(Dis)Solving the ZLB equilibrium through Income Policies*

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August 21, 2019

Abstract

We investigate the possibility to reflate an economy experiencing a long-lasting ZLB episode with subdued or negative inflation, by imposing a minimum level of wage inflation. Our proposed income policy relies on the same mechanism behind past disinflationary policies, but it works in the opposite direction. It is formalized in terms of a downward nominal wage rigidity (DNWR) such that wage inflation cannot be lower than a fraction of the intended inflation target. This specification of the DNWR allows to dissolve the ZLB steady state equilibrium in an OLG model featuring "secular stagnation" and in a infinite-life representative agent model, where this equilibrium can emerge due to deflationary expectations. It always exists a level of the inflation target, and so a minimum wage inflation, that eradicates the ZLB. Our analysis suggests an alternative supply-side policy solution to Japan, which has not managed to raise low inflationary expectations, despite large expansionary fiscal and monetary policies.

Keywords: Zero lower bound, Wage indexation, Income policy, Inflation expectations.

JEL classification: E31, E52, E64.

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*We are grateful for the useful comments to Carlos Carvalho, Giovanni Crea, Neil Mehrotra and Martín Uribe.

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1 Introduction

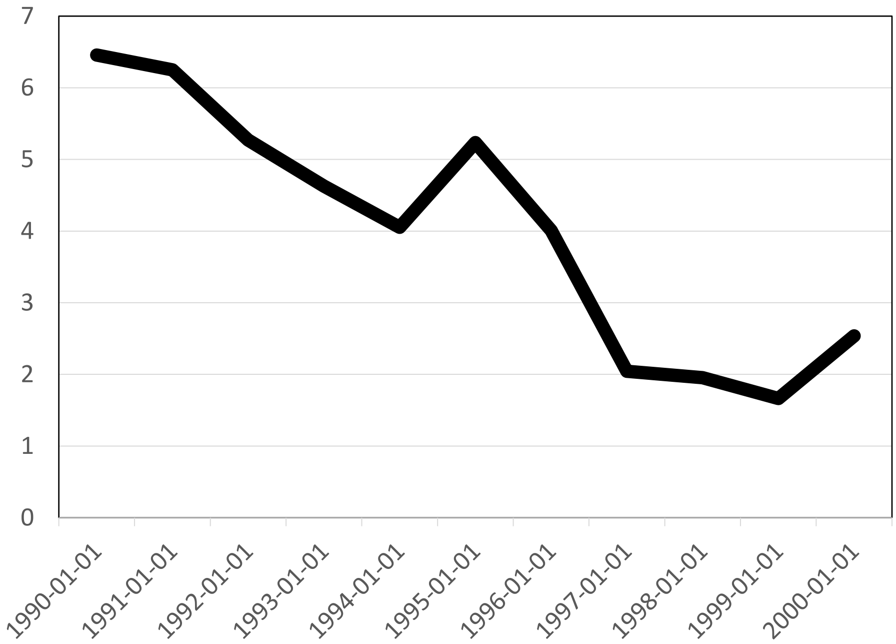
Inflation rate in Italy was about 6% at the beginning of the '90s. It needed to decrease by about 4% in few years to satisfy the inflation Maastricht criterion - inflation at most 1.5% above the average of the three lowest rates among the EU members. Figure 1 shows that Italy met the challenge. The Protocol signed by the employers and trade-union organizations on 23 July 1993 was the cornerstone for the structural reduction of inflation. The protocol marked the definite dismantling of the automatic indexation to past inflation mechanism in collective contracts, because unions finally accepted to have the price inflation expected (and targeted) by the government as a common reference for the indexation of national collective contracts.[1] The aim of this policy was to engineer a disinflation with the smallest output/employment costs, meanwhile safeguarding the purchasing power of wages.[2] The main channel that led to the successful disinflation was the realignment of inflation expectations to the target level chosen by government. Fabiani et al. (1998) and Destefanis et al. (2005) find evidence that after the signing of the agreement wage setters switched to more forward-looking behavior and that the target inflation rate had impact on inflation expectations. The problem of Italy was a problem of "de-indexing" the economy by de-indexing the wage bargaining process and thus breaking the wage-price inflation spiral. This type of income policies were very popular at the time, and many examples show that they could be a very efficient way to disinflate the economy.[3]

[1] The 'Protocol on Incomes and Employment Policy, on Contractual Arrangements, on Labor Policies and on Support for the Production System' (*Protocollo sulla politica dei redditi e dell'occupazione, sugli assetti contrattuali, sulle politiche del lavoro e sul sostegno al sistema produttivo*) was drafted by the presidency of the Council of Ministers on 3 July 1993, under Prime Minister Carlo Azeglio Ciampi. The wage contracts indexation was based on the targeted inflation rate (*tasso d'inflazione programmata*) and not on actual or past inflation. While the automatic mechanism (the so-called *scala mobile*) had ceased to operate already in 1992, wage setting was still very much backward-looking.

[2] Contracts were renegotiated every two years, thus workers could regain any difference between actual and target inflation only after two years. Income policies of this kind need to be accompanied by a coherent institutional effort in terms of contractionary monetary and fiscal policy. This had been the case in Italy.

[3] There are other examples of successful income policies that contain wage inflation to curb price inflation. In Australia, the Hawke government in March 1983 promoted Accord Mark I with the unions to restrain wage increases, in order to fight a period of high unemployment and high inflation. The Accord lasts 13 years and was renegotiated several times (Accords Mark I-VII). As a result of the improvement in industrial relations, a corporatist model emerged where the Australian Council of Trade Unions (ACTU) was regularly consulted over government decisions and was represented on economic policymaking bodies

Figure 1: Inflation rate (CPI, %) in Italy in the '90s. Source: FRED.



What has all this to do with the zero lower bound (ZLB) problem and deflation or subdued inflation? The current macroeconomic scenario is starkly different from the prevailing one in the '80s and the '90s. In that period, the main concern of the policy makers was to quell the inflationary spiral triggered by the oil shocks of the '70s. Now, central bankers are struggling to hit the inflation target and some advanced countries are still stuck in a liquidity trap, more than ten years after the global financial crisis. We argue that, although current problems are different from past ones, the solutions could be similar. Past disinflationary policies show that de-indexing the economy is an effective way to tackle inflation. The other side of the coin could be that "re-indexing" the economy is an effective way to tackle deflation. The idea is that all these plans were thought to stop the *upward* inertia in the behavior of inflation (or the so-called wage-price spiral). The problem in a ZLB (or in the path the lead to the ZLB) derives from the same logic, but

such as the board of the Reserve Bank of Australia. In the 1990s, the Dutch corporatist model (the so-called Polder model) gained popularity because of good social and economic performance. The Polder model is based on consulting between the government and the social partners, involving them in the design and implementation of socio-economic policies (see Visser and Hemerijck, 1997). Similar model are in place in Belgium and in Finland and other Scandinavian countries.

it is a spiral *downward* rather than upward. This paper simply argue that policy should use the very same measures the other way round, that is, in the opposite direction.

This work puts forward a policy proposal able to avoid a "secular stagnation" and/or to eliminate a ZLB/deflationary equilibrium. We propose to simply impose a lower bound on wage inflation: an income policy based on a downward nominal wage rigidity (DNWR) such that wage inflation cannot be lower than a fraction of the intended inflation target. We show that with this simple DNWR constraint, it will always exists a level of inflation target that eradicates the ZLB equilibrium.

We show how our policy proposal works in two very different frameworks using the models in two influential papers in this literature: Eggertsson et al. (2019) (EMR, henceforth) and Schmitt-Grohé and Uribe (2017) (SGU henceforth). EMR is an overlapping generation (OLG) model of secular stagnation, where a ZLB equilibrium arises when the natural interest rate is negative. SGU is an infinite-life representative agent model, where a ZLB can arise due to expectations of deflation. Both paper feature a DNWR constraint. We show that tweaking this constraint to allow for "reflationary income policies" eliminate the ZLB equilibrium, provided that the inflation target is sufficiently high. If wage inflation is sufficiently high, then there is no possibility for agents to coordinate on a deflationary or a secular stagnation equilibrium. Expectations of a deflation (or low inflation) and ZLB are not consistent with rational expectations. Our mechanism has the same flavour of the Italian case, but upside-down. Note that in equilibrium the DNWR does not bind, hence it is not the case that it is mechanically imposed in equilibrium. Moreover, both price and wage inflation are equal to the intended target and there is full employment in the unique equilibrium that survives. The DNWR acts as a coordination device that destroys the bad ZLB equilibrium.

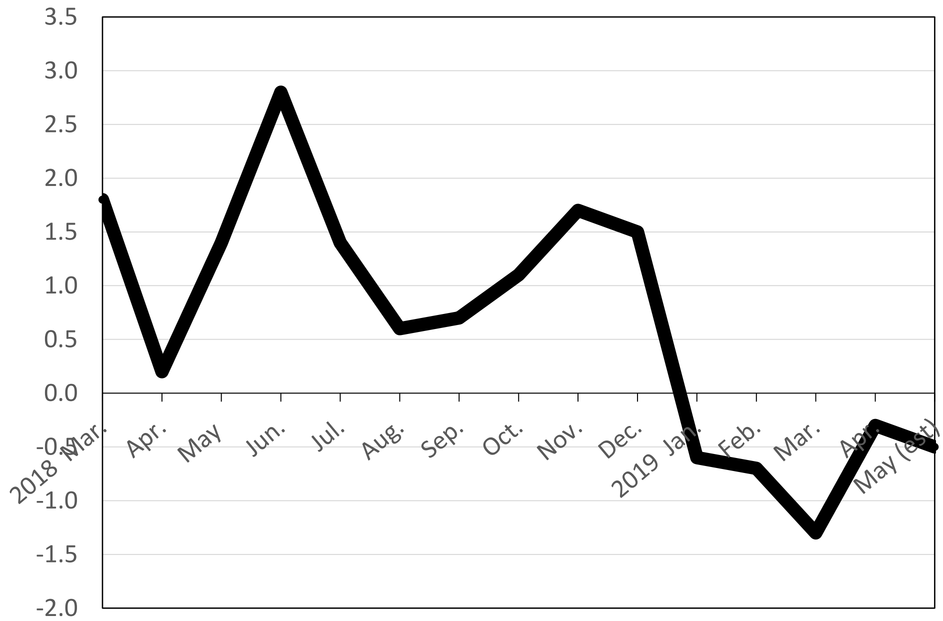
EMR discusses the effects of an increase in the inflation target in their model, but while it allows for a better outcome, it cannot exclude a secular stagnation equilibrium. Hence they propose other possible demand-side solutions (especially fiscal policy). Our policy, instead, is a supply-side solution, as all the income policies. Our modification of the DNWR in those two models moves the aggregate supply curve, not aggregate demand. We believe this proposal to be a natural way of thinking about the ZLB problem. First, our approach recognizes the ZLB, and the often corresponding deflation or too low inflation problem, as a "nominal" problem. Second, once one sees the problem in this way, it is natural to think about it as a "reflation" problem, that is just the opposite of a disinflation. Many successful disinflationary policies in the '80s and '90s de-indexed the economy, using a set of policies (mainly income policies and some degree of corporatism) to engineer a reduction of inflation. Our proposal is just to adopt the same set of policies with the opposite goal: to re-index the economy in order to engineer a reflation.

