The Role of Nonhomothetic Preferences and Rent Sharing for Trade Structure and Welfare in an Open Economy*

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Abstract

We develop a framework for studying how differences in the level and dispersion of per-capita income affect trade structure and welfare in a two-country model. Thereby, we assume that consumers have PIGL preferences, which admit a representative consumer and thus facilitate the aggregation of heterogeneous consumer demand even if Engel curves are not linear. We embed the resulting demand system into a textbook model of the home-market effect, with two sectors of production and one input factor. We associate the basic outside good with a necessity and the sophisticated differentiated good with a luxury, and we assume that heterogeneity of income arises due to heterogeneity of households in their effective labor supply. Using this model, we show that in line with the home-market effect countries have a trade surplus in the good for which they have relatively higher domestic demand, making the country with a higher level and dispersion of per-capita income a net exporter of the sophisticated good. However, the structure of trade is irrelevant for welfare in the open economy. This changes when considering rent sharing in the sophisticated goods sector, and we show that in this case feedback effects of trade on the level and dispersion of income can lead to losses from trade in the country exporting the basic good.

Keywords: Nonhomothetic preferences; Rent sharing; Trade structure; Welfare effects of trade

JEL Classification: F12, F15, C33

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1 Introduction

In recent years, economists have put nonhomothetic preferences back on the research agenda in order to explore demand-side explanations for phenomena usually attributed to supply-side effects. A research area, in which the consideration of nonhomothetic preferences has turned out to be particularly promising, is the field of international trade, where differences in the level and dispersion of income have found to be empirically important determinants of the international exchange of goods (see, for instance, Bergstrand, 1989; Hunter, 1991; Francois and Kaplan, 1996; Caron et al., 2014).[1] From a theory point of view, analyzing demand-side factors can be challenging of course, if not only the level but also the dispersion of income matters for expenditures. This follows from the important insight that the dispersion of income is relevant for aggregate expenditures only if preferences do not have Gorman form, which in turn can make the aggregation of individual demand functions an onerous task. However, one can avoid the aggregation problem even in the case of non-linear Engel curves, when relying on a class of preferences put forward by Muellbauer (1975, 1976). Muellbauer (1975) introduces the term "price-independent generalized-linear" (PIGL) preferences to refer to this class, which (though more general) shares an important feature with the Gorman class that is particularly attractive for the aggregation of consumer demand. PIGL preferences admit a (positive) representative consumer, who is characterized by an expenditure level for which the value (expenditure) shares of consumption equal the value shares of the aggregate economy.[2]

In this paper, we employ PIGL preferences to shed light on how differences in the level and dispersion of per-capita income affect trade structure and welfare, and we analyze how feedback effects of trade on income distribution due to rent sharing between firms and workers influence the expenditure structure, the pattern of specialization, and welfare in the open economy. To give local demand and thus differences in expenditure levels a role, we set up a two-country model along the lines of Helpman and Krugman (1985) that features a home-market effect. We thereby employ the structure proposed by these authors and consider two sectors of production and a single factor input (labor). One of the two sectors produces a homogeneous good with a linear technology under perfect competition. Production in the other sector is subject to increasing returns to scale and delivers differentiated goods that are sold under monopolistic competition. Adding the additional assumption that the homogeneous good is freely traded, while exports of differentiated goods are subject to iceberg trade costs, then gives the textbook model of home-market effect usually considered in the literature – with the mere difference that we allow for heterogeneous producers of the differentiated goods and account for fixed as well as variable costs of exporting, as suggested by Melitz (2003).[3]

[1]Nonhomothetic preferences have also sparked interest in recent macroeconomic research to explain, for instance, how distributions of income and wealth are linked to economic growth (Foellmi and Zweimüller, 2006) and how economic growth induces structural change by increasing (dispersed) household income (Boppart, 2014).

[2]Only if the thus defined expenditure level corresponds to the mean of expenditures, PIGL preferences have Gorman form. Therefore, PIGL preferences are more general than the Gorman class and, as put forward by Muellbauer (1975), they are actually the most general class of preferences that admits a well-defined representative consumer.

[3]The home-market effect has strong empirical support (see, for instance, Davis and Weinstein, 1999, 2003; Head and Ries, 2001), although details of the model, such as the assumption of zero trade costs of the outside good, seem to be more controversial (cf. Davis, 1998). Crozet and Trionfetti (2008) give an overview of the respective literature.

To the extent that PIGL preferences do not have Gorman form, we can give the two goods a natural interpretation from consumer theory and associate the homogeneous good with a basic necessity and the differentiated good with a sophisticated luxury (in the spirit of Francois and Kaplan, 1996). This implies that the Engel curve of the former is concave, whereas the Engel curve of the latter is convex in the expenditure level. Furthermore, to make our results directly comparable with findings from other models featuring a home-market effect, we consider a subgroup of PIGL preferences that contains the widely used Cobb-Douglas preferences as a special case. Except for the Cobb-Douglas case, aggregate expenditures for the consumption of sophisticated goods depend on the level and dispersion of per-capita income. Dispersion of income exists in our model due to differences in the (effective) labor supply of households, and we show that the impact of these differences on market demand is captured by a dispersion measure that is a negative monotonic transformation of the well-known Atkinson index and therefore carries a nice economic interpretation.

Due to the described non-linearity of Engel curves in the case of PIGL preferences, demand for sophisticated goods is larger in the country that features a higher level and dispersion of per-capita income, which, following the reasoning from the literature on home-market effects, is the country that has a trade surplus of sophisticated goods in the open economy. Larger differences of countries in their expenditure structure lead to a stronger specialization on the production of goods for which a country has relatively higher local demand. This raises inter- and reduces intra-industry trade. Therefore, the model considered here is consistent with Linder's (1961) hypothesis that more equal per-capita income levels of two economies provide larger scope for (intra-industry) trade in those goods, for which local demand is an important determinant of production.[4] As put forward by Davis (1998), the home-market effect is more pronounced at lower trade costs, making intra-industry trade less important if the two economies become more integrated. In the limiting case of Cobb-Douglas preferences expenditures for basic and sophisticated goods are independent of personal income levels, and hence there is no inter-industry trade, provided that the two countries have the same market size. Relying on a utilitarian social welfare function, there are gains from trade in this model, which are independent of the trade structure in the open economy and thus the same for the two economies.

We study the robustness of our finding that welfare effects are invariant to changes in the trade structure by introducing rent sharing in the sophisticated goods sector. Assuming that rent sharing only exists in one sector acknowledges the rich evidence on (persistent) inter-industry pay gaps (see Krueger and Summers, 1988; Blanchflower et al., 1996; Katz and Autor, 1999), whereas associating the sector featuring rent sharing with the sophisticated goods industry captures the widespread view that employer characteristics are important determinants of these pay gaps (see Dickens and Katz, 1987; Abowd et al., 2012). We associate rent sharing with collective bargaining between firms and firm-level unions and assume that bargaining is efficient and, for given aggregates, does not influence the size of production

[4] Empirical evidence in favor of the Linder (1961) hypothesis has been reported, for instance, by Thursby and Thursby (1987), Bergstrand (1989, 1990), and Hallak (2010). Francois and Kaplan (1996), Dalgin et al. (2008), Bernasconi (2013), and Vollmer and Martínez-Zarzoso (2016) show that bilateral trade is not only affected by differences in the level of per-capita income but also by differences of the two trading partners in their income distributions.

surplus. This allows us to decouple effects associated with the distribution of production surplus (which are of interest here) from efficiency losses associated with a reduction in the size of production surplus due to lower employment of unionized than non-unionized firms. Rent sharing induces firms in the sophisticated goods sector to pay a union wage premium, which makes the level and dispersion of per-capita income endogenous in our model. To be more specific, the level and dispersion of per-capita income depend on the share of workers employed in the production of sophisticated goods and they are no longer independent of each other. For instance, a higher ex ante dispersion of (effective) labor supply leads to a higher ex post level of per-capita income, because a higher income dispersion generates a larger market for sophisticated goods, implying that a larger fraction of workers benefits from the union wage premium. In the open economy, rent sharing produces a feedback effect of trade on the endogenous level and dispersion of per-capita income with notable consequences for the effects of trade in our model. Whereas the existence of feedback effects on the level and dispersion of per-capita income leaves our insights regarding the trade structure unaffected, it generates an asymmetry in the welfare effects of trade and makes losses from trade possible for the country specializing in the production of the basic good.

Emphasizing the role of differences in the level and dispersion of per-capita income, our model contributes to a growing literature that points to demand-side explanations for observed patterns of international trade. Based on the Linder (1961) hypothesis, Markusen (1986) has developed a first theoretical model to explain the impact of per-capita income differences on the structure of international trade in a setting with intra- and inter-industry trade. Bergstrand (1990) and Markusen (2013) extend the analysis and show that adding nonhomothetic preferences makes otherwise traditional trade models better suited to accord with empirical evidence.[5] In a recent contribution, Simonovska (2015) uses nonhomothetic preferences to explain the positive relationship between prices of tradable goods and per-capita income. Considering Stone-Geary utility functions with linear Engel curves, these models rely on preferences, which are nonhomothetic but still have Gorman form. Hence, market demand is independent of the distribution of income and an aggregation problem does not exist.

Stockey (1991) considers a fairly general structure of nonhomothetic preferences in a setting with vertically differentiated products to shed light on the trade structure between rich and poor countries and to explain empirical evidence on product cycles, which suggests that new, high quality products are first consumed in rich countries and only at later stages also consumed in poor countries. Fieler (2011) considers preferences that do not have Gorman form to explain the role of per-capita income for trade structure in a multi-country Ricardian model along the lines of Eaton and Kortum (2002), and she uses this model to show that a technology shock in China has different effects on countries with differing per-capita income levels. Caron et al. (2014) employ the same preference structure to improve the predictions of the Heckscher-Ohlin Vanek model regarding the factor content of trade and show that their correction is quantitatively important.[6] Foellmi et al. (2018) consider a model with nonhomothetic preferences and

[5]Bergstrand (1989) shows how the gravity equation has to be adjusted in order to account for differences in per-capita income along with differences in factor endowments as key determinants of bilateral trade. Hunter (1991) provides early empirical evidence that accounting for per-capita income differences may explain missing trade in empirical work based on the Heckscher-Ohlin models.

[6]Both Fieler (2011) and Caron et al. (2014) build on a generalized CES preference structure, in which the demand elas-

consumption indivisibilities (captured by purchases of 0 or 1 units of differentiated goods) to shed light on the role of per-capita income differences as a determinant of 'export zeros' observed in the world trade matrix. In all of these models, income distribution would matter for aggregate demand. However, utilizing the assumption of symmetric households the authors do not address the role of income distribution for the trade patterns and thus avoid problems associated with the aggregation of heterogeneous consumer demand.

Matsuyama (2000) imposes nonhomothetic '0-1' preferences into a Ricardian model of North-South trade with a continuum of goods and shows that acknowledging the nonhomotheticity of preferences changes the insights from an otherwise identical Dornbusch et al. (1977) model regarding the role of technological advancement, population growth, and income redistribution in the South on the terms-of-trade and welfare in the two economies. Fajgelbaum et al. (2011) build on the preference structure proposed by Flam and Helpman (1987) and assume that households purchase one unit of a vertically differentiated good and allocate the rest of their expenditures on the consumption of a homogeneous outside good. Assuming that quality of the differentiated good and quantity of the homogeneous good are complements makes their preferences nonhomothetic, because the impact of income on indirect utility depends on the chosen quality of the differentiated good. To allow for monopolistic competition between firms producing horizontally differentiated varieties of the same quality level, Fajgelbaum et al. (2011) augment their discrete choice problem with a stochastic utility term (similar to McFadden, 1978), and they use this framework to provide a reasoning for the empirical observation that richer countries export goods of higher quality (see Hallak, 2010). Accounting for income differences while at the same time considering preferences that do not have Gorman form, these contributions face the problem of aggregating heterogeneous household demand, and they solve this problem by making consumer choices discrete and assuming that households do not purchase more than one unit of the available products. Employing the class of PIGL preferences introduced by Muellbauer (1975, 1976) we aggregate heterogeneous household demand relying on a (positive) representative consumer. As a result, we do not need to restrict the chosen consumption bundle beyond the usual requirement that it must be affordable under the household's budget constraint, and we therefore complement previous work on how differences in the level and dispersion of per-capita income shape trade in an open economy, by emphasizing the role of personal income for household expenditures through its impact on the intensive margin of consumption.

Furthermore, employing a rent-sharing mechanism gives so far unexplored feedback effects of trade on the level and dispersion of per-capita income, and we show that these effects, while irrelevant for the trade structure in the open economy, are important for understanding the welfare effects of trade. Addressing such feedback effects relates our analysis to a sizable literature dealing with the distributional consequences of trade in models featuring labor market imperfection. An important strand of this literature has used the idea of rent sharing to link wages to profits with the purpose to make them firm-specific

ticities of income and prices are constant and proportional (as suggested by Pigou's Law). Matsuyama (2017) considers even more general isoelastically nonhomothetic CES preferences, which allow to decouple the effects generated by income elasticity differences and those generated by price elasticity differences, and he uses this framework to show how trade liberalization and economic growth affect the patterns of structural change, innovation, and trade in the presence of Engel's Law.

in settings of heterogeneous producers. These models are employed for explaining how asymmetric effects of trade on firms translate into asymmetric effects on workers. Examples from this literature include Egger and Kreickemeier (2009, 2012), Helpman et al. (2010), and Amiti and Davis (2012). Other models do not produce firm-specific wages, despite a heterogeneity of producers. Combining the assumption of isoelastic demand for goods with the assumption of Nash bargaining between firms and workers, Eckel and Egger (2009) find in line with our results that wages in the Melitz (2003) model are the same for all producers if firms and workers engage in collective bargaining. Helpman and Itskhoki (2010), Felbermayr and Prat (2011), and Felbermayr et al. (2011) show that this result extends to models of individual bargaining with search frictions. To abstract from involuntary unemployment (which would unnecessarily complicate our analysis), we follow Bastos and Kreickemeier (2009) and assume that workers who do not find a job in the sophisticated goods industry are employed in the production of basic goods at the market-clearing wage. Finally, considering an efficient bargaining framework in the context of international trade, our analysis is related to Mezzetti and Dinopoulos (1991) and Blanchard and Giavazzi (2003).

