Endogenous ambiguity in cheap talk

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Abstract

This paper provides a rationale for ambiguous language, understood as language generated according to an incompletely known communication rule. We consider a cheap talk game in which a (possibly ambiguity averse) sender (S) able to randomize according to unknown probabilities faces an ambiguity averse receiver (R). We show that under fairly general circumstances, there exist equilibria featuring ambiguous (i.e. Ellsbergian) communication strategies that allow both S and R to obtain a higher ex ante payoff than any non-Ellsbergian equilibrium. Ambiguity, by triggering worse-case decision-making, allows to shift R's response to information towards S's favorite action. R also benefits because equilibria featuring ambiguous communication strategies involve a larger amount of information transmission.

Keywords: cheap talk, ambiguity.

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1 Introduction

Ambiguous language, understood as language generated according to an incompletely known communication rule, is ubiquitous[1]. One potential explanation for this state of affairs is that language ambiguity is advantageous to the parties involved in communication[2]. Our paper offers support for this view. We show that under fairly general circumstances, a (possibly ambiguity averse) sender (S) and an ambiguity averse receiver (R) can jointly benefit from the use of an ambiguous (Ellsbergian) communication strategy. This holds true in the sense that for any influential equilibrium featuring a non-Ellsbergian communication strategy, there always exists an Ellsbergian equilibrium ensuring both S and R a strictly higher expected payoff. S gains because ambiguity mitigates conflict by shifting R's response to information. R also benefits because her decreased ability to respond to information is (more than) compensated by more information.

We model ambiguous communication as randomization by S conditional on a privately observed draw from an Ellsberg urn containing an unknown share ρ of red balls[3]. Consider the following example. Let S privately observe a state $\omega \in [0,1]$ drawn from a known distribution. Pick t, c_1, c_2 s.t. $0 < c_1 < t < c_2 < 1$ and let $I_1 = [0,t]$ and $I_2 = (t,1]$.

[1]Our definition of ambiguous language coincides with the definition of vagueness given by pragmat(ic)ist philosopher C.S Peirce: *"A proposition is vague when there are possible states of things concerning which it is intrinsically uncertain whether, had they been contemplated by the speaker, he would have regarded them as excluded or allowed by the proposition. By intrinsically uncertain we mean not uncertain in consequence of any ignorance of the interpreter, but because the speaker's habits of language were indeterminate".* Lipman (2009) (from whom the above citation is taken) offers a more concrete formulation of the above: *"A related but different critique is that words are vague because we don't know the strategies of other people. If 2 does not know 1's pure strategy, then 2 does not know the intended meaning of the words 1 uses*".

[2]See Lipman (2009), entitled "*Why is language vague?*" or Cohen (2006), entitled "*Why ambiguity*?". Wright (1976), cited in the former, writes: "..*the utility and point of the classifications expressed by many vague predicates would be frustrated if they were supplied with sharp boundaries.*".

[3]Lipman (2009) explicitly considers the possibility of S conditioning her messaging on both the state of the world *and* some privately observed payoff-irrelevant signal. In such a case: "*Since 2 does not observe the signal, 2 does not know how 1 is using words.*". The author however concludes negatively: "..*an equilibrium of this form is at least Pareto inefficient. That is, the best possible outcome can be achieved in an equilibrium where 1 does not condition his choice of words on the payoff-irrelevant signal*". The key difference to our setup is that the distribution of the payoff irrelevant signal is assumed by Lipman to be objectively describable.

Now, let S use the following communication strategy. For $i = 1, 2$, if $\omega \in I_i$ and S draws a red ball, she sends message m_i^A if $\omega \leq c_i$ and m_i^B if $\omega > c_i$. If $\omega \in I_i$ and S draws a non-red ball, she reverses the strategy followed after a red draw, i.e. sends m_i^B if $\omega \leq c_i$ and m_i^A if $\omega > c_i$. After determining whether $\omega \in I_1$ or $\omega \in I_2$, S thus randomizes with unknown probabilities between two reciprocal messaging rules on I_i. Note that if S is known to follow this strategy, the message subscript i allows R to learn whether $\omega \leq t$ or $\omega > t$ but leaves her Knighteanly uncertain as to whether $\omega \leq c_i$ or $\omega > c_i$.

To relate the above description to everyday language, let the state ω be S's assessment of a theater play. Let $m_1^A, m_1^B, m_2^A, m_2^B$ be the respective words {*great*, *brilliant*,*mediocre*,*bad*}. One can reasonably postulate that anyone would ascribe to any element of α={*great*, *brilliant*} a more positive meaning than to any element of β={*mediocre*,*bad*} and would use these words accordingly. In contrast, there appears to be no established convention on how to rank *mediocre* versus *bad* or *great* versus *brilliant*. While the two binary sets α and β are clearly ordered with respect to each other, elements within these respective sets are thus themselves not clearly ordered. These *local orders* might be said to constitute an individual's linguistic style, which is idiosyncratic, arbitrary and to some extent privately known (at least to strangers). We posit that these local orders are subject to Knightean uncertainty, i.e. decided according to a process that is not objectively describable.

We show that the above described type of ambiguity of language (which one could describe as *incomplete orders*) has real consequences in that it beneficially affects S's ability to influence R's decision making if the latter is ambiguity averse. Ambiguity triggers worse case decision making by R, implying subjective overweighting of low probability events. This may serve as an implicit commitment device allowing R to act more in line with S's preferences, thus allowing finer information transmission to both participants' benefit.

Formally, the novelty of our analysis resides is the study of Ellsbergian strategies within the classical Crawford and Sobel (1982) cheap talk game (CS in what follows). In so-doing, we build on the work of Azrieli & Teper (2011), Bade (2010) as well as Riedel & Sass (2011) who have formalized the notions of ambiguous strategies and equilibrium. A large set of papers study applications to finance, tournaments, contract theory or mechanism design. Bose & Renou (2014), which is closest to the present paper, examines mediated communication featuring an Ellsbergian device that allows to beneficially generate Knightean uncertainty. The class of Ellsbergian communication strategies that we introduce builds on this idea. In the cheap talk literature, a set of papers model ambiguity as noise by which a given sent message gives rise to distribution over observed messages (describable as diverse interpretations). In Board, Blume and Kawamura (2007), noise is exogenous whereas in Blume and Board (2014), S voluntarily adds noise. In both cases, noise beneficially mitigates conflict. Our insight is similar; the main difference being that noise is Knightean in our setting. We refer to Lipman (2009) for a more speculative reflection on ambiguous language that among other aspects discusses the potential role of bounded rationality. Finally, Kellner and Le Quement (2014)) examines cheap talk with an ambiguous prior distribution of the state. We find that in equilibrium, communication exhibits features that one can interpret as two different modes of ambiguous communication. The paper thus provides a competing rationale for ambiguous language as compared to the present paper.

2 Model

There are two players, a sender S and a receiver R. The state ω is privately known to S and drawn from a commonly known distribution endowed with the continuously differentiable cdf F and density f on the support $[0,1]$. S privately draws a ball from an Ellsberg urn containing balls of n different colors numbered 1 to n. Let ρ_i denote the proportion of color i balls. The vector $\rho = (\rho_1, ., \rho_n)$ is Knighteanly unknown to S and R. Let θ be a random variable taking value θ_i if the drawn ball has color n. The variable θ is thus privately observed by S. Let Δ^ρ denote the set of all vectors $\rho = (\rho_1, ., \rho_n)$ satisfying $\sum_{i=1}^{n} \rho_i = 1$. S picks a message $m \in M$, where M is a rich message space. R picks an action $a \in \mathbb{R}$. The timing of the game is as follows. S observes ω and draws a ball from the urn. S sends some m. R picks a after observing m. Given a and ω, the utility function of $J \in \{S, R\}$ is denoted $U^J(a, \omega)$ and

$$U^S(a, \omega) = G(\omega + \beta(\omega) - a);\ U^R(a, \omega) = G(\omega - a),$$

where $G(x)$ is a concave and single peaked function of x and $\beta(\omega) > 0$, $\forall \omega \in [0,1]$.

For two bias function $\beta(\omega)$ and $\beta'(\omega)$ s.t. $\beta'(\omega) > \beta(\omega)\ \forall\omega$, we write $\beta' > \beta$. For given β and $\varepsilon > 0$, let $\beta + \varepsilon$ denote β' s.t. $\beta'(\omega) = \beta(\omega) + \varepsilon\ \forall\omega$. Letting subscripts denote partial derivatives, we also assume the following for every $j \in \{R, S\} : U_1^j = 0$ for some a, $U_{11}^j < 0$, $U_{12}^j > 0$. Both S and R are ambiguity averse and apply the Max-Min decision rule (Gilboa (1987), Gilboa and Schmeidler (1989)).

A standard communication strategy is given by a family of signaling rules for S denoted by $q(m\,|\omega)$. Such a family defines a distribution over M for each value of ω and is thus a mapping $[0, 1] \to \Delta^M$, where Δ^M is the set of distributions over M. An Ellsbergian communication strategy is given by a (finite, for simplicity) set $(Q_1, ., Q_n)$ of standard communication strategies. One interpretation of how S effectively plays such a strategy is that she conditions her choice of Q_i on the value of θ, i.e. on the color of the ball drawn from the available Ellsberg urn. One may thus write an Ellsbergian communication strategy as a set $(q(m\,|\omega, \theta_1), ., q(m\,|\omega, \theta_n))$. A strategy of R is a decision rule $\delta(a\,|m)$ specifying a mixed action for any $m \in M$. Letting Δ^a denote the set of distributions over $\mathbb{R}$, a strategy of R is a mapping $M \to \Delta^a$.

We define an equilibrium concept that is an analogue of Perfect Bayesian equilibrium for the case where S can use an Ellsbergian strategy. Our equivalents of the sequential rationality and consistent beliefs conditions reduce to the following requirements here. The strategy of S is sequentially rational, given the strategy of R and S's beliefs. The strategy of R follows the Max-Min rule conditional on beliefs. As to belief formation, S learns the true state and R performs prior by prior Bayesian updating conditional on knowledge of S's equilibrium strategy[4]. Note that S faces no ambiguity at any information set where she is called upon to act.

Formally, a strategy profile $\{(q(m\,|\omega, \theta_1), ., q(m\,|\omega, \theta_n)), \delta(a\,|m)\}$ is thus an equilibrium iff the following conditions hold. First, $\int_M q(m\,|\omega, \theta)dm = 1\ \forall\,(\omega, \theta) \in [0, 1] \times \{\theta_1, ., \theta_n\}$, where any m^* in the support of $q(m\,|\omega, \theta)$ solves

$$\max_{m \in M} \int_{a \in \mathbb{R}} U^S(a, \omega)\delta(a\,|m)da.$$

[4]A consensus has yet to emerge on the right modelling of updating of ambiguity averse preferences. We refer to Siniscalchi (2011) as well as Hanany and Klibanoff (2007, 2009) for a discussion of this issue.

Second, for each m, δ^* solves

$$\max_{\delta^* \in \Delta^a} \min_{\rho \in \Delta^\rho} \int_0^1 \left(\int_{a \in \mathbb{R}} U^R(a,\omega) \delta^*(a \,| m) da \right) p(\omega \,| m, \rho) d\omega,$$

where

$$p(\omega \,| m, \rho) = \frac{\sum_{\theta \in \{\theta_1,..,\theta_n\}} p(\theta \,|\rho) q(m \,|\omega, \theta) f(\omega)}{\int_0^1 \sum_{\theta \in \{\theta_1,..,\theta_n\}} p(\theta \,|\rho) q(m \,| t, \theta) f(t) dt}.$$

We follow Sobel (2013) in distinguishing between informative, influential and payoff-relevant communication. Communication is informative if it affects beliefs, i.e. if $p(. \,| m, \rho)$ is not constant on the equilibrium path for all ρs. Communication is influential if it affects actions, i.e. if $\delta(. \,| m)$ is not constant on the equilibrium path. Communication is payoff-relevant if at least one agent's ex ante expected payoff differs from that implied by R's ex ante payoff maximizing action. Denote by respectively $\pi^S\left(\beta, \widetilde{E}\right)$ and $\pi^R\left(\widetilde{E}\right)$ the (ex ante) expected payoff of S and R given the decision rule implemented in equilibrium $\widetilde{E}$. The notion of ex ante expected payoffs is unproblematic as we examine equilibria featuring no ex ante Knightean uncertainty regarding the implemented decision rule.

