Falling Interest Rates and Credit Misallocation: Lessons from General Equilibrium

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Abstract

What is the effect of declining interest rates on the efficiency of resource allocation and overall economic activity? We study this question in a setting in which entrepreneurs with different productivity invest in capital, subject to financial frictions. We show that a fall in the interest rate has an ambiguous effect on aggregate output. In partial equilibrium, a lower interest rate raises aggregate investment both by relaxing financial constraints and by prompting relatively less productive entrepreneurs to invest. In general equilibrium, however, this higher demand for capital raises its price and crowds out investment by the more productive entrepreneurs. When this crowding-out effect is strong enough, a fall in the interest rate becomes contractionary. Moreover, in a dynamic setup, such reallocation effects among entrepreneurs interact with the classic balance-sheet channel, giving rise to a *boom-bust* impulse response of output to a fall in the interest rate. We provide evidence in support of our mechanism using data from the US and Spain.

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1 Introduction

One distinctive feature of recent decades has been the sustained and significant decline in real interest rates across the globe. Although the conventional wisdom holds that declining interest rates should stimulate economic activity –at least over the short-term,– there are mounting concerns that declining rates may have undesirable side effects, such as endangering financial stability (Rajan, 2015; Martinez-Miera and Repullo, 2017; Brunnermeier and Koby, 2018; Bolton et al., 2021) or slowing-down the pace of technological innovation and long-term growth (Benigno and Fornaro, 2014; Liu et al., 2019; Benigno et al., 2020).

Perhaps the most common worry among economists and policy makers is that declining rates may foster the proliferation of unproductive economic activities. This view is consistent with recent evidence, which suggests that periods of fast credit growth fueled by low interest rates have been characterized by low aggregate productivity growth (Reis, 2013; Gopinath et al., 2017; García-Santana et al., 2020). Despite its intuitive appeal, this view leaves many open questions. Can declining interest rates foster socially unproductive activities? If so, under which conditions? Can this effect be strong enough to hamper economic activity and growth?

In this paper, we develop a tractable framework to address these questions. We consider an economy populated by entrepreneurs who can invest in capital. We make two crucial assumptions. First, entrepreneurs have heterogeneous productivity, i.e., they differ in their effectiveness at using capital to produce consumption goods. Second, entrepreneurs face financial frictions, i.e., they cannot pledge the entire surplus from their activities to outsiders. To keep matters simple, we focus throughout on a small-open economy that takes the world interest rate as given. We then ask a simple question: how does a fall in the interest rate affect the allocation of capital across entrepreneurs, aggregate investment and output?[1]

The conventional view holds that lower interest rates stimulate economic activity by raising both entrepreneurs' willingness and ability to invest. Our framework challenges this view, by highlighting that in a world of heterogeneous productivity and financial frictions, it may foster investment by the wrong mix of agents. The reason is that declining interest rates make it attractive for relatively less productive entrepreneurs to invest. This raises the equilibrium price of capital and crowds-out investment by more productive (albeit financially constrained) entrepreneurs; that is, capital is reallocated from more to less productive entrepreneurs. We formalize this *reallocation effect* of declining interest rates and show that it mitigates their stimulative effect on output. Moreover, we show that this effect can be strong enough to make

[1] The main lessons from our analysis extend to a closed-economy setup, where interest rates are endogenous and interest rate changes are driven by economic fundamentals (see Section 3.1).

aggregate output decline in response to a fall in the interest rate!

Our mechanism relies on the two central assumptions highlighted above: heterogeneous productivity and financial frictions. When individual entrepreneurs invest, they do not internalize the effect of their investment on the price of capital. This gives rise to a pecuniary externality. In the absence of heterogeneity or financial frictions, the return to investment is equalized across entrepreneurs and this externality has no first-order effect on output or welfare. With heterogeneity and financial frictions, however, this is no longer the case. Simply put, the less productive entrepreneurs invest too much because they do not internalize the crowding-out effect of their investment on that of the more productive entrepreneurs.

We characterize the determinants of the reallocation effect, and show that its strength depends crucially on the elasticity of the capital supply and on the severity of the financial friction. In particular, when the supply of capital is sufficiently inelastic and the financial friction is severe enough, the reallocation effect is strong enough to make a fall in the interest rate be contractionary. Importantly, this possibility never arises if investment is instead dictated by a benevolent social planner, who is subject to the same financial constraints that limit entrepreneurial investment but who has the power to prevent certain entrepreneurs from investing. By being able to control the strength of the general-equilibrium reallocation effect, the social planner ensures that a fall in the interest rate is always expansionary.

Our findings extend to a dynamic setting in which entrepreneurial net worth is endogenous. This is important because the macro-finance literature has traditionally emphasized the balance sheet channel (e.g. Kiyotaki and Moore (1997)), by which rising asset prices are expansionary through their impact on entrepreneurial wealth. In particular, an increase in asset prices boosts the net worth of entrepreneurs with larger holdings of capital thereby relaxing their financial constraints. As more productive entrepreneurs employ more capital in our setting, one would intuitively expect the balance-sheet channel to amplify the stimulative effect of declining rates. We analyze the economy's response to a permanent decline in the interest rate and show that this intuition is not fully correct. The reason is that balance-sheet effects are driven by unexpected changes in the price of capital and are thus inherently transitory, whereas the general-equilibrium forces that drive reallocation are permanent. As a result, the response of output to a fall in the interest rate may feature a transitory boom, followed by a permanent bust: the balance-sheet channel temporarily boosts investment by the more productive entrepreneurs, but its effect gradually wears-off as the contractionary reallocation effect begins to dominate.

Our theory is consistent with recent evidence on the macroeconomic effects of credit booms driven by low interest rates. In particular, it sheds light on the experience of several Southern European economies (e.g. Spain and Italy) during the early 2000s, when a reduction in interest rates coincided with local booms in credit and asset prices and, at the same time, with a decline in aggregate TFP and an increase in productivity dispersion across firms. Interpreted through the lens of our framework, this happened partly because – although they boosted investment and consumption – lower interest rates had an adverse effect on resource allocation by favoring the investment of unproductive firms. One implication of our theory is that this unproductive investment can be socially undesirable despite having a positive NPV from a private standpoint.

To test the theory, we analyze the effects of interest rate changes across geographical regions in the United States and Spain. The key prediction of the theory is that the stimulative effect of declining interest rates should be weaker when the reallocation effect induced by the increase in the price of capital is stronger. Testing this in the data requires taking a stance on a specific type of capital that may be relevant in practice. For this, note that our theory applies to any input of production that requires investment (i.e. that needs to be employed before final cash flows are realized) and whose supply is somewhat inelastic, e.g. commercial real-state, skilled labor, housing.[2] For data availability reasons, we focus on land (as a proxy for real-estate). In particular, we measure the response of sectoral investment at the local level (MSA for the case of the US, municipality for the case of Spain) to changes in the interest rate, and analyze how this response correlates with the elasticity of land supply and with the land-intensity of the sector. The main predictions of the theory are that, in response to a decline in the interest rate; (i) the investment of land-intensive sectors should expand less in regions where land supply is less elastic, and; (ii) the productivity of land-intensive sectors should decline (and the within-sector misallocation should increase) more in regions where the land supply is less elastic. Preliminary results using firm-level data from COMPUSTAT find strong support for prediction (i). We are currently in the process of evaluating prediction (ii) using Spanish firm-level data.

In spirit, our paper is closely related to recent work that studies the dismal productivity performance that often characterizes credit booms in practice (Reis, 2013; Gopinath et al., 2017; Doerr, 2018; García-Santana et al., 2020; Gorton and Ordonez, 2020). Within this work, the papers closest to us are Reis (2013) and Gopinath et al. (2017). Like us, both papers show that lower interest rates can lead to a decline in TFP in the presence of financial frictions, as some firms – not necessarily the most productive ones – take advantage of cheaper credit. Differently from them, however, our focus is on how general-equilibrium effects crowd-out the

[2]We view commercial real-state and housing as two separate types of capital. While commercial-real state is a direct input of production, housing is an indirect input, since in equilibrium firms must compensate their employees for housing costs.

activity of the most productive firms and lead, not just to greater misallocation, but even to a fall in aggregate output.[3] We also show how this negative effect of lower interest rates on TFP interacts dynamically with traditional balance-sheet effects (e.g. Kiyotaki and Moore (1997)), according to which lower interest rates – by boosting asset prices – transfer resources towards the economy's most productive agents. A related line of work emphasizes "zombie" lending, broadly defined as relatively unproductive firms that remain active in low-interest rate environments (Caballero et al., 2008; Adalet McGowan et al., 2018; Tracey, 2019; Schivardi et al., 2020). In contrast, in our model, all investment has a positive net present value from a private standpoint, but some investments are nonetheless excessive from a social standpoint.

Our paper is also related to the growing literature on macroeconomics with heterogeneous agents. Much of this literature has studied how heterogeneity shapes an economy's response to monetary policy. Although this research focuses mostly on heterogeneity among households (e.g. Cloyne et al. (2020), Kaplan et al. (2018), Slacalek et al. (2020)), a growing body of work also analyzes heterogeneity among firms (e.g. Anderson and Cesa-Bianchi (2020), Cloyne et al. (2018), Manea (2020), Ottonello and Winberry (2020)).[4] Relative to this work, whose main objective is to understand the transmission of monetary policy, we focus on the effects of interest rate changes on the allocation of resources between high- and low-productivity firms, and how these effects are shaped by general-equilibrium considerations.

Finally, from a more conceptual standpoint, our model is closely related to previous work that stresses inefficiencies in the allocation of factors of production due to financial constraints. One recurring theme in this work is that individual firms do not internalize the effect of their demand on factor prices, which may lead to inefficient outcomes in the presence of financial constraints. Ventura and Voth (2015), Martin et al. (2018) and Asriyan et al. (2019) provide recent examples of this work.

The paper is organized as follows. Section 2 presents the model and the main equilibrium conditions. Section 3 characterizes the equilibrium effects of declining interest rates, as well as the normative properties of equilibria. Section 4 presents a dynamic extension of the baseline model. Section 5 (in progress) provides supporting evidence, and Section 6 concludes.

[3] Reis (2013) and Benigno and Fornaro (2014) show that financial integration with the rest of the world can induce a general-equilibrium reallocation of resources from tradable to non-tradable sector, a feature not present in our setting.

[4] Relatedly, Leahy and Thapar (2019) study how age-structure shapes the impact of monetary policy: they find that monetary policy is most potent in regions with a larger share of middle-aged due to their higher propensity for entrepreneurship. On the other hand, Caggese and Pérez-Orive (2020) show how lower interest rates may become less expansionary in economies where intangible investments become more important.

2 Baseline Model

We first consider an economy that lasts for two periods, $t = 0, 1$. There are two goods: a perishable consumption good (c) and capital (k). There are two sets of risk-neutral agents, entrepreneurs and capitalists, each of unit mass.

Preferences. The preferences of all agents are given by:

$$U = E_0[c_1],$$

where c_1 is the consumption at $t = 1$ and $E_0[\cdot]$ is the expectations operator at $t = 0$.

Endowments. Each entrepreneur is endowed with w units of the consumption good at $t = 0$, while capitalists have no endowment.