The remainder of the paper is organized as follows. In Section 2, we set up the basic structure of our model and discuss the closed economy equilibrium under the assumption that both the basic and the sophisticated goods sector pay the same, market-clearing wage. In Section 3, we consider trade between two countries that are symmetric in all respects, except for the level and dispersion of per-capita income. There, we also discuss how differences in the level and dispersion of per-capita income affect the trade structure and welfare in the open economy. In Section 4, we introduce rent sharing between firms and unions in the sophisticated goods sector and show how feedback effects of trade on the level and dispersion of per-capita income affect the trade structure and the welfare effects of trade in our model. Section 5 concludes with a summary of our results.

2 The closed economy

We consider a static economy that is populated by an exogenous mass of H (single person) households. Each household inelastically supplies a units of labor input in a competitive labor market. Households differ in their supply of labor (for instance, due to differences in abilities), leading to different levels of income and heterogeneity in consumption expenditures. Assuming that preferences do not have Gorman form, the distribution of consumption expenditures is instrumental for the aggregate demand of two types of goods: basic goods, G, which are homogeneous, and sophisticated goods, S, which are differentiated. Workers are mobile between the sectors producing basic and sophisticated goods.

2.1 Preferences and household demand

To establish a link between the distribution of consumption expenditures and aggregate demand, we deviate from the widely used class of Gorman type preferences, for which (except for uninteresting corner solutions) such a link does not exist, and consider instead the more general class of price-independent generalized linear (so-called "PIGL") preferences introduced by Muellbauer (1975, 1976). These preferences can be represented by an indirect utility function of the following form

$$v(\mathbf{P}, e^i) = \frac{1}{\varepsilon}\left[\frac{e^i}{a(\mathbf{P})}\right]^{\varepsilon} + b(\mathbf{P}), \tag{1}$$

where $\mathbf{P}$ is a price vector, e^i is expenditure of household i and ε is a constant. As pointed out by Boppart (2014), these preferences give the most general class of utility functions that avoid an aggregation problem, because there exists a representative expenditure level such that a household with this expenditure level has the same value (or expenditure) shares of consumption as the aggregate economy.[7] For $\varepsilon = 1$ the indirect utility function has Gorman form and under the additional assumption that $b(\cdot)$ is independent of the price vector $\mathbf{P}$ preferences are homothetic. We consider a subclass of PIGL preferences and assume that households have preferences over sophisticated goods, X_S, and basic goods, X_G, which are represented by an indirect utility function of the following form:

$$v(P_G, P_S, e^i) = \frac{1}{\varepsilon}\left(\frac{e^i}{P_S}\right)^{\varepsilon} - \frac{\beta}{\varepsilon}\left(\frac{P_G}{P_S}\right)^{\varepsilon}, \tag{2}$$

where P_S, P_G are prices of sophisticated and basic goods, respectively, and $\varepsilon \in [0, 1)$, $\beta > 0$ is assumed. As explained by Boppart (2014) and formally shown in the appendix, in contrast to more general forms of PIGL preferences, the preferences considered here allow for a closed form representation of the direct utility function.

Applying Roy's identity to indirect utility function (2), we can derive Marshallian demand functions for X_G^i and X_S^i, according to

$$X_G^i = \beta\left(\frac{e^i}{P_G}\right)^{1-\varepsilon} \quad \text{and} \quad X_S^i = \frac{e^i}{P_S}\left[1 - \beta\left(\frac{e^i}{P_G}\right)^{-\varepsilon}\right], \tag{3}$$

respectively. In the limiting case of $\varepsilon = 0$ preferences in Eq (2) are Cobb-Douglas and Engel curves are linear in the expenditure level.[8] If $\varepsilon > 0$, preferences do not have Gorman form. Then, the Engel curve of the basic good is concave making this good a necessity, with its value share of consumption decreasing in expenditure level. In contrast, the Engel curve for the sophisticated good is convex making this good a luxury, with its value share of consumption increasing in expenditure level. Furthermore, in order to

[7] The term of generalized linearity has been introduced by Muellbauer (1975) to emphasize that the preferences are more general than the Gorman class which features consumption levels that are linear in expenditures, rendering (marginal) value shares of consumption independent of the overall expenditure level. This property does not extend to other preference classes. However, generalized linear preferences accord with the weaker condition that the *ratio* of marginal value shares of any two goods are independent of the overall expenditure level. The notion of price independency is used by Muellbauer (1975) to express that the representative expenditure level for which an individual household chooses the same value shares of consumption as the aggregate economy is the same for all permissible prices.

[8] In the limiting case of $\varepsilon = 0$ the preferences in Eq.(1) are PIGLOG, producing value shares of consumption that are (affine-)linear in the logarithm of expenditures (see Pollak and Wales, 1992, for a discussion). For the subclass of PIGL preferences in Eq. (2) the limiting case of $\varepsilon = 0$ establishes an indirect utility function of the form $v(P_G, P_S, e^i) = \ln\left[e^i/\left(P_G^{\beta} P_S^{1-\beta}\right)\right]$, which corresponds to the case of an indirect (log-transformed) Cobb-Douglas utility function.

ensure that both goods are purchased by household i, it must be true that $e^i/P_G > \beta^{1/\varepsilon}$ and we impose a parameter constraint below that establishes the intended result that all households purchase basic as well as sophisticated goods.

The finding that Engel curves for basic and sophisticated goods are differently shaped is the result of assuming that the respective goods enter the utility function asymmetrically. This asymmetry is justified in our model, because we assume that basic goods are homogeneous, whereas sophisticated goods are differentiated and can be aggregated to the composite discussed above according to

$$X_S^i = \left[\int_{\omega\in\Omega} x_S^i(\omega)^{\frac{\sigma-1}{\sigma}} d\omega\right]^{\frac{\sigma}{\sigma-1}}, \tag{4}$$

where $\sigma > 1$ is the constant elasticity of substitution between the varieties of sophisticated goods from set Ω. The price corresponding to the composite X_S^i is an index of the prices of differentiated varieties, $p_S(\omega)$, and it is defined by the condition that $P_S X_S^i$ is equal to the household's overall expenditures for sophisticated goods, $\int_{\omega\in\Omega} p_S(\omega)x_S^i(\omega)d\omega$. As formally shown in the appendix, the respective price index features constant elasticity and is given by $P_S \equiv \left[\int_{\omega\in\Omega} p_S(\omega)^{1-\sigma} d\omega\right]^{\frac{1}{1-\sigma}}$. Using Roy's identity, we can then derive individual demand for a single variety of the sophisticated good, ω, according to

$$x_S^i(\omega) = \frac{e^i}{P_S}\left(\frac{p_S(\omega)}{P_S}\right)^{-\sigma}\left[1-\beta\left(\frac{e^i}{P_G}\right)^{-\varepsilon}\right]. \tag{5}$$

Aggregating over all households, gives market demand functions

$$X_G = \int_0^H X_G^i di = \beta\frac{H\bar{e}}{P_G}\left(\frac{\bar{e}}{P_G}\right)^{-\varepsilon}\psi, \tag{6}$$

$$x_S(\omega) = \int_0^H x_S^i(\omega)di = \frac{H\bar{e}}{P_S}\left(\frac{p_S(\omega)}{P_S}\right)^{-\sigma}\left[1-\beta\left(\frac{\bar{e}}{P_G}\right)^{-\varepsilon}\psi\right], \tag{7}$$

where $\bar{e} \equiv \frac{1}{H}\int_0^H e^i di$ is the average expenditure level and $\psi \equiv \frac{1}{H}\int_0^H \left(\frac{e^i}{\bar{e}}\right)^{1-\varepsilon} di$ is a measure of economy-wide expenditure dispersion. The dispersion measure lies between 0 and 1 and has an intuitive economic interpretation, because it is a negative monotonic transformation of the well-known Atkinson index, which can be expressed as $A = 1 - \psi^{\frac{1}{1-\varepsilon}}$ (cf. Atkinson, 1970). The Atkinson index captures inequality aversion and in the context of social welfare it allows to infer the gain to be achieved from a redistribution scheme that makes income (or in our case expenditure) levels identical. Accordingly, higher levels of ψ indicate either lower inequality or a lower social aversion against inequality, and the limiting case of $\psi = 1$ is reached if either all households have the same level of expenditures or if $\varepsilon = 0$ makes preferences homothetic and thus expenditure differences irrelevant for aggregate demand.

The PIGL preferences in Eq. (2) are particularly attractive for our purposes, because the concavity of ψ allows for a meaningful ranking of expenditure distributions. As formally shown by Atkinson (1970), if there are two expenditure distributions, $F(e)$ and $F^*(e)$ differing by a mean-preserving spread, $F(e)$

corresponds to a higher level of ψ than $F^*(e)$ if F second-order stochastically dominates F^*. From Eqs. (6) and (7) we can then conclude that for a given mean of expenditures, $\bar{e}$, aggregate demand for basic (necessity) goods is higher and aggregate demand for sophisticated (luxury) goods is lower if expenditures are less dispersed. This is a consequence of the shape of the Engel curves. Since the Engel curve for basic goods is concave, redistributing income from households with high expenditures to households with low expenditures increases average demand for necessities. The opposite is true for sophisticated goods, featuring convex Engel curves. The permissible extent of inequality is limited in our model by the requirement that all households, including those with the lowest expenditure level, purchase both types of goods (see below).

2.2 Endowments, technology, and production

We assume that labor is the only production input and supplied by workers in a competitive labor market at the common wage rate w per efficiency unit of labor. Workers differ in their abilities, a, and thus in the efficiency units of labor provided to the firm. We assume that abilities are distributed among workers over interval $[\underline{a}, \overline{a}]$ according to a continuously differentiable cumulative distribution function $H(a)$: $H = \int_{\underline{a}}^{\overline{a}} dH(a)$. Income of a worker endowed with a units of labor is then given by aw. With a common wage rate per efficiency unit of labor for all workers and costless hiring, firms are indifferent between employing workers with high or low levels of a, and the average labor income of workers is given by λw, where $\lambda \equiv \int_{\underline{a}}^{\overline{a}} a dH(a)$ is average labor endowment of workers and λH therefore the economy-wide supply of labor.

The technology in the basic goods sector is linear in labor input and we assume that one efficiency unit of labor produces one unit of output: $L_G = Q_G$, where L_G is total labor input in efficiency units and Q_G is (total) output in sector G. Due to perfect competition, the price of the basic good is then linked to the wage per efficiency unit by $P_G = w$. Output in the sophisticated goods industry is also linear in labor input and given by $q_S(\omega) = \varphi(\omega) l_S(\omega)$, where $q_S(\omega)$, $l_S(\omega)$ are production output and labor input of a firm producing variety ω and $\varphi(\omega)$ is a firm-specific productivity parameter. To start production, firms in the sophisticated goods sector must invest f units of the basic good and after this investment they produce under monopolistic competition, with each firm supplying a unique variety (as a result of price competition and the existence of fixed costs). Facing market demand (7), they set their prices as a markup over marginal production costs, $p_S(\omega) = [\sigma/(\sigma - 1)]w/\varphi(\omega)$. Hence, despite the assumption of PIGL preferences price markups are constant and the same for all producers in our model, similar to the seminal contributions of Krugman (1980) and Melitz (2003) who consider homothetic utility functions.

2.3 Firm entry and general equilibrium

Whereas entry of firms into the sector of basic goods is free, we follow Melitz (2003) and assume that entry into the sector of sophisticated goods is costly and associated with a two-stage problem. At stage one, firms make an investment of f_e efficiency units of the basic good into a lottery, which allows them to draw a productivity level $\varphi(\omega)$ from a common distribution $G(\varphi)$. The investment into the lottery allows for a single draw and is immediately sunk. Conditional on their productivity level, firms then decide in a second stage on whether to start production or not. Production is attractive for a firm if its productivity draw promises non-negative profits. Denoting by $r(\omega) = p_S(\omega)q_S(\omega)$ the revenues of firm ω, the maximum attainable profits of the firm under constant markup pricing are given by $\pi(\omega) = r(\omega)/\sigma - P_G f$, and paying the additional fixed cost necessary to start production, $P_G f$, is attractive for firm ω if $\pi(\omega) \geq 0$. Since revenues increase with productivity, this establishes an indifference condition separating active from inactive firms. The revenue that renders a firm indifferent between production and non-production is denoted by r_d and constant in terms of labor efficiency units: $r_d/P_G = \sigma f$. Firms with a productivity higher than that of the indifferent firm, denoted φ_d, make positive profits when choosing to produce, because with constant markup pricing the revenue ratio of two firms increases in their productivity ratio with constant elasticity $\sigma - 1$: $r(\omega)/r_d = [\varphi(\omega)/\varphi_d]^{\sigma-1}$. Combining $r_d/P_G = \sigma f$ with the restriction imposed by profit maximization that (risk-neutral) firms choose to invest into the lottery if and only if the expected return on the investment is non-negative, i.e. if $\int_{\varphi_d}^{\infty} \pi(\varphi) dG(\varphi) \geq P_G f_e$, we get a condition that renders potential entrants indifferent between making and not making the investment into the productivity lottery:

$$f \left\{ \int_{\varphi_d}^{\infty} \left(\frac{\varphi}{\varphi_d} \right)^{\sigma-1} dG(\varphi) - [1 - G(\varphi_d)] \right\} = f_e \tag{8}$$

This free entry condition implicitly determines cutoff productivity φ_d and establishes the well-known (though somewhat peculiar) result that under iso-elastic demand the two-stage entry process put forward by Melitz (2003) makes r_d/P_G and φ_d independent of the general equilibrium outcome for economy-wide aggregates. Since relative to the least productive firm, all firm-level variables can be expressed as functions of the relative productivity ratio $\varphi(\omega)/\varphi_d$, we can omit ω from now on and index all firms by their productivity level.

