfare. Such a mimicking strategy would be incentive compatible on the separating interval since A_2 achieves first best. Similarly, on the pooling interval, A_2 cannot do any better than merely revealing its private information as the action rule of A_1 remains fixed at x_1^{pa}.

However, instead of committing to a fixed action on the pooling interval, A_1 can instead commit to a message contingent action rule $x_1^{fa}(\theta)$ for every $\theta \in [0,1]$. The following series of claims must be valid for the commitment mechanism to be optimal for A_1.

Claim 1 *On the separating interval, the commitment rule mirrors the simultaneous protocol, i.e.,* $\forall \theta \in [0, \bar{\theta}] : x_1^{fa}(\theta) = \bar{x}_1(\theta)$.

This follows directly from noting that both agents achieve first best joint action $\bar{\phi}_\theta^i$ on this interval. To see this, the best response of A_2 to $\bar{x}_1(\theta)$ is $\bar{x}_2(\theta)$. Further, the pair of actions $(\bar{x}_2(\theta), \bar{x}_1(\theta))$ is an unique maxima and therefore is incentive compatible for A_2.

Claim 2 *On the pooling interval there is no single flat segment such that* $\forall \theta \in m_{pool} : x_1^{fa}(\theta) = z \geq \bar{x}_1(\bar{\theta})$.

Suppose $x_1^{fa}(\theta) = \bar{x}_1(\bar{\theta})$. Then $\forall \theta \in m_{pool} : x_2(\theta) = \bar{k}$. This cannot be optimal since A_1 can always do better by committing a bit more and satisfying A_2's IC. Instead, suppose $x_1^{fa}(\theta) = z > \bar{x}_1(\bar{\theta})$. Say, for the sake of argument that $z = x_1^{pa}$, i.e. A_1 mimics the action under sequential protocol. This again cannot be optimal since agent A_2's action is less than $\bar{k}$ on the interval $(\bar{\theta}, \bar{\theta}_{pa})$. A_1 can instead always allocate lesser to the project and induce A_2 to contribute $\bar{k}$. Given the imperfect substitutability of actions, this increases the expected payoff of agent A_1 by minimizing miscoordination (from over-allocation) on the pooling interval.

Claim 3 *If $x_1^{fa}(\theta)$ is strictly increasing on any interval (θ_1, θ_2) within m_{pool}, then A_2 must allocate $\bar{k}$ for all types in this interval.*

Claim 3 follows from previous arguments. Again, there are a number of different ways in which A_2's IC can be satisfied on the increasing interval, i.e. multiple pairs (x_1, x_2) satisfy $\forall\theta \in (\theta_1, \theta_2) : \phi^2(x_2, x_1) = \bar{\phi}^2_\theta$. However, of all these pairs that satisfy first best for A_2, the one that minimizes A_1's miscoordination loss is the one in which A_2 contributes $x_2 = \bar{k}$. If this weren't true, A_1 could increase its welfare by decreasing her actions and inducing a higher action from A_2.

Claim 4 *On m_{pool}, there cannot be a flat segment followed by a strictly increasing interval.*

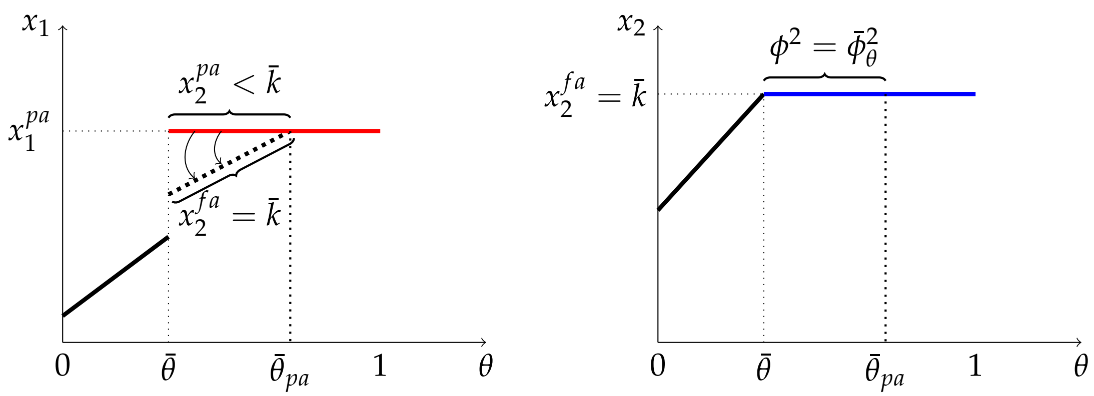
Claim 4 is true since on a flat segment A_1's action is independent of communication. This implies that either A_2's IC is satisfied for all types in that interval or there is inefficiency for some types. If it is the former, then A_1 can improve its payoff by previous arguments (see Claim 3) and extracting $\bar{k}$ from A_2. If it is the latter on the other hand, for types that do not achieve first best, A_2 can always deviate to the strictly increasing interval and benefit from greater actions of A_1, thereby violating IC constraint for truth-telling.

Figure 5 illustrates the consequence of Claim 1 - Claim 4. Specifically, A_1 instead of committing to a single flat action on $(\bar{\theta}, \bar{\theta}_{pa})$, pivots and provides lesser thereby extracting $\bar{k}$ from A_2. A_2 still achieves first best levels $\bar{\phi}^2_\theta$ while for A_1 there is over-allocation resulting in greater miscoordination. However, the miscoordination is lesser compared to the sequential protocol in that $\bar{\phi}^1_\theta < \phi^1(x_1^{fa}(\theta), \bar{k}) < \phi^1(x_1^{pa}, x_2^{pa}(\theta, x_1^{pa}))$. This implies that miscoordination is minimized under the commitment rule leading to an increase in expected utility for A_1. In light of these arguments, the commitment problem can be reformulated as the following:

$$
\begin{aligned}
&\underset{x_1^{fa}(\theta)\in V}{\operatorname{argmax}} \int_{\bar{\theta}}^{\bar{\theta}_{fa}} U\left(\phi^1\left(x_1^{fa}(\theta), \bar{k}\right), \theta\right) dF + \int_{\bar{\theta}_{fa}}^{1} U\left(\phi^1\left(x_1^{fa}(\bar{\theta}_{fa}), \bar{k}\right), \theta\right) dF \text{ such that} \\
&\forall\theta', \theta'' \in [0,1] : U\left(\phi^2\left(\bar{k}, x_2^{fa}(\theta')\right), \theta', b\right) \geq U\left(\phi^2\left(x_2^{fa}(\theta', x_1^{fa}(\theta'')), x_1^{fa}(\theta'')\right), \theta', b\right) \\
&x_1^{fa}(\bar{\theta}_c) \equiv \underset{x_1\in V}{\operatorname{argmax}}\, U\left(\phi^2(\bar{k}, x_1), \theta, b\right)
\end{aligned}
$$

Two important properties of the optimal commitment rule becomes clear from the above reformulation. First, there is *maximal contribution* from A_2 on the interval m_{pool}. Second, A_1 *caps actions* at $\bar{\theta}_{fa}$ by allocating up to $x_1^{fa}(\bar{\theta}_{fa})$ but no more on the interval $(\bar{\theta}_{fa}, 1]$.

Proposition 7 *The optimal action rule for A_1 is given by the following:*



(a) A_1's action

(b) A_2's action

Figure 5: A_1 can commit to an action that is strictly lower than x_1^{pa} on the interval $(\bar{\theta}, \bar{\theta}_{pa})$. Notice this is possible since A_2 can always increase its action to $\bar{k}$ and achieve first best $\bar{\phi}_\theta^2$.

1. $\forall \theta \in [0, \bar{\theta}] : x_1^{fa}(\theta) = \bar{x}_1(\theta)$
2. $\forall \theta \in (\bar{\theta}, \bar{\theta}_{fa}] : x_1^{fa}(\theta) \equiv \arg\max_{x_1} U(\phi^2(\bar{k}, x_1), \theta, b)$
3. $\forall \theta \in (\bar{\theta}_{fa}, 1] : x_1^{fa}(\theta) = x_1^{fa}(\bar{\theta}_{fa})$

Proof. See Appendix A.9 ■

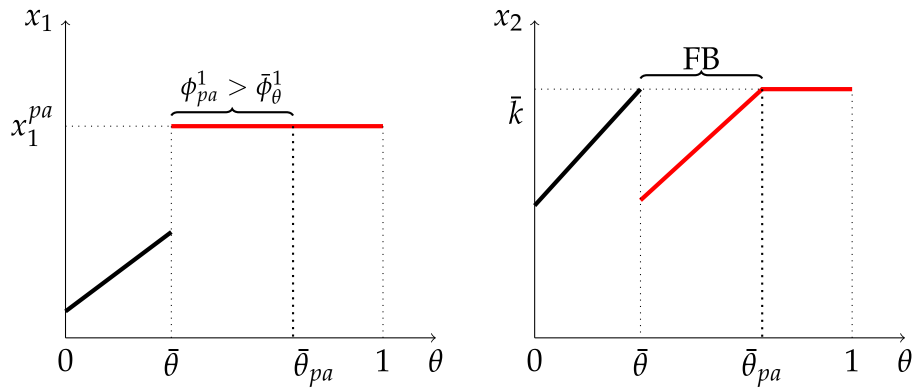
The optimal action rule mimics the sequential (simultaneous) protocol on the separating interval $[0, \bar{\theta}]$. On the pooling interval, the rule provides the first best levels of coordination function for A_2 up to some (higher) threshold $\bar{\theta}_{fa}$ and then is unchanged beyond. The optimal rule exhibits two key features. First, it is discontinuous at exactly $\bar{\theta}$ and nowhere else. Second, on the interval $(\bar{\theta}, \bar{\theta}_{fa}]$ where A_1's actions are strictly increasing, agent A_2's action is constant and fixed at $\bar{k}$. That is, out of all possible incentive compatible commitment rules the one that maximizes A_1's expected utility is the one that induces the highest action from A_2.[17]

Proposition 8 *The optimal commitment rule improves ex-ante welfare of both agents compared to the case of sequential protocol.*