Our analysis proceeds as follows. Section 3 states central (and with one minor exception) preexisting results for the non-Ellsbergian case. Section 4 analyzes the general model and contains our main result concerning the existence of Pareto improving Ellsbergian equilibria. Section 5 examines the special case of the Uniform-Quadratic setup and studies specific analytically tractable subclasses of Ellsbergian communication strategies, with an eye to examining questions left unanswered within the general setup.

3 The non-Ellsbergian case

Our utility functions are a special case of those assumed in CS, which allows us to directly invoke existing comparative static results. Consider two sender utility functions featuring respectively β and β' with $\beta' > \beta$. These can be generated from a common function $U^S(a,\omega,b)$ satisfying the assumptions made in sections 2 and 5 of CS. I.e. $U^S(a,\omega,b)$ is s.t. b is a scalar parameter measuring interest misalignment, $U^S_{13} \geq 0$ everywhere and $U^S(a,\omega,0) = U^R(a,\omega)$.

Remark 1 *Consider G, β and β' s.t. $\beta' > \beta$. There is a function $U^S(a, \omega, b)$ s.t: a) $U^S(a, \omega, 1) = G(\omega + \beta(\omega) - a)$, b) $U^S(a, \omega, 2) = G\left(\omega + \beta'(\omega) - a\right)$, c) $U^S(a, \omega, 0) = G(\omega - a)$, d) $U^S_{13}(a, \omega, b)$ is strictly positive everywhere.*

Proof: see Appendix A.

We know from CS that absent Ellsbergian strategies, any equilibrium is equivalent to an equilibrium featuring a non-Ellsbergian partitional communication strategy (NPCS). Let there be labelled messages $\{m_0\}_{r=0}^{N-1}$. A NPCS is described by a vector of thresholds $t_0 = 0 < t_1 < . < t_N = 1$ s.t. sender type 0 sends m_0 and all types in $(t_i, t_{i+1}]$ send m_i $\forall i$. We call an equilibrium featuring such a strategy an NPCE and call N its fineness. We call an NPCE non-degenerate if each action is induced by a set of types with positive measure. Denote by $a^*_{ne}(t_{i-1}, t_i)$ R's optimal action given $\omega \in (t_{i-1}, t_i]$. The profile of thresholds $\{t_r\}_{r=1}^{N-1}$ constitutes an NPCE iff

$$U^S(a^*_{ne}(t_{i-1}, t_i), t_i) = U^S(a^*_{ne}(t_i, t_{i+1}), t_i),\ i = 1, ., N-1. \tag{1}$$

The fact that the class of NPCEs is sufficient implies that randomization by S is not useful in the non-Ellsbergian case. For every equilibrium involving randomization (for example by S conditioning her messaging on some payoff-irrelevant private signal), there is indeed a NPCE implementing the same decision rule.

CS states the following monotonicity condition **M:** If t and $\widetilde{t}$ are two solutions of (1) with $t_0 = \widetilde{t}_0$ and $\widetilde{t}_1 > t_1$, then $\widetilde{t}_i > t_i$, for any $i \geq 2$. CS and Szalay (2012) provide different sufficient conditions for M. We assume that condition **M** holds and next recall a set of basic properties of the model. Given N, let $\Gamma(N)$ denote the set of βs s.t. there exists an N-partitions equilibrium.

Proposition 1 ***Aspects of the Crawford & Sobel (1982) characterization.***

Assume that S is restricted to using non-Ellsbergian strategies.

1. Given β, there is a finite $\overline{N}(\beta)$ s.t $\forall N \leq \overline{N}(\beta)$ $(N > \overline{N}(\beta))$, there is a unique (there is no) N-partitions equilibrium. We call the unique N-partitions equilibrium $E(\beta, N)$ and denote its threshold profile by $\{t_r(\beta, N)\}_{r=1}^{N-1}$.

2. $\overline{N}(\beta) \geq \overline{N}(\beta')$ if $\beta' > \beta$.

3. $\pi^R(E(\beta,N)) > \pi^R(E(\beta',N))$ *for* $\beta < \beta' \in \Gamma(N)$.

4. $\pi^S(\beta, E(\beta, N-1)) < \pi^S(\beta, E(\beta,N))$ *and* $\pi^R(E(\beta, N-1)) < \pi^R(E(\beta,N))$ *for* $\beta \in \Gamma(N)$.

Point 2 states that a higher bias leads to a weakly lower maximal number of equilibrium partitions. Point 3 states that given N, R's expected payoff decreases as bias increases. Point 4 states that given β, both S and R favor equilibria with more partitions. We now show that given N, a less biased S obtains a higher expected payoff. This result does not appear in CS and is an equivalent of Point 3 for S.

Lemma 1 $\pi^S(\beta, E(\beta,N)) > \pi^S(\beta+\varepsilon, E(\beta+\varepsilon,N))$ *for* β *s.t.* $\beta, \beta+\varepsilon \in \Gamma(N)$.

Proof: see Appendix A.

4 Main analysis

This section is organized as follows. We first introduce the class of Ellsbergian partitional communication strategies and present equilibrium conditions for corresponding equilibria. We then show our main result concerning the existence of Pareto improving Ellsbergian equilibria conditional on there existing an influential non-Ellsbergian equilibrium. We finish by noting the existence of a new type of (Ellsbergian) babbling equilibrium displaying new properties w.r.t. the classical babbling equilibrium.

Simple Ellsberg randomization, defined below, is a key building block of the Ellsbergian partitional communication strategies that we shall focus on.

Definition 1 ***Simple Ellsberg randomization*** φ

Let $(\underline{\omega}, \overline{\omega}] \subseteq [0,1]$, $c \in (\underline{\omega}, \overline{\omega}]$ *and* $m, m' \in M$. *The simple Ellsberg randomization* $\varphi(\underline{\omega}, \overline{\omega}, c, m, m')$ *is defined as follows. Given* $\theta = \theta_1$, *send* m *with probability* 1 *if* $\omega \in (\underline{\omega}, c)$ *and* m' *with probability* 1 *if* $\omega \in [c, \overline{\omega}]$. *If* $\theta \neq \theta_1$, *send* m' *with probability* 1 *if* $\omega \in (\underline{\omega}, c)$ *and* m *with probability* 1 *if* $\omega \in [c, \overline{\omega}]$.

A simple Ellsberg randomization is thus a randomization with unknown probabilities over two reciprocal partitional strategies on $(\underline{\omega}, \overline{\omega}]$. We credit Bose and Renou (2014) with proposing this particular type of method of generating ambiguous beliefs. We now introduce the Ellsbergian communication strategy that shall be the focus of our analysis

Definition 2 ***Ellsbergian partitional communication strategy (EPCS)***

Let there be two profiles of thresholds $t_0 = 0 < t_1 < . < t_{N-1} < t_N = 1$ *and* $\{c_r\}_{r=0}^{N-1}$ *s.t* $c_i \in (t_i, t_{i+1}]$, $i = 0, ., N-1$. *If* $\omega \in (t_i, t_{i+1}]$, *S applies* $\varphi\left(t_i, t_{i+1}, c_i, m_i^A, m_i^B\right)$, $i = 0, ., N-1$. *An EPCS is thus summarized by* $\left\{\{t_r\}_{r=1}^{N-1}, \{c_r\}_{r=0}^{N-1}\right\}$.

An EPCS simply adds Ellsbergian randomization to a NPCS and posits a simple two-steps procedure. S first determines the partition $(t_i, t_{i+1}]$ in which ω is located. She then applies the randomization $\varphi\left(t_i, t_{i+1}, c_i, m_i^A, m_i^B\right)$. For a given EPCS featuring $\{t_r\}_{r=1}^{N-1}$, we still refer to N as the number of partitions (or fineness). We call an equilibrium featuring an EPCS an Ellsbergian PCE (i.e. EPCE).

Slightly abusing notation, denote by $a_{ne}^*(I)$ R's best response to $\omega \in I$, where I is an interval of $[0, 1]$. We assume the following.

Assumption 1 *Let* $0 \leq \underline{\omega} < \overline{\omega} \leq 1$ *and* $c \in (\underline{\omega}, \overline{\omega}]$. *Let* $I_1 = (\underline{\omega}, c)$ *and* $I_2 = [c, \overline{\omega}]$. *Given* $i \in \{1, 2\}$, *let* $\{j\} = \{1, 2\} \setminus \{i\}$. *For any* $i \in \{1, 2\}$,

$$E\left[U^R(a_{ne}^*(I_i), \omega) \left|\omega \in I_i\right.\right] > E\left[U^R(a_{ne}^*(I_i), \omega) \left|\omega \in I_j\right.\right]. \tag{2}$$

The assumption implies a form of ex ante equivalence of all closed subsets of $[0, 1]$ in that none yields disproportionately high payoffs. Technically speaking, it ensures well-behaved expected utility curves, which yields a simple characterization of R's best responses as given in the next Lemma. We know that A.1 is satisfied in the Uniform-Quadratic model and is thus compatible with condition **M**.

Lemma 2 *In an equilibrium featuring the EPCS* $\left(\{t_r\}_{r=1}^{N-1}, \{c_r\}_{r=0}^{N-1}\right)$, *R's best response to* m_i^A *and* m_i^B *is identical. Denote it by* $a_e^*(t_i, t_{i+1}, c_i)$.

a) $a_e^*(t_i, t_{i+1}, c_i)$ *satisfies:*

$$\begin{aligned} & \int_{t_i}^{c_i} U^R(a_e^*(t_i, t_{i+1}, c_i), \omega) f\left(\omega \left|\omega \in (t_i, c_i)\right.\right) d\omega \\ = \; & \int_{c_i}^{t_{i+1}} U^R(a_e^*(t_i, t_{i+1}, c_i), \omega) f\left(\omega \left|\omega \in [c_i, t_{i+}]\right.\right) d\omega. \end{aligned}$$

b) $a_e^*(t_i, t_{i+1}, c_i)$ *is a continuous and strictly increasing function of* c_i *and*

$$a_e^*(t_i, t_{i+1}, t_i) < a_{ne}^*(t_i, t_{i+1}) < a_e^*(t_i, t_{i+1}, t_{i+1}).$$

Proof: see Appendix B.

Figure 1 below illustrates this Lemma and shows how R's best response shifts to the right as c_i moves towards t_{i+1}. We set $t_i = 0$, $t_{i+1} = .75$ and consider in turn $c_i = .2$ and $c_i = .6$. Continuous curves correspond to $E\left[U^R(a,\omega)\,|\omega \in (0,.2)\right]$ and $E\left[U^R(a,\omega)\,|\omega \in [.2,.75]\right]$. Dashed curves are equivalents for respectively $\omega \in (0,.6)$ and $\omega \in [.6,.75]$.

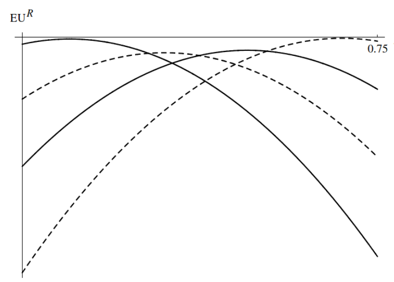


Figure 1.