Profit income generated by active producers of sophisticated goods is used to pay for the lottery fixed costs of successful and unsuccessful entrants. Thus, provided that firm ownership is equally shared by households and provided that there is no redistribution of income, individual consumption expenditure equals labor income, $e^i = wa^i$, and it is heterogeneous due to an exogenous heterogeneity of abilities. The workers achieving the lowest income has an ability level of $\underline{a}$ and an expenditure level of $\underline{a}w$. Due to $w = P_G$, the minimum expenditure level necessary for the consumption of both goods then establishes a lower bound of abilities, $\underline{a} > \beta^{1/\varepsilon}$, which is assumed to be fulfilled throughout our analysis in order to ensure that the value share of consumption attributed to sophisticated goods is positive for all households. With this insight at hand, we can then determine the mass of firms producing sophisticated goods in general equilibrium, M, by applying the goods market clearing condition derived from Eq. (7):

$$H\bar{e} \left[1 - \beta \left(\frac{\bar{e}}{P_G} \right)^{-\varepsilon} \psi_a \right] = M r(\varphi_d) \int_{\varphi_d}^{\infty} \left(\frac{\varphi}{\varphi_d} \right)^{\sigma-1} \frac{dG(\varphi)}{1 - G(\varphi_d)}, \tag{9}$$

where $r(\varphi_d) = r_d$ has been used. Accounting for $r(\varphi_d)/w = \sigma f$, $\bar{e} = w\lambda$ and substituting the free entry condition (8), then establishes

$$M = \frac{H\lambda[1-\beta\lambda^{-\varepsilon}\psi_a]}{\sigma f}\left[1+\frac{f_e/f}{1-G(\varphi_d)}\right]^{-1}, \tag{10}$$

where $\psi_a \equiv \int_{\underline{a}}^{\overline{a}} \left(\frac{a}{\lambda}\right)^{1-\varepsilon} dH(a)$ is an inverse measure of ability dispersion, which in the case of a competitive labor market equals the transformed Atkinson index measuring expenditure dispersion: $\psi = \psi_a$. The share of workers employed in the sophisticated goods industry, h_S, is linked to the mass of firms producing there and given by

$$h_S = \frac{Mr(\varphi_d)}{H\lambda w}\frac{\sigma-1}{\sigma}\int_{\varphi_d}^{\infty}\left(\frac{\varphi}{\varphi_d}\right)^{\sigma-1}\frac{dG(\varphi)}{1-G(\varphi_d)} = \frac{\sigma-1}{\sigma}\left[1-\beta\lambda^{-\varepsilon}\psi_a\right], \tag{11}$$

where the second equality sign follows from substituting $r(\varphi_d) = \sigma P_G f$ and Eq. (10). Using Eq. (10) and the constant markup rule $p_s(\varphi) = [\sigma/(\sigma-1)]w/\varphi$ in the definition of the price index for sophisticated goods, we can further compute the price index according to

$$P_S = \frac{w}{\varphi_d}\frac{\sigma}{\sigma-1}\left\{\frac{H\lambda[1-\beta\lambda^{-\varepsilon}\psi_a]}{\sigma f}\right\}^{\frac{1}{1-\sigma}}, \tag{12}$$

where the bracket expression gives an *augmented* mass of varieties of sophisticated goods that would be necessary to achieve price index P_S under the assumption that all of these varieties are sold at the price charged by the least productive firm, $p_S(\varphi_d)$. Since on average firms have higher productivity than φ_d, the average price charged by producers is lower than $p_S(\varphi_d)$ and the augmented mass of varieties is therefore larger than the mass of varieties available in the market. According to Eq. (12), the price index for sophisticated goods decreases with a higher cutoff productivity level and a higher augmented mass of available varieties. For $\varepsilon > 0$, the latter depends on the level and dispersion of per-capita income. Both higher per-capita income and higher income dispersion increase expenditures for sophisticated goods, causing firm entry, because wealthier households increase their demand for luxuries if preferences do not have Gorman form.

For our welfare analysis, we follow Atkinson (1970) and associate social welfare with an additively separable and symmetric function of individual income levels. More specifically, we postulate a Bergson-Samuelson social welfare function that is equal to average indirect utilities of households. Accounting for Eqs. (2) and (12), we obtain

$$V(P_G, P_S, \overline{e}, \hat{\psi}) \equiv \frac{1}{\varepsilon}\left(\frac{P_G}{P_S}\right)^{\varepsilon}\left[\left(\frac{\overline{e}}{P_G}\right)^{\varepsilon}\hat{\psi} - \beta\right], \tag{13}$$

where $\overline{e} = \lambda w$ and $\hat{\psi} \equiv \frac{1}{H}\int_0^H \left(\frac{e^i}{\overline{e}}\right)^{\varepsilon}$ is an income dispersion index different from ψ, which equals the index of ability dispersion $\hat{\psi}_a \equiv \int_{\underline{a}}^{\overline{a}} \left(\frac{a}{\lambda}\right)^{\varepsilon} dH(a)$, due to our assumption of a competitive labor market. Giving equal weight to all households, we take a utilitarian perspective. Social welfare under this perspective is different, however, to the welfare achieved by the household with a representative expenditure level of $e_r \equiv \bar{e}\psi^{-1/\varepsilon}$. As pointed out above, the (price-invariant) representative level of expenditure is defined by Muellbauer (1975) to ensure that an individual household with this expenditure level has the same value shares of consumption as the aggregate economy. This establishes a *representative consumer* under PIGL preferences whose interpretation is not too different from the positive representative consumer in the Gorman class of preferences. As explained in detail by Muellbauer (1976), the representative household thus defined lacks, however, a normative interpretation from welfare functions based on ethical judgements, such as the utilitarian welfare function considered in Eq. (13).

From a utilitarian perspective, the market outcome is not socially optimal for two reasons. With concave indirect utility (due to $\varepsilon > 0$), households have inequality-aversion and, hence, a social planner can increase welfare through Dalton's (1920) principle of transfer, which states that with concave utility functions (and symmetry of households in the perception of the social planner) a transfer from a wealthier individual to a poorer one that does not change their income ranking reduces inequality and increases social welfare. This effect can be seen in Eq. (13), when acknowledging that a mean-preserving spread of income (or expenditures) lowers $\hat{\psi}$ and thus social welfare $V(\cdot)$. The incentive of the social planner to harmonize income is counteracted, however, by a distortion of the resource allocation arising because households devote part of their expenditures to basic goods, which makes, all other things equal, the number of firms entering the lottery too small from a social planner's point of view. As pointed out by Dhingra and Morrow (2016) this allocational inefficiency exists because the markups charged in the two industries differ. Introducing a transfer from poor to rich people would increase demand for sophisticated goods and therefore provide a (partial) remedy for the misallocation of resources.

In the limiting case of $\varepsilon = 0$ there is no inequality aversion, leaving the misallocation of resources due to distorted entry as the only source of inefficiency. In this case, the social planner can increase welfare by making entry into the sector of sophisticated goods more attractive (cf. Benassy, 1996). Things are more complicated if $\varepsilon > 0$, because it is not clear a priori, which of the two counteracting effects of higher inequality dominates in this case. To gain further insights into the relative strength of the counteracting effects, we can evaluate the social welfare function $V(\cdot)$ at $\varepsilon = 1/2$, which establishes $\psi = \hat{\psi}$. This specific case is of interest here, because for a given mean, changes in the distribution of income have an effect on social welfare only through changes in dispersion indices ψ and $\hat{\psi}$ – which are the same by construction if $\varepsilon = 1/2$. The social welfare effects of lower income inequality (a higher ψ) are then given by

$$\frac{dV(P_G, P_S, \bar{e}, \psi)}{d\psi} \equiv \sqrt{\frac{P_G}{P_S}\frac{\lambda}{(\sigma-1)^2}}\left[2\sigma - 1 - \frac{1-\left(\beta/\sqrt{\lambda}\right)^2}{1-(\beta/\sqrt{\lambda})\psi}\right]. \tag{14}$$

From Eq. (14), positive welfare effects of lower income inequality are more likely ceteris paribus if σ is larger.[9] This is intuitive, because higher levels of σ reduce the price markups charged by monopo-

[9]Evaluated at $\sigma = 1$, Eq. (14) establishes $dV(P_G, P_S, \lambda, \psi)/d\psi >, =, < 0$ if $\beta >, =, < \lambda^{1/2}\int_{\underline{a}}^{\overline{a}}(a/\lambda)^{1/2}dH(a)$. Evaluating condition $\underline{a} > \beta^{1/\varepsilon}$ at $\varepsilon = 1/2$, establishes $\beta < \int_{\underline{a}}^{\overline{a}} a^{1/2}dH(a)$ and thus $dV(P_G, P_S, \lambda, \psi)/d\psi < 0$ in the limiting case

listically competitive firms in the sophisticated goods industries, which lowers the problem of resource misallocation due to distorted market entry. Also, lower income inequality increases welfare if β is sufficiently small. In the limiting case of $\beta = 0$ the model degenerates to a one-sector economy, in which only the sophisticated goods sector is active and thus aggregate demand is independent of the distribution of income. Whereas changes in the distribution of income exert counteracting effects on social welfare if $\beta, \varepsilon > 0$, a higher mean of income for given dispersion indices (for instance, due to a proportional increase of all abilities) unambiguously increases welfare.

3 The open economy

In the open economy, we consider trade between two countries that are symmetric in all respects except for the level as well as the dispersion of consumption expenditures. Trade in basic goods is free of costs, and hence wages per efficiency unit of labor, w, are the same in both countries, provided that production is diversified in both countries. In contrast, trade in sophisticated goods is subject to iceberg trade costs, implying that $\tau > 1$ units of the good must be shipped in order for one unit to arrive in the foreign country. In addition, exporting is subject to a fixed cost and requires the investment of f_x units of basic goods. We assume that these fixed costs are the same for all producers and that they are sufficiently high to make exporting only attractive for the most productive producers, in accordance with the rich empirical evidence on selection into export status (see, for instance, Bernard and Jensen, 1995; Clerides et al., 1998; Mayer and Ottaviano, 2008). We discuss the parameter domain supporting diversification and selection below.

Revenues of the least productive firms in the two economies (which are non-exporters by assumption) are linked by the zero-profit conditions, $r(\varphi_d) = \sigma P_G f$, $r^*(\varphi_d^*) = \sigma P_G f$, which can be combined to

$$\rho \left(\frac{P_S^*}{P_S}\right)^{\sigma-1} = \left[\frac{p_S(\varphi_d^*)}{p_S(\varphi_d)}\right]^{\sigma-1} = \left(\frac{\varphi_d}{\varphi_d^*}\right)^{\sigma-1}, \quad \rho \equiv \frac{H^*\lambda^*\left[1-\beta\left(\lambda^*\right)^{-\varepsilon}\psi_a^*\right]}{H\lambda\left[1-\beta\left(\lambda\right)^{-\varepsilon}\psi_a\right]} \tag{15}$$

where an asterisk is used to distinguish foreign from domestic variables and $\rho > 0$ captures relative differences of the two countries in their expenditures for sophisticated goods, provided that the two countries have the same total labor endowment (and thus the same aggregate income), $H\lambda = H^*\lambda^*$. We do not elaborate on differences in total labor endowment, because the effects of such differences are well known from Helpman and Krugman (1985) and because they are the same whether preferences are homothetic or not. If labor endowment is the same in the two economies, differences in expenditures and thus $\rho \neq 1$ can materialize only if preferences do not have Gorman form ($\varepsilon > 0$). As outlined in the closed economy, $\varepsilon > 0$ makes Engel curves non-linear implying that wealthier people have higher value shares of consumption of sophisticated goods, because these goods are luxuries from the perspective of households. Accordingly, we have $\rho > (<) 1$ if either $\lambda^* > (<) \lambda$ or $\psi > (<) \psi^*$.

Exporters make non-negative profits in the foreign country and, since revenues are increasing in

of $\sigma = 1$. In contrast, $dV(P_G, P_S, \lambda, \psi)/d\psi > 0$ holds for sufficiently high levels of σ.

productivity and fixed costs are the same for all producers, the least-productive domestic exporter with cutoff productivity φ_x is therefore characterized by the condition that its export profits are zero. This gives

$$\frac{r(\varphi_x)}{t\sigma}\rho\left(\frac{P_S^*}{P_S}\right)^{\sigma-1} = P_G f_x, \tag{16}$$

where $t \equiv \tau^{\sigma-1}$ is introduced to facilitate notation. Combining the two indifference conditions (15) and (16) then establishes a link between the domestic exporter and the foreign non-exporter productivity cutoff, $tf_x/f = (\varphi_x/\varphi_d^*)^{\sigma-1}$. Using the zero export profit condition for the foreign economy, we can derive a similar condition linking the foreign exporter and the domestic non-exporter productivity cutoff $tf_x/f = (\varphi_x^*/\varphi_d)^{\sigma-1}$. Considering further that aggregate profits of firms must equal the expenditures for the fixed costs of entering the productivity lottery country, we can further derive the free entry conditions at home and abroad, according to

$$f\left\{\int_{\varphi_d}^{\infty}\left(\frac{\varphi}{\varphi_d}\right)^{\sigma-1} dG(\varphi) - [1-G(\varphi_d)]\right\} + f_x\left\{\int_{\varphi_x}^{\infty}\left(\frac{\varphi}{\varphi_x}\right)^{\sigma-1} dG(\varphi) - [1-G(\varphi_x)]\right\} = f_e \tag{17}$$

and

$$f\left\{\int_{\varphi_d^*}^{\infty}\left(\frac{\varphi}{\varphi_d^*}\right)^{\sigma-1} dG(\varphi) - [1-G(\varphi_d^*)]\right\} + f_x\left\{\int_{\varphi_x^*}^{\infty}\left(\frac{\varphi}{\varphi_x^*}\right)^{\sigma-1} dG(\varphi) - [1-G(\varphi_x^*)]\right\} = f_e, \tag{18}$$

respectively. These two free entry conditions are derived in the appendix and, as formally shown there, they characterize a unique open economy equilibrium with $\varphi_d = \varphi_d^*$, provided that the two goods are produced in both economies. Symmetry in the cutoff productivity levels of non-exporters leads to symmetry in the cutoff productivity levels of exporters, $\varphi_x = \varphi_x^*$, and hence we can infer from the zero export profit condition that $tf_x/f > 1$ implies $\varphi_x > \varphi_d$, $\varphi_x^* > \varphi_d^*$, establishing the intended result of selection of high-productivity firms into exporting at home and abroad. Using $tf_x/f = (\varphi_x/\varphi_d)^{\sigma-1}$ in free entry condition (17) gives an implicit relationship between trade cost parameter t and cutoff productivities φ_d, φ_x, which is again derived and discussed in the appendix:

$$\frac{d\varphi_d}{dt} = -\frac{1}{\sigma-1}\frac{\varphi_d}{t}\frac{b(t)}{a(t)+b(t)} < 0 \quad \text{and} \quad \frac{d\varphi_x}{dt} = \frac{1}{\sigma-1}\frac{\varphi_x}{t}\frac{a(t)}{a(t)+b(t)} > 0, \tag{19}$$

where

$$a(t) \equiv \int_{\varphi_d}^{\infty}\left(\frac{\varphi}{\varphi_d}\right)^{\sigma-1}\frac{dG(\varphi)}{1-G(\varphi_d)}, \qquad b(t) \equiv \frac{f_x}{f}\int_{\varphi_x}^{\infty}\left(\frac{\varphi}{\varphi_x}\right)^{\sigma-1}\frac{dG(\varphi)}{1-G(\varphi_d)} \tag{20}$$

are auxiliary expressions introduced to simplify notation. These auxiliary expressions are not arbitrarily chosen. In a model with heterogeneous firms along Melitz (2003) important effects are captured by changes in the ratio of average and minimum revenues (cf. Bas et al., 2017), and $a(t)$, $b(t)$ capture these ratios for domestic and exporting sales, respectively. For the subsequent analysis it is useful to note that $(\varphi_x/\varphi_d)^{\sigma-1} = tf_x/f$ gives the ranking $a(t) > b(t)$.

