Asset Pricing with Heterogeneous Agents: Estimation and Inference on International Stock Markets.*

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Abstract

We find that cross-sectional moments of countries' consumption growth are useful factors to explain variation in expected returns across international stock market indices. The skewness alone explains 17% of that variation. The first four consumption moments explain a proportion of variation that is similar to the three global Fama-French factors. We also address the weak-identification issue common in linear macro-factor models. Some consumption-based factors happen to be weakly identified and that might constitute a threat to statistical inference for the risk prices in multi-factor models. We find in particular that having one weakly-identified factor in a multi-factor model yields unbounded confidence intervals for all prices of risk.

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1 Introduction

This paper investigates if cross-sectional higher moments of countries' consumption growth are pricing factors that can explain heterogeneity in risk premium across international stock markets. We build on the heterogeneous agents' consumption-based asset pricing literature (Constantinides and Duffie (1996); Constantinides and Ghosh (2017)). In our framework, each country representative agent faces the country specific consumption shocks which (s)he would like to smooth out by investing on international financial markets[1]. Thus, country stock indices can be viewed as assets in the consumer's investable universe. The question that arises is whether actual expected excess returns on country stock indices are reward for exposure to cross-sectional higher moments of countries' consumption growth.

Cross-sectional first and second moments of consumption growth (mean and dispersion) have previously been considered in the literature as pricing factors for returns on international equity and foreign exchange markets (Sarkissian (2003); Darrat et al. (2011)). The cross-sectional third moment has recently been considered for explaining the equity premium on the US stock market (Constantinides and Ghosh (2017)). Here, we extend these previous studies by considering, on one hand, the cross-sectional skewness and kurtosis of consumption growth and, on the other hand, the developed countries stock markets[2].

There are reasons to believe that higher-order cross-sectional moments of consumption growth across countries might be plausible pricing factors for international stock indices.

[1]Indeed, by holding the country specific market portfolio, the consumer would remain exposed to the systematic risk which is negatively correlated with aggregate consumption growth. Foreign investment allows to achieve a more diversified portfolio than the ones offered by each country specific market portfolio. There is an increase of international financial markets integration. Nowadays, domestic investors can internationally diversify their portfolio by investing into foreign stock markets without further costs.

[2]We choose to focus on country specific stock market indices to avoid sample selection bias that could come from stock data availability and also to reduce the noise from the diversifiable risk components in our analysis.

A left-skewed distribution of countries' consumption growth portraits that country growth rates worse than the global average are extreme. This happens when consumption in few countries falls sharply or grows poorly in a way to pull down the world average growth rate. This is a bad news for the global economy, and more so when the underperforming growth rates come from systemically important countries. Thus, the average global investor dislikes stock markets which tend to decline when the cross-sectional skewness of country consumption growth rates falls, i.e., positive covariation with skewness. Therefore (s)he would require a larger premium for investing in these stock indices. Likewise, a leptokurtic cross-sectional distribution of countries' consumption growth signals that enough countries have their consumption growth rates sufficiently lagged from the global average. This again represents a bad news for the world economy, against which the average global investor hedges herself by requiring an additional reward on stock indices that tend to decline simultaneously with increasing cross-sectional kurtosis of countries' consumption growth rates, i.e., negative covariation with kurtosis.

We first observe that developed countries display an heterogeneity in their stock market exposure to the different cross-sectional moments of the consumption growth. For example, the Norwegian stock market index shows a positive risk exposure to the cross-sectional mean of consumption growth while US stock market index appears to be negatively exposed to the cross-sectional skewness of consumption growth. It means that the Norwegian stock market might reward investors for holding its market portfolios because it performs poorly at the same time the world average consumption growth decreases, while the US stock market rewards investors for holding its market portfolio because it performs poorly when the skewness of the cross-section of countries consumption growth increases, meaning when a higher proportion of countries has a consumption growth realization above the average[3]. Indeed, the consumption growth for Norway is positively

[3]This explanation only makes sense if the price of the skewness risk is negative. With a positive price of skewness risk, the US stock market portfolio will appear as an insurance contract and the consumer

correlated with the world consumption growth while the US consumption growth is negatively correlated with world consumption skewness.

Second, we find that the higher order cross-sectional moments of the consumption growth have an important contribution in explaining the country levels of the equity risk premium. The skewness factor alone explains 17% of the variability of country portfolio expected returns and it has a higher explanatory power than consumption mean and variance factors for the variability in countries expected stock market excess return. Furthermore, the four consumption-based factors model with the first four cross-sectional moments of consumption growth explains 26% of the variability in countries expected stock market excess return and yields the smallest mean absolute pricing errors. It achieves an explanatory power similar to the Fama-French global three factors model.

Our econometric modeling considers the recent developments in empirical assets pricing which aim at ruling out the spurious factors and providing credible statistical inference for the prices of risk. Indeed, Kleibergen (2009); Kleibergen and Zhan (2015) highlight the rank deficiency that occurs at the first stage of the Fama and MacBeth (1973) regressions and which shows a weak identification of the prices of risk in the second stage of the regression. We observe[4] a high absolute variability of countries stock markets exposures with respect to the first cross-sectional moment of consumption growth, which range from 0.07 to 2.39. However, the variability in their exposures to the higher order cross-sectional moments of consumption (variance, skewness and kurtosis) is lower; the betas vary respectively from -0.6 to 0.3, from -1.1 to 0.5 and from -0.1 to 0.2, for the variance, the skewness and the kurtosis of the cross-sectional distribution of consumption growth[5]. The lack of variability in the betas from the first pass regression even though some betas

will be paying a premium for holding the contract.

[4]See 7.

[5]The relative variability (the variability of the standardized beta) is similar across the risk factors ranging from -1.5 to 2.5, from -2.0 to 1.6, from -1.3 to 1.8 and from -1.9 to 2.4 for respectively the mean, the variance, the skewness and the kurtosis factors.

are statistically different from zero is what makes the identifiability of the prices of risk difficult (Kleibergen and Zhan (2019)). The tests of identification of risk prices for the different consumption-based pricing factors reveal that the consumption growth cross-sectional mean and skewness are strongly identified[6], while the cross-sectional variance and skewness are weakly identified. This observation helps to better understand our regressions output and implies to be cautious in the interpretation of the results.

The remainder of the paper is organized as follows: In section 2, we provide a brief literature review in order to connect this paper to the heterogeneous agents models literature. Section 3 provides the theoretical background to establish the link between the equity risk premium and the cross-sectional higher order moments of consumption growth in a heterogeneous agents model. It also highlights the difference with the representative agent model. Section 4 presents the empirical approach. It presents the data and descriptive statistics. It describes the different econometric techniques (GMM, Fam-MacBeth regression and Bayesian MCMC) that are used in the estimation and it also provides the results of our estimations. Section 5 concludes. An appendix is provided in section 6 with Tables, Figures and details about the calculus.

2 Literature review

The understanding of the driving forces behind the observed levels and fluctuations of asset prices is still a challenging and very prolific area of research in finance. Since the seminal paper of Markowitz (1952) that sets the building blocks of Modern Portfolio Theory, followed by the design of the capital asset pricing model (CAPM) by Sharpe (1964), Lintner (1965) and Mossin (1966), and the seminal contribution of the consumption-

[6]This result echoes to the recent debate about the identification of the risk premia in consumption-based factors models (Kleibergen and Zhan (2019), Kroencke (2020)). The different measures of assets exposure to consumption in the U.S yield different strength of identification of the consumption price of risks. We add some international evidence about the identification of the consumption prices of risk.

based CAPM of Lucas (1978) that connects asset prices to economic fundamentals (such as consumer preferences, aggregate consumption growth, etc.), researchers have learned a lot about those driving forces by confronting finance theories to observed macroeconomic and financial market data. Some puzzles have been raised from the discrepancy between the predictions of the proposed models and the observed asset prices. One of the well known puzzles is the equity premium and risk free rate puzzle that was pointed out in the standard consumption-based CAPM model, henceforth CCAPM, with a representative agent endowed with time separable utility function and log-normal consumption[7].

In response to these puzzles, a strand of the literature has proposed to move from the representative agent framework to the heterogeneous agents models where the cross-sectional dispersion in agents consumption matters in the determination of equilibrium risk premium (Constantinides and Duffie (1996), Sarkissian (2003)). More recently, Constantinides and Ghosh (2017) have shown that beyond the cross-sectional dispersion, the cross-sectional skewness in households consumption growth also matters for equilibrium asset prices. Indeed, in their framework economic agents have recursive preferences (Kreps and Porteus (1978); Epstein and Zin (1989)) and households face labor income uninsurable risks that lead to left skewness in the cross-sectional consumption growth distribution, which is counter-cyclical and pushes investors to ask for a high compensation to hold risky assets. In an otherwise representative agent model, the log-normally distributed aggregate consumption they considered in their model would have generated the equity premium puzzle, however as they showed, the cross-sectional distribution in households consumption enables to create enough consumption risks and to alleviate the puzzle.

This paper investigates the cross-sectional differences in countries discount rates for

[7]The puzzle arises from the fact that the observed equity premium is too high and the observed risk free rate is too low to be explained by a standard CCAPM model where the representative agent has a reasonable coefficient of relative risk aversion (below 10), given the smooth observed consumption growth (Mehra and Prescott (1985); Rietz (1988); Weil (1989); Mehra (2003) and see Mehra (2003) for a review).

risky investment projects. It aims at explaining the differences in countries risk premiums observed on financial markets. This question is connected to the literature that explains the country differences in currency excess return observed on the foreign exchange market. For example, Sarkissian (2003) used the framework proposed by Constantinides and Duffie (1996) to analyze the impact of imperfect consumption risk sharing in a multi-country world on the formation of time-varying risk premiums in foreign exchange market and on their cross-sectional differences. He found that the heterogeneous agent CCAPM model that accounts for the world consumption dispersion, enables to better explain the currency risk premiums observed on foreign exchange market than the representative agent CCAPM do. However, the empirical exercise also reveals some difficulties of the model to fully explain the forward premium puzzle. Some possible solutions suggested by the author was to account for within country consumption dispersion and to use wealth-based measure for world growth and dispersion in the asset pricing model instead of only consumption based ones. This paper will go in that direction by considering the consumption per capita weighted cross-sectional moments of country consumption growth instead of the equally weighted moments.[8]

Constantinides and Ghosh (2017) extend the Constantinides and Duffie (1996) framework by allowing agents to have identical recursive preferences and the idiosyncratic consumption shocks to be driven by a Poisson mixture of normal distribution. They show that this household consumption growth specification enables to generate countercyclical higher order moments of the cross-sectional consumption growth distribution even though the aggregate consumption process is log-normally distributed. The state variable (i.e. the household consumption risk) that drives the cross-sectional consumption growth distribution determines the market equity risk premium. Schmidt (2016) has shown that the idiosyncratic distribution of household consumption is affected by tail events which

[8]The analysis presented along the paper is made with the weighted cross-sectional moments. We also consider the equally weighted cross-sectional moments and the results are similar.

also affect the equity premium in a heterogeneous agents model with incomplete markets. The cross-sectional kurtosis is considered in this paper to capture the tail events in the cross-sectional consumption growth distribution.

This paper also relates to the literature on international equity markets. Indeed, in segmented financial markets, there exist some specific markets risk premiums justified by the costs that the investor could incurred to retrieve his funds or the capital lost that could happen due to country specific political risks (Bekaert et al. (2016)). In such a context, the country specific investor will face some idiosyncratic risks that (s)he would like to smooth out by investing internationally on financial markets; however by doing so (s)he will incurred the hosting countries specific political risks which (s)he can not diversify. In this paper, we choose to simplify the investor problem by ignoring the market segmentation. We behave as if the financial markets are fully integrated and there is no undiversifiable country specific risks incurred by investing abroad. Even though, this might sound as a strong assumption, it is still valuable to understand how the cross-sectional distribution of investor idiosyncratic risks might affect the equity risk premium and this paper contributes in doing that. Furthermore, we have recently seen a gradual increase in the international financial markets integration that facilitates the ownership of foreign assets by domestic investors without incurring much costs.

3 Theoretical background

3.1 International heterogeneous agents CCAPM

We assume the world is populated by n country representative consumers indexed by i. Each country's consumer receives a labor income and (s)he can trade on the international financial market. The proceedings from the labor income and the financial assets[9] is used to buy the amount of the single numeraire good that is consumed. Following, Constantinides and Ghosh (2017); Constantinides and Duffie (1996), we banned the consumer from being able to insure against income idiosyncratic shocks through the market. Thus, the markets are incomplete, meaning that for example there are no traded state contingent goods to hedge against the possible consumption decline in country j due to state budget disagreement. The countries representative consumers have identical recursive preferences:

$$V_{i,t} = \left[(1-\delta) C_{i,t}^{1-1/\psi} + \delta \left(\mathrm{E}_t \left(V_{i,t+1}^{1-\gamma} \right)^{\frac{1-1/\psi}{1-\gamma}} \right) \right]^{\frac{1}{1-1/\psi}} \tag{1}$$

where δ is the subjective discount factor, γ is the relative risk aversion (RRA) coefficient and ψ is the elasticity of inter-temporal substitution (EIS). The heterogeneity amount the consumers comes from their specific consumption dynamics given by:

$$C_{it} = I_{it} + D_t = \delta_{it} C_t \tag{2}$$

where I_{it} is the consumer i labor income, C_t is the aggregate (world) consumption and D_t is the dividend paid by the market portfolio. δ_{it} captures the country idiosyncratic risk which comes from the labor income (or the aggregate output) of the country and evolves as follows.

$$\delta_{i,t} = \delta_{i,t-1} \exp\left(\eta_{i,t} \sigma \sqrt{d_t} - \sigma^2 \frac{d_t}{2} \right) \tag{3}$$

where d_t is conditionally distributed Poisson process driving the occurrence of the consumer i idiosyncratic shocks at time t. Following Constantinides and Duffie (1996), d_t can be viewed as the cross-country variation of countries consumption growth at time t. It can also summarize the combination of cross-sectional higher order moments conditional on information at time t

[9] The financial assets are made by all kinds of existing traded assets (e.g. securities, bonds, derivatives, etc.)

that influences the dynamics of the consumer consumption risk and which drive asset prices. $\eta_{i,t}$ is an i.i.d standard normal capturing the idiosyncratic shock occurring in country i at time t. The consumer i consumption growth is then given by:

$$\Delta c_{i,t+1} = \eta_{i,t+1}\sigma\sqrt{d_{t+1}} - \sigma^2\frac{d_{t+1}}{2} + \Delta c_{t+1} \tag{4}$$

Its dynamic is driven by the country's income idiosyncratic shock and the world aggregate consumption dynamics. The aggregate consumption growth is i.i.d normal with mean μ and variance σ_c^2: $\Delta c_{t+1} = \mu + \sigma_c\epsilon_{t+1}$, $\epsilon_{t+1} \sim$ i.i.d$N(0,1)$. As shown in Epstein and Zin (1989), the Stochastic Discount Factor (SDF) of consumer i is:

$$M_{i,t+1} = \exp\left(\theta\log\delta - \frac{\theta}{\psi}\Delta c_{i,t+1} + (\theta-1)\, r_{i,c,t+1}\right) \tag{5}$$

where $\theta = \frac{1-\gamma}{1-1/\psi}$ and $r_{i,c,t+1}$ is the log-return on the aggregate wealth portfolio of consumer i. The wealth portfolio delivers consumption as dividend each period. We assume that the Poisson distribution followed by d_t is governed by the unique state variable of the model ω_t, which drives the consumers income shocks: Prob$(d_t = n)$=$e^{-\omega_t}\omega_t^n/n!$, $n = 0, 1, ...\infty, \mathbb{E}(d_t) = \omega_t$. Following Constantinides and Ghosh (2017) to ease the computational exposure, we define the scaled state variable $x_t \equiv \left(e^{\gamma(\gamma-1)\sigma^2/2} - 1\right)\omega_t$. We assume that x_t follows an auto-regressive gamma process of order 1, ARG(1) (Gourieroux and Jasiak (2006)).

$$x_{t+1} = \nu\xi + \rho x_t + \varepsilon_{x,t+1}, \tag{6}$$

where $\nu > 0$, $\xi > 0$, and $\rho > 0$, $\varepsilon_{x,t+1}$ is a martingale difference sequence, $\mathbb{E}\left(x_{t+1}|x_t\right) = \nu\xi + \rho x_t$, and var $\left(x_{t+1}|x_t\right) = \nu\xi^2 + 2\rho\xi x_t$.

As shown by Constantinides and Ghosh (2017), the above consumption dynamics delivers an autarchy equilibrium where country representative consumer consumption growth and SDF are independent on its consumption level. The SDF of consumer i depends on the return on its wealth portfolio which is unobservable. In order to compute the SDF, we assume that the consumer wealth-consumption ratio is an affine function of the state variable:

$$z_{i,c,t} = z_{c,t}\left(\omega_t\right) = A_0 + A_1 x_t \tag{7}$$

Using the Campbell and Shiller (1988) approximation, we can expressed the log-return on its wealth portfolio as follows:

$$r_{i,c,t+1} = r_{c,t+1}(\omega_t) = \kappa_0 + \kappa_1 z_{c,t+1}(\omega_{t+1}) - z_{c,t}(\omega_t) + \Delta c_{i,t+1}$$

Substituting the expression of the SDF and the wealth portfolio return in the standard asset pricing Euler equation, we can solve for the unknown coefficients A_0 and A_1, which are obtained by solving the non-linear system of equations (36).

The consumers common SDF can be expressed in term of the state variable as follows:

$$\begin{aligned} M_{i,t+1} = \exp & \left(\theta \log \delta - \gamma \left(\eta_{i,t+1}\sigma\sqrt{d_{t+1}} - \sigma^2 \frac{d_{t+1}}{2} + \Delta c_{t+1}\right) + (\theta - 1)\left(\kappa_0 + \kappa_1\left[A_0 + A_1 x_{t+1}\right]\right)\right. \\ & \left. - (\theta - 1)\left[A_0 + A_1 x_t\right]\right) \end{aligned} \tag{8}$$

Equation (8) shows that only shocks that affect the distribution of the aggregate consumption will be priced in equilibrium. There is a distinction between the time series distribution of the aggregate consumption and the evolution of cross-sectional distribution of the countries consumption. Both dimensions are sources of aggregate risk for country's representative consumers. On one hand, a drop in the aggregate level of consumption increases the marginal utility of the consumption good for every consumer; the effect is higher for a higher risk aversion coefficient. On the other hand, an increase in the cross-sectional dispersion of the consumption (combined with the idiosyncratic risk) will put each consumer at a higher risk of getting a low consumption, which also raises the SDF.

The log risk free rate can be obtained by taking the conditional expectation of the log SDF.

$$r_t^f = \Gamma_0^f + \Gamma_1^f x_t \tag{9}$$

where

$$\begin{aligned}
\Gamma_0^f &= -\theta \log \delta + \gamma\mu - \frac{1}{2}\gamma^2\sigma_c^2 - (\theta - 1)\left(\kappa_0 + (\kappa_1 - 1)\, A_0\right) - \lambda\nu\xi - \frac{1}{2}\lambda^2\nu\xi^2 \\
\Gamma_1^f &= -\left(\lambda\rho\,(1 + \lambda\xi) - (\theta - 1)A_1\right)
\end{aligned}$$

The first three terms in the expression of the risk free rate are very common especially when $\theta = 1$ as it is the case for the CRRA utility function. They show that the risk free rate will increase when the preference for the present is higher (δ closed to zero) or the consumption growth is positive; and the risk free rate will decrease when the consumption growth is more volatile (for precautionary saving motives). The new term that depends on the moment of the consumer risk (x_t) will reduce the risk free rate as expected. When the consumption risk increases, agents choose to increase their savings buffer thus lowering the risk free rate. As highlighted by Constantinides and Ghosh (2017), the model predicts that the unconditional mean of the risk free rate will decrease with an increase in the consumer risk and its unconditional variance will increase; consistently with the observed data.