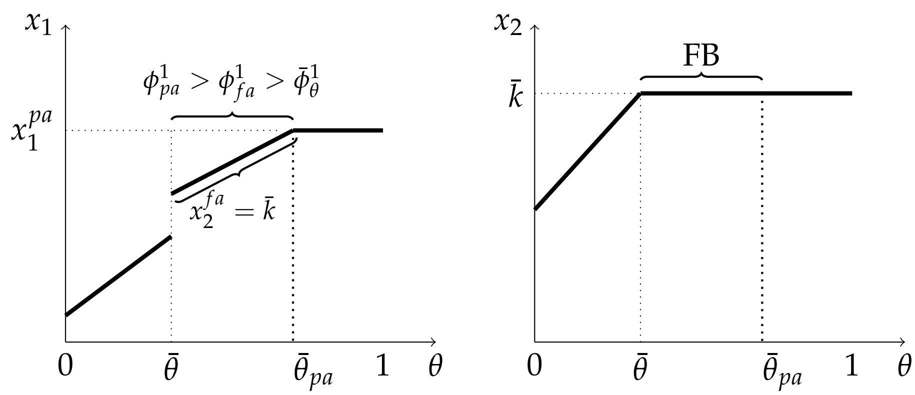
Proof. See Appendix A.10 ■

The intuition is the following. Notice that A_1's commitment problem is equivalent to choosing a cutoff threshold $\bar{\theta}_{fa}$ up to which there is strictly increasing actions. In other words, agent A_1's problem is equivalent to choosing a cutoff $\bar{\theta}_{fa}$ and a corresponding cap on contributions $x_2^{fa}(\bar{\theta}_{fa})$ such that A_2 takes the maximal action $\bar{k}$ and achieves first best up to $\bar{\theta}_{fa}$. Suppose A_1

[17]That is, $\forall \theta \in (\bar{\theta}, \bar{\theta}_{fa}]$, $\phi^2(\bar{k}, x_1) = \phi^2(\bar{k} - \epsilon, x_1 + \gamma) = \bar{\phi}_\theta^2$, implies that $U(\phi^1(x_1, \bar{k}), \theta) > U(\phi^1(x_1 + \gamma, \bar{k} - \epsilon), \theta)$.



(a) A_1 action under sequential protocol (b) A_2 action under sequential protocol



(c) A_1 action under commitment rule (d) A_2 action under commitment rule

Figure 6: Under the ex ante commitment, A_1 can pivot and induce $\bar{k}$ from A_2 (see 6(b),6(d)). This way, the miscoordination loss is mitigated for A_1 (6(c)).

mimics the sequential protocol actions (see Figure 6) on the separating interval and chooses the cutoff $\bar{\theta}_{pa}$ and a cap $x_1^{fa}(\bar{\theta}_{pa}) = x_1^{pa}(\bar{\theta}_{pa}) = x_1^{pa}$ such that it satisfies maximal action on $(\bar{\theta}, \bar{\theta}_{pa}]$ (meaning $x_2^{fa}(\theta) = \bar{k}$ on this interval).

Since A_1 induces A_2 to contribute $\bar{k}$, and commits to taking the residual action required to satisfy her IC constraint (Figure 6(d)), the marginal utility for A_2 is strictly increasing at $(\bar{\theta}_{pa}, x_1^{fa}(\bar{\theta}_{pa}))$. This implies that the threshold is greater than $\bar{\theta}_{pa}$, and ipso facto, the cap on contributions with commitment is also higher. That is, $\bar{\theta}_{fa} > \bar{\theta}_{pa}$ and $x_1^{fa}(\bar{\theta}_{fa}) > x_1^{fa}(\bar{\theta}_{pa})$ (see Figure 7). For A_2, as argued previously, welfare is strictly increasing in the cutoff threshold and therefore the optimal commitment rule is welfare improving since A_2 achieves first best on $[0, \bar{\theta}_{fa}]$ and on the interval $(\bar{\theta}_{fa}, 1]$, the cap under the commitment rule is greater than under sequential decision-making.

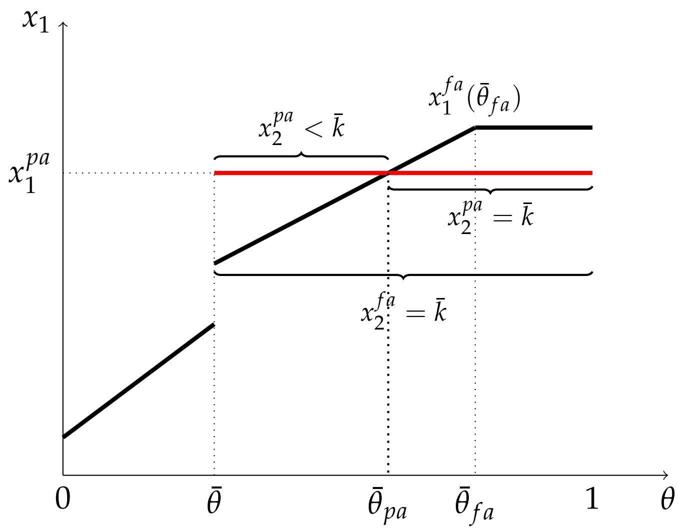


Figure 7: The optimal mechanism exhibits two key features. On the interval $(\bar\theta, 1]$, there is maximal contribution from A_2 ($x_2^{fa} = \bar k$). Further, A_1 places a cap on actions given by $x_1^{fa}(\bar\theta_{fa})$.

Extensions

Lying costs

The equilibrium in both protocols exhibits some level of lying by the informed agent. Experimental evidence suggests that there is an intrinsic propensity to say the truth even when the information conveyed is *soft* (Gneezy, 2005; Hurkens and Kartik, 2009), suggesting an aversion to lying. In international alliances, misrepresentation of information by a national leader could lead to distrust and reputational loss, especially when it is possible for the uninformed members to learn about the true state of the world ex-post.

Introducing lying costs changes the incentives of the informed agent drastically. Suppose, for sake of exposition, lying costs are minimized when the messages are truthful (i.e. $\mu(\theta) = \theta$). Then, the presence of lying costs eliminates all but the most informative equilibrium under both simultaneous and sequential protocols. The intuition is that there is now a lying cost associated with wrongful reporting for no marginal benefit in utility. (On the interval $[0, \bar\theta]$, $U_1\left(\phi^2(x_2(\theta,\theta), x_1(\theta)), \theta, b\right) = 0$ implying that truthful reporting is indeed a solution.) That is, by lying, A_2 does no better than under truthful reporting but incurs a *wasteful cost* by pooling with the other types and sending $\tilde m = 1$. This implies that there is an unique separating equilibrium on $[0, \bar\theta]$ such that $\mu(\theta) = \theta$.

What is left to consider is the equilibrium messaging on the pooling interval, $m_{pool} = (\bar\theta, 1]$. One way to interpret my results is by considering them as the limit case of a game with lying costs. As the intensity of lying costs goes to zero, the equilibrium messaging is truthful on $[0, \bar\theta]$

and all other types send the message $m = 1$. Specifically, when the intensity of lying is very small, there is an incentive to (almost) costlessly exaggerate information beyond $\bar{\theta}$, resulting in the maximally exaggerated message at the limit, $\lim_{\theta \downarrow \bar{\theta}} \mu(\theta) = 1$. On the other hand, when the lying costs are sufficiently high, there is full separation as the incentives to exaggerate are counteracted by the lying costs.

The interesting case is when the lying costs are sufficiently high but not prohibitively so. It is then possible for alternate equilibrium messaging strategies to emerge. For example, agent A_2 could *bunch* state space and send the same (possibly inflated) message for every type in this partition, resulting in clustering of A_2's private information (Chen, 2011) on the interval m_{pool}. In this case the intensity of the cost parameter, the bias and degree of substitutability would together determine the indifference condition that characterizes such a *clustering equilibria*.[18]

Verifiable Information Disclosure

So far, the analysis has focused mainly on transmission of *soft information*. In many projects the nature of information is verifiable (Grossman (1981); Milgrom (1981)). The informed agent can provide verifiable information about project quality, for example. Alternatively, the project contract might specify evidence provision as a requirement. When information can be verified, the incentives for communication change completely. There is unraveling in the sense that A_2 would always find it optimal to reveal every state truthfully, leading to full information transmission even in the presence of action constraints. This is straightforward to observe. On the pooling interval, for the highest state $\theta = 1$, A_2 is better off revealing. This way, $x_1(1) > x_1(m_{pool})$ and since there are under-allocation concerns for A_2 on this interval ($U_1(\phi^2(\bar{k}, x_1(m_{pool})), 1, b) > 0$), it follows that revealing the highest state by providing verifiable evidence improves the agent's utility. However, this argument holds for all states below as well and there is complete unraveling.

9 Conclusion

The paper investigates the role of communication and commitment when there are (one-sided) information asymmetries between agents. When agents' decisions are substitutable and they face action constraints, under simultaneous decision-making protocol, there is only partial information revelation (communication barriers) in equilibrium. There is a positive relationship

[18] Chen (2011) finds clustering and inflated messaging in a completely different setup. In Chen's work, there is a small prior probability that an informed sender is honest (always reports truthfully) and the uninformed receiver is naive (always believes the message). This leads to message inflation and clustering at the top end of the message spectrum.

between amount of information revealed and efficiency, in that welfare of both agents are strictly increasing in the extent of information shared. With sequential decision making, the total amount of information conveyed remains unchanged but the two agents' welfare improves.

With ex-ante commitments, the uninformed agent commits to minimizing the mis-coordination losses up to a threshold and also caps actions beyond this threshold of information. The optimal commitment mechanism increases the payoff to both agents compared to the other protocols. The analysis provides an informational and efficiency rationale for the use of binding commitments in international alliances.

There are potentially other incentive problems associated with the presence of constraints that are worth exploring. For example, when there is two sided incomplete information, constraints might exacerbate the communication barriers between agents. In fact, as information is more dispersed, the inefficiencies emerging from constraints might worsen leading to decreased welfare. Alternatively, when players instead have a coordination motive with strategic complementarity in actions, constraints might still play a similar role in constraining the credibility of information. Another avenue for future research is to endogenize the investment in the action set. Though constraints were assumed to be exogenous in this paper, it could very well be that agents invest in actions ex-ante at some marginal cost. SInce the domain of actions available to each player determines the extent of information revealed, this investment decision might differ according to what the underlying decision-making protocol is. All such scenarios require a more detailed analysis, and are left for future work.