Finally, note that we naturally chose two influential ZLB frameworks with a DNWR to present our analysis, given that we impose a DNWR. However, our solution would work also if the economy is trapped in a ZLB/deflationary equilibrium without a binding DNWR to start with, and, hence, it does not feature unemployment in this equilibrium.

The policy proposal is utmost relevant for Japan today, because it is tailored for an economy experiencing a long-lasting ZLB episode, which has not come to an end despite huge and prolonged monetary and fiscal interventions. The prime minister of Japan, Shinzo Abe, has long sought to influence wage negotiations to push for increases in nominal wages coherent with the inflation target. The wage negotiations between the Japan Business Federation (*Keidanren*) and the Trade Union Confederation (*Rengo*) occur during the "spring offensive" called *Shuntō*, which is very influential because it sets the context for bargaining between individual companies and unions. However, in contrast with the Italian experience of consultation (i.e., *Concertazione*), the government does not take part in the negotiations, so the outcome fell well short of Mr. Abe's call for a 3% increase. Average wages (i.e., total cash earnings) increased by 0.1% in 2015, 0.6% in 2016 and 0.4% in 2017 according to data from the Japanese Ministry of Health, Labor and Welfare. Figure 2 shows the behavior from 2018 onward of nominal average wage increases (month-to-month in the preceding year). While the bargaining in 2018 was promising, average nominal wage growth turned *negative* in every month of 2019, hitting in March the lowest level of -1.3%.[4] Dismal wage increases, despite a tight labor market,

[4]See e.g., "Shinzo Abes campaign to raise Japanese wages loses steam", FT online, 22 January 2019.

Figure 2: Percentage Nominal Wage Increase in Japan (month-to-month in the preceding year). Source: Ministry of Health, Labor and Welfare



have become the biggest drag on the Japan efforts to reflate the economy. According to our proposal, Mr. Abe should try to enforce wage inflation through, for example, the use of profit tax levy or subsidies (Wallich and Weintraub, 1971; Okun, 1978). Moreover, according to commentaries and statements from government officials, the idea of a lack in consumption demand is behind the call for the wage increase. However, we argue this is the wrong way of looking at the problem: our solution is a supply side one. Expectations are such that the economy is trapped in a low inflation equilibrium, and a DNWR based on a minimal wage inflation is the supply side cure. A once-for-all increase in the level (vs. the rate of growth) of the minimum wage or of the consumption tax, as recently proposed by the government,[5] would not work. The cure is about engineering a reflation through a national agreement (as in the Italian experience) between employers and union associations and the government to determine a sustained wage inflation, and

[5]See, e.g., "Labor ministry panel suggests hiking minimum wage by ¥27 to push Japan average above ¥900", The Japan Times online, 31 July 2019. The consumption tax was already raised from 5% to 8% in April 2014. Now, the Japanese government plans to raise it to 10%. See, e.g., "Abe sticks with plan to raise Japan's consumption tax despite weak tankan results", The Japan Times online, 1 July 2019.

about changing the deflationary psychology (as for the Brazilian Real Plan, see below); it is not about a wage or price level increase.

The Brazilian Real Plan in the late 1990s is another well-known example of the benefits of de-indexing an economy suffering an inflationary spiral by affecting inflationary expectations. The Brazilian experience is quite different from the Italian one in the '90s. However, it is very relevant to our paper because it is an illustrative example of how de-indexing could be a powerful tool to coordinate inflation expectation and so to shift the economy from a high-inflation equilibrium to a low-inflation one. In the models we analyze, there are two fundamental steady states: a ZLB one and one with full employment and inflation at the target. Our proposal shows how "indexing the economy" could be a powerful tool to coordinate inflation expectation and shift the economy from a ZLB/low-inflation equilibrium to one with full employment and inflation at the target.

The Brazilian economy was plagued by extraordinary high inflation levels in the '80s and several policy measures to bring inflation under control failed poorly. It was clear to Brazilian economists that indexation was the problem: wages were fully indexed and adjusted more than once in a year.[6] In July 1994, the Brazilian Minister of Finance Cardoso put in place the so-called *Plano Real* in order to stabilize the economy. A key feature of the plan was the monetary reform that introduced a new currency, i.e., Real Unity of Value (*Unidade Real de Valor or URV*), that was originally pegged 1:1 to the dollar. Initially, the new currency only served as unit of account, while the official currency, *cruzeiro*, was still used as mean of exchange. However, most contracts were denominated and indexed in the new currency, which was more stable than the *cruzeiro*. Contrary to the several previous attempts to bring down inflation, the Real Plan did not impose any control on prices and wages, but Brazilian consumers learned the possibility of price stability. As a consequence, inflationary expectations dropped and the inflationary spiral was arrested. The Plan succeeded for the psychological effect on inflation expectations

[6] *"Brazilian economists have long recognized that in a setting of full, compulsory indexation, orthodox monetary restraint is not a satisfactory answer to inflation. The idea that inflation has inertia, by virtue of the indexation law and practice, implies the need for an alternative stabilization strategy, namely, "heterodoxy." The issue is not only to control demand, but, more important, to coordinate a stop to wage and price increases, which feed on one another."* (Dornbusch, 1997, p. 373)

and on the inflationary culture. Annual inflation decreased from 909.7% in 1994 to 14.8% in 1995 and then to 9.3% in 1996 and 4.3% in 1997.

The paper proceeds as follows. Section 2 presents how our policy would work in the EMR model, while Section 3 does the same in the SGU model. Section 4 concludes.

2 Reflation in the EMR OLG model

In sections 2.1 and 2.2, we carefully spell out the EMR model. Once the reader has grasped the logic of the equilibria in the EMR model, then it would be straightforward to understand our main result and the implications of our policy proposal in section 2.3.

2.1 The EMR OLG model

EMR study an economy with overlapping generations of agents who live three periods, firms and a central bank in charge of monetary policy (Appendix A.1 spells out the details and the derivations of the model). Population grows at a rate g_t and there is no capital in the economy.

Young households borrow up to an exogenous debt limit D_t by selling a one-period riskless bond to middle-aged households, which supply inelastically their labor endowment $\bar{L}$ for a wage W_t and get the profits Z_t from running a firm. Only middle-aged households work and run a firm. Generations exchange financial assets in the loan market, and in equilibrium the total amount of funds demanded by young households equals the one supplied by middle-aged ones. Old agents simply dissave and consume their remaining wealth. As in any OLG model, the equilibrium real interest rate is endogenously determined and clears the asset market. It coincides with the natural interest rate, i.e., r^f when output is at potential, i.e., Y^f.

The production technology of firms exhibits decreasing returns to labor, L_t, which is the only input of production. The labor market operates under perfect competition. However, workers are unwilling to supply labor for a nominal wage, W_t, lower than a minimum level, so that

$$W_t = \max\left\{W_t^*, \alpha P_t \bar{L}^{\alpha-1}\right\}, \tag{1}$$

where W_t^* is the lower bound on the nominal wage, α measures the degree of decreasing returns to labor and P_t is the price level. The DNWR is key in the model to generate a ZLB equilibrium. As in Schmitt-Grohé and Uribe (2016), we make the simple assumption that the minimum level is proportional to the nominal wage in the previous period:

$$W_t^* = \delta W_{t-1}, \tag{2}$$

where $\delta \leq 1$.[7] The labor market does not necessarily clear because of downwardly rigid wages. If labor market clearing requires a wage W_t larger than δW_{t-1}, the DNWR constraint is not binding, thus the nominal wage is flexible and the aggregate labor demand equals the economy's labor endowment, i.e., $L_t = \bar{L}$. On the contrary, if labor supply exceeds labor demand at the wage $W_t = \delta W_{t-1}$, the wage cannot decrease further because of the DNWR constraint, so that involuntary unemployment arises, i.e., $L_t < \bar{L}$.

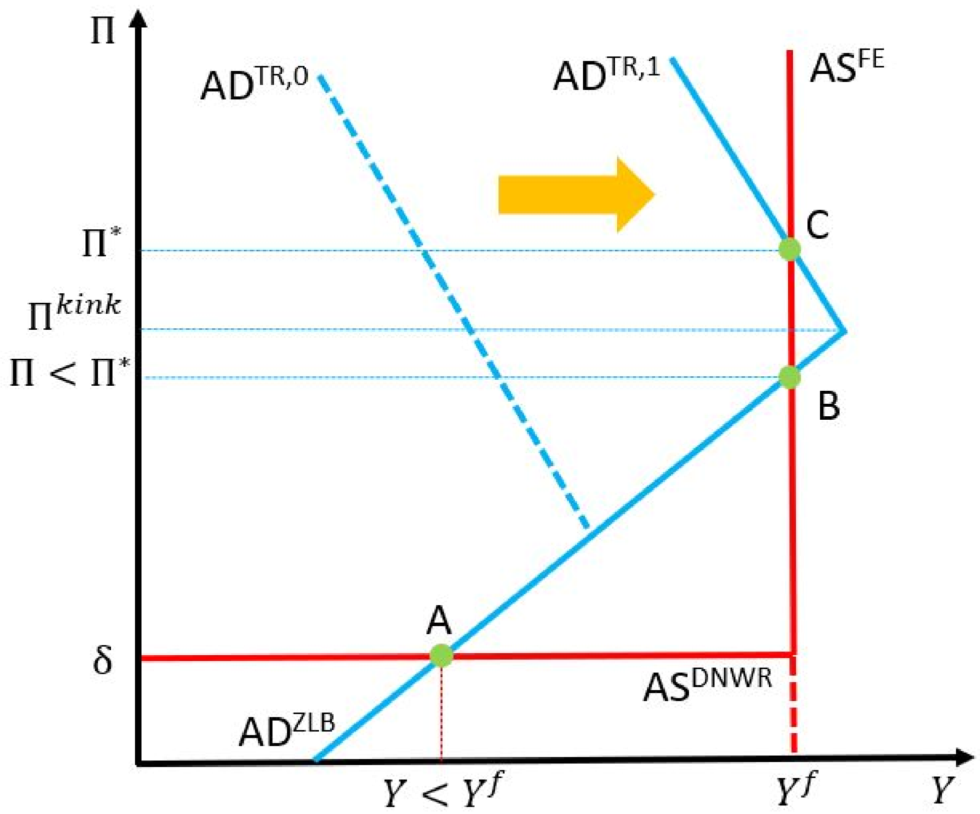
The model is closed with a standard Taylor rule that responds only to inflation and it is subject to the ZLB constraint, that is

$$1 + i_t = \max\left[1, \left(1 + r_t^f\right) \Pi^* \left(\frac{\Pi_t}{\Pi^*}\right)^{\phi_\pi}\right], \tag{3}$$

where $\phi_\pi > 1$, Π^* is the gross inflation target, and r_t^f is the natural real interest rate, that is, the unique level of real interest rate compatible with full employment in the OLG model.