The Max-Min best response $a_e^*(t_i, t_{i+1}, c_i)$ hedges against ambiguity by equalizing R's expected payoff under priors (t_i, c_i) and $[c_i, t_{i+1}]$. It thus lies strictly between the two peaks $a_{ne}^*(t_i, c_i)$ and $a_{ne}^*(c_i, t_{i+1})$. Graphically, it is easy to see that $a_e^*(t_i, t_{i+1}, c_i)$ increases as c_i increases because both $E\left[U^R(a,\omega)\,|\omega \in (t_i, c_i)\right]$ and $E\left[U^R(a,\omega)\,|\omega \in [c_i, t_{i+1}]\right]$ move to the right. To see that S can use Ellsbergian randomization to shift R's best response to $(t_i, t_{i+1}]$ to the right, note first that for $c_i = t_{i+1}$ by definition

$$a_{ne}^*(t_i, t_{i+1}) < a_e^*(t_i, t_{i+1}, t_{i+1}) < a_{ne}^*(t_{i+1}, t_{i+1}),$$

i.e. the Max-Min action is larger than the expected utility maximizing action $a_{ne}^*(t_i, t_{i+1})$. The following comparison to a non-Ellsbergian scenario helps to understand how S can exploit R's ambiguity aversion. Consider the hypothetical case in which the randomization probability ρ_1 is known (with $\rho_1 \in (0,1)$) and let $c_i = t_{i+1}$. When R receives m_i^A or m_i^B, R assigns probability 1 to the event $\omega \in (t_i, t_{i+1})$ and 0 to $\omega = t_{i+1}$, because the latter has zero mass. It follows that her best response to m_i^A or m_i^B is $a_{ne}^*(t_i, t_{i+1})$. Now, go back to our setup in which ρ_1 is Knighteanly unknown and let R receive m_i^A or m_i^B. The

Max-Min payoff attached to any action $a < a_e^*(t_i, t_{i+1}, t_{i+1})$ is the worst case payoff corresponding to $\omega = t_{i+1}$. In other words, for this range of actions, R radically overweights the event $\omega = t_{i+1}$ relative to an expected utility maximizer who weights events by their probability. Note that the essence of this argument continues to hold true for values of c_i that are high but less close to t_{i+1}. A similar argument would hold for a more general class of models of ambiguity aversion (α-Max-Min or the smooth ambiguity model).

The following Lemma shows that Ellsbergian partitional equilibrium conditions have a similar form as in CS.

Lemma 3 *An equilibrium featuring the EPCS* $\left(\{t_r\}_{r=1}^{N-1}, \{c_r\}_{r=0}^{N-1}\right)$ *exists iff*

$$U^S(a_e^*(t_{i-1}, t_i, c_{i-1}), t_i) = U^S(a_e^*(t_i, t_{i+1}, c_i), t_i),\ i = 1, ., N-1. \tag{3}$$

Proof: $\forall i \in \{1, ., N-1\}, m_i^A$ and m_i^B trigger an identical best response, so S is indifferent between m_i^A and m_i^B for any $\omega \in (t_i, t_{i+1}]$. We thus only need to consider deviations across messages carrying different subscripts. Condition (3) ensures that $\forall (t_i, t_{i+1}]$, S weakly prefers any element of $\{m_i^A, m_i^B\}$ to any other equilibrium message. Note that (3) is identical to the NPCE condition (1), except R's best response is now $a_e^*(t_i, t_{i+1}, c_i)$ instead of $a_{ne}^*(t_i, t_{i+1})$.■

The following Lemma is the key to our main result, Proposition 2.

Lemma 4 *Given* β *and* $N \geq 2$, *if* $E(\beta, N)$ *exists and is non-degenerate, there is an* $\bar{\varepsilon} > 0$ *s.t. for any* $\varepsilon \leq \bar{\varepsilon}$, *there exists an EPCE* $\widetilde{E}$ *summarized by* $\left(\{t_i\}_{i=1}^{N-1}, \{c_i\}_{i=0}^{N-1}\right)$ *s.t.:*

a) $t_i = t_i(\beta - \varepsilon, N)$ *and* $a_e^*(t_{i-1}, t_i, c_{i-1}) = a_{ne}^*(t_{i-1}, t_i) + \varepsilon$, $i = 1, ., N-1$.

b) $\pi^S\left(\beta, \widetilde{E}\right) = \pi^S\left(\beta - \varepsilon, E\left(\beta - \varepsilon, N\right)\right)$.

c) $\pi^R\left(\widetilde{E}\right) > \pi^R\left(E\left(\beta, N\right)\right)$.

Proof: See Appendix C.

If the non-Ellsbergian equilibrium $E(\beta, N)$ exists and is non-degenerate, there thus exists an Ellsbergian equilibrium $\widetilde{E}$ that quasi-replicates the non-Ellsbergian equilibrium $E(\beta - \varepsilon, N)$ of a game in which S is replaced by a sender with a lower bias $\beta - \varepsilon$. While

$\widetilde{E}$ features the same profile of thresholds $\{t_i(\beta-\varepsilon,N)\}_{i=1}^{N-1}$ as $E(\beta-\varepsilon,N)$, R's action for each partition is shifted upwards given

$$a_e^*(t_{i-1},t_i,c_{i-1}) = a_{ne}^*(t_{i-1}(\beta-\varepsilon,N),t_i(\beta-\varepsilon,N))+\varepsilon,\ i=1,.,N-1. \tag{4}$$

$\widetilde{E}$ is thus not outcome-equivalent to $E(\beta-\varepsilon,N)$. The two equilibria are however virtually payoff-equivalent for S in the sense that $\pi^S\left(\beta,\widetilde{E}\right)$ is equal to the payoff obtained in $E(\beta-\varepsilon,N)$ by a sender with bias $\beta-\varepsilon$. As to R, transiting from $E(\beta,N)$ to $\widetilde{E}$ implies a trade-off. While Proposition 1 implies that she prefers the new threshold profile conditional on best responding to partitions, her response to partitions now shifts away from the optimal one (see (4)). An Envelope Theorem argument ensures that the trade-off is resolved positively for ε low enough: Given the FOC holding at $a_{ne}^*(t_{i-1}(\beta-\varepsilon,N),t_i(\beta-\varepsilon,N))$, the cost of marginally shifting a at that point is 0. For ε small, R thus achieves a substantial gain at a negligible cost by transiting from $E(\beta,N)$ to $\widetilde{E}$.

We may now state our main result, which follows from the combined application of Proposition 1, Lemma 1 and Lemma 4.

Proposition 2 *If there exists a non-degenerate and influential non-Ellsbergian equilibrium, there exists an Ellsbergian equilibrium $\widetilde{E}$ that ensures both S and R a strictly higher ex ante payoff than that obtained in their ex-ante favored non-Ellsbergian equilibrium.*

Proof**:** We know from Proposition 1 that absent Ellsbergian strategies, S and R's strictly preferred equilibrium is the finest NPCE $E\left(\beta,\overline{N}(\beta)\right)$ (assuming without loss of generality that it is non-degenerate). Lemma 4 shows that given β, there exists an EPCE ensuring S the payoff

$$\pi^S\left(\beta-\varepsilon,E\left(\beta-\varepsilon,\overline{N}(\beta)\right)\right),$$

for $\varepsilon>0$ and small enough. It follows immediately from Lemma 1 that this equilibrium improves on $\pi^S\left(\beta,E\left(\beta,\overline{N}(\beta)\right)\right)$. Lemma 4 states that this same equilibrium ensures R a payoff strictly larger than $\pi^R\left(E\left(\beta,\overline{N}(\beta)\right)\right)$ for ε.small enough. ■

We have not been able to identify simple conditions guaranteeing the existence of an influential Ellsbergian equilibrium for parameter values such that only babbling is feasible absent Ellsbergian strategies. We therefore have chosen to omit such a result. We conclude our main analysis with a remark on the role of communication in our setup. Sobel

(2013) writes for the non-Ellsbergian case: "*In order for communication to be payoff-relevant for R it must be both informative and influential.*" and "*Relative to babbling, payoff-relevant communication must increase R's expected utility but may make S worse off.*". We now show that both of these properties break down once one allows for Ellsbergian strategies; equilibrium communication can be payoff-relevant without being either informative or influential and it can be payoff-relevant while making S better-off and R worse-off. Let us call *classical babbling equilibrium* an equilibrium in which communication is non-informative and S uses a non-Ellsbergian communication strategy.

Proposition 3 *There exists an equilibrium featuring non-informative, non-influential and payoff-relevant communication ensuring S (resp. R) an ex ante payoff strictly larger (resp. smaller) than the payoff obtained in the classical babbling equilibrium.*

Proof: $\exists \bar{\varepsilon} > 0$, s.t. for any $\varepsilon \leq \bar{\varepsilon}$, one can find a $c \in [0,1]$ s.t. $a_e^*(0,1,c) = a_{ne}^*(0,1) + \varepsilon$. Also given $\beta(\omega) > 0 \; \forall \omega \in [0,1]$, there is a $\bar{\delta} > 0$ s.t. $\forall \delta \leq \bar{\delta}$,

$$\int_0^1 U^S \left(a_{ne}^*(0,1) + \delta, \omega \right) f(\omega) d\omega > \int_0^1 U^S \left(a_{ne}^*(0,1), \omega \right) f(\omega) d\omega.$$

As to R, note that she necessarily looses whenever $a_e^*(0,1,c)$ shifts away from $a_{ne}^*(0,1)$.■

Communication, in the above equilibrium, does not generate multiple sets of posteriors along the equilibrium path and is thus non-informative, implying that it is also non-influential. Communication however generates a set of posteriors which is different from R's prior and which leads R to pick an action that is different from her ex ante optimal action. We call this equilibrium an Ellsbergian babbling equilibrium.

5 The Uniform-Quadratic case

The following section considers the so-called Uniform-Quadratic setup. Our general analysis leaves many questions unanswered and we now tackle these for this particular setup. First, we have no notion of what constitutes the optimal subclass of EPCEs. Another unanswered question relates to the size of ex ante payoff improvements that can be achieved through Ellsbergian strategies. A final question is whether Ellsbergian communication can help generate influential communication in cases where it is impossible absent Ellsbergian strategies.

We focus on two subclasses of EPCEs that are both intuitive and analytically tractable. The first involves equally sized partitions while the second involves a maximal use of Ellsbergian randomization. We shall see that 1) whenever there exists an influential NPCE, both classes typically contain equilibria featuring more partitions than the finest NPCE as well as Pareto-dominating the latter 2) payoff improvements achieved through Ellsbergian communication can be significant and 3) Ellsbergian communication can generate the possibility of influential communication.

Assume ω is uniformly distributed on $[0,1]$ and

$$U^S(\omega,a,b) = -(a-(\omega+b))^2,\ U^R(\omega,a) = -(a-\omega)^2.$$

CS (1982) show the following results. Given $N \geq 2$, $\exists b_{ne}(N) = \frac{1}{2N(N-1)}$ s.t. $\forall b \leq b_{ne}(N)$, there exists a unique N-partitions NPCE. An influential NPCE thus exists iff $b \leq \frac{1}{4}$. Given b, there is a $N_{ne}(b) = \left\langle \frac{1}{2b}\left(b+\sqrt{b\,(b+2)}\right)\right\rangle$ s.t. $\forall N \leq N_{ne}(b)$, there exists a unique N-partitions NPCE (where $\langle x \rangle$ denotes the highest integer smaller than x).

For purely didactic purposes, we briefly recall main results obtained in our main analysis, reformulating these within the Uniform-Quadratic setup. Note first that the best response characterized in Lemma 2 reads

$$a_e^*(t_i,t_{i+1},c_i) = \frac{t_i+t_{i+1}+c_i}{3}.$$

We know from Lemma 4 that for a given $N \geq 2$, there is an $\overline{\varepsilon}$ s.t. for any $\varepsilon < \overline{\varepsilon}$, a sender with $b < b_{ne}(N)$ can achieve the payoff $\pi^S(b-\varepsilon, E\,(b-\varepsilon,N))$ by using an Ellsberg strategy. For $\varepsilon > 0$ small enough, the EPCS involved is given by $\left\{\{t_i(b-\varepsilon,N)\}_{i=1}^{N-1}, \{c_i\}_{i=0}^{N-1}\right\}$ s.t.

$$c_i = \frac{t_i(b-\varepsilon,N)+t_{i+1}(b-\varepsilon,N)}{2} + 3\varepsilon,\ i = 0,.,N-1.$$

The expected payoff of S in the corresponding EPCE is

$$\begin{aligned}
&\sum_{0}^{N-1}\int_{t_i(b-\varepsilon,N)}^{t_{i+1}(b-\varepsilon,N)}\left(\frac{t_i(b-\varepsilon,N)+t_{i+1}(b-\varepsilon,N)+c_i}{3}-(\omega+b)\right)^2 d\omega\\
=&\sum_{0}^{N-1}\int_{t_i(b-\varepsilon,N)}^{t_{i+1}(b-\varepsilon,N)}\left(\frac{t_i(b-\varepsilon,N)+t_{i+1}(b-\varepsilon,N)}{2}-\omega-(b-\varepsilon)\right)^2 d\omega\\
=&\ \pi^S(b-\varepsilon,E(b-\varepsilon,N))>\pi^S(b,E(b,N)).
\end{aligned}$$

For ε sufficiently small, R also favours the constructed equilibrium over $E(b,N)$. We now turn to the specific classes of SPEs and MAEs.

