Pricing Perpetual Turbo Warrants - An application to the Hong Kong exchange market*

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Abstract

This article deals with the pricing of turbo warrants and perpetual turbo warrants. (Non perpetual) turbo warrants have already been treated in several articles, but no application has been run to check the validity of the pricing formula. On another hand, perpetual turbo warrants studies are scarce as they are recent financial instruments. In this work, an application to Hong Kong exchange market is provided for the (non perpetual) turbo warrants. Then, exotic portfolios are constructed for replicating the perpetual turbo warrants and then to propose a price. Due to a lack of historic data of perpetual turbos, only numerical simulated results are presented for perpetual turbo warrants. Properties are presented under the Geometric Brownian Motion model.

Keywords: Down and Out Call option, Down and In look-back option, Perpetual Turbo Warrants options, Hong Kong exchange market.

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1 Introduction

Turbo warrants are very recent products which appeared firstly in Germany in 2001 and were since widely exchanged in Europe and Hong Kong. Société Générale bank has been one of the first banks to emit these instruments. These products are commonly used and issued by both investors and financial institutes as they give specific means for speculation. These instruments are usually managed in a strictly bullish or bearish speculations.

Results on pricing finite maturity Turbo warrants are found in a number of articles, however pricing perpetual turbo warrant has not been specifically treated. The classical pricing formulas for derivatives (of Black-Scholes type) provide the value of a derivative product with a finite maturity. However, perpetual Turbo warrants are products with an infinite maturity. Consequently, the aim of this article is come up with a method to price perpetual Turbo warrants.

In order to better understand how to price turbo warrant in general, the pricing of the finite maturity turbo warrants are first presented. Since turbo warrants are very exotic products, classical pricing methods such as for vanilla options cannot be used. However, exact formulas for pricing this instrument exist in the literature.

However, in the literature, we did not find any application to real data to check the validity of the formula. Consequently, in this paper, the validity of these formulas is checked on historical data from Hong Kong Exchange markets. Empirical results are satisfying.

This article then focuses on perpetual (i.e. infinite maturity) Turbo warrants. Inspired by results on finite maturity turbo warrants, we found an appropriate replicating portfolio composed by exotic products and we used this portfolio to price perpetual turbo warrant. The properties of our pricing model are first tested on simulated underlying assets. Secondly, our model will be tested on historical data.

Section 2 exposes the state of the art. In section 3, we recall the case of finite maturity turbo warrants, their replicating portfolio as well as their pricing formula under Geometric Brownian Motion (GBM) model are brought to light. These results will help us then explain the perpetual case. In Section 4, we propose several implementations of the finite maturity turbo warrant pricing formula. We also present our pricing formula for perpetual turbo warrant. section 5 provides an application on historical data. And finally, section 6 concludes.

2 State of the art

At our knowledge, there is no article in the literature that proposes a pricing formula for perpetual Turbo warrants. In this section, we then recall the state of the art of two king of financial options that will serv as a basis for the perpetual Turbo warrants: the turbo warrant and the perpetual options.

2.1 Turbo warrants

Turbo warrants first appeared in Germany at the end of 2001. They have experienced enormous growth in Northern Europe and Hong Kong. Turbo warrants are down-and-out barrier options in which the rebate is another exotic option. Eriksson [2006] was the first to derive the valuation formula for a turbo warrant, based on the geometric Brownian motion (GBM) of the underlying asset.

The turbo warrants pricing has been studied by [Wong and Chan, 2008] using the CEV model and a stochastic volatility model. The authors obtained analytical solutions for turbo warrants under the stochastic volatility model to examine the behaviour and sensitivity of turbo warrants to implied volatility. Wong and Lau [2008] use a jump-diffusion model. They derived analytical solutions of turbo warrants based on the double exponential jump diffusion model, using a Laplace transform to study the sensitivity of the turbo warrant to jump parameters. Eriksson [2006] used the classical GBM model. Le et al. [2014] examined the turbo warrants pricing in the hybrid stochastic and local volatility model provided by Choi et al. [2013]. An finally, Ji-Hun and Chang-Rae [2016] focused on the pricing of turbo warrants within the framework of the SEV model.