[7] This assumption is consistent with the empirical evidence in Schmitt-Grohé and Uribe (2016). A more general specification would allow the DNWR to depend on the level of employment or unemployment as in EMR and SGU, respectively. Our results will be unaffected by this alternative assumption. Hence, without loss of generality, we prefer to start with the simplest case for better intuition. EMR assume $W_t^* = \gamma W_{t-1} + (1-\gamma)\,\alpha P_t \bar{L}^{\alpha-1}$, such that the minimum nominal wage is the weighted average of the past wage level and the "flexible" level corresponding to full employment, i.e., $\alpha P_t \bar{L}^{\alpha-1}$. We present this case in Appendix A.2. Moreover, we will present the somewhat similar case in which the minimum wage depends on unemployment as in SGU in the next section.

Figure 3: Aggregate demand and supply curves in the EMR model



2.2 Steady State Equilibrium in the EMR OLG model

Figure 3 conveniently shows the steady state relationships implied by this model, using an aggregate demand (AD) and aggregate supply (AS) diagram (see A.1 for the derivation). Both curves are characterized by two regimes and thus they both exhibit a kink.

Whether or not the DNWR constraint, (1), is binding defines the two regimes in the AS curve. The AS curve is vertical at the full employment level, $Y^f = \bar{L}^\alpha$, when (1) is not binding and $W = \alpha P \bar{L}^{\alpha-1}$.[8] Otherwise, $W_t = W_t^* = \delta W_{t-1} \geq \alpha P_t \bar{L}^{\alpha-1}$. This is a situation in which steady state wage and price inflation are equal to δ, while the level of the real wage is $\frac{W_t}{P_t} \geq \alpha \bar{L}^{\alpha-1}$. The AS is thus flat at $\Pi^W = \Pi = \delta$ for $\forall L \leq \bar{L}$, and the level of employment (and output) is demand determined along the AS^{DNWR}.

Whether or not the ZLB constraint, (3), is binding defines the two regimes for the AD curve. When the ZLB is not binding and monetary policy follows the Taylor rule,

[8] Note that we can suppress the time subscripts t, because we are just considering steady state relationships, where variables are constant.

the AD curve in steady state is given by

$$Y_{AD}^{TR} = D + \left(\frac{1+\beta}{\beta}\right)\left(\frac{1+g}{1+r^f}\right)\left(\frac{\Pi^*}{\Pi}\right)^{\phi_\pi - 1} D, \tag{4}$$

where β is the subjective discount factor. Assuming the Taylor principle is satisfied (i.e., $\phi_\pi > 1$), equation (4) defines a negative relationship between steady state inflation and output. When the inflation rate is higher than the target, the nominal interest rate increases more than inflation, resulting in a higher real interest rate ($r > r^f$ for $\Pi > \Pi^*$ in (3)) that increases savings and contracts demand. However, when the ZLB is binding, the steady state AD becomes

$$Y_{AD}^{ZLB} = D + \left(\frac{1+\beta}{\beta}\right)(1+g)\,\Pi D, \tag{5}$$

which defines a positive relationship between steady state inflation and output. The higher is inflation, the lower the real interest rate in this case, because the nominal interest rate is stuck at zero, and $1 + r = 1/\Pi$. We denote Π^{kink} the inflation rate at which (4) and (5) crosses, that is

$$\Pi^{kink} = \left[\frac{1}{(1+r^f)}\right]^{\frac{1}{\phi_\pi}} \Pi^{*\,\frac{\phi_\pi - 1}{\phi_\pi}}. \tag{6}$$

Π^{kink} determines when the ZLB becomes binding.

To prepare ground for the intuition of our main result, Figure 3 depicts how the AD curve moves with the inflation target. An increase in the inflation target shifts out the downward sloping AD^{TR} part of the AD curve (and increases the absolute value of its negative slope), but it does not affect the upward sloping AD^{ZLB} part, as evident from equations (4) and (5).[9] As a result, a higher inflation target shifts out the kink in the AD, hence Π^{kink} is an increasing function of Π^*.

The crossing between the AS and the AD curves identifies a steady state. A "secular stagnation" equilibrium arises when $r^f < 0$, as Figure 3 shows. For a negative natural interest rate, there can be two different cases (leaving aside a limit, non-generic case),

[9]Figure 3 follows Figure 6 Panel A in EMR and the discussion therein in Section VI, p. 25. As EMR, we depict AD^{TR} as linear in Figure 3 for clarity, despite it being non-linear (the curve has an asymptote at $Y = D$). None of the results obviously depends on this.

depending on the level of the inflation target. In the first case (see the dashed line $AD^{TR,0}$), AD^{TR} does not cross AS^{FE} so that there is a unique steady state at point A, given by the intersection between AD^{ZLB} and AS^{DNWR}. Hence, this is a demand-determined and stagnant steady state (secular stagnation), where $i = 0$, $\Pi^W = \Pi = \delta$ and $Y < Y^f$. In the second case (see the solid line $AD^{TR,1}$), there are three different steady states: (A) the *ZLB-U* equilibrium just described that features ZLB, steady state inflation lower than the target and unemployment: $i = 0, \Pi = \delta < \Pi^*, Y \leq Y^f$; (B) a *ZLB-FE* equilibrium that occurs at the intersection of the AD^{ZLB} and the AS^{FE}, and it features ZLB, steady state inflation lower than the target and full employment: $i = 0, \Pi = \frac{1}{1+r^f} \leq \Pi^*, Y = Y^f$;[10] (C) a *TR-FE* equilibrium that occurs at the intersection of the AD^{TR} and the AS^{FE}; it features a positive nominal interest rate, steady state inflation equal to the target and full employment: $i > 0, \Pi = \Pi^*, Y = Y^f$.

EMR study these equilibria.[11] Moreover, they consider which type of policies could avoid the secular stagnation steady state *ZLB-U*, which always exists for $r^f < 0$. The only possibility to eradicate this equilibrium is through policies that make the natural interest positive. An increase of public debt could do that, because it absorbs the extra savings that drag the equilibrium real interest rate down, eventually restoring a positive r^f. However, in their quantitative exercise, EMR shows that starting from a value of $r^f = -1.47\%$ and a debt-to-GDP ratio of 118%, the debt-to-GDP ratio needs to almost double to 215% to reach a value r^f of 1% and then to cancel the secular stagnation equilibrium. Hence, while a minimum level of debt which eliminates this equilibrium always exists, this value might be very high and not necessarily sustainable and/or achievable.[12] EMR looks at alternative options to raise r^f to positive values, because in their model monetary

[10] As mentioned by EMR, this equilibrium is similar to the deflationary steady state analyzed in Benhabib et al. (2001).

[11] They show that the equilibria *ZLB-U* and *TR-FE* are determinate, while the equilibrium *ZLB-FE* is indeterminate. While they show it for their DNWR specification (see footnote 7), these results still hold in the simpler specification of this Section. Results are available upon request.

[12] For example, Japan has been in a liquidity trap for about two decades, despite a debt-to-DGP ratio above 200%. In EMR words (p.41): *"Such a large level of debt raises questions about the feasibility of this policy, for we have not modeled any costs or limits on the governments ability to issue risk-free debt-an assumption that may be strained at such high levels. While these results suggest that several reforms would tend to increase the natural rate of interest, the menu of options does not paint a particularly rosy picture relative to the alternative of raising the inflation target of the central bank."*

policy is powerless. As explained earlier, an increase in the inflation target moves AD^{TR}, but move neither the AD^{ZLB} nor the AS. Hence, if the natural real interest rate is negative, a *ZLB-U* always exists no matter what the inflation target is.

In the next section we present our proposal such that an appropriate choice of the inflation target is always able to dissolve the secular stagnation equilibrium.

2.3 Dissolving the ZLB Equilibrium

We now present a policy proposal able to avoid a secular stagnation even if $r^f < 0$. As explained in the Introduction, the secular stagnation equilibrium *ZLB-U* vanishes with our policy proposal. We demonstrate our proposal by a simple modification of equation (2) that defines the minimum level of wages W_t^* in the DNWR constraint (1) to

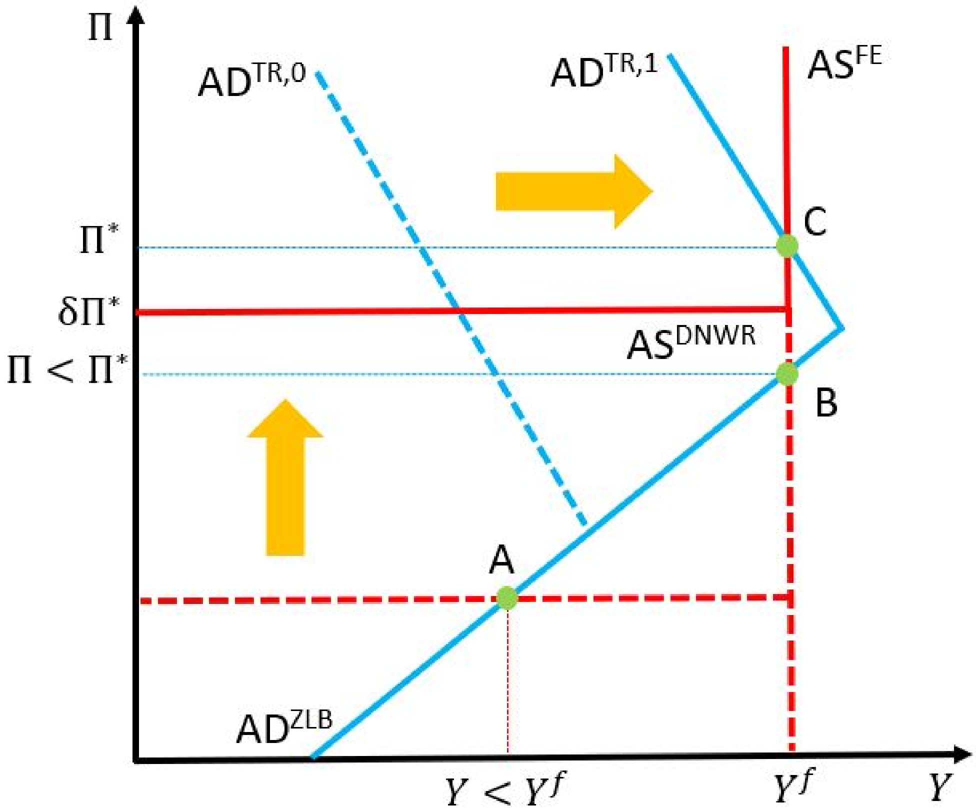
$$W_t^* = \delta \Pi^* W_{t-1}. \tag{7}$$

From an economic point of view, (7) implies that wage inflation cannot be lower than a certain fraction δ of the inflation target, Π^*. Hence, δ could be thought as the minimum degree of indexation of the wage growth rate to the inflation target. (7) captures the idea behind the disinflationary policies in Italy. Wage inflation is anchored to a target inflation rate, Π^*. However, while there the goal was to put a *ceiling* on the pressure for wage increases to *decrease* the rate of inflation, here the goal is to put a *floor* on wage deflation to *increase* the rate of inflation.