5.1 Symmetric partitions equilibria

Let $D(b,N)$ be the optimal decision rule of S, conditional on (at most) N different actions being taken with positive probability. It is defined as follows. Set $t_i=\frac{i}{N}$, $i=0,.,N$ and for $\omega\in(t_i,t_{i+1}]$ (including t_i if $i=0$), pick action $\frac{t_i+t_{i+1}}{2}+b$. The following result lists key properties of the subclass of EPCEs that implement $D(b,N)$ for $N\geq 2$ (we term these *symmetric partitions equilibria,* abbreviated SPE).

Remark 2 *a)* $\forall b\leq\frac{1}{12}$ *there is a finite* $N_s(b)\geq 2$ *s.t.* $\forall N\in\{2,.,N_s(b)\}$ *, there exists an SPE implementing* $D(b,N)$*. If* $b>\frac{1}{12}$*, there is no equilibrium implementing* $D(b,N)$ $\forall N\geq 2$.

b) $\forall b\leq\frac{1}{18}$ *(so that* $N_s(b)\geq 3$*), S and R obtain a strictly higher expected payoff in an SPE implementing* $D(b,N')$ *than in an SPE implementing* $D(b,N)$*, for* $N_s(b)\geq N'>N\geq 2$.

c) $\forall b\in\left(\frac{1}{18},\frac{1}{12}\right]$ *,* $N_s(b)<N_{ne}(b)$*;* $\forall b\in\left(\frac{1}{30},\frac{1}{18}\right]$ *,* $N_s(b)=N_{ne}(b)$*;* $\forall b\leq\frac{1}{30}$*,* $N_s(b)>N_{ne}(b)$.

d) $\forall b\leq\frac{1}{18}$*, S and R obtain a strictly higher expected payoff in an SPE implementing* $D(b,N')$ *than in* $E(b,N)$*, for* $N_s(b)\geq N'\geq N\geq 2$.

Proof: see Appendix D.

Points a) and b) are reminiscent of the CS characterization for the Uniform-Quadratic case. Point c) shows that for b sufficiently small, there exists an SPE featuring more partitions than the finest NPCE. Point d) shows that for b small enough, a given N-partitions NPCE is Pareto dominated by any SPE featuring weakly more than N partitions. This is trivially true for S. As to R, this reveals that the loss implied by her distorted best responses in SPEs is more than compensated by a more favorable partitions profile.

5.2 Maximal ambiguity equilibria

We now explore a subclass of EPCEs featuring what may be coined a maximal use of Ellsbergian randomization (we term these *maximal ambiguity equilibria,* abbreviated MAE). Given $\{t_i\}_{i=1}^{N-1}$, the involved communication strategy is constructed by setting $c_i = t_{i+1}$ $\forall i = 1, ., N$, where $N \geq 2$. We here compare the set of MAEs with the sets of NPCEs and SPEs.

Remark 3 *a)* $\forall b \leq \frac{1}{3}$*, there is a finite* $N_m(b) \geq 2$ *s.t.* $\forall N \in \{2, ., N_m(b)\}$*, there exists a (unique) N-partitions MAE. If* $b > \frac{1}{3}$*, there is no N-partitions MAE* $\forall N \geq 2$.

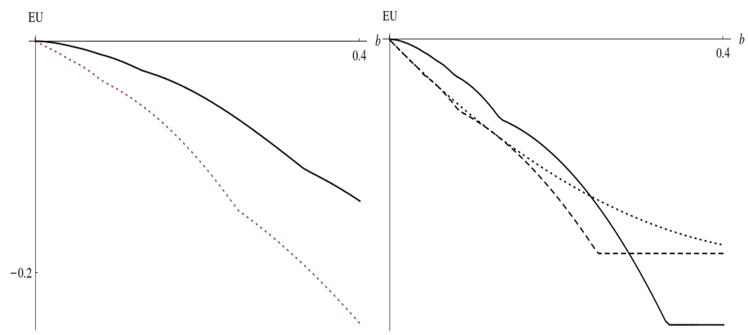
b) $\forall b \leq \frac{1}{4}, N_m(b) \geq N_{ne}(b); \forall b \leq \frac{1}{12}, N_m(b) \geq N_s(b) + 1.$

c) $\forall b \leq \frac{1}{12}$*, S and R obtain a strictly higher expected payoff in the unique MAE featuring* $N_s(b) + 1$ *partitions than in an SPE implementing* $D(b, N)$*, for* $N \in \{2, ., N_s(b)\}$.

Proof: See Appendix E.

Point b) shows that for b small enough, the finest MAE is finer than the finest NPCE and SPE. Point c) states that if there exists any SPE, there exists an MAE that is preferred by S and R to any influential SPE. Given the welfare comparison of SPEs and NPCEs for $b \leq \frac{1}{18}$ appearing in our previous remark, it follows that conditional on $b \leq \frac{1}{18}$, there exists an MAE preferred by S and R to any SPE and any NPCE.

Though MAEs exist for $b \leq \frac{1}{3}$ and NPCEs for $b \leq \frac{1}{4}$, we do not have a comparison of MAEs with NPCEs for $b \in \left(\frac{1}{18}, \frac{1}{4}\right]$. Our comparative analysis of MAEs thus remains incomplete. Figure 2 and 3 complement our results with a numerical analysis. They show that for $b \in \left(\frac{1}{18}, \frac{1}{4}\right]$, S and R obtain a strictly higher expected payoff in the finest MAE equilibrium than in the finest NPCE equilibrium, just as for $b \leq \frac{1}{18}$. The continuous curve in Figure 2 shows, for every b, S's expected payoff in the finest MAE. The dotted curve gives S's payoff in the finest NPCE. Figure 3 is an equivalent for R's expected payoff. The black curve shows, for every b, R's expected payoff in the finest MAE. The dashed curve in Figure 3 represents R's payoff in the finest NPCE. Note that the payoff improvement achieved by S and R through MAEs is substantial. The dotted curve in Figure 3 shows the highest payoff achievable by R with mediated communication (Goltsman et al. (2009)). For all but very high bs, R prefers the best MAE to the best possible mediated communication equilibrium[5]. We find this latter fact noteworthy, given that mediation is a strong instrument.



Figures 2 and 3.

5.3 Overcoming babbling

We conclude by showing that MAEs can beneficially help overcome the non-Ellsbergian babbling equilibrium $E(b,1)$.

Remark 4 *Let $b \in \left(\frac{1}{4}, \frac{1}{3}\right]$ so that only the classical babbling equilibrium $E(b,1)$ exists absent Ellsberg strategies. There exists a (unique) 2-partitions MAE (call it $M(b,2)$).*
- $\forall b \in \left(\frac{1}{4}, \frac{1}{3}\right]$, $\pi^S(b, M(b,2)) > \pi^S(b, E(b,1))$.
- $\forall b < \frac{1}{12}(1+\sqrt{6}) \simeq 0.28$, $\pi^R(M(b,2)) > \pi^R(E(b,1))$.

Proof: We know from the proof of Remark 3 that $M(b,2)$ exists iff $b \leq \frac{1}{3}$. In $M(b,2)$, t_1 solves:

$$\frac{t_1+2}{3} - t_1 - b = t_1 + b - \frac{2t_1}{3} \Leftrightarrow t_1 = \frac{2}{3} - 2b.$$

[5]Mediated communication assumes the existence of a third party, the mediator. S communicates with the mediator who then communicates with R. The mediator can commit to a communication rule and Goltsman et al. (2009) consider the rule that is ex ante optimal for R.

$\pi^S\left(b, M(b,2)\right)$ is thus:

$$-\int_0^{\frac{2}{3}-2b}\left(\left(\frac{2\left(\frac{2}{3}-2b\right)}{3}\right)-(\omega+b)\right)^2 d\omega-\int_{\frac{2}{3}-2b}^{1}\left(\left(\frac{\frac{2}{3}-2b+2}{3}\right)-(\omega+b)\right)^2 d\omega,$$

which is equal to $\frac{8}{3}b^3-\frac{25}{9}b^2+\frac{11}{27}b-\frac{1}{27}$. This expression is strictly larger than $\pi^S(b,E(b,1)) = -b^2-\frac{1}{12}, \forall b \in \left(\frac{1}{4},\frac{1}{3}\right]$. We have $\pi^R\left(M(b,2)\right)=-\frac{1}{27}+\frac{2}{9}b-\frac{4}{3}b^2$ while $\pi^R(E(b,1))=-\frac{1}{12}$. The first expression is larger than the second iff $b \leq \frac{1}{12}(1+\sqrt{6}) = 0.28.$■

6 Conclusion

This paper rationalizes ambiguous language by showing that under fairly general circumstances, a sender and an ambiguity averse receiver can both benefit from the use of ambiguous communication strategies. Future research ought to examine how our insights generalize. First, one could relax some of our key assumptions (e.g. Condition **M** and Assumption 1) and check whether our main results survive. Second, one ought to consider more complex Ellsbergian strategies and clarify whether they allow to increase equilibrium payoffs. Alternatively, one might consider ambiguous communication within the context of other communication protocols (for example mediation).

7 Appendix A

7.1 Proof of Remark 1

The proof is constructive. Define

$$U^S(a,\omega,b)=\begin{cases} G(a-\omega-\left(\beta(\omega)+(b-1)\left[\beta'(\omega)-\beta(\omega)\right]\right)) \text{ if } b\geq 1 \\ G\left(a-\omega-\left(\beta(\omega)b^{\frac{\beta'(\omega)-\beta(\omega)}{\beta(\omega)}}\right)\right) \text{ if } b\leq 1. \end{cases}$$

This function clearly satisfies a), b) and c). As to d), note that $U_1^S(a,\omega,b)=G'(.)$ and that

$$U_{13}^S(a,\omega,b)=\begin{cases} -G''(\omega+\beta(\omega)-a)\left[\beta'(\omega)-\beta(\omega)\right] \text{ if } b\geq 1 \\ -G''(\omega+\beta(\omega)-a)\left[\beta'(\omega)-\beta(\omega)\right]b^{\frac{\beta'(\omega)-\beta(\omega)}{\beta(\omega)}} \text{ if } b\leq 1. \end{cases}$$

Given that G'' is negative everywhere and that $\beta'(\omega) - \beta(\omega) > 0$, it follows that $U_{13}^S > 0$ everywhere. Note also that $U_1^S(a, \omega, b)$ is indeed continuously differentiable in b given that $\lim_{b \to 1-} U_{13}^S(a, \omega, b) = \lim_{b \to 1+} U_{13}^S(a, \omega, b) = 1$.■

7.2 Proof of Lemma 1

In what follows, we abuse notation and denote the utility function of S by $U^S(a, \omega, \beta)$, thus explicitly referring to the bias function β. Note that β is not a scalar parameter as in the original CS setup. We have:

$$\pi^S\left(\beta, E(\beta, N)\right) = \sum_{i=0}^{N-1} \int_{t_i(\beta,N)}^{t_{i+1}(\beta,N)} U^S(a_{ne}^*(t_i(\beta, N), t_{i+1}(\beta, N)), \omega, \beta) f\left(\omega\right) d\omega.$$

Let us define

$$\frac{d\pi^S\left(\beta, E(\beta, N)\right)}{d\beta} = \lim_{\varepsilon \to 0} \frac{\pi^S\left(\beta + \varepsilon, E(\beta + \varepsilon, N)\right) - \pi^S\left(\beta, E(\beta, N)\right)}{\varepsilon}.$$