2.2 Perpetual options

Section in progress.

3 Technical reminder of finite maturity turbo warrants

3.1 Definition of Turbo warrants

The classical turbo warrants have a Finite Maturity, denoted T, and are European style contracts. Let S_t denotes the underlying asset price at time t. Let K denotes the strike price, and H the barrier. Let us define τ_H the time when S_t has crossed H for the first time: $\tau_H = \inf(t \geq 0; S_t \leq H)$. Let define:

$$m_{a,b} = \inf\{S_t, a \leq t \leq b\}, \tag{1}$$

$$M_{a,b} = \sup\{S_t, a \leq t \leq b\}. \tag{2}$$

A turbo call (respectively put) warrant pays the owner $(S_T - K)^+$ (respectively $(K - S_T)^+$) at date T if S_t has not crossed H. If $t = \tau_H$ the first warrant contract is cancelled, and another contract for the rebate begins. This new contract is generally a European call(respectively put) warrant on the minimum (respectively maximum) of the underlying asset's price, with the same strike price but with a shorter Maturity date, called Rebate Maturity, T_0: m_{τ_H,τ_H+T_0} (respectively M_{τ_H,τ_H+T_0}. Hence, the payoff of a such a contract is $(m_{\tau_H,\tau_H+T_0} - K)^+$ (respectively $(K - M_{\tau_H,\tau_H+T_0})^+$).

It should be noted that in all the call (respectively put) contracts $H \geq K$ (respectively $H \leq K$)and the initial underlying price is normally above (respectively below) the Barrier. The owner of the call contract has a bullish speculation as the payoff increases when the price goes up, but has a little rebate if the price goes down to hit the barrier.

For simplicity, only call turbo warrant contracts will be considered in the following. Same results are applicable in the case of a put turbo warrant.

3.2 Replicating Portfolio for a Call Turbo-Warrant

When time $t \leq \tau_H$, the price of a Turbo warrant is not only given by the price of the first contract but also by the expectation that the rebate part may occur. In order to price this turbo warrant, it should be noted that its price variations can be replicated by a portfolio of simpler instruments. Le et al. [2014] explain that a turbo warrant can be separated into two structured products: a Down and Out Call option and a Down and In look-back option. The price of the turbo warrant is simply the sum of the prices of these two elements.

Down and Out Call option

The first product is a Down and Out Call (denoted DOC) option. This is an exotic barrier option. Its pay-off is the same as for a call option but this contract is only active until the underlying asset price is above a Barrier H. When the price hits the barrier, this contract is cancelled. The DOC's price when it is activated is give by Equation 3:

$$DOC(t, s) = E_Q[e^{-r(T-t)}(S_T - K)^+ 1_{\tau_H > T} | S_t = s], \tag{3}$$

where r denotes the risk free interest rate. E_Q denotes the expectation following a risk neutral probability Q.

For replicating the turbo warrant, the DOC option needs to have the same underlying asset, Maturity T, Strike K, and barrier H as the turbo warrant.

Down and In look-back option

The second product is a Down and In Look-back (denoted DIL) option. The Payoff of such an option is based on the historic underlying price. In our case, the pay-off is calculated according to the minimum price that the underlying asset has reached between τ_H and $\tau_H + T_0$. The DIL option is hence a call option on the minimum value of the underlying asset with respect to the same strike but with a smaller maturity denoted T_0. Its price is given by Equation 4:

$$DIL(t, s; T_0) = E_Q[e^{-r(\tau_H + T_0 - t)}(m_{\tau_H, \tau_H + T_0} - K)^+ 1_{\tau_H \leq T} | S_t = s], . \tag{4}$$

This DIL option can also be decomposed into simpler instruments by noticing that this option has the same pay-off than a look-back (denoted LB) option if the underlying asset price hits the barrier. Consequently, the DIL is decomposed into two parts: the first one is the LB option itself, which is priced separately from the event of hitting the barrier, and the second one is the expectation of such an event happening.