From Eq. (19) we see that with selection a decline in the trade cost parameter leads to an increase in the cutoff productivity of the marginal producer, which is a non-exporter in this case. The reason for this effect lies in the entry mechanism postulated by Melitz (2003) and adopted for our model. Because export profits increase ceteris paribus with lower trade costs and because this increase induces higher average profits of active producers, the probability of a successful productivity draw which allows to start production must decrease in order to restore the requirement of zero expected profits for potential entrants into the productivity lottery imposed by the free entry condition. This explains why cutoff productivity φ_d must increase if trade costs decrease.[10] In contrast, productivity cutoff φ_x unambiguously declines if trade costs fall, because, as outlined above, lower trade cost make exporting more attractive. Putting together, changes in the two cutoff productivity levels give the intuitive result that the share of exporters, $[1 - G(\varphi_x)]/[1 - G(\varphi_d)]$, increases monotonically if t falls.

These results have been derived under the assumption that both countries produce basic as well as sophisticated goods. To analyze for which parameter combinations such a diversification equilibrium exists, we proceed in two steps. We first assess under which conditions both countries produce the sophisticated good provided that they also produce the basic good. We then study under which conditions both countries produce the basic good provided that they also produce the sophisticated good. The overlap of the two domains then gives the parameter combinations supporting a diversification equilibrium. If both countries produce the basic good, the market clearing conditions for sophisticated goods at home and abroad are given by

$$H\lambda[1 - \beta\lambda^{-\varepsilon}\psi_a] = Mf\sigma\left[a(t) + \mu b(t)\right] \tag{21}$$

and

$$H^*\lambda^*[1 - \beta(\lambda^*)^{-\varepsilon}\psi_a^*] = Mf\sigma\left[\mu a(t) + b(t)\right], \tag{22}$$

respectively, where $\mu \equiv M^*/M$ is the ratio of foreign to domestic producers. Combining Eqs. (21) and (22), we obtain an implicit relationship between expenditure ratio ρ and firm ratio μ, which is given by

$$\rho = \frac{\mu a(t) + b(t)}{a(t) + \mu b(t)}. \tag{23}$$

Acknowledging $a(t) > b(t)$, the right-hand side of (23) increases in μ and an interior solution with $\mu \in (0, \infty)$ is established if $\rho \in \left(\underline{\rho}(t), \overline{\rho}(t)\right)$, with $\underline{\rho}(t) \equiv b(t)/a(t) < 1$ and $\overline{\rho}(t) \equiv a(t)/b(t) > 1$.

[10] In a model without fixed costs of exporting, even the least productive firm serves foreign households, and hence average profits are pinned down by the zero profit of the least productive firm. Accordingly, the probability of a successful productivity draw and hence φ_d would be unaffected if trade cost parameter t falls.

Noting that $\underline{\rho}'(t) < 0$, $\overline{\rho}'(t) > 0$ and that $\lim_{t\to\infty} \underline{\rho}(t) = 0$, $\lim_{t\to\infty} \overline{\rho}(t) = \infty$,[11] we can conclude that for sufficiently high trade costs an interior solution with $\mu \in (0, \infty)$ exists for any possible expenditure ratio $\rho > 0$.

Provided that both countries produce sophisticated goods, there is production of basic goods if demand for labor from the sophisticated goods industry is lower than supply of labor in the respective economy, which is the case if

$$h_S \equiv \frac{Mr(\varphi_d)[a(t) + b(t)]}{H\lambda w} \frac{\sigma - 1}{\sigma} < 1, \qquad h_S^* \equiv \frac{M^* r(\varphi_d)[a(t) + b(t)]}{H^* \lambda^* w} \frac{\sigma - 1}{\sigma} < 1 \tag{24}$$

hold at home and abroad, respectively. Accounting for $r(\varphi_d) = \sigma P_G f$, $M^* = \mu M$ and substituting M from the market clearing condition in Eq. (21), we can compute (acknowledging $H\lambda = H^*\lambda^*$)

$$1 - \beta\lambda^{-\varepsilon}\psi_a < \frac{\sigma}{\sigma - 1} \min\left\{ \frac{\overline{\rho}(t) - 1}{\overline{\rho}(t) - \rho}, \frac{1 - \underline{\rho}(t)}{\rho - \underline{\rho}(t)} \right\}. \tag{25}$$

In the appendix, we show that (25) is fulfilled for high trade costs, and we can thus conclude from the analysis above that a diversification equilibrium is achieved in our model if t is sufficiently large.

Focussing on a diversification equilibrium, we can solve Eq. (23) for

$$\mu = \frac{\rho a(t) - b(t)}{a(t) - \rho b(t)} = \frac{1}{\underline{\rho}(t)} \frac{\rho - \underline{\rho}(t)}{\overline{\rho}(t) - \rho}, \tag{26}$$

where the second equality sign makes use of the definitions of $\overline{\rho}(t)$ and $\underline{\rho}(t)$. Acknowledging $a(t) > b(t)$, it follows from Eq. (26) that $\mu >, =, < 1$ if $\rho >, =, < 1$. Differentiating μ gives

$$\frac{d\mu}{d\rho} = \frac{a(t)^2 - b(t)^2}{[a(t) - \rho b(t)]^2} > 0, \qquad \frac{d\mu}{dt} = -\frac{\left(\rho^2 - 1\right)[a'(t)b(t) - a(t)b'(t)]}{[a(t) - \rho b(t)]^2} \tag{27}$$

and thus $d\mu/dt >, =, < 0$ if $1 >, =, < \rho$. The derivatives in Eq. (27) indicate that firms in our model are market-seeking in the sense that for positive trade costs, a larger fraction of firms enters the country featuring higher expenditures for sophisticated goods. The market-seeking effect becomes more pronounced with lower trade costs. In the limiting case of Cobb-Douglas preferences ($\varepsilon = 0$), we have $\rho = \mu = 1$.

The ratio of the foreign relative to the domestic mass of producers, $\mu = M^*/M$ plays an important role in our model for the pattern of trade in the open economy. To see this, we can first note that home's

[11] The derivative of $\overline{\rho}(t)$ is given by $\overline{\rho}'(t) = [a'(t)b(t) - a(t)b'(t)]/b(t)^2$, implying that

$$a'(t)b(t) - a(t)b'(t) = -b(t)\left[\frac{G'(\varphi_d)}{1 - G(\varphi_d)} + \frac{\sigma - 1}{\varphi_d} a(t)\right]\frac{d\varphi_d}{dt} + a(t)\left[\frac{G'(\varphi_x)}{1 - G(\varphi_d)}\frac{f_x}{f} + \frac{\sigma - 1}{\varphi_x} b(t)\right]\frac{d\varphi_x}{dt} > 0$$

establishes $\overline{\rho}'(t) > 0$. Furthermore, $\underline{\rho}'(t) < 0$ follows from the observation that $\underline{\rho}(t) = 1/\overline{\rho}(t)$.

exports and imports are given by

$$EX_S = M \int_{\varphi_x}^{\infty} \frac{r^*(\varphi)}{t} \frac{dG(\varphi)}{1 - G(\varphi_d)} = Mb(t)\sigma P_G f, \tag{28}$$

$$IM_S = M^* \int_{\varphi_x^*}^{\infty} \frac{r(\varphi)}{t} \frac{dG(\varphi)}{1 - G(\varphi_d^*)} = \mu Mb(t)\sigma P_G f, \tag{29}$$

respectively, where Eqs. (15), (16), and (20) have been used. This reveals that home is a net importer (net exporter) of sophisticated goods if $\mu > (<)1$. Acknowledging the link between μ and ρ from above, we can conclude that differences in the level and dispersion of per-capita income are important determinants of the structure between two economies if preferences do not have the Gorman form. Further insights on the link between trade costs and trade structure can be obtained from the Grubel-Lloyd index, which is a measure for the share of intra-industry trade and is defined as follows

$$GLI = 1 - \sum_j \frac{|EX_j - IM_j|}{\sum_j (EX_j + IM_j)}, \tag{30}$$

where $j \in \{G, S\}$ is an industry index. To pin down the extent of trade in the basic goods sector, we assume that households in the case of indifference purchase the domestic product. Then, $\rho > 1$ establishes $IM_G = 0$ and $EX_G = IM_S - EX_S$, where the latter follows form the balance of payments condition. As a consequence, we have $\sum_j (EX_j + IM_j) = 2IM_S$. In contrast, $\rho < 1$ establishes $EX_G = 0$ and $IM_G = EX_S - IM_S$, leading to $\sum_j (EX_j + IM_j) = 2EX_S$. Substituting into the Grubel-Lloyd index, we obtain

$$GLI = \begin{cases} \frac{EX_S}{IM_S} = \frac{1}{\mu} & \text{if} \quad \rho > 1 \\ 1 & \text{if} \quad \rho = 1 \\ \frac{IM_S}{EX_S} = \mu & \text{if} \quad \rho < 1 \end{cases}. \tag{31}$$

The main insights regarding the role of ρ and t for the trade structure in our model are summarized by the following proposition.

Proposition 1 *The country with the relatively higher expenditures for sophisticated goods is a net exporter of these goods in the open economy. The share of intra-industry trade, measured by the Grubel-Lloyd index, increases in the similarity of countries in terms of consumption expenditures. If expenditures differ between the two economies, the share of intra-industry trade decreases monotonically if trade cost parameter t falls.*

Proof Follows from substituting $\mu = (\rho t - 1)/(t - \rho)$ into Eqs. (28), (29), and (31).

The results in Proposition 1 point to a home-market effect, which in its most general interpretation states that a country exports on average those goods for which it has the larger domestic market. Helpman and Krugman (1985) show that such a home-market effect exists in a two sector model not too different from ours, in which a differentiated good is produced with increasing returns to scale whereas a homogenous good is produced with constant returns to scale. Provided that the homogeneous good is freely traded, the existence of trade costs for differentiated goods leads to a home-market effect under love-of-variety preferences for differentiated goods, implying that the country with a larger endowment of labor (the only factor of production in this model) exports this good because it features higher local demand for it. As pointed out by Davis (1998), this home-market effect is reinforced if trade costs fall, making countries more dissimilar in their production structure (and, in consequence, intra-industry trade less important). Our results extend these insights to a setting, in which countries are symmetric in terms of labor endowment (and thus aggregate income) and differences in expenditure for sophisticated goods exist, because preferences do not have Gorman form and because the two countries differ in the level and dispersion of per-capita income. In this case, the country featuring higher per-capita income or a higher dispersion of income has higher domestic demand for and thus a trade surplus in the differentiated good, which is a luxury from the perspective of households.[12]

The trade structure effects in Proposition 1 are well in line with the Linder (1961) hypothesis, which postulates that intra-industry (manufacturing) trade is higher between countries featuring more similar per-capita income levels. Markusen (1986) has provided a first formal account to show this hypothesis using preferences that are nonhomothetic but still have Gorman form. Fajgelbaum et al. (2011) have further extended the discussion regarding the role of personal income by considering preferences that do not have Gorman form, and have shown that in such an environment it is not only the level but also the dispersion of per-capita income that shapes the direction and pattern of trade. Whereas the Linder (1961) hypothesis is often used as an argument that *overall* trade is higher between countries that are more similar in terms of per-capita income, this conclusion is not immediate, because while higher similarity in per-capita income increases intra-industry trade, it lowers, at the same time, inter-industry trade as pointed out by Hunter (1991). To assess, which of the two effects dominates, we can note from above that total (intra- plus inter-industry) trade is given by $2EX_S$ if $\rho < 1$ and by $2IM_S$ if $\rho > 1$. Acknowledging from Eqs. (**??**), (43) and zero-profit condition $(1-\alpha)r(\varphi_d) = \sigma f$, that M

(28), and (29), we obtain the following corollary to Proposition 1.

Corollary 1 *Total trade is lower if countries are more similar in the level and dispersion of per-capita income.*

Proof See the appendix.

The result in Corollary 1 is directly linked to the home-market effect, which, as put forward by Crozet and Trionfetti (2008), implies that "a country whose share of world demand for a good is larger than average will have – ceteris paribus – a more than proportionally larger-than-average share of world production of that good" (p. 309). Following this reasoning, an increase in the relative expenditures for sophisticated goods of home will lead to a domestic increase and a foreign decrease in the production of this good.

[12]The effects considered here are different from Krugman (1980) who considers a home-market effect, arising from exogenous differences in the preferences of two economies.

Provided that the increase in relative expenditures for sophisticated goods is due to an increase in the level and dispersion of per-capita income of home, foreign demand will be unaffected, implying higher exports of sophisticated goods from home and higher exports of the basic good from abroad, provided that home has been a net exporter of sophisticated goods initially ($\rho < 1$), as put forward by the corollary.

To complete the discussion in this section, we finally determine the effects of trade on welfare. For this purpose, we first compute the price index for sophisticated goods. Accounting for $P_S = p(\varphi_d)\{M[a(t)+\mu b(t)]\}^{\frac{1}{1-\sigma}}$ and noting from the market clearing condition for sophisticated goods in Eqs. (21) that $M\sigma f[a(t)+\mu b(t)] = H\lambda[1-\beta\lambda^{-\varepsilon}\psi_a]$, we can solve for the price index of home according to $P_S = (\varphi_d^A/\varphi_d)P_S^A$. Since the augmented mass of varieties of sophisticated goods is the same in the open as in the closed economy and unaffected by changes in the trade cost parameter, changes in price index can only materialize in this model if the cutoff productivity level changes. Substituting the price index into social welfare function $V = V(P_G, P_S, \overline{e}, \psi)$ in (13), then gives

$$V = \left(\frac{\varphi_d}{\varphi_d^A}\right)^{\varepsilon} V_A. \tag{32}$$

We summarize the impact of trade on welfare in the following proposition.

Proposition 2 *There are gains from trade of equal size in both countries and these gains increase monotonically if trade costs fall.*

Proof Follows from Eq. (32).