Technology. Each capitalist has access to a production technology that converts $\chi(k)$ units of the consumption good into k units of capital at $t = 0$, where χ is quasi-convex and weakly increasing in k.[5] Capital can be used for production by entrepreneurs. Specifically, each entrepreneur has access to a production technology that converts one unit of capital at $t = 0$ into A units of the consumption good at $t = 1$. We refer to A as entrepreneurial productivity and assume that it is distributed independently across entrepreneurs, with distribution G that has an associated density g with full support on the interval $[0, 1]$.

Markets. The economy is small and open and there is an international financial market where agents can borrow and lend consumption goods at a world interest rate R. Here, we introduce a central friction of our analysis by supposing that an entrepreneur can always walk away with a fraction $1 - \lambda$ of her output at $t = 1$. This pledgeability friction will endogenously limit the borrowing and investment that each entrepreneur can undertake. There is also a competitive capital market, where agents can trade capital at a unit price q in period $t = 0$. Note that, as the economy ends at $t = 1$, capital is no longer valuable after production at that date.

2.1 Optimization and equilibrium

Since agents can borrow and lend consumption goods in the international financial market at the interest rate R, only the clearing of the capital market is crucial for equilibrium. To characterize this market clearing condition, we analyze next the demand and supply of capital.

[5] The case where capitalists are simply endowed with $\bar{K}$ units of capital is captured by the following cost function: $\chi(k) = 0$ for $k \leq \bar{K}$ and $\chi(k)$ is infinite thereafter.

Capital demand. Let b_A and k_A respectively denote total borrowing and capital demand by an entrepreneur with productivity A. Given prices $\{q, R\}$, the entrepreneur makes her optimal borrowing and investment decisions to maximize her expected consumption:

$$\max_{\{b_A, k_A\}} \quad A \cdot k_A - R \cdot b_A, \tag{1}$$

subject to budget, borrowing and feasibility constraints:

$$q \cdot k_A \leq \; w + b_A, \tag{2}$$

$$R \cdot b_A \leq \; \lambda \cdot A \cdot k_A, \tag{3}$$

$$0 \leq k_A \; . \tag{4}$$

Optimization leads to the following capital demand:

$$k_A(q, R) \begin{cases} = 0 & \text{if } \frac{A}{q} < R \\ \in \left[0, \frac{1}{q - \frac{\lambda \cdot A}{R}} \cdot w\right] & \text{if } R = \frac{A}{q} \\ = \frac{1}{q - \frac{\lambda \cdot A}{R}} \cdot w & \text{if } \frac{\lambda \cdot A}{q} < R < \frac{A}{q} \\ = \infty & \text{if } R \leq \frac{\lambda \cdot A}{q} \end{cases}, \tag{5}$$

which has an associated level of borrowing:

$$b_A(q, R) = q \cdot k_A(q, R) - w. \tag{6}$$

Equation (5) states that when the interest rate, R, is smaller than the return to capital, A/q, the entrepreneur demands no capital as it is more profitable to invest in financial markets, while she is indifferent when both returns are equalized. When the interest rate, instead, is smaller than the return to capital but greater than its pledgeable return, the entrepreneur demands capital until her borrowing constraint binds.[6] Finally, when the interest rate is below the pledgeable return to capital, the entrepreneur's demand of capital is unbounded.

The demand function $k_A(q, R)$ is decreasing in both q and R. For an entrepreneur who is unconstrained, i.e., $A \leq q \cdot R$, lower values of q or R raise the return to investing in capital. For an entrepreneur who is constrained, lower values of q or R relax the borrowing constraint and enable her to expand her borrowing and her purchases of capital. The demand function $k_A(q, R)$

[6] Note that $q - \lambda \cdot A \cdot R^{-1}$ represents the "down payment" that is required to purchase a unit of capital: the price of each unit is q, but a part $\lambda \cdot A \cdot R^{-1}$ can be financed by borrowing against the unit's future output.

is also weakly increasing in λ, because a higher pledgeability of output enables constrained entrepreneurs to expand their borrowing and thus their purchases of capital.

We denote the aggregate demand for capital by entrepreneurs by:

$$K^D(q, R) \equiv \int_0^1 k_A(q, R) \cdot dG(A). \tag{7}$$

Capital supply. Given the price of capital, each capitalist chooses his supply of capital to maximize profits:

$$K^S(q) = \arg\max_{k \geq 0} \; q \cdot k - \chi(k), \tag{8}$$

where we assume that a solution to Equation (8) exists.[7] Since all capitalists are identical, we use $K^S(q)$ to denote both the individual and the aggregate supply of capital, which is weakly increasing in q.

Market clearing. The price of capital, q, ensures that the capital market clears:

$$K^S(q) = K^D(q, R). \tag{9}$$

Aggregate domestic output is given by:

$$Y(q, R) = \int_0^1 A \cdot k_A(q, R) \cdot dG. \tag{10}$$

Throughout, our main objective is to characterize the effect of changes in the interest rate on aggregate output and, in particular, on the aggregate stock of capital and its allocation among entrepreneurs. As we shall see, this latter allocative effect will drive the changes in the aggregate productivity of capital, defined as the ratio $Y(q, R)/K^S(q)$.

Equilibrium. An equilibrium consists of prices $\{q, R\}$ and allocations $\{\{k_A, b_A\}_A, K^S, Y\}$, such that, given prices, $\{k_A, b_A\}_A$ satisfy Equations (5) and (6), K^S satisfies Equation (8), the capital market clears according to Equation (9), and Y satisfies Equation (10).

3 Equilibrium effects of changes in interest rates

We are now ready to analyze the equilibrium of the economy. In particular, we are interested in understanding the aggregate effects of changes in the interest rate R. For now, we interpret

[7] E.g. if $\chi(\cdot)$ is increasing and convex with $\chi(0) = \chi'(0) = 0$, then $K^S(q) = \chi'^{-1}(q)$ and is increasing in q.

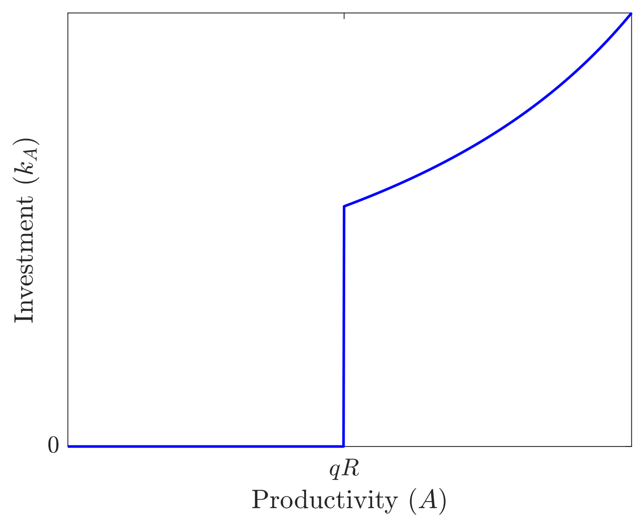


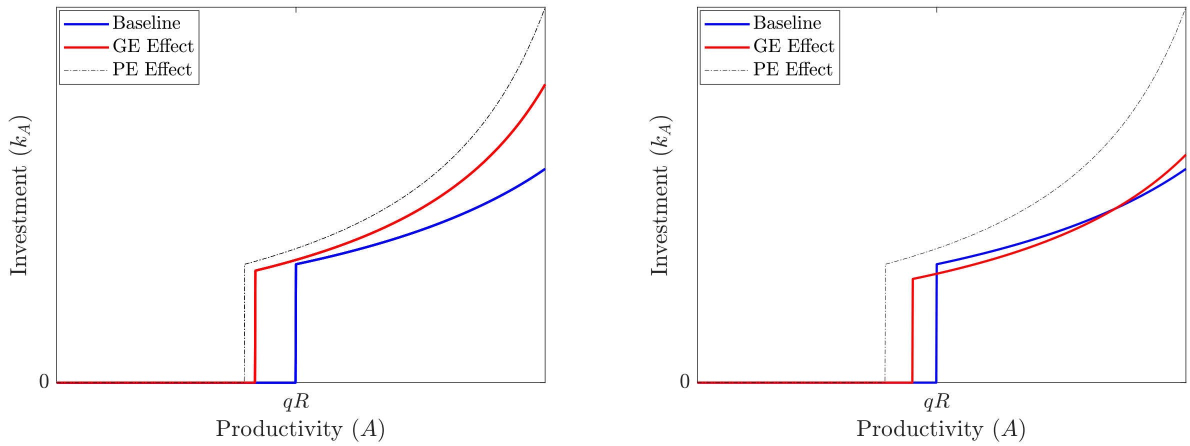
Figure 1: **Distribution of capital across entrepreneurs given prices** $\{q, R\}$**.**

changes in R as being induced by exogenous factors, such as changes in the world interest rate or in capital inflows. We later show that out results go through when R is endogenously determined and its decline is driven by changes in model fundamentals.

Figure 1 depicts the distribution of capital across entrepreneurs for given prices $\{q, R\}$. To determine the aggregate effects of a change in R, we need to understand how this distribution of capital responds to a such change. In what follows, we refer to those entrepreneurs who find it optimal not to invest (i.e., $A < q \cdot R$) as "infra-marginal", to those that invest until their borrowing constraint binds (i.e, $A > q \cdot R$) as "supra-marginal", and to those that are indifferent (i.e., $A = q \cdot R$) as "marginal" entrepreneurs.

At first sight, the effect of a change in R on investment and output seems trivial. It follows immediately from Equation (5) that, for a given price q, all entrepreneurs raise their demand of capital when R falls. Supra-marginal entrepreneurs demand more capital because a lower value of R raises the present value of pledgeable output thus relaxing their borrowing constraints. Moreover, some infra-marginal entrepreneurs start investing because a lower value of R raises the present value of investment. This partial-equilibrium effect of a decline in R is depicted through a shift from the solid-blue to the dashed curve in panels (a) and (b) of Figure 2.

As long as the supply of capital is not perfectly elastic, however, the effects of a decline in R do not end here. There is also a general-equilibrium effect because the price of capital q must increase to insure capital market clearing. This general-equilibrium effect makes capital less attractive and reduces investment along all margins, but it cannot be so strong as to raise the



(a) Weak GE Effects

(b) Strong GE Effects

Figure 2: **Partial- and general-equilibrium effects of a fall in the interest rate on k_A.**

productivity of the marginal entrepreneur, $q \cdot R$: otherwise, it follows from Equation (5) that all entrepreneurs would reduce their demand of capital, which is a contradiction. This implies that a decline in the interest rate must necessarily raise the investment of some infra-marginal entrepreneurs, although it may reduce the investment of some supra-marginal entrepreneurs.

Formally, the change in the investment of a supra-marginal entrepreneur with productivity A is given by:

$$\frac{dk_A}{dR} = \frac{\left|\frac{dq}{dR}\right| - \frac{\lambda \cdot A}{R^2}}{q - \frac{\lambda \cdot A}{R}} \cdot k_A. \tag{11}$$

Equation (11) shows that the change in investment induced by a change in the interest rate has a partial- and a general-equilibrium component. The second term in the numerator represents the partial-equilibrium effect, by which a decline in R increases the pledgeable return to capital and thus entrepreneurs' ability to invest. The first term in the numerator captures instead the general-equilibrium effect, which raises the price of capital thereby reducing entrepreneurial demand of capital.