From an analytical point of view, comparing Figure 4 with Figure 3 reveals how this simple modification changes the results in the previous section. The main point is that (7) *makes the AS curve to shift with the inflation target*, because the AS^{DNWR} curve is now equal to $\delta\Pi^*$, rather than simply δ, as in the EMR case. Hence, an increase in the inflation target shifts the AS^{DNWR} curve upward. As the AD curve is unchanged with respect to the previous section, raising the inflation target shifts out AD^{TR}, as in Figure 3. We are now in the position to state our main result in the following proposition.

Proposition 1. *Assume $r^f < 0$ and $\delta < 1$. Then, if $\Pi^* > \frac{1}{\delta(1+r^f)}$, there exists a unique,*

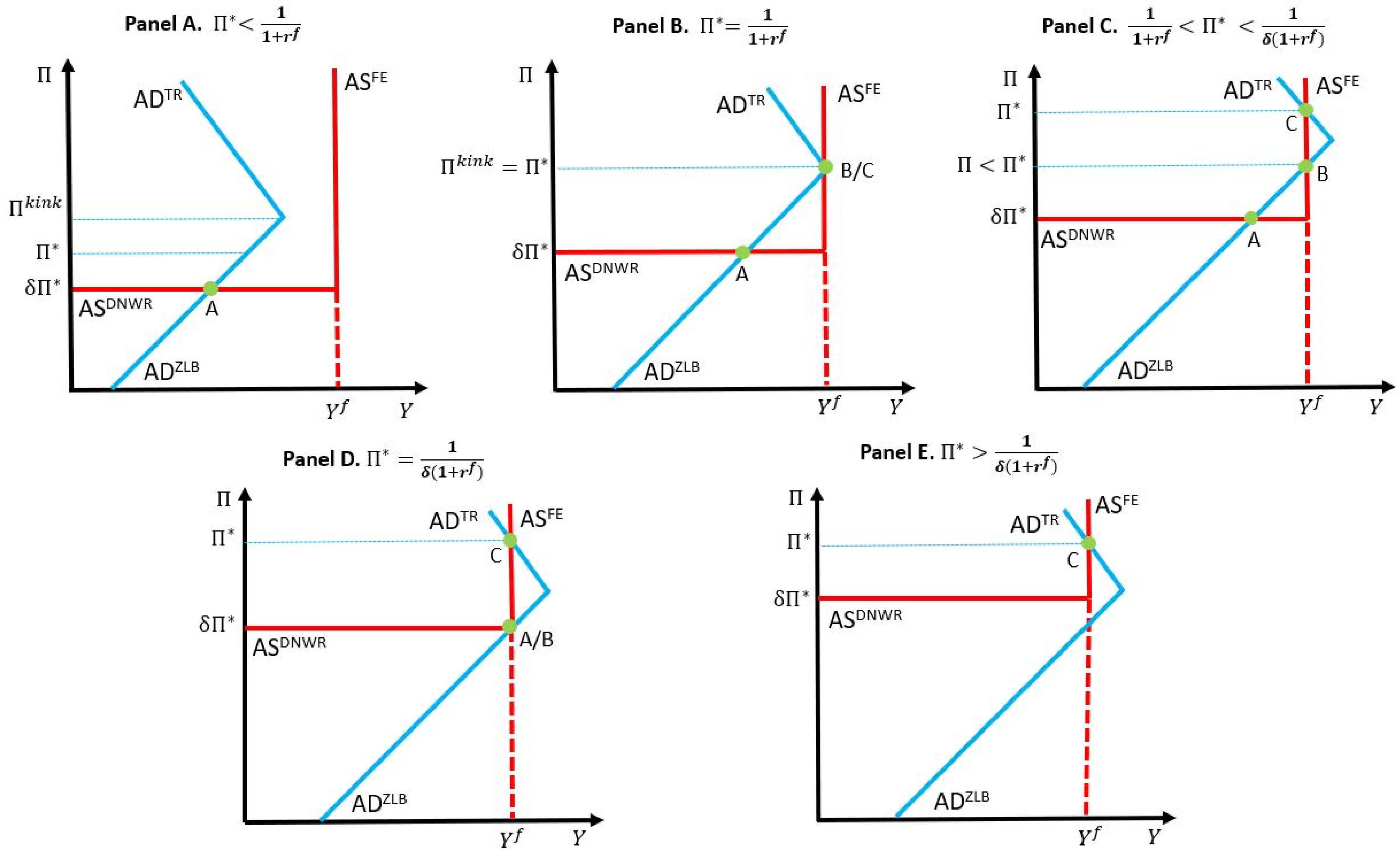
Figure 4: Raising the inflation target in the EMR model with our DNWR



locally determinate, $TR - FE$ equilibrium, where the ZLB is not binding, the inflation rate is equal to the target and output is at full employment, i.e., $i > 0$, $\Pi = \Pi^*$, $Y = Y^f$.

In other words, it always exists a sufficiently high level of the inflation target, Π^*, such that the unique and locally determinate equilibrium features full employment and inflation at the target without binding ZLB. While the formal proof of Proposition 1 is in the Appendix A.1.3, Figure 5 displays the intuition very clearly. It shows five different panels, each for different ranges of values of the inflation target. As the inflation target increases, the economy moves from Panel A to Panel E. The key thing to note is that while the AD curve moves as described in the previous section, now also the AS^{DNWR} shifts upward. For sufficiently high inflation target, the economy reaches the situation in Panel E, where only the $TR - FE$ equilibrium exists. Therefore, the secular stagnation equilibrium, $ZLB - U$, disappears if $\Pi^* \geq \left[\delta(1 + r^f)\right]^{-1}$.

Figure 5: All possible steady state equilibria in the EMR model with our DNWR



Let's turn to define the different equilibria in the Figure. As the level of the inflation target increases, five different cases (two of which are non generic) emerges: Panel A: if $\Pi^* < 1/(1+r^f)$, only the $ZLB-U$ equilibrium exists at point A; Panel B: if $\Pi^* = \frac{1}{1+r^f}$, two equilibria exist: a $ZLB-U$ equilibrium at point A and an equilibrium at point B/C that is a combination between $ZLB-FE$ and $TR-FE$, where output is at full employment, the nominal interest rate prescribed by the Taylor rule is exactly zero and the inflation rate is equal to the target; Panel C: if $\frac{1}{1+r^f} < \Pi^* < \frac{1}{\delta(1+r^f)}$, three equilibria exist: $ZLB-U$ at point A, $ZLB-FE$ at point B and $TR-FE$ at point C; Panel D: if $\Pi^* = \frac{1}{\delta(1+r^f)}$, two equilibria exist: $TR-FE$ at point C and an equilibrium at point A/B that is a combination between $ZLB-U$ and $ZLB-FE$, where output is at full employment, the ZLB is binding (and i is off the Taylor rule) and the inflation rate is lower than the target, $\Pi = \delta\Pi^* < \Pi^*$; Panel E: if $\Pi^* > \frac{1}{\delta(1+r^f)}$, only the $TR-FE$ equilibrium exists at point C.

Contrary to EMR where monetary policy is powerless, now monetary policy can wipe out the ZLB equilibrium by choosing an adequate inflation target. Alternatively, for a given r^f, one could choose δ to reach a particular inflation target. Hence, interpreting our proposed solution in (7) as an income policy, for given values of r^f and of the intended inflation target, the condition $\delta > \left[\Pi^*(1+r^f)\right]^{-1}$ gives the necessary value of δ that determines the degree of indexation of nominal wages to the inflation target. Using the number in EMR, if $r^f = -1.47\%$, then δ should be equal to 0.995 or 0.976 to reach an inflation target of 2% or 4%, respectively.

Finally, there is another important implication of our proposed policy with respect to EMR, that we summarize in the next proposition.

Proposition 2. *Assume $r^f < 0$ and $\delta < 1$ and that the economy is trapped in a secular stagnation equilibrium, $ZLB-U$ (Panel A). Then, an increase in the inflation target is always beneficial, in the sense that steady state output and inflation increase, irrespective if this increase is sufficient or not to escape the secular stagnation.*

Any, however small, increase in the target shifts upwards the AS^{DNWR}, and thus it moves the secular stagnation equilibrium along the AD^{ZLB} increasing the level of output and inflation. This is depicted in Figure 5, where the $ZLB - U$ equilibrium A in Panel A moves up in Panels B, C and D. This does not happen in the EMR specification. In Figure 3 both AD^{ZLB} and AS^{DNWR} curve do not change with the inflation target. As a result, a mild increase in the target does not affect the secular stagnation equilibrium $ZLB - U$ at point A, capturing Krugman's (2014) idea of "timidity trap". Only sufficiently large changes in the target makes the $TR - FE$ equilibrium to appear.[13] Our model has a similar flavour, but has a quite different implication: while it is still true that the policy is subject to a "timidity trap" to escape the secular stagnation, in the sense that the inflation target should be sufficiently high to avoid it, an increase in the target is always beneficial.

3 Reflation in the SGU infinite-life model

We now turn to a different model and to a different DNWR specification to show that our proposed policy works as well in this framework. The logic is very similar in this case, so , we still convey it mostly by using figures and put most of the derivations in Appendix A.3.[14]

3.1 Steady State Equilibrium in the SGU infinite-life model

SGU employs a simple flexible-price, infinite-life representative agent model to study the dynamics leading to a liquidity trap and a jobless recovery. With respect to the model in the previous section, they also employ a different specification of the DNWR constraint

$$\frac{W_t}{W_{t-1}} \geq \gamma\left(u_t\right) = \gamma_0\left(1 - u_t\right)^{\gamma_1} = \gamma_0\left(\frac{L_t}{\bar{L}}\right)^{\gamma_1}. \tag{8}$$

[13] *"Small changes in the inflation target have no effect, capturing Krugman's observation of the "law of the excluded middle" or "timidity trap" when trying to explain why the Japanese economy might not respond to a higher inflation target announced by the Bank of Japan unless it was sufficiently aggressive."* (EMR, p.3).

[14] Compared to the original model in SGU, we abstract from growth, from the shocks and from fiscal policy. Our results are unaffected by this modification.