This corresponds to the marginal effect on the payoff of S of a change in her bias function from $\beta(\omega)$ to $\beta'(\omega) = \beta(\omega) + \varepsilon$, for any ω. So let us examine:

$$\frac{d\pi^S\left(\beta, E(\beta, N)\right)}{d\beta} \tag{5}$$

$$= \sum_{i=0}^{N-1} \frac{d\left(\int_{t_i(\beta,N)}^{t_{i+1}(\beta,N)} U^S(a_{ne}^*(t_i(\beta, N), t_{i+1}(\beta, N)), \omega, \beta) f\left(\omega\right) d\omega\right)}{d\beta} \tag{6}$$

$$= \sum_{i=0}^{N-1} \left(\begin{array}{c} \int_{t_i(\beta,N)}^{t_{i+1}(\beta,N)} \frac{dU^S(a_{ne}^*(t_i(\beta,N), t_{i+1}(\beta,N)), \omega, \beta)}{d\beta} f\left(\omega\right) d\omega \\ +U^S(a_{ne}^*(t_i(\beta, N), t_{i+1}(\beta, N)), t_{i+1}(\beta, N), \beta) f\left(t_{i+1}(\beta, N)\right) \frac{\partial t_{i+1}(\beta,N)}{\partial \beta} \\ -U^S(a_{ne}^*(t_i(\beta, N), t_{i+1}(\beta, N)), t_i(\beta, N), \beta) f\left(t_i(\beta, N)\right) \frac{\partial t_i(\beta,N)}{\partial \beta} \end{array} \right)$$

$$= \sum_{i=0}^{N-1} \int_{t_i(\beta,N)}^{t_{i+1}(\beta,N)} \frac{dU^S(a_{ne}^*(t_i(\beta, N), t_{i+1}(\beta, N)), \omega, \beta)}{d\beta} f\left(\omega\right) d\omega \tag{7}$$

$$+ \sum_{i=1}^{N-1} \left(\left[\begin{array}{c} -U^S(a_{ne}^*(t_i(\beta, N), t_{i+1}(\beta, N)), t_i(\beta, N), \beta) \\ +U^S(a_{ne}^*(t_{i-1}(\beta, N), t_i(\beta, N)), t_i(\beta, N), \beta) \end{array} \right] f\left(t_i(\beta, N)\right) \frac{\partial t_i(\beta, N)}{\partial \beta} \right) \tag{8}$$

$$+U^S(a_{ne}^*(t_0(\beta, N), t_1(\beta, N)), t_0(\beta, N), \beta) f\left(t_0(\beta, N)\right) \frac{\partial t_0(\beta, N)}{\partial \beta} \tag{9}$$

$$-U^S(a_{ne}^*(t_{N-1}(\beta, N), t_N(\beta, N)), t_N(\beta, N), \beta) f\left(t_N(\beta, N)\right) \frac{\partial t_N(\beta, N)}{\partial \beta}. \tag{10}$$

Note first that the second line of the above expression is equal to 0 given that for every $i \in \{1, ., N-1\}$,

$$U^S(a^*_{ne}(t_{i-1}(\beta,N), t_i(\beta,N)), t_i(\beta,N), \beta) - U^S(a^*_{ne}(t_i(\beta,N), t_{i+1}(\beta,N)), t_i(\beta,N), \beta) = 0.$$

Note furthermore that by definition $\frac{\partial t_0(\beta,N)}{\partial \beta} = \frac{\partial t_N(\beta,N)}{\partial \beta} = 0$, given that $t_0(\beta,N) = 0$ and $t_N(\beta,N) = 1$. We now show that for every $i \in \{0, ., N-1\}$,

$$\int_{t_i(\beta,N)}^{t_{i+1}(\beta,N)} \frac{dU^S(a^*_{ne}(t_i(\beta,N), t_{i+1}(\beta,N)), \omega, \beta)}{d\beta} f(\omega)\, d\omega < 0.$$

Note that for every $i \in \{0, ., N-1\}$,

$$\begin{aligned} & \int_{t_i(\beta,N)}^{t_{i+1}(\beta,N)} \frac{dU^S(a^*_{ne}(t_i(\beta,N), t_{i+1}(\beta,N)), \omega, \beta)}{d\beta} f(\omega)\, d\omega \\ = & \int_{t_i(\beta,N)}^{t_{i+1}(\beta,N)} \frac{\partial U^S(a^*_{ne}(t_i(\beta,N), t_{i+1}(\beta,N)), \omega, \beta)}{\partial \beta} f(\omega)\, d\omega + \\ & \int_{t_i(\beta,N)}^{t_{i+1}(\beta,N)} \frac{\partial U^S(a^*_{ne}(t_i(\beta,N), t_{i+1}(\beta,N)), \omega, \beta)}{\partial a} f(\omega)\, d\omega \left(\begin{array}{c} \frac{\partial a^*_{ne}(t_i(\beta,N), t_{i+1}(\beta,N))}{\partial t_i} \frac{\partial t_i(\beta,N)}{\partial \beta} + \\ \frac{\partial a^*_{ne}(t_i(\beta,N), t_{i+1}(\beta,N))}{\partial t_{i+1}} \frac{\partial t_{i+1}(\beta,N)}{\partial \beta} \end{array} \right). \end{aligned}$$

Note first that

$$\frac{\partial U^S(a^*_{ne}(t_i(\beta,N), t_{i+1}(\beta,N)), \omega, \beta)}{\partial \beta} = -\frac{\partial U^S(a^*_{ne}(t_i(\beta,N), t_{i+1}(\beta,N)), \omega, \beta)}{\partial a}.$$

The above is true because we have assumed that $U^S(a, \omega, \beta(\omega)) = G(a - \omega + \beta(\omega))$ for some concave and single peaked function G. Note now that

$$\int_{t_i(\beta,N)}^{t_{i+1}(\beta,N)} \frac{\partial U^S(a^*_{ne}(t_i(\beta,N), t_{i+1}(\beta,N)), \omega, \beta)}{\partial a} f(\omega)\, d\omega > 0. \tag{11}$$

To see this, note first that $a^*_{ne}(t_i(\beta,N), t_{i+1}(\beta,N))$ by definition satisfies:

$$\int_{t_i(\beta,N)}^{t_{i+1}(\beta,N)} \frac{\partial U^R(a^*_{ne}(t_i(\beta,N), t_{i+1}(\beta,N)), \omega, \beta)}{\partial a} f(\omega)\, d\omega = 0. \tag{12}$$

Second, note that we have assumed that $U^S(a, \omega, 0) = U^R(a, \omega)$ and $U^S_{13} > 0$. It follows that (12) implies (11). Intuitively, R's favorite action conditional on $\omega \in (t_i(\beta,N), t_{i+1}(\beta,N)]$

is smaller than S's favoured action, thus implying that the derivative of S's expected payoff function w.r.t. a at the chosen a^*_{ne} must be > 0. Finally, note that

$$\frac{\partial t_i(\beta,N)}{\partial\beta} < 0, \; \frac{\partial t_{i+1}(\beta,N)}{\partial\beta} < 0,$$
$$\frac{\partial a^*_{ne}(t_i(\beta,N),t_{i+1}(\beta,N))}{\partial t_i} > 0, \; \frac{\partial a^*_{ne}(t_i(\beta,N),t_{i+1}(\beta,N))}{\partial t_{i+1}} > 0.$$

■

8 Appendix B

8.1 Three claims

We first state three claims that hold independently of Assumption 1.

Claim 1 ***Concavity and single peakedness***

Note that $E\left[U^R(a,\omega)\,|\omega\in[\underline{\omega},\overline{\omega}]\right]$ *is a concave and single peaked function with a unique maximizer* $a^*_{ne}(\underline{\omega},\overline{\omega})$.

Proof of Claim 1: Omitted.■

Claim 2 ***Shift in maximum***

For any $\underline{\omega},\overline{\omega},\Delta_1,\Delta_2$ *s.t.*$\Delta_1\geq 0$ *and* $\Delta_2\geq 0$ *(with strict inequality for at least one of the two),* $0\leq\underline{\omega}<\overline{\omega}\leq 1$ *and* $0\leq\underline{\omega}+\Delta_1<\overline{\omega}+\Delta_2\leq 1$, *it holds true that* $a^*_{ne}(\underline{\omega},\overline{\omega})<a^*_{ne}(\underline{\omega}+\Delta_1,\overline{\omega}+\Delta_2)$.

Proof of Claim 2: We prove the statement for $\Delta_1=0$ and $\Delta_2>0$. The proof for remaining cases is similar. By definition, it is true that $\int_{\underline{\omega}}^{\overline{\omega}}U_1^R(a^*_{ne}(\underline{\omega},\overline{\omega}),\omega)f(\omega)\,d\omega=0$ (Fact Z). Now, given the concavity of $\int_{\underline{\omega}}^{\overline{\omega}+\Delta_2}U^R(a,\omega)f(\omega)\,d\omega$, we simply need to prove that $\int_{\underline{\omega}}^{\overline{\omega}+\Delta_2}U_1^R(a^*_{ne}(\underline{\omega},\overline{\omega}),\omega)f(\omega)\,d\omega>0$. Note that

$$\begin{aligned}&\int_{\underline{\omega}}^{\overline{\omega}+\Delta_2}U_1^R(a^*_{ne}(\underline{\omega},\overline{\omega}),\omega)f(\omega)\,d\omega\\ =&\int_{\underline{\omega}}^{\overline{\omega}}U_1^R(a^*_{ne}(\underline{\omega},\overline{\omega}),\omega)f(\omega)\,d\omega+\int_{\overline{\omega}}^{\overline{\omega}+\Delta_2}U_1^R(a^*_{ne}(\underline{\omega},\overline{\omega}),\omega)f(\omega)\,d\omega>0.\end{aligned}\quad(13)$$

The first integral in (13) is equal to 0, so we simply need to prove that the second integral is strictly positive. Now, given the assumption that $U_{12}^R > 0$, note that Fact Z trivially implies that $U_1^R(a_{ne}^*(\underline{\omega},\overline{\omega}),\omega) > 0$, for any $\omega \geq \overline{\omega}$ (with stricty inequality for $\omega \geq \overline{\omega}$), which in turn implies that $\int_{\overline{\omega}}^{\overline{\omega}+\Delta_2} U_1^R(a_{ne}^*(\underline{\omega},\overline{\omega}),\omega) f(\omega)\, d\omega > 0$.■

Claim 3 ***Single crossing condition***

If a^* *is s.t.* $E\left[U^R(a^*,\omega)\,|\omega \in [\underline{\omega}+\Delta_1,\overline{\omega}+\Delta_2]\right] = E\left[U^R(a^*,\omega)\,|\omega \in [\underline{\omega},\overline{\omega}]\right]$, *then for any* $a > a^*$, $E\left[U^R(a^*,\omega)\,|\omega \in [\underline{\omega}+\Delta_1,\overline{\omega}+\Delta_2]\right] > E\left[U^R(a^*,\omega)\,|\omega \in [\underline{\omega},\overline{\omega}]\right]$.

Proof of Claim 3: Given $U_{12} > 0$, it must be true that

$$\frac{\partial E\left[U^R(a,\omega)\,|\omega \in [\underline{\omega}+\Delta_1,\overline{\omega}+\Delta_2]\right]}{\partial a} > \frac{\partial E\left[U^R(a,\omega)\,|\omega \in [\underline{\omega},\overline{\omega}]\right]}{\partial a}, \forall a > a^*,$$

which implies that for any $a > a^*$,

$$E\left[U^R(a,\omega)\,|\omega \in [\underline{\omega}+\Delta_1,\overline{\omega}+\Delta_2]\right] > E\left[U^R(a,\omega)\,|\omega \in [\underline{\omega},\overline{\omega}]\right].$$

■

8.2 Proof of Lemma 2

Step 1 Consider a mixed action $\widetilde{a}$ of R given by a distribution $\widetilde{g}$ over $[0,1]$. Denote by $\overline{a}\left(\widetilde{a}\right)$ the pure action satisfying $\overline{a}\left(\widetilde{a}\right) = \int_0^1 a\widetilde{g}(a)da$. Recall that the payoff function U^R is concave. It follows by Jensen's inequality that the expected payoff of $\widetilde{a}$ is weakly smaller than that of $\overline{a}\left(\widetilde{a}\right)$ for any prior distribution F of the state ω, i.e.

$$\int_0^1 \left(\int_0^1 U^R(a,\omega)\widetilde{g}(a)da\right) f(\omega)d\omega \leq \int_0^1 U^R(\overline{a}\left(\widetilde{a}\right),\omega)f(\omega)d\omega.$$

R is a Max-Min decision maker, i.e. chooses the (possible mixed action) a^* (given by the distribution g^*) that maximizes

$$\min_f \int_0^1 \left(\int_0^1 U^R(a,\omega)g^*(a)da\right) f(\omega)d\omega.$$

Suppose the optimal Max-Min action assigns positive probability to multiple pure actions, i.e.that g^* is not degenerate. We know that the pure action $\overline{a}\left(a^*\right) = \int_0^1 ag^*(a)da$

does weakly better for any prior distribution f of the state. It follows that two cases are possible. Either a^* and $\overline{a}(a^*)$ are both solutions to the Max-Min problem or $\overline{a}(a^*)$ is while a^* is not. Consequently, we may without loss of generality focus on pure actions in searching for the Max-Min solution.