A Appendix

A.1 Proof of Proposition 1

Sufficiency

Let $\bar{\phi}^2_\theta = \phi^2(\bar{x}_2(\theta), \bar{x}_1(\theta))$ be the first best levels of contribution for A_2 when the state is θ. When HTIC condition is satisfied, it implies that for every other $\theta \in [0, 1)$, $\bar{x}_2(\theta) < \bar{k}$ by single crossing property of the utility function ($U_{12} > 0$). But if this is the case, when A_2 sends a truthful message $m = \theta$, the optimal action under both constrained and unconstrained (no action constraints) optimization coincide remains the same. This means that for every $\theta \in [0, 1]$, $x^*_2(\theta) = \bar{x}_2(\theta)$. Since A_1 does not face any constraints, it also implies that $x^*_1(\theta) = \bar{x}_1(\theta)$ and $\bar{\phi}^2_\theta = \phi^2(x^*_2(\theta), x^*_1(\theta))$. This ensures there is no inefficiency and A_2 always achieves first best levels of coordination function for every θ. Hence, there exists an equilibrium in which there is full information revelation.

Necessity

Suppose HTIC is violated ($\bar{x}_2(1) > \bar{k}$) but there is full information revelation by A_2. Then, by definition, there exists a non-empty set $G = \{\theta : \bar{x}_2(\theta) > \bar{k}\}$. When HTIC is violated, the action under unconstrained best response does not coincide with the equilibrium actions that are bounded by the action constraint, ie $\forall \theta \in G : x^*_2(\theta) = \bar{k} < \bar{x}_2(\theta)$ under truthful revelation. Now take a $\theta^{'} \in G$. If A_2 reports $\theta^{'}$, the optimal actions are $x^*_2(\theta^{'}) = \bar{k}$ and $x^*_1(\theta^{'})$ solves $\max_{x_1 \in V} U\left(\phi^1(x_1, \bar{k}), \theta^{'}\right)$. However, given imperfect substitutability, $\phi^1(x^*_1(\theta^{'}), \bar{k}) < \phi^2(\bar{k}, x^*_1(\theta^{'}))$ $< \phi^2(\bar{x}_2(\theta^{'}), \bar{x}_1(\theta^{'})) \equiv \bar{\phi}^2_{\theta^{'}}$. But, because HTIC is violated, the coordination function under truth-telling is $\phi^2(\bar{k}, x^*_1(\theta^{'}))$ which is clearly not optimal for A_2 in the sense that $U_1\left(\phi^2(\bar{k}, x^*_1(\theta^{'})), \theta^{'}, b\right)$ > 0. From continuity, there exists an ϵ such that if A_2 deviates and sends a message $m = \theta^{'} + \epsilon$, it induces equilibrium actions $x^*_1(\theta^{'} + \epsilon) > x^*_1(\theta^{'})$ and $x^*_2(\theta^{'}, \theta^{'} + \epsilon) = \bar{k}$. This way A_2 can guarantee a higher payoff since $\phi^2(\bar{k}, x^*_1(\theta^{'} + \epsilon)) > \phi^2(\bar{k}, x^*_1(\theta^{'}))$ and,

$$U(\phi^2(\bar{k}, x^*_1(\theta^{'} + \epsilon)), \theta^{'}, b) > U(\phi^{2^*}(\bar{k}, x^*_1(\theta^{'})), \theta^{'}, b)$$

However, this means that A_2 has an incentive to deviate and send an exaggerated message, precluding truthful communication. This is a contradiction. **QED**

A.2 Proof of Lemma 1

Suppose there exists an equilibrium in messaging strategy such that some types in $G = (\bar{\theta}, 1]$ send a different message from $m = 1$. Since I consider monotonic equilibria, wlog, $\exists \theta^{'} \in G$ such that types in $(\bar{\theta}, \theta^{'}]$ send a message $m^{'}$ and types in $(\theta^{'}, 1]$ send a message $m^{''}$, where $m^{'} < m^{''}$. Then, from single crossing ($U_{12} > 0$) it follows that $x_1^*(m^{'}) \equiv \underset{x_1 \in V}{\text{argmax}}\, \mathbb{E}_\theta U\left(\phi^1(x_1, x_2^*(\theta, m^{'})), \theta\right)$ and $x_1^*(m^{''}) \equiv \underset{x_1 \in V}{\text{argmax}}\, \mathbb{E}_\theta U\left(\phi^1(x_1, x_2^*(\theta, m^{''})), \theta\right)$ are such that $x_1^*(m^{'}) < x_1^*(m^{''})$. By a similar argument, $x_1^*(\theta^{'}) \equiv \underset{x_1 \in V}{\text{argmax}}\, U\left(\phi^1(x_1, x_2^*(\theta^{'})), \theta^{'}\right)$ must be such that $x_1^*(m^{'}) > x_1^*(\theta^{'}) > x_1^*(m^{''})$. $x_1^*(\theta^{'})$ is simply A_1's equilibrium action when A_2's message is truthful (i.e. $m = \theta^{'}, p(\theta^{'}|m) = 1$).

But if this were the case, at $m = \theta^{'}$, $\tilde{x}_2(\theta^{'}) > \bar{k} \implies x_2^*(\theta^{'}) = \bar{k}$. Further, for A_2 the utility is increasing at $m = \theta^{'}$, i.e. $U_1\left(\phi^2(\bar{k}, x_1^*(\theta^{'})), \theta^{'}, b\right) > 0$. This is driven by Assumption 3, since A_1 chooses an action x_1 to achieve $\bar{\phi}^1_{\theta^{'}} < \bar{\phi}^2_{\theta^{'}}$. However, if $U_1\left(\phi^2(\bar{k}, x_1^*(\theta^{'})), \theta^{'}, b\right) > 0$ and $U_{11} < 0$, it implies that the following holds:

$$U\left(\phi^2(\bar{k}, x_1^*(\theta^{'})), \theta^{'}, b\right) > U\left(\phi^2(\bar{k}, x_1^*(m^{'})), \theta^{'}, b\right)$$

The payoff to A_2 from sending a truthful message at $\theta^{'}$ is greater than from pooling with some lower types and sending the message $m^{'}$. Given Assumption 5, it holds that $\underline{k} < x_2^*(\theta^{'}, m^{''})$. A_2's equilibrium action from sending a pooling message $m^{''}$ when the true state is $\theta^{'}$ is always within the available domain of actions. But if this were true, there are two possibilities.
If $x_2^*(\theta^{'}, m^{''}) = \bar{k}$, then it holds that

$$U\left(\phi^2(\bar{k}, x_1^*(m'')), \theta^{'}, b\right) > U\left(\phi^2(\bar{k}, x_1^*(m^{'})), \theta^{'}, b\right)$$

If $x_2^*(\theta^{'}, m^{''}) < \bar{k}$, then $\phi^2(x_2^*(\theta^{'}, m^{''}), x_1^*(m'')) = \bar{\phi}^2_{\theta^{'}}$ meaning that A_2 achieves first best levels of the coordination function in which case,

$$U\left(\bar{\phi}^2_{\theta^{'}}, \theta^{'}, b\right) > U\left(\phi^2(\bar{k}, x_1^*(m^{'})), \theta^{'}, b\right)$$

As a result, A_2 with private information $\theta^{'}$ would always deviate and send the higher pooling message $m^{''}$. This argument holds for higher partitions and when types in G are pooled with types in $[0, \bar{\theta}]$. This completes the proof. **QED**

A.3 Proof of Proposition 2

Consider the following construction of PRTE for a threshold θ^*:

- If $\theta \leq \theta^*$, $m = \theta$; if $\theta > \theta^*$, $m = 1$.
- If $m \leq \theta^*$, $p(\theta \mid m = \theta) = 1$; if $m = 1$, $p(\theta \mid m) = f(\theta)$
- When $m \leq \theta^*$: $x_2^*(m) = \bar{x}_2(m)$ and $x_1^*(m) = \bar{x}_1(m)$
- When $m = 1$:

  $x_2^*(\theta, m) \equiv \arg\max_{x_2 \in V} U\left(\phi^2(x_2, x_1^*(m)), \theta, b\right)$

  $x_1^*(m) \equiv \arg\max_{x_1 \in V} \int_{\theta^*}^{1} U\left(\phi^1(x_1, x_2^*(\theta, m)), \theta\right) f(\theta) d\theta$
- When $m \in (\theta^*, 1)$: $p(\theta^* | m) = 1$.

The first condition says that for all states in $[0, \theta^*]$, A_2 communicates truthfully, and for any state above, pools by sending an exaggerated message $m = 1$. The second condition describes the formation of posterior beliefs. For any message on $[0, \theta^*]$, A_1 believes it to be separating and for messages $m = 1$, the posterior is just the conditional prior on the state space. The third and fourth statements indicate the equilibrium actions conditional on the message and posterior beliefs of A_1. The final condition rules out any profitable off-equilibrium path deviations. For off-equilibrium path messages $m \in (\theta^*, 1)$, A_1 assigns the belief $\theta = \theta^*$, that is the deviation comes from the highest possible truth-telling type.

Then, for an equilibrium with cutoff θ^* to exist, there should be no profitable deviations for any type. To check this, consider the types in $(0, \theta^*]$ and $(\theta^*, 1]$. For any $\theta \in (0, \theta^*]$, A_2 does not have an incentive to deviate from truth telling since it achieves first best levels $\bar{\phi}^2_\theta$, ie $x_2^*(\theta) = \bar{x}_2(\theta) \leq \bar{k}$.

For types $\theta \in (\theta^*, 1]$, the payoff from sending $m = 1$ is still higher than sending any other off-equilibrium path message. There are possible two cases to consider.

Case i): $x_2^*(\theta, m) < \bar{k}$

In this case, A_2 achieves first best in that the agent can do no better than under $m = 1$.

Case ii): $\bar{x}_2(\theta, m) > \bar{k}$

Here, the informed agent is constrained by the bound meaning there is some under-allocation for A_2 (meaning $U_1 > 0$). Notice that $x_1^*(m) > x_1^*(\theta^*)$ which means that A_1's action is higher upon receiving the pooling message $m = 1$ resulting in a discontinuity at θ^*. However, since $\phi_2^2(\bar{k}, x_1) > 0$ and $U_1 > 0$ for A_2 at the bound, a higher action from A_1 reduces the inefficiency from miscoordination. Given that $x_1^*(m) > x_1^*(\theta^*)$, it follows that $U\left(\phi^2(\bar{k}, x_1^*(m)), \theta, b\right) > U\left(\phi^2(\bar{k}, x_1^*(\theta^*)), \theta, b\right)$ for all such θ. This concludes the proof. **QED**

A.4 Proof of Proposition 3

Take any PRTE with threshold θ^*. I make the following claim.