Let us now consider a risky project in country i that generates levered consumption dividends:

$$\Delta d_{i,t+1} = \mu_d + \beta_{d,i}\Delta c_{i,t+1} + \sigma_d \varepsilon_{i,d,t+1} \tag{10}$$

where $\varepsilon_{i,d,t+1} \sim$ i.i.d$N(0,1)$ and it is independent of all the other shocks. $\beta_{d,i}$ captures the exposure of the dividend growth to consumption growth.

To compute the equity return, we follow the same procedure used for the wealth portfolio. We assume that the price-dividend ratio is an affine function of the state variable.

$$z_{i,m,t} = z_{m,t}\left(\omega_t\right) = A_{0m} + A_{1m}x_t$$

Substituting expression of the log-price dividend ratio in the standard Euler equation for the equity return allows to solve for the unknown coefficient and to obtain the expected stock market return as follows. The expected stock market return for consumer i is given by:

$$\mathbb{E}_t r_{i,m,t+1} = \Gamma^m_{0,i} + \Gamma^m_{1,i} x_t \tag{11}$$

where

$$\Gamma^m_{0,i} = \kappa_{0m} + \left(\kappa_{1m} - 1\right) A_{0m} + \left(\kappa_{1m} A_{1m} + \frac{\beta_{d,i}\sigma_d^2}{2\left(e^{\gamma(\gamma-1)\sigma^2/2} - 1\right)}\right)\nu\xi + \mu_d$$

$$\Gamma^m_{1,i} = \rho\left(\kappa_{1m} A_{1m} + \frac{\beta_{d,i}\sigma_d^2}{2\left(e^{\gamma(\gamma-1)\sigma^2/2} - 1\right)}\right) - A_{1m}$$

Finally, the market risk premium for consumer i can be deduced by subtracting the risk free rate in equation (9) from the expected stock market return in equation (11) to obtain:

$$rp_{i,m,t} = \mathbb{E}_t r_{i,m,t+1} - r_{f,t} = \Gamma_{0,i} + \Gamma_{1,i} x_t \tag{12}$$

where

$$\Gamma_{0,i} = \Gamma^m_{0,i} - \Gamma^f_0$$

$$\Gamma_{1,i} = \Gamma^m_{1,i} - \Gamma^f_1$$

At this stage, the reader may wonder how does the risk premium relate to the cross-sectional moments of consumption growth? And how do they explain the variation in average market returns across countries? The answer to the first question is given by the presence of the country consumption risk (x_t) in equation (12) that defines the risk premium and the answer to the second question of the reader is provided by the the risk exposure of the country dividend growth to the country consumption risk ($\beta_{d,i}$). Indeed, the cross-sectional moments of the countries consumption growth distribution are determined by the country consumption risk x_t as we can see from subsection 6. We can see that an increase in the consumption risk reduces the average consumption growth, increases the consumption growth cross-sectional volatility, reduces the consumption growth cross-sectional skewness and increases the consumption growth cross-sectional kurtosis. Furthermore, the higher the exposure of a country to the consumption risk, the higher its risk premium since the consumer in that country will face a higher risk that the returns on its risky asset will fall at the same time its consumption is at the bottom.

3.2 Model's estimation

We estimate the structural parameters of the model described above using the cross-sectional moments of the countries consumption growth, returns on country's market index and the US. risk free asset. We first compute the aggregate SDF which is obtained as the cross-sectional expectation of the individual stochastic discount factor in equation 8.

$$
\begin{aligned}
M_{t+1} &= E\left(M_{i,t+1}|I_{t+1}\right) \\
&= \exp\left(\theta\log\delta - \gamma\Delta c_{t+1} + \gamma(\gamma+1)\sigma^2\frac{d_{t+1}}{2} + (\theta-1)\left(\kappa_0+\kappa_1\left[A_0+A_1x_{t+1}\right]\right)\right. \\
&\left.-(\theta-1)\left[A_0+A_1x_t\right]\right)
\end{aligned} \tag{13}
$$

We remind that $\sigma^2 d_{t+1}$ is the cross-sectional variance of countries consumption growth as given by equation (4), which is obtained by taking the expectation conditional on the information at $t+1$ such that the remaining randomness is the one across countries. Let denote by $\hat{\sigma}\left(\Delta c_{i,t+1}\right)$ a consistent estimate of $\sigma^2 d_{t+1}$. As $x_t \equiv \left(e^{\gamma(\gamma-1)\sigma^2/2}-1\right)\omega_t$ and as we relate the cross-sectional centered moments of countries consumption growth to ω_t[10], we can compute an estimate $\hat{x}_t$ of x_t using these moments. We can now compute an estimate $\hat{M}_{t+1}$ of the aggregate SDF to price the set of countries market portfolio in our hands.

$$
\begin{aligned}
\hat{M}_{t+1} = \exp&\left(\theta\log\delta - \gamma\Delta c_{t+1} + \frac{1}{2}\gamma(\gamma+1)\hat{\sigma}\left(\Delta c_{i,t+1}\right) + (\theta-1)\left(\kappa_0+\kappa_1\left[A_0+A_1\hat{x}_{t+1}\right]\right)\right. \\
&\left.-(\theta-1)\left[A_0+A_1\hat{x}_t\right]\right)
\end{aligned} \tag{14}
$$

The log-linearization coefficient κ_0 and κ_1 are obtained simultaneously with the affine coefficients of the wealth-consumption ratio (7) by solving the non-linear equation system and a fixed point problem that determines the mean wealth-consumption ratio.

[10] See equation (39) - (42) in appendix.

$$
\begin{cases}
\theta A_0 & = \theta \log \delta + \theta \kappa_0 + \theta \kappa_1 A_0 + (1-\gamma)\,\mu + \frac{(1-\gamma)^2}{2}\sigma^2 + (1+\theta\kappa_1 A_1)\,\nu\xi \left(1 + \frac{1}{2}\left[1+\theta\kappa_1 A_1\right]\xi\right) \\
\theta A_1 & = \rho\,(1+\theta\kappa_1 A_1)\,(1 + \left[1+\theta\kappa_1 A_1\right]\xi) \\
\kappa_1 & = \frac{e^{\bar{z}_c}}{1+e^{\bar{z}_c}} \\
\kappa_0 & = \log\left(1+e^{\bar{z}_c}\right) - \kappa_1 \bar{z}_c \\
\bar{z}_c & = A_0 + A_1 \left(\frac{\nu\xi}{1-\rho}\right)
\end{cases}
\tag{15}
$$

The last line is obtained by computing the unconditional expectation of x_t from equation (6). The estimated aggregate SDF allows to connect asset returns to the deep parameters of the economy: Those driving the preferences of the consumer and the ones driving the cross-sectional distribution of consumption growth. We stack all the parameters in one vector

$$
\Theta = (\delta, \gamma, \psi, \mu, \sigma, \nu, \xi, \rho) \tag{16}
$$

We use the developed countries market returns denote by R_s as test assets to estimate Θ. In order to put more discipline in the estimation of the parameters, we also add the US t-bill rate as the risk free asset (R_b) and some unconditional moments of cross-sectional consumption growth distribution. The unconditional moments of consumption growth are the following: The mean of the cross-sectional average of countries consumption growth, the mean and variance of cross-sectional variance of countries consumption growth, the mean and variance of cross-sectional skewness of countries consumption growth. Let denote by $e_t(\Theta)$ the vector of moment conditions conditional on Θ, which combines model's pricing errors and differences between empirical and theoretical moments of cross-sectional distribution of country consumption growth:

$$e_t(\Theta) = \begin{pmatrix} 1 - \hat{M}_t \left[R'_{s,t} \; R'_{b,t}\right]' \\ \mu - cg_t \\ \text{m\_cross\_var} - varcg_t \\ \text{v\_cross\_var} - \left[varcg_t^2 - \overline{varcg_t}^2\right] \\ \text{m\_cross\_skew} - skewcg_t \\ \text{v\_cross\_skew} - \left[skewcg_t^2 - \overline{skewcg_t}^2\right] \end{pmatrix}$$

Where theoretical unconditional moments of cross-sectional distribution of country consumption growth (m_cross_var, v_cross_var, m_cross_skew and v_cross_skew) are defined in appendix 6. The moment conditions used for the GMM estimation are given by:

$$E\left[e_t(\Theta)\right] = 0$$

We have 24 moment conditions to estimate 8 parameters and our model is over-identified. We use the identity matrix as the weighting matrix in the GMM in order to estimate Θ. By doing so, we take a stand for the unbiasness in the usual bias-efficiency trade-off (Cochrane (2005)).

4 Empirical Analysis

4.1 Data and Descriptive statistics

The data used for the empirical analysis are sourced from Datastream for consumption (PFCE), CPI, asset prices, and from the FRED database for OECD recession indicator. We use assets data from Morgan Stanley Capital International (MSCI) stock market indexes for 18 countries for which we also get the quarterly data on consumption[11]: Australia, Austria, Belgium, Canada, Denmark, France, Germany, Hong Kong, Italy, Japan, Netherlands, Norway, Singapore, Spain, Sweden, Switzerland, the UK, and the US. The data cover the period from 1970Q1 to 2018Q4. The indexes are value-weighted and adjusted for dividend reinvestment. We compute the real per capita consumption growth by subtracting inflation and population growth from the aggregate consumption growth. The excess return are computed by subtracting the US 1-month Tbill rate from the indexes returns[12]. The cross-sectional moments of consumption are weighted using the country consumption levels expressed in dollars as weights.

The descriptive statistics presented in Table 2 show an annualized average excess return of 4.3% on the world portfolio with a standard deviation of 16.7%. There is a huge dispersion in the country portfolio excess returns; the minimum average annualized excess return is 0.17% observed in Italy and the maximum in 9.1% in Hong Kong. As usual, consumption growth is much smoother than equity returns. The dispersion is less pronounced for the average consumption growth across country. The annualized world average consumption growth is 2% with a standard deviation of 1.4%. The minimum value for the country average consumption growth is 1% observed in Switzerland and the maximum is 4.9%, observed in New Zealand.

It is interesting to look at the correlation matrix between the excess returns on country market portfolios, the recession indicator and the cross-sectional moments of consumption growth. We observe that the country market excess returns are negatively correlated with recession indicator and positively correlated with world excess market return, meaning that country excess return drops during recession period and it increases with the world

[11]We also use consumption data of Finland, Greece, Portugal, South Africa, South korea. We do not have a long sample data for consumption of Hong Kong.

[12]We used the MSCI return index denominated in US dollar instead of in local currency. By doing so we adopt the view point of a US investor and we can easily compare foreign assets. We also considered the MSCI return index denominated in local currency for the same exercise and the results were very similar.

market portfolio return. Furthermore, as expected there is a negative correlation between the cross-sectional mean of consumption growth and recession indicator; and a positive correlation between the recession indicator and the cross-sectional variance. Thus, the cross-sectional average of country consumption growth drops during recessions, while the dispersion between countries consumption growth increases.

More surprisingly, we observe a positive correlation between cross-sectional skewness and recession indicator, meaning that the distribution of country consumption growth is more likely to be positively skewed during recessions compared to normal times. This observation contrasts with one made by Constantinides and Ghosh (2017) in the US where they found a negative correlation between recessions and cross-sectional household consumption growth skewness. In their case, the household consumption growth distribution is more negatively skewed during recessions compared to normal times since a higher proportions of households are more likely to experience a drop in their consumption during recession periods compared to normal times.

Then, what could explain the different behavior of the cross-sectional consumption growth distribution we observe internationally during recession? A possible answer can come from the fact that our sample is made by developed countries whose consumption growth rates are more stable. To make the parallel with the US case, this will correspond to look at the consumption growth distribution in the upper quantile of the household consumption distribution. The richest country are more likely to be less affected by recessions and during those periods, to have a consumption growth that will not differ too much from the one they have during normal times, but the less rich country are the one more likely to experience a negative consumption growth. Thus, given that among the countries we sample there is a higher proportion of rich and more stable countries, then we expect to see a positive shift of the cross-sectional consumption growth distribution (to the right) during recession period because the few number of countries experiencing a negative consumption growth will lower the cross-sectional average consumption growth. However, the median country consumption growth driven by the higher proportion of rich countries will be above the mean, hence an increase of skewness in the country cross-sectional consumption growth distribution during recessions.

Finally, We observe that the cross-sectional skewness of consumption growth is positively correlated with the cross-sectional mean; it is negatively correlated with both the cross-sectional variance and the cross-sectional kurtosis. This observation seems to align more with the commonly admitted business cycles co-movements where during recessions, average consumption growth drops, cross-sectional dispersion increases, cross-sectional skewness becomes more negative and cross-sectional kurtosis increases. However, since the cross-sectional skewness is positively correlated with the recession indicator, the correlations between the cross-sectional skewness and the other cross-sectional moments go in the opposite direction compared to the common business cycles co-movements.

Thus, there are two antagonistic forces driving the co-movements between the business cycles and the cross-sectional consumption growth moments; one that comes from the sample of developed countries under study as explained in the previous section and the other commonly admitted co-movements where the cross-sectional dispersion of consumption growth increases during recessions, cross-sectional consumption mean drops, while cross-sectional skewness decreases and cross-sectional kurtosis increases. Therefore, the analysis needs to be pushed further to determine how the cross-sectional moments of consumption growth relate to the countries expected excess returns.

4.2 Linear Factor Model

Assuming the absence of arbitrage and the law of one price on financial markets, we know that there exist a positive SDF that enables to price any assets (Cochrane (2005)). The return on a given asset i in excess of the return on the risk free asset should satisfy the standard Euler asset pricing equation given by:

$$\mathbb{E}_t \left[M_{t+1} R^e_{i,t+1} \right] = 0 \tag{17}$$

Moreover, we assume that the SDF is a linear combination of K underlying factors, that is:

$$-\frac{M_{t+1}}{\mathbb{E}(M_{t+1})} = k + b' f_{t+1} \tag{18}$$

where b is the vector of coefficients that define the SDF as a linear function of the factors. Plugging the SDF equation (18) in the Euler equation (17), we can compute the expected excess return as follows:

$$\begin{aligned} \mathbb{E}_t \left[R^e_{i,t+1} \right] &= cov_t \left[-\frac{M_{t+1}}{\mathbb{E}(M_{t+1})}, R^e_{i,t+1} \right] \\ &= \mathbb{E}_t \left[R^e_{i,t+1} \left(f_{t+1} - \mu_f \right)' b \right] \end{aligned} \tag{19}$$

where $\mu_f = \mathbb{E}\left[f_{t+1} \right]$. We can estimate the parameters of the model (b, μ_f) by GMM[13] using the following moment conditions:

$$e(z_t, \theta) = \begin{bmatrix} R^e_t - R^e_t \left(f_t - \mu_f \right)' b \\ f_t - \mu_f \end{bmatrix} \otimes Z_t \tag{20}$$

where $\otimes$ refers to the Kronecker product and Z_t is the vector of instruments.

The standard Euler asset pricing equation enables to obtain the beta formulation of

[13] See appendix 6 for more details.

expected return as follows:

$$\begin{aligned}\mathbb{E}\left[R^e_{i,t+1}\right] &= cov\left[-\frac{M_{t+1}}{\mathbb{E}(M_{t+1})}R^e_{i,t+1}\right] \\ &= b^{'}cov(f_{t+1}, R^e_{i,t+1}) \\ &= \lambda^{'}\beta_i \end{aligned} \tag{21}$$

where $\lambda = \Sigma_{ff}b$ and $\beta_i = \Sigma_{ff}^{-1}\Sigma_{fi}$ with $\mu_f = \mathbb{E}\left[f_{t+1}\right]$, $\Sigma_{ff} = \mathbb{E}\Big[\left(f_{t+1} - \mu_f\right)\left(f_{t+1} - \mu_f\right)^{'}\Big]$ and $\Sigma_{fi} = \mathbb{E}\big[\left(f_{t+1} - \mu_f\right)R^e_{i,t+1}\big]$. λ and β_i are respectively the factors risk premium and the quantities of factors risk embedded in the asset i.

We follow the Fama and MacBeth (1973) cross sectional regressions procedure to estimate the quantities of risk and the risk premium attached to each factor. In short, in a first step we do a time series regression of the excess returns on the factors to obtain an estimate of β_i for each asset.

$$R^e_{i,t} = a_i + \beta_i^{'}f_t + \epsilon_{i,t} \tag{22}$$

In a second step, for each time period t, we do a cross-sectional regression[14] of excess return on the beta to get a time series estimate of risk premium vector λ and a time series estimates of the pricing errors α_i. From the beta formulation in equation (21), the excess return on asset i for each time period can be expressed as follows:

$$R^e_{i,t} = \lambda_t^{'}\beta_i + \alpha_{i,t},\ \ i = 1, 2, ..., N \text{ for each } t. \tag{23}$$

[14]There is always a question of whether to include the intercept or not in this regression. This has to do with the trade-off between statistical efficiency and the clarity and economically interpretability of the results often faced in empirical asset pricing (Cochrane (2005), p. 293). Indeed, as highlighted by Savov (2011), including the intercept gives the model more freedom but it can lead to poorly estimated factor premia when there is little variation in betas. Furthermore, even-though it improves the goodness of fits, including the intercept also implies a paradoxical risk-free rate that has non-zero excess return relative to itself.

In the case where the expected asset returns are fully spanned by the betas of the specified risk factors, the average pricing error $\alpha_i \equiv \mathbb{E}(\alpha_{i,t})$ should be equal to zero. Otherwise, $\alpha_i \neq 0$ and the model is considered as misspecified.

Figure 9 shows the evolution of cross-sectional moments of consumption growth that we considered as pricing factors. The OECD and Non OECD countries recession periods are also represented by the blue bars. The cross-sectional mean of consumption growth displays negative peaks during recessions especially during the first and second oil crises respectively in 1973-1974 and 1979-1980, the 1990 oil price shock and the 2008 financial crisis. The cross-sectional variance of consumption growth displays some positive peaks during the recession periods mentioned previously except the 2008 financial crisis. However, we do observe a positive peak of the cross-sectional variance of consumption growth during the 1997 Asian financial crisis where the cross-sectional mean of consumption growth does not seem to be negatively affected. After that period, we observe that the cross-sectional variance has become more stable compared to the past. The cross-sectional skewness of consumption growth presents a different path than the cross-sectional mean. During the first and second oil crises it displays a positive peak but for subsequent recessions we observe negative peaks. Similarly, the cross-sectional kurtosis follows a different path than the cross-sectional variance. Overall, we can see that the different cross-sectional moments of consumption growth do not follow exactly the same path and they can be useful in capturing different international economic issues.

Table 7 reports the risk exposures of the tested assets to the consumption risk factors in the four models that we consider: the standard consumption-based capital asset pricing model, denoted CCAPM where the cross-sectional mean of consumption growth is the unique pricing factor; the two factors model considered by Darrat et al. (2011) denoted DLP with cross-sectional mean and variance of consumption growth as the pricing factors; the Skewness only model with the cross-sectional skewness of consumption growth as the unique pricing factor and the four factors model denoted HWCCAPM with the first four cross-sectional moments of consumption growth (mean, variance, skewness and kurtosis) as pricing factors. We report the p-value for testing the rank deficiency of the $\mathbf{B}$ matrix of betas from the first stage of the FM regressions (see Kleibergen and Zhan (2019)). It shows that the CCAPM model is not rank deficient, which also means that there is enough variability in the returns exposures to risk factors in order to (strongly) identify the risk price ($\lambda_{\Delta c}$). The test also rejects the weak identification of the risk price in the Skewness model. However, the DLP model and the HWCCAPM both happen to be weakly identified[15].