The existence of gains from trade are not a priori clear in our setting, because the market outcome for the closed economy is not socially optimal and we know from the literature of second best that in such a case lifting a constraint may aggravate the distortion of the market outcome and thereby lead to welfare loss (see, for instance, Markusen, 1981; Newbery and Stiglitz, 1984, for two prominent contributions in the context of trade). The analysis above reveals that concerns about losses from trade are not justified in our setting. Since the engine for gains from trade is a decline in the price index of sophisticated goods in response to trade liberalization and since lower trade costs induce the price index to fall irrespective of preference parameter ε there are gains from trade for preferences with and without Gorman form.[13] Furthermore, the entry mechanism in our model implies that price indices adjust to compensate for differences in the expenditure level for sophisticated goods, according to Eq. (15). Since expenditures are exogenous, relative price indices do not change, provided that cutoff productivities are the same in the two economies, which is the case as long as the fixed costs of entry, production, and exporting (and thus prices of the basic good) are the same. This leads to the notable result that the welfare gains are the same in the two economies and independent of the trade structure. In the next section, we analyze to what extent the effects in this section change when allowing for feedback effects of trade on the level and distribution of per-capita income.

[13] To verify that gains from trade also exist in the limiting case of $\varepsilon = 0$, one can use the indirect utility function for the Cobb-Douglas case discussed in fn 8.

4 Feedback effects of trade on labor income

In our baseline specification, workers are paid their marginal value product and keeping labor productivity constant trade affects welfare through adjustments of prices and thus real labor income, but leaves the distribution of labor income unaffected. The parsimonious model outlined in the previous section is therefore well suited for studying how *ex ante* differences in the level and dispersion of per-capita income affect the structure of trade, but it does not allow to address the widespread concern that gains from trade are not fairly distributed and that trade is one important factor explaining a widening in the dispersion of income observed over the last decades. To allow for feedback effects of trade on income inequality, we introduce a model of rent sharing and consider a framework of collective bargaining between firms and firm-level unions in the sophisticated goods industry. Workers who do not find a job in the sophisticated goods industry are employed in the basic goods sector offering the market-clearing wage w (cf. Bastos and Kreickemeier, 2009).[14] Following Blanchard and Giavazzi (2003), we assume that firms and unions jointly set wages and employment under efficient bargaining. We begin our analysis by describing how the bargaining framework affects the closed economy equilibrium and discuss the changes for the open economy afterwards.

4.1 Bargaining in the closed economy

Using the goods demand in Eq. (7) we can write revenues of a firm as $r(\omega) = D^{1/\sigma} x_S(\omega)^{1-1/\sigma}$, where $D \equiv H\bar{e}\left(1-\beta(\bar{e}/P_G)^{-\varepsilon}\psi\right)/P_S^{1-\sigma}$ is a demand shifter that is common to all producers. Then, accounting for $x_S(\omega) = \varphi(\omega) l_S(\omega)$, profits for the firm are given by

$$\pi(\omega) = D^{\frac{1}{\sigma}} \varphi(\omega)^{1-\frac{1}{\sigma}} l_S(\omega)^{1-\frac{1}{\sigma}} - w_S(\omega) l_S(\omega) - P_G f. \tag{33}$$

Firms and unions jointly set wages and employment to maximize the generalized Nash product $\{[w_S(\omega) - w] l_S(\omega)\}^{\alpha} \left[\pi(\omega) - \overline{\pi}\right]^{1-\alpha}$, where $\alpha \in (0,1)$ is the bargaining power of the union, $[w_S(\omega) - w] l_S(\omega)$, $\pi(\omega) - \overline{\pi}$ are the union's and the firm's contribution to the Nash product, and $\overline{\pi} = -P_G f$ is the profit obtained by the firm in the case of disagreement.[15] The solution to the the firm-union maximization problem is given by the *contract curve*

$$x_S(\omega) = \varphi(\omega) l_S(\omega) = D p_S(\omega)^{-\sigma}, \quad \text{with} \quad p_S(\omega) \equiv \frac{\sigma}{\sigma-1} \frac{w}{\varphi(\omega)}, \tag{34}$$

[14] We choose a model without involuntary unemployment to ensure that households have sufficiently high income for purchasing sophisticated goods even in the absence of a generous unemployment compensation system (which would be of no further interest for our analysis). This makes the collective bargaining framework considered here more suited for our analysis than an otherwise in many respects similar framework of individual bargaining of the Stole and Zwiebel (1996)-type with search frictions (necessary to generate a bilateral monopoly between workers and firms), as put forward, for instance, by Helpman and Itskhoki (2010) and Felbermayr and Prat (2011).

[15] With risk-neutral workers, the union's contribution can be derived from a utilitarian objective function with the labor return in the case of disagreement given by w (cf. Oswald, 1985). Furthermore, we follow the standard approach and associate the bargaining pair with a *closed shop*, implying that all workers employed by a firm are member of the same union.

and the *rent-sharing curve*

$$w_S(\omega) = w + \alpha \left[\frac{r(\omega)}{l_S(\omega)} - w \right] = \nu w, \quad \text{with} \quad \nu \equiv \frac{\sigma - 1 + \alpha}{\sigma - 1} > 1 \tag{35}$$

capturing the union wage premium.

The bargaining outcome is efficient because it lies on the contract curve, leaving no scope for Pareto improvements of the two bargaining parties. In our setting, employment along the contract curve is independent of the negotiated wage and, for given aggregates, it is the same as in the benchmark without unions. As an immediate consequence of this, the price charged by a firm is independent of whether the firm pays the negotiated or the market-clearing wage. Bargaining in our setting therefore influences how the production surplus is distributed between firms and workers but not the size of this surplus. This is a consequence of unions putting equal weight on wage and employment increases in their negotiations with firms. It is a further notable feature of our model that firms in the sophisticated goods sector pay the same wage rate irrespective of their productivity level: $w_S(\omega) \equiv w_S$.[16] Due to a uniform wage in the sophisticated goods industry, heterogeneity of any two firms is again fully described by differences in their productivity levels and we can therefore drop ω and use productivity to index firms similar to the benchmark in Section 2. Revenues and profits of firms are then given by $r(\varphi) = Dp_S(\varphi)^{1-\sigma}$ and $\pi(\varphi) = (1-\alpha)r(\varphi)/\sigma - P_G f$, respectively.

Zero profits of the least-productive firm, $(1-\alpha)r(\varphi_d) = \sigma P_G f$, imply that revenues of this firm are higher than in the benchmark without unions, because receiving a lower fraction of the surplus of production the least productive firm requires a higher level of surplus to cover for the fixed cost of production. The free-entry condition remains to be given by Eq. (8), and hence the existence of unions does not change the cutoff productivity level φ_d. Turning to the general equilibrium, we can first note that with random allocation of abilities to firms, per-capita income is given by

$$\bar{e} = w(1-h_S)\int_{\underline{a}}^{\overline{a}} a dH(a) + w_S h_S \int_{\underline{a}}^{\overline{a}} a dH(a) = w\lambda \left[1 + h_S(\nu - 1)\right], \tag{36}$$

where $\lambda \equiv \int_{\underline{a}}^{\overline{a}} a dH(a)$ is the average labor endowment of workers and h_S is the fraction of workers employed in the sophisticated goods industry, which is linked to the mass of producers by Eq. (11). Wage dispersion can be computed according to

$$\psi = \left(\frac{w}{\bar{e}}\right)^{1-\varepsilon} \left[1 + h_S\left(\nu^{1-\varepsilon} - 1\right)\right] \int_{\underline{a}}^{\overline{a}} a^{1-\varepsilon} dH(a) = \frac{1 + h_S\left(\nu^{1-\varepsilon} - 1\right)}{\left[1 + h_S(\nu - 1)\right]^{1-\varepsilon}} \psi_a, \tag{37}$$

where $\psi_a = \int_{\underline{a}}^{\overline{a}} (a/\lambda)^{1-\varepsilon} dH(a)$ measures the dispersion of abilities. In the limiting case of $\alpha = 0$,

[16] Barth and Zweimüller (1995) have pointed out that wage differentiation between producers requires differences in the elasticity of revenues with respect to employment and Eckel and Egger (2009) show that such differences do not prevail if demand is isoelastic. The finding that wage negotiations do not lead to firm-specific wages in models featuring isoelastic demand and productivity differences of firms is not specific to the assumption of collective bargaining. For instance, Helpman and Itskhoki (2010) and Felbermayr and Prat (2011) show that the result extends to a model with individual bargaining of the Stole and Zwiebel (1996)-type and involuntary unemployment due to search frictions.

unions have no bargaining power and producers of sophisticated goods pay the market clearing wage $w_S = w$. In this case, we have $\nu = 1$ and thus $\psi = \psi_a$, implying that the wage dispersion is the same as in the benchmark without unions. If $\alpha > 0$ gives unions bargaining power, wage dispersion is more pronounced and thus $\psi < \psi_a$. The extent of wage dispersion depends on the share of workers employed in the sophisticated goods industry, h_S, and the effect of h_S on ψ is nonmonotonic.[17]

Market clearing for sophisticated goods gives

$$H\lambda w\left[1-\beta\lambda^{-\varepsilon}\psi_a\right] + Mr(\varphi_d)B\int_{\varphi_d}^{\infty}\left(\frac{\varphi}{\varphi_d}\right)^{\sigma-1}\frac{dG(\varphi)}{1-G(\varphi_d)}$$
$$= Mr(\varphi_d)\int_{\varphi_d}^{\infty}\left(\frac{\varphi}{\varphi_d}\right)^{\sigma-1}\frac{dG(\varphi)}{1-G(\varphi_d)}, \tag{38}$$

where $B \equiv \frac{\sigma-1}{\sigma}\left[(\nu-1)-(\nu^{1-\varepsilon}-1)\beta\lambda^{-\varepsilon}\psi_a\right] < 1$ captures the impact on the expenditures of sophisticated goods from changes in the level and dispersion of per-capita income due to the existence of a union wage premium. To be more specific, total revenues of firms are equal to $H\lambda w h_S[\sigma/(\sigma-1)]$, and hence the additional expenditures that are generated by rent sharing between firms and unions in the sophisticated goods industry are given by $H\lambda w h_S B[\sigma/(\sigma-1)]$. Substituting $\sigma P_G f = (1-\alpha)r(\varphi_d)$ and accounting for the free entry condition (8), Eq. (38) can be solved for the mass of producers

$$M = \frac{1-\alpha}{1-B}\frac{H\lambda\left[1-\beta\lambda^{-\varepsilon}\psi_a\right]}{\sigma f}\left[1+\frac{f_e/f}{1-G(\varphi_d)}\right]^{-1}, \tag{39}$$

where $1-\alpha < 1-B$ implies that the existence of unions reduces the mass of firms producing sophisticated goods. From Eq. (11), we further obtain

$$h_S = \frac{\sigma-1}{\sigma}\frac{1-\beta\lambda^{-\varepsilon}\psi_a}{1-B}, \tag{40}$$

which reveals that the fraction of workers employed in the sophisticated goods industry is larger than in the benchmark without unions.[18] This result may be surprising at a first glance as we know from above that fewer firms produce sophisticated goods. However, the decline in the mass of producers is counteracted and dominated by an increase in average firm size following from the zero profit condition of the marginal producer. For an intuition, it is worth noting from Eqs. (36) and (37) that the level and dispersion of per-capita income are higher with unions than without unions. This causes higher demand for sophisticated goods leading to higher employment than in the benchmark discussed in Section 2. To see this, note that total expenditures for sophisticated goods are given by $H\lambda w[1-\beta\lambda^{-\varepsilon}\psi_a]/(1-B)$

[17] In the limiting case $h_S = 0$, there is no employment in the sophisticated goods industry making the union wage premium irrelevant and establishing $\psi = \psi_a$. In the limiting case of $h_S = 1$ all workers are employed in the sophisticated goods industry and receive the union wage premium, again resulting in $\psi = \psi_a$. Dispersion index ψ is u-shaped and reaches a minimum at

$$\hat{h}_S \equiv \frac{(1-\varepsilon)(\nu-1)-(\nu^{1-\varepsilon}-1)}{\varepsilon(\nu-1)(\nu^{1-\varepsilon}-1)} \in (0,1).$$

[18] Note that $h_S < 1$ is guaranteed because $1-B > \frac{\sigma-1}{\sigma}[1-\beta\lambda^{-\varepsilon}\psi_a]$ holds for all possible α.

and are therefore higher with unions ($B > 0$) than without unions ($B = 0$).

Using the mass of firms from Eq. (39), we can determine the price index according to

$$P_S = \frac{\sigma}{\sigma - 1}\frac{w}{\varphi_d}\left\{\frac{1-\alpha}{1-B}\frac{H\lambda\left[1-\beta\lambda^{-\varepsilon}\psi\right]}{\sigma f}\right\}^{\frac{1}{1-\sigma}} \tag{41}$$

which, for $\alpha > 0$, is larger than the price index in Eq. (12). The increase in the price index has a negative effect on welfare, which is, however counteracted by an increase in the average labor income. If preferences are nonhomothetic ($\varepsilon > 0$), there is also an increase in wage dispersion, which counteracts the positive effect of a higher per-capita income level. However, accounting for $(\bar{e}/P_G)^{\varepsilon}\hat{\psi} = \lambda^{\varepsilon}\hat{\psi}_a[1 + h_S(\nu^{\varepsilon} - 1)]$ and noting from above that the share of workers employed in the sophisticated goods industry increases in union wage premium ν, it follows that the dispersion effect does not dominate, leaving two counteracting effects. In the limiting case of $\varepsilon = 0$, preferences are Cobb-Douglas and the dispersion of income is irrelevant for welfare. In this case, welfare is unambiguously lower with than without unions. This is intuitive, because we know from Section 2 that with Cobb-Douglas preferences, the mass of firms choosing to produce in the sophisticated goods industry is below the social optimum. Since the mass of firms is further reduced if firms receive a smaller fraction of the production surplus, the existence of unions unambiguously lowers social welfare in this case. Welfare effects are more involved if the preferences are nonhomothetic and, as formally shown in the appendix, it cannot be ruled out in this case that welfare is higher with than without unions. This completes the discussion of the closed economy.