The DNWR implies that the lower bound on wage inflation depends on the level of unemployment, u, or on the employment ratio $L/\bar{L}$. When $L = 0$ (or $u = 1$) the lower bound is zero, then it increases with employment with elasticity γ_1, and at full employment wage inflation cannot be lower than γ_0. SGU imposes the following important assumption on γ_0: $\beta < \gamma_0 \leq \Pi^*$, where β is the subjective discount factor of the representative agent. For simplicity, we assume $\gamma_0 = \Pi^*$, as SGU do in their quantitative calibration. The DNWR (8) implies the following complementary slackness condition

$$\left(\bar{L} - L_t\right)\left[W_t - \gamma_0 \left(1 - u_t\right)^{\gamma_1} W_{t-1}\right] = 0 \tag{9}$$

that ties down quite strictly the type of equilibrium under unemployment. If $L_t < \bar{L}$, then in steady state it follows $W_t/W_{t-1} \equiv \Pi^W = \Pi = \gamma_0 \left(1 - u_t\right)^{\gamma_1} < \gamma_0 = \Pi^*$. Hence, steady state inflation is below the target whenever there is positive unemployment.

Similar to the previous model, thus there are two regimes characterizing the AS in steady state. First, AS is vertical at full employment: $Y_{AS}^{FE} = Y^f = \bar{L}^{\alpha}$. Second, the AS^{DNWR} is upward sloping in the presence of unemployment due to the binding DNWR constraint:

$$Y_{AS}^{DNWR} = \left[\left(\frac{\Pi}{\gamma_0}\right)^{\frac{1}{\gamma_1}} \bar{L}\right]^{\alpha}. \tag{10}$$

The two branches of the AS meet at the kink when $\Pi_{AS}^{kink} = \gamma_0$, hence, at the inflation target under our simplifying assumption $\gamma_0 = \Pi^*$.

The demand side is shaped by a monetary policy rule with a ZLB

$$1 + i_t = \max\left\{1, 1 + i^* + \alpha_\pi \left(\Pi_t - \Pi^*\right) + \alpha_y \ln\left(\frac{Y_t}{Y^f}\right)\right\} \tag{11}$$

where $1 + i^* = \Pi^*/\beta$. In steady state (11) becomes

$$\ln Y_{AD}^{TR} = \ln Y^f - \frac{\beta\alpha_\pi - 1}{\beta\alpha_y}\left(\Pi - \Pi^*\right) \tag{12}$$

for $1 + i > 1$. This equation yields a negative steady state relationship between output and inflation, if monetary policy is active ($\beta\alpha_\pi > 1$), as in EMR model.

The main difference between an OLG model, as in EMR, and a representative agent model, as in SGU, lies in the steady state determination of the equilibrium/natural real interest rate. Given the Euler equation, the inverse of the subjective discount factor β pins down the natural real interest rate in a representative agent model, so the latter does not depend on the supply and demand of assets in the economy as in an OLG economy. This has important implications for the shape of the AD, because the AD^{TR} is downward sloping as in the EMR model, but AD^{ZLB} is now horizontal in this model, rather than upward sloping. If the ZLB is binding, the steady state inflation rate must equal to β, because $i = 0$ and $1 + r = 1/\beta$, whatever the level of steady state output. AD is therefore flat at $\Pi = \beta$, and steady state output is determined by the AS.

Figure 6: Aggregate demand and supply curves in the SGU model

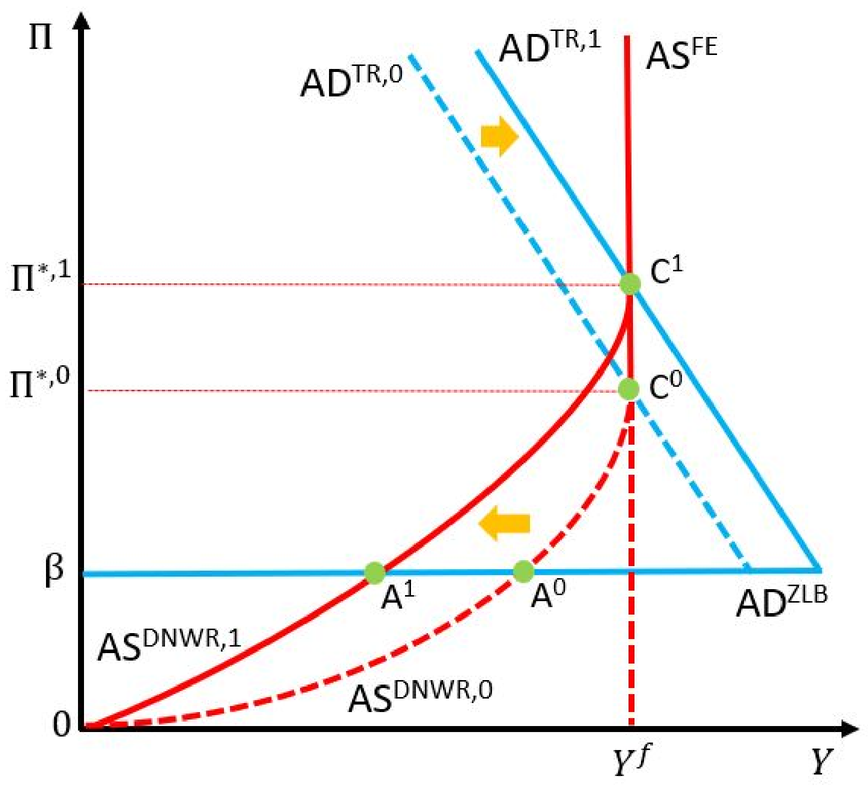


Figure 6 shows the $AS - AD$ diagram for the SGU model. The assumption in SGU $\beta < \gamma_0 \leq \Pi^*$ guarantees that it does not exists an intersection between AS^{FE} and AD^{ZLB}. Moreover, there cannot be also an intersection between AD^{TR} and AS^{DNWR}.[15] Given

[15]For any $\Pi \leq \Pi^*$, $Y_{AS}^{DNWR} \leq Y^f \leq Y_{AD}^{TR}$, which goes through the point (Y^f, Π^*).

these assumptions, there are always two equilibria.[16] As in the previous section, point A^0 is a $ZLB-U$ type of equilibrium, where both the ZLB and the DNWR constraints are binding, while point C^0 is a $TR-FE$ one, where none of the two constraints is binding, the economy is at full employment and inflation at target.[17] The Figure also shows what happens when the inflation target increases: AD^{TR} shifts out, as before, but now AS^{DNWR} moves to the left. A higher target increases $\gamma_0 = \Pi^*$, hence makes the AS^{DNWR} steeper (see (10)). It follows that raising the inflation target is detrimental in this model for a liquidity trap equilibrium. As the steady state inflation is always equal to β on the AD^{ZLB}, an increase in the target enlarges the inflation gap, Π/Π^*, and the binding DNWR dictates higher unemployment in equilibrium.

3.2 Dissolving the ZLB equilibrium

We now adapt our policy proposal to this model. Recall that the idea is to reflate the economy by using the DNWR constraint to impose *a floor to the rate of growth of nominal wages that depends on the inflation target.* (8) does not do that because wage inflation is bounded by zero, when employment is zero. To see how our policy proposal would also work in this model, let's simply modify the DNWR (8) in a similar vein as (7)

$$\frac{W_t}{W_{t-1}} \geq \delta\Pi^* + \gamma\left(u_t\right) = \delta\Pi^* + \gamma_0\left(1-u_t\right)^{\gamma_1}, \tag{13}$$

assuming now that $\beta < \delta\Pi^* + \gamma_0 \leq \Pi^*$, which is the equivalent assumption to $\beta < \gamma_0 \leq \Pi^*$ in the SGU case. Accordingly the AS^{DNWR} becomes

$$Y_{AS}^{DNWR} = \left[\left(\frac{\Pi - \delta\Pi^*}{\gamma_0}\right)^{\frac{1}{\gamma_1}} \bar{L}\right]^{\alpha}. \tag{14}$$

[16]There are no restrictions on γ_1. So we can distinguish three cases: if $\gamma_1 > \alpha$, the AS^{DNWR} is convex as depicted in Figure 6; it is concave for $\gamma_1 < \alpha$; and it is a straight line when $\gamma_1 = \alpha$. Whether the AS^{DNWR} is convex or concave (or a straight line) does not affect our results qualitatively, but the $ZLB-U$ equilibrium A^0 is associated with a larger negative output gap when AS^{DNWR} is concave (or a straight line).

[17]Although point A^0 in Figure 6 features $Y < Y^f$, $\Pi < \Pi^*$ and $i = 0$, it is not determinate, contrary to the corresponding equilibrium in the EMR OLG model. Rather, it is indeterminate as B in Figure 3. Furthermore, the $ZLB-U$ equilibrium in the SGU model does not reflect the idea of secular stagnation as described in Summers (2015) that entails $r^f < 0$. Therefore, we define it *deflationary* equilibrium ($\Pi = \beta < 1$), instead of *secular stagnation* one.