Claim: $\forall \theta^{'} \in (0, \theta^*), \exists\, \epsilon > 0 : \forall \theta \in (\theta^{'} - \epsilon, \theta^{'}]$,

$$U\left(\phi^2\left(x_2^*(\theta), x_1^*(\theta)\right), \theta, b\right) = U\left(\phi^2\left(x_2^*(\theta, m_{(\theta'-\epsilon,\theta']}), x_1^*(m_{(\theta'-\epsilon,\theta']})\right), \theta, b\right)$$

Where the message $m_{(\theta'-\epsilon,\theta']}$ simply implies that the type is in the interval $(\theta^{'} - \epsilon, \theta^{'}]$. The claim just states that for any separating type $\theta^{'}$, it is possible to find a pooling interval of types $m_{pool} = m_{(\theta'-\epsilon,\theta']}$ such that the indifference condition holds for all types within this interval, i.e. each of the types in the pooling interval is indifferent between the separating message and the pooling one. The indifference (IC) condition merely requires that A_2 is able to achieve $\bar{\phi}^S_\theta$ which is possible as long as best responses are within the constraints.

To show this, all we need to check for are the indifference conditions of the boundary types $\theta^{'} - \epsilon$ and $\theta^{'}$,

$$U\left(\phi^2\left(x_2^*(\theta^{'}), x_1^*(\theta^{'})\right), \theta^{'}, b\right) = U\left(\phi^2\left(x_2^*(\theta^{'}, m_{(\theta'-\epsilon,\theta']}), x_1^*(m_{(\theta'-\epsilon,\theta']})\right), \theta^{'}, b\right)$$
$$U\left(\phi^2\left(x_2^*(\theta^{'}-\epsilon), x_1^*(\theta^{'}-\epsilon)\right), \theta^{'}-\epsilon, b\right) = U\left(\phi^2\left(x_2^*(\theta^{'}-\epsilon, m_{(\theta'-\epsilon,\theta']}), x_1^*(m_{(\theta'-\epsilon,\theta']})\right), \theta^{'}-\epsilon, b\right)$$

The latter condition follows from noting that any upward deviation is always within the domain of available actions (from Assumption 5). That is, $x_1^*(\theta^{'}-\epsilon) > x_1^*(m_{(\theta'-\epsilon,\theta']})$ from single crossing ($U_{12} > 0$) and $x_2^*(\theta^{'}-\epsilon) < x_2^*(\theta^{'}-\epsilon, m_{(\theta'-\epsilon,\theta']})$ due to imperfect substitutability. However, $\phi^2\left(x_2^*(\theta^{'}-\epsilon), x_1^*(\theta^{'}-\epsilon)\right) = \phi^2\left(x_2^*(\theta^{'}-\epsilon, m_{(\theta'-\epsilon,\theta']}), x_1^*(m_{(\theta'-\epsilon,\theta']})\right) = \bar{\phi}^2_{\theta'-\epsilon}$ meaning that A_2 achieves first best levels of coordination function for the type $\theta^{'} - \epsilon$ irrespective of whether the message is a separating or pooling one.

The former condition states that the type $\theta^{'}$ would pool with lower types and be indifferent from separating. To see this, notice that $x_2^*(\theta^{'}) = k^{'} < \bar{k}$ under a separating (truthful) message. By continuity, there must exist a ϵ-deviation such that the $x_2^*(\theta^{'}, m_{(\theta'-\epsilon,\theta']}) \in (k^{'}, \bar{k}]$. If this were not true, then $\lim_{\epsilon\to 0} x_2^*(\theta^{'}, m_{(\theta'-\epsilon,\theta']}) = k^{'} < \bar{k}$, a contradiction. As before, since $x_2^*(\theta^{'}) < x_2^*(\theta^{'}, m_{(\theta'-\epsilon,\theta']})$ it follows (from Assumption 3 and SC) that $x_1^*(\theta^{'}) > x_1^*(m_{(\theta'-\epsilon,\theta']})$ but $\phi^2\left(x_2^*(\theta^{'}), x_1^*(\theta^{'})\right) = \phi^2\left(x_2^*(\theta^{'}, m_{(\theta'-\epsilon,\theta']}), x_1^*(m_{(\theta'-\epsilon,\theta']})\right) = \bar{\phi}^2_{\theta'}$. If not, A_2 can always increase actions up to the point where it achieves first best. Therefore, there is always the possibility of pooling within any PRTE. This completes the proof. **QED**

A.5 Proof of Lemma 2

Take a pooling message $m_{pool} = (\theta^*, 1]$ associated with the PRTE θ^*. Suppose, A_1's response $x_1^*(m_{pool})$ is such that $\forall\theta \in (\theta^*, 1) : x_2^*(\theta, m_{pool}) < \bar{k}$ and $x_2^*(1, m_{pool}) = \bar{k}$. This means that A_2 achieves $\bar{\phi}_\theta^S$ for every type in the interval m_{pool}. Then, evaluating the FOC of the A_1 gives,

$$\int_{\theta^*}^{1} U_1\left(\phi^1\left(x_1^*(m_{pool}), x_2^*(\theta, m_{pool})\right), \theta\right)\phi_1^1 f(\theta)d\theta \tag{4}$$

When A_2 achieves first best, it must be that $\phi^2\left(x_2^*(\theta, m_{pool}), x_1^*(m_{pool})\right) = \bar{\phi}_\theta^2$. But this implies that there is miscoordination for A_1 in that $\phi^1\left(x_1^*(m_{pool}), x_2^*(\theta, m_{pool})\right) > \bar{\phi}_\theta^1$. This further entails that $U_1\left(\phi^1\left(x_1^*(m_{pool}), x_2^*(\theta, m_{pool})\right), \theta\right) < 0$ on the interval $(\theta^*, 1]$. From this, it follows that equation 4 is less than zero. This means that A_1's action cannot be such that the constraint is not binding for A_2, for every types in m_{pool}.

Since $x_2^*(\theta^*) \leq \bar{k}$, from continuity property, it follows that $\exists\theta_{sim}^* \in (\theta^*, 1] : \forall\theta \in (\theta^*, \theta_{sim}^*), x_2^*(\theta) \leq \bar{k}$ and $\forall\theta \in [\theta_{sim}^*, 1], x_2^*(\theta) = \bar{k}$. This completes the proof. **QED**

A.6 Proof of Proposition 4

Let $W_1(\theta^*)$ and $W_2(\theta^*)$ be the ex-ante welfare of the two agents respectively. I will write them down in terms of the cutoff threshold θ^*.

A_1 **Welfare:**

$$W_1(\theta^*) = \int_0^{\theta^*} U\left(\phi^1\left(x_1^*(t), x_2^*(t)\right), t\right) f(t)dt + \int_{\theta^*}^{1} U\left(\phi^1\left(x_1^*(m_{pool}^{\theta^*}), x_2^*(t, m_{pool}^{\theta^*})\right), t\right) f(t)dt$$

Taking the derivative of A_1's welfare with respect to θ^*,

$$\frac{dW_1(\theta^*)}{d\theta^*} = \left[U\left(\phi^1\left(x_1^*(\theta^*), x_2^*(\theta^*)\right), \theta^*\right) - U\left(\phi^1\left(x_1^*(m_{pool}^{\theta^*}), x_2^*(\theta^*, m_{pool}^{\theta^*})\right), \theta^*\right)\right] f(\theta^*) > 0$$

for any $\theta^* \leq \bar{\theta}$ since $\phi^1\left(x_1^*(\theta^*), x_2^*(\theta^*)\right) = \bar{\phi}_{\theta^*}^1$, the first best levels of coordination. Further, there is a discontinuous jump at θ^* following a pooling message, implying that

$\left|\phi^1\left(x_1^*(\theta^*), x_2^*(\theta^*)\right) - \phi^1\left(x_1^*(m_{pool}^{\theta^*}), x_2^*(\theta^*, m_{pool}^{\theta^*})\right)\right| > 0$ at θ^*.

A_2 **Welfare:**

Take any two cutoff equilibria $\theta^1, \theta^2 \leq \bar{\theta}$, call them $PRTE_1$ and $PRTE_2$, such that $\theta^1 < \theta^2$ (wlog). Let the corresponding pooling messages associated with the PRTE be $m_{pool}^1 = (\theta^1, 1]$ and $m_{pool}^2 = (\theta^2, 1]$ respectively. I will establish that A_2 is better off with the more informative equilibrium θ^2. Similar to arguments made in Lemma 2, for cutoff equilibria θ^1, θ^2 there exists a corresponding θ_{sim}^1 and θ_{sim}^2 such that $x_2^*(\theta_{sim}^1, m_{pool}^1) = x_2^*(\theta_{sim}^2, m_{pool}^2) = \bar{k}$.
From SC property, A_1s action must be higher for the pooling message m_{pool}^2 corresponding to the threshold θ^2, i.e. $x_1^*(m_{pool}^2) > x_1^*(m_{pool}^1)$. If this is true, then $\theta_{sim}^1 < \theta_{sim}^2$. Suppose not, and $\theta_{sim}^1 > \theta_{sim}^2$. Then, $x_2^*(\theta_{sim}^2, m_{pool}^1) < x_2^*(\theta_{sim}^1, m_{pool}^1) = \bar{k}$. But $x_2^*(\theta_{sim}^2, m_{pool}^1) \geq x_2^*(\theta_{sim}^2, m_{pool}^2) = \bar{k}$. This is a contradiction. Therefore the claim holds. In order to prove the result for A_2, I consider two possible scenarios.