Finally, following Cochrane (2005), p. 246, λ and α_i can be estimated as the time average of the cross-sectional regressions estimates.

$$\hat{\lambda} = \frac{1}{T}\sum_{t=1}^{T}\lambda_t, \qquad \hat{\alpha}_i = \frac{1}{T}\sum_{t=1}^{T}\hat{\alpha}_{it}. \tag{24}$$

and their sampling errors are given by:

$$\sigma^2\left(\hat{\lambda}\right) = \frac{1}{T^2}\sum_{t=1}^{T}\left(\hat{\lambda}_t - \hat{\lambda}\right)^2, \qquad \sigma^2\left(\hat{\alpha}_i\right) = \frac{1}{T^2}\sum_{t=1}^{T}\left(\hat{\alpha}_{it} - \hat{\alpha}_i\right)^2. \tag{25}$$

The joint significance of the pricing errors can be tested using the chi-squared asymptotic distribution as follows:

$$\alpha'\hat{\Omega}^{-1}\alpha \sim \chi^2\left(N-K\right) \tag{26}$$

where

$$\hat{\Omega} = \frac{1}{T^2}\sum_{t=1}^{T}\left(\hat{\alpha}_{it} - \hat{\alpha}_i\right)\left(\hat{\alpha}_{it} - \hat{\alpha}_i\right)'$$

[15]We also considered solo-factor models with cross-sectional variance and kurtosis. The rank deficiency test shows that both models are weakly identified. The results of these tests are not reported in the paper but are available upon request. These two cross-sectional factors (variance and kurtosis) might be responsible of the weak identification of DLP and HWCCAPM models.

The formula above for testing the joint significance of αs does not account for the fact that risk exposures of assets to the pricing factors (βs) have been estimated in the first stage and it also assumes that the pricing errors are uncorrelated through time (even though they can be cross-sectionally correlated). In order to correct this two limitations, we use Shanken (1992)'s correction for the first and Newey and West (1987)'s Heterosckedasticity and Auto-Correlation adjusted variance-covariance matrix for the second.

We also provide two additional measures of fit that are commonly used in the literature (Campbell and Vuolteenaho (2004); Yogo (2006); Darrat et al. (2011)), the mean absolute pricing error (MAE) which is the average of the absolute value of the pricing errors and the pseudo r-squared ($\bar{R}^2$) which measures the percentage of variability in asset expected excess returns explained by the risk factors.

$$MAE = \frac{1}{N}\sum_{i=1}^{N}\left|\frac{1}{T}\sum_{t=1}^{T}e_{it}\right| \text{ and } \bar{R}^2 = 1 - \left(\frac{\bar{e}'\bar{e}}{\left(\bar{R}^e - \frac{1}{N}\sum_i \bar{R}_i^{\,e}\right)'\left(\bar{R}^e - \frac{1}{N}\sum_i \bar{R}_i^{\,e}\right)}\right) \tag{27}$$

where e_{it} is the pricing error of asset i at time t, $\bar{e}$ is the vector of time average of pricing errors.

The skewness only model yields a mean absolute pricing errors that is lower than the one achieved by the standard CCAPM model and which is not statistically different from the one obtained in the DLP model with two factors (the cross-sectional mean and dispersion). As a pricing factor, cross-sectional skewness only explains 16.7%[16] of the variability in the country expected excess returns. The high value of R-squared achieved by the skewness model shows how powerful is the cross-sectional skewness factor in explaining the cross-sectional variability in average excess return. This R-squared can be contrasted to 5.6% yield by the standard CCAPM, the 5.9% of the DLP model and

[16] This value attains 32% when the MSCI return index expressed in local currency are used. The value obtained for the CCAPM, DLP and HWCCAPM models are respectively 1%, 5% and 50%.

the 26% achieved by the fourth cross-sectional moments factor model. On the other side the CAPM model and the 3 Fama-French model yield R-squared of 9.2% and 29% respectively.

Figure 10 shows the average (absolute) pricing errors by country for the different consumption-based factor models. For the majority of countries, the Skewness model generates smaller pricing errors compared to the CCAPM model and even performs better than the two factors model considered by Darrat et al. (2011). The consumption-based four factors model yields the smallest pricing errors which for some countries such as U.S or Norway are considerably smaller compared to the average excess returns.

The skewness price of risk is negative even-though not statistically significant. In fact, none of the consumption-based factors appear to be significant once we adjust the t-stat. by the Shanken's correction or we use the weak identification robust statistic to build the confidence sets. However, the market factor is strongly identified and positively priced in the CAPM model. In the global three factors model, the factors are also strongly identified; the HML factor is negatively priced and the price of the Size factor is not statistically different from zero.

4.3 Weak identification robust test for linear factor model

The FM t-test used to test the significance of the risk prices in Table 10 is valid when the sample size is large and the matrix of factor risk exposures, $\mathbf{B}$ is not rank deficient. the result in table 7, we see that the later assumption might not be satisfied. Kleibergen and Zhan (2019) propose an extension to the GRS test that jointly test the equality of the risk prices to a specified value and the correct specification of the factor model without assuming a rank condition on $\mathbf{B}$. Furthermore, this testing procedure allows to avoid the pretest bias that could arise from first testing the rank deficiency of $\mathbf{B}$ and then use the FM test if the null hypothesis of the previous test is rejected. Under the null hypothesis

$\mathrm{H}_0 : \lambda_f = \lambda_{f,0}$, rewriting equation 23 in the matrix form, the model can be expressed as follows:

$$\begin{aligned} \mathbf{R}_t &= \alpha\left(\lambda_{f,0}\right) + \mathbf{B}\left(\bar{f}_t + \lambda_{f,0}\right) + \epsilon_t \\ E\left(\mathbf{R}_t\right) &= \mathbf{B}\lambda_{f,0}, \end{aligned} \tag{28}$$

Where $\bar{f}_t = f_t - \mu_f$ is the demeaned factor and the sample counterpart of μ_f is $\bar{f} = \frac{1}{T}\sum_{t=1}^{T} f_t$. As presented by Kleibergen and Zhan (2019), the procedure for testing the good specification of the model given the pre-specified value of the risk price i.e $\mathrm{H}_0 : \alpha\left(\lambda_{f,0}\right) = \alpha - \mathbf{B}\left(\lambda_{f,0} - \mu_f\right)$ consist of regressing each excess returns on a constant term and $\bar{f}_t + \lambda_{f,0}$, then test the joint significance of constant terms at the pre-specified value of $\lambda_{f,0}$ using the GRS test. The test statistic for that is given by:

$$\text{GRS-FAR}\left(\lambda_{f,0}\right) = \frac{T}{1 + \lambda_{f,0}'\hat{Q}^{-1}\lambda_{f,0}}\hat{\alpha}\left(\lambda_{f,0}\right)'\hat{\Sigma}^{-1}\hat{\alpha}\left(\lambda_{f,0}\right) \tag{29}$$

where $\hat{\alpha}\left(\lambda_{f,0}\right) = \bar{\mathbf{R}} - \hat{\mathbf{B}}\lambda_{f,0}$, $\hat{\mathbf{B}} = \frac{1}{T}\sum_{t=1}^{T}\mathbf{R}_t\left(\bar{f}_t\right)'\hat{Q}^{-1}$, $\hat{Q} = \frac{1}{T}\sum_{t=1}^{T}\bar{f}_t\bar{f}_t'$ and $\hat{\Sigma}$ is a consistent estimate of the variance-covariance matrix of residuals from the time series regressions of returns on factors[17].

We invite the reader to notice that for a correctly specified factor pricing model, the tested factor's price of risk must be equal to mean of the factor ($\lambda_{f,0} = \mu_f$) and the tested pricing errors are equal to the model pricing errors ($\alpha\left(\lambda_{f,0}\right) = \alpha$). In that situation, the GRS-FAR is equal to the GRS statistic adjusted for the fact that assets risk exposures

[17]The GRS-FAR can equivalently be written as follows:

$$\text{GRS-FAR}\left(\lambda_{f,0}\right) = \frac{T}{1 - \lambda_{f,0}'\hat{Q}\left(\lambda_{f,0}\right)^{-1}\lambda_{f,0}}\tilde{\alpha}\left(\lambda_{f,0}\right)'\hat{\Sigma}^{-1}\tilde{\alpha}\left(\lambda_{f,0}\right)$$

where $\tilde{\alpha}\left(\lambda_{f,0}\right) = \bar{\mathbf{R}} - \tilde{\mathbf{B}}\left(\lambda_{f,0}\right)\lambda_{f,0}$, $\tilde{\mathbf{B}}\left(\lambda_{f,0}\right) = \frac{1}{T}\sum_{t=1}^{T}\mathbf{R}_t\left(\bar{f}_t + \lambda_{f,0}\right)'\hat{Q}\left(\lambda_{f,0}\right)^{-1}$ and $\hat{Q}\left(\lambda_{f,0}\right) = \frac{1}{T}\sum_{t=1}^{T}\left(\bar{f}_t + \lambda_{f,0}\right)\left(\bar{f}_t + \lambda_{f,0}\right)'$

(βs) are estimated.

As the sample size goes to infinity, the GRS-FAR statistics converges to a chi-squared distribution with $N-1$. In finite sample, under the normality assumption, the $100 \times (1-\alpha)\%$ confidence set of λ_f denoted $CS_{\lambda_f}(\alpha)$ consist of all values of $\lambda_{f,0}$ for which the GRS-FAR test does not reject using the $100 \times (1-\alpha)\%$ critical value that results from the F-distribution.

$$CS_{\lambda_f}(\alpha) = \left\{ \lambda_{f,0} : \frac{T-k-N+1}{T(N-1)} \times \text{GRS-FAR}\left(\lambda_{f,0}\right) \leq F_\alpha\left(N-1, T-k-N+1\right) \right\} \tag{30}$$

The confidence set of the risk prices are given in Table 9. Globally, the prices of the consumption risk factors are not statistically different from zero as their 95% confidence intervals contain zero. The price of market portfolio risk is statistically different from zero and positive in the CAPM model. The weak identification robust confidence intervals for the prices of factor risk in the global 3 factors model also contain zero. Contrary to previous findings on the statistical inference about the price of consumption risk factor in the U.S where the 95% confidence intervals are unbounded with NIPA consumption data on non-durable goods and services (see Kleibergen (2009); Kleibergen and Zhan (2015, 2019)), Figure 6 shows that the 95% confidence interval of the risk price of cross-sectional mean of consumption growth is bounded. It is also the case for the cross-sectional skewness of consumption growth. However, for the cross-sectional variance and kurtosis, the confidence intervals are unbounded which corroborates the weak-identification of the prices of risk detected in Table 8. Furthermore, in multi-factor models, the presence of a weakly identified parameter seems to contaminate the other parameters. Indeed, as Table 9 and Figure 7 show, combining any weakly identified factor with a strongly identified one leads to a weakly identified model where the confidence sets of the prices of risk are unbounded.

Taking the 3 Fama-French pair by pair, we can reject the null hypothesis that the market and the Book-to-Market ratio factors are jointly priceless. The same happens for the combination of the market and the size factors. However, the 95% confidence set for the risk prices of the Book-to-Market ratio and the size factor contains zero, meaning that we can not reject the null hypothesis of zero price for those risk factors. In the case of the consumption-based factors, all the joint confidence sets of risk prices contain zero and are unbounded.

4.4 Estimation of the structural parameters using the countries returns

In this section, we reconsider the stochastic discount factor that was derived in the modeling part (section 3.2). Our aim is to use the cross-section of country's market portfolio returns and the cross-sectional moments of international consumption growth to back out the latent variable driving the agent's consumption growth risk and the structural parameters of the model .

Let denote by $\Theta = (\delta, \gamma, \mu, \omega, \sigma)$. Considering the sdf defined by equation 14 and given the 18 observed country market returns, we use the Euler pricing formula for each asset and the moments of the cross-sectional distribution to estimate the structural parameters of the model. We estimate the model at the quarterly frequency over the period from 1970:Q1 to 2018:Q4.

Table 1 shows the results of the estimation. The subjective discount factor $\delta = 0.872$, the risk aversion coefficient $\gamma = 1.56$ and the EIS $\psi = 0.85$. The EIS is higher than the inverse of the risk aversion coefficient, which corresponds to a preference for early resolution of uncertainty. The autocorrelation of country consumption risk is $\rho = 0.996$ meaning that it is highly persistent. This persistence goes in pair with the persistence of the risk-free rate for which the sample auto-correlation is 0.96.

The model estimated parameters allow to back out the model stochastic discount factor which is represented on figure 1. The estimated sdf $\hat{M}$ shoots up during some recessions periods and predicts an increase in country consumption risk during those periods. Table 12 also shows that $\hat{M}$ is positively related to the cross-sectional variance and kurtosis of countries consumption growth, and negatively related to the cross-sectional skewness. We also plot the realized expected returns and their counterparts predicted by the model given the estimated parameters. The model can fairly predict the expected returns on country market portfolio as shown on figure 2, which means that the original idea of Constantinides and Ghosh (2017) about household cross-sectional consumption distribution as a driver of asset prices in the U.S also applies internationally and the country cross-sectional consumption distribution can explain the variation of expected returns on international financial markets.

Table 1: Structural parameters estimation

Par.	δ	γ	ψ	μ	σ	ν	ξ	ρ	MAE
Est.	0.872	1.56	0.85	0.03	1.42	4.5e-04	-9.74e-06	0.996	0.60
									pseudo-R^2
Std.Er.	0.007	0.009	0.007	0.007	0.008	8.4e-05	1.7e-05	0.0073	-0.33

The pseudo-R^2 is computed as 1 minus the mean squared pricing errors divided by the variance of expected asset returns. The J-stat is 14.25 and the model is not rejected at the 5% level of significance. The asymptotic critical value of J-stat is given by 95% quantile of the chi-squared with 16 degrees of freedom.

4.5 Non-parametric extraction of the SDF

In the previous section, we assume the existence of the stochastic discount factor and that it was a linear function of some factors such as the cross-sectional moments of the country consumption growth, for which we estimate the coefficients. In this section, we go the other way round by first (non-parametrically) estimating the stochastic discount factor that has been used for pricing asset and then relates that to our consumption-based factors. We use a Bayesian Markov chain Monte Carlo (MCMC) method to non-parametrically extract the ex-post stochastic discount factor that has been used in a unified global financial market to price the set of observed country specific market portfolios. Our aim is to relate this model free SDF to the cross-sectional moments of consumption growth in order see how this model free ex-post sdf co-moves with the cross-sectional consumption moments. We closely follow the method provided by Gallant and Hong (2007); Gallant and Tauchen (2018). We start by admitting the existence of a SDF that is used to price all the existing assets on the market[18]. Our objective is to estimate the ex-post sdf denotes by $\{\theta_t\}_{t=1,\ldots,T}$ that has been used to price the set of observed stocks, here the 18 country specific market portfolios and the risk free asset, here the US 3-month Tbill rate. The conditional pricing errors are given by:

$$e_{t,t-1}(\theta_t) = 1 - \theta_t \begin{pmatrix} R_{s,t} \\ R_{b,t} \end{pmatrix} \tag{31}$$

where 1 denotes a vector of 1's of length nineteen. The following instruments are used to obtain the unconditional moment restrictions:

$$V_t = \begin{pmatrix} R_{s,t} - 1 \\ R_{b,t} - 1 \\ \Delta c_t^W \\ TS_t \\ cay_t \\ 1 \end{pmatrix} \tag{32}$$

[18]The existence of a (positive) sdf is guaranteed by the (absence of arbitrage) law of one price. By admitting the law of one price and the absence of arbitrage, we can use stochastic discount factors without implicitly assuming anything about utility functions, aggregation, complete markets, and so on. (Cochrane (2005))

owhere Δc_t^W is the world consumption growth rate, TS_t is the US term spread computed as the difference between the 10-years and the 1-month T-bill rates and cay_t is the US log consumption -aggregate wealth ratio obtained from Martin Lettau's website. The unconditional moment restriction are given by:

$$m\left(x_t, x_{t-1}, \theta_t\right) = V_{t-1} \otimes e_{t,t-1}\left(\theta\right) \tag{33}$$

where $\otimes$ denotes the Kronecker product.

The Bayesian inference assumes that the parameter of interest is a random variable and there exist a joint distribution of the observable and the parameters $(x_t; \theta_t)$. By the Bayes's rule,

$$p(x, \theta) = p(x|\theta)p(\theta) = p(\theta|x)p(x) \tag{34}$$

Thus, the posterior distribution $\pi(\theta) = p(\theta|x)$ of the sdf $\{\theta_t\}_{t=1,\ldots,T}$ is proportional to the product of two components.

$$\pi(\theta) \propto \ell(\theta)p(\theta) \tag{35}$$

Where $\ell(\theta) = p(x|\theta)$ is the likelihood which gives the chances that the sdf θ_t has been used to price assets given the observed returns. $p(\theta)$ is the prior probability of θ. The prior probability about θ comes from the information contained in the US bond yield curve. Indeed, we assume that the ex-post sdf we are estimating has been used to price the risk free asset in US and we provide the connection that follows between the sdf and the bond yield curve in appendix 6 as well as the prior probability.

Figure 13 presents the evolution of the ex-post SDF from 1970Q1 to 2018Q4. On the same graph, the OECD-NonOECD recession indicator is also represented. We observe that the ex-post SDF captures the business cycles; the inter-temporal marginal rate of substitution shoots up before or during recession periods. This means that given the expected drop in consumption, investors are willing to pay a higher price for the risk free asset to hedge against the drop in stock prices and in respond to the increase in the marginal utility.

Table 12 presents the regression of the ex-post estimated SDF on the consumption-based factors. Unfortunately, none of the factors is significant in predicting the ex-post SDF. This disappointing result could be explain by our implementation of the SDF extraction which could be improved or the non-linearity of the functional form of the relationship between the extracted sdf and the consumption-based factor.

5 Conclusion

Heterogeneous agents consumption-based capital asset pricing model has recently received a renewal of attention in order to explain the market level and the cross-sectional variation in the equity risk premium. This model is motivated by the presence of uninsurable idiosyncratic income risk faced by economic agents and, it emphasizes on the cross-sectional distribution of this idiosyncratic risk as an important factor to explain asset prices. In this paper, we investigate these hypothesis in an integrated world financial market with country representative heterogeneous agents. More specifically, we examine whether the higher order cross-sectional moments of country consumption growth can serve as pricing factors in an international heterogeneous agents CCAPM. We find that higher order cross-sectional moments as pricing factors can explain the cross-sectional variation in country average risk premium. The cross-sectional skewness and kurtosis which have not previously been considered previously bring an additional explanatory power and enable to reduce the pricing errors compared to the already known cross-sectional mean and variance factors. We also test the weak-identification of consumption-based factors and rely on a weak-identification robust method for inference. We experiment that in linear multi-factor models, the presence of a weakly identified factor "contaminates" the others and yields to unbounded confidence intervals for the prices of risk. Our approach follows the advice of Kroencke (2020) to look at factors from a broader perspective when analysing consumption-based factors for pricing assets and not just limit ourselves to model specification tests. Finally, we non-parametrically extract a model-free ex-post stochastic discount factor that has been used to price the observed country market portfolios but failed to linearly relate that ex-post SDF to the cross-sectional moments of countries consumption growth. As future research, it would be interesting to look at the importance of these factors for pricing disaggregate international stocks or mutual funds.