Figure 7: Raising the inflation target in the SGU model with our DNWR

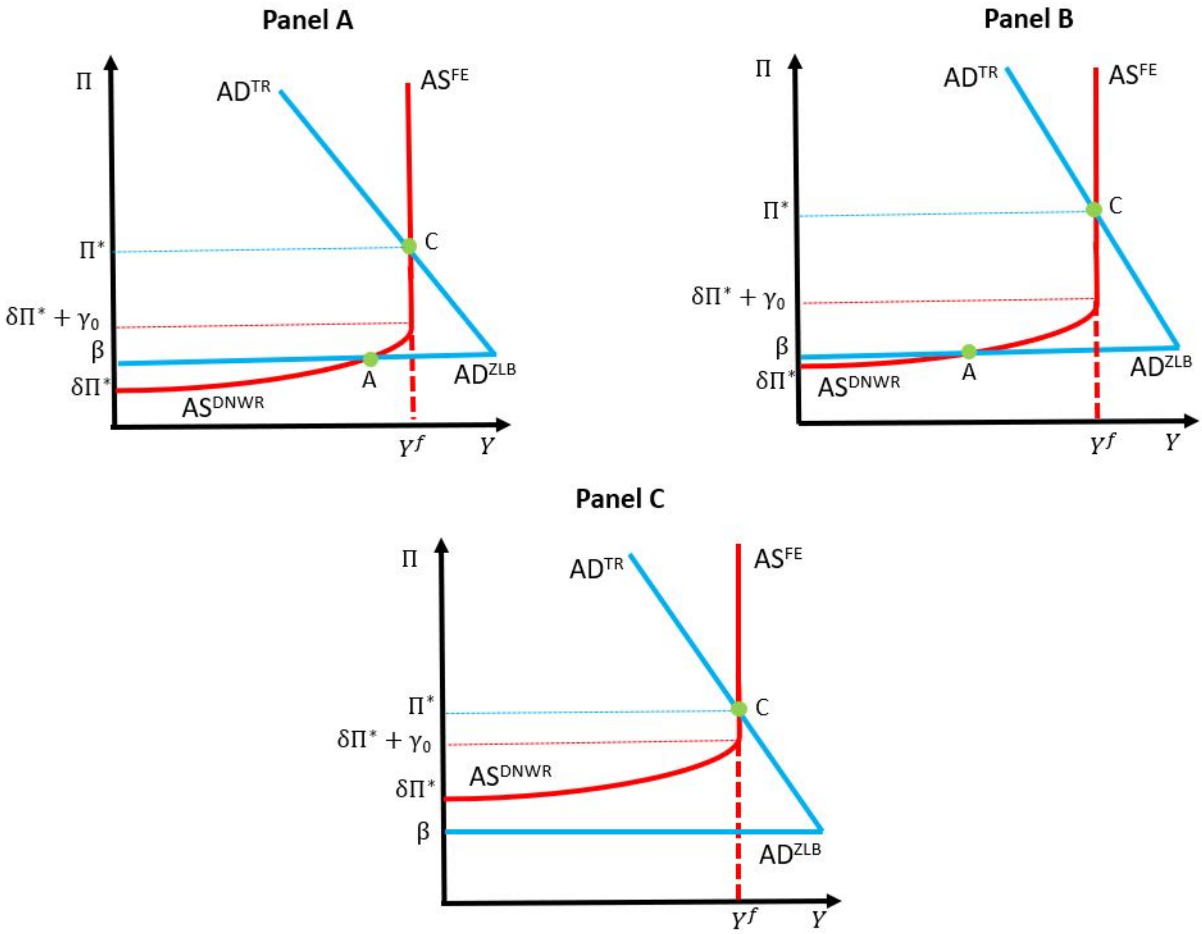


Figure 7 shows how this modification yields similar implications as in the previous case. Panel A displays the two equilibria, $ZLB-U$ and $TR-FE$, with our modified DNWR. The other two panels show what happens when the inflation target increases. Panel B shows that a too timid increase in the target has perverse effects: unemployment goes up in the $ZLB-U$ equilibrium, for the same level of deflation, $\Pi = \beta$. Krugman's (2014) timidity trap is *enhanced*: an increase in the target worsens the deflationary equilibrium. As explained above, this follows directly from the assumption on the DNWR constraint: a larger inflation gap calls for a higher unemployment. This is an important warning to remember regarding the implementation of our policy proposal. If an increase in the target causes the indexation policy to force the wages to increase by more, but agents do not adjust their inflation expectations upwards, then a deflationary equilibrium still exists, but higher unemployment is needed to support it. This result is the opposite of Proposition 2 in section 2.3. However, this stark difference is not due to the different

DNWR. Indeed, Appendix A.2 shows that Proposition 2 is robust to the case in which the DNWR constraint depends on employment (as in the original EMR's work).

The crucial difference between these two models lies on the demand side, and more precisely, on AD^{ZLB}. The latter is upward sloping and steeper than the AS^{DNWR} in an OLG model, because an increase in steady state inflation decreases the real interest rate, spurring demand, when the ZLB is binding. In an infinite-life representative agent economy, instead, the real interest rate is not endogenously determined, but it is given by $1/\beta$. It follows that steady state inflation is given ($\Pi = \beta$) in a ZLB equilibrium. This has two key implications. First, there is no positive effect on demand of an increase in the inflation target in a ZLB equilibrium. Second, price inflation is given, so inflation expectations do not adjust to the intended increase in wage inflation in the ZLB equilibrium. In other words, wage inflation has to be equal to price inflation, that is, equal to β in the ZLB equilibrium. Hence, any attempt to increase wage indexation by linking the increase in the nominal wages to a higher inflation target has to be compensated by higher unemployment, given the DNWR (13). The liquidity trap gets worse, because the policy is trying to force an increase in wage inflation, but agents don't believe prices could increase. Prices are actually decreasing in equilibrium. The increase in the inflation target is too timid, hence unless firms change their expectations by moving to the other $TR-FE$ equilibrium, the ZLB equilibrium survives and actually worsen.

Panel C shows that for a sufficiently high inflation target, however, deflationary expectations cannot be supported in equilibrium. From an a analytical point of view, this happens for $\Pi^* > \beta/\delta$. Intuitively, by forcing the increase in wage inflation above a certain threshold, there is no level of unemployment that support the ZLB equilibrium. As the effect on inflation expectations of the Brazilian Real Plan induced the switch from one inflationary equilibrium to a stable inflation one, our DNWR constraint acts as a coordination device for agents on the now unique $TR-FE$ equilibrium. It is reasonable to think that the switch might actually happen before reaching the limit of $u = 1$ as in this simple framework. At a certain point the level of unemployment would become unsustainable, so that agents would be compelled to coordinate on higher inflation expectations, that is, on the $TR-FE$ equilibrium. We can rearrange the condition that guarantees a unique equilibrium of the type $TR-FE$ as $\delta > \beta/\Pi^*$. This provides the degree of wage indexation necessary to achieve a specific inflation target, for a given discount factor. If $\beta = 0.95$, an inflation target of 2% (4%) requires δ greater than 0.93 (0.91) to be sustained.[18]

We conclude by stating two propositions that parallel those in the previous section for the OLG model.

Proposition 3. *Assume $\beta < \delta\Pi^* + \gamma_0 \leq \Pi^*$. Then, if $\Pi^* > \beta/\delta$, there exists a unique, locally determinate, $TR-FE$ equilibrium, where the ZLB is not binding, the inflation rate is equal to the target and output is at full employment, i.e., $i > 0$, $\Pi = \Pi^*$, $Y = Y^f$.*

Proposition 4. ***Enhanced Timidity Trap.*** *Assume $\beta < \delta\Pi^* + \gamma_0 \leq \Pi^*$ and that the economy is trapped in a deflationary equilibrium, $ZLB-U$ (Panel A). Then, an increase in the inflation target is always detrimental, in the sense that steady state output decreases in a ZLB equilibrium, unless this increase is sufficient to escape deflation.*

4 Conclusions

We have presented here a policy proposal to reflate economies experiencing a long-lasting ZLB episode with subdued or negative inflation. The ZLB problem is a "nominal" problem, in the sense that, for any level of the real interest rate, there is always a minimum inflation level that prevents a liquidity trap. As de-indexing the economy has been proved an effective way to tackle high inflation in past historical episodes, we suggest to apply the same mechanism, but in the opposite direction, to engineer inflation. More precisely, our policy of "re-indexing" the economy consists in imposing a minimum wage inflation that delivers the necessary inflation rate to escape from the ZLB.

In order to prove the validity of our proposed income policy, we have studied the ZLB problem through the lens of the OLG model of EMR and the infinite-life representative agent model of SGU, which both feature a ZLB equilibrium and downwardly rigid nominal wages. Our proposal is to impose *a floor on wage inflation that depends on a fraction*

[18] If we assume a deterministic trend in productivity as in SGU, the necessary level of δ to sustain any given inflation target declines.

of the inflation target through the downward nominal wage rigidity. This is exactly the opposite of the ceiling on wage inflation imposed in some past disinflationary policies. Under our assumption, the ZLB equilibrium disappears in both models. Note that in equilibrium the DNWR does not bind, hence it is not mechanically imposed in equilibrium. Moreover, both price and wage inflation are equal to the intended target and there is full employment in the unique equilibrium that survives. The DNWR acts as a coordination device that destroys the bad ZLB equilibrium. This result is robust to the specification of the downward nominal wage rigidity, and it requires a sufficiently high inflation target. Indeed, if the lower bound on wage inflation is not high enough, the economy is trapped in the Krugman's (2014) "timidity trap".

The timidity trap highlights the differences between the OLG and the infinite-life model, leading to different implications of our policy proposal according to the model. If the economy lies in the ZLB equilibrium and the inflation target is increased by a small and insufficient amount, the OLG economy moves to a better equilibrium featuring higher steady state output and inflation, despite the ZLB. Indeed, a higher target transmits to price inflation via wage indexation and this in turn reduces the real interest rate, stimulating demand and output. In other terms, raising the inflation target when nominal wage growth is indexed to it mitigates the ZLB problem in the OLG model, even if the increase is not sufficient to lift the economy out of the liquidity trap. This novel result is overturned in an infinite-life model, because the equilibrium real interest rate is fixed and thus the inflation level is equal to discount factor in the ZLB equilibrium. Indeed, the higher wage inflation produced by raising the inflation target does not translate in higher price inflation, and, given the DNWR constraint, the ZLB equilibrium features even lower output and inflation because of a larger inflation gap.

Finally, three issues would deserve further investigation. First, our simplified models do not exhibit a transitional dynamics from the ZLB equilibrium to an equilibrium with full employment and inflation at the target. More complicated and realistic models (for example, with capital) would entail a transitional dynamics between these two equilibria. Although the transitional dynamics constitutes an interesting future direction of our research and the associated costs cannot be disregarded, we don't think this could really affect our results. Indeed, our policy proposal is thought for economies that are stuck in a ZLB equilibrium, where output is chronically lower than the potential and inflation never hits the targeted level. Japan is the most prominent example. In a such a scenario, it is very hard to think that the gains in terms of output and inflation of escaping from the ZLB could be lower than the cost associated with the transitional dynamics. Second, we abstract from the presence of shocks. However, a tight DNWR constraint would impede the short-run adjustment of the economy to shocks, especially supply shocks, requiring a flexible real wage. This lack of flexibility would obviously impose short-run costs to the economy. Third, the pass-through from wage to price inflation could be affected by international competition in an open economy context, if the goods market is not longer perfectly competitive but national and foreign firms supply different varieties of goods. Indeed, firms could only partially transmit the higher labor costs to prices to preserve their competitiveness. If the exchange rate is flexible, a devaluation of the national currency can compensate for the higher prices, preserving the market shares of firms in the international markets. Moreover, a depreciated currency can contribute to boost inflation via the higher cost of imported goods. On the contrary, in the case of a monetary union (or a currency area in general), coordination among the member states is necessary to implement our policy proposal. Otherwise, countries that implement our income policy would suffer an appreciation in real terms with respect to those that do not, with negative consequences in terms of current account imbalances.

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A Appendix

A.1 Appendix to EMR

A.1.1 Model

The maximization problem of the representative household is

$$\max_{C_{t+1}^m, C_{t+2}^o} E_t \left\{ \ln C_t^y + \beta \ln C_{t+1}^m + \beta^2 \ln C_{t+2}^o \right\}$$

s.t.

$$C_t^y = B_t^y \tag{A1}$$

$$C_{t+1}^m = Y_{t+1} - (1 + r_t) B_t^y - B_{t+1}^m \tag{A2}$$

$$C_{t+2}^o = (1 + r_{t+1}) B_{t+1}^m \tag{A3}$$

$$(1 + r_t) B_t^y = D_t, \tag{A4}$$

where $Y_t = \frac{W_t}{P_t} L_t + \frac{Z_t}{P_t}$.[19] C_t^y, C_{t+1}^m and C_{t+2}^o denote the real consumption of the generations, while B_t^y and B_{t+1}^m are respectively the real value of bonds sold by young households and bought by middle-aged ones. Equation (A4) represents the debt limit, which is assumed to be binding for the young generation.[20] The optimality condition for the maximization problem is the standard Euler equation

$$\frac{1}{C_t^m} = \beta (1 + r_t) E_t \frac{1}{C_{t+1}^o}. \tag{A5}$$

Generations exchange financial assets in the loan market, whose equilibrium condition is

$$(1 + g_t) B_t^y = B_t^m. \tag{A6}$$

The loan demand on the left-hand side of (A6) can be denoted with L_t^d and alternatively expressed as

$$L_t^d = \left(\frac{1 + g_t}{1 + r_t} \right) D_t \tag{A7}$$

by using (A4) to substitute for B_t^y. Combining (A2), (A3), (A4) and (A5) yields the loan supply

$$L_t^s = \frac{\beta}{1 + \beta} (Y_t - D_{t-1}). \tag{A8}$$

The market clearing real interest rate which equates (A7) and (A8) is

$$(1 + r_t) = \frac{(1 + g_t)(1 + \beta) D_t}{\beta (Y_t - D_{t-1})} \tag{A9}$$

and it coincides with the natural interest rate

$$\left(1 + r_t^f\right) = \frac{(1 + g_t)(1 + \beta) D_t}{\beta (Y^f - D_{t-1})} \tag{A10}$$

[19] Labor demand L_t does not necessarily equate labor supply $\bar{L}$, as explained above.