Equation (11) shows that the investment of supra-marginal entrepreneurs may decline when the interest rate falls, provided that general-equilibrium effects are strong and financial frictions severe enough. As $\lambda \to 0$, for instance, only the general equilibrium effect is present and the investment of all supra-marginal entrepreneurs must (weakly) increase in the interest rate. Panels (a) and (b) in Figure 2, respectively, depict the general-equilibrium effect of a fall in the interest rate on entrepreneurial investment, through a shift from the dashed to the solid-red curve. In panel (a), general-equilibrium effects are weak and all supra-marginal entrepreneurs invest more when the interest rate falls; in panel (b), instead, general-equilibrium effects are strong and some supra-marginal entrepreneurs invest less when the interest rate declines.

The effect of changing interest rates on output is shaped by the behavior of investment across entrepreneurs. Formally, we can combine Equations (9) and (10) to obtain:

$$\frac{dY}{dR} = \underbrace{\int_{q\cdot R}^{1} (A - q \cdot R) \cdot \frac{dk_A}{dR} \cdot dG(A)}_{\text{Capital-reallocation effect } \equiv \mathcal{R}} + \underbrace{q \cdot R \cdot \frac{dK^S(q)}{dR}}_{\text{Capital-supply effect } \equiv \mathcal{K}}, \tag{12}$$

where dk_A/dR is given by Equation (11). The first term in Equation (12), which we denote by $\mathcal{R}$, captures the capital-reallocation effect: the change in output driven by changes in the investment of supra-marginal entrepreneurs. As we have already noted, this effect can be positive or negative depending on the relative strength of the general- and partial- equilibrium effects. In what follows, we say that the capital-reallocation effect is *stronger* when $\mathcal{R}$ is more positive. The second term in Equation (12), which we denote by $\mathcal{K}$, captures instead the capital-supply effect: the change in output driven by adjustments in the aggregate capital stock. This effect is always (weakly) negative, since lower interest rates raise the demand for capital and thus the equilibrium stock of capital. In what follows, we say that the capital-supply effect is *stronger* when $\mathcal{K}$ is more negative.

Equation (12) illustrates the role of both heterogeneity and financial frictions in shaping the aggregate response of output to changes in the interest rate. In the absence of heterogeneity, all entrepreneurs would have the same productivity; in the absence of financial frictions, only the most productive entrepreneur would invest. In either case, the capital-reallocation effect would disappear and the response of output to the interest rate would be negative and driven only by the capital-supply effect, i.e., on the economy's ability to adjust the capital supply to the shifting demand. With heterogeneous productivity and financial frictions, however, the response of output to changes in the interest rate depends not just on the behavior of aggregate investment but also on its reallocation across entrepreneurs. In fact, the capital reallocation effect can be so strong that falling interest rates may become contractionary!

To illustrate this, consider a simple example in which there is no borrowing and lending (i.e., $\lambda = 0$) and the capital stock is fixed (i.e., $K^S(q) = \bar{K}$). The lack of borrowing and lending means that the investment of supra-marginal entrepreneurs is equal to w/q, and thus independent of the interest rate: this maximizes the strength of the reallocation effect, $\mathcal{R}$. The fixed capital supply, in turn, eliminates the capital supply effect, $\mathcal{K}$, altogether. In this case, a decline of the interest rate must necessarily reduce aggregate output. By boosting the investment of infra-marginal entrepreneurs, a lower interest rate raises the equilibrium price of capital and thus reduces supra-marginal investment (which is more productive).

This is of course a stark example but, as the next proposition shows, the result is more general and does not rely on such extreme scenarios.

Proposition 1 *Consider two economies that have the same equilibrium allocations and are identical in all respects except the capital supply schedule. Let ε denote the elasticity of capital supply with respect to the price of capital q in equilibrium. Then, in the economy with lower ε:*

- *The capital-reallocation effect, $\mathcal{R}$, is stronger;*
- *The capital-supply effect, $\mathcal{K}$, is weaker;*
- *The response of output to a change in the interest rate, dY/dR, is less negative.*

Moreover, for low enough ε, dY/dR is positive if also λ is below a threshold.

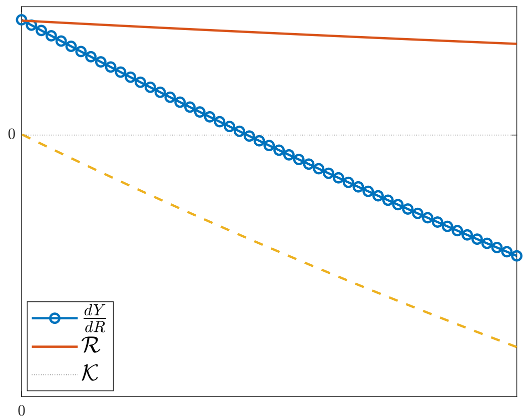
Proposition 1 states that the response of output to the interest rate is decreasing in the elasticity of the capital supply, ε, which governs the strength of the general-equilibrium effect.[8] This is illustrated in Figure 3, which plots dY/dR against ε for low and high values of λ, respectively. Both panels show that dY/dR increases as ε decreases. Lower values of ε weaken the capital-supply effect and, by reinforcing the general-equilibrium response of the price of capital, strengthen the capital-reallocation effect. When λ is low, moreover, the reallocation effect becomes positive and – for low values of ε – a fall in the interest rate leads to a decline in aggregate output (see panel (a)).

3.1 Robustness

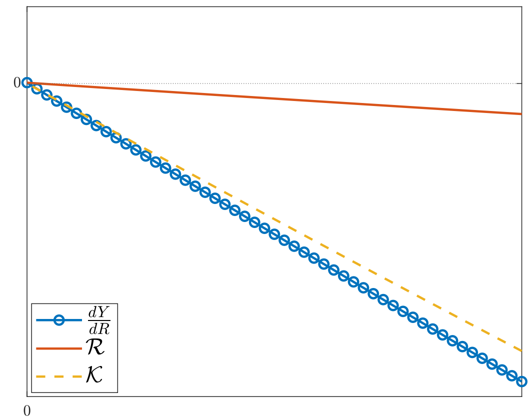
Proposition 1 shows that the interaction of heterogeneous productivity and financial frictions gives rise to capital-reallocation effects in general equilibrium, which can mitigate or even overturn the expansionary effects of declining interest rates. This result was derived in a fairly stylized environment. As we discuss next, however, our findings extend to more general settings.

Unconstrained firms. We have assumed throughout that all entrepreneurs are subject to financial frictions. Yet, one may wonder how our results change if some agents do not face financial frictions. Indeed, there is an increased interest among macroeconomics to understand the potentially heterogeneous reaction of constrained vs. unconstrained firms to monetary

[8] The parameterization of the cost function $\chi(\cdot)$ used for Figure 3 is provided in the proof of Proposition 1.



(a) Low λ



(b) High λ

Figure 3: **Effect of the elasticity of capital supply on dY/dR.**

policy (e.g. Cloyne et al. (2018); Ottonello and Winberry (2020)). To this end, in Appendix B.1, we introduce unconstrained firms into our baseline setup and show that nothing substantial changes provided that in equilibrium these firms operate side-by-side with some constrained entrepreneurs. This requires that the unconstrained firms not be too productive; otherwise, they would absorb the entire capital stock and the economy would behave as if frictionless. Moreover, because the unconstrained firms freely adjust their demand in response to changes in the interest rate, the presence of such firms can actually strengthen the capital-reallocation effects by prompting larger adjustments in the price of capital, further raising dY/dR.

Diminishing returns at entrepreneur level. In our baseline model, entrepreneurs operate a linear production technology. As a result, entrepreneurial investment has the "bang-bang" property whereby (generically) an entrepreneur either finds it unattractive to invest or wants to invest as much as possible in capital. In Appendix B.2, we show that our results remain unchanged if there are diminishing returns at entrepreneur level, a feature commonly used in the firm-dynamics literature (e.g. Hopenhayn (1992)). The key difference is that, now, the set of marginal entrepreneurs has positive measure and, if Inada conditions are satisfied, it includes all entrepreneurs below a certain threshold on productivity A. In equilibrium, these entrepreneurs are unconstrained because – given their lower marginal product of capital – they operate at a smaller scale than the constrained, more productive entrepreneurs.[9] Under similar conditions as in our baseline model, moreover, a fall in the interest rate generates reallocation

[9]Note that, since now there is a mass of marginal entrepreneurs who are unconstrained in equilibrium, this extension is conceptually similar to the previous one where some firms are assumed to be unconstrained.

of capital from more to less productive entrepreneurs, thus reducing output.

Closed economy and savings' gluts. We have considered an economy that is small and open, which takes the world interest rate as given. One may wonder what would change in a closed economy, where the interest rate is determined endogenously. To this end, Appendix B.3 shows that our results remain unchanged if the fall in the interest rate is endogenously triggered by agents' increased desire to save (i.e., a savings' glut). In a closed-economy version of our baseline model, such an increase in savings is captured either through a shift in preferences (higher patience) or in endowments (capitalists become richer). Thus, our findings are consistent with one of the most popular hypotheses to explain the sustained decline in interest rates over the past several decades (e.g. Bernanke et al. (2005); Caballero et al. (2008)).

Dynamics of wealth accumulation. Finally, our baseline model is essentially static, as it features a two-period economy where entrepreneurial wealth is exogenously specified. In a dynamic world, however, such wealth would naturally be endogenous to (i) productivity, as more productive entrepreneurs may accumulate wealth faster; and (ii) asset prices, due to the well-known balance-sheet effects á la Kiyotaki and Moore (1997). In Section 4, we derive our results in a dynamic version of our model and show how they are affected by wealth accumulation and balance sheet effects.

3.2 Normative properties

We now turn to the normative properties of equilibrium. In particular, we analyze the extent to which the level of aggregate output, as well as its response to changes in the interest rate, are (in)efficient.

Consider the problem of a social planner whose objective is to maximize aggregate consumption at $t = 1$.[10] The planner is constrained to only choosing investment $\{k_A\}$ for entrepreneurs, but agents make all the other decisions on their own. This implies, in particular, that the planner must respect individual budget and financial constraints. To simplify the exposition, we further assume that $\chi(\cdot)$ is strictly convex.

Formally, the social planner solves the following maximization problem:

$$\max_{\{k_A\}} \int A \cdot k_A \cdot dG(A) - R \cdot [\chi(K^S) - w], \tag{13}$$

[10]Since preferences are linear, such an objective is equivalent to maximizing the equally-weighted aggregate welfare. We thus abstract from distributional considerations.

subject to:

$$R \cdot (q \cdot k_A - w) \leq \lambda \cdot A \cdot k_A \quad \text{and} \quad 0 \leq k_A \quad \forall A, \tag{14}$$

and to capitalists' optimization and market clearing:

$$\chi^{-1}(q) = K^S = \int k_A \cdot dG(A). \tag{15}$$

The objective function of the planner in (13) says that aggregate consumption at $t = 1$ is equal to aggregate output net of repayments to international lenders, which are given by R times the difference between the cost of capital production and aggregate endowment at $t = 0$. Equations (14) and (15) state that the planner must respect individual budget and financial constraints, feasibility, and market clearing. In particular, the planner is not able to make transfers so as to overcome financial frictions.