6 Appendix

Appendix A: Solution of the Theoretical CCAPM.

The consumers common SDF can be expressed in term of the state variable as follows:

$$
\begin{aligned}
M_{i,t+1} &= \exp\left(\theta \log\delta - \gamma\left(\eta_{i,t+1}\sigma\sqrt{d_{t+1}} - \sigma^2\frac{d_{t+1}}{2} + \Delta c_{t+1}\right) + (\theta-1)\left(\kappa_0 + \kappa_1 z_{c,t+1}(\omega_{t+1}) - z_{c,t}\left(\omega_t\right)\right)\right) \\
&= \exp\left(\theta\log\delta - \gamma\mu + (\theta-1)\left(\kappa_0 + (\kappa_1 - 1)A_0 + \kappa_1 A_1 \nu\xi\right) + (\theta-1)\left(\rho\kappa_1 - 1\right)A_1 x_t \right. \\
&\left. + (\theta-1)\kappa_1 A_1 \varepsilon_{x,t+1} - \gamma\left(\eta_{i,t+1}\sigma\sqrt{d_{t+1}} - \sigma^2\frac{d_{t+1}}{2} + \sigma_c\varepsilon_{c,t+1}\right)\right)
\end{aligned}
$$

The log risk free rate can be obtained by taking the conditional expectation of the log SDF.

$$
\begin{aligned}
r_t^f &= -\log\mathbb{E}_t\left(M_{i,t+1}\right) \\
&\approx -\theta\log\delta + \gamma\mu - \frac{1}{2}\gamma^2\sigma_c^2 - (\theta-1)\left(\kappa_0 + (\kappa_1-1)A_0\right) - \lambda\nu\xi - \frac{1}{2}\lambda^2\nu\xi^2 \\
&- \left(\lambda\rho\left(1+\lambda\xi\right) - (\theta-1)A_1\right)x_t
\end{aligned}
$$

where

$$
\lambda = \frac{e^{\frac{\gamma(\gamma+1)}{2}\sigma^2} - 1}{e^{\frac{\gamma(\gamma-1)}{2}\sigma^2} - 1} + (\theta-1)\kappa_1 A_1
$$

The standard asset pricing Euler equation for the consumer i, in particular for the return on the wealth portfolio gives:

$$
\mathbb{E}_t\left[M_{i,t+1}R_{i,c,t+1}\right] = 1 \iff \mathbb{E}_t\left[\exp\left(m_{i,t+1} + r_{i,c,t+1}\right)\right] = 1
$$

where $M_{i,t+1}$ is given by equation (5) and

$$r_{i,c,t+1} = \log(R_{i,c,t+1}) = \log\left(\frac{P_{i,c,t+1} + C_{i,t+1}}{P_{i,c,t}}\right)$$

which implies that

$$\begin{aligned} e^{\theta(A_0+A_1x_t)} &= \mathbb{E}_t \exp\left(\theta\log\delta + (1-\gamma)\Delta c_{i,t+1} + \theta\left(\kappa_0 + \kappa_1\left(A_0 + A_1x_{t+1}\right)\right)\right) \\ &\approx e^{\theta\log\delta+\theta\kappa_0+\theta\kappa_1A_0+(1-\gamma)\mu+\frac{(1-\gamma)^2}{2}\sigma_c^2+[1+\theta\kappa_1A_1]\mathbb{E}(x_{t+1}|x_t)+\frac{1}{2}[1+\theta\kappa_1A_1]^2\text{var}(x_{t+1}|x_t)} \end{aligned}$$

Thus, with $\mathbb{E}\left(x_{t+1}|x_t\right) = \nu\xi + \rho x_t$ and $\text{var}\left(x_{t+1}|x_t\right) = \nu\xi^2 + 2\rho\xi x_t$, we deduce that:

$$\begin{cases} \theta A_0 &= \theta\log\delta + \theta\kappa_0 + \theta\kappa_1A_0 + (1-\gamma)\mu + \frac{(1-\gamma)^2}{2}\sigma^2 + (1+\theta\kappa_1A_1)\nu\xi\left(1 + \frac{1}{2}\left[1+\theta\kappa_1A_1\right]\xi\right) \\ \\ \theta A_1 &= \rho\left(1+\theta\kappa_1A_1\right)\left(1+\left[1+\theta\kappa_1A_1\right]\xi\right) \end{cases} \tag{36}$$

The coefficients A_0 and A_1 can be obtained by solving the non-linear system of equations (36).

To compute the equity return, we follow the same procedure used for the wealth portfolio. We assume that the price-dividend ratio is an affine function of the state variable.

$$z_{i,m,t} = z_{m,t}\left(\omega_t\right) = A_{0m} + A_{1m}x_t$$

Substituting expression of the log-price dividend ratio in the standard Euler equation for the equity return allows to solve for the unknown coefficient and to obtain the expected stock market return as follows.

$$\mathbb{E}_t\left[M_{i,t+1}R_{i,m,t+1}\right] = 1 \iff \mathbb{E}_t\left[\exp\left(m_{i,t+1} + r_{i,m,t+1}\right)\right] = 1$$

where $M_{i,t+1}$ is given by equation (5) and

$$r_{i,m,t+1} = \log(R_{i,m,t+1}) = \log\left(\frac{P_{i,m,t+1} + D_{i,t+1}}{P_{i,m,t}}\right)$$

which implies that

$$\begin{aligned} e^{z_{m,t}(\omega_t)} =& \mathbb{E}_t \exp\left(\theta \log \delta - \frac{\theta}{\psi} \Delta c_{i,t+1} + (\theta - 1)\, r_{i,c,t+1} + \kappa_{0m} + \kappa_{1m} z_{m,t+1}(\omega_{t+1}) + \Delta d_{i,t+1}\right) \\ \approx & e^{\theta \log \delta + (\theta-1)\kappa_0 + (\theta-1)(\kappa_1 - 1)A_0 + \kappa_{0m} + \kappa_{1m} A_{0m} + \mu_d - (\theta-1)A_1 x_t + \frac{1}{2}\sigma_d^2 + (\beta_d - \gamma)\mu + \frac{1}{2}(\beta_d - \gamma)^2 \sigma_c^2} \\ & \times e^{\lambda_m (\nu\xi + \rho x_t) + \frac{1}{2}\lambda_m^2 \left(\nu\xi^2 + 2\rho\xi x_t\right)} \end{aligned}$$

where

$$\lambda_m = \frac{e^{(\beta_d - \gamma)(\beta_d - \gamma - 1)\frac{\sigma^2}{2}} - 1}{e^{\frac{\gamma(\gamma-1)}{2}\sigma^2} - 1} + \left(\left(\theta - 1\right)\kappa_1 A_1 + \kappa_{1m} A_{1m}\right)$$

Thus, A_{0m} and A_{1m} should satisfy:

$$\begin{cases} A_{0m} &= \theta \log \delta + (\theta - 1)\kappa_0 + (\theta - 1)\,(\kappa_1 - 1)A_0 + \kappa_{0m} + \kappa_{1m} A_{0m} + \mu_d \\ & + \frac{1}{2}\sigma_d^2 + (\beta_d - \gamma)\,\mu + \frac{1}{2}\,(\beta_d - \gamma)^2\,\sigma_c^2 + \lambda_m \nu\xi\,\left(1 + \frac{1}{2}\lambda_m \xi\right) \\ A_{1m} &= \rho\lambda_m\,(1 + \lambda_m \xi) - (\theta - 1)A_1 \end{cases} \tag{37}$$

The expected stock market return for consumer i is given by:

$$\begin{aligned} \mathbb{E}_t r_{i,m,t+1} &= \kappa_{0m} + \kappa_{1m} \mathbb{E}_t z_{m,t+1}(\omega_{t+1}) - z_{m,t}\,(\omega_t) + \mathbb{E}_t \Delta d_{i,t+1} \\ &= \kappa_{0m} + (\kappa_{1m} - 1)\,A_{0m} + \left(\kappa_{1m} A_{1m} + \frac{\beta_d \sigma_d^2}{2\left(e^{\gamma(\gamma-1)\sigma^2/2} - 1\right)}\right)\nu\xi + \mu_d \\ &+ \left(\rho\left(\kappa_{1m} A_{1m} + \frac{\beta_d \sigma_d^2}{2\left(e^{\gamma(\gamma-1)\sigma^2/2} - 1\right)}\right) - A_{1m}\right) x_t \end{aligned}$$

Appendix B: Derivation of the cross-sectional moments of consumption growth distribution.

The derivation of the cross-sectional moments of the consumption growth distribution closely follows Constantinides and Ghosh (2017). This derivation uses the following identity:

$$e^{-\omega}\sum_{k=0}^{\infty} e^{kn}\omega^n/n! = e^{-\omega}\sum_{k=0}^{\infty}\left(e^k\omega\right)^n/n! = e^{-\omega}e^{e^k\omega}. \tag{38}$$

Differentiating one, two, three and four times with respect to k and setting $k = 0$, we obtain:

$$\begin{aligned}
e^{-\omega}\sum_{k=0}^{\infty} n\omega^n/n! &= \omega \\
e^{-\omega}\sum_{k=0}^{\infty} n^2\omega^n/n! &= \omega^2 + \omega \\
e^{-\omega}\sum_{k=0}^{\infty} n^3\omega^n/n! &= \omega^3 + 3\omega^2 + \omega \\
e^{-\omega}\sum_{k=0}^{\infty} n^4\omega^n/n! &= \omega^4 + 6\omega^3 + 7\omega^2 + \omega
\end{aligned}$$

The country relative consumption growth defined in equation (2) satisfies:

$$\begin{aligned}
\ln\left(\frac{C_{i,t+1}/C_{t+1}}{C_{i,t}/C_t}\right) &= \ln\left(\delta_{i,t+1}\right) - \ln\left(\delta_{i,t}\right) \\
&= \eta_{i,t+1}\sigma\sqrt{d_{t+1}} - \sigma^2\frac{d_{t+1}}{2}
\end{aligned}$$

Thus, the cross-sectional centered first to fourth moments are given by:

$$
\begin{aligned}
\mu_1\left[\ln\left(\frac{C_{i,t+1}/C_{t+1}}{C_{i,t}/C_t}\right)|\omega_{t+1}\right] &= \mathbb{E}\left(\eta_{i,t+1}\sigma\sqrt{d_{t+1}}-\sigma^2\frac{d_{t+1}}{2}|\omega_{t+1}\right)\\
&= \mathbb{E}\left(\mathbb{E}\left(\eta_{i,t+1}\sigma\sqrt{d_{t+1}}-\sigma^2\frac{d_{t+1}}{2}|d_{t+1}\right)|\omega_{t+1}\right)\\
&= -\left(\sigma^2/2\right)\omega_{t+1}
\end{aligned} \tag{39}
$$

$$
\begin{aligned}
\mu_2\left[\ln\left(\frac{C_{i,t+1}/C_{t+1}}{C_{i,t}/C_t}\right)|\omega_{t+1}\right] &= \mathbb{E}\left(\left[\eta_{i,t+1}\sigma\sqrt{d_{t+1}}-\sigma^2\frac{d_{t+1}}{2}-\mu_1\left[\ln\left(\frac{C_{i,t+1}/C_{t+1}}{C_{i,t}/C_t}\right)|\omega_{t+1}\right]\right]^2|\omega_{t+1}\right)\\
&= \left(\sigma^2+\sigma^4/4\right)\omega_{t+1}
\end{aligned} \tag{40}
$$

$$
\begin{aligned}
\mu_3\left[\ln\left(\frac{C_{i,t+1}/C_{t+1}}{C_{i,t}/C_t}\right)|\omega_{t+1}\right] &= \mathbb{E}\left(\left[\eta_{i,t+1}\sigma\sqrt{d_{t+1}}-\sigma^2\frac{d_{t+1}}{2}-\mu_1\left[\ln\left(\frac{C_{i,t+1}/C_{t+1}}{C_{i,t}/C_t}\right)|\omega_{t+1}\right]\right]^3|\omega_{t+1}\right)\\
&= -\left(\left(3/2\right)\sigma^4+\left(1/8\right)\sigma^6\right)\omega_{t+1}
\end{aligned} \tag{41}
$$

$$
\begin{aligned}
\mu_4\left[\ln\left(\frac{C_{i,t+1}/C_{t+1}}{C_{i,t}/C_t}\right)|\omega_{t+1}\right] &= \mathbb{E}\left(\left[\eta_{i,t+1}\sigma\sqrt{d_{t+1}}-\sigma^2\frac{d_{t+1}}{2}-\mu_1\left[\ln\left(\frac{C_{i,t+1}/C_{t+1}}{C_{i,t}/C_t}\right)|\omega_{t+1}\right]\right]^4|\omega_{t+1}\right)\\
&= \left(3\sigma^4+\left(3/2\right)\sigma^6+\left(1/16\right)\sigma^8\right)\omega_{t+1}+\left(3\sigma^4+\left(3/2\right)\sigma^6+\left(3/16\right)\sigma^8\right)\omega_{t+1}^2
\end{aligned} \tag{42}
$$

Unconditional first and second moments of cross-sectional consumption growth moments.

The mean of cross-sectional variance of consumption growth is given by:

$$\text{m\_cross\_var} = E(\text{cross\_var}) = \left(\sigma^2 + {\sigma^4}/{4}\right) E(\omega_t) = \left(\sigma^2 + {\sigma^4}/{4}\right) \frac{\nu\xi}{(1-\rho)\left(e^{\gamma(\gamma-1)\sigma^2/2} - 1\right)}$$

The variance of cross-sectional variance of consumption growth is given by:

$$\text{v\_cross\_var} = \text{Var}(\text{cross\_var}) = \left(\sigma^2 + {\sigma^4}/{4}\right)^2 \text{Var}(\omega_t) = \left(\frac{\sigma^2 + {\sigma^4}/{4}}{e^{\gamma(\gamma-1)\sigma^2/2} - 1}\right)^2 \left(\frac{\nu\xi^2}{(1-\rho^2)}\right)\left[1 + \frac{2\nu}{1-\rho}\right]$$

The mean of cross-sectional skewness of consumption growth is given by:

$$\text{m\_cross\_skew} = E(\text{cross\_skew}) = \left(-{3\sigma^4}/{2} - {\sigma^6}/{8}\right) E(\omega_t) = -\left({3\sigma^4}/{2} + {\sigma^6}/{8}\right) \frac{\nu\xi}{(1-\rho)\left(e^{\gamma(\gamma-1)\sigma^2/2} - 1\right)}$$

The variance of cross-sectional skewness of consumption growth is given by:

$$\text{v\_cross\_skew} = \text{Var}(\text{cross\_skew}) = \left(\frac{{3\sigma^4}/{2} + {\sigma^6}/{8}}{e^{\gamma(\gamma-1)\sigma^2/2} - 1}\right)^2 \left(\frac{\nu\xi^2}{(1-\rho^2)}\right)\left[1 + \frac{2\nu}{1-\rho}\right]$$

GMM estimation of the linear factor models

Following Cochrane (2005); Yogo (2006), the linear factor model can be expressed in terms of pricing errors and the GMM (Hansen (1982)) can be applied to estimate the coefficients (b) that define the SDF as a linear combination of the pricing factors. From the asset pricing Euler equation (17) and the linear SDF formulation (18), the pricing errors at time t can be expressed as a $(N+F)K \times 1$ vector of moment function as follows:

$$e(z_t, \theta) = \begin{bmatrix} R_t^e - R_t^e \left(f_t - \mu_f\right)' b \\ f_t - \mu_f \end{bmatrix} \otimes Z_t$$

where $\theta = (b', \mu_f')'$ is a $(2F) \times 1$ vector of parameters, Z_t is a $K \times 1$ vector of instruments, f_t is a $F \times 1$ vector of factors, R_t^e is a $N \times 1$ vector of excess returns and $z_t = (R_t^{e'}, f_t', Z_t')'$ stacks all the variables together. From the asset pricing Euler equation (17), the moment function satisfies the moment restriction $\mathbb{E}\left[e\left(z_t, \theta_0\right)\right] = 0$, for some $\theta_0 \in \Theta$. The parameters are estimated by 2-Step GMM and obtained by minimizing the criterion function:

$$S_T(\theta; \bar{\theta}_T(\theta)) = T.\left(e_T(z_t, \theta)\right)' W_T(\bar{\theta}_T(\theta))\left(e_T(z_t, \theta)\right)$$

Where $e_T(z_t, \theta)$ is the time average of $e(z_t, \theta)$, $W_T(\bar{\theta}_T(\theta))$ is a positive semi-definite weighting matrix and $\bar{\theta}_T(\theta)$ is the value of the parameters used to compute W_T. In the first step, we use the identity matrix as the weighting matrix, $W_T = I_{(N+F)K}$. For the second step, we use the optimal weighting matrix computed with $\bar{\theta}_T(\theta)$ fixed to the value estimated in the first step. The optimal weighting matrix is the inverse of the asymptotic variance-covariance matrix of $\sqrt{T} e_T(z_t, \theta)$ and it can be consistently estimated using a kernel based estimator (Newey and West (1987); Andrews (1991)) by:

$$\widehat{\Omega}(\theta) = \widehat{\Omega}_0 + \sum_{j=1}^{m} \bar{k}_j \left(\widehat{\Omega}_j + \widehat{\Omega}_j'\right) \tag{43}$$

where $\widehat{\Omega}_j = T^{-1} \sum_{t=j+1}^{T} \left(e(z_t, \theta) - e_T(z_t, \theta)\right)\left(e(z_{t-j}, \theta) - e_T(z_t, \theta)\right)'$ and $\bar{k}_j$ is a kernel function (e.g: Bartlett, Quadratic Spectral, Parzen, Truncated, etc.). We use the Bartlett kernel with bandwidth m,$\bar{k}_j = \begin{cases} (1 - \frac{j}{m+1})\ if\, 0 \leq j \leq m \\ 0\ \ if\ \ Not \end{cases}$. Newey and West (1994) showed how to automatically select the bandwidth or number of lag ($m = parameter \times T^{1/3}$) to include in the summation in equation (43).

We test over-identifying restrictions of the model by using the Hansen (1982) J-test. The degree of over-identification is $(N+F)\,K-2F$.

Appendix B1: The likelihood of θ.