[20] This assumption holds for $D_{t-1} < \frac{1}{1+(1+\beta)\beta} Y_t$.

at the potential level of output Y^f.

Each middle-aged household runs a firm that is active for just one period in a perfectly competitive market. The production technology of firms is given by

$$Y_t = L_t^{\alpha} \tag{A11}$$

where $0 < \alpha < 1$. Profits are

$$Z_t = P_t Y_t - W_t L_t \tag{A12}$$

and they are maximized, under the technological constraint (A11), if the real price of labor equals its marginal productivity,

$$\frac{W_t}{P_t} = \alpha L_t^{\alpha-1}. \tag{A13}$$

Wages are subject to the DNWR constraint (1) that we report again here

$$W_t = \max\left\{W_t^*, \alpha P_t \bar{L}^{\alpha-1}\right\}, \tag{A14}$$

where the lower bound on the nominal wage, W_t^*, is given by (2). Finally, the standard Fisher equation holds:

$$1 + r_t = (1 + i_t) E_t \Pi_{t+1}^{-1}, \tag{A15}$$

where E_t denotes the expectation operator.

A.1.2 Steady State Equilibrium

A competitive equilibrium is a set of quantities $\{C_t^y, C_t^m, C_t^o, B_t^y, B_t^m, Y_t, Z_t, L_t\}$ and prices $\{P_t, W_t, r_t, i_t\}$ that solve (1), (3), (A1), (A2), (A3), (A4), (A5), (A6), (A11), (A12), (A13) and (A15), given $\{D_t, g_t\}$ and initial values for W_{-1} and B_{-1}^m. Here we study the steady state equilibrium, which can be represented by aggregate demand and supply.

AS is characterized by two regimes, which depend on equation (1) through the steady state inflation rate. For $\Pi \geq \delta$, AS can be derived from equations (1), (A11) and (A13):

$$Y_{AS}^{FE} = \bar{L}^{\alpha} = Y^f.$$

Otherwise, the aggregate supply is given by

$$\Pi = \delta.$$

The regime of AD depends on the lower bound on the nominal interest rate expressed in equation (3). For a positive nominal interest rate $(1 + i > 1)$, we get the following AD by combining equations (3), (A9) and (A15):

$$Y_{AD}^{TR} = D + \left(\frac{1+\beta}{\beta}\right)\left(\frac{1+g}{1+r^f}\right)\left(\frac{\Pi^*}{\Pi}\right)^{\phi_\pi - 1} D.$$

A different AD is derived from the equations above, when the central bank hits the ZLB $(1 + i = 1)$:

$$Y_{AD}^{ZLB} = D + \left(\frac{1+\beta}{\beta}\right)(1+g)\,\Pi D$$

The inflation rate at which the ZLB becomes binding is computed from the two arguments on the right-hand side of (3):

$$\Pi^{kink} = \left[\frac{1}{(1+r^f)}\right]^{\frac{1}{\phi_\pi}} \Pi^{*\frac{\phi_\pi - 1}{\phi_\pi}}.$$

A.1.3 Proof of Proposition 1

Here we study the calibrations of the inflation target associated with the 5 panels in Figure 5. We start from the first and the last panel, which imply a unique equilibrium (a $ZLB-U$ equilibrium in Panel A and a $TR-FE$ equilibrium in Panel E). Then we derive the other cases. A proof of the Proposition 1 follows from the analysis of the case $\Pi^* > \frac{1}{\delta(1+r^f)}$. As explained in the main text, there are three possible steady state equilibria in the EMR OLG model (see Figure 3):

(A) $ZLB-U$ that occurs at the intersection of the AD^{ZLB} and the AS^{DNWR}, and it features

$$Y = D + \left(\frac{1+\beta}{\beta}\right)(1+g)\,\delta\Pi^* D \leq Y^f$$

$$i = 0$$

$$\Pi = \delta\Pi^* < \Pi^*;$$

(B) $ZLB-FE$ that occurs at the intersection of the AD^{ZLB} and the AS^{FE}, and it features

$$Y = Y^f$$

$$i = 0$$

$$\Pi = \frac{1}{1+r^f} \leq \Pi^*;$$

(C) $TR-FE$ that occurs at the intersection of the AD^{TR} and the AS^{FE}, and it features

$$Y = Y^f$$

$$i > 0$$

$$\Pi = \Pi^*.$$

Panel A. $\Pi^* < \frac{1}{1+r^f}$. The second term in the max operator of equation (3) is lower than 1 for $\Pi = \Pi^*$, so $i = 0$ and a $TR-FE$ equilibrium is impossible. As the resulting inflation level is $\Pi < \Pi^* < \frac{1}{1+r^f}$ because of the ZLB, even a $ZLB-FE$ equilibrium cannot exist and the only possible equilibrium is of the type $ZLB-U$.

Panel E. $\Pi^* > \frac{1}{\delta(1+r^f)}$. Even if the inflation level reaches its lower bound $\Pi = \delta\Pi^*$, $r = r^f$ (and so $Y = Y^f$) can be achieved without hitting the ZLB. This can be verified by substituting r for r^f and Π for $\delta\Pi^*$ in the Fisher equation (A15). As the ZLB is not binding ($i > 0$), $ZLB-U$ and $ZLB-FE$ equilibria cannot emerge and the unique equilibrium is of the type $TR-FE$.

Panel B. $\Pi^* = \frac{1}{1+r^f}$. There exists an equilibrium with inflation at the target and output at the potential in this case. In fact, the term $\left(1+r^f\right)\Pi^*\left(\frac{\Pi}{\Pi^*}\right)^{\phi_\pi}$ in equation (3) is 1 for $\Pi = \Pi^*$. This equilibrium features accordingly $Y = Y^f$ (because $r = r^f$), $i = 0$ and $\Pi = \Pi^* = \frac{1}{1+r^f}$, so it is a combination between $ZLB-FE$ and $TR-FE$ equilibria. Anyway, this is not the unique equilibrium, but there still exists a $ZLB-U$ equilibrium because $\Pi^* < \frac{1}{\delta\left(1+r^f\right)}$.

Panel C. $\frac{1}{1+r^f} < \Pi^* < \frac{1}{\delta\left(1+r^f\right)}$. Given $\frac{1}{1+r^f} < \Pi^*$, the second term in the max operator of the Taylor rule (3) is greater than 1 for $\Pi = \Pi^*$, so the ZLB is not binding in correspondence of the inflation target and the natural interest rate. As a consequence, a $TR-FE$ equilibrium exists, but it is not the unique equilibrium given that $\Pi^* < \frac{1}{\delta\left(1+r^f\right)}$. Even $ZLB-FE$ and $ZLB-U$ equilibria emerge and, in particular, the $ZLB-FE$ equilibrium differs from the type $TR-FE$ ($\frac{1}{1+r^f} = \Pi < \Pi^*$).

Panel D. $\frac{1}{1+r^f} < \Pi^* = \frac{1}{\delta\left(1+r^f\right)}$. For $\Pi = \delta\Pi^*$ and $r^f = r$, the Fisher equation (A15) implies binding ZLB ($i = 0$). So, even if the DNWR is at work, for a zero nominal interest rate is possible to achieve $Y = Y^f$. This means that, along with a $TR-FE$ equilibrium (which still exists because $\Pi^* > \frac{1}{1+r^f}$), an equilibrium with binding ZLB survives. Given $i = 0$, it follows from the Fisher equation

$$\Pi = \delta\Pi^* = \frac{1}{1+r^f}.$$

Therefore this equilibrium is a combination between $ZLB-U$ and $ZLB-FE$ equilibria.

A.2 Downward Nominal Wage Rigidity à la EMR

A.2.1 Steady State Equilibrium

We assume a different specification of the DNWR:

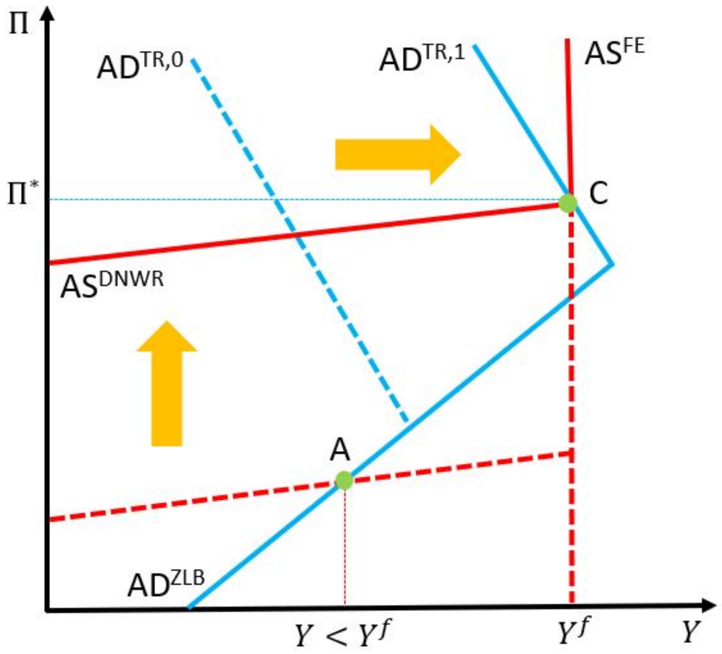
$$W_t^* = \gamma\Pi^* W_{t-1} + (1-\gamma)\,\alpha P_t \bar{L}^{\alpha-1} \tag{A16}$$

The model is the same outlined in Appendix A.1, apart from this assumption which alters aggregate supply. For $\Pi \geq \Pi^*$, AS is still given by $Y_{AS}^{FE} = Y^f$, while it becomes

$$Y_{AS}^{DNWR} = \left[\frac{1-\gamma\frac{\Pi^*}{\Pi}}{1-\gamma}\right]^{\frac{\alpha}{1-\alpha}} Y^f \tag{A17}$$

for $\Pi < \Pi^*$. This equation is derived from (A11), (A13) and (A16). It is represented by an upward sloping curve in Figure 8. If inflation falls below the target, wages cannot adjust to clear the labor market because of DNWR (A16), and involuntary unemployment determines a level of output lower than the potential one. This results in a positive relation between steady state inflation and output which is a direct consequence of a too high real wage: as inflation increases, the real wage approaches the level consistent with full employment, reducing the output gap. Although the segment of the AS corresponding to binding DNWR is not longer flat like in Section 2, the central mechanism behind our result still holds (Figure 8). Even if the DNWR depends on the "flexible" nominal

Figure 8: Raising the inflation target with our DNWR á la EMR



wage, $\alpha P_t \bar{L}^{\alpha-1}$, the AS curve moves with the inflation target and so raising Π^* shifts the AS^{DNWR} upward. We can accordingly establish a proposition similar to Proposition 1 in Section 2 and Proposition 2 continues to hold.