Scenario (a): *When* $\theta_{sim}^1 < \theta^2$. That is, $\theta^1 < \theta_{sim}^1 < \theta^2 < \theta_{sim}^2$. The welfare to A_2 under the two PRTE's is given by,

$$W_2(\theta^1) = \int_0^{\theta^1} U\left(\phi^2\left(x_2^*(t), x_1^*(t)\right), t, b\right) f(t)dt + \int_{\theta^1}^1 U\left(\phi^2\left(x_2^*(t, m_{pool}^1), x_1^*(m_{pool}^1)\right), t, b\right) f(t)dt$$

$$W_2(\theta^2) = \int_0^{\theta^2} U\left(\phi^2\left(x_2^*(t), x_1^*(t)\right), t, b\right) f(t)dt + \int_{\theta^2}^1 U\left(\phi^2\left(x_2^*(t, m_{pool}^2), x_1^*(m_{pool}^2)\right), t, b\right) f(t)dt$$

Under $PRTE_1$, A_2's equilibrium action is within the bound for the interval $(0, \theta_{sim}^1]$. Since $\theta_{sim}^1 < \theta_{sim}^2$, A_2's action is also within the bound over the interval $(0, \theta_{sim}^1]$ under $PRTE_2$. Therefore, what is left to be checked are those states in which the constraints are binding for A_2. In

$PRTE_1$, this corresponds to the interval $(\theta^1_{sim}, 1]$. On the same interval, I compare the expected (ex-ante) utility under $PRTE_2$. I will refer to this utility as the residual welfare that results from inefficiency, $W_2^{RES}(\theta^1)$ and $W_2^{RES}(\theta^1)$ respectively.

$$W_2^{RES}(\theta^1) = \int_{\theta^1_{sim}}^{\theta^2_{sim}} U\left(\phi^2\left(\bar{k}, x_1^*(m^1_{pool})\right), t, b\right) f(t)dt + \int_{\theta^2_{sim}}^{1} U\left(\phi^2\left(\bar{k}, x_1^*(m^1_{pool})\right), t, b\right) f(t)dt$$

$$W_2^{RES}(\theta^2) = \int_{\theta^1_{sim}}^{\theta^2} U\left(\phi^2\left(x_2^*(t), x_1^*(t)\right), t, b\right) f(t)dt + \int_{\theta^2}^{\theta^2_{sim}} U\left(\phi^2\left(x_2^*(t, m^2_{pool}), x_1^*(m^2_{pool})\right), t, b\right) f(t)dt + \int_{\theta^2_{sim}}^{1} U\left(\phi^2\left(\bar{k}, x_1^*(m^2_{pool})\right), t, b\right) f(t)dt$$

Taking the expression $W_2^{RES}(\theta^1)$ and expanding the first term, we get,

$$\int_{\theta^1_{sim}}^{\theta^2} U\left(\phi^2\left(\bar{k}, x_1^*(m^1_{pool})\right), t, b\right) f(t)dt + \int_{\theta^2}^{\theta^2_{sim}} U\left(\phi^2\left(\bar{k}, x_1^*(m^1_{pool})\right), t, b\right) f(t)dt$$

Comparing the above expression with the first two terms of $W_2^{RES}(\theta^2)$,

$$\int_{\theta^1_{sim}}^{\theta^2} U\left(\phi^2\left(x_2^*(t), x_1^*(t)\right), t, b\right) f(t)dt + \int_{\theta^2}^{\theta^2_{sim}} U\left(\phi^2\left(x_2^*(t, m^2_{pool}), x_1^*(m^2_{pool})\right), t, b\right) f(t)dt >$$
$$\int_{\theta^1_{sim}}^{\theta^2} U\left(\phi^2\left(\bar{k}, x_1^*(m^1_{pool})\right), t, b\right) f(t)dt + \int_{\theta^2}^{\theta^2_{sim}} U\left(\phi^2\left(\bar{k}, x_1^*(m^1_{pool})\right), t, b\right) f(t)dt$$

This follows from pair-wise comparison of the terms,

$$\int_{\theta^1_{sim}}^{\theta^2} U\left(\phi^2\left(x_2^*(t), x_1^*(t)\right), t, b\right) f(t)dt > \int_{\theta^1_{sim}}^{\theta^2} U\left(\phi^2\left(\bar{k}, x_1^*(m^1_{pool})\right), t, b\right) f(t)dt \tag{5}$$

$$\int_{\theta^2}^{\theta^2_{sim}} U\left(\phi^2\left(x_2^*(t, m^2_{pool}), x_1^*(m^2_{pool})\right), t, b\right) f(t)dt > \int_{\theta^2}^{\theta^2_{sim}} U\left(\phi^2\left(\bar{k}, x_1^*(m^1_{pool})\right), t, b\right) f(t)dt \tag{6}$$

Similarly comparing the last term of $W_2^{RES}(\theta^1)$ and $W_2^{RES}(\theta^2)$,

$$\int_{\theta^2_{sim}}^{1} U\left(\phi^2\left(\bar{k}, x_1^*(m^2_{pool})\right), t, b\right) f(t)dt > \int_{\theta^2_{sim}}^{1} U\left(\phi^2\left(\bar{k}, x_1^*(m^1_{pool})\right), t, b\right) f(t)dt \tag{7}$$

The inequality 5 follows from noting that on the interval $(\theta^1_{sim}, \theta^2]$, A_2 achieves $\bar{\phi}^2_t$ under the higher threshold equilibrium.

$$\forall t \in (\theta^1_{sim}, \theta^2] : U\left(\phi^2\left(x_2^*(t), x_1^*(t)\right), t, b\right) > U\left(\phi^2\left(\bar{k}, x_1^*(m^1_{pool})\right), t, b\right)$$

Similarly, inequality 6 is true since on the interval $(\theta^2, \theta^2_{sim}]$, A_2 induces A_1 to allocate more with message m^2_{pool} and correspondingly changes its action to achieve first best $\bar{\phi}^2_t$.

$$\forall t \in (\theta^2, \theta^2_{sim}] : U\left(\phi^2\left(x_2^*(t, m^2_{pool}), x_1^*(m^2_{pool})\right), t, b\right) > U\left(\phi^2\left(\bar{k}, x_1^*(m^1_{pool})\right), t, b\right)$$

The last inequality 7 follows from noting that since $x_1^*(m^1_{pool}) < x_1^*(m^2_{pool})$, it is valid that $\phi^2\left(\bar{k}, x_1^*(m^1_{pool})\right) < \phi^2\left(\bar{k}, x_1^*(m^2_{pool})\right)$ and because there is a positive spillover at the bound for A_2, ie $U_1 \mid_{t \in (\theta^2_{sim}, 1]} > 0$,

$$\forall t \in (\theta^2_{sim}, 1] : U\left(\phi^2\left(\bar{k}, x_1^*(m^2_{pool})\right), t, b\right) > U\left(\phi^2\left(\bar{k}, x_1^*(m^1_{pool})\right), t, b\right)$$

Comparing the terms pairwise therefore yields the required result, $W_2^{RES}(\theta^2) > W_2^{RES}(\theta^1)$.

Scenario (b): *When* $\theta^1_{sim} > \theta^2$. That is, $\theta^1 < \theta^2 < \theta^1_{sim} < \theta^2_{sim}$.
In this case, as earlier, I will look at states in which there is inefficiency generated by information pooling and compare the residual welfare.

$$W_2^{RES}(\theta^1) = \int_{\theta^1_{sim}}^{\theta^2_{sim}} U\left(\phi^2\left(\bar{k}, x_1^*(m^1_{pool})\right), t, b\right) f(t)dt + \int_{\theta^2_{sim}}^{1} U\left(\phi^2\left(\bar{k}, x_1^*(m^1_{pool})\right), t, b\right) f(t)dt$$

$$W_2^{RES}(\theta^2) = \int_{\theta^1_{sim}}^{\theta^2_{sim}} U\left(\phi^2\left(x_2^*(t, m^2_{pool}), x_1^*(m^2_{pool})\right), t, b\right) f(t)dt + \int_{\theta^2_{sim}}^{1} U\left(\phi^2\left(\bar{k}, x_1^*(m^2_{pool})\right), t, b\right) f(t)dt$$

Pairwise comparison yields,

$$\int_{\theta^1_{sim}}^{\theta^2_{sim}} U\left(\phi^2\left(x_2^*(t, m^2_{pool}), x_1^*(m^2_{pool})\right), t, b\right) f(t)dt > \int_{\theta^1_{sim}}^{\theta^2_{sim}} U\left(\phi^2\left(\bar{k}, x_1^*(m^1_{pool})\right), t, b\right) f(t)dt \tag{8}$$

$$\int_{\theta^2_{sim}}^{1} U\left(\phi^2\left(\bar{k}, x_1^*(m^2_{pool})\right), t, b\right) f(t)dt > \int_{\theta^2_{sim}}^{1} U\left(\phi^2\left(\bar{k}, x_1^*(m^1_{pool})\right), t, b\right) f(t)dt \tag{9}$$

The inequalities 8 and 9 follow from arguments made earlier. Specifically, on $(\theta^1_{sim}, \theta^2_{sim}]$ A_2 is able to achieve $\bar{\phi}^2_t$ with the cutoff equilibrium θ^2 and is therefore strictly better off compared to the equilibrium threshold θ^1. In the interval $(\theta^2_{sim}, 1]$, there is inefficiency from miscoordination in that $\phi^2(.) < \bar{\phi}^2_t$. However, since A_2 induces a higher action from A_1 under θ^2 equilibrium, $x_1^*(m^2_{pool}) > x_1^*(m^1_{pool})$, it follows that $\phi^2\left(\bar{k}, x_1^*(m^1_{pool})\right) < \phi^2\left(\bar{k}, x_1^*(m^2_{pool})\right) < \bar{\phi}^2_t$ and given $U_1 > 0$ on this interval,

$$\forall t \in (\theta^2_{sim}, 1] : U\left(\phi^2\left(\bar{k}, x_1^*(m^2_{pool})\right), t, b\right) > U\left(\phi^2\left(\bar{k}, x_1^*(m^1_{pool})\right), t, b\right)$$

Therefore, $W_2^{RES}(\theta^2) > W_2^{RES}(\theta^1)$. This completes the proof. **QED**

A.7 Proof of Proposition 5

Consider the following set of messaging strategies, beliefs and action profiles under ex-post commitment protocol:

1. If $\theta \leq \theta^*$, $m = \theta$; if $\theta > \theta^*$, $m = 1$.
2. If $m \leq \theta^*$, $p(\theta \mid m = \theta) = 1$; if $m = 1$, $p(\theta \mid m) = f(\theta)$
3. When $m \leq \theta^*$: $x_1^*(m) = \bar{x}_1(m)$ and $x_2^*(\theta, x_1^*(m)) = \underset{x_2 \in V}{\text{argmax}}\, U\left(\phi^2\left(x_2, x_1^*(m)\right), \theta, b\right) \equiv \bar{x}_2(m)$
4. When $m = 1$:
   - $x_1^*(m) \equiv \arg\max_{x_1 \in V} \int_{\theta^*}^{1} U\left(\phi^1(x_1, x_2^*(\theta, x_1)), \theta\right) f(\theta)d\theta$
   - $x_2^*(\theta, x_1^*(m)) \equiv \arg\max_{x_2 \in V} U\left(\phi^2(x_2, x_1^*(m)), \theta, b\right)$
5. When $m \in (\theta^*, 1)$: $p(\theta^*|m) = 1$.