The price of this LB option at time τ_H is:

$$LB(\tau_H, S_{\tau_H}; T_0) = E_{Q,\tau_H}[e^{-rT_0}(m_{\tau_H, \tau_H + T_0} - K)^+]. \tag{5}$$

Following Domingues [2012], it can be deducted that:

$$LB(\tau_H, S_{\tau_H}; T_0)) = LB(0, H, T_0). \tag{6}$$

The expectation of the event of hitting the barrier happening (adjusted for risk free interest rate) is:

$$E_Q[e^{-r(\tau_H - t)} 1_{\tau_H \leq T} | S_t = s],$$

so that:

$$DIL(t, s; T_0) = LB(0, H, T_0)E_Q[e^{-r(\tau_H - t)}1_{\tau_H \leq T}|S_t = s]. \tag{7}$$

The look-back (LB) part can be seen according to a look-back call with a floating strike (LC_{fl}) which is a call in which the strike is a moving parameter. Domingues [2012] detail how to obtain the look-back part, and we conclude that:

$$DIL(t, s, T_0) = [LC_{fl}(0, H, K, T_0) - LC_{fl}(0, H, H, T_0)]E[\exp^{-r(\tau_H - t)} 1_{\tau_H \leq T}|S_t = s], \tag{8}$$

where $LC_{fl}(t, S_t, K, T_0)$ is the price of look-back floating strike call with strike equals to K and a maturity T_0.

Knowing that the turbo warrant is the sum of this two products, the price of a turbo warrant can be writen as:

$$\begin{aligned} TC(t, s) &= DOC(t, s) + DIL(t, s), & (9)\\ &= DOC(t, s) + LB(0, H, T_0)E_Q[\exp^{-r(\tau_H - t)} 1_{\tau_H \leq T}|S_t = s] & (10)\\ &= DOC(t, s) + [LC_{fl}(0, H, K, T_0) - LC_{fl}(0, H, H, T_0)]DR(t, s), & (11) \end{aligned}$$

where,

$$DR(t, s) = E[exp^{-r(\tau_H - t)}1_{\tau_H \leq T}|S_t = s].$$

This provides a replicating portfolio for the turbo warrant. To examine the characteristics of each element (DOC and DIL) each component is represented in Figure 1. We notice that the DIL price is positive even when the price is above the barrier. This price increases when the underlying price gets closer to the barrier. The DOC part is maximum if the underlying price is far above the barrier and it decreases when the underlying price approaches the barrier to become null from the moment it hits it.

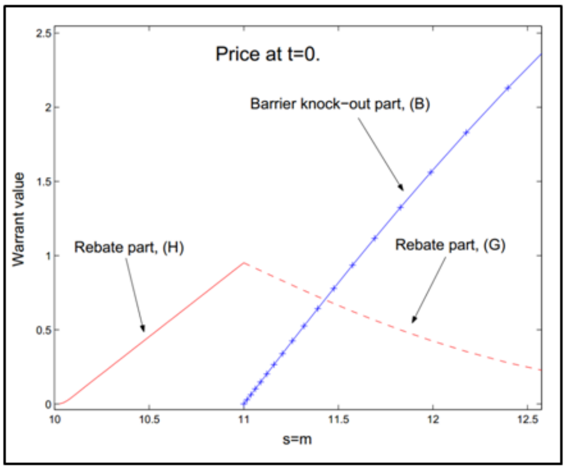

Figure 1: DOC and DIL value according to the underlying asset's price

3.3 Pricing of Finite Maturity Turbo warrants

The Geometric Brownian Motion (GBM) is used as a model of the underlying asset and describes its price as:

$$dS_t = (r - d)S_t dt + \sigma S_t dW_t, \tag{12}$$

where W_t is a standard Brownian Motion, r is the risk free interest rate, and d is the dividend yield. Usual hypotheses are assumed: r and σ are constant, r is the same for all maturities, the short selling of securities is permitted, there are no transactions costs, the assets are perfectly divisible, there are no riskless arbitrage opportunities, time t is continuous.

The pricing formulas are given in Domingues [2012]: The exact expressions of each part of Equation 11 is obtained by using Laplace Transformation, trying to have simpler differential equations and then inverting the Laplace Transformation (see Domingues [2012] for the exact demonstration).