To understand the solution to the planner's problem, consider the (social) net present value (NPV_A^{SP}) of unit of investment, k_A, by entrepreneur with productivity A:

$$\mathrm{NPV}_A^{SP} \equiv \frac{A}{R} - q - \left[\chi''\left(K^S\right) \cdot \int \gamma_{\widehat{A}} \cdot k_{\widehat{A}} \cdot dG(\widehat{A})\right] \tag{16}$$

where $\gamma_{\widehat{A}}$ denotes the multiplier on the borrowing constraints of entrepreneurs with productivity $\widehat{A}$. The first observation is that, since NPV_A^{SP} is linear and increasing in A, there exists a marginal entrepreneur $\widetilde{A}$ with $\mathrm{NPV}_{\widetilde{A}}^{SP} = 0$, such that only entrepreneurs with productivity above $\widetilde{A}$ invest and they do so until their borrowing constraints bind. The second observation is that, since the term in brackets is positive, the planner perceives a higher social cost (or a lower social benefit) of investment than individual entrepreneurs, who only compare A/R to q. This is because the planner internalizes that each additional unit of investment by entrepreneurs with productivity A raises the equilibrium price of capital (as $\chi''(K^S) > 0$) and thus tightens the borrowing constraints of all entrepreneurs with productivity above $\widetilde{A}$. Since the borrowing constraints bind for all such entrepreneurs (as $\gamma_A > 0 \; \forall A > \widetilde{A}$), this entails a first-order welfare loss. Consequently, the planner restricts investment by some entrepreneurs by setting $\widetilde{A} > q \cdot R$.

The following proposition summarizes the above discussion and also states its main implication for the response of output to changes in interest rates. Let superscripts $\{CE\}$ and $\{SP\}$ indicate the competitive equilibrium and the social planner's allocations respectively.

Proposition 2 *If $\widetilde{A}$ denotes the productivity of the marginal entrepreneur at the social planner's allocation, then:*

$$\widetilde{A} > R \cdot q^{CE} > R \cdot q^{SP},$$

i.e., relative to the competitive equilibrium, the planner restricts investment by some supra-marginal entrepreneurs, which depresses asset prices. Moreover:

$$\frac{dY^{SP}}{dR} < 0,$$

i.e., at the planner's allocation, a fall in the interest rate is never contractionary.

The first part of Proposition 2 follows directly from our previous discussion. It only adds that, by forbidding some entrepreneurs from investing altogether, the planner reduces the price of capital relative to the competitive equilibrium thus allowing more productive entrepreneurs to expand their investment. The second part of Proposition 2 follows directly from the first. To see this, simply note that the only reason for which a fall in the interest rate can reduce output is that it reallocates capital from supra- to infra-marginal entrepreneurs (see Equation (12)). But the planner can always keep these reallocation effects under control by adjusting the productivity of the marginal entrepreneur, $\widetilde{A}$, when the interest rate changes.

These results are related to the recent literature on zombie firms, which emphasizes that low interest rates incentivize unproductive investment (Caballero et al., 2008; Adalet McGowan et al., 2018; Tracey, 2019; Schivardi et al., 2020). As in that literature, declining interest rates can lead to excessive investment in our model. Differently from that literature, however, all investment in our model has a positive net present value from a private standpoint. It is nonetheless excessive from a social standpoint because individual entrepreneurs do not internalize the crowding-out effect that their investment has on more productive entrepreneurs. It is this externality that is at the heart of our results.

4 Dynamics of wealth accumulation

In Section 3, we characterized the effects of changes in the interest rate under the assumption that entrepreneurial wealth was exogenous. This raises the question of whether our results extend to a dynamic economy, in which entrepreneurial wealth accumulates endogenously through internal reinvestment. In particular, productive entrepreneurs may stand to gain from a fall in the interest rate, as they benefit both through lower costs of borrowing and potentially through higher asset prices and their positive balance-sheet effects (e.g. Kiyotaki and Moore (1997)). We turn to this question next.

4.1 Dynamic model

Suppose that time t is continuous. To simplify notation, we omit time subscripts whenever possible. As in the baseline model of Section 3, the economy is populated by a continuum of entrepreneurs with mass one. However, in the dynamic model, entrepreneurs may not only differ in their productivity A, but also in their wealth w. Individual productivity A is stochastic and it fluctuates over time according to an idiosyncratic Poisson process with common arrival rate $\theta > 0$. If hit by the Poisson shock, the individual entrepreneur draws a new productivity from aggregate distribution G. Otherwise, the entrepreneur keeps her productivity unchanged. Individual wealth w instead evolves endogenously according to the individual return on wealth and the rate at which individual entrepreneurs consume their wealth. Absent the Poisson shock, the law of motion of entrepreneurial wealth is

$$\dot{w} = y + \dot{q} \cdot k - r \cdot b - c, \tag{17}$$

where $y = A \cdot k$ is the output rate obtained from operating capital stock $k \geq 0$; $\dot{q}$ is the rate of change of the price of capital $q > 0$; $r > 0$ is the interest rate on debt/savings $b = q \cdot k - w$; and $c \geq 0$ is the consumption rate.

As in the baseline model, entrepreneurial may be financially constrained. To simplify the exposition, here we introduce financial constraints by assuming that an entrepreneur can walk away with a portion $1 - \lambda \in (0, 1)$ of her capital stock immediately after issuing debt. From this, we obtain the standard collateral constraint as in Moll (2014):

$$b \leq \lambda \cdot q \cdot k. \tag{18}$$

Entrepreneurs have logarithmic preferences for consumption and discount future consumption at rate $\rho > r$.[11] For simplicity, we assume throughout that the capital supply is fixed and equal to $\bar{K}$. Thus, capitalist optimization is irrelevant. This is without loss of generality, but it allows us to focus on our main objective, i.e., to show that the same capital-reallocation effects that we identified in the baseline model are also present in the dynamic economy.

[11] Entrepreneurs must be impatient relative to international lenders because otherwise their wealth would grow away from the collateral constraint.

4.2 Equilibrium

In any period t, entrepreneurs choose their consumption c, their capital stock k, and their debt b, given the path of asset prices and the interest rate. The optimal choice of entrepreneurs is:

$$k = \begin{cases} \frac{1}{1-\lambda} \cdot \frac{w}{q} & \text{if } A + \dot{q} \geq r \cdot q \\ 0 & \text{otherwise} \end{cases}, \tag{19}$$

and

$$b = q \cdot k - \omega \tag{20}$$

$$c = \rho \cdot w. \tag{21}$$

Equation (19) says that, just as in the baseline model, there is a threshold entrepreneur who is indifferent between saving or investing in capital. The only difference is that this threshold is now given by $r \cdot q - \dot{q}$, as part of the return to capital accrues in the form of capital gains. Entrepreneurs above the threshold (i.e., supra-marginals) borrow and invest as much as possible whereas those below (i.e., infra-marginals) save at interest rate r. Equation (21) says that, due to logarithmic preferences, entrepreneurs consume a portion ρ of their wealth each period.

From substituting the optimal choice into law of motion (17), it follows that individual wealth w evolves according to:

$$\dot{w} = \begin{cases} \left[\left(\frac{A+\dot{q}}{q} - \lambda \cdot r\right) \cdot \frac{1}{1-\lambda} - \rho\right] \cdot w & \text{if } A + \dot{q} \geq r \cdot q \\ (r - \rho) \cdot w & \text{otherwise} \end{cases}. \tag{22}$$

Equation (22) captures the endogeneity of wealth accumulation in this dynamic economy. In particular, note that more productive entrepreneurs accumulate wealth at a faster pace.

To characterize the equilibrium, it is convenient to aggregate the behavior of all entrepreneurs with the same productivity A. To do so, we can define the aggregate wealth of those entrepreneurs as:

$$W_A \equiv \int w \cdot f(A, w) \cdot dw, \tag{23}$$

where $f(A, w)$ is the population share of entrepreneurs with productivity A and wealth w. Combining Equations (22) and (23), and taking into account the stochastic evolution of individual productivity, it follows that W_A evolves according to:

$$\dot{W}_A = \begin{cases} \left[\left(\frac{A+\dot{q}}{q} - \lambda \cdot r\right) \cdot \frac{1}{1-\lambda} - \rho - \theta\right] \cdot W_A + \theta \cdot g\left(A\right) \cdot W & \text{if } A + \dot{q} \geq r \cdot q \\ \left(r - \rho - \theta\right) \cdot W_A + \theta \cdot g\left(A\right) \cdot W & \text{otherwise} \end{cases} \tag{24}$$

where g is the density related to cumulative probability function G and $W > 0$ is aggregate wealth, that is,

$$W \equiv \int W_A \cdot dA. \tag{25}$$

The per-capita investment of entrepreneurs with productivity A is then given by:

$$k_A = \begin{cases} \frac{1}{1-\lambda} \cdot \frac{W_A}{q} \cdot \frac{1}{g(A)} & \text{if } A + \dot{q} \geq r \cdot q \\ 0 & \text{otherwise} \end{cases}. \tag{26}$$

Finally, using Equation (23), we can express market clearing as:

$$\bar{K} = \int_{A \geq r \cdot q - \dot{q}} k_A \cdot dG(A), \tag{27}$$

and aggregate output as:

$$Y = \int_{A \geq r \cdot q - \dot{q}} A \cdot k_A \cdot dG(A). \tag{28}$$

An equilibrium consists of paths of prices $\{q, R\}$ and of allocations $\{\{W_A, k_A\}_A, W, Y\}$ such that Equations (24)-(28) are satisfied in all periods.

4.3 Reallocation effects in steady state

We begin by analyzing the steady-state effects of a fall in the interest rate r. The key feature of the steady state is that prices and aggregate variables are constant over time, i.e., $\dot{q} = 0$ and $\dot{W}_A = 0$ for all A. This implies, from Equation (24), that the steady-state wealth of entrepreneurs with productivity A is a constant share of aggregate wealth W:

$$W_A = \begin{cases} \frac{\theta}{\theta + \rho + \frac{1}{1-\lambda} \cdot \left(\lambda \cdot r - \frac{A}{q}\right)} \cdot g\left(A\right) \cdot W & \text{if } A \geq r \cdot q \\ \frac{\theta}{\theta + \rho - r} \cdot g\left(A\right) \cdot W & \text{otherwise} \end{cases}. \tag{29}$$

Equation (29) shows that, in this dynamic economy, there is a natural link between productivity and wealth. In particular, per-capita wealth $W(A)/g(A)$ is strictly increasing in A for all supra-marginal entrepreneurs. The reason is that entrepreneurs with a higher productivity have higher return on wealth. In a related vein, the equation also shows that wealth accumulation tilts per-capita wealth away from a uniform distribution according to the first factor on the RHS. The factor can then be interpreted as the endogenous distribution of wealth in steady state in a dynamic setup.