Notice that the main point of departure from the simultaneous protocol arises from A_2's equilibrium action $x_2^*(\theta, x_1^*(m))$ that takes into account A_1's action in the second stage, post communication. Clearly, on the interval of separation $[0, \theta^*]$ A_1 can do no better than allocate $\bar{x}_2(\theta)$. This is driven by the concavity of $U(.)$ in that there is an unique $\bar{\phi}^1_\theta$ for every θ and this corresponds to the pair of actions $(\bar{x}_2(\theta), \bar{x}_1(\theta))$. Now, on the pooling interval A_1 takes into account that A_2 can now observe the action committed to by A_1 and best respond to them. Jointly, $(x_2^*(\theta, x_1^*(m)), x_1^*(m)))$ must maximize the expected payoffs of the agents.
Finally, I check to see if the informed agent would want to deviate from the equilibrium messaging strategy. Suppose A_2 deviates and sends an out-of-equilibrium message $m \in (\theta^*, 1)$. Then, A_1 assigns the belief that it comes from the type θ^* and plays the corresponding action $x_1^*(m) = \bar{x}_1(\theta^*)$. The types $m^*_{pool} = (\theta^*, 1]$ are at least as better off sending the pooling message $m = 1$. To see this, if $x_1^*(m) = \bar{x}_1(\theta^*)$, then there exists a threshold, say $\theta_{out} \leq \bar{\theta}$, such that $\phi^2\left(\bar{k}, \tilde{x}_1(\theta^*)\right) = \bar{\phi}^2_{\theta_{out}}$. This is true since $\phi^2\left(\bar{k}, \tilde{x}_1(\bar{\theta})\right) = \bar{\phi}^2_{\bar{\theta}}$ and by continuity there should exist such a type θ_{out}. Given this, A_2 with information in $(\theta^*, \theta_{out}]$ cannot do any better from deviating, since under the pooling message, they induce a higher action from A_1. This means every type $t \in (\theta^*, \theta_{out}]$ achieves first best $\bar{\phi}^2_t$ under the pooling message. That is, $\phi^2\left(x_2^*(t, x_1^*(m^*_{pool})), x_1^*(m^*_{pool})\right) = \phi^2\left(\bar{k}, x_1^*(\theta^*)\right) = \bar{\phi}^2_t$. But notice that every type in $(\theta_{out}, 1]$ would prefer sending the message $m = 1$ instead of the out-of-equilibrium one. This is driven by the miscoordination concerns that manifest as a result of the constraints. Specifically,

$$\forall \theta \in (\theta_{out}, 1] : \phi^2\left(\bar{k}, x_1^*(\theta^*)\right) < \phi^2\left(x_2^*(\theta, x_1^*(m^*_{pool})), x_1^*(m^*_{pool})\right) \leq \bar{\phi}^2 \bar{\theta}$$

Therefore, all types in $(\theta_{out}, 1]$ are strictly worse off by deviating. This completes the proof. **QED**

A.8 Proof of Proposition 6

I will continue to focus on the ex-ante efficient equilibrium ($\bar{\theta}$) for comparison of welfare. Again, on the separating interval $[0, \bar{\theta}]$, both the protocols provide the same ex ante welfare to both agents. So it is sufficient to focus on the pooling interval, henceforth $m_{pool} = (\bar{\theta}, 1]$. Let A_1's action after m_{pool} be x_1^{sim} and x_1^{pa} under simultaneous and sequential protocols respectively. To compare equilibrium welfare, it is essential to prove Lemma 3.

Lemma 3: $x_1^{pa} > x_1^{sim}$

Under simultaneous protocol, agent A_1's equilibrium action x_1^{sim} is given by the following FOC (from Lemma 2),

$$\int_{\bar{\theta}}^{\bar{\theta}_{sim}(x_1^{sim})} U_1\left(\phi^1\left(x_1^{sim}, x_2^{sim}(t, m_{pool})\right), t\right)\phi_1^1 dF + \int_{\bar{\theta}_{sim}(x_1^{sim})}^{1} U_1\left(\phi^1\left(x_1^{sim}, \bar{k}\right), t\right)\phi_1^1 dF = 0 \tag{10}$$

A_1's equilibrium action is given by the FOC from differentiating equation 3. That is, x_1^{pa} solves,

$$\int_{\bar{\theta}}^{\bar{\theta}_{pa}(x_1)} U_1\left(\phi^1\left(x_1, x_2^{pa}(t, x_1)\right), t\right). \left[\phi_1^1 + \phi_2^1 . \frac{dx_2}{dx_1}\right] dF + \int_{\bar{\theta}_{pa}(x_1)}^{1} U_1\left(\phi^1\left(x_1, \bar{k}\right), t\right)\phi_1^1 dF = 0 \tag{11}$$

Evaluating the above equation 11 at x_1^{sim} gives,

$$\begin{aligned} &\int_{\bar{\theta}}^{\bar{\theta}_{pa}(x_1^{sim})} U_1\left(\phi^1\left(x_1^{sim}, x_2^{pa}(t, x_1^{sim})\right), t\right). \left[\phi_1^1 + \phi_2^1 . \frac{dx_2}{dx_1}\right]\Bigg|_{x_1 = x_1^{sim}} dF + \\ &\int_{\bar{\theta}_{pa}(x_1^{sim})}^{1} U_1\left(\phi^1\left(x_1^{sim}, \bar{k}\right), t\right)\phi_1^1 dF \end{aligned} \tag{12}$$

But at $x_1 = x_1^{sim}$, it holds that $\bar{\theta}_{pa}(x_1^{sim}) = \bar{\theta}_{sim}(x_1^{sim})$ and $x_2^{pa}(t, x_1^{sim}) = x_2^{sim}(t, m_{pool})$. The second expression follows from the fact that the agent's equilibrium action mimics the simultaneous protocol action $x_2^{sim}(t, m_{pool})$ as there is an unique type θ for which $\phi^2(\bar{k}, x_1^{sim}) = \bar{\phi}_\theta$. Further, this implies that when $x_1 = x_1^{sim}$ under sequential protocol, the cutoff after which A_2 always allocates $\bar{k}$ corresponds with $\bar{\theta}_{sim}(x_1^{sim})$, resulting in the first equality. Substituting these expressions into equation 12 and rearranging gives,

$$\begin{aligned} &\int_{\bar{\theta}}^{\bar{\theta}_{sim}(x_1^{sim})} U_1\left(\phi^1\left(x_1^{sim}, x_2^{pa}(t, x_1^{sim})\right), t\right)\phi_1^1 dF + \int_{\bar{\theta}_{sim}(x_1^{sim})}^{1} U_1\left(\phi^1\left(x_1^{sim}, \bar{k}\right), t\right)\phi_1^1 dF \\ &+ \int_{\bar{\theta}}^{\bar{\theta}_{sim}(x_1^{sim})} U_1\left(\phi^1\left(x_1^{sim}, x_2^{pa}(t, x_1^{sim})\right), t\right).\phi_2^1 . \frac{dx_2^{pa}}{dx_1}\Bigg|_{x_1 = x_1^{sim}} dF \end{aligned}$$

However, the first two expressions are equal to the LHS of equation 10, and therefore equal to zero. The only expression left is the last one given by,

$$\int_{\bar{\theta}}^{\bar{\theta}_{sim}(x_1^{sim})} U_1\left(\phi^1\left(x_1^{sim}, x_2^{pa}(t, x_1^{sim})\right), t\right).\phi_2^1.\left.\frac{dx_2^{pa}}{dx_1}\right|_{x_1=x_1^{sim}} dF$$

Notice that $U_1\left(\phi^1\left(x_1^{sim}, x_2^{pa}(t, x_1^{sim})\right), t\right) < 0$ for A_1 on this interval since A_2 always moderates its action in order to achieve $\bar{\phi}_t^2$, but this results in over-allocation for agent A_1, ie $\phi^1\left(x_1^{sim}, x_2^{pa}(t, x_1^{sim})\right) > \bar{\phi}_t^1$. Given Assumption 2, $\phi_2^1 > 0$ and from Assumption 3, $\frac{dx_2}{dx_1} < 0$ implying that the above integral is always positive.

$$\int_{\bar{\theta}}^{\bar{\theta}_{sim}(x_1^{sim})} U_1\left(\phi^1\left(x_1^{sim}, x_2^{pa}(t, x_1^{sim})\right), t\right).\phi_2^1.\left.\frac{dx_2^{pa}}{dx_1}\right|_{x_1=x_1^{sim}} dF > 0 \tag{13}$$

Since the expected utility for A_1 in the sequential protocol is increasing at x_1^{sim} and $U_{11} < 0$, it follows that $x_1^{pa} > x_1^{sim}$. This completes the proof of the lemma.

Given $x_1^{pa} > x_1^{sim}$, it is straightforward to see that equilibrium welfare is higher under sequential protocol. By mimicking x_1^{sim}, A_1's expected utility is the same as in the simultaneous protocol, on the pooling interval. However, from equation 13, we have established that the expected utility is increasing at $x_1 = x_1^{sim}$. More formally, the following equations hold:

$$\mathbb{E}_\theta\left[U\left(\phi^1\left(x_1, x_2^{pa}(\theta, x_1)\right), \theta\right)\right]\Big|_{x_1=x_1^{sim}} = \mathbb{E}_\theta\left[U\left(\phi^1\left(x_1^{sim}, x_2^{sim}(\theta, m_{pool})\right), \theta\right)\right]$$

$$\left.\frac{d\mathbb{E}_\theta\left[U\left(\phi^1\left(x_1, x_2^{pa}(\theta, x_1)\right), \theta\right)\right]}{dx_1}\right|_{x_1=x_1^{sim}} > 0$$

These above two equations guarantee that agent A_1, by mimicking the simultaneous protocol action can guarantee an expected payoff equal to that under the simultaneous protocol and therefore does better by increasing its action such that $x_1^{pa} = x_1^{sim}$.