Proposition 5. *Assume $r^f < 0$ and $\gamma < 1$. Then, if $\Pi^* > \frac{1}{1+r^f}$, there exists a unique, locally determinate, $TR - FE$ equilibrium, where the ZLB is not binding, the inflation rate is equal to the target and output is at full employment, i.e., $i > 0$, $\Pi = \Pi^*$, $Y = Y^f$.*

Proof:

There are three possible steady state equilibria in the EMR OLG model with DNWR (A16):

(A) $ZLB - U$ that occurs at the intersection of the AD^{ZLB} and the AS^{DNWR}, and it features

$$Y = \left[\frac{1 - \gamma\frac{\Pi^*}{\Pi}}{1-\gamma}\right]^{\frac{\alpha}{1-\alpha}} Y^f < Y^f$$

$$i = 0$$

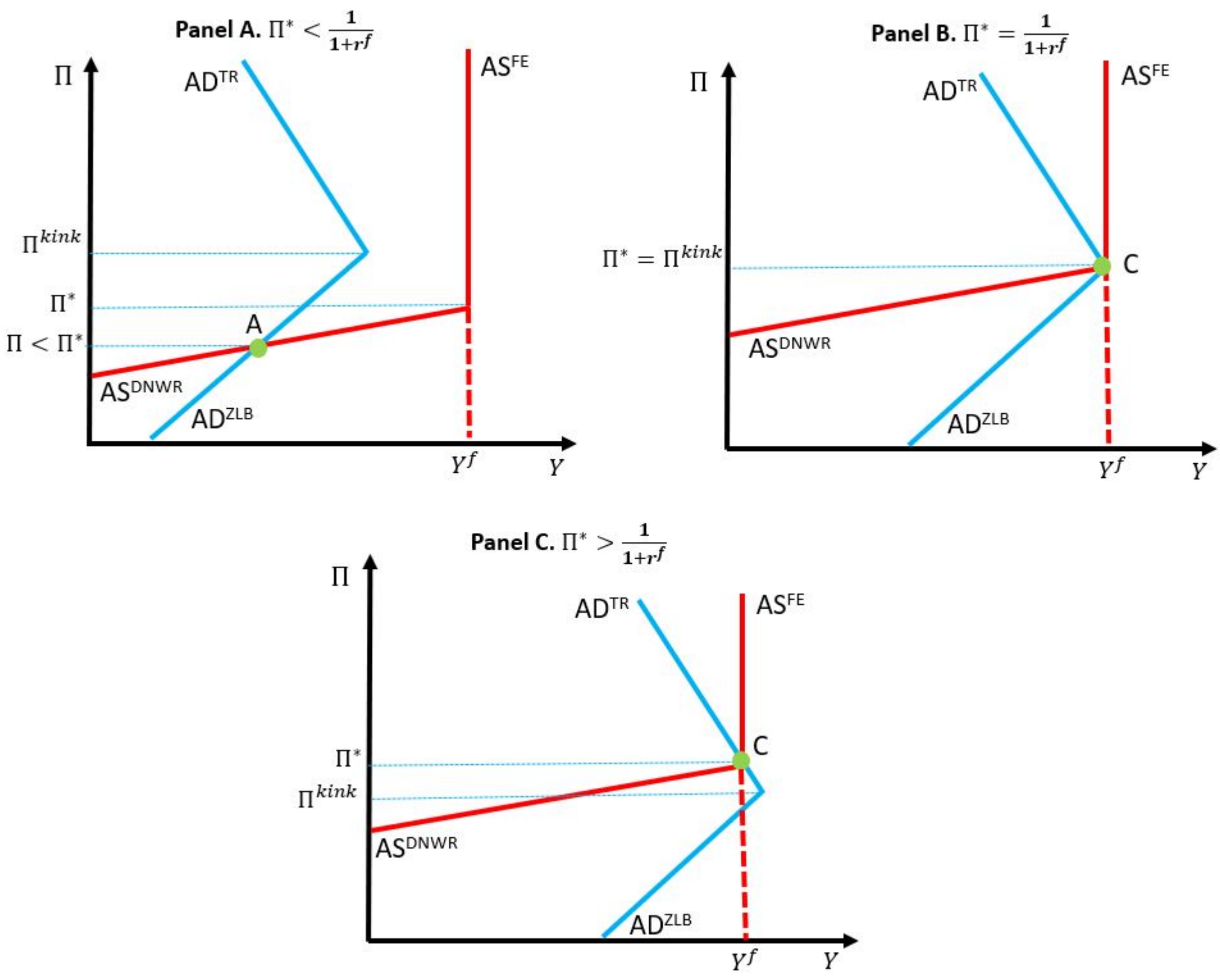
$$\Pi = \frac{1}{1+r} < \Pi^*$$

(B) $ZLB - FE$ that is identical to the equilibrium in the proof of Proposition 1;

(C) $TR - FE$ that is identical to the equilibrium in the proof of Proposition 1.

If $r^f < 0$, three different cases can emerge and they are all depicted in Figure 9. AD can intersect AS on its upward sloping segment AS^{DNWR} and the resulting unique equilibrium is a $ZLB - U$ (Panel A); AD can intersect AS on its vertical segment AS^{FE}

Figure 9: All possible steady state equilibria with our DNWR á la EMR



and the unique equilibrium is a combination between a $ZLB - FE$ and a $TR - FE$ equilibrium, because $Y = Y^f$, $\Pi = \Pi^* = \frac{1}{1+r^f}$ and $i = 0$ (Panel B); AD can intersect AS on its vertical segment AS^{FE} and the only equilibrium is a $TR - FE$ (Panel C). Now, we study the parameterizations of Π^* corresponding to these three cases. A proof of Proposition 5 follows from the analysis of the case $\Pi^* > \frac{1}{1+r^f}$.

Panel A. $\Pi^* < \frac{1}{1+r^f}$. The proof is the same of Proposition 1.

Panel B. $\Pi^* = \frac{1}{1+r^f}$. The second term in the max operator of equation (3) is 1 (binding ZLB) in correspondence of an inflation level equal to the target Π^*. So, the unique equilibrium is a combination between a $ZLB - FE$ and a $TR - FE$ equilibrium, given that $Y = Y^f$ (in fact, $r = r^f$), $i = 0$ and $\Pi = \Pi^* = \frac{1}{1+r^f}$.

Panel C. $\Pi^* > \frac{1}{1+r^f}$. The ZLB is never binding in this case, because the term $\left(1 + r^f\right) \Pi^* \left(\frac{\Pi}{\Pi^*}\right)^{\phi_\pi}$ in the monetary policy rule (3) is greater than 1 for $\Pi = \Pi^*$. Therefore, the only possible equilibrium is a $TR - FE$.

A.3 Appendix to SGU

A.3.1 Model

Unless otherwise mentioned, the notation is identical to that of the model in Appendix A.1.1. The representative household seeks to maximize the utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma} - 1}{1-\sigma} \right)$$

s.t.

$$P_t C_t + B_t = W_t L_t + Z_t + (1 + i_{t-1}) B_{t-1}$$

$$lim_{j \to \infty} E_t \left[\prod_{s=0}^{j} (1 + i_{t+s})^{-1} \right] B_{t+j+1} \geq 0.$$

C_t denotes the real consumption expenditure, while B_t is the value of risk-free bonds in nominal terms. The optimality conditions for the household's problem is the Euler equation

$$C_t^{-\sigma} = \beta(1 + i_t) E_t \left[\frac{C_{t+1}^{-\sigma}}{\Pi_{t+1}} \right] \tag{A18}$$

and the no-Ponzi-game constraint

$$lim_{j \to \infty} E_t \left[\prod_{s=0}^{j} (1 + i_{t+s})^{-1} \right] B_{t+j+1} = 0$$

which holds with equality. The problem of the representative firm is the same illustrated in Appendix A.1.1, while the DNWR described in the main text is:

$$\frac{W_t}{W_{t-1}} \geq \gamma_0 \left(\frac{L}{\bar{L}} \right)^{\gamma_1}. \tag{A19}$$

The aggregate resource constraint imposes

$$Y_t = C_t \tag{A20}$$

and the aggregate rate of unemployment is:

$$u_t = \frac{\bar{L} - L_t}{\bar{L}} \tag{A21}$$

A.3.2 Steady State Equilibrium

A competitive equilibrium is a set of processes $\{Y_t, C_t, L_t, u_t, \Pi_t, W_t, i_t\}$ that solve (9), (11), (A11), (A13), (A18), (A19), (A20) and (A21), given the initial value for W_{-1}. We study the steady state equilibrium by analyzing aggregate demand and supply, which are characterized by two regimes. For $\Pi \geq \gamma_0 = \Pi^*$, AS is obtained from (9), (A11) and

(A19):

$$Y_{AS}^{FE} = \bar{L}^{\alpha} = Y^{f}.$$

By combining the same equations, AS becomes

$$Y_{AS}^{DNWR} = \left[\left(\frac{\Pi}{\gamma_0} \right)^{\frac{1}{\gamma_1}} \bar{L} \right]^{\alpha}$$

when $\Pi < \gamma_0 = \Pi^*$. Now, we turn to aggregate demand. For a positive nominal interest rate,

$$1 + i = \frac{\Pi^*}{\beta} + \alpha_\pi \left(\Pi - \Pi^* \right) + \alpha_y \ln \left(\frac{Y}{Y^f} \right)$$

and AD can be computed from the Taylor rule by substituting $1 + i$ for its steady state value $\frac{\Pi}{\beta}$:

$$\ln Y_{AD}^{TR} = \ln Y^f - \frac{\beta \alpha_\pi - 1}{\beta \alpha_y} \left(\Pi - \Pi^* \right).$$

It can be alternatively expressed as:

$$Y_{AD}^{TR} = \frac{Y^f}{e^{\Phi(\Pi - \Pi^*)}}$$

where $\Phi = \frac{(\beta \alpha_\pi - 1)}{\beta \alpha_y}$. If the ZLB binds $(1 + i = 1)$, AD turns

$$\Pi = \beta$$

and it is computed by following the same steps as above.