4.2 Bargaining in the open economy

For the open economy, we adopt the assumptions of Section 3 and consider trade between two countries that are fully symmetric except for the level and dispersion of per-capita income. Furthermore, we assume that trade in the basic goods industry is free of, whereas exports in the sophisticated goods sector are subject to iceberg transport costs and a fixed cost of market penetration, which leads to selection of the most-productive firms into export status. Then, revenues of the least productive producers are again linked by the respective zero profit conditions in the two economies, $(1-\alpha)r(\varphi_d) = \sigma P_G f$, $(1-\alpha)r^*(\varphi_d^*) = \sigma P_G f$. However, the resulting link between cutoff productivity levels and the price indices in the two economies are now more complicated, because the expenditures for sophisticated goods are no longer exogenous but are influenced by adjustments in the share of workers used in the production of sophisticated goods who receive a wage premium on their labor input due to the rent sharing of firms and unions. Combining the two zero profit conditions establishes

$$\rho\zeta\left(\frac{P_S^*}{P_S}\right)^{\sigma-1} = \left[\frac{p_S(\varphi_d^*)}{p_S(\varphi_d)}\right]^{\sigma-1} = \left(\frac{\varphi_d}{\varphi_d^*}\right)^{\sigma-1}, \quad \text{where} \quad \zeta \equiv \frac{1 + \frac{\sigma}{\sigma-1}h_S^* B^*/[1-\beta(\lambda^*)^{-\varepsilon}\psi_a^*]}{1 + \frac{\sigma}{\sigma-1}h_S B/[1-\beta\lambda^{-\varepsilon}\psi_a]} \tag{42}$$

is an augmenting factor that captures the additional relative expenditure dispersion due to the existence of unions, $B^* \equiv \frac{\sigma-1}{\sigma}\left[(\nu-1) - (\nu^{1-\varepsilon}-1)\beta(\lambda^*)^{-\varepsilon}\psi_a^*\right] < 1$, and ρ is defined in Eq. (15). With the additional assumption that the two countries have the same total labor supply, $H\lambda = H^*\lambda^*$, we get a clear link between the ranking of the two countries in terms of expenditure parameters ρ and B: $B^* > (<) B$ if $\rho > (<) 1$.

Since firms take aggregate variables as given, the modification of the zero profit condition due to the assumption of a unionized labor market does not affect firm entry into production and into exporting, and hence (17)-(20) are the same as in the benchmark without unions. However, the presence of unions changes the market clearing conditions for sophisticated goods at home and abroad, which, provided that a diversification equilibrium exists, are given by

$$H\lambda w \left[1 - \beta\lambda^{-\varepsilon}\psi_a\right] + Mr(\varphi_d)B[a(t) + b(t)] = Mr(\varphi_d)[a(t) + \mu b(t)], \tag{43}$$

and

$$H^*\lambda^* w \left[1 - \beta(\lambda^*)^{-\varepsilon}\psi_a^*\right] + Mr(\varphi_d)B^*\mu[a(t) + b(t)] = Mr(\varphi_d)[\mu a(t) + b(t)], \tag{44}$$

respectively. Combining Eqs. (43) and (44), gives

$$\rho = \frac{[\mu a(t) + b(t)] - \mu[a(t) + b(t)]B^*}{[a(t) + \mu b(t)] - [a(t) + b(t)]B}, \tag{45}$$

Under the sufficient condition $\alpha < \sigma/2$ (assumed from now on), the right-hand side of Eq. (45) is positive for any possible realization of μ, and in this case there is production of sophisticated goods in both countries if $\rho \in \left(\underline{\rho}(t), \overline{\rho}(t)\right)$, with the two bounds given by $\underline{\rho}(t) = \{[a(t)/b(t)](1-B) - B\}^{-1}$ and $\overline{\rho}(t) = [a(t)/b(t))](1-B^*) - B^*$, respectively. Due to $B, B^* > 0$, the interval of permissible levels of ρ and thus the interval of permissible differences in the expenditures for sophisticated goods is smaller ceteris paribus with than without unions. However, the lower and upper bound of the ρ interval have similar properties as in the benchmark model. In particular, $\underline{\rho}'(t) < 0$, $\overline{\rho}'(t) > 0$ and $\lim_{t\to\infty} \underline{\rho}(t) = 0$, $\lim_{t\to\infty} \overline{\rho}(t) = \infty$ are robust to giving unions in the sophisticated goods industry bargaining power.[19] Therefore, provided that there is basic goods consumption in both countries, there is also sophisticated goods production in the two economies if trade costs are sufficiently high. Basic goods production requires that the two conditions in Eq. (24) are fulfilled. Substituting the market clearing conditions from Eqs. (43) and (44), the parameter constraint supporting basic goods production is given by

$$1 - \beta\lambda^{-\varepsilon}\psi_a < \frac{\sigma}{\sigma - 1} \min\left\{\frac{\overline{\rho}(t)(1 - B) - (1 - B^*)}{\overline{\rho}(t) - \rho}, \frac{[(1 - B^*) - \underline{\rho}(t)(1 - B)]}{\rho - \underline{\rho}(t)}\right\} \tag{46}$$

and, as formally shown in the appendix, it is fulfilled for sufficiently high levels of t.

[19] The ranking $\underline{\rho}(t) < 1 < \overline{\rho}(t)$ is no longer guaranteed for arbitrary low levels of t. To ensure that $\underline{\rho}(t), \overline{\rho}(t)$ give meaningful bounds for the permissible range of ρ, we focus on trade cost parameters that are sufficiently high to maintain the original ranking $\underline{\rho}(t) < 1 < \overline{\rho}(t)$.

Solving Eq. (45) for μ, we obtain

$$\mu = \frac{[\rho a(t) - b(t)] - \rho[a(t) + b(t)]B}{[a(t) - \rho b(t)] - [a(t) + b(t)]B^*} = \frac{1}{\underline{\rho}(t)} \frac{\rho - \underline{\rho}(t)}{\overline{\rho}(t) - \rho}, \tag{47}$$

where the second equality sign follows from the definitions of $\underline{\rho}(t)$ and $\overline{\rho}(t)$. From Eq. (47), we can infer that $\mu >,=,< 1$ if $\rho(1-B) >,=,< 1-B^*$, and in the appendix we show that $d\mu/dt >,=,< 0$ if $1-B^* >,=,< \rho(1-B)$. Noting that $\rho(1-B) >,=,< 1-B^*$ is equivalent to $\rho >,=,< 1$, then establishes the following result.

Proposition 3 *The results upon the role of ρ and t for trade structure outlined in Proposition 1 extend to the model variant with unions.*

Proof See the appendix.

To determine the welfare effects of trade, we first compute the augmented mass of available varieties of the sophisticated good and the share of workers active in the sophisticated goods sector at home:[20]

$$M[a(t) + \mu b(t)] = \frac{H\lambda[1 - \beta\lambda^{-\varepsilon}\psi_a]}{\sigma f} \frac{(1-\alpha)[\overline{\rho}(t) - \rho + \rho(1-B) - (1-B^*)]}{\overline{\rho}(t)(1-B) - (1-B^*)} \tag{48}$$

and

$$h_S = \frac{\sigma - 1}{\sigma} \frac{[1 - \beta\lambda^{-\varepsilon}\psi_a][\overline{\rho}(t) - \rho]}{\overline{\rho}(t)(1-B) - (1-B^*)}, \tag{49}$$

where autarky values of these variables correspond to the limiting case $t = \infty$. Changes in both of these variables are relevant. For instance, changes in the augmented mass of available varieties influence the price index of sophisticated goods, according to

$$P_S = \frac{\sigma}{\sigma - 1} \frac{w}{\varphi_d} \left\{ \frac{H\lambda[1 - \beta\lambda^{-\varepsilon}\psi_a]}{\sigma f} \frac{(1-\alpha)[\overline{\rho}(t) - \rho + \rho(1-B) - (1-B^*)]}{\overline{\rho}(t)(1-B) - (1-B^*)} \right\}^{\frac{1}{1-\sigma}}. \tag{50}$$

Similar to the model without unions, higher trade costs lower the cutoff productivity level, which increases the price index of sophisticated goods ceteris paribus. With unions, there are, however, additional effects through changes in the augmented mass of available consumer goods. As noted in the closed economy, rent sharing makes production of sophisticated goods less attractive, leading to a fall in the mass of firms producing these goods and thus to an increase in price index P_S. If $\rho(1-B) < 1-B^*$ makes the home country an exporter of sophisticated goods, trade provides at least a partial remedy for the negative effect of unions on firm entry, reflected in a higher mass of augmented varieties of sophisticated goods and thus a lower price index. In this case, higher trade cost have an additional positive effect on the price

[20] To solve for $M[a(t)+\mu b(t)]$, we substitute the zero-profit condition of the least productive producer, $(1-\alpha)r(\varphi_d) = \sigma P_G f$ into market clearing condition (43) and acknowledge Eq. (A.30) from the appendix. Furthermore, h_S can be computed, combining Eqs. (24) and (43).

index, with a further detrimental effect on social welfare, according to Eq. (13). From Eq. (49), the negative welfare effect is reinforced by a fall in the share of workers receiving the union wage premium, because a decline in h_S lowers per-capita income and thus $(\bar{e}/P_G)^\varepsilon \hat{\psi} = \lambda^\varepsilon \hat{\psi}_a[1 + h_S(\nu^\varepsilon - 1)]$.

Things are different if $\rho(1-B) > 1-B^*$ makes the home country an importer of sophisticated goods. Whereas a higher trade cost again increases price index P_S due to a fall in the cutoff productivity level φ_d, trade now reinforces the negative effect of unions on firm entry and higher trade cost therefore provide a remedy for this additional distortion by increasing the mass of augmented varieties of sophisticated goods and lowering price index P_S. It is in general not clear which of the two effects dominates and hence higher trade costs may increase or decrease price index P_S. If the price index falls, higher trade costs unambiguously increase domestic welfare because the share of workers employed in the sophisticated goods industry and thus the share of workers receiving a union wage premium is positively linked to trade costs in the importing country according to Eq. (49). Thus, allowing for feedback effects on the level and distribution of per-capita income can change the welfare effects of trade, and we summarize the main insights regarding these effects in the following proposition.

Proposition 4 *In the model with union wage setting, gains from trade are guaranteed for the country that exports the sophisticated goods, whereas welfare losses are possible for the country importing sophisticated goods. In the case of symmetric countries both trading partners benefit from trade liberalization.*

Proof See the appendix.

The existence of sector-specific wage payments will generate losers from globalization in the country specializing in the production of basic goods, which promises lower wages. This provides a demand-based explanation for anti-globalization attitudes of workers observed at least in some industrialized economies. However, the insight that the price index of sophisticated goods can increase in the process of globalization is even more disconcerting, because it implies that all workers in the country that specializes in the production of the basic good may be worse off through trade liberalization. Such losses are only possible, however, if preferences do not have Gorman form ($\varepsilon > 0$), which suggests that giving up the assumption of homothetic preferences may change the rather optimistic view shared by many economists that trade, while not necessarily benefitting all households symmetrically, at least makes households better off on average.

The analysis in this section provides a nuanced picture about the distributional effects of trade. On the one hand, it follows from Proposition 4 that trade can be detrimental for the poorer country, thereby augmenting pre-existing differences in the per-capita income of countries. Hence, acknowledging nonhomothetic preferences, may further fan the flames in the already heated debate on whether the international distribution of trade surplus is just. On the other hand, depending on the size of the sophisticated goods industry in the closed economy, trade may increase or decrease the dispersion of per-capita income according to the Atkinson index. This is, because the ex ante dispersion of per-capita income is not decisive for the ex post pattern of trade when allowing for rent sharing in the sophisticated goods industry. As pointed out in the discussion of the closed economy, the dispersion of per-capita income is non-monotonic in the fraction of workers receiving the union wage premium. Therefore, it is possible that trade reduces income inequality within both economies, if prior to trade liberalization a large fraction of workers has been employed for producing sophisticated goods in the country net exporting these goods ex post, while only a small fraction of workers has been employed for producing sophisticated goods in the country net importing these goods ex post.

5 Conclusion

We have developed a two-country model, in which differences in the level and dispersion of per-capita income affect the trade structure, because nonhomothetic preferences make Engel curves non-linear. To solve the aggregation problem of consumer demand we rely on PIGL preferences, which admit a representative consumer even if the preferences do not have Gorman form. We use this demand structure in an otherwise traditional trade model with two sectors of production and one factor input, featuring home-market effects. Thereby, we follow the common approach and associate one product with a homogeneous outside good that is produced with linear technology, sold under perfect competition, and freely traded between the two economies. The other good is differentiated, produced under increasing returns to scale, and sold by monopolistically competitive producers that differ in their productivities. Exports of this good are subject to variable and fixed trade costs. We associate the homogeneous outside good with a basic necessity and the differentiated good with a sophisticated luxury, implying that Engel curves for the former are concave, whereas Engel curves for the latter are convex in household expenditure. Assuming that households differ in their effective labor supply and thus their expenditure levels, we show that, all other things equal, a country becomes a net exporter of the sophisticated good if it exhibits a higher level and dispersion of per-capita income than its trading partner. Lacking feedback effects of trade on the level and dispersion of per-capita income, this parsimonious model generates the somewhat counterintuitive result that the trade structure in the open economy is irrelevant for welfare, which due to a fall in the price index of sophisticated goods is stimulated by trade in both economies.

To account for such feedback effects, we augment our model with rent sharing in the sophisticated goods industry, due to efficient bargaining between firms and firm-level unions. Rent sharing makes the level and dispersion of per-capita income endogenous and interdependent. The extension to imperfectly competitive labor markets provides a more nuanced picture about the welfare effects of trade, while leaving our insights from the baseline model regarding the trade structure in the open economy unchanged. The net exporter of sophisticated goods benefits from an increase in per-capita labor income, because the stronger specialization on the production of sophisticated goods implies that more workers receive the wage premium negotiated by unions. Whereas the impact of trade on the dispersion of per-capita income is a priori not clear, increases of per-capita income paired with gains from a lower price index of sophisticated goods unambiguously make the net exporter of the sophisticated good better off in the open economy. Things are different for the net importer of this good, who may lose from trade due to a decline in per-capita income – because fewer workers are employed for producing the sophisticated good – and due to a less favorable change in the price index of sophisticated goods – because a relatively large fraction of sophisticated goods is imported and thus subject to trade costs.

The specific form of nonhomothetic preferences considered in this paper proves useful, because it gives a demand structure that is simple and therefore makes it accessible for a broad range of models of international trade discussed in recent years. Thereby, insights from our rent-sharing model suggest that the consideration of nonhomothetic preferences is particularly important for understanding how feedback effects of trade on domestic expenditures change our insights upon gains from trade in an environment of labor market imperfection. For instance, models of heterogeneous firms and firm-specific wages put forward by the recent literature on trade and labor markets may provide a much richer picture about welfare effects, when the respective changes of income prominently discussed in these models are directly relevant for social welfare in a setting with non-linear Engel curves. We think that the model outlined here provides an interesting point of departure for conducting such analyses and we hope that research along these lines can help reconciling the positive assessment of globalization in academic research with the more controversial discussion in the general public.