Let consider the normalized average moment condition defined on the probability space $(\chi\times\Theta,\mathcal{C}^o,P^o)$ defined by:

$$Z\left(x,\theta\right)=\sqrt{n}S_n^{-1/2}\left(\theta\right)\left(U_v\otimes U_e\right)'\bar{m}\left(x,\theta\right) \tag{44}$$

where U_e (resp. U_v) is a set of orthogonal vectors used to diagonalize $\Sigma_e=Var\left[e_{t,t-1}\right]$ (resp. $\Sigma_v=Var\left[V_t\right]$) and $S_n(\theta)$ is a diagonal matrix defined below by equation (46). Indeed, given the vector

$$H_t\left(\theta\right)=\left(U_v\otimes U_e\right)'m\left(x_t,x_{t-1},\theta_t\right) \tag{45}$$

with elements $h_{i,t}\left(\theta\right)$, we have that:

$$\begin{aligned}Var\left[H_t\left(\theta\right)\right]&=\left(U_v\otimes U_e\right)'\Sigma_m\left(U_v\otimes U_e\right)\\&=\left(U_v\otimes U_e\right)'\left(\Sigma_v\otimes\Sigma_e\right)\left(U_v\otimes U_e\right)=D_v\otimes D_e\end{aligned}$$

Thus, $Var\left[H_t\left(\theta\right)\right]$ can be estimated by a diagonal matrix $S_n(\theta)$ with elements:

$$s_i(\theta)=\frac{1}{n}\sum_{t=2}^{n}\left[h_{i,t}(\theta)-\frac{1}{n}\sum_{t=2}^{n}h_{t,i}(\theta)\right]^2 \tag{46}$$

There exist a probability space $(\chi\times\Theta,\mathcal{C}^*,P^*)$ on which the conditional density of x given θ is $f(x|\theta)=\phi\left[Z\left(x,\theta\right)\right]$ (Gallant (2015)), where $\phi(.)$ denotes the pdf of the Gaussian distribution. Thus, the likelihood of θ is proportional to the euclidean norm of $Z(x,\theta)$.

$$\ell(\theta) \propto \exp(-Z(x,\theta)'Z(x,\theta)) \tag{47}$$

Appendix B2: Bond Yield curve and Prior probability of θ.

Let's assume we observe the ex-post SDF $\{\theta_t\}_{t=1,..,T}$ that has been used to price the existing assets. For the risk free asset, the present value of the contract that pays 1 unit of consumption n-period later is given by:

$$PV_{t,n}(1) = \exp\left[E\left(\sum_{s=1}^{n} sdf_{t+s-1,t+s}\right) + \frac{1}{2}Var\left(\sum_{s=1}^{n} sdf_{t+s-1,t+s}\right)\right] \tag{48}$$

where $sdf_{t-1,t} = \log(\theta_t)$. Now, let's assume that the $w_t = \left(\log(\theta_t), \Delta c_t^W\right)$ follows an Vector Auto-regression process of order 1 (VAR(1)), then we can write:

$$w_t = d_0 + Dw_{t-1} + u_t \tag{49}$$

The coefficients (d_0, D, Σ_u) of this VAR(1) model can be estimated by least squares. Plugging the estimates in the bond present value equation (48), we can deduce the yield curve as a linear function of sdf as follows:

$$\begin{aligned}
Y^*_{n,t} &= -\log[PV_{t,n}(1)]/n \\
&= \frac{-1}{n}\left[\sum_{s=1}^{n}(n+1-s)\hat{D}^{s-1}\hat{d}_0 + \left(\sum_{s=1}^{n}\hat{D}^s\right)w_t\right]_{(1)} \\
&- \left[\frac{1}{2n}\sum_{s=1}^{n}\left(\sum_{u=1}^{s}\hat{D}^{u-1}\right)\hat{\Sigma}_d\left(\sum_{u=1}^{s}\hat{D}^{u-1}\right)'\right]_{(1,1)}
\end{aligned}$$

and we can compute the prior probability of θ using the mean and variance of the 1-year and 30-year bond yields as follows:

$$p(\theta) = \prod_{t=1}^{n} \phi\left[\left(Y^*_{1,t} - 0.00896\right)/0.01\right] \phi\left[\left(Y^*_{30,t} - 0.02\right)/0.01\right]$$

Appendix B3: Metropolis-Hastings MCMC inference of θ.

The algorithm proceeds as follows (see Gamerman and Lopes (2006), chapter 6.):

1. Initialize the iteration counter $i = 1$ and set an arbitrary initial value $\theta^{(0)}$ e.g. $\theta^0_t \sim i.i.d\aleph(0,4)$

2. Move the chain to a new value θ^{i+1} generated from the density $q(\theta^i, .)$.

3. Here we use a random walk to move to get a candidate ϕ: e.g. $\phi = \theta^i_t + i.i.d\aleph(0,4)$

4. Compute the prior probability $p(\phi)$ and likelihood $\ell(\phi)$ as described above.

5. Compute the posterior probability of $\pi(\phi)$

6. Evaluate the acceptance probability of the move $\alpha = min\{1, \frac{\pi(\phi)}{\pi(\theta^i)}\}$. If the move is accepted, $\theta^{i+1} = \phi$. If it is not accepted,$\theta^{i+1} = \theta^i$ and the chain does not move.

7. Change the counter from i to $i+1$ and return to step 2 until convergence is reached.

Appendix C: Data, descriptive statistics and additional tables.

Table 2: Summary statistics: Excess return on the MSCI country index

Statistic	N	Mean	St. Dev.	Min	Pctl(25)	Pctl(75)	Max
World	196	1.07	8.34	−27.59	−1.66	5.49	21.87
Pacific_U	196	1.00	10.88	−37.74	−4.52	7.81	29.13
EAFE_U	196	1.02	9.31	−25.68	−2.61	6.31	26.51
Europe_U	196	1.13	9.30	−26.30	−2.85	6.77	29.39
Australia	196	0.91	12.12	−54.88	−4.83	8.25	28.63
Austria	196	0.71	13.02	−56.62	−5.28	6.86	42.87
Belguim	196	1.35	11.38	−46.21	−3.79	7.08	29.09
Canada	196	0.96	10.08	−40.04	−4.23	6.39	26.78
Denmark	196	1.81	10.14	−33.05	−3.19	8.04	36.35
France	196	1.08	12.11	−48.76	−4.03	8.04	32.81
Germany	196	1.07	11.69	−45.94	−5.16	8.44	32.55
Hong_Kong	196	2.27	17.75	−70.09	−4.92	12.32	54.52
Italy	196	0.04	13.24	−37.16	−7.47	7.91	52.18
Japan	196	0.98	11.41	−40.29	−6.26	7.90	30.77
Netherlands	196	1.60	10.05	−37.51	−3.16	8.03	29.18
Norway	196	1.28	15.02	−52.48	−5.61	9.63	44.50
Spain	196	0.80	12.40	−44.73	−6.21	8.14	51.37
Sweden	196	1.79	12.27	−36.61	−4.56	9.35	36.90
Switzerland	196	1.44	9.61	−27.28	−3.20	7.82	32.42
UK	196	1.06	10.64	−36.18	−4.31	6.77	58.44
Singapore	196	1.32	14.79	−46.81	−4.81	8.60	65.32
USA	196	1.22	8.32	−31.64	−2.68	6.21	19.53

This table presents the summary statistics of the quarterly excess returns (in percentage) on the MSCI country/region indexes. The data span the period from 1970Q1 to 2018Q4. The return are expressed in dollars and they are in excess of US. 1-month Tbill rate. The Pacific index captures large and mid cap representation across 5 developed markets in the Pacific region: Australia, Japan, Hong Kong, New Zealand, Singapore. The EAFE index is for Europe, Australasia and Far East and it includes all developed markets outside of US and Canada.

Table 3: Summary statistics: Consumption growth by country

Statistic	N	Mean	St. Dev.	Min	Pctl(25)	Pctl(75)	Max
World	196	2.04	1.40	−2.64	1.27	3.09	6.39
Australia	196	1.87	1.70	−2.31	0.88	3.00	6.44
Austria	196	1.92	2.18	−5.79	0.49	3.04	9.29
Belguim	196	1.77	1.87	−1.79	0.43	2.77	8.82
Canada	196	1.86	1.86	−4.56	0.97	2.98	7.30
Finland	196	2.25	2.88	−5.29	0.76	3.89	10.07
France	196	1.67	1.54	−1.87	0.62	2.66	6.02
Germany	196	1.89	1.87	−1.88	0.51	3.24	7.70
Greece	196	1.86	3.56	−12.76	0.76	3.89	9.29
Italy	196	1.67	2.39	−4.82	0.47	3.13	7.83
Japan	196	2.10	2.41	−4.41	0.70	3.28	10.00
Luxembourg	196	1.92	2.27	−5.17	0.30	3.38	7.57
Netherlands	196	1.44	2.27	−5.37	0.13	2.76	7.20
NewZealand	196	4.88	5.91	−3.78	0.35	8.75	18.26
Norway	196	2.26	2.43	−4.71	0.89	3.74	9.61
Portugal	196	2.17	4.03	−6.73	−0.19	3.59	15.34
South.Africa	196	1.20	3.07	−10.45	−0.69	2.93	9.40
South.Korea	196	4.69	4.16	−14.19	2.27	7.50	13.59
Spain	196	1.69	2.71	−5.93	0.06	3.31	7.52
Sweden	196	1.37	2.35	−6.11	0.29	2.63	7.94
Switzerland	196	1.03	1.50	−4.54	0.29	1.64	6.47
UK	196	2.34	2.66	−5.25	0.97	3.86	9.11
US	196	2.06	1.82	−3.17	1.17	3.14	6.88
Average	196	2.09	1.42	−2.49	1.18	3.11	5.73
Median	196	1.98	1.29	−2.49	1.18	2.82	5.14

This table presents the summary statistics on the real per capita growth rate (in percentage) of the quarterly private final consumption expenditure. The data span the period from 1970Q1 to 2018Q4.

Table 4: Correlation matrix of Recession. world excess return and consumption factors

	Recession	$R^{ex}_{w,t}$	$\Delta c_{w,t}$	$d_{w,t}$	$skew_{w,t}$	$kur_{w,t}$	$\Delta c_{w,t-1}$	$d_{w,t-1}$	$skew_{w,t-1}$	$kur_{w,t-1}$
Australia	**-0.18**	**0.66**	-0.01	0.02	-0.07	0.00	-0.03	-0.06	-0.13	0.01
Austria	**-0.19**	**0.50**	**0.16**	-0.06	-0.13	0.01	0.09	-0.12	-0.13	-0.09
Belgium	**-0.14**	**0.70**	0.13	0.08	**-0.15**	-0.01	0.05	-0.03	**-0.19**	0.01
Canada	**-0.22**	**0.81**	0.11	0.02	0.01	0.02	0.05	-0.13	-0.03	0.00
Denmark	**-0.22**	**0.52**	0.07	-0.03	-0.03	-0.02	-0.02	-0.13	-0.03	-0.05
France	**-0.21**	**0.72**	0.09	0.07	-0.12	-0.04	0.04	-0.08	**-0.16**	-0.04
Germany	**-0.20**	**0.69**	0.00	0.02	-0.12	0.03	-0.08	-0.07	-0.12	-0.01
Hong_Kong	**-0.16**	**0.57**	0.09	0.02	-0.02	0.05	-0.02	0.00	-0.04	-0.01
Italy	**-0.15**	**0.61**	0.08	0.07	-0.12	-0.01	0.04	-0.02	**-0.15**	0.01
Japan	**-0.23**	**0.71**	**0.15**	0.00	-0.10	0.00	0.05	-0.06	-0.12	0.05
Netherlands	**-0.24**	**0.78**	0.04	0.04	-0.09	-0.04	-0.07	-0.08	-0.11	-0.02
Norway	**-0.25**	**0.53**	**0.20**	-0.02	-0.01	-0.07	**0.16**	-0.11	-0.13	-0.09
Spain	-0.13	**0.63**	0.06	0.01	-0.04	0.00	0.02	-0.14	-0.02	0.08
Sweden	-0.11	**0.66**	0.00	0.04	0.04	0.05	-0.08	-0.06	-0.02	0.08
Switzerland	**-0.14**	**0.73**	0.03	-0.01	-0.13	-0.02	-0.06	**-0.14**	**-0.14**	-0.02
UK	-0.11	**0.73**	0.01	-0.03	-0.08	0.08	-0.10	-0.10	-0.12	0.11
Singapore	**-0.17**	**0.61**	0.09	-0.04	0.04	0.01	-0.03	-0.09	-0.05	0.00
USA	-0.09	**0.92**	0.01	0.03	-0.08	0.11	-0.07	-0.08	-0.12	0.07
Recession	1	**-0.18**	**-0.32**	**0.21**	0.14	0.01	**-0.22**	**0.20**	**0.15**	-0.03
$R^{ex}_{w,t}$	**-0.18**	1	0.08	0.03	-0.10	0.09	-0.02	-0.09	-0.12	0.09
$\Delta c_{w,t}$	**-0.32**	0.08	1	0.07	0.04	0.06	**0.91**	0.07	0.01	0.08
$d_{w,t}$	**0.21**	0.03	0.07	1	-0.10	-0.01	0.10	**0.76**	-0.06	0.00
$skew_{w,t}$	0.14	-0.10	0.04	-0.10	1	**0.14**	0.11	-0.05	**0.81**	0.10
$kur_{w,t}$	0.01	0.09	0.06	-0.01	**0.14**	1	0.08	0.04	0.14	**0.71**
$\Delta c_{w,t-1}$	**-0.22**	-0.02	**0.91**	0.10	0.11	0.08	1	0.08	0.03	0.05
$d_{w,t-1}$	**0.20**	-0.09	0.07	**0.76**	-0.05	0.04	0.08	1	-0.10	-0.03
$skew_{w,t-1}$	**0.15**	-0.12	0.01	-0.06	**0.81**	0.14	0.03	-0.10	1	**0.14**
$kur_{w,t-1}$	-0.03	0.09	0.08	0.00	0.10	**0.71**	0.05	-0.03	**0.14**	1

Recession denotes the OECD and Non OECD recession indicator provided by OECD which tracks the business cycles in 35 OECD and Non-member economies. $R^{ex}_{w,t}$ is the world excess return, $\Delta c_{w,t}$ is the cross-sectional average of the country consumption growth, $d_{w,t}$ is the cross-sectional variance of country consumption growth, $skew_{w,t}$ is the cross-sectional skewness of country consumption growth. In bold font shows the the significance at 5% level.

Table 5: Summary statistics: Cross-sectional Mean, Dispersion, Skewness and Kurtosis

Panel A: Equally weighted moments.

Statistic	N	Mean	St. Dev.	Min	Pctl(25)	Pctl(75)	Max
Cross_mean	196	2.09	1.42	−2.49	1.18	3.11	5.73
Cross_var	196	6.96	4.76	0.52	2.89	10.58	20.15
Cross_skew	196	0.46	1.15	−3.14	−0.11	1.16	2.73
Cross_kurt	196	4.81	2.71	1.70	2.80	6.27	13.36

Panel B: Consumption per capita weighted moments.

W_Cross_mean	196	2.04	1.40	−2.64	1.27	3.09	6.39
W_Cross_var	196	3.09	2.05	0.18	1.52	3.99	11.16
W_Cross_skew	196	0.37	1.63	−5.13	−0.66	1.45	4.73
W_Cross_kurt	196	5.45	7.16	−1.59	0.43	7.53	41.95

This table presents the summary statistics of the cross sectional moments of the real per capita growth rate (in percentage) of the quarterly private final consumption expenditure. Panel A presents the equally weighted cross-sectional moment and Panel B presents the same moments where the weights are the proportion of the country consumption to world consumption. The data span the period from 1970Q1 to 2018Q4.

Table 6: Expected returns by periods (Skewness and Mean)

	Skew>0	Skew<0	Diff.	t-stat (diff.)	Mean>0	Mean<0	Diff.	t-stat (diff.)
Australia	0.01	0.03	-0.02	-1.42	0.02	0.05	-0.03	-0.58
Austria	0.00	0.04	-0.04	-2.47	0.02	-0.06	0.08	0.93
Belguim	0.00	0.05	-0.04	-3.03	0.03	-0.04	0.07	0.96
Canada	0.02	0.02	0.00	-0.38	0.02	0.01	0.01	0.21
Denmark	0.02	0.04	-0.02	-1.15	0.03	-0.02	0.05	0.85
France	0.01	0.04	-0.04	-2.54	0.02	0.01	0.01	0.21
Germany	0.00	0.04	-0.04	-2.76	0.02	0.00	0.02	0.31
Hong_Kong	0.02	0.05	-0.03	-1.18	0.04	0.01	0.03	0.38
Italy	-0.01	0.04	-0.05	-2.78	0.02	-0.02	0.04	0.69
Japan	0.00	0.03	-0.02	-1.64	0.02	-0.03	0.04	0.97
Netherlands	0.01	0.04	-0.04	-2.88	0.03	0.00	0.03	0.47
Norway	0.02	0.04	-0.02	-1.18	0.03	0.01	0.02	0.28
Spain	0.01	0.04	-0.02	-1.41	0.02	0.01	0.02	0.30
Sweden	0.03	0.04	-0.01	-0.55	0.03	0.01	0.03	0.64
Switzerland	0.00	0.04	-0.04	-2.97	0.02	0.00	0.02	0.59
UK	0.01	0.04	-0.03	-2.11	0.03	0.00	0.02	0.54
Singapore	0.02	0.02	-0.01	-0.34	0.02	0.01	0.02	0.23
USA	0.02	0.03	-0.02	-1.58	0.02	0.01	0.01	0.30
R.FF_US	0.01	0.01	0.01	5.45	0.01	0.01	0.01	2.40
factor.Mrkt	0.01	0.03	-0.02	-1.85	0.02	0.01	0.02	0.29
factor.Cross_mean	0.02	0.02	0.00	-0.81	0.02	-0.01	0.04	10.31
factor.Cross_var	0.03	0.03	0.00	1.23	0.03	0.03	0.00	0.39
factor.Cross_skew	0.02	-0.01	0.03	17.72	0.00	0.01	0.00	-1.66
factor.Cross_kurt	0.07	0.04	0.02	2.44	0.06	0.02	0.04	3.85

This table summarizes the change in expected returns by periods of positive or negative cross-sectional average (or skewness) of countries consumption growth. It also shows the correlation between episodes of positive or negative cross-sectional average (or skewness) of countries consumption growth and the U.S risk free rate (R.FF_US) and the cross-sectional moment of countries consumption growth.