In the case of the look-back call, the price is:

$$\begin{aligned} LC_{fl}(t, S_t, m_{0,t}, T) &= S_t e^{-d\tau}\Phi[d_{bs1}(S_t, m_{0,t})] - m_{0,t}e^{-r\tau}\Phi[d_{bs}(S_t, m_{0,t})] \\ &+ \frac{S_t}{\delta}e^{-r\tau}(\frac{S_t}{m_{0,t}})^{-\delta}\Phi[d_{bs}(m_{0,t}, S_t)] - e^{-d\tau}\Phi[-d_{bs1}(S_t, m_{0,t})], \end{aligned} \tag{13}$$

and in the case of the down and Out Call, the price is:

$$\begin{aligned} DOC(t, s) &= S_t e^{-d\tau}\Phi[dbs1(S_t, H)] - He^{-r\tau}\Phi[d_{bs}(S_t, H)] \\ &- \left(\frac{H}{S_t}\right)^{\delta-1}\left\{\frac{H^2}{S_t}e^{-d\tau}\Phi\left[d_{bs1}\left(\frac{H^2}{S_t}, H\right)\right] - He^{-r\tau}\Phi\left[d_{bs}\left(\frac{H^2}{S_t}, H\right)\right]\right\} \\ &+ (H-K)e^{-r\tau}\Phi[d_{bs}(S_t, H)] - \left(\frac{H}{S_t}\right)^{\delta-1}\Phi[d_{bs}(S_t, H)], \end{aligned} \tag{14}$$

$$DR(t, s) = \left(\frac{H}{S_t}\right)^{\alpha+}\Phi\left[\theta\frac{ln(\frac{H}{S_t}+\beta\tau)}{\sigma\sqrt{\tau}}\right] + \left(\frac{H}{S_t}\right)^{\alpha-}\Phi\left[\theta\frac{ln(\frac{H}{S_t}-\beta\tau)}{\sigma\sqrt{\tau}}\right], \tag{15}$$

where $\Phi[.]$ is the normal cumulative distribution function and

$$d_{bs}(Y, X) = \frac{ln(\frac{Y}{X}) + r - d - \frac{\sigma^2}{2}\tau}{\sigma\sqrt{\tau}}, \tag{16}$$

$$d_{bs1}(Y, X) = d_{bs}(Y, X) + \sigma\sqrt{\tau}, \tag{17}$$

$$\delta = \frac{2(r-d)}{\sigma^2}, \tag{18}$$

$$\theta = sign[ln(\frac{S_t}{H})], \tag{19}$$

$$\beta = \sqrt{(r - d - \frac{\sigma^2}{2})^2 + 2r\sigma^2}, \tag{20}$$

$$\alpha_{\pm} = \frac{r - d - \frac{\sigma^2}{2} \pm \beta}{\sigma^2}. \tag{21}$$

4 Methodology

4.1 Implementation of Turbo Warrants Pricing

4.1.1 Data description

Data on turbo warrants are taken from the from ©Hong Kong Exchanges and Clearing Limited. They represent daily prices on the Hong Kong exchange market (HKEx):

called Callable Bull Bear Contract (CBBC). The underlying asset's prices are taken from Yahoo! Finance. Figure 2 shows the underlying asset's price and the CBBC price. Both of them are normalized with respect to the mean of each data.