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A Theoretical appendix

A.1 A closed form representation of the direct utility function

Applying Roy's identity to Eq.(2) gives the Marshallian demand functions in Eq. (3). These demand functions can be used to solve for

$$\frac{P_G}{P_S} = \frac{X_S^i}{\left(\frac{X_G^i}{\beta}\right)^{\frac{1}{1-\varepsilon}} - X_G^i} \quad \text{and} \quad \frac{e^i}{P_S} = \frac{P_G}{P_S}\left(\frac{X_G^i}{\beta}\right)^{\frac{1}{1-\varepsilon}}. \tag{A.1}$$

Substitution into Eq. (2), then gives the direct utility function

$$u(X_G^i, X_S^i) = \frac{1}{\varepsilon}\left(X_S^i\right)^{\varepsilon} \frac{\left(\frac{X_G^i}{\beta}\right)^{\frac{\varepsilon}{1-\varepsilon}} - \beta}{\left[\left(\frac{X_G^i}{\beta}\right)^{\frac{1}{1-\varepsilon}} - X_G^i\right]^{\varepsilon}}. \tag{A.2}$$

This completes the proof.

A.2 Derivation of price index P_S

Acknowledging Eq. (4), households choose X_G^i, $x_S^i(\omega)$ to maximize utility Eq. (A.2), subject to their budget constraint $P_G X_G^i + \int_{\omega\in\Omega} p_S(\omega)x_S^i(\omega) \leq e^i$. The first-order conditions for the respective Lagrangian problem establish

$$\frac{x_S^i(\omega)^{-\frac{1}{\sigma}}}{\left(X_S^i\right)^{\frac{\sigma-1}{\sigma}}}\left[\left(\frac{X_G^i}{\beta}\right)^{\frac{1}{1-\varepsilon}} - X_G^i\right] = \frac{p_S(\omega)}{P_G} \tag{A.3}$$

This establishes for any two varieties of sophisticated goods ω and $\hat{\omega}$ a link for consumption expenditures according to $p_S(\omega)x_S^i(\omega) = x_S^i(\hat{\omega})p_S(\hat{\omega})^{\sigma}p_S(\omega)^{1-\sigma}$. Integrating over ω, then establishes

$$\int_{\omega\in\Omega} p_S(\omega)x_S^i(\omega)d\omega = x_S^i(\hat{\omega})p_S(\hat{\omega})^{\sigma}\int_{\omega\in\Omega} p_S(\omega)^{1-\sigma}d\omega. \tag{A.4}$$

Using the latter together with $X_G^i = \beta\left(\frac{e^i}{P_G}\right)^{1-\varepsilon}$ from Eq. (3) in the binding budget constraint, we obtain

$$e^i\left[1 - \beta\left(\frac{e^i}{P_G}\right)^{-\varepsilon}\right] = x_S^i(\hat{\omega})p_S(\hat{\omega})^{\sigma}\int_{\omega\in\Omega} p_S(\omega)^{1-\sigma}d\omega. \tag{A.5}$$

Evaluating (A.3) for $\hat{\omega}$ and substituting for $x_S^i(\hat{\omega})p_S(\hat{\omega})^{\sigma}$, Eq. (A.5) can be solved for

$$e^i\left[1 - \beta\left(\frac{e^i}{P_G}\right)^{-\varepsilon}\right] = X_S^i\left[\int_{\omega\in\Omega} p_S(\omega)^{1-\sigma}d\omega\right]^{\frac{1}{1-\sigma}}, \tag{A.6}$$

making $P_S \equiv \left[\int_{\omega\in\Omega} p_S(\omega)^{1-\sigma}d\omega\right]^{\frac{1}{1-\sigma}}$ a valid price index for the composite X_S^i, because total expenditures of household i devoted to sophisticated goods are given by $P_S X_S^i$. This completes the proof.

A.3 Derivation and discussion of Eqs. (17) and (18)

Total profits of firms in the home country are given by

$$\Pi_t = M_e \int_{\varphi_d}^{\infty} \frac{r(\varphi)}{\sigma} dG(\varphi) - M_e P_G f \int_{\varphi_d}^{\infty} dG(\varphi) + M_e \int_{\varphi_x}^{\infty} \frac{r_x(\varphi)}{\sigma} dG(\varphi) - M_e P_G f_x \int_{\varphi_x}^{\infty} dG(\varphi),$$

where M_e is the mass of firms entering the productivity lottery and $r_x(\varphi) \equiv t^{-1}\rho\,(P_S/P_S^*)^{1-\sigma}\, r(\varphi)$ are the export revenues of a domestic firm with productivity φ. Solving the integrals gives the expected profit for a potential entrant

$$\frac{\Pi_t}{M_e} = \frac{r(\varphi_d)}{\sigma} \int_{\varphi_d}^{\infty} \left(\frac{\varphi}{\varphi_d}\right)^{\sigma-1} dG(\varphi) - [1 - G(\varphi_d)]P_G f + \frac{r(\varphi_x)}{t\sigma}\rho \left(\frac{P_S}{P_S^*}\right)^{1-\sigma} \int_{\varphi_x}^{\infty} \left(\frac{\varphi}{\varphi_x}\right)^{\sigma-1} dG(\varphi) - [1 - G(\varphi_x)]P_G f_x \tag{A.7}$$

Using $r(\varphi_d) = \sigma P_G f$ from the condition that the marginal producer makes zero profits and setting $\Pi_t/M_e = P_G f_e$ gives the free entry condition in Eq. (17). The free entry condition for the foreign economy in Eq. (18) can derived in analogy.

Combining the free entry conditions in Eqs. (17) and (18) gives an implicit relationship between φ_d and φ_d^* according to

$$\Gamma(\varphi_d, \varphi_d^*) \equiv \left\{\int_{\varphi_d}^{\infty} \left(\frac{\varphi}{\varphi_d}\right)^{\sigma-1} dG(\varphi) - [1 - G(\varphi_d)]\right\} - \frac{f_x}{f}\left\{\int_{\varphi_x^*}^{\infty} \left(\frac{\varphi}{\varphi_x^*}\right)^{\sigma-1} dG(\varphi) - [1 - G(\varphi_x^*)]\right\} - \left\{\int_{\varphi_d^*}^{\infty} \left(\frac{\varphi}{\varphi_d^*}\right)^{\sigma-1} dG(\varphi) - [1 - G(\varphi_d^*)]\right\} + \frac{f_x}{f}\left\{\int_{\varphi_x}^{\infty} \left(\frac{\varphi}{\varphi_x}\right)^{\sigma-1} dG(\varphi) - [1 - G(\varphi_x)]\right\} = 0, \tag{A.8}$$

where $\varphi_x/\varphi_d^* = \varphi_x^*/\varphi_d = (tf_x/f)^{\frac{1}{\sigma-1}}$ are acknowledged from the main text. Partially differentiating Γ gives

$$\frac{\partial \Gamma}{\partial \varphi_d} = -\frac{\sigma-1}{\varphi_d}\left[\int_{\varphi_d}^{\infty} \left(\frac{\varphi}{\varphi_d}\right)^{\sigma-1} dG(\varphi) - \frac{1}{t}\int_{\varphi_x^*}^{\infty} \left(\frac{\varphi}{\varphi_d}\right)^{\sigma-1} dG(\varphi)\right] < 0 \tag{A.9}$$

and

$$\frac{\partial \Gamma}{\partial \varphi_d^*} = \frac{\sigma-1}{\varphi_d^*}\left[\int_{\varphi_d^*}^{\infty} \left(\frac{\varphi}{\varphi_d^*}\right)^{\sigma-1} dG(\varphi) - \frac{1}{t}\int_{\varphi_x}^{\infty} \left(\frac{\varphi}{\varphi_d^*}\right)^{\sigma-1} dG(\varphi)\right] > 0. \tag{A.10}$$

Then, noting that $\Gamma(\cdot) = 0$ if $\varphi_d = \varphi_d^*$ and accounting for the monotonicity of $\Gamma(\cdot)$ in φ_d and φ_d^*, it must be true that in any equilibrium with (i) diversification of production and (ii) partitioning of firms by their export status in both countries, we have $\varphi_d = \varphi_d^*$. This completes the proof.

A.4 Derivation and discussion of Eq. (19)

Using $\varphi_x = (tf_x/f)^{\frac{1}{\sigma-1}}\varphi_d$ and the free entry condition in Eq. (17) establishes an implicit relationship between φ_d and t according to

$$\hat{\Gamma}(\varphi_d, t) \equiv \left\{\int_{\varphi_d}^{\infty}\left(\frac{\varphi}{\varphi_d}\right)^{\sigma-1} dG(\varphi) - [1 - G(\varphi_d)]\right\} + \frac{f_x}{f}\left\{\int_{\varphi_x}^{\infty}\left(\frac{\varphi}{\varphi_x}\right)^{\sigma-1} dG(\varphi) - [1 - G(\varphi_x)]\right\} - \frac{f_e}{f} = 0. \tag{A.11}$$

Partial differentiation of Γ gives

$$\frac{\partial\hat{\Gamma}}{\partial\varphi_d} = -\frac{\sigma-1}{\varphi_d}\left[\int_{\varphi_d}^{\infty}\left(\frac{\varphi}{\varphi_d}\right)^{\sigma-1} dG(\varphi) + \frac{f_x}{f}\int_{\varphi_x}^{\infty}\left(\frac{\varphi}{\varphi_x}\right)^{\sigma-1} dG(\varphi)\right] < 0 \tag{A.12}$$

and

$$\frac{\partial\hat{\Gamma}}{\partial t} = -\frac{f_x}{f}\frac{1}{t}\int_{\varphi_x}^{\infty}\left(\frac{\varphi}{\varphi_x}\right)^{\sigma-1} dG(\varphi) < 0. \tag{A.13}$$

Applying the implicit function theorem to $\Gamma(\varphi_d, t) = 0$ then establishes

$$\frac{d\varphi_d}{dt} = -\frac{\partial\hat{\Gamma}/\partial t}{\partial\hat{\Gamma}/\partial\varphi_d} = -\frac{\varphi_d}{t}\frac{1}{\sigma-1}\frac{\frac{f_x}{f}\int_{\varphi_x}^{\infty}\left(\frac{\varphi}{\varphi_x}\right)^{\sigma-1} dG(\varphi)}{\int_{\varphi_d}^{\infty}\left(\frac{\varphi}{\varphi_d}\right)^{\sigma-1} dG(\varphi) + \frac{f_x}{f}\int_{\varphi_x}^{\infty}\left(\frac{\varphi}{\varphi_x}\right)^{\sigma-1} dG(\varphi)} < 0 \tag{A.14}$$

Substituting $a(t)$ and $b(t)$ from Eq. (20), we obtain $d\varphi_d/dt$ in Eq. (19). Totally differentiating $\varphi_x = (tf_x/f)^{\frac{1}{\sigma-1}}\varphi_d$ further implies

$$\frac{d\varphi_x}{dt} = \frac{\varphi_x}{t}\frac{1}{\sigma-1} + \frac{\varphi_x}{\varphi_d}\frac{d\varphi_d}{dt}, \tag{A.15}$$

which, substituting $d\varphi_d/dt$ from above, gives $d\varphi_x/dt$ in Eq. (19). This completes the proof.

A.5 Derivation and discussion of constraint (25)

Combining (24) for home with the market clearing condition in Eq. (21) and accounting for $r(\varphi_d) = \sigma P_G f$ and Eq. (23), we can compute

$$\frac{\sigma-1}{\sigma}\left[1 - \beta\lambda^{-\varepsilon}\psi_a\right] < \frac{a(t) - b(t)}{a(t) - \rho b(t)} = \frac{\overline{\rho}(t) - 1}{\overline{\rho}(t) - \rho} \equiv \hat{g}_0(t), \tag{A.16}$$

where the equality sign makes use of the definition of $\overline{\rho}(t)$. Combining (24) for abroad with the market clearing condition in Eq. (21) and accounting for $r(\varphi_d) = \sigma P_G f$, $M^* = \mu M$ and Eq. (23), we can compute

$$\frac{\sigma-1}{\sigma}\left[1 - \beta\lambda^{-\varepsilon}\psi_a\right] < \frac{a(t) - b(t)}{\rho a(t) - b(t)} = \frac{1 - \underline{\rho}(t)}{\rho - \underline{\rho}(t)} \equiv \hat{g}_1(t), \tag{A.17}$$

where the equality sign makes use of the definition of $\underline{\rho}(t)$ and the observation that $\underline{\rho}(t)\overline{\rho}(t) = 1$. The parameter constraint in Eq. (25) then follows from (A.16) and (A.17). Accounting for $\lim_{t\to\infty} \hat{g}_0(t) = 1$ and $\lim_{t\to\infty} \hat{g}_1(t) = 1/\rho$, we can conclude that conditions (A.16) and (A.17) are fulfilled in the closed economy. Noting further that

$$\hat{g}_0'(t) = -\frac{(\rho-1)\left[a'(t)b(t) - a(t)b'(t)\right]}{\left[a(t) - \rho b(t)\right]^2}, \qquad \hat{g}_1'(t) = \frac{(\rho-1)\left[a'(t)b(t) - a(t)b'(t)\right]}{\left[\rho a(t) - b(t)\right]^2}, \tag{A.18}$$

where $a'(t)b(t) - a(t)b'(t) > 0$ holds, according to fn. (11), and accounting for $\hat{g}_0(t) >, =, < \hat{g}_1(t)$ if $\rho >, =, < 1$, we can safely conclude that the parameter constraint in (25) is fulfilled for sufficiently high t. This completes the proof.

A.6 Proof of Corollary 1

Consider first $\rho < 1$, which implies that home is a net exporter of sophisticated goods, according to Proposition 1. As shown in the main text, total exports plus imports of home are then given by $2EX_S$, which equals exports plus imports of the foreign economy due to balanced trade. From Eqs. (21), (28), we obtain

$$EX_S = P_G H \lambda [1 - \beta\lambda^{-\varepsilon}\psi_a] \frac{b(t)}{a(t) + \mu b(t)} = P_G H \lambda [1 - \beta\lambda^{-\varepsilon}\psi_a] \frac{\overline{\rho}(t) - \rho}{\overline{\rho}(t)^2 - 1}, \tag{A.19}$$

where the second equality sign follows from substituting Eq. (26) and accounting for the definition of $\overline{\rho}(t)$. In a similar vein, we can note that $\rho > 1$ makes home an importer of the sophisticated good, with the total exports and imports given by IM_S. Combining Eqs. (21), (29), we can compute

$$IM_S = P_G H \lambda [1 - \beta\lambda^{-\varepsilon}\psi_a] \frac{\mu b(t)}{a(t) + \mu b(t)} = P_G H \lambda [1 - \beta\lambda^{-\varepsilon}\psi_a] \frac{\underline{\rho}(t)[\rho - \underline{\rho}(t)]}{1 - \underline{\rho}(t)^2}, \tag{A.20}$$

where the second equality sign follows from substituting Eq. (26) and accounting for the definition of $\underline{\rho}(t)$. Noting that $dEX_S/d\rho < 0$ while $dIM_S/d\rho > 0$ then establishes Corollary 1. This completes the proof.