Step 2 Point a) follows immediately from Claims 1-3 and Assumption 1 given above. Given $0 \leq \underline{\omega} \leq \overline{\omega} \leq 1$, note that $E\left[U^R(a,\omega)\,|\omega \in (\underline{\omega},\overline{\omega}]\right]$ is a concave and single peaked function with a unique maximizer $a^*_{ne}(\underline{\omega},\overline{\omega})$. The functions $E\left[U^R(a,\omega)\,|\omega \in (\underline{\omega},c)\right]$ and $E\left[U^R(a,\omega)\,|\omega \in [c,\overline{\omega}]\right]$ cross once at some value $\widetilde{a}$ on the interval $(a^*_{ne}(\underline{\omega},c), a^*_{ne}(c,\overline{\omega}))$. It follows immediately that $\widetilde{a}$ is the unique Max-Min solution.

Step 3 This proves Point b). We simply state and prove the following Lemma.

Lemma 5 ***Comparative statics result***

Assume that Assumption 1 holds. Let $0 \leq \underline{\omega} < \overline{\omega} \leq 1$. *Let* $a^*(c)$ *be the unique value a s.t.*

$$E\left[U^R(a^*(c),\omega)\,|\omega \in (\underline{\omega},c)\right] = E\left[U^R(a^*(c),\omega)\,|\omega \in [c,\overline{\omega}]\right]. \tag{14}$$

It follows that $a^*(c)$ *is continuous and strictly increasing in c on* $[\underline{\omega},\overline{\omega}]$.

Proof of above Lemma:

Let $\underline{\omega} \leq c < c' \leq \overline{\omega}$. Consider the three functions $E\left[U^R(a,\omega)\,|\omega[(\underline{\omega},c)\right]$, $E\left[U^R(a,\omega)\,|\omega \in [c,c')\right]$ and $E\left[U^R(a,\omega)\,|\omega \in [c',\overline{\omega}]\right]$. We know that $a^*_{ne}(\underline{\omega},c) < a^*_{ne}(c,c') < a^*_{ne}(c',\overline{\omega})$. Furthermore, using Assumption 1, the unique crossing point a_1 of

$$E\left[U^R(a,\omega)\,|\omega \in (\underline{\omega},c)\right] \text{ and } E\left[U^R(a,\omega)\omega \in [c,c')\right]$$

belongs to $(a^*_{ne}(\underline{\omega},c), a^*_{ne}(c,c'))$. Similarly, by Assumption 1, the unique crossing point a_3 of

$$E\left[U^R(a,\omega)\,|\omega \in [c,c')\right] \text{ and } E\left[U^R(a,\omega)\,|\omega \in [c',\overline{\omega}]\right]$$

belongs to $(a^*_{ne}(c,c'), a^*_{ne}(c',\overline{\omega}))$. It also follows that the unique crossing point a_2 of

$$E\left[U^R(a,\omega)\,|\omega \in (\underline{\omega},c)\right] \text{ and } E\left[U^R(a,\omega)\,|\omega \in [c',\overline{\omega}]\right]$$

belongs to (a_1,a_3). We thus have $a_1 < a_2 < a_3$.

Now, let us first compare $E\left[U^R(a,\omega)\,|\omega \in (\underline{\omega},c)\right]$ and $E\left[U^R(a,\omega)\,|\omega \in [c,\overline{\omega}]\right]$. Note that for every a, there is some $\alpha \in (0,1)$ s.t.

$$E\left[U^R(a,\omega)\,|\omega \in [c,\overline{\omega}]\right] = \alpha E\left[U^R(a,\omega)\,|\omega \in [c,c')\right] + (1-\alpha)E\left[U^R(a,\omega)\,|\omega \in [c',\overline{\omega}]\right].$$

We know that for any $a < a_1$,

$$\max\left\{E\left[U^R(a,\omega)\,|\omega \in [c,c')\right], E\left[U^R(a,\omega)\,|\omega \in [c',\overline{\omega}]\right]\right\} < E\left[U^R(a,\omega)\,|\omega \in (\underline{\omega},c)\right].$$

It follows that for $a < a_1$, $E\left[U^R(a,\omega)\,|\omega \in (\underline{\omega},c)\right] > E\left[U^R(a,\omega)\,|\omega \in [c,\overline{\omega}]\right]$. We also know from step 1 that for any $a \geq a_2$,

$$\begin{aligned} E\left[U^R(a,\omega)\,|\omega \in [c,c']\right] &> E\left[U^R(a,\omega)\,|\omega \in [\underline{\omega},c)\right] \cap \\ E\left[U^R(a,\omega)\,|\omega \in [c',\overline{\omega}]\right] &\geq E\left[U^R(a,\omega)\,|\omega \in (\underline{\omega},c)\right]. \end{aligned}$$

It follows that for $a \geq a_2$, $E\left[U^R(a,\omega)\,|\omega \in (\underline{\omega},c)\right] < E\left[U^R(a,\omega)\,|\omega \in [c,\overline{\omega}]\right]$. We may conclude that $E\left[U^R(a,\omega)\,|\omega \in (\underline{\omega},c)\right]$ and $E\left[U^R(a,\omega)\,|\omega \in [c,\overline{\omega}]\right]$ cross somewhere on $[a_1,a_2)$.

Let us now compare $E\left[U^R(a,\omega)\,|\omega \in (\underline{\omega},c')\right]$ and $E\left[U^R(a,\omega)\,|\omega \in [c',\overline{\omega}]\right]$. Note that for every a, there is some $\widetilde{\alpha} \in (0,1)$ s.t.

$$E\left[U^R(a,\omega)\,|\omega \in (\underline{\omega},c')\right] = \widetilde{\alpha} E\left[U^R(a,\omega)\,|\omega \in (\underline{\omega},c)\right] + (1-\widetilde{\alpha})E\left[U^R(a,\omega)\,|\omega \in [c,c']\right].$$

We know that for any $a \leq a_2$,

$$\begin{aligned} E\left[U^R(a,\omega)\,|\omega \in (\underline{\omega},c)\right] &\geq E\left[U^R(a,\omega)\,|\omega \in [c',\overline{\omega}]\right] \cap \\ E\left[U^R(a,\omega)\,|\omega \in [c,c')\right] &> E\left[U^R(a,\omega)\,|\omega \in [c',\overline{\omega}]\right]. \end{aligned}$$

It follows that for $a \leq a_2$, $E\left[U^R(a,\omega)\,|\omega \in [\underline{\omega},c')\right] > E\left[U^R(a,\omega)\,|\omega \in [c',\overline{\omega}]\right]$. We also know that for any $a > a_3$,

$$\max\left\{E\left[U^R(a,\omega)\,|\omega \in [\underline{\omega},c)\right], E\left[U^R(a,\omega)\,|\omega \in [c,c')\right]\right\} < E\left[U^R(a,\omega)\,|\omega \in [c',\overline{\omega}]\right].$$

It follows that for $a > a_3$, $E\left[U^R(a,\omega)\,|\omega \in [\underline{\omega},c')\right] < E\left[U^R(a,\omega)\,|\omega \in [c',\overline{\omega}]\right]$. We may conclude that $E\left[U^R(a,\omega)\,|\omega \in [\underline{\omega},c')\right]$ and $E\left[U^R(a,\omega)\,|\omega \in [c',\overline{\omega}]\right]$ cross somewhere on $(a_2,a_3]$.

Having now proved that $E\left[U^R(a,\omega)\,|\omega\in[\underline{\omega},c)\right]$ and $E\left[U^R(a,\omega)\,|\omega\in[c,\overline{\omega}]\right]$ cross somewhere on $[a_1,a_2)$ while $E\left[U^R(a,\omega)\,|\omega\in[\underline{\omega},c')\right]$ and $E\left[U^R(a,\omega)\,|\omega\in[c',\overline{\omega}]\right]$ cross somewhere on $(a_2,a_3]$, it follows that $a^*(c)<a^*(c')$. The continuity of $a^*(c)$ in c follows from the continuity of $E\left[U^R(a,\omega)\,|\omega\in[\underline{\omega},c)\right]$ and $E\left[U^R(a,\omega)\,|\omega\in[c,\overline{\omega}]\right]$ in c.

Finally, to see that $a^*_{ne}(t_i,t_{i+1})<a^*_e(t_i,t_{i+1},t_{i+1})$, note that it follows immediately from Claims 1-3 and Assumption 1 that the value of a ensuring equality of $E\left[U^R(a,\omega)\,|\omega\in[\underline{\omega},\overline{\omega})\right]$ and $E\left[U^R(a,\omega)\,|\omega=\overline{\omega}]\right]$ is strictly larger than $a^*_{ne}(\underline{\omega},\overline{\omega})$. If similarly follows that the value of a ensuring equality of $E\left[U^R(a,\omega)\,|\omega=\underline{\omega}]\right]$ and $E\left[U^R(a,\omega)\,|\omega\in[\underline{\omega},\overline{\omega}]\right]$ is strictly smaller than $a^*_{ne}(\underline{\omega},\overline{\omega})$.■

9 Appendix C

Step 1 We here prove Lemma 4. This step proves Point a). In what follows, as in Appendix A, we abuse notation and denote the utility function of S by $U^S(a,\omega,\beta)$, thus explicitly referring to the bias function β. Note that β is not a scalar parameter as in the original CS setup. First, note that given $t_1(\beta,N),.,t_{N-1}(\beta,N)$, there is some maximal $\bar{\varepsilon}$ s.t. for any $\varepsilon\le\bar{\varepsilon}$, one can pick a profile $\{c_i\}_{i=0}^{N-1}$ s.t. for every $i\in\{0,.,N-1\}$:

$$a^*_e(t_i(\beta,N),t_{i+1}(\beta,N),c_i)=a^*_{ne}(t_i(\beta,N),t_{i+1}(\beta,N))+\varepsilon.$$

By continuity of a^*_e and t_i, for any given N, there exists a $\overline{\delta}$ s.t. for every $\delta\le\overline{\delta}$ and every $\varepsilon\le\frac{\bar{\varepsilon}}{2}$, there is some profile $\{c_i\}_{i=0}^{N-1}$ compatible with $\{t_i(\beta-\delta,N)\}_{i=1}^{N-1}$ s.t. for every $i\in\{0,.,N-1\}$:

$$a^*_e(t_i(\beta-\delta,N),t_{i+1}(\beta-\delta,N),c_i)=a^*_{ne}(t_i(\beta-\delta,N),t_{i+1}(\beta-\delta,N))+\varepsilon.$$

Choosing $\delta=\varepsilon\le\min\left\{\overline{\delta},\frac{\bar{\varepsilon}}{2}\right\}$, there thus exists some profile $\{c_i\}_{i=0}^{N-1}$ s.t. for every $i\in\{0,.,N-1\}$:

$$a^*_e(t_i(\beta-\varepsilon,N),t_{i+1}(\beta-\varepsilon,N),c_i)=a^*_{ne}(t_i(\beta-\varepsilon,N),t_{i+1}(\beta-\varepsilon,N))+\varepsilon.$$

Given the U^S assumed , having picked such a profile $\{c_i\}_{i=0}^{N-1}$, note that for every $i \in \{0,.,N-1\}$ and ω :

$$\begin{aligned}& U^S\left(a_e^*(t_i(\beta-\varepsilon,N),t_{i+1}(\beta-\varepsilon,N),c_i),\omega,\beta\right)\\ = \; & U^S\left(a_{ne}^*(t_i(\beta-\varepsilon,N),t_{i+1}(\beta-\varepsilon,N)),\omega,\beta-\varepsilon\right).\end{aligned}$$

It follows that if for every $i \in \{1,.,N-1\}$

$$\begin{aligned}& U^S\left(a_{ne}^*(t_{i-1}(\beta-\varepsilon,N),t_i(\beta-\varepsilon,N)),t_i(\beta-\varepsilon,N),\beta-\varepsilon\right)\\ = \; & U^S\left(a_{ne}^*(t_i(\beta-\varepsilon,N),t_{i+1}(\beta-\varepsilon,N)),t_i(\beta-\varepsilon,N),\beta-\varepsilon\right),\end{aligned}$$

as implied by the existence of the NPCE $E(\beta-\varepsilon,N)$ for a sender bias given by $\beta-\varepsilon$, then it must be true that for every $i \in \{1,.,N-1\}$,

$$\begin{aligned}& U^S\left(a_e^*(t_{i-1}(\beta-\varepsilon,N),t_i(\beta-\varepsilon,N),c_{i-1,i}),t_i(\beta-\varepsilon,N),\beta\right)\\ = \; & U^S\left(a_e^*(t_i(\beta-\varepsilon,N),t_{i+1}(\beta-\varepsilon,N),c_i),t_i(\beta-\varepsilon,N),\beta\right),\end{aligned}$$

which implies that the equilibrium $\widetilde{E}$ exists.