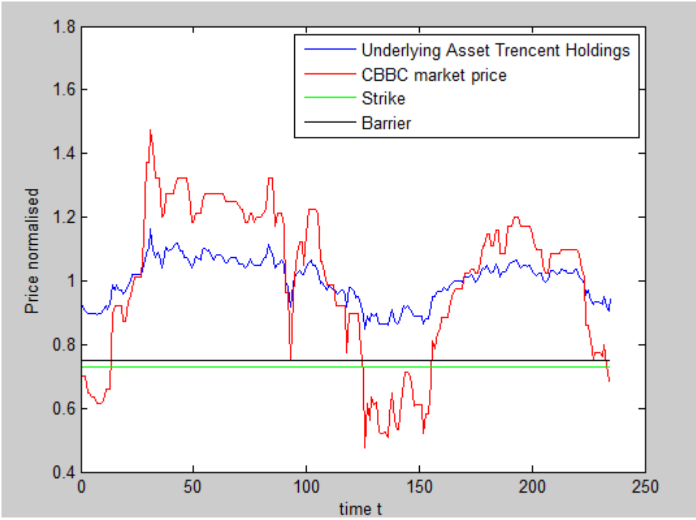

Figure 2: Data of Trencent Holdings

It can be noticed that the CBBC has the same fluctuation than the underlying asset but with higher variations. This is due to the gearing effect of a Turbo warrant. In fact, turbo warrant are used because of their important leverage effect: the price of a turbo warrant is lower than a call option (with same parameters) for example, it has the same pay-off when the speculation is right. Operators use it to maximize their profit for lesser premium. In the case of Trencent Holdings (Chinese Company), the underlying asset does not touch the Barrier (see Figure 2), giving us more data to work on until maturity.

The annualized short term US treasury bills rate is used as a proxy of the risk free interest rate.

4.1.2 Pricing Formula Implementation under GBM model

As for the volatility, we use the initial implied volatility [1]. In our case, the implied volatility is computed using Newton-Raphson algorithm by minimizing the difference between the real CBBC price and the calculated price using GBM model.

We work also with the *delayed implied volatility*, meaning that we take at time t the implied volatility that had our CBBC in time $t - N * days$.

The question of which volatility to consider is still very important, but the results we found using the implied volatility at time 0 are very satisfying.

Matlab software is used.

[1]We recall that the implied volatility is the volatility that once put in the GBM pricing model leading to the exact market price for the calculated CBBC price.

4.2 Perpetual Turbo warrant

4.2.1 Definition of a Perpetual Turbo warrant

Perpetual turbo warrant is a kind of turbo warrant that does not have a finite maturity. This implies also an American character, because if it has not an American style, this product would pay always only the rebate. Those characteristics naturally apply only to the active part of the turbo warrant as the rebate is always based on a short maturity (in general the 3-days maturity) and if not the contract would become useless (as it is not exchangeable after it has touched the barrier and wouldn't reach maturity).

4.2.2 Pricing of Perpetual turbo warrant

To price the perpetual Turbo warrant, a replicating portfolio of two exotic options deducted from the finite case is constructed. This portfolio contains an American Down and Out call (ADOC) which is a DOC with an infinite maturity and a simple DIL. The DIL part does not change because it is related to the probability that the underlying hits the barrier which is independent from the maturity Consequently, the price of a perpetual turbo warrant is equal to:

$$PTW(t, s) = ADOC(t, s) + DIL(t, s). \tag{22}$$

Down-and-In Lookback Call option price

Considering the definition of the perpetual turbo warrant, the pricing of the rebate part is exactly the same as for the finite maturity product. The following formula is then used:

$$DIL(t, s) = [LC_{fl}(0, H, K, T_0) - LC_{fl}(0, H, H, T_0)]DR(t, s). \tag{23}$$

The DR part can be calculated by Monte Carlo method: we simulate a large number of GBM models and see the implicit time in which the underlying hits the Barrier. We then calculate $\exp^{-r(\tau_H - t)}$ and find the expectation $E[\exp^{-r(\tau_H - t)} | S_t = s]$ where τ_H is that implicit time, by calculating the mean of a large number of simulations.

The Down-and-Out Call option price

Regarding the DOC in the case of finite maturity case, we find two major differences:

1. American character: the PTW is always exercisable. This fact imposes a new condition on the price as the value is the maximum between the expected payoff and the spot payoff.

2. Perpetual character: the PTW does not have a finite maturity. This implies the use of new pricing techniques that take into account all possible times of barrier touch

The problem of the perpetual DOC has been widely explored in the literature and in particular by M. Jeanblanc [2009]. In their work, the price for a Parisian perpetual DOC presented (an evolution of the perpetual DOC) that is deactivated only if underlying asset is below the barrier level for at least a certain time (called delay D). The perpetual DOC we are considering here is then a Parisian perpetual DOC with a 0 delay. We will use those results for our perpetual turbo warrant.