Together with the definition of aggregate wealth, Equation (29) implies that:

$$\int_{r\cdot q}^{1} \frac{\theta}{\theta+\rho+\frac{1}{1-\lambda}\cdot\left(\lambda\cdot r-\frac{A}{q}\right)}\cdot dG(A) = 1-\frac{\theta}{\theta+\rho-r}\cdot[G(r\cdot q)-G(0)], \tag{30}$$

while the market clearing condition can be written as:

$$\bar{K} = \left[\int_{r\cdot q}^{1} \frac{\theta}{\theta+\rho+\frac{1}{1-\lambda}\cdot\left(\lambda\cdot r-\frac{A}{q}\right)}\cdot dG(A)\right]\cdot\frac{1}{1-\lambda}\cdot\frac{W}{q}. \tag{31}$$

Equations (29)-(31) determine $\{W_A\}$, W, and q, and jointly illustrate the steady-state effects of a fall in the interest rate, r. Equation (29) shows that, for given aggregate wealth W, a fall in r reduces the wealth of infra-marginal entrepreneurs and raises the wealth of supra-marginal entrepreneurs: in a nutshell, lower interest rates transfer wealth from creditors to debtors. This also implies that the productivity of the marginal entrepreneur, $r \cdot q$, falls as saving became relatively less attractive than investing in capital. Thus, if aggregate wealth W were fixed, the price of capital would rise along with the increase in demand (see Equation (31)). But aggregate wealth is itself endogenous, and it may rise or fall when r declines. Intuitively, if the losses inflicted on infra-marginal entrepreneurs by a lower value of r are not compensated by the gains of supra-marginal entrepreneurs (e.g. if λ is small), then aggregate wealth will decline when r falls. As a result, the net effect on the price of capital, q, is ambiguous. What is unambiguous, however, is that – just as in the baseline model – aggregate wealth in terms of capital (W/q) must fall to ensure market clearing.

Figure 4 summarizes the above discussion. Panel (a) shows the steady-state shares of aggregate wealth held by entrepreneurs as a function of their productivity, for two levels of the interest rate, $r_H > r_L$. The figure shows how a fall in the interest rate from r_H to r_L raises the share of wealth of supra-marginal entrepreneurs and reduces the productivity of the marginal entrepreneur. Panel (b) instead shows how the steady-level of aggregate wealth in terms of capital, W/q, declines as the interest rate falls from r_H to r_L.

But how does all of this affect aggregate output? To address this question, we can combine

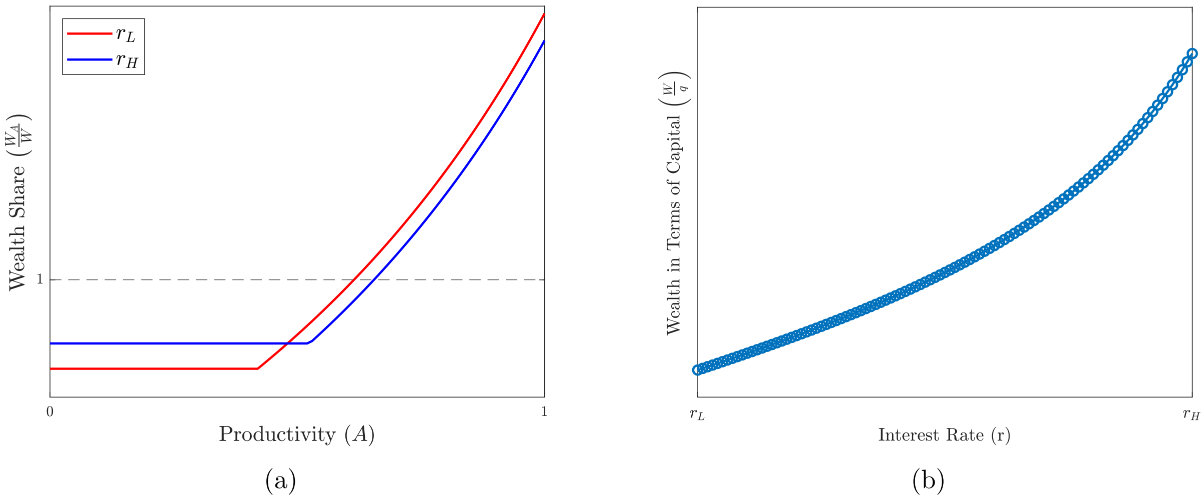


Figure 4: **Steady-state effect of a fall in the interest rate on wealth shares and total wealth in terms of capital.**

Equations (27) and (28) to obtain:

$$\frac{dY}{dr} = \int_{r \cdot q}^{1} (A - r \cdot q) \cdot \frac{dk_A}{dr} \cdot dG(A). \tag{32}$$

Equation (32) is the equivalent of Equation (12) for the dynamic setup. Since the capital stock is fixed at $\bar{K}$, there is no capital-supply effect. There is, however, a capital-reallocation effect, which depends on the change in the equilibrium level of investment of supra-marginal entrepreneurs. As in the baseline model, this effect can be positive or negative because it captures both the reallocation of capital from supra- to infra-marginal entrepreneurs (which reduces aggregate output) and the reallocation of capital among supra-marginal entrepreneurs (why may increase or reduce aggregate output).

Figure 5 illustrates the effect of a fall in r for a specific parametrization of the dynamic economy. Panel (a) shows the per-capita investment of entrepreneurs as a function of their productivity, for two levels of the interest rate, $r_H > r_L$. The figure shows how a fall in the interest rate from r_H to r_L reduces the investment of all supra-marginal entrepreneurs and raises that of infra-marginal entrepreneurs. Due to this reallocation, Panel (b) shows that the steady-level of output declines monotonically as the interest rate falls from r_H to r_L.

In the baseline model, Proposition 1 characterized conditions under which $dY/dr > 0$: the reason is that the reallocation among supra-marginals could be weakened arbitrarily by reducing λ. In the dynamic setup, however, this is no longer possible because the redistribution among

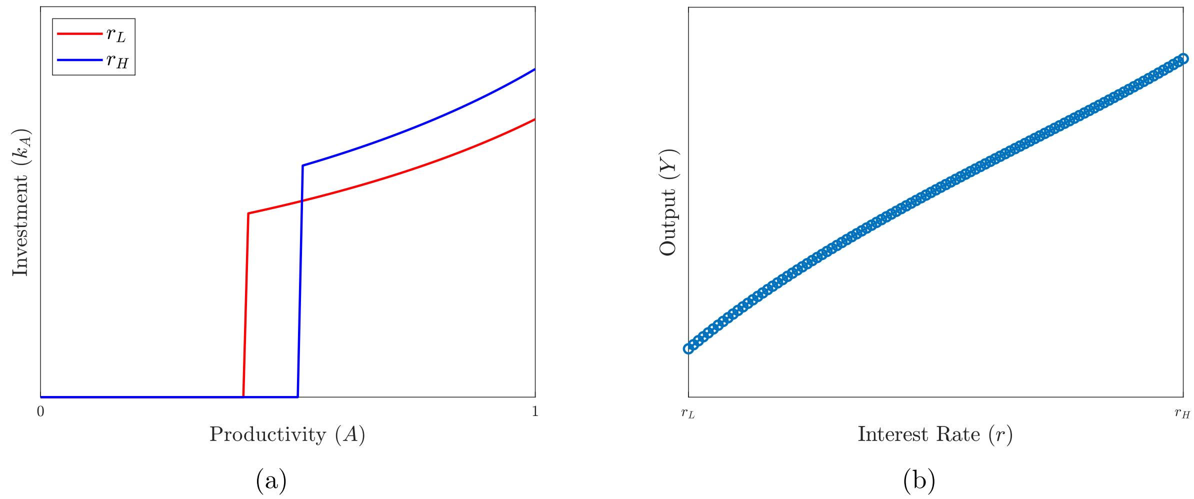


Figure 5: **Steady-state effect of a fall in the interest rate on entrepreneurial investment and aggregate output.**

supra-marginals operates through the dynamic accumulation of wealth (even if λ equals zero). Although we have been unable to provide general conditions under which $dY/dr > 0$ in the dynamic economy, we have not found a parametrization where this does not hold.

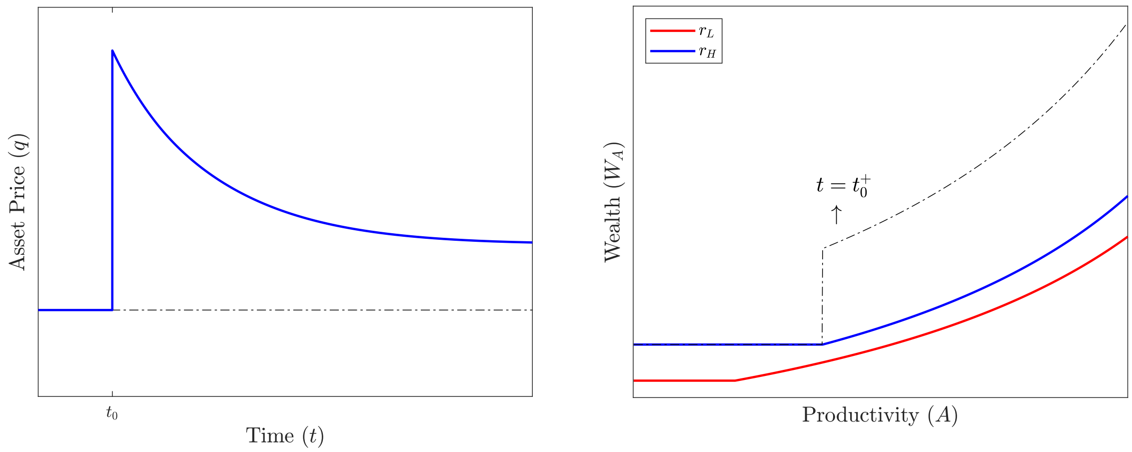
4.4 Balance-sheet effects and transition dynamics

The previous section showed that a fall in r can lead to a steady-state decline in output due to reallocation of capital from supra-marginal to infra-marginal entrepreneurs. As we show next, however, this convergence may be non-monotonic due to balance-sheet effects.

To see this, suppose that the economy is in steady state at time t_0 when it experiences an unexpected, permanent fall in the interest rate from r_H to r_L. For concreteness, we focus throughout on the parametrization in which the steady-state value of q rises in response to the fall in r. As is standard in the literature (e.g. Kiyotaki and Moore (1997)), changes in q can give rise to balance-sheet effects provided that credit contracts are non-contingent. In our environment, as a result, changes in q will affect the net worth of supra-marginal entrepreneurs, proportionally to their capital holdings.

To see this, note that on impact a fall in r leads to an instantaneous increase in the price of capital from q_{t_0} to $q_{t_0^+}$, where the subindexes t_0 and t_0^+ respectively denote the instants before and after the shock.[12] As a result, the wealth of supra-marginal entrepreneurs jumps in

[12]If the price of capital q did not rise, there would an excess demand of capital since the permanent fall in r would induce infra-marginal entrepreneurs to invest.



(a) (b)

Figure 6: **Balance-sheet effects induced by a fall in the interest rate.**

proportion to their capital holdings, while the wealth of infra-marginals is unaffected:

$$W_{A,t_0^+} = \begin{cases} \left(\frac{q_{t_0^+}}{q_{t_0}} - \lambda\right) \cdot \frac{1}{1-\lambda} \cdot W_{A,t_0} & \text{if } A \geq r \cdot q_{t_0} \\ W_{A,t_0} & \text{otherwise} \end{cases} . \tag{33}$$

From then on, the evolution of aggregate variables $\{\{W_A, k_A\}_A, W, q, Y\}$ is characterized as before by Equations (24)-(28).