For A_2, $x_1^{pa} > x_1^{sim}$ implies that $\bar{\theta}_{pa} > \bar{\theta}_{sim}$. That is, for a greater measure of types on the pooling interval, the constraint is not binding, $\forall t \in (\bar{\theta}, \bar{\theta}_{pa}] : x_2^{pa}(t, x_1^{pa}) \leq \bar{k} \implies \phi^2\left(x_2^{pa}(t, x_1^{pa}), x_1^{pa}\right) = \bar{\phi}_t^2$. As before, I will write down the residual welfare on the interval $(\bar{\theta}_{sim}, 1]$ under both protocols.

$$W_2^{sim}(\bar{\theta}) = \int_{\bar{\theta}_{sim}}^{\bar{\theta}_{pa}} U\left(\phi^2\left(\bar{k}, x_1^{sim}\right), t, b\right) f(t)dt + \int_{\bar{\theta}_{pa}}^{1} U\left(\phi^2\left(\bar{k}, x_1^{sim}\right), t, b\right) f(t)dt$$

$$W_2^{pa}(\bar{\theta}) = \int_{\bar{\theta}_{sim}}^{\bar{\theta}_{pa}} U\left(\bar{\phi}_t^2, t, b\right) f(t)dt + \int_{\bar{\theta}_{pa}}^{1} U\left(\phi^2\left(\bar{k}, x_1^{pa}\right), t, b\right) f(t)dt$$

Pairwise comparison yields,

$$\int_{\bar{\theta}_{sim}}^{\bar{\theta}_{pa}} U\left(\bar{\phi}_t^2, t, b\right) f(t)dt > \int_{\bar{\theta}_{sim}}^{\bar{\theta}_{pa}} U\left(\phi^2\left(\bar{k}, x_1^{sim}\right), t, b\right) f(t)dt \tag{14}$$

$$\int_{\bar{\theta}_{pa}}^{1} U\left(\phi^2\left(\bar{k}, x_1^{pa}\right), t, b\right) f(t)dt > \int_{\bar{\theta}_{pa}}^{1} U\left(\phi^2\left(\bar{k}, x_1^{sim}\right), t, b\right) f(t)dt \tag{15}$$

Clearly, equation 14 follows from noting that A_2 achieves $\bar{\phi}_t^2$ under the ex-post commitment on the interval $(\bar{\theta}_{sim}, \bar{\theta}_{pa}]$ and therefore cannot do better. Equation 15 holds because on the interval where there is under-allocation, ie $(\bar{\theta}_{pa}, 1]$, $\phi^2\left(\bar{k}, x_1\right) < \bar{\phi}_t^2 \implies U_1(.) > 0$ for A_2. Since $x_1^{pa} > x_1^{sim}$, from Assumption 2 it follows that A_2 is better off under sequential protocol for all types in $(\bar{\theta}_{pa}, 1]$. Therefore, $W_2^{pa}(\bar{\theta}) > W_2^{sim}(\bar{\theta})$. This completes the proof. **QED**

A.9 Proof of Proposition 7

The key to proving this is to look at all pairs of actions (x_1, x_2) that achieve the first best for A_2, in order to satisfy its IC constraint. Given Assumption 1 and Assumption 2, for any $\theta \in [0, 1]$, there are different contribution pairs (x_1, x_2) such that $\phi^2(x_2, x_1) = \bar{\phi}_\theta^2$. I proceed by constructing the set of ϕ^1 that corresponds with all admissible pairs (x_1, x_2) such that for any θ, $\phi^2(x_2, x_1) = \bar{\phi}_\theta^2$. The following defines this admissible set:

$$\forall \theta \in [0, 1], (x_2, x_1) \in V : \mathcal{A}_\theta = \left\{\phi^1(x_1, x_2) : \phi^2(x_2, x_1) = \bar{\phi}_\theta^2\right\}$$

Therefore, the commitment rule becomes one of choosing an appropriate pair from $\mathcal{A}_\theta$ such that it maximizes the expected utility of A_1.

Claim 1: From previous arguments, on the interval $[0, \bar{\theta}]$ the incentive compatible action rule that maximizes A_1's expected utility is the one that mimics the unconstrained action $\bar{x}_1(\theta)$.

Specifically, the action pair $(\bar{x}_1(\theta), \bar{x}_2(\theta))$ is such that $\phi^1(\bar{x}_1(\theta), \bar{x}_2(\theta)) \in \mathcal{A}_\theta$ and $\phi^1(\bar{x}_1(\theta), \bar{x}_2(\theta)) = \bar{\phi}^1_\theta \equiv \underset{\phi^1}{\text{argmax}}\, U(\phi^1, \theta)$. This proves Claim 1.

To show claims 2,3 and 4, I will impose further structure on the set $\mathcal{A}_\theta$ for the interval m_{pool}. From continuity property of $\phi^1(.)$ and $\phi^2(.)$, the set $\mathcal{A}_\theta$ is compact. Further, let $\sup \mathcal{A}_\theta = \phi^1_{sup}(\theta)$ and $\inf \mathcal{A}_\theta = \phi^1_{inf}(\theta)$.

Definition 3 *Let $x_1^{inf}(\theta)$ be such that $\phi^2\left(\bar{k}, x_1^{inf}(\theta)\right) = \bar{\phi}^2_\theta$.*

Lemma 4 $\forall \theta \in m_{pool} : \phi^1(x_1^{inf}(\theta), \bar{k}) = \phi^1_{inf}(\theta)$

Proof. Note that x_2 varies from $\underline{k}$ to $\bar{k}$ and x_1 is just the residual contribution that ensures $\phi^2(.) = \bar{\phi}^2_\theta$. Applying total differentiation to ϕ^2, we get the following:

$$d\phi^2 = \frac{\partial \phi^2}{\partial x_2}.dx_2 + \frac{\partial \phi^2}{\partial x_1}.dx_1$$

Since $\phi^2(.) = \bar{\phi}^2_\theta$, a constant in $\mathcal{A}_\theta$, $d\phi^2 = 0$. Substituting this in the above equation and rearranging,

$$\left|\frac{dx_1}{dx_2}\right| = \frac{\frac{\partial \phi^2}{\partial x_2}}{\frac{\partial \phi^2}{\partial x_1}} > 1$$

Similarly,

$$d\phi^1 = \frac{\partial \phi^1}{\partial x_2}.dx_2 + \frac{\partial \phi^1}{\partial x_1}.dx_1$$

$$\frac{d\phi^1}{dx_2} = \frac{\partial \phi^1}{\partial x_2} + \frac{\partial \phi^1}{\partial x_1}.\frac{dx_1}{dx_2} = \frac{\partial \phi^1}{\partial x_2} - \left|\frac{dx_1}{dx_2}\right|.\frac{\partial \phi^1}{\partial x_1} \tag{16}$$

$$\implies \frac{d\phi^1}{dx_2} < \left[\frac{\partial \phi^1}{\partial x_1} - \left|\frac{dx_1}{dx_2}\right|.\frac{\partial \phi^1}{\partial x_1}\right] = \frac{\partial \phi^1}{\partial x_2}.\left[1 - \left|\frac{dx_1}{dx_2}\right|\right] < 0 \tag{17}$$

Equation 17 follows from imperfect substitutability in that $\frac{\partial \phi^1}{\partial x_1} > \frac{\partial \phi^1}{\partial x_2}$. Lemma 4 establishes that ϕ^1 is decreasing in the actions of A_2. This implies that the infimum of the set $\mathcal{A}_\theta$ corresponds with the pair of actions in which A_2 takes the maximal action $\bar{k}$ and A_1, the residual $x_1^{inf}(\theta)$. ∎

Lemma 5 $\forall \theta \in m_{pool} : \phi^1_{inf}(\theta) > \bar{\phi}^1_\theta$

Proof. From lemma 4 it is clear there is an ordering over ϕ^1. Specifically, $\phi^1_{sup}(\theta) > ...... > \phi^1_{inf}(\theta)$. Suppose $\phi^1_{inf}(\theta) > \bar{\phi}^1_\theta$ were not true. Then, either $\phi^1_{sup}(\theta) > ..... > \bar{\phi}^1_\theta > ... > \phi^1_{inf}(\theta)$ or $\bar{\phi}^1_\theta > \phi^1_{sup}(\theta) > ..... > \phi^1_{inf}(\theta)$. If the former was true, then A_2 can achieve first best by truthfully revealing the state θ. That is, A_2 could have revealed truthfully up to some higher threshold $\bar{\theta}$, which violates the most informative threshold equilibrium $\bar{\theta}$. The latter cannot be true because of the imperfect substitutability assumption and a positive conflict of interest. Therefore it must hold that $\phi^1_{sup}(\theta) > ...... > \phi^1_{inf}(\theta) > \bar{\phi}^1_{\theta}$. ■

From Lemma 5, it is clear that on the interval m_{pool}, there is over-allocation for agent A_1 as long as A_2 achieves first best. However, precisely for this reason, it implies that $U_1 < 0$ and therefore the following holds:

$$\forall \theta \in m_{pool} : \phi^1_{inf}(\theta) \equiv \underset{\phi^1 \in \mathcal{A}_\theta}{\operatorname{argmax}} \, U(\phi^1, \theta) \tag{18}$$

That is, of all contribution pairs (x_1, x_2) that satisfy A_2's IC constraint for truth-telling, the one that maximizes A_1's utility is the one that minimizes this miscoordination problem, which coincides with $x_2 = \bar{k}$. I proceed now to prove Claim 2,3 and 4.

Claim 2: Suppose the claim weren't true and say A_1, wlog, allocates $x_1^{fa}(\theta) = z, \forall \theta \in m_{pool}$. There are two possible cases to consider.

Case i) $z = x_1^{fa}(\bar{\theta}) = \tilde{x}_1(\bar{\theta})$

In this case, A_2 allocates $x_2 = \bar{k}$ for every possible type in m_{pool}. If this is so, then $\forall \theta \in m_{pool}$: $\phi^1(\tilde{x}_1(\bar{\theta}), \bar{k}) = \bar{\phi}^1_{\bar{\theta}} < \bar{\phi}^1_{\theta}$. This implies that the expected marginal utility of A_1 is less than zero and given $U_{11} < 0$, there is an incentive for A_1 to increase her actions. Therefore, $z \neq x_1^{fa}(\bar{\theta})$.

Case ii) $z > x_1^{fa}(\bar{\theta})$

If this were true, then there exists some types such that A_2 allocates less than $\bar{k}$ and still achieves first best. That is,

$$\exists T \subset m_{pool}, \forall t \in T : x_2^{fa}(t, z) < \bar{k}$$

Such a set T must exist from the continuity property of $U(.)$ and $\phi^i(.)$. Specifically, when $z > x_1^{fa}(\bar{\theta})$, then there is always a cutoff type θ_z (from Lemma 2) such that $x_2^{fa}(\theta_z, z) = \bar{k}$. However, this implies that for all types $t \in (\bar{\theta}, \theta_z)$, it must be that $x_2^{fa}(t, z) < \bar{k}$. That is $T = (\bar{\theta}, \theta_z)$ exists. But if this set exists, then A_1 is not maximizing its expected utility since it can always reduce actions and make A_2 contribute more, specifically, $\bar{k}$. To see this, consider the following alternate action rule:

$$\forall t \in T : x_1^{fa}(t) = x_1^{inf}(t) \text{ such that } \phi^1(x_1^{inf}(t), \bar{k}) = \phi^1_{inf}(t) \in \mathcal{A}_t$$

$$\forall t \in m_{pool} \setminus T : x_1^{fa}(t) = z$$

Clearly, on the interval subset T, A_1 now achieves a greater expected utility since $\forall t \in T$, $U\left(\phi^1(x_1^{inf}(t), \bar{k}), t\right) > U\left(\phi^1(z, x_2^{fa}(t,z)), t\right)$. Further, this action rule is also incentive compatible in that A_2 cannot do better by misreporting. Therefore, there cannot be a flat segment on m_{pool} such that A_1 commits to a communication independent action. This proves Claim 2.