Step 2 This proves Point b). Using the fact that $U^S(a+\varepsilon,\omega,\beta) = U^S(a,\omega,\beta-\varepsilon)$, the expected payoff of S in $\widetilde{E}$ is given by:

$$\begin{aligned}& \sum_{i=0}^{N-1}\int_{t_i(\beta-\varepsilon,N)}^{t_{i+1}(\beta-\varepsilon,N)} U^S\left(a_e^*(t_i(\beta-\varepsilon,N),t_{i+1}(\beta-\varepsilon,N),c_i),\omega,\beta\right)f(\omega)d\omega\\ = \; & \sum_{i=0}^{N-1}\int_{t_i(\beta-\varepsilon,N)}^{t_{i+1}(\beta-\varepsilon,N)} U^S\left(a_{ne}^*(t_i(\beta-\varepsilon,N),t_{i+1}(\beta-\varepsilon,N)),\omega,\beta-\varepsilon\right)f(\omega)d\omega\\ = \; & \pi^S\left(\beta-\varepsilon,E\left(\beta-\varepsilon,N\right)\right) > \pi^S\left(\beta,E\left(\beta,N\right)\right).\end{aligned}$$

Step 3 Note first that

$$\pi^R\left(\widetilde{E}\right) = \sum_{i=0}^{N-1}\int_{t_i(\beta-\varepsilon,N)}^{t_{i+1}(\beta-\varepsilon,N)} U^R(a_{ne}^*(t_i(\beta-\varepsilon,N),t_{i+1}(\beta-\varepsilon,N))+\varepsilon,\omega)f\left(\omega\right)d\omega.$$

We have:

$$\frac{d\pi^R\left(\widetilde{E}\right)}{d\varepsilon} = \left(\sum_{i=0}^{N-1}\frac{d\left(\int_{t_i(\beta-\varepsilon,N)}^{t_{i+1}(\beta-\varepsilon,N)} U^R(a_{ne}^*(t_i(\beta-\varepsilon,N),t_{i+1}(\beta-\varepsilon,N))+\varepsilon,\omega)f\left(\omega\right)d\omega\right)}{d\varepsilon}\right).$$

By Leibniz rule, the above can be rewritten as

$$\sum_{i=0}^{N-1}\left(\begin{array}{c} \int_{t_i(\beta-\varepsilon,N)}^{t_{i+1}(\beta-\varepsilon,N)} \frac{dU^R(a_{ne}^*(t_i(\beta-\varepsilon,N),t_{i+1}(\beta-\varepsilon,N))+\varepsilon,\omega)f(\omega)}{d\varepsilon}d\omega \\ +U^R(a_{ne}^*(t_i(\beta-\varepsilon,N),t_{i+1}(\beta-\varepsilon,N))+\varepsilon,t_{i+1}(\beta-\varepsilon,N))f\left(t_{i+1}(\beta-\varepsilon,N)\right)\frac{dt_{i+1}(\beta-\varepsilon,N)}{d\varepsilon} \\ -U^R(a_{ne}^*(t_i(\beta-\varepsilon,N),t_{i+1}(\beta-\varepsilon,N))+\varepsilon,t_i(\beta-\varepsilon,N))f\left(t_i(\beta-\varepsilon,N)\right)\frac{dt_i(\beta-\varepsilon,N)}{d\varepsilon} \end{array}\right).$$

Note that:

$$\begin{aligned} & \int_{t_i(\beta-\varepsilon,N)}^{t_{i+1}(\beta-\varepsilon,N)} \frac{dU^R(a_{ne}^*(t_i(\beta-\varepsilon,N),t_{i+1}(\beta-\varepsilon,N))+\varepsilon,\omega)f(\omega)}{d\varepsilon}d\omega\Big|_{\varepsilon=0} \\ = \; & \int_{t_i(\beta-\varepsilon,N)}^{t_{i+1}(\beta-\varepsilon,N)} \frac{\partial U^R(a_{ne}^*(t_i(\beta,N),t_{i+1}(\beta,N)),\omega)}{\partial a}f(\omega)\,d\omega \\ & \times\left(1-\frac{\partial a_{ne}^*(t_i(\beta,N),t_{i+1}(\beta,N))}{\partial t_i}\frac{\partial t_i(\beta,N)}{\partial\beta}-\frac{\partial a_{ne}^*(t_i(\beta,N),t_{i+1}(\beta,N))}{\partial t_{i+1}}\frac{\partial t_{i+1}(\beta,N)}{\partial\beta}\right). \end{aligned}$$

Given the FOC characterizing $a_{ne}^*(t_i(\beta,N),t_{i+1}(\beta,N))$, it follows that for every $i \in \{0,.,N-1\}$,

$$\int_{t_i(\beta,N)}^{t_{i+1}(\beta,N)} \frac{\partial U^R(a_{ne}^*(t_i(\beta,N),t_{i+1}(\beta,N)),\omega)}{\partial a}f(\omega)\,d\omega = 0$$

It follows that

$$\begin{aligned} \frac{d\pi^R\left(\widetilde{E}\right)}{d\varepsilon}\Bigg|_{\varepsilon=0} &= \sum_{i=0}^{N-1}\left(\begin{array}{c} -U^R(a_{ne}^*(t_i(\beta,N),t_{i+1}(\beta,N)),t_{i+1}(\beta,N))f\left(t_{i+1}(\beta,N)\right)\frac{\partial t_{i+1}(\beta,N)}{\partial\beta} \\ +U^R(a_{ne}^*(t_i(\beta,N),t_{i+1}(\beta,N)),t_i(\beta,N))f\left(t_i(\beta,N)\right)\frac{\partial t_i(\beta,N)}{\partial\beta} \end{array}\right) \\ &= \sum_{i=1}^{N-1}\left[\begin{array}{c} -U^R(a_{ne}^*(t_{i-1}(\beta,N),t_i(\beta,N)),t_i(\beta,N)) \\ +U^R(a_{ne}^*(t_i(\beta,N),t_{i+1}(\beta,N)),t_i(\beta,N)) \end{array}\right]f\left(t_i(\beta,N)\right)\frac{\partial t_i(\beta,N)}{\partial\beta} \end{aligned}$$

Given Condition M, $\frac{\partial t_i(\beta,N)}{\partial\beta} < 0$ for every $i \in \{1,.,N-1\}$. Furthermore, for every $i \in \{1,.,N-1\}$,

$$\begin{aligned} U^R(a_{ne}^*(t_i(\beta,N),t_{i+1}(\beta,N)),t_i(\beta,N)) - U^R(a_{ne}^*(t_{i-1}(\beta,N),t_i(\beta,N)),t_i(\beta,N)) &< \\ U^S(a_{ne}^*(t_i(\beta,N),t_{i+1}(\beta,N)),t_i(\beta,N),\beta) - U^S(a_{ne}^*(t_{i-1}(\beta,N),t_i(\beta,N)),t_i(\beta,N),\beta) &= 0. \end{aligned}$$

The inequality appearing on the first line holds true because $a_{ne}^*(t_{i-1}(\beta,N),t_i(\beta,N)) < a_{ne}^*(t_i(\beta,N),t_{i+1}(\beta,N))$, $U^S(a,\omega,0) = U^R(a,\omega)$ and $U_{13}^S > 0$ everywhere. The equality appearing on the second line holds true by definition because $\{t_i(\beta,N)\}_{i=1}^N$ is a non-Ellsbergian equilibrium partitional communication strategy.■

10 Appendix D

Step 1 We prove Remark 2 in what follows. This proves Point a). It is trivially true that if there exists an EPCE implementing $D(b,N)$, then there exists an EPCE featuring the following communication strategy. First, set $\{t_i\}_{i=1}^{N-1}$ by letting $t_i = \frac{i}{N}$ for $i = 1,.,N$. Second, let c_i satisfy

$$\frac{t_i + t_{i+1} + c_i}{3} = \frac{t_i + t_{i+1}}{2} + b, i = 0,.,N-1. \tag{15}$$

Condition (15) is feasible iff given any $i \in \{0,.,N-1\}$,

$$\frac{t_i + t_{i+1}}{2} + b \leq \frac{t_i + 2t_{i+1}}{3} \Leftrightarrow b \leq \frac{t_{i+1} - t_i}{6} = \frac{1}{N} \Leftrightarrow b \leq \frac{1}{6N}.$$

Assuming that the above condition is satisfied, the constructed strategy constitutes an equilibrium iff for any $i \in \{1,.,N-1\}$,

$$= -\left(\frac{t_{i-1} + t_i + c_{i-1}}{2} - t_i - b\right)^2 = -\left(\frac{t_i + t_{i+1} + c_i}{2} - t_i - b\right)^2,$$

the above being equivalent to

$$\left(\left(\frac{2i+1}{2N} + b\right) - \frac{i}{N} - b\right)^2 = \left(\left(\frac{2i-1}{2N} + b\right) - \frac{i}{N} - b\right)^2 \Leftrightarrow \left(\frac{1}{2N}\right)^2 = \left(-\frac{1}{2N}\right)^2,$$

which is always true. The obtained condition $6Nb \leq 1$ means that $\forall b$, there is a finite $N_s(b) = \left\langle \frac{1}{6b} \right\rangle$ s.t. for any $N \leq N_s(b)$, there exists an SPE implementing $D(b,N)$.

Step 2 This proves Point b). We show in next step that for $b \leq \frac{1}{18}$, $N_s(b) \geq 3$. Note that

$$\begin{aligned} \pi^R(D(b,N)) &= -\sum_{i=1}^{N} \int_{\frac{i-1}{N}}^{\frac{i}{N}} \left(\omega - \left(\frac{\frac{i-1}{N} + \frac{i}{N}}{2} + b\right)\right)^2 d\omega \\ &= -\sum_{i=1}^{N} \left(-\frac{1}{12}\left(\frac{i-1}{N} - \frac{i}{N}\right)^3 - \left(\frac{i-1}{N} - \frac{i}{N}\right) b^2\right) = -b^2 - \frac{1}{12N^2}. \end{aligned}$$

It can similarly be shown that $\pi^S(b, D(b,N)) = -\frac{1}{12N^2}$. It is immediate that $\pi^S(b, D(b,N))$ and $\pi^R(D(b,N))$ are increasing in N.

Step 3 This proves Point c). Note that $N_s(b) \geq 5$ for $b \leq \frac{1}{30}$ while $N_{ne}(b) \geq 5$ for $b \leq \frac{1}{40}$. Furthermore,

$$\frac{1}{6b} - \left(\frac{1}{2b}\left(b + \sqrt{b\,(b+2)}\right)\right) \geq 1 \text{ if } b \leq \frac{1}{4} - \frac{1}{12}\sqrt{7} \in \left(\frac{1}{40}, \frac{1}{30}\right).$$

It follows that for any $b \leq \frac{1}{30}$, $N_s(b) > N_{ne}(b)$. Now, note that $N_s(b) \geq 3$ for $b \leq \frac{1}{18}$ while $N_{ne}(b) \geq 3$ for $b \leq \frac{1}{12}$. It follows that for $b \in \left(\frac{1}{18}, \frac{1}{12}\right]$, $N_{ne}(b) > N_s(b)$. Note finally that

$$b = \frac{1}{24} \Rightarrow \frac{1}{6b} = \frac{1}{2b}\left(b + \sqrt{b\,(b+2)}\right) = 4.$$

It follows immediately that for $b \in \left(\frac{1}{30}, \frac{1}{24}\right]$, $N_s(b) = N_{ne}(b) = 4$, while for $b \in \left(\frac{1}{24}, \frac{1}{18}\right]$, $N_s(b) = N_{ne}(b) = 3$.