Table 7: Beta with countries market portfolios excess returns (consumption-based factor models)

	CCAPM		Skewness		DLP				HWCCAPM							
	$\beta_{\Delta c}$	t-stat	β_{Skew}	t-stat	$\beta_{\Delta c}$	t-stat	β_{Var}	t-stat	$\beta_{\Delta c}$	t-stat	β_{Var}	t-stat	β_{Skew}	t-stat	β_{Kurt}	t-stat
Australia	0.14	0.19	-0.11	-0.20	0.13	0.19	0.03	0.07	0.12	0.17	0.02	0.05	-0.15	-0.29	0.06	0.38
Austria	1.79	1.80	-0.84	-1.50	1.85	1.86	-0.50	-1.27	1.86	1.93	-0.57	-1.47	-1.06	-2.00	0.14	0.94
Belguim	1.42	1.52	-0.93	-2.15	1.41	1.49	0.14	0.35	1.43	1.60	0.07	0.22	-1.03	-2.74	0.10	0.88
Canada	0.77	1.22	0.30	0.64	0.77	1.22	0.01	0.03	0.75	1.19	0.03	0.09	0.27	0.53	0.02	0.16
Denmark	0.92	1.09	-0.16	-0.42	0.97	1.15	-0.42	-1.15	0.96	1.13	-0.44	-1.17	-0.29	-0.82	0.08	1.03
France	0.99	1.48	-0.82	-2.11	0.99	1.47	0.03	0.07	1.02	1.56	-0.04	-0.10	-0.88	-2.36	0.05	0.39
Germany	0.54	0.78	-0.70	-1.64	0.54	0.79	-0.02	-0.04	0.53	0.82	-0.06	-0.21	-0.85	-2.02	0.19	1.76
Hong_Kong	1.31	1.38	-0.22	-0.32	1.29	1.36	0.20	0.31	1.28	1.33	0.18	0.27	-0.29	-0.41	0.08	0.63
Italy	0.91	1.08	-0.81	-1.76	0.90	1.06	0.09	0.19	0.92	1.09	0.03	0.06	-0.88	-1.67	0.08	0.66
Japan	1.51	2.27	-0.76	-1.54	1.51	2.23	0.05	0.15	1.53	2.26	-0.01	-0.02	-0.85	-1.82	0.08	0.79
Netherlands	0.72	1.10	-0.44	-1.20	0.72	1.12	-0.03	-0.10	0.73	1.16	-0.06	-0.21	-0.53	-1.61	0.08	0.84
Norway	2.39	2.54	0.17	0.27	2.43	2.58	-0.32	-0.51	2.44	2.60	-0.31	-0.52	0.09	0.12	-0.05	-0.23
Spain	1.00	1.39	-0.17	-0.39	1.03	1.42	-0.21	-0.48	1.02	1.41	-0.23	-0.54	-0.27	-0.68	0.08	0.85
Sweden	0.37	0.50	0.51	0.95	0.37	0.50	-0.01	-0.01	0.30	0.41	0.04	0.11	0.37	0.76	0.21	2.09
Switzerland	0.49	0.88	-0.72	-1.79	0.51	0.90	-0.11	-0.43	0.51	0.99	-0.16	-0.68	-0.84	-2.28	0.13	1.16
UK	0.37	0.53	-0.27	-0.67	0.38	0.54	-0.09	-0.21	0.35	0.53	-0.10	-0.25	-0.40	-1.18	0.18	1.45
Singapore	0.98	0.97	0.48	0.79	1.02	0.99	-0.31	-0.57	0.99	0.94	-0.28	-0.51	0.39	0.63	0.05	0.31
USA	0.07	0.12	-0.42	-1.17	0.05	0.10	0.10	0.33	0.04	0.08	0.07	0.24	-0.50	-1.65	0.14	1.64
Pseudo R^2	0.06		0.17		0.06				0.26							
P-value. $B = 0$	0.00		0.03		0.18				0.76							

This table shows the estimated β's from the first step Fama-MacBeth regressions. The test assets are the 18 country market portfolios plus the US risk free asset, over 1970Q1 to 2018Q4 (quarterly data). The pseudo-R^2 is a goodness of fit measure that captures the percentage of the variation of asset excess returns that is explained by the risk factors. The p-value is based on the F-test of $H_0 : rank\left(\iota_N \vdots \beta\right) = 1$ (or equivalently $\mathbf{B} = 0$); see Kleibergen and Zhan (2019). In each case, we subtract the risk free rate from the asset return such that we can directly test for $\mathbf{B} = 0$. The results are reported for the single factor models (CCAPM, CAPM and Skewness) and the consumption multi-factor models (DLP and HCCAPM).

Table 8: Beta with countries market portfolios (CAPM and global 3 factors model)

	CAPM		Global 3 factors model					
	β_{Mrkt}	t-stat	β_{Mrkt}	t-stat	β_{HML}	t-stat	β_{SMB}	t-stat
Australia	1.03	9.10	0.97	10.36	0.29	3.74	0.45	2.16
Austria	0.85	4.39	1.35	8.71	0.87	6.73	0.35	1.72
Belgium	0.99	9.66	1.13	9.12	0.45	3.22	-0.41	-1.99
Canada	0.98	12.12	1.02	13.98	-0.03	-0.25	0.35	2.90
Denmark	0.75	7.11	0.98	10.96	0.25	2.80	0.22	1.56
France	1.07	14.42	1.20	18.59	0.27	3.24	-0.24	-2.02
Germany	1.05	9.85	1.34	11.48	0.21	2.34	-0.18	-1.37
Hong_Kong	1.18	8.07	0.95	13.58	-0.18	-1.01	0.63	1.78
Italy	1.02	10.99	1.22	22.54	0.56	5.73	-0.10	-0.35
Japan	1.00	11.92	0.90	9.55	-0.13	-1.11	0.54	2.24
Netherlands	1.02	14.26	1.12	15.25	0.32	4.89	-0.28	-2.97
Norway	1.02	6.27	1.22	9.99	0.56	3.84	0.56	2.95
Spain	0.95	8.30	1.25	16.22	0.42	3.42	-0.27	-1.24
Sweden	1.14	10.36	1.34	20.34	-0.25	-1.78	0.57	2.40
Switzerland	0.88	17.95	0.92	23.21	0.39	4.62	-0.42	-3.27
UK	0.99	12.62	0.97	15.29	0.21	3.48	-0.35	-4.19
Singapore	1.14	8.55	1.06	10.55	-0.02	-0.18	0.48	1.37
USA	0.92	26.69	0.90	27.85	-0.11	-1.91	-0.32	-3.55
Pseudo R^2	0.09		0.29					
P-value. B=0	0.00		0.00					

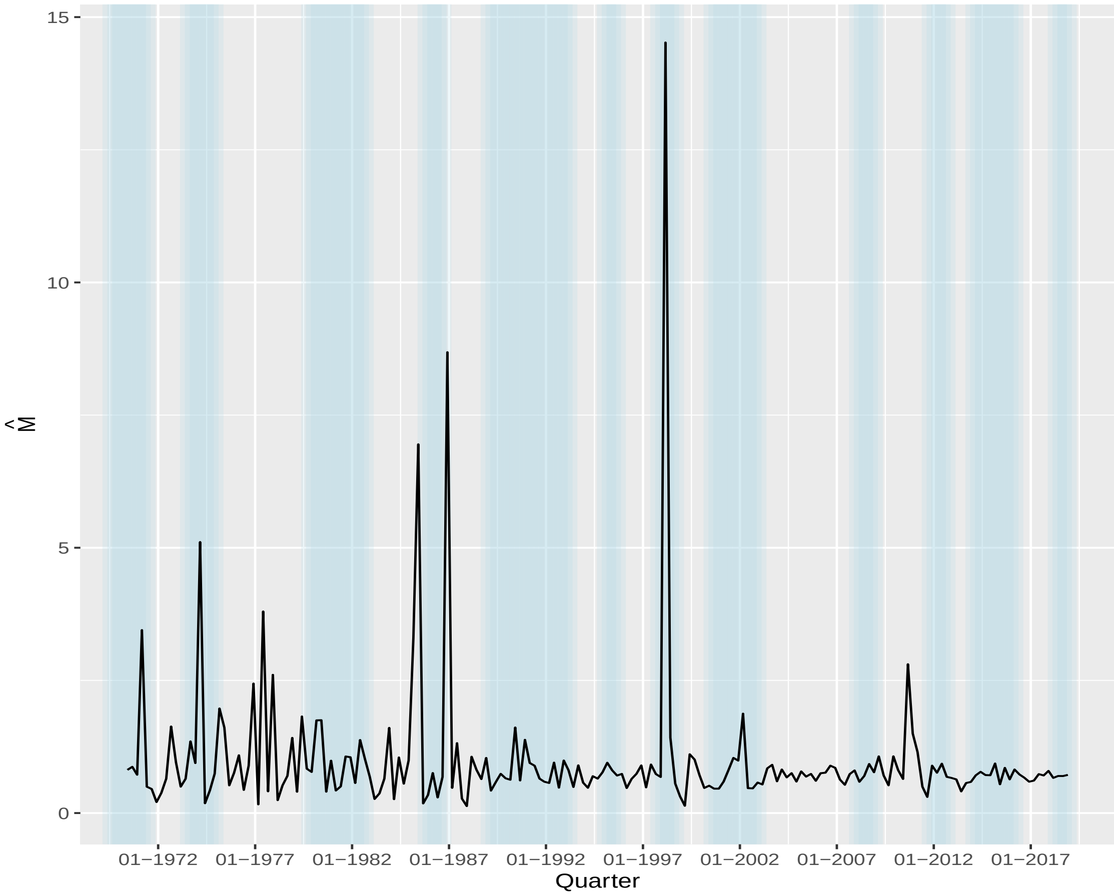
This table shows the estimated β's from the first step Fama-MacBeth regressions. The t-stats. are computed using the Newey-West standard errors with 6 lags. The test assets are the 18 country market portfolios plus the US risk free asset over 1970Q1 to 2018Q4 (quarterly data). The pseudo-R^2 is a goodness of fit measure that captures the percentage of the variation of asset excess returns that is explained by the risk factor. The p-value is based on the F-test of $H_0 : rank\left(\iota_N \vdots \beta\right) = 1$ (or equivalently $\mathbf{B} = 0$); see Kleibergen and Zhan (2019). In each case, we subtract the risk free rate from the asset return such that we can directly test for $\mathbf{B} = 0$. The results are reported for the single factor models (CAPM) with the world MSCI index as the market factor, and the global three factors model for which we obtained the data from Kenneth French Website. For the later factor model, the data range from 1990Q3 to 2018Q4.

Table 9: Estimation results: Fama-MacBeth regressions (with-out intercept in the second pass)

	Consumption-based Models				CAPM and 3 factors Models		
	CCAPM	Skewness	DLP	HWCCAPM		CAPM	3 Global FF
λ_{CCAPM}	0.93		0.98	0.79	λ_{CAPM}	1.20	1.27
	(1.72)		(1.72)	(1.67)		(1.83)	(2.11)
	[1.44]		[1.38]	[1.03]		[1.83]	[2.10]
	(-1.42, 1.40)		(-∞, ∞)	(-∞, ∞)		(0.90, 1.43)	(-0.9, 1.3)
$\lambda_{CrossVar}$			0.66	0.87	λ_{HML}		-1.02
			(0.66)	(0.86)			(-1.79)
			[0.53]	[0.53]			[-1.70]
			(-∞, ∞)	(-∞, ∞)			(-30.5, 0.5)
$\lambda_{CrossSkew}$		-1.02		0.62	λ_{SMB}		0.06
		(-1.23)		(1.28)			(0.11)
		[-1.05]		[0.79]			[0.10]
		(-1.66, 3.33)		(-∞, ∞)			(-116.0, 2.0)
$\lambda_{CrossKurt}$				7.89			-
				(2.28)			
				[1.40]			
				(-∞, ∞)			
R^2	5.60%	16.69%	5.88%	25.97%		9.16%	28.98%
MAE	0.36	0.35	0.36	0.33		0.36	0.41
Rank test	3.34	1.79	1.34	0.71		905.56	3.82
p-value	(2.69e-05)	(0.03)	(0.18)	(0.76)		(0.00)	(0.00)
Alpha test	16.71	18.56	15.13	8.38		26.16	58.71
p-value	(0.47)	(0.35)	(0.52)	(0.87)		(0.07)	(0.00)

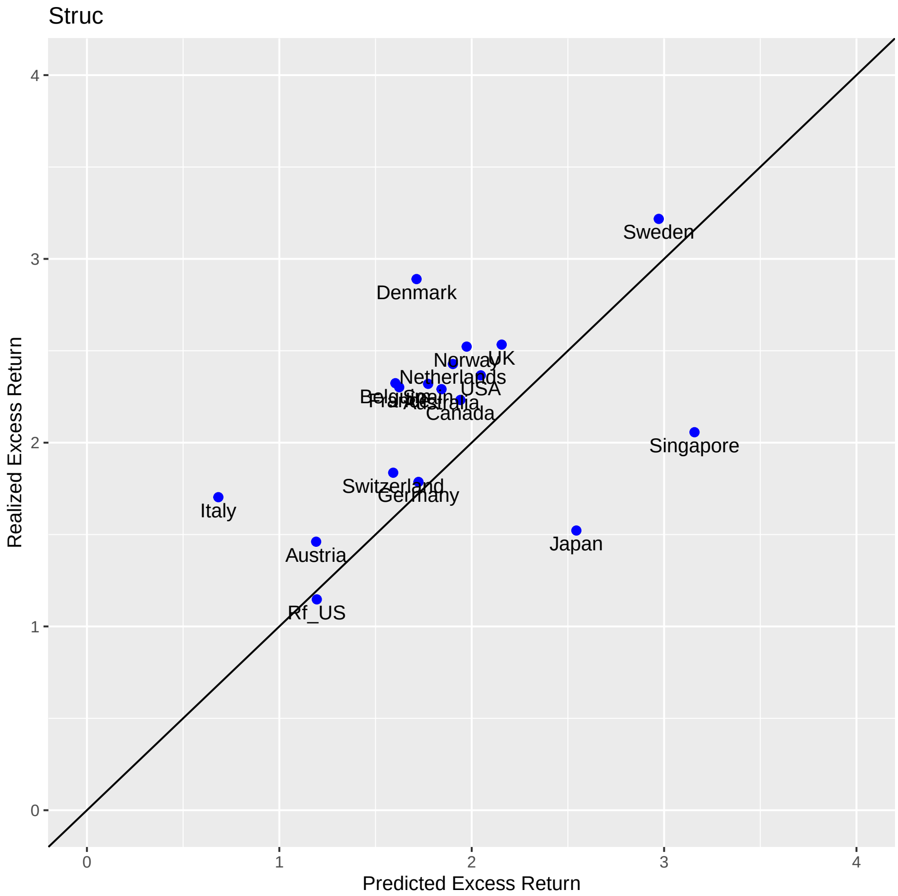
This table shows the results of the Fama-MacBeth regressions. The t-stats. are computed using the Newey-West standard errors with 6 lags. In the first step. we regress for each country the excess return on the factors to obtained the risk exposures (betas). In the second step. we run a cross-sectional regression of the country excess returns at each time period on the betas (without intercept) to obtain the risk price λs. The 3 Fama-French global factor model (Global FF-3factors) uses the data from 1990M1 to 2018M12. The Excess return in the CAPM is the MSCI World return minus the US 3-month Tbill rate. The Newey-West t-statistics with 6 lags (in (blue)) are given in brackets just below the estimates and the Shanken t-statistics (in [red]). which adjust for the fact that the betas are estimated, are given below the Newey-West t-statistics. The 95% weak identification robust confidence set for the price of risk is given in brackets below the Shanken t-statistics. Alpha test is computed using equation (26) and the p-value is given below in brackets. R^2 is the r-squared from the cross-sectional regression (with intercept) of average excess return on the betas.

Figure 1: Structural model implied stochastic discount factor



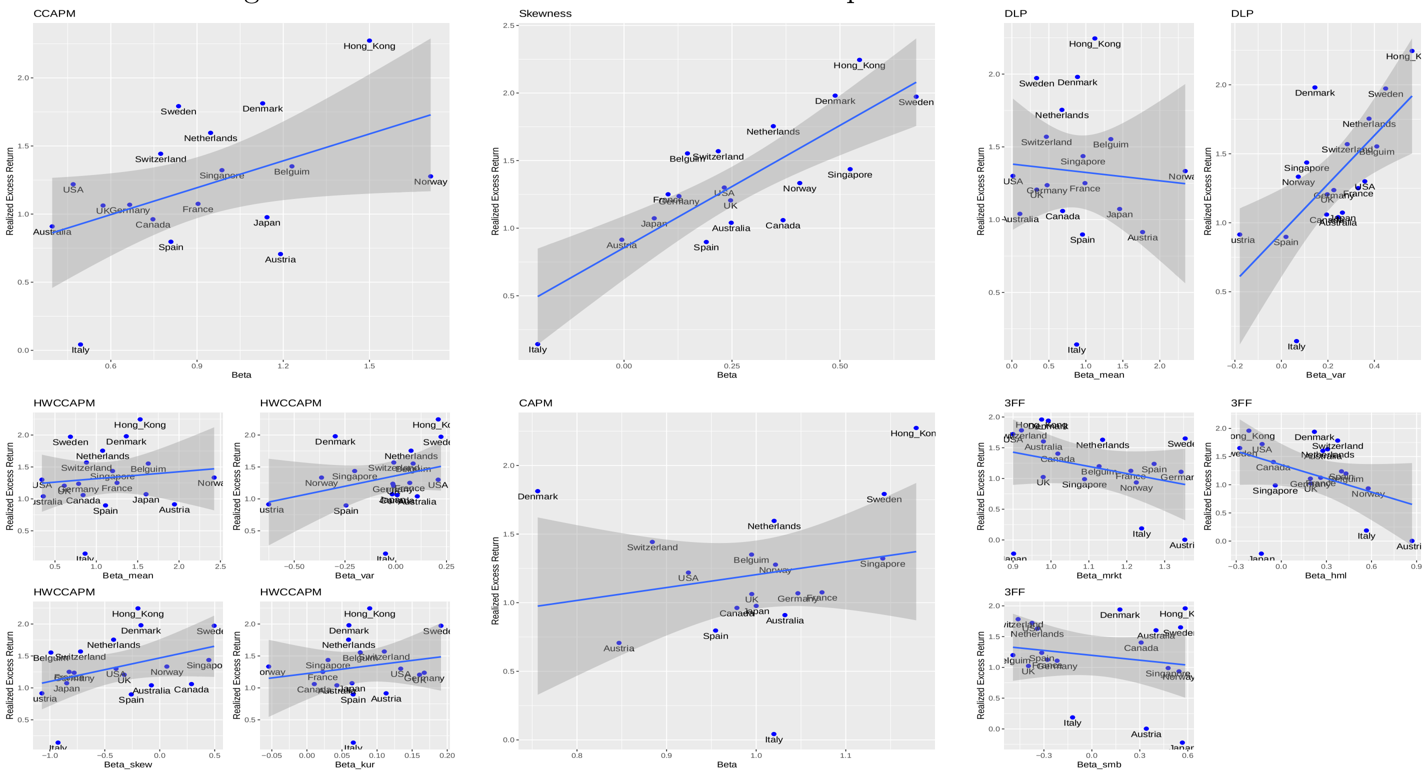
This figure represents the evolution of the structural model estimated stochastic discount factor (see equation 14 from 1970Q2 to 2018Q4). OECD and NON OECD recessions are represented by the blue bars.

Figure 2: Realized and (structural model) predicted expected returns



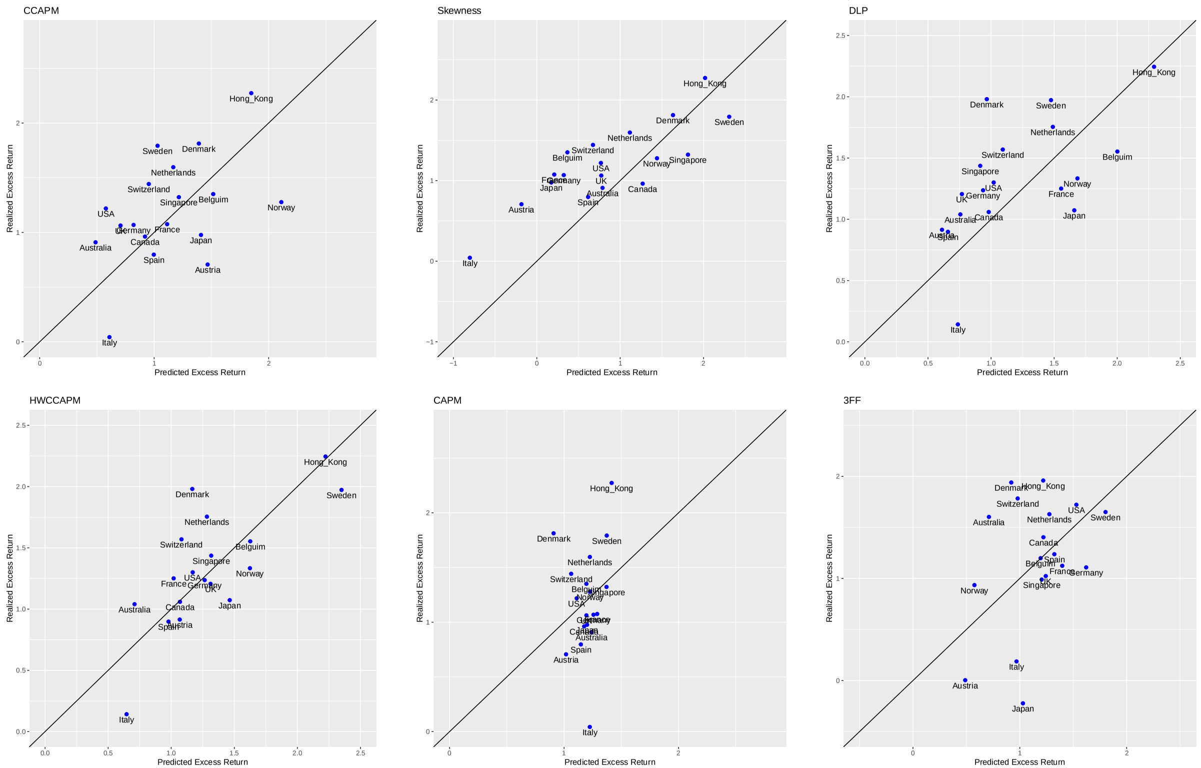
This figure shows the realized and the (structural model) predicted expected return on country market indices. Rf_US represents the average T-bill rate in US considered as the risk free rate.