Claim 3: Suppose instead there was a strictly increasing interval $(\theta_1, \theta_2) \in m_{pool}$ such that $\exists t \in (\theta_1, \theta_2) : x_2^{fa}(t, x_1^{fa}(t)) < \bar{k}$. Then, given IC must be satisfied, $\phi^1\left(x_1^{fa}(t), x_2^{fa}(t, x_1^{fa}(t))\right) \in \mathcal{A}_t$. But clearly, from Lemma 4, Lemma 5 and Equation 18 A_1 can always instead choose to allocate $x_1^{inf}(t)$ such that $x_2^{fa}(t, x_1^{inf}(t)) = \bar{k}$. This satisfies IC of A_2 since $\phi^1(x_1^{inf}(t), \bar{k}) \in \mathcal{A}_t$ and increases the payoff to A_1 since $U\left(\phi^1(x_1^{inf}(t), \bar{k}), t\right) > U\left(\phi^1(x_1^{fa}(t), x_2^{fa}(t, x_1^{fa}(t))), t\right)$. This proves Claim 3.

Claim 4: Suppose, instead there exists a flat segment followed by a strictly increasing segment in m_{pool}. Say, wlog, the flat segment is on $(\theta_1, \theta_2]$ such that $\forall t \in (\theta_1, \theta_2] : x_1^{fa}(t) = z$, and let the strictly increasing segment be on (θ_2, θ_3). From Claim 3, it holds that A_2 must take an action $\bar{k}$ on this interval and further, IC constraint must be satisfied in that $\forall t \in (\theta_2, \theta_3) : \phi^2(\bar{k}, x_1^{inf}(t)) = \bar{\phi}_t^2$. Take the type θ_2. For this type it must be that the IC is satisfied on the flat segment, i.e. $\phi^2(x_2^{fa}(\theta_2, z), z) = \bar{\phi}_{\theta_2}^2$. If not, A_2 can always deviate and report $t \in (\theta_2, \theta_3)$ and increase its expected payoff. This implies that z must be such that $x_2^{fa}(\theta_2, z) = \bar{k}$, since otherwise A_1 is not payoff maximizing, again from previous arguments. When A_2 contributes $\bar{k}$, A_1's residual contribution must be in the set $\mathcal{A}_{\theta_2}$ and equal to,

$$z = x_1^{inf}(\theta_2) \text{ such that } \phi^1(x_1^{inf}(\theta_2), \bar{k}) = \phi_{inf}^1(\theta_2) \in \mathcal{A}_{\theta_2}$$

But clearly, if $z = x_1^{inf}(\theta_2)$, then for all types $t \in (\theta_1, \theta_2)$, it must also hold that $x_2^{fa}(t,z) < \bar{k}$, from single crossing condition. However, if $x_2^{fa}(t,z) < \bar{k}$, then A_1 can always decrease its actions on this interval, and extract more from A_2 whilst satisfying the IC constraint of A_2 (Lemma 4 and Lemma 5). This proves Claim 4.

Together, the four claims imply the following rules hold:

1. Claim 1 $\implies$ On the separating interval $[0, \bar{\theta}]$, the optimal ex-ante action rule mimics the simultaneous protocol actions, $\bar{x}_1(\theta)$.
2. Claim 2 and Claim 3 $\implies$ There is an interval $(\bar{\theta}, \bar{\theta}_{fa}) \in m_{pool}$ in which agent A_1's action decisions are dependent on communication and given by $x_1^{fa}(\theta) = x_1^{inf}(\theta)$, while

$x_2^{fa}(\theta) = \bar{k}$ such that $\phi^2(\bar{k}, x_1^{inf}(\theta)) = \bar{\phi}_\theta^2$ and $\phi^1(x_1^{inf}(\theta), \bar{k}) \in \mathcal{A}_\theta$.

3. Finally, Claim 4 $\implies$ On the interval $[\bar{\theta}_{fa}, 1]$, A_1's action is independent of communication and is equal to $x_1 = x_1^{inf}(\bar{\theta}_{fa}) = x_1^{fa}(\bar{\theta}_{fa})$.

This completes the proof. **QED**

A.10 Proof of Proposition 8

Consider the following commitment strategy. The uninformed agent A_1 commits to an ex-ante action rule on m_{pool} such that:

$$\forall t \in (\bar{\theta}, \bar{\theta}_{pa}) : x_1^{fa}(t) = x_1^{inf}(t)$$

$$\forall t \in [\bar{\theta}_{pa}, 1] : x_1^{fa}(t) = x_1^{pa}(m_{pool}) \equiv x_1^{pa}$$

The above action rule exactly replicates the ex-post commitment protocol in that it provides A_2 first best joint coordination $\bar{\phi}_t^2$ on the interval $(\bar{\theta}, \bar{\theta}_{pa}]$. Clearly, this action rule is IC for A_2 and provides the same expected welfare compared to sequential protocol case. Further, on the interval $[\bar{\theta}_{pa}, 1]$, A_1's welfare is same as under the sequential protocol.
However, $\forall t \in (\bar{\theta}, \bar{\theta}_{pa})$, A_1 actually does better since A_2's action is maximal ($\bar{k}$) on this interval and this minimizes the inefficiency from miscoordination, as shown in Lemma 4 and Lemma 5 in the proof of Proposition 7. That is,

$$\forall t \in (\bar{\theta}, \bar{\theta}_{pa}) : U\left(\phi^1\left(x_1^{inf}(t), \bar{k}\right), t\right) > U\left(\phi^1\left(x_1^{pa}, x_2(t, x_1^{pa})\right), t\right)$$

Therefore by following an IC commitment rule that is strictly increasing on $(\bar{\theta}, \bar{\theta}_{pa})$ and flat on $[\bar{\theta}_{pa}, 1]$, A_1 achieves a higher ex-ante welfare while A_2 is indifferent compared to the case of sequential decision making.
Now consider the sequence of actions $\left\{x_1^{inf}(t)\right\}_{t \in (\bar{\theta}, \bar{\theta}_{pa})}$ and checking the marginal utility of A_1 for each type t,

$$U_1\left(\phi^1\left(x_1^{pa}, x_2(t, x_1^{pa})\right), t\right) < U_1\left(\phi^1\left(x_1^{inf}(t), \bar{k}\right), t\right) \tag{19}$$

Equation 19 follows from noting that utility of A_1 is decreasing in ϕ^1 on this interval and since $U_{11} < 0$ and $\phi^1\left(x_1^{pa}, x_2(t, x_1^{pa})\right) < \phi^1\left(x_1^{inf}(t), \bar{k}\right) = \phi_{inf}^1(t)$. Now, on the interval $[\bar{\theta}_{pa}, 1]$, since $x_1^{fa}(t) = x_1^{pa}$, the ex ante commitment rule provides the same expected marginal utility as sequential protocol for A_1. Summing the marginal utilities under the ex-ante action rule with the sequence $\left\{x_1^{inf}(t)\right\}_{t\in(\bar{\theta},\bar{\theta}_{pa})}$ and $\left\{x_1^{pa}\right\}_{t\in[\bar{\theta}_{pa},1]}$, it is clear that,

$$\int_{\bar{\theta}}^{\bar{\theta}_{pa}} U_1\left(\phi^1\left(x_1^{inf}(t),\bar{k}\right),t\right)\cdot\left[\phi_1^1+\phi_2^1.\frac{dx_2}{dx_1}\right]dF+\int_{\bar{\theta}_{pa}}^{1} U_1\left(\phi^1\left(x_1^{pa},\bar{k}\right),t\right)\phi_1^1 dF \tag{20}$$

Since $x_2^{fa}(t)=\bar{k}$, it follows that $\frac{dx_2}{dx_1}=0$ under this action rule. The above equation simplifies to,

$$\int_{\bar{\theta}}^{\bar{\theta}_{pa}} U_1\left(\phi^1\left(x_1^{inf}(t),\bar{k}\right),t\right).\phi_1^1 dF+\int_{\bar{\theta}_{pa}}^{1} U_1\left(\phi^1\left(x_1^{pa},\bar{k}\right),t\right)\phi_1^1 dF>0 \tag{21}$$

The above inequality follows from Equation 19. Therefore, the marginal utility of A_1 from following the above ex-ante commitment rule is less than zero, and given $U_{11}<0$, this further implies that under a fully autonomous protocol, A_1 can satisfy A_2's first best above this threshold. From Equation 21, it therefore follows that:

$$\bar{\theta}_{fa}>\bar{\theta}_{pa}$$

A_1's welfare

The increase in agent A_1's welfare is driven by the fact that the mimicking strategy provided A_1 a higher expected utility under ex-ante commitment and further, marginal utility is increasing at $\bar{\theta}_{pa}$ (from Equation 21). Therefore, the expected utility from the commitment rule where $\bar{\theta}_{fa}>\bar{\theta}_{pa}$ is greater compared to sequential protocol.

A_2's welfare

On the interval $[0,\bar{\theta}_{fa}]$, A_2 achieves first best levels of joint action in that $\forall t\in[0,\bar{\theta}_{fa}]:\phi^2(.)=\bar{\phi}_t^2$ under the equilibrium ex-ante commitment rule. Further, on $t\in(\bar{\theta}_{fa},1]$, it must be that $x_1^{fa}(t)=x_1^{fa}(\bar{\theta}_{fa})>x_1^{fa}(\bar{\theta}_{pa})$, since $x_1^{fa}(\bar{\theta}_{pa})=x_1^{pa}$ and $\bar{\theta}_{fa}>\bar{\theta}_{pa}$. However, on this interval, there is under-allocation of actions for A_2 and therefore it must hold that $U\left(\phi^2\left(\bar{k},x_1^{fa}(\bar{\theta}_{fa}),t\right),t\right)>U\left(\phi^2\left(\bar{k},x_1^{pa},t\right),t\right)$. Therefore, the overall expected ex-ante welfare is greater with the commitment mechanism. This completes the proof. **QED**

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