The transition of the economy to its new steady state is depicted in Figures 6 and 7. Panel (a) of Figure 6 shows the evolution of the price of capital q. As the figure shows, q rises on impact and then gradually declines to its new, higher steady-state value. Consistent with Equation (33), panel (b) shows that, on impact, this rise in q boosts the wealth of all supra-marginal entrepreneurs (see shift from the solid blue curve to the dashed curve). Figure 7 then depicts the resulting non-monotonic evolution of output. On impact, output increases due to balance-sheet effects, which benefit the most productive supra-marginal entrepreneurs and enable a reallocation of capital in their favor. From then on, output declines monotonically to its new, lower steady-state value. Thus, balance-sheet effects are a one-time transfer to supra-marginal entrepreneurs, which materialize in the instant that the agents in our economy are surprised by the fall in r. The reallocation effects that we have emphasized throughout are instead permanent, just as the fall in r, and thus become dominant eventually.[13]

[13]For the interested reader, in Appendix B.4 we show that the insight – i.e., that balance-sheet effects mask

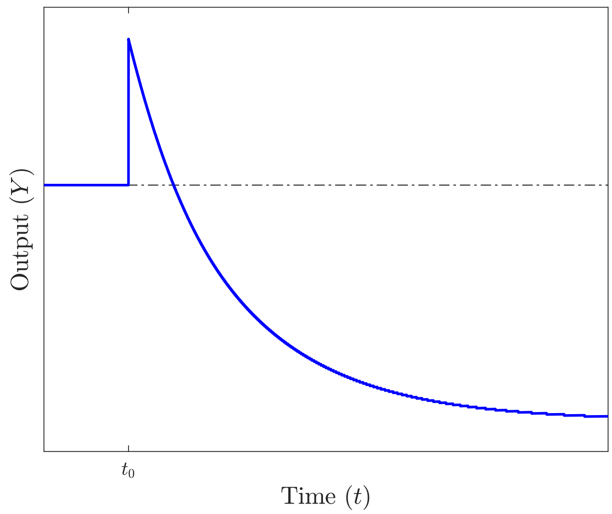


Figure 7: **Evolution of output after a fall in the interest rate.**

5 Supporting evidence

Work in progress ...

6 Conclusions

What is the effect of declining interest rates on the efficiency of resource allocation and overall economic activity? We study this question in a setting where entrepreneurs with different productivities invest in capital, subject to financial frictions. We show that a fall in the interest rate has an ambiguous effect on aggregate output. In partial equilibrium, a lower interest rate raises aggregate investment both by relaxing borrowing constraints and by prompting relatively unproductive entrepreneurs to increase investment. In general equilibrium, however, this higher demand for capital raises its price. This crowds out investment by relatively more productive entrepreneurs, reallocating it to less productive ones. Contrary to the conventional wisdom, we show that a fall in interest rates can be contractionary if this general equilibrium is strong enough, which occurs when: (i) the supply of capital is sufficiently inelastic and (ii) the financial frictions are severe enough. We show that such contractionary effects arise because of an externality, whereby less productive entrepreneurs fail to internalize the effect of their capital demand on its price. We also show that such these effects can be temporarily masked by balance sheet effects resulting from the effect of interest rates on the price of capital. Empirical evidence

the reallocation effects induced by lower interest rate only temporarily – can also be obtained in the classic setting of Kiyotaki and Moore (1997).

using firm-level data from the US and Spain supports of our model mechanism.

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A Proofs

Proof of Proposition 1. The capital market clearing condition is:

$$K^S(q) = \int_{q\cdot R}^{1} \frac{R}{q\cdot R - \lambda\cdot A}\cdot g(A)\cdot dA\cdot w, \tag{34}$$

and aggregate output, Y, is given by:

$$Y = \int_{q\cdot R}^{1} A\cdot \frac{R}{q\cdot R - \lambda\cdot A}\cdot g(A)\cdot dA\cdot w. \tag{35}$$

The derivative of aggregate output, Y, with respect to the interest rate, R, is:

$$\begin{aligned} \frac{dY}{dR} = & \int_{q\cdot R}^{1} \frac{A}{(q\cdot R - \lambda\cdot A)^2}\cdot\left(\left|\frac{dq}{dR}\right| - \frac{\lambda\cdot A}{R^2}\right)\cdot g(A)\cdot dA\cdot w \\ & + \frac{R}{1-\lambda}\cdot g(q\cdot R)\cdot w\cdot\left(R\cdot\left|\frac{dq}{dR}\right| - q\right), \end{aligned} \tag{36}$$

which, all else equal, is increasing in $|dq/dR|$. To arrive at Equation (12), we totally differentiate the capital market clearing condition and replace the last term in Equation (36).

Assume $\chi(\cdot)$ is locally twice differentiable, so that the capital supply elasticity is given by:

$$\varepsilon = \frac{q}{\chi''\left(\chi'^{-1}(q)\right)\cdot\chi'^{-1}(q)}. \tag{37}$$

From the capital market clearing condition (34), therefore, we have:

$$\left|\frac{dq}{dR}\right| = \frac{q\cdot\int_{q\cdot R}^{1}\frac{1}{(q\cdot R-\lambda\cdot A)^2}\cdot\frac{\lambda\cdot A}{R^2}\cdot g(A)\cdot dA\cdot w + \frac{q\cdot R}{1-\lambda}\cdot g(q\cdot R)\cdot w}{\varepsilon\cdot K^S(q) + q\cdot\int_{q\cdot R}^{1}\frac{1}{(q\cdot R-\lambda\cdot A)^2}\cdot g(A)\cdot dA\cdot w + \frac{R}{1-\lambda}\cdot g(q\cdot R)\cdot w}. \tag{38}$$

Thus, observe that, all else equal, $\left|\frac{dq}{dR}\right|$ is decreasing in the capital supply elasticity, ε, in equilibrium. It follows that dY/dR is decreasing in ε. Below, we will verify that it is indeed possible to change ε (by adjusting $\chi(\cdot)$) without affecting the equilibrium allocations.

Next, dY/dR is continuous in λ and, when $\lambda = 0$, aggregate output is given by:

$$Y = \left(\int_{q\cdot R}^{1} A\cdot\frac{g(A)}{1-G(q\cdot R)}\cdot dA\right)\cdot K^S(q), \tag{39}$$

where:

$$K^S(q) = \frac{w}{q} \cdot (1 - G(q \cdot R)). \tag{40}$$

From the capital market clearing condition, it is clear that $q \cdot R$ must rise with R. Hence, it follows that:

$$\frac{d\left(\int_{q \cdot R}^{1} A \cdot \frac{g(A)}{1-G(q \cdot R)} \cdot dA\right)}{dR} > 0, \tag{41}$$

that is, aggregate TFP is increasing in R. Moreover, since:

$$\frac{dY}{dR} = \left(\int_{q \cdot R}^{1} A \cdot \frac{g(A)}{1 - G(q \cdot R)} \cdot dA\right) \cdot K^S(q) + \left(\int_{q \cdot R}^{1} A \cdot \frac{g(A)}{1 - G(q \cdot R)} \cdot dA\right) \cdot \frac{dK^S(q)}{dR}, \tag{42}$$

we have that $dY/dR > 0$ provided that the capital supply elasticity, ε, is small enough, i.e., ε is below some $\bar{\varepsilon} > 0$. Fix such an $\varepsilon < \bar{\varepsilon}$. By continuity, there exists a threshold $\bar{\lambda}_\varepsilon$ such that $dY/dR > 0$ for all $\lambda < \bar{\lambda}_\varepsilon$.

Finally, to produce Figure 3, we consider the following parameterization of the capital supply schedule (which is equivalent to parmeterizing the cost of capital production). Suppose that the interest rate is equal to R and let γ be defined as follows:

$$\bar{K} = \int_{\gamma \cdot R}^{1} \frac{1}{\gamma - \frac{\lambda \cdot A}{R}} \cdot w \cdot dG(A). \tag{43}$$

Consider the following family of capital supply schedules:

$$K^S(q; \varepsilon) = \max\left\{0,\ \bar{K} \cdot \left(1 + \varepsilon \cdot \frac{q - \gamma}{\gamma}\right)\right\},\ \varepsilon \in [0, \infty). \tag{44}$$

Note that, at the interest rate R, the equilibrium allocations are independent of ε, the elasticity of the capital supply. In particular, $K^S = \bar{K}$ and $q = \gamma$. But, as the interest rate changes, the equilibrium allocations will change since γ is a parameter and q will no longer equal γ. ■

Proof of Proposition 2. Consider the problem of the social planner:

$$\max_{\{k_A\}} \int A \cdot k_A \cdot dG(A) - R \cdot \left(\chi\left(\int k_A \cdot dG(A)\right) - w\right)$$

subject to:

$$R \cdot \left(\chi'\left(\int k_A \cdot dG(A)\right) \cdot k - w\right) \leq \lambda \cdot A \cdot k_A \quad (\gamma_A \cdot g(A)),$$

$$0 \leq k_A \quad (\omega_A \cdot g(A)).$$

In parentheses, we denote the multipliers on the constraints. We also suppose that the cost of capita production, $\chi(\cdot)$, is strictly convex (see below for the case of inelastic capital supply).

The first-order condition to the planner's problem are given by:

$$\frac{A}{R} - q - \chi''\left(K^S\right) \cdot \int \gamma_{\widehat{A}} \cdot k_{\widehat{A}} \cdot dG\left(\widehat{A}\right) = \gamma_A \cdot \left(q - \frac{\lambda \cdot A}{R}\right) - \omega_A \quad \forall A, \tag{45}$$

which together with the Kuhn-Tucker conditions characterize the solution to the problem. Since at the planner's allocation it must be that $q = \chi'\left(K^S\right) \geq \frac{\lambda}{R}$, it follows that there exists $0 < \widetilde{A} \leq 1$ such that $\omega_A = 0$ for all $A < \widetilde{A}$. There are thus two possibilities.

Case 1: $\widetilde{A} = 1$. In this case, $\omega_A > 0 = \gamma_A$ for $A < 1$ and, therefore, $\widetilde{A} > q^{CE} \cdot R$. Further, market clearing requires that $q^{SP} = \frac{\lambda}{R}$ and the capital stock and output are given by:

$$Y = K^{SP} = \chi^{-1}\left(\frac{\lambda}{R}\right),$$

and note that $q^{SP} < q^{CE}$.