The Parisian Perpetual DOC is priced as follows, when the underlying asset follows a GBM model:

$$ADOP(S_0, H, D) = \left(1 - \frac{\Psi(-\kappa\sqrt{D})}{\Psi(\kappa\sqrt{D})} e^{2l\kappa}\right) \frac{1}{\sigma\kappa} \left(\frac{S_0}{H_c}\right)^{\gamma} \left(\frac{\gamma H_c}{\gamma - 1} - \frac{r}{\gamma} K\right) \quad (24)$$

with $\kappa = \sqrt{2r + \nu^2}$, $\gamma = \frac{-\nu + \sqrt{2r+\nu^2}}{\sigma}$, $\nu = \frac{1}{\sigma}(r - d - \frac{\sigma^2}{2})$ and $l = \frac{1}{\sigma} ln(\frac{H}{S_0})$. The exercise boundary H_c is defined implicitly by:

$$H_c - K = \frac{1}{\sigma\kappa} \left(1 - \frac{\Psi(-\kappa\sqrt{D})}{\Psi(\kappa\sqrt{D})} \left(\frac{H}{H_c}\right)^{\frac{2\kappa}{\sigma}}\right) \left(\frac{\gamma H_c}{\gamma - 1} - \frac{r}{\gamma} K\right). \quad (25)$$

4.2.3 Implementation of perpetual Turbo warrant pricing model

The minor difficulty is to solve the implicit equation for the dynamic barrier H_c. It is easy-done on VBA thanks to the built-in solver but it is a little more complex on MatLab, as the solutions are not always real and the appropriate one has to be selected manually. That is not a big deal for a constant volatility (as it is only one equation) but poses real issues on an evolving volatility (as there is one equation each time).

The major difficulty has been to find reliable data on perpetual turbo warrants. As for the time of the publication we did not manage to have any data on any underlying asset for this product. We can still evaluate the goodness of this model but we have no way of proposing an expectation neither for the error nor for the parameters of the model (interest rate, dividend rate and volatility). We have created an Excel Template for the Perpetual with a constant volatility but not one for the evolving volatility as in this environment there are still same issues over the convergence of the volatilities.

Following the lack of reliable data, we decided to price perpetual turbo warrant over underlying assets that are used also for finite maturity turbo warrants on which we have official data. This allows us to compare the behaviour of the perpetual with the finite maturity one.

5 Empirical results

5.1 Results for Turbo Warrants

5.1.1 Empirical Results under GBM model

Figure 3 compares the theoretical prices of the turbo warrant under the GBM model with the real market prices.

The theoretical formula gives us a very good approximation of the real CBBC price as the error is around 11% on average.

5.1.2 Pricing Formula Implementation under extended model

We have also implemented an Excel VBA Template that allows a fast evaluation of the price of the Turbo Warrant. It offers the possibility to use a constant volatility or

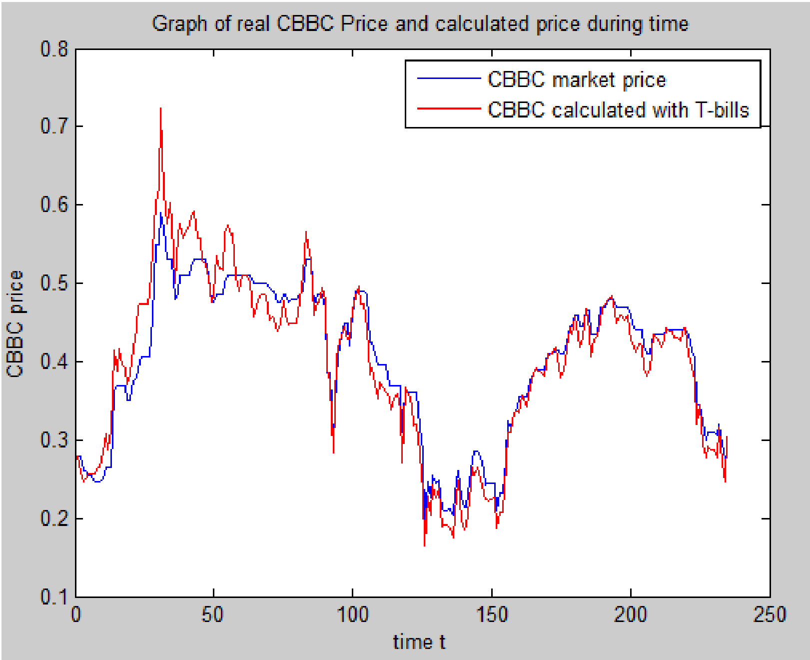

Figure 3: Real CBBC price and calculated price under GBM model in HKD

to propose a dynamic one. For the constant volatility model, it estimates the implied volatility of the turbo by minimizing the difference between the issue price and the one calculated using the historical volatility. To have an evolution of the implied volatility, we fix the number N of days we look back and minimize the difference of the price N days before and the one calculated.