Figure 3: Betas vs realized returns in Consumption-based factors models



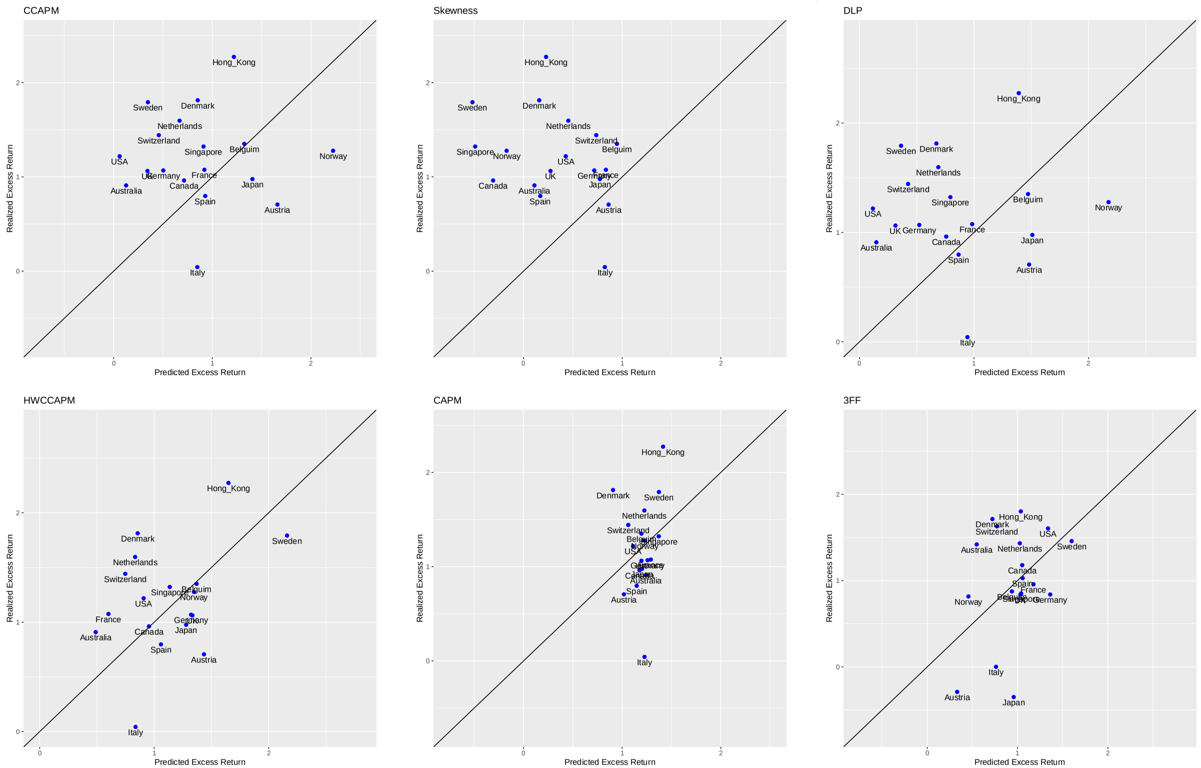
This figures represent betas (risk exposures) vs. realized expected excess returns in the consumption-based factor models. CCAPM denotes the model with only the cross-sectional mean of consumption growth as pricing factor, Skewness is for the model with only the cross-sectional skewness as pricing factor, DLP denotes the model with cross-sectional mean and variance as pricing factors, HWCCAPM is for the model with the four first cross-sectional moments of consumption growth as pricing factors.

Figure 4: Realized vs predicted returns in Consumption-based factors models (Fama-MacBeth regressions with intercept)



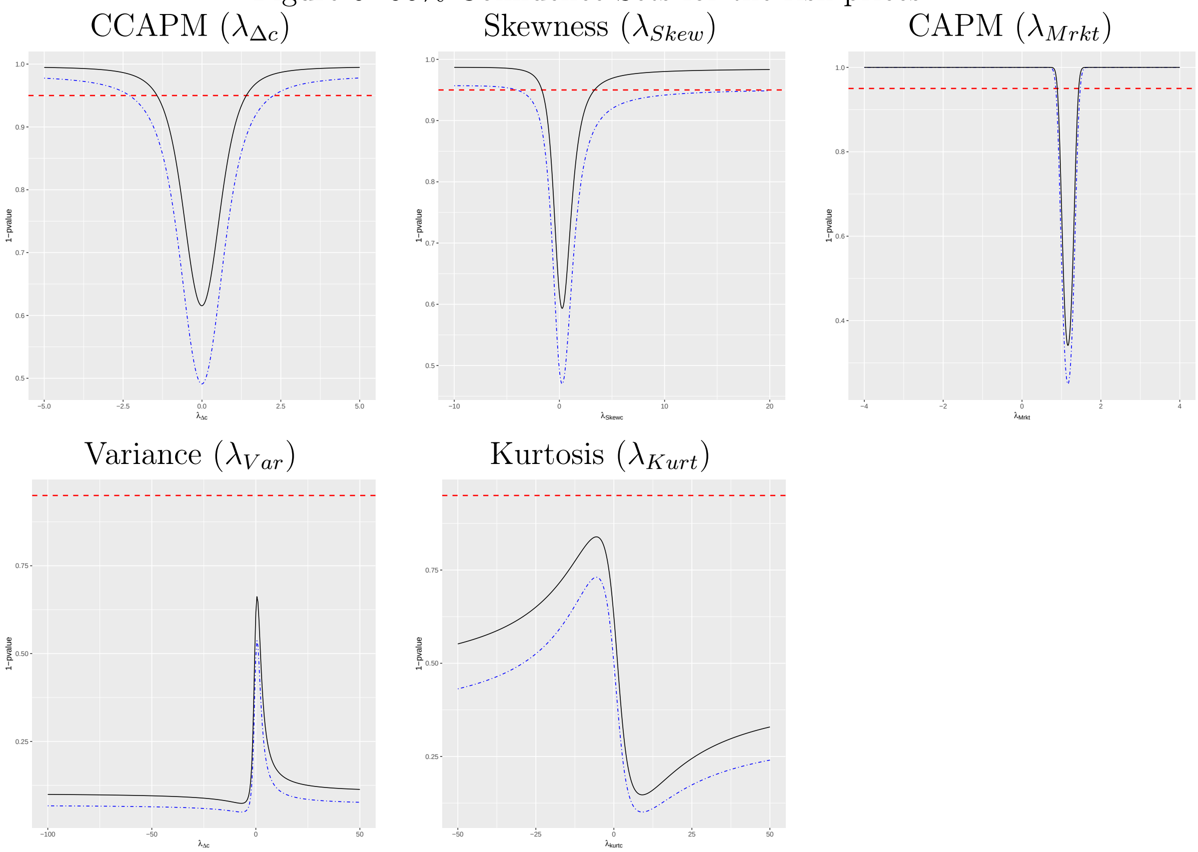
This figure represents betas and the predicted expected excess returns in the consumption-based factor models. The factors are the cross-sectional consumption mean, the cross-sectional consumption variance, the cross-sectional consumption skewness and the cross-sectional consumption kurtosis. The second step of the Fama-MacBeth regression is made with intercept.

Figure 5: Realized vs predicted returns in Consumption-based factors models (Fama-MacBeth regressions without intercept)



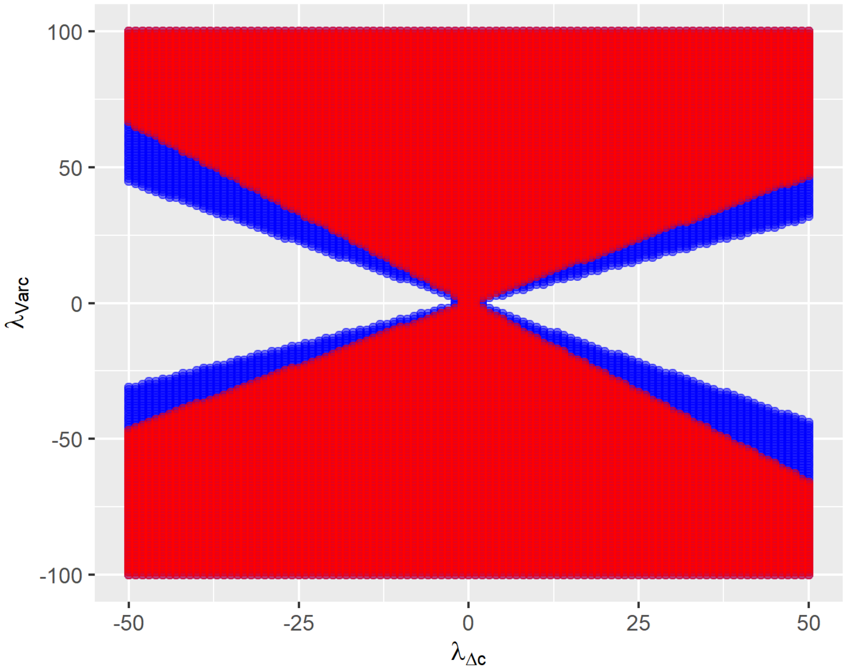
This figure represents betas and the predicted expected excess returns in consumption-based factor models. The factors are the cross-sectional consumption mean, the cross-sectional consumption variance, the cross-sectional consumption skewness and the cross-sectional consumption kurtosis. The second step of the Fama-MacBeth regression is made without intercept.

Figure 6: 95% Confidence Sets for the risk prices



This figure represents one minus p-value plot for the GRS_FAR (black line) and for the F_GRS_FAR (dash-dotted line) - see section 4.3. The dashed horizontal line is the 95% rejection threshold. Each plot corresponds to a single factor model. In the upper panel, from left to right we have respectively the standard CCAPM with the cross-sectional mean of consumption growth as factor, the skewness only model with the cross-sectional skewness of consumption growth as factor and the CAPM model with the world market excess return as factor. In the bottom panel, from left to right we have respectively the variance only with the cross-sectional variance of consumption growth and the the kurtosis only with the cross-sectional kurtosis of consumption growth as factor

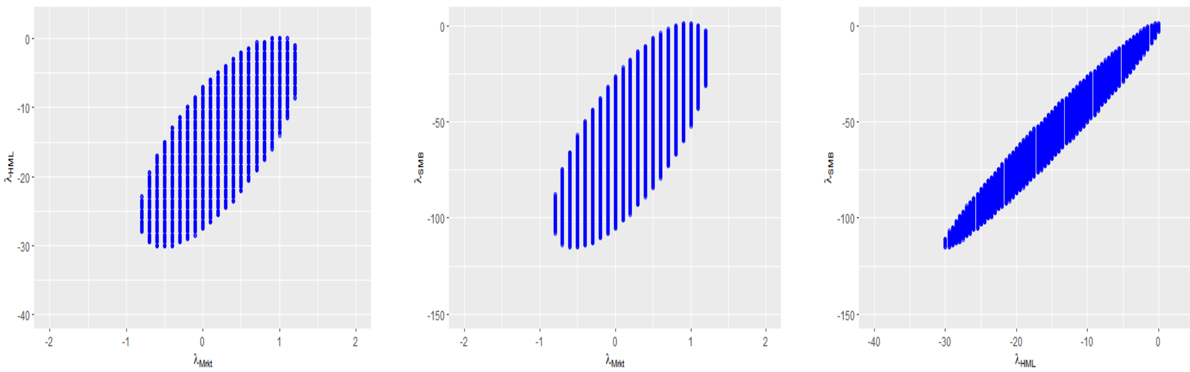
Figure 7: 95% joint confidence sets for the risk prices in the DLP model



This figure represents the 95% joint confidence set for the cross-sectional mean and variance risk prices. The red dots correspond to the confidence set obtained by using the chi-squared asymptotic distribution and the blue dots correspond to the confidence set obtained by using the finite sample Fischer distribution (see section 4.3).

Figure 8: 95% joint confidence Sets for the risk prices in the 3 Fama-French factors model

(λ_{Mrkt},λ_{HML}) (λ_{Mrkt},λ_{SMB}) (λ_{HML},λ_{SMB})



This figure represents from left to right the 95% joint confidence sets of the risk prices for respectively the market vs. High minus Low factors, the market vs. Small minus Big factors and the High minus Low vs. Small minus Big factors. The red dots correspond to the confidence set obtained by using the chi-squared asymptotic distribution and the blue dots correspond to the confidence set obtained by using the finite sample Fischer distribution (see section 4.3).

Table 10: Estimation results: Fama-MacBeth regressions (with intercept)

Panel A: Consumption-based Model									
	$Cste$	λ_{CCAPM}	$\lambda_{CrossVar}$	$\lambda_{CrossSkew}$	$\lambda_{CrossKurt}$	MAE	$\bar{R}^2$	α-test	Avrge.|t-stat|
CCAPM	1.22	-0.02				0.36	5.6%	22.8	0.64
	(1.88)	(-0.06)						(0.16)	(0.27)
Cross Skewness (Only)	1.32			0.36		0.35	16.7%	22.8	0.66
	(1.83)			(0.76)				(0.16)	(0.26)
Heterogeneous World CCAPM (DLP)	1.22	-0.001	0.14			0.36	5.9%	22.61	0.68
	(1.90)	(-0.003)	(0.16)					(0.12)	(0.25)
Heterogeneous World CCAPM	0.93	0.27	0.56	0.50	3.06	0.33	26.0%	15.32	0.65
	(1.16)	(0.28)	(0.53)	(0.95)	(0.98)			(0.36)	(0.26)
Panel B: CAPM and Fama-French 3 factors Model									
		λ_{CAPM}	λ_{HML}	λ_{SMB}		MAE	$\bar{R}^2$		
CAPM	0.29	0.92				0.36	9.2%	25.83	0.64
	(0.16)	(0.50)						(0.08)	(0.27)
Global FF-3factors	1.98	-0.68	-0.87	-0.40		0.41	29.0%	52.3	1.03
	(2.17)	(-0.59)	(-1.19)	(-0.55)				(0.00)	(0.16)

This table shows the results of the Fama-MacBeth regressions. In the first step we regress for each country the excess return on the factors to obtained the risk exposures (betas). In the second step we cross-sectionally regress the country excess returns at each time period on the betas to obtain the risk price λs. The 3 Fama-French global factor model (Global FF-3factors) uses the data from 1990M1 to 2018M12. The Excess return in the CAPM is the MSCI World return minus the US 3-month Tbill rate. The Newey-West t-statistics with 6 lags are given in brackets below the estimates. α-test is computed using a modified version of equation (26) and the p-value is given below in brackets.

Table 11: Results of the GMM estimation

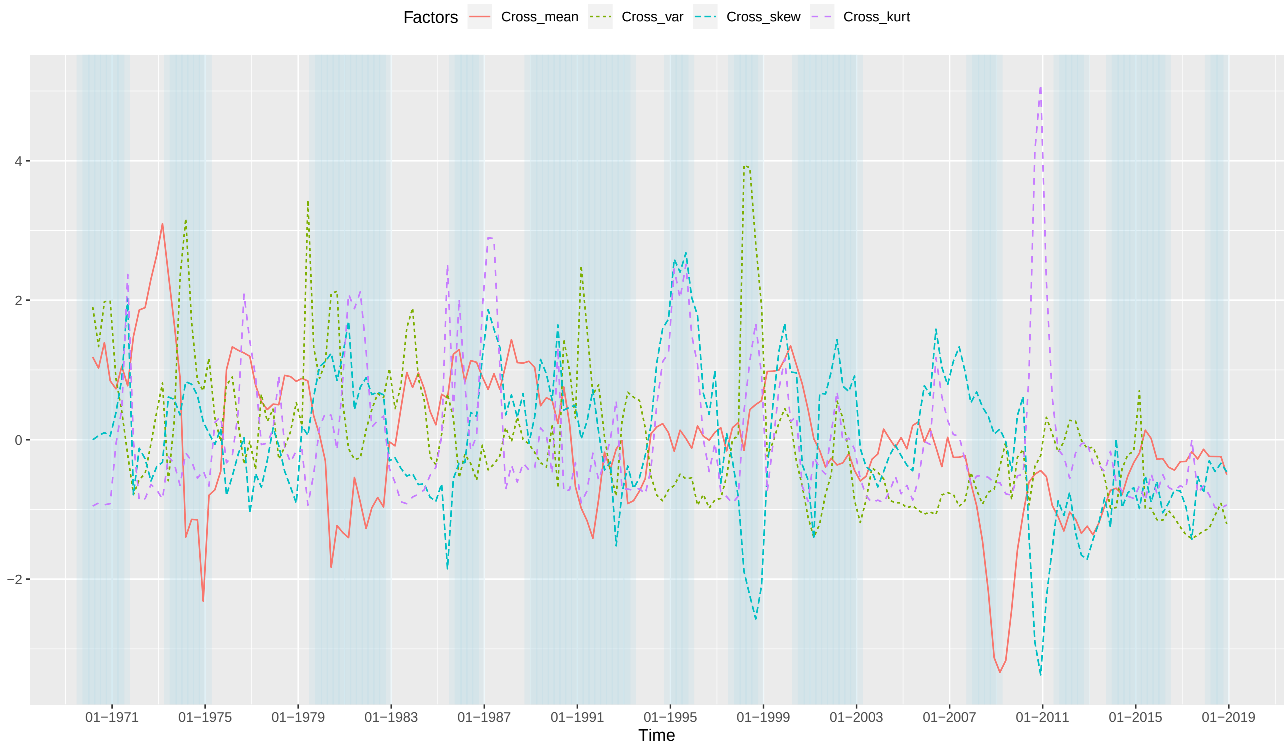
Panel A: Consumption-based Model										
	b_{CMean}	b_{CVar}	b_{CSkew}	b_{CKurt}	μ_{CMean}	μ_{CVar}	μ_{CSkew}	μ_{CKurt}	MAE	$\bar{R}^2$
CCAPM	34.25				0.0065				0.41	7.2%
	(2.10)				(1.31)					
Skewness			39.85				-0.02		0.52	-49.7%
			(4.76)				(-3.50)			
DLP	26.88	41.42			0.013	0.019			0.39	18.6%
	(1.27)	(2.72)			(3.54)	(5.64)				
HWCCAPM	30.4	30.1	13.1	10.3	0.014	0.025	0.001	0.05	0.26	54.3%
	(1.38)	(1.22)	(0.55)	(1.75)	(3.23)	(6.89)	(0.29)	(3.83)		
Panel B: CAPM and Fama-French 3 factors Model										
	b_{Mrkt}	b_{HML}	b_{SMB}	μ_{Mrkt}	μ_{HML}	μ_{SMB}				
CAPM	1.74			0.01					0.36	5%
	(1.42)			(1.60)						
3FF	1.76	-4.19	-5.47	0.013	0.012	0.004			0.47	-42.6%
	(0.95)	(-1.19)	(-0.83)	(2.02)	(3.14)	(1.28)				

This table shows the results of the GMM estimation. We use the identity matrix as the weighting matrix. The pseudo R-squared is the proportion of variation in the expected excess returns explained by the model. MAE is the mean absolute error.