Case 2: $\widetilde{A} < 1$. In this case, $\gamma_A > 0 = \omega_A$ for all $A > \widetilde{A}$ and after some algebra we have:

$$\widetilde{A} = q^{SP} \cdot R + \frac{\chi''\left(K^S\right) \cdot \int_{\widetilde{A}}^{1} \left(A - q^{SP} \cdot R\right) \cdot \frac{1}{\left(q^{SP} - \frac{\lambda \cdot A}{R}\right)^2} \cdot w \cdot dG\left(A\right)}{1 + \chi''\left(K^S\right) \cdot \int_{\widetilde{A}}^{1} \frac{1}{\left(q^{SP} - \frac{\lambda \cdot A}{R}\right)^2} \cdot w \cdot dG\left(A\right)}, \tag{46}$$

where

$$q^{SP} = \chi'\left(K^S\right) = \chi'\left(\int_{\widetilde{A}}^{1} \frac{1}{q^{SP} - \frac{\lambda \cdot A}{R}} \cdot w \cdot dG\left(A\right)\right). \tag{47}$$

Clearly, again, $\widetilde{A} > q^{SP} \cdot R$ since the RHS in Equation (46) is positive. Note that, from Equation (47), the capital price q^{SP} is depressed below q^{CE} since the entrepreneurs who invest do so until their financial constraints bind, but there are fewer such entrepreneurs.

Next, note that when R falls, the planner can always ensure the same equilibrium allocations (with unchanged equilibrium price of capital, q^{SP}), in which case all supra-marginal entrepreneurs' financial constraints become slack. Moreover, it is clear that the planner would never want to reduce both $K^S(q^{SP})$ and Y^{SP} in response to a fall in R, since the aggregate productivity of capital (TFP) is higher than R times the marginal cost of producing a unit of capital, $\chi'(K^S)$, both before and after the fall in R. Finally, it follows that the decline in R must be expansionary. For instance, the planner can always increase $\widetilde{A}$ and k_A for $A > \widetilde{A}$ so that K^S is unchanged but TFP increases. ■

B Complementary Appendix

B.1 Unconstrained firms

Consider a variation of our baseline economy where, in addition to entrepreneurs, there is a mass of firms that do not face financial constraints, i.e., they are unconstrained. For concreteness, we assume that these firms are owned and operated by the capitalists. In particular, assume that these firms have a production technology that can converts k units of capital in $t = 0$ into $f(k)$ units of the consumption good in $t = 1$, where f is twice differentiable with $f(0) = 0$, $f'(\cdot) > 0$ and $f''(\cdot) < 0$.

For simplicity, suppose that the capital supply is inelastic and entrepreneurs cannot pledge their output to creditors, i.e., $\lambda = 0$. Lastly, to ensure that some entrepreneurs operate side-by-side with these firms, we assume that $f'(\bar{K}) < 1$; that is, the marginal product of capital in these firms – if they were to employ the entire stock of capital – is lower than that of the most productive entrepreneur.

In this economy, there are two types of equilibria depending on whether the interest rate, R, is above or below the threshold $\widetilde{R}$.[14] If $R > \tilde{R}$, then the unconstrained firms are inactive, $R \cdot q > f'(0)$, and the analysis of the previous section applies. If $R < \tilde{R}$, the unconstrained firms are active and $R \cdot q = f'(K^T(q, R)))$, where $K^T(q, R) > 0$ denotes the aggregate units of capital employed by these firms; it only depends on (and is decreasing in) $R \cdot q$. In this case, market clearing requires that:

$$\bar{K} = K^T(q, R) + \int_{q \cdot R}^{1} \frac{w}{q} \cdot dG\left(A\right), \tag{48}$$

It follows that in equilibrium a fall in R raises the price of capital q (and weakly reduces $R \cdot q$). The response of aggregate output to changes in the interest rate is given by:

$$\frac{dY}{dR} = \int_{R \cdot q}^{1} (A - R \cdot q) \cdot \frac{d}{dR}\left(\frac{w}{q}\right) \cdot dG(A), \tag{49}$$

which note only features the capital-reallocation effect (as capital supply is fixed). Moreover, since w/q decreases in response to a fall in R, we have that $dY/dR > 0$.

[14] If $\lim_{k \to 0} f'(k) = \infty$, then $\widetilde{R} = \infty$. Otherwise, $\tilde{R}$ is implicitly defined by:

$$\int_{\widetilde{R} \cdot q}^{\bar{A}} \frac{w}{q} \cdot dG\left(A\right) \equiv \bar{K},$$

with $q = f'(0) \cdot \widetilde{R}^{-1}$. At this interest rate, the entrepreneurs are just able to absorb the entire capital stock $\bar{K}$.

What is going on? When the unconstrained firms are active, the price of capital is determined by their marginal product of capital, since $R \cdot q = f'(K^T)$. Thus, any decline in the interest rate must be compensated by an increase in the price of capital, which implies that the demand of capital by supra-marginal entrepreneurs must fall. Ultimately, a decline in the interest rate simply redistributes capital from supra-marginal entrepreneurs to the less productive unconstrained firms, thereby reducing output. Although this example with fixed capital stock and no credit whatsoever is stark, it is straightforward to prove the analogue of Proposition 1 for this setting: namely, as the capital supply becomes less elastic, dY/dR increases and becomes positive provided that λ is below a threshold.

B.2 Diminishing returns at entrepreneur level

Consider a variation of our baseline economy in which entrepreneurial technology has diminishing returns. In particular, suppose that an entrepreneur with productivity A can convert k of capital at $t = 0$ into $A \cdot h(k)$ units of the consumption good at $t = 1$, where $h(\cdot)$ satisfies $h'(\cdot) > 0$, $h''(\cdot) < 0$ and $\lim_{k \to 0} h'(k) \to \infty$. As before, A is distributed according to the cdf G on interval $[0, 1]$ that has an associated density g. For simplicity, assume that the capital supply is inelastic and entrepreneurs cannot pledge their output to creditors, i.e., $\lambda = 0$.

Optimal investment of entrepreneur with productivity A is given by:

$$k_A(q, R) = \min \left\{ h'^{-1}\left(\frac{R \cdot q}{A}\right), \frac{w}{q} \right\}. \tag{50}$$

Therefore, it follows that there is a cut-off productivity give by:

$$\widetilde{A} = \frac{R \cdot q}{h'\left(\frac{w}{q}\right)},$$

such that all entrepreneurs with productivity $A > \widetilde{A}$ are constrained by their resources while the rest are unconstrained.

In this economy, the market clearing condition for capital is given by:

$$\bar{K} = \int_0^{\widetilde{A}} h'^{-1}\left(\frac{R \cdot q}{A}\right) \cdot g(A) \cdot dA + \frac{w}{q} \cdot (1 - G(\widetilde{A})), \tag{51}$$

and aggregate output is:

$$Y = \int_0^{\widetilde{A}} A \cdot h'^{-1}\left(\frac{R \cdot q}{A}\right) \cdot g(A) \cdot dA + \int_{\widetilde{A}}^1 A \cdot \frac{w}{q} \cdot g(A) \cdot dA. \tag{52}$$

Combining Equations (51) and (52), we have:

$$\frac{dY}{dR} = \int_0^{\widetilde{A}} \left(A - \int_{\widetilde{A}}^1 x \cdot \frac{g(x)}{1 - G(x)} \cdot dx\right) \cdot \frac{d}{dR}\left(h'^{-1}\left(\frac{R \cdot q}{A}\right)\right) \cdot dA. \tag{53}$$

From Equation (50) and the market clearing condition, we have that $d\left(h'^{-1}\left(R \cdot q/A\right)\right)/dR < 0$ since h is concave and $R \cdot q$ is increasing in R. Moreover, it is clear that $A < \int_{\widetilde{A}}^1 x \cdot \frac{g(x)}{1-G(x)} \cdot dx$ for $A \leq \widetilde{A}$. Hence, it follows that dY/dR is positive.

What is the intuition? In partial equilibrium, as R falls the investment of unconstrained entrepreneurs expands while the investment of constrained entrepreneurs is unchanged. This implies that, in general equilibrium, the price of capital must rise. Ultimately, exactly as in our baseline economy with linear technology, the decline in R simply reallocates capital from entrepreneurs with $A > \widetilde{A}$ to entrepreneurs with $A < \widetilde{A}$, thereby reducing output. Again, although this example with fixed capital stock and no credit whatsoever is stark, it is straightforward to prove the analogue of Proposition 1 for this setting: namely, as the capital supply becomes less elastic, dY/dR increases and becomes positive provided that λ is below a threshold.

B.3 Closed economy: endogenous interest rates and savings gluts

Throughout our main analysis, we considered a small open economy that experienced an exogenous fall in the world interest rate. In this appendix, we show that none of our main insights would change if the economy were closed, but where the fall in the interest rate instead were the result of a savings glut, i.e., an increase in the economy's desired savings.

Suppose now that the economy is closed, that the agents preferences are given by:

$$U = E_0\{c_0 + \beta \cdot c_1\} \tag{54}$$

for some $\beta \in (0, 1)$, and that the capitalists have an endowment $w^C > 0$ of the consumption good at $t = 0$. Given these adjustments, we next show that the main results from our baseline setting can be obtained by raising the desired savings in this economy.

Proposition 3 *The effects of a fall in the interest rate, R, as described in Proposition 1 are isomorphic to those of an increase in w^C and/or β.*

In what follows, we illustrate the proof of this result. First, note that the equilibrium interest rate, R, must be greater than β^{-1}. Otherwise, there would be a positive credit demand but no savings, as all agents who do not invest in capital would want to consume; hence, the credit market would not clear.

Second, observe that, given prices $\{q, R\}$, the aggregate savings of the savers (i.e., the capitalists and entrepreneurs with productivity $A < q \cdot R$ are given by:

$$S(q,R) \begin{cases} = w^C + q \cdot K^S(q) - \chi(K^S(q)) + w \cdot G(q \cdot R) & \text{if } R > \beta^{-1}, \\ \in [0,\ w^C + q \cdot K^S(q) - \chi(K^S(q)) + w \cdot G(q \cdot R)] & \text{if } R = \beta^{-1} \end{cases}. \tag{55}$$

Equation (55) states that if $R > \beta^{-1}$, then the savers save all their resources, which are given by their endowments of the consumption good, the market value of capital, less the costs of producing the capital. If $R = \beta^{-1}$, then the savers are indifferent between saving and consuming these resources. As a result, the credit market clearing condition is given by:

$$S(q,R) = \int_{q \cdot R}^{1} b_A(q,R) \cdot dG(A), \tag{56}$$

which together with Equations (5), (6), (8), (9) and (10), characterizes the equilibrium.

Lastly, observe that the aggregate credit demand can be expressed as:

$$\int_{q \cdot R}^{1} b_A(q,R) \cdot dG(A) = q \cdot K^S(q) - w \cdot (1 - G(q \cdot R)), \tag{57}$$

since the entrepreneurs, who invest in capital, use all of their endowment plus borrowing to finance purchases of capital.

Therefore, we can immediately see that there are two possibilities in equilibrium.

Case 1. Consider a candidate equilibrium where the interest rate, R, is equal to β^{-1}. For this to be an equilibrium, it must be that:

$$w^C + q \cdot K^S(q) - \chi(K^S(q)) + w \cdot G(q \cdot \beta^{-1}) \geq q \cdot K^S(q) - w \cdot (1 - G(q \cdot R)), \tag{58}$$

which holds if and only if:

$$w^C + w \geq \chi(K^S(q)), \tag{59}$$

where the equilibrium price of capital, q, satisfies clears the capital market:

$$K^S(q) = \int_{q\cdot R}^{1} k_A(q, \beta^{-1}) \cdot dG(A). \tag{60}$$

It is therefore immediate that in this case the effects of an increase β on the aggregate capital and output are isomorphic to those of a fall in R analyzed in Section 3. Moreover, observe that this candidate is an equilibrium if w^C and/or β are large enough.