A.7 Unions and social welfare

In the main text, we argue that social welfare is lower with than without unions if $\varepsilon = 0$. Evaluating the direct utility function in Eq. (A.2) at $\varepsilon = 0$, gives

$$\lim_{\varepsilon\to 0} u(X_G^i, X_S^i) = \beta \ln\left(\frac{X_G^i}{\beta}\right) + (1-\beta)\ln\left(\frac{X_S^i}{1-\beta}\right) \tag{A.21}$$

This is a standard Cobb-Douglas utility function, which in monotone transformation can be written in textbook form as

$$\tilde{u}(X_G^i, X_S^i) = \left(\frac{X_G^i}{\beta}\right)^{\beta} \left(\frac{X_S^i}{1-\beta}\right)^{1-\beta}. \tag{A.22}$$

Evaluating the consumption levels in Eq. (3) at $\varepsilon = 0$ and substituting the resulting expressions into $\tilde{u}(X_G^i, X_S^i)$, we get the indirect utility function $v(P_G, P_S, e_i) = e_i/[P_G^{\beta} P_S^{1-\beta}]$. Evaluated at the average income then gives utilitarian welfare $V = \overline{e}/[P_G^{\beta} P_S^{1-\beta}]$. We can now note from Section 4.1 that $\varepsilon = 0$

gives

$$B = \frac{\alpha}{\sigma}(1-\beta) \quad \text{and} \quad h_S(\nu - 1) = \frac{\frac{\alpha}{\sigma}(1-\beta)}{1-\frac{\alpha}{\sigma}(1-\beta)}. \tag{A.23}$$

Then, accounting for Eqs. (36) and (41), it follows that social welfare is higher (lower) with than without unions if $\tilde{V}(\alpha) > (<) 1$, where

$$\tilde{V}(\alpha) \equiv \left(\frac{1-\alpha}{1-\frac{\alpha}{\sigma}(1-\beta)}\right)^{\frac{1-\beta}{\sigma-1}} \frac{1}{1-\frac{\alpha}{\sigma}(1-\beta)} \tag{A.24}$$

and $\tilde{V}(0) = 1$. Differentiation gives

$$\tilde{V}'(\alpha) = \frac{1-\beta}{\sigma}\left(\frac{1-\alpha}{1-\frac{\alpha}{\sigma}(1-\beta)}\right)^{\frac{1-\beta}{\sigma-1}} \left(\frac{1}{1-\frac{\alpha}{\sigma}(1-\beta)}\right)^2 \tilde{V}_1(\beta), \tag{A.25}$$

where

$$\tilde{V}_1(\beta) \equiv 1 - \frac{\sigma - 1 + \beta}{(1-\alpha)(\sigma-1)} < 0. \tag{A.26}$$

This proves that welfare is lower with than without unions if preferences are Cobb-Douglas.

We now turn to the case of nonhomothetic preferences with $\varepsilon > 0$. In this case, accounting for Eqs. (13), (41) and acknowledging $(\overline{e}/P_G)^{\varepsilon}\hat{\psi} = \lambda^{\varepsilon}\hat{\psi}_a[1 + h_S(\nu^{\varepsilon} - 1)]$ from the main text, it follows welfare is larger (lower) with than without unions if $\hat{V}(\alpha) > (<) \lambda^{\varepsilon}\hat{\psi}_a - \beta$, where :

$$\hat{V}(\alpha) \equiv \left(\frac{1-\alpha}{1-B}\right)^{\frac{\varepsilon}{\sigma-1}} \left\{\lambda^{\varepsilon}\hat{\psi}_a\left[1 + h_S(\nu^{\varepsilon} - 1)\right] - \beta\right\} \tag{A.27}$$

and $\hat{V}(0) = \lambda^{\varepsilon}\hat{\psi}_a - \beta$. Differentiating $\hat{V}(\alpha)$ and evaluating the derivative at $\alpha = 0$, gives

$$\begin{aligned}
\hat{V}'(0) &= -\frac{\beta\varepsilon}{\sigma}\left\{\lambda^{-\varepsilon}\psi_a\left(\lambda^{\varepsilon}\hat{\psi}_a - \beta\right)\frac{\sigma-\varepsilon}{\sigma-1} - \left[1 - \beta\lambda^{-\varepsilon}\psi_a\right]\right\} \\
&= -\frac{\beta\varepsilon}{\sigma}\left\{\left(\psi_a\hat{\psi}_a - \beta\lambda^{-\varepsilon}\psi_a\right)\frac{\sigma-\varepsilon}{\sigma-1} - \left[1 - \beta\lambda^{-\varepsilon}\psi_a\right]\right\}
\end{aligned} \tag{A.28}$$

which is negative for a sufficiently high ε. Hence, there exist parameterizations of our model for which $\hat{V}'(0) > 0$. This completes the proof.

A.8 Derivation and discussion of constraint (46)

Combining Eq. (43) with the constraint for a positive production level of basic goods at home from (24), we can compute for

$$\frac{\sigma-1}{\sigma}\left[1 - \beta\lambda^{-\varepsilon}\psi_a\right] < \frac{a(t) + \mu b(t)}{a(t) + b(t)} - B. \tag{A.29}$$

Acknowledging Eq. (45), we can compute

$$\frac{a(t)+\mu b(t)}{a(t)+b(t)} = 1 + \frac{[\rho(1-B)-(1-B^*)]b(t)}{[a(t)-\rho b(t)]-[a(t)+b(t)]B^*} = 1 + \frac{\rho(1-B)-(1-B^*)}{\overline{\rho}(t)-\rho}, \tag{A.30}$$

where the second equality sign follows from the definition of $\overline{\rho}(t)$. Substituting Eq. (A.30) into (A.29) then gives

$$\frac{\sigma-1}{\sigma}\left[1-\beta\lambda^{-\varepsilon}\psi_a\right] < 1-B+\frac{\rho(1-B)-(1-B^*)}{\overline{\rho}(t)-\rho} \equiv \hat{g}_2(t) \tag{A.31}$$

which is fulfilled if $\rho \geq 1$. To see this, note that with $H\lambda = H^*\lambda^*$, $\rho > 1$ implies $\rho(1-B) > 1-B^*$, so that $\hat{g}_2(t) > 1-B$, which noting that $1-B > \frac{\sigma-1}{\sigma}[1-\beta\lambda^{-\varepsilon}\psi_a]$ is sufficient for the production of basic goods in the home country. In contrast, $\rho < 1$ and thus $\rho(1-B) < 1-B^*$ imply $\hat{g}_2(t) < 1-B$. However, since $\hat{g}_2'(t) > 0$ and $\lim_{t\to\infty} = 1-B$ hold in this case, we can safely conclude that the condition in (A.31) is fulfilled for sufficiently high t.

In a next step, we combine Eq. (44) with the constraint for a positive production level of basic goods abroad from (24) and compute

$$\frac{\sigma-1}{\sigma}\left[1-\beta(\lambda^*)^{-\varepsilon}\psi_a^*\right] < \frac{a(t)+b(t)/\mu}{a(t)+b(t)} - B^*. \tag{A.32}$$

Acknowledging Eq. (45), we can compute

$$\frac{a(t)+b(t)/\mu}{a(t)+b(t)} = 1 + \frac{[(1-B^*)-\rho(1-B)]b(t)}{[\rho a(t)-b(t)]-\rho[a(t)+b(t)]B} = 1 + \frac{[(1-B^*)-\rho(1-B)]}{\rho/\underline{\rho}(t)-1}, \tag{A.33}$$

where the second equality sign follows from the definition of $\underline{\rho}(t)$. Substituting Eq. (A.33) into (A.32), we obtain

$$\frac{\sigma-1}{\sigma}\left[1-\beta(\lambda^*)^{-\varepsilon}\psi_a^*\right] < 1-B^*+\frac{(1-B^*)-\rho(1-B)}{\rho/\underline{\rho}(t)-1} \equiv \hat{g}_3(t). \tag{A.34}$$

For $\rho \leq 1$ and thus $1-B^* \geq \rho(1-B)$, we have $\hat{g}_3(t) \geq 1-B^*$, which noting that $1-B^* > \frac{\sigma-1}{\sigma}[1-\beta(\lambda^*)^{-\varepsilon}\psi_a^*]$ is sufficient for (A.34). In contrast, we have $\hat{g}_3(t) < 1-B^*$ if $\rho > 1$ and thus $1-B^* < \rho(1-B)$, and in this case it is not a priori clear that (A.34) holds. However, acknowledging that $\rho > 1$ gives $\hat{g}_3'(t) > 0$ while $\lim_{t\to\infty}\hat{g}_3(t) = 1-B^*$ holds for any ρ, it follows that (A.34) must be fulfilled for a sufficiently high level of t. Finally, using the definition of ρ, we can rewrite (A.34) as follows:

$$\frac{\sigma-1}{\sigma}\left[1-\beta\lambda^{-\varepsilon}\psi_a\right] < \frac{[(1-B^*)-\underline{\rho}(t)(1-B)]}{\rho-\underline{\rho}(t)}, \tag{A.35}$$

which together with (A.31) establishes (46). This completes the proof.

A.9 Proof of Proposition 3

To show Proposition 3, we first differentiate μ with respect to t. For this purpose, we introduce the two auxiliary expressions

$$\tilde{g}_0(t) \equiv \frac{\rho a(t) - b(t)}{a(t) + b(t)}, \qquad \tilde{g}_1(t) \equiv \frac{a(t) - \rho b(t)}{a(t) + b(t)}. \tag{A.36}$$

Differentiation of $\tilde{g}_0(t)$ and $\tilde{g}_1(t)$ gives

$$\tilde{g}_0'(t) = \tilde{g}_1'(t) = \frac{(\rho + 1)[a'(t)b(t) - a(t)b'(t)]}{[a(t) + b(t)]^2} > 0. \tag{A.37}$$

Furthermore, noting from Eq. (47) that $\mu = [\tilde{g}_0(t) - \rho B]/[\tilde{g}_1(t) - B^*]$, we can compute

$$\frac{d\mu}{dt} = \frac{\tilde{g}_0'(t)(1 - \mu)}{\tilde{g}_1(t) - B^*}, \tag{A.38}$$

which, accounting for $\mu >, =, < 1$ if $\rho(1 - B) >, =, < 1 - B^*$ establishes $d\mu/dt >, =, < 0$ if $1 - B^* >, =, < \rho(1 - B)$. Due to $\varphi_d = \varphi_d^*$ and $\varphi_x = \varphi_x^*$ it must still be true that the country hosting more sophisticated goods producers is the exporter of sophisticated goods. Noting further that the Grubel-Lloyd index remains to be given by Eq. (31) then establishes Proposition 3 and completes the proof.

A.10 Properties of price index (50) and proof of Proposition 4

Differentiating Eq. (50) with respect to t gives $dP_S/dt = [P_S/(\sigma - 1)]K(\rho)$, with

$$K(\rho) \equiv -\frac{d\varphi_d}{dt}\frac{\sigma - 1}{\varphi_d} - \frac{\overline{\rho}'(t)B\,[\rho(1 - B) - (1 - B^*)]}{[\overline{\rho}(t) - \rho B - (1 - B^*)]\,[\overline{\rho}(t)(1 - B) - (1 - B^*)]} \tag{A.39}$$

Acknowledging $d\varphi_d/dt < 0$, it follows that $dP_S/dt > 0$ if $\rho(1 - B) \leq 1 - B^*$. Due to differentiability, it is also clear that $dP_S/dt > 0$ extends to small positive differences of $\rho(1 - B)$ and $1 - B^*$. To see what happens if these differences are large, we can substitute $\overline{\rho}'(t) = (1 - B^*)[a'(t)b(t) - a(t)b'(t)]/b(t)^2$, and the derivative from fn. 11, and obtain

$$\begin{aligned} K(\rho) = {} & \frac{d\varphi_d}{dt}\frac{\sigma - 1}{\varphi_d}\left\{\frac{a(t)}{b(t)}\left[1 + \frac{a(t)}{b(t)}\right]\frac{B(1 - B^*)\,[\rho(1 - B) - (1 - B^*)]}{[\overline{\rho}(t) - \rho B - (1 - B^*)]\,[\overline{\rho}(t)(1 - B) - (1 - B^*)]} - 1\right\} \\ & + \frac{1}{b(t)}\left\{\frac{G'(\varphi_d)}{1 - G(\varphi_d)}\frac{d\varphi_d}{dt} - \frac{a(t)}{b(t)}\frac{G'(\varphi_x)}{1 - G(\varphi_d)}\frac{f_x}{f}\frac{d\varphi_x}{dt}\right\} \\ & \times \frac{B(1 - B^*)\,[\rho(1 - B) - (1 - B^*)]}{[\overline{\rho}(t) - \rho B - (1 - B^*)]\,[\overline{\rho}(t)(1 - B) - (1 - B^*)]}, \end{aligned} \tag{A.40}$$

where

$$\frac{\sigma - 1}{\varphi_x}\frac{d\varphi_x}{dt} = -\frac{a(t)}{b(t)}\frac{\sigma - 1}{\varphi_d}\frac{d\varphi_d}{dt} \tag{A.41}$$

has been used from Eq. (19). Finally, noting that

$$\lim_{t\to\infty}\frac{a(t)}{b(t)}\left[1+\frac{a(t)}{b(t)}\right]\frac{B(1-B^*)\left[\rho(1-B)-(1-B^*)\right]}{\left[\overline{\rho}(t)-\rho B-(1-B^*)\right]\left[\overline{\rho}(t)(1-B)-(1-B^*)\right]} \\ =\frac{B}{1-B}\left(\frac{\rho(1-B)}{1-B^*}-1\right) \tag{A.42}$$

is larger than one if $\rho B(1-B)>1-B^*$, we can safely conclude that $\rho B\geq 1$ is sufficient to ensure that $dP_S/dt<0$ holds for high levels of t. Noting further that $dh_S/dt >,=,<0$ if $\rho(1-B)>,=,<1-B^*$ and acknowledging $(\overline{e}/P_G)^{\varepsilon}\hat{\psi}=\lambda^{\varepsilon}\hat{\psi}_a[1+h_S(\nu^{\varepsilon}-1)]$ then establishes Proposition 4 and completes the proof.