Table 12: Predictability of the ex-post sdf by the cross-sectional moments of consumption growth

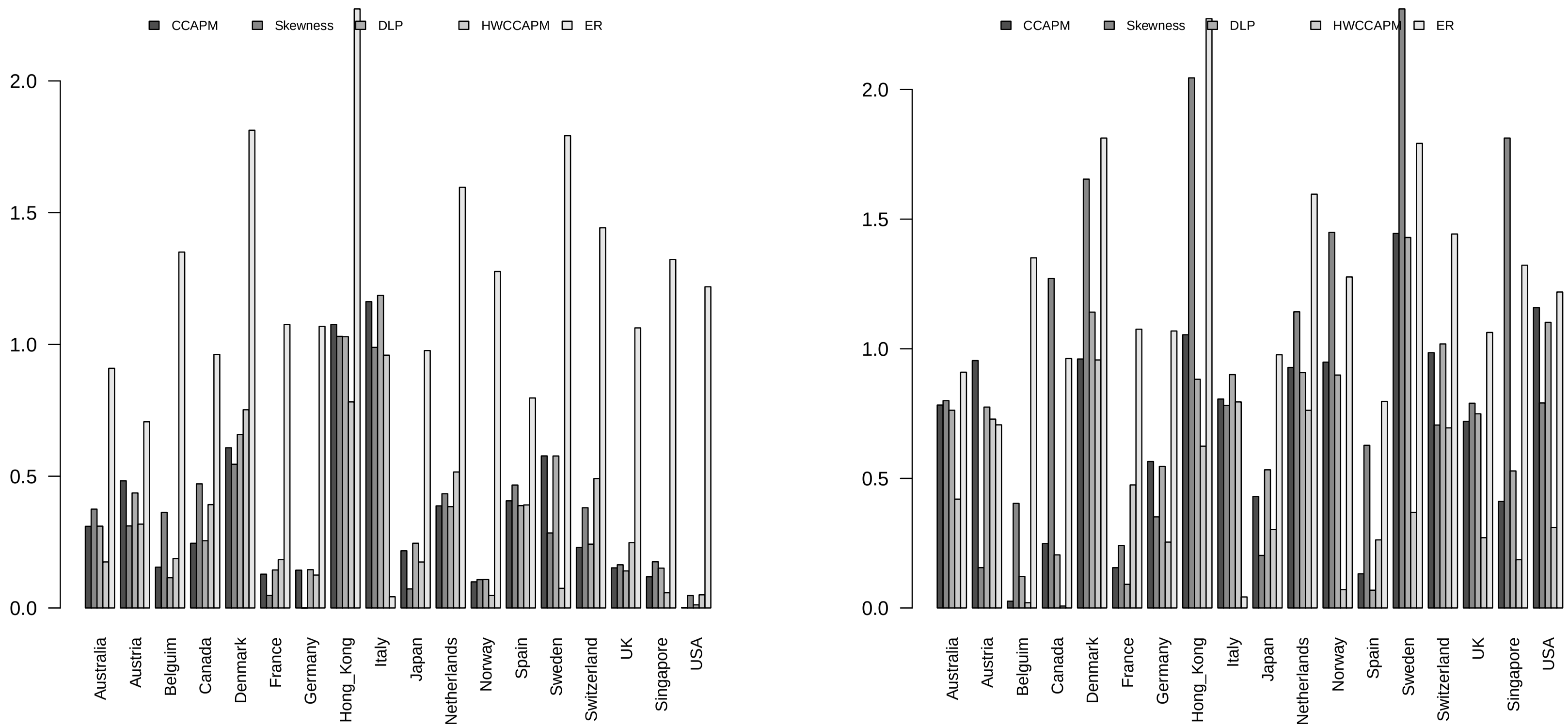
	Dependent variable: Estimated sdf	
	Strutural Model sdf	Ex-post sdf
W_Cross_mean	−0.014	−0.060
	(0.066)	(0.049)
W_Cross_var	0.244***	0.007
	(0.046)	(0.034)
W_Cross_skew	−0.100*	0.062
	(0.057)	(0.043)
W_Cross_kurt	0.041***	−0.013
	(0.013)	(0.010)
Constant	0.076	−0.017
	(0.219)	(0.162)
Observations	195	196
R^2	0.181	0.025
Adjusted R^2	0.163	0.005
Residual Std. Error	1.286	0.956
F Statistic	10.465***	1.227
Note:	*p<0.1; **p<0.05; ***p<0.01	

Figure 9: Evolution of the cross-sectional moments of consumption growth



This figure shows the evolution of the cross-sectional (centred) moments (mean, variance, skewness and kurtosis) of international consumption growth from 1970Q1 to 2018Q4. The cross-sectional moments are weighted by country consumption measured in dollars. The variables on the figure have been standardized.

Figure 10: Absolute pricing errors in consumption-based factor models



This figure shows the absolute pricing errors by asset (country market index) resulting from the estimation of the consumption-based factor models. CCAPM denotes the model with only the cross-sectional mean of consumption growth as pricing factor, Skewness is for the model with only the cross-sectional skewness as pricing factor, DLP denotes the model with cross-sectional mean and variance as pricing factors, HWCCAPM is for the model with the four first cross-sectional moments of consumption growth as pricing factors and ER denotes the average excess return that models wants to explain. For the left hand side picture, the second step of the Fama-MacBeth regressions is with an intercept (See Table 10) and for the right hand side picture it is without the intercept (See Table 9).

Figure 11: Implied Beta vs. Realized returns (GMM estimation)

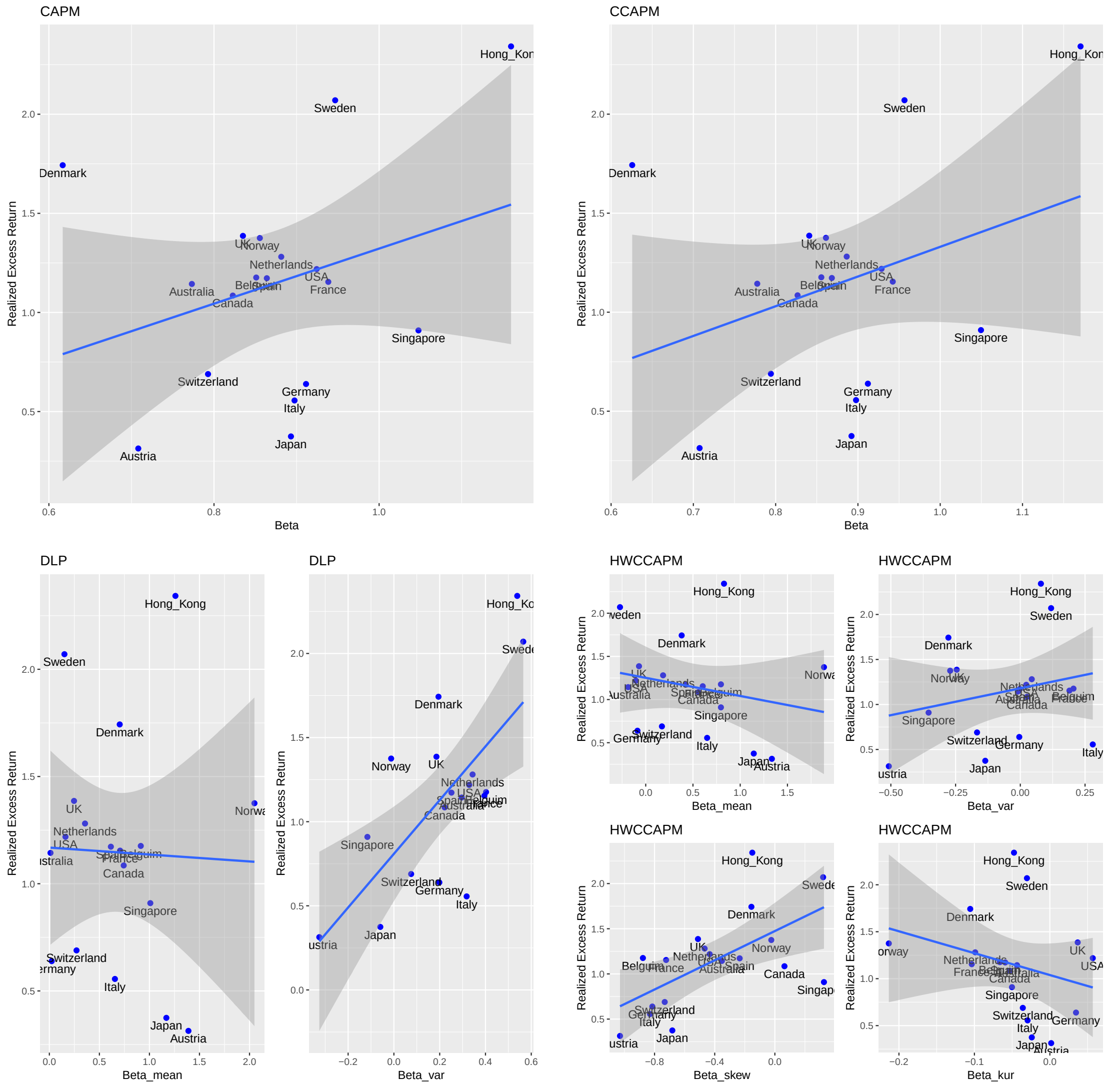


Figure 12: Predicted returns vs. Realized returns (GMM estimation)

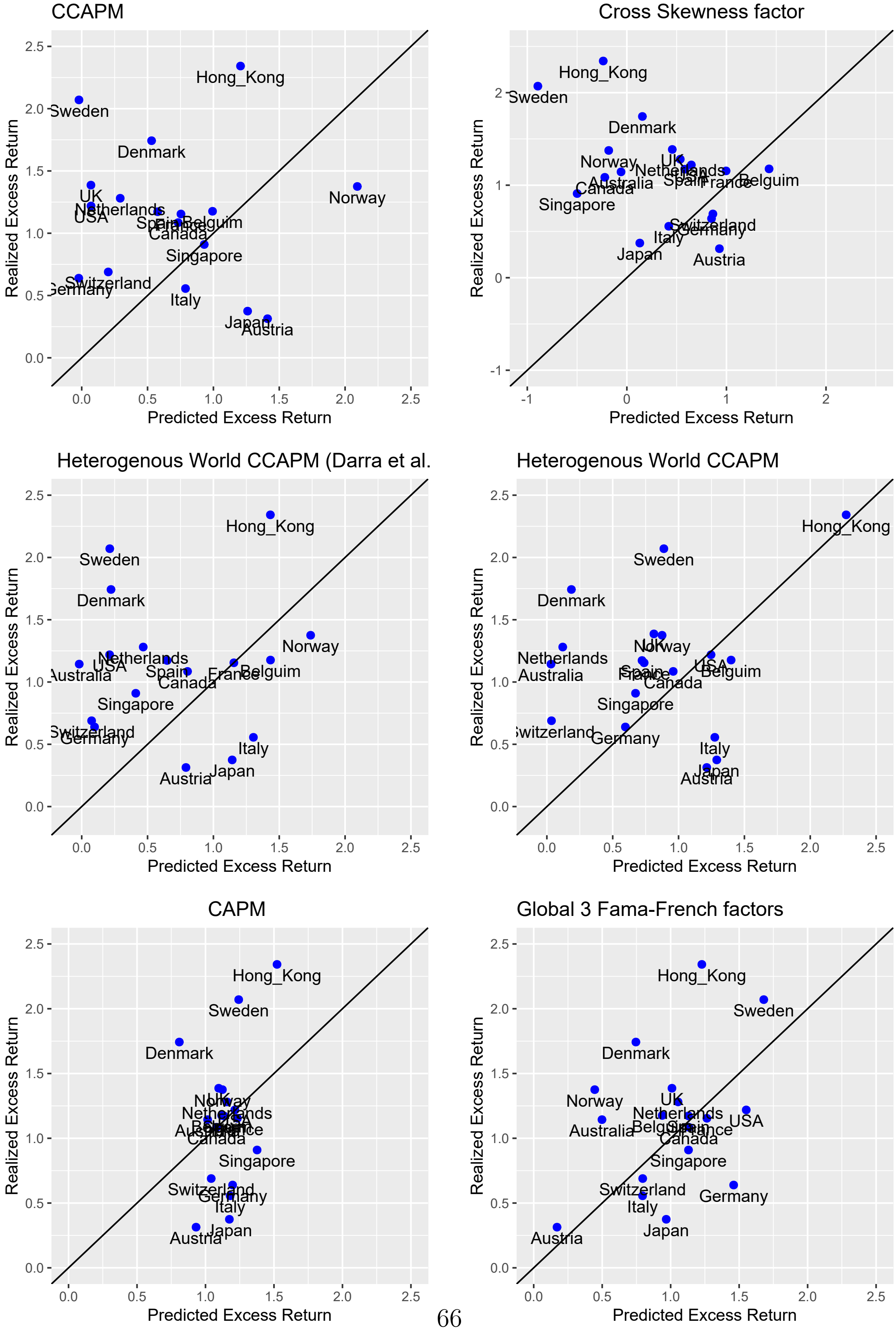
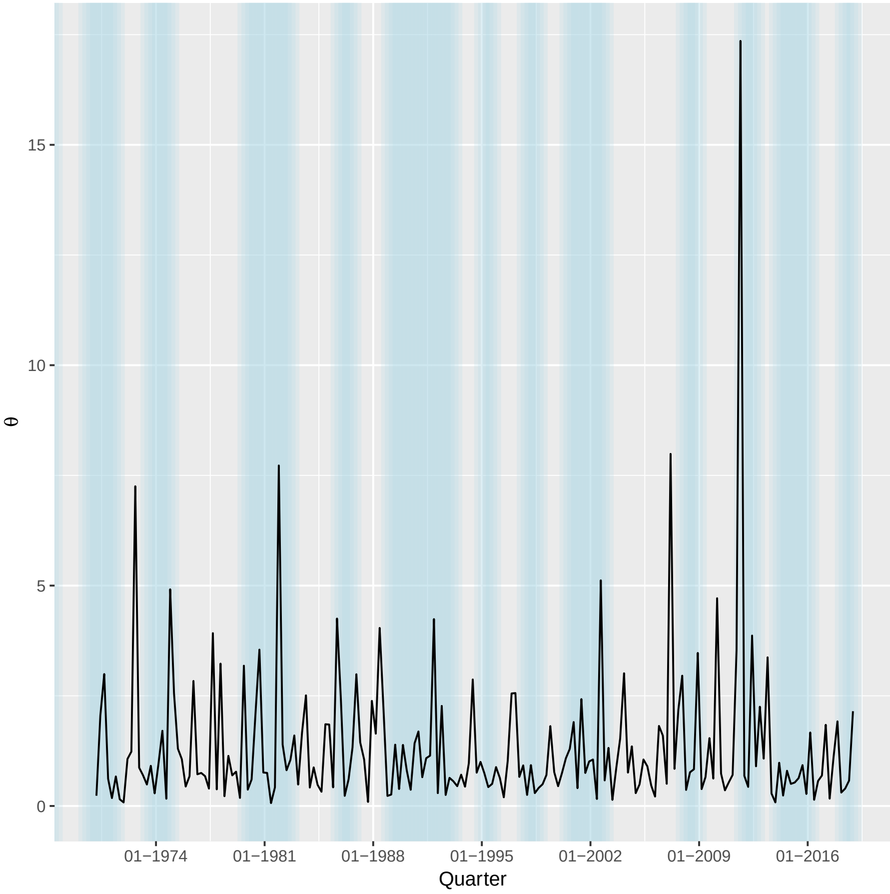
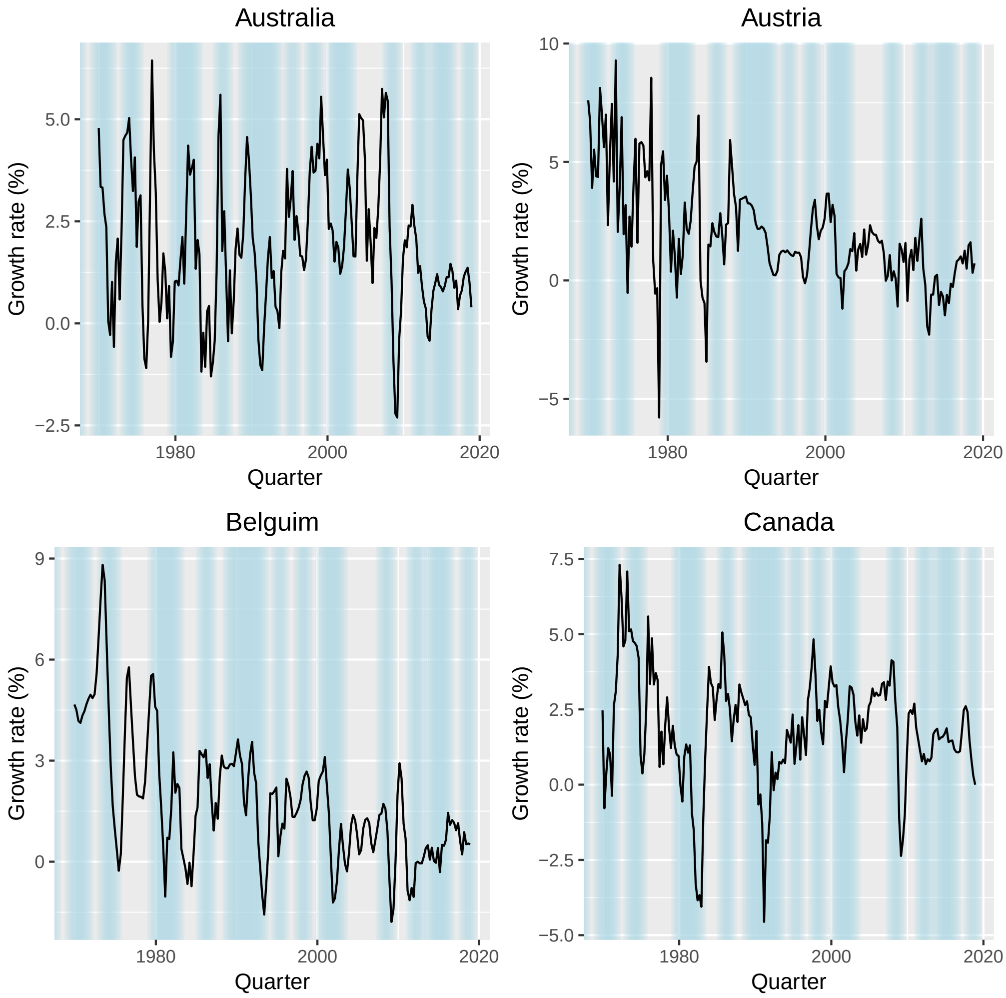


Figure 13: Model free ex-post stochastic discount factor



This figure shows the ex-post model-free sdf extracted from the panel of country market portfolio returns. The blue vertical bars represent the OECD and Non OECD countries recession indicator.

Figure 14: Time series evolution of (selected) countries consumption growth



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