While overlooking the non-convergence of some values (that are possible using the faster but more imprecise Excel solver), the results using an evolving volatility are not so much better of the one using a constant one, and in the latter we have around 1/200 of the calculation time needed. We advise to use the more precise one only when you disregard the cost (and time) of calculation in favour of absolute precision.

WORK IN PROGRESS TO PROVIDE FIGURES
[picture of the constant case]
[picture of the multi-volatility case]

5.2 Results for Perpetual Turbo Warrants

5.2.1 Preliminary empirical results

Using pricing formulas in Equation 22 and Equation 24, the price of the perpetual turbo warrant is obtained for the case of Trencent Holdings:

From our simulation shown in **Figure 4** we found that:

1. The perpetual Turbo warrant follows the fluctuation of the underlying asset

2. The closer the underlying asset's price gets to the barrier, the closer the price of the PTW gets to the normal one.

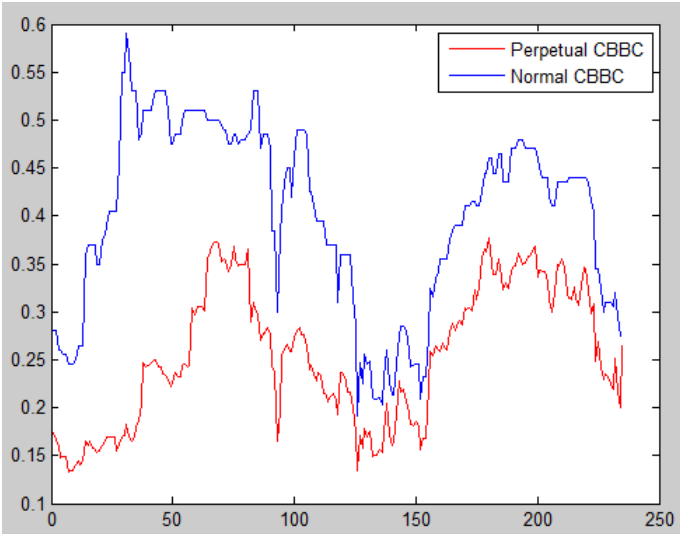

Figure 4: Perpetual and normal Turbo prices in HKD

Let us explain some of the results, the closer the underlying asset's gets to the barrier, the higher the probability to hit it gets, and the prices of finite and perpetual turbo warrant become the closest because the down and out part loses value and the DIL part gains value. Since the DOC part is the essential difference between the two instruments, their prices become closer when the DOC part loses value.

The American aspect of the turbo makes it more identical to the fluctuation of the underlying asset. The fact that the perpetual CBBC is represented cheaper than the normal one may be a source of doubt of the results, we actually think that it is only due to a difference in the entitlement ratio which is the number of warrant necessary to buy one share. It is 100 for the Trencent Holdings normal turbo warrant, we believe that if there was a perpetual turbo on Trencent Holdings it would have a smaller Entitlement Ratio.

5.2.2 Numerical results: sensitivité analysis

Work in progres

5.2.3 Application to real historical data on perpetual turbo warrants

Work in progres

6 Conclusion

In this article, we analysed the pricing of turbo warrants under GBM model and we came up with a solution to price the infinite Turbo warrant by finding an appropriate replicating portfolio and treating each element apart. In the case of the finite maturity turbo, results are compared with real data and we were satisfied with them. In the infinite maturity case, we compared finite and infinite Turbos and tried to explain the differences. We still have to validate results but no real data has been found. This article is one of a few (if not the first one) to treat the pricing of infinite Turbo warrant, it can be a start to further research.

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