Case 2. Consider a candidate equilibrium where the interest rate, R, is above β^{-1}. This candidate is in turn equilibrium if at $R = \beta^{-1}$, the inequality (59) is violated, i.e., if w^C and/or β are small. Hence, in this case, the equilibrium prices $\{q, R\}$ are such that:

$$w^C + w = \chi(K^S(q)), \tag{61}$$

and

$$K^S(q) = \int_{q\cdot R}^{1} k_A(q, R) \cdot dG(A). \tag{62}$$

Here, a rise in w^C raises the capital price (as $\chi(K^S(q))$ is increasing in q) and lowers the interest rate (to offset the effect of a higher q that depresses capital demand). Hence, the effects of an increase in w^C on the aggregate capital and output are isomorphic to those of a fall in R analyzed in Section 3.

Lastly, note that if the equilibrium is initially in Case 2, then an increase in w^C eventually moves the equilibrium into Case 1.

B.4 Our mechanism in the Kiyotaki-Moore model

In this Appendix, we show that the capital-reallocation effects induced by falling interest rates that we emphasized through the main text are also present in the class macro-finance model of Kiyotaki and Moore (1997). As in our dynamic setup setup of Section 4, however, in that model as well these effects will be temporarily masked by the balance-sheet channel.

Time is now infinite, $t = 0, 1, ...$ Assume, for simplicity, that all entrepreneurs in the modern sector have the same productivity $A \in (0, 1)$, and that the capital stock is fixed at $\bar{K} > 0$. Thus, aggregate output in any period t depends solely on the allocation of capital between the modern and traditional sectors:

$$Y_t = A \cdot K_t + a \cdot f(\bar{K} - K_t), \tag{63}$$

where K_t denotes the aggregate stock of capital employed in the modern sector at time t.

We focus on equilibria in which the traditional sector is active in all periods and, hence, its demand for capital is given by:

$$\frac{a \cdot f'\left(\bar{K} - K_{t+1}\right) + q_{t+1}}{q_t} = R, \tag{64}$$

i.e., the return to capital within the traditional sector must equal the interest rate.

As in the static model, we introduce a financial friction by assuming that – in any period – an entrepreneur can walk away with a fraction $1 - \lambda$ of her resources, which now include her output and the market value of her capital. It thus follows that entrepreneurs face the following borrowing constraint:

$$R \cdot B_t \leq \lambda \cdot (A + q_{t+1}) \cdot K_{t+1}, \tag{65}$$

where B_t and K_{t+1} respectively denote entrepeneurial borrowing and investment in period t.[15] Note that, since all entrepreneurs are identical, B_t and K_{t+1} also represent aggregate borrowing and investment in the modern sector.

In any period t, the net worth of entrepreneurs equals the sum of their output and the market value of their capital minus repayments to creditors: $A \cdot K_t + q_t \cdot K_t - R \cdot B_{t-1}$. We assume that entrepreneurs consume a fraction $1 - \rho$ of this net worth in every period, where $\rho \cdot R < 1$.[16] This ensures, in the spirit of Kiyotaki and Moore (1997), that the financial constraint holds with equality in all periods. As a result, the modern-sector demand for capital is given by:

$$K_{t+1} = \frac{1}{q_t - \lambda \cdot \frac{A + q_{t+1}}{R}} \cdot \rho \cdot (1 - \lambda) \cdot (A + q_t) \cdot K_t, \tag{66}$$

where we make parametric assumptions to ensure that both sectors are active in a neighborhood of the steady state.[17]

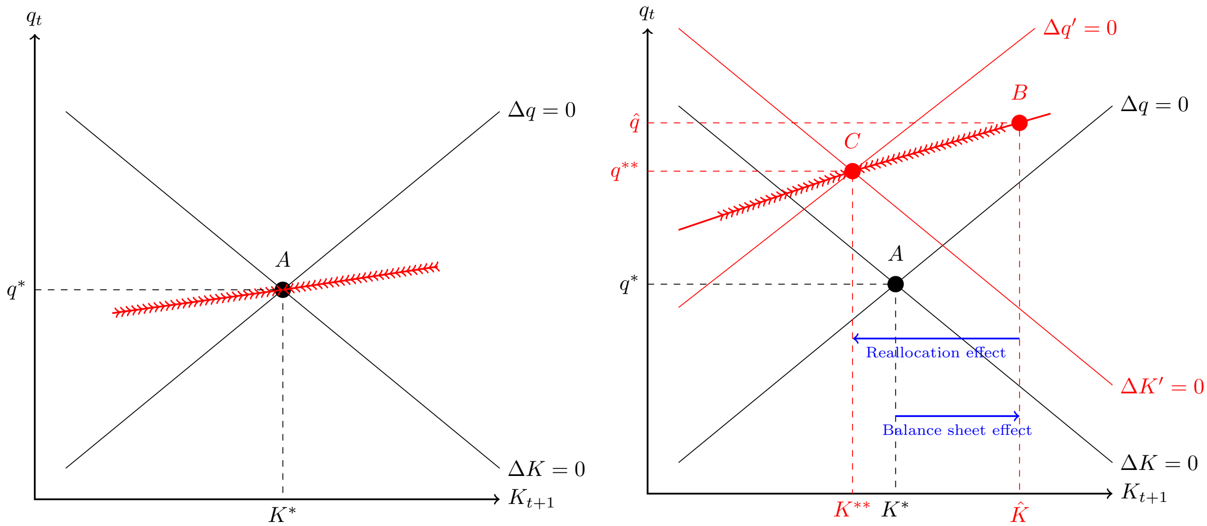
Thus, given an initial value for $K_0 > 0$ and a no bubbles condition on the price of capital,

[15] In Kiyotaki and Moore (1997), the output of investment is not pledgeable but the resale value of capital is fully pledgeable. Although our results would also go through under that specification, we have chosen the current specification in order to preserve symmetry with the baseline model of Section 2.

[16] E.g., it is sufficient to assume that entrepreneurs have log-preferences, i.e., $U^E = \sum_{t=0}^{\infty} \rho^t \cdot log(c_t)$. Note that the preferences of other agents (i.e., capitalists and traditional investors) are irrelevant for the evolution of q_t, K_t and Y_t.

[17] In particular, if K_0 is close to steady state, this requires that:

$$\frac{a \cdot f'(0)}{R - 1} > \frac{R \cdot \rho \cdot (1 - \lambda) + \lambda}{R - R \cdot \rho \cdot (1 - \lambda) - \lambda} \cdot A > \frac{a \cdot f'(\bar{K})}{R - 1}.$$



(a) Convergence to steady-state

(b) Effects of unexpected fall in R

Figure 8: **Equilibrium dynamics and balance sheet effects.** The figure illustrates a phase diagram for the joint evolution of the price of capital and the stock of capital in the modern sector. The saddle path of the system is depicted by a red curve with arrows pointing to the steady state: the left panel depicts the dynamics before the unexpected decline in the interest rate, whereas the right panel depicts the dynamics after it.

Equations (64) and (66) fully characterize the equilibrium of this economy. Panel (a) of Figure 8 portrays the equilibrium dynamics with the help of a phase diagram in the (K_{t+1}, q_t)-space. The $\Delta q = 0$ locus depicts all the combinations of K_{t+1} and q_t for which Equation (64) is satisfied with $q_t = q_{t+1}$. The locus is upward sloping because a higher level of modern-sector investment, K_{t+1}, is associated with a higher productivity of capital in the traditional sector and – since capital is priced by this sector – with a higher level of q_t. The $\Delta K = 0$ locus depicts instead all the combinations of K_{t+1} and q_t for which Equation (64) is satisfied with $K_t = K_{t+1}$. The locus is downward sloping because a higher level of modern-sector investment, K_{t+1}, is only affordable to constrained entrepreneurs if the equilibrium price of capital, q_t, is lower. As the figure shows, the system displays saddle-path dynamics. From an initial condition $K_0 < K^*$, both K and q increase monotonically as the economy transitions to the steady state and modern-sector entrepreneurs accumulate net worth. The opposite dynamics follow from an initial condition $K_0 > K^*$.

The right-hand panel of Figure 8 portrays the response to a permanent and unanticipated decline in R in a given period t_0. In response to a lower R, both loci shift upwards. The $\Delta q = 0$ locus shifts up because the traditional sector's willingness to pay for capital increases alongside the net present value of dividends; the $\Delta K = 0$ also shifts up because entrepreneurs' ability to pay for capital increases as lower interest rates relax their borrowing constraint. The presence of financial frictions, however, mitigates the shift in the $\Delta K = 0$ locus. Thus, as the figure shows, a decline in R triggers an increase in the steady-state price of capital to q^{**}, and a reduction in the capital employed in the modern sector to K^{**}. Hence, a reduction in the interest rate leads to a fall in the steady-state level of output despite the presence of dynamics.

This does not mean, however, that balance sheet effects do not play a role. Indeed, on impact, in response to a decline in the interest rate, the value of capital increases from $q^* \cdot K^*$ to $q_{t_0} \cdot K^*$ while entrepreneurial debt payments - which are pre-determined - remain unaffected and equal to $R \cdot B^*$.[18] Therefore:

$$K_{t+1} = \begin{cases} \frac{1}{q_t - \lambda \cdot \frac{A+q_{t+1}}{R}} \cdot \rho \cdot ((1-\lambda) \cdot (A+q^*) + q_t - q^*) \cdot K^* & \text{if } t = t_0 \\ \frac{1}{q_t - \lambda \cdot \frac{A+q_{t+1}}{R}} \cdot \rho \cdot (1-\lambda) \cdot (A+q_t) \cdot K_t & \text{if } t > t_0 \end{cases}. \tag{67}$$

The evolution of q_t is still given by Equation (64). This means that the adjustment of K to the new steady-state is not monotonic. As the right-hand panel of Figure 8 shows, K_{t+1} rises to $\widehat{K}$ on impact: this, as stated in the figure, is the balance sheet effect. The expansion of the modern sector is short-lived, though, since from that period onwards the economy evolves along the saddle-path towards its new steady state, which features a higher price of capital but a lower capital stock in the modern sector and thus a lower level of output. This decline from $\widehat{K}$ to K^{**} is, as stated in the figure, due to the reallocation effect: the higher demand of capital by the traditional sector keeps capital prices high, and these slowly erode the net worth of modern-sector entrepreneurs. As a result, the dynamic behavior of aggregate output in this economy resembles closely that of the dynamic economy in Section 4, illustrated in Figure 7.

The key takeaway is that the same reallocation forces that we analyzed in our baseline model of Section 2 are also at work in a dynamic environment. Moreover, these forces are persistent in response to a permanent decline in the interest rate, while the balance-sheet effects that are often highlighted in the literature are transitory. To be sure, an unexpected decline in the interest rate does have an initial balance-sheet effect that benefits productive entrepreneurs and reallocates capital towards them, raising average productivity and output. But this effect is by nature temporary: the reason is that it represents a one-time shock to the level of entrepreneurial net worth, but it does not affect the dynamic evolution of net worth thereafter.

[18] As in Kiyotaki and Moore (1997), these balance sheet effects require that entrepreneurs' debt payments are not indexed to the price of capital.