Step 4 This proves Point d). It is a priori clear that for any N and $N' \geq N$, $\pi^S(b, D(b, N')) > \pi^S(b, E(b, N))$. Recall that $\pi^R(D(b, N)) = -b^2 - \frac{1}{12N^2}$ and that $\pi^R(D(b, N))$ is thus increasing in N. On the other hand, $\pi^R(E(b, N))$ is $-\frac{1}{12N^2} - \frac{b^2(N^2-1)}{3}$. So

$$\begin{aligned} & \pi^R(D(b, N)) - \pi^R(E(b, N)) \\ = \;& -b^2 - \frac{1}{12N^2} - \left(-\frac{1}{12N^2} - \frac{b^2(N^2-1)}{3}\right) = \frac{1}{3}b^2\left(N^2 - 4\right). \end{aligned}$$

Note that the above expression is weakly (strictly) positive for any $N \geq (>)2$. It follows that R always strictly gains from the transition from $E\,(b, N)$ to an equilibrium implementing $D(b, N')$, given $N' \geq N \geq 2$.■

11 Appendix E

11.1 Points a) and b)

Step 1 We prove Remark 3 in what follows. This proves Point a). Let $0 < t_1 < . < t_N = 1$ and set, for every i, $c_i = t_{i+1}$. For such an Ellsbergian strategy to be I.C., we need that $\forall$ $i = 1, ., N-1$,

$$\left(\frac{t_i + 2t_{i+1}}{3} - t_i - b\right)^2 = \left(\frac{t_{i-1} + 2t_i}{3} - t_i - b\right)^2 \Leftrightarrow t_{i+1} = \frac{3}{2}t_i - \frac{1}{2}t_{i-1} + 3b. \quad (16)$$

Solving the above linear difference equation, we obtain a unique solution parameterized by t_1:

$$t_i^* = 2^{1-i}\left[6b - 3b2^{i+1} + 3bi2^i - t_1 + 2^i t_1\right]. \tag{17}$$

Now, pick an N. Setting $t_N = 1$, solving for the (unique) implied t_1, it follows that there is a unique threshold profile $\left\{\widetilde{t}_r(b,N)\right\}_{r=1}^{N-1}$ constituting an N-partitions MAE. Thresholds satisfy, for $r = 1, ., N$

$$\widetilde{t}_r(b,N) = 2^{1-r}\left(6b - 3b2^{r+1} + 3b(r)2^r + (-1+2^r)\left(-\frac{\left(12b - 12\left(2^N\right)b - 2^N + 6\left(2^N\right)Nb\right)}{2\left(2^N\right) - 2}\right)\right).$$

We may now look for the maximal value of b compatible with the existence of an N-partitions MAE. Call this $b_m(N)$. To find it, simply assume $t_1 = 0$ in (17) and find the b that solves $t_N^* = 1$ given $t_1 = 0$:

$$t_N^* = 1 \Leftrightarrow 2^{1-i}\left[6b - 3b2^{i+1} + 3bi2^i\right] = 1 \Rightarrow b_m(N) = \frac{1}{2^{1-N}\left[6 - 3\left(2^{N+1}\right) + 3N2^N\right]}.$$

Similarly, $N_m(b)$ is the largest positive integer i s.t. $2^{1-i}\left(6b - 3b2^{i+1} + 3bi2^i\right) < 1$.

Step 2 This proves Point b). Note that

$$\frac{b_m(N)}{b_{ne}(N)} = \frac{2^N N\left(N-1\right)}{3N2^N - 2^N 6 + 6} > 1 \text{ for } N \geq 2.$$

Thus, $b_m(N) > b_{ne}\left(N\right), \forall N \geq 2$. It follows that $\forall\ b \leq b_m(2)$, there exists an MAE as fine as the finest NPCE. Note also that

$$b_m(N+1) - b_s(N) = \frac{1}{6N}\frac{2^N - 1}{2^N N - 2^N + 1} > 0\,, \forall N \geq 2.$$

11.2 Point c)

Step 1 We start by proving Point c) for S. Given $b \leq b_s(N)$, we slightly abuse notation by denoting by respectively $\pi^S(b, D(b,N))$ and $\pi^R(D(b,N))$ the expected payoff of respectively S and R in an equilibrium implementing $D(b,N)$. Given $b \leq b_m(N)$, denote by respectively $\pi^S(b, M(b,N))$ and $\pi^R(M(b,N))$ the expected payoff of S and R in the unique N-partitions MAE. Note that $\forall N \geq 2$, $\pi^S(b, D(b,N)) = -\frac{1}{12N^2}$.and note the following three further facts:

Fact 1: $\forall N \geq 2$, $\pi^S(b_s(N), M(b_s(N), N)) = \pi^S(b_s(N), D\,(b_s(N), N))$.

Fact 2: $\forall N \geq 2$, $\pi^S(b_s(N), M(b_s(N), N+1)) > \pi^S(b_s(N), D(b_s(N), N))$.

Fact 3: $\forall N \geq 2$ and $b \in (b_s(N+1), b_s(N)]$, $\pi^S(b, M(b, N+1))$ is strictly decreasing in b.

Consider thus any $b \leq b_s(2)$. We know that there is an $N \geq 2$ s.t. $b \in (b_s\,(N+1)\,, b\,(N)]$ and $b \leq b_m(N+1)$. Furthermore, it follows from Facts 1,2 and 3 that

$$\pi^S(b, M(b, N+1)) > \pi^S(b, D(b, N)).$$

We simply prove Facts 1, 2 and 3 in what follows.

Step 2 This step proves Fact 1. Note that the condition defininining $b_s(N)$ (recalling that $t_i = \frac{i}{N})$) is

$$\frac{t_i + t_{i+1}}{2} + b = \frac{t_i + 2t_{i+1}}{3} \Leftrightarrow b = \frac{t_{i+1} - t_i}{6} = \frac{1}{N} \Leftrightarrow b = \frac{1}{6N}.$$

In other words, for the highest possible value of b compatible with the existence of an Ellsbergian symmetric partitions equilibrium, the unique SPE is actually an MAE.

Step 3 This step proves Fact 2. Simply note that

$$\begin{aligned} & \pi^S(b_s(N), M(b_s(N), N+1)) - \pi^S(b_s(N), D(b_s(N), N)) \\ = \quad & \frac{4(2^N - 1)(17\left(2^N\right) - 13)}{189(2^{N+1} - 1)N^3} > 0. \end{aligned}$$

Step 4 This step proves Fact 3. Let $\left\{\widetilde{t}_r(b, N)\right\}_{r=1}^{N-1}$ be the profile of thresholds characterizing the unique Ellsbergian maximal ambiguity N-partitions equilibrium. Using the explicit formula derived for thresholds, note that for $r = 1, ., N$,

$$\frac{\partial \widetilde{t}_r(b, N)}{\partial b} = \frac{6\left(2^N N(1 - 2^r) - 2^r r(1 - 2^N)\right)}{2^r\left(2^N - 1\right)} < 0.$$

Let us now explicity consider the derivative $\frac{\partial \pi^S(b, M(b, N))}{\partial b}$. We have:

$$\pi^S(b, M(b, N)) = \sum_{i=0}^{N-1} \int_{\widetilde{t}_i(b,N)}^{\widetilde{t}_{i+1}(b,N)} U^S(a_e^*(\widetilde{t}_i(b, N), \widetilde{t}_{i+1}(b, N), \widetilde{t}_{i+1}(b, N)), \omega, b) f\,(\omega)\, d\omega.$$

Thus,

$$\begin{aligned}
&\frac{d\pi^S(b,M(b,N))}{db}\\
=&\sum_{i=0}^{N-1}\frac{d\left(\int_{\widetilde{t}_i(b,N)}^{\widetilde{t}_{i+1}(b,N)}U^S(a_e^*(\widetilde{t}_i(b,N),\widetilde{t}_{i+1}(b,N),\widetilde{t}_{i+1}(b,N)),\omega,b)f(\omega)\,d\omega\right)}{db}\\
=&\sum_{i=0}^{N-1}\left(\begin{array}{c}\int_{\widetilde{t}_i(b,N)}^{\widetilde{t}_{i+1}(b,N)}\frac{dU^S(a_e^*(\widetilde{t}_i(b,N),\widetilde{t}_{i+1}(b,N),\widetilde{t}_{i+1}(b,N)),\omega,b)}{db}f(\omega)\,d\omega\\
+U^S(a_e^*(\widetilde{t}_i(b,N),\widetilde{t}_{i+1}(b,N),\widetilde{t}_{i+1}(b,N)),\widetilde{t}_{i+1}(b,N),b)f\left(\widetilde{t}_{i+1}(b,N)\right)\frac{\partial\widetilde{t}_{i+1}(b,N)}{\partial b}\\
-U^S(a_e^*(\widetilde{t}_i(b,N),\widetilde{t}_{i+1}(b,N),\widetilde{t}_{i+1}(b,N)),\widetilde{t}_i(b,N),b)f\left(\widetilde{t}_i(b,N)\right)\frac{\partial\widetilde{t}_i(b,N)}{\partial b}\end{array}\right)\\
=&\sum_{i=0}^{N-1}\int_{\widetilde{t}_i(b,N)}^{\widetilde{t}_{i+1}(b,N)}\frac{dU^S(a_e^*(\widetilde{t}_i(b,N),\widetilde{t}_{i+1}(b,N),\widetilde{t}_{i+1}(b,N)),\omega,b)}{db}f(\omega)\,d\omega,
\end{aligned}$$

where the last equality follows from standard arguments. We use the fact that $\forall i\in\{1,.,N-1\}$,

$$U^S(a_e^*(\widetilde{t}_{i-1}(b,N),\widetilde{t}_i(b,N),\widetilde{t}_i(b,N)),\widetilde{t}_i(b,N),b)-U^S(a_e^*(\widetilde{t}_i(b,N),\widetilde{t}_{i+1}(b,N),\widetilde{t}_{i+1}(b,N)),\widetilde{t}_i(b,N),b)=0,$$

as well as the fact that by definition

$$\frac{\partial\widetilde{t}_0(b,N)}{\partial b}=\frac{\partial\widetilde{t}_N(b,N)}{\partial b}=0,$$

given that $\widetilde{t}_0(b,N)=0$ and $\widetilde{t}_N(b,N)=1$. We now show that for every $i\in\{0,.,N-1\}$,

$$\int_{\widetilde{t}_i(b,N)}^{\widetilde{t}_{i+1}(b,N)}\frac{dU^S(a_e^*(\widetilde{t}_i(b,N),\widetilde{t}_{i+1}(b,N),\widetilde{t}_{i+1}(b,N)),\omega,b)}{db}f(\omega)\,d\omega<0.$$

Note that for every $i\in\{0,.,N-1\}$,

$$\begin{aligned}
&\int_{\widetilde{t}_i(b,N)}^{\widetilde{t}_{i+1}(b,N)}\frac{dU^S(a_e^*(\widetilde{t}_i(b,N),\widetilde{t}_{i+1}(b,N),\widetilde{t}_{i+1}(b,N)),\omega,b)}{db}f(\omega)\,d\omega\\
=&\int_{\widetilde{t}_i(b,N)}^{\widetilde{t}_{i+1}(b,N)}\frac{\partial U^S(a_e^*(\widetilde{t}_i(b,N),\widetilde{t}_{i+1}(b,N),\widetilde{t}_{i+1}(b,N)),\omega,b)}{\partial b}f(\omega)\,d\omega+\\
&\int_{\widetilde{t}_i(b,N)}^{\widetilde{t}_{i+1}(b,N)}\frac{\partial U^S(a_e^*(\widetilde{t}_i(b,N),\widetilde{t}_{i+1}(b,N),\widetilde{t}_{i+1}(b,N)),\omega,b)}{\partial a}f(\omega)\,d\omega\\
&\times\left(\begin{array}{c}\frac{\partial a_e^*(\widetilde{t}_i(b,N),\widetilde{t}_{i+1}(b,N),\widetilde{t}_{i+1}(b,N))}{\partial\widetilde{t}_i}\frac{\partial\widetilde{t}_i(b,N)}{\partial b}+\\
\frac{\partial a_e^*(\widetilde{t}_i(b,N),\widetilde{t}_{i+1}(b,N),\widetilde{t}_{i+1}(b,N))}{\partial\widetilde{t}_{i+1}}\frac{\partial\widetilde{t}_{i+1}(b,N)}{\partial b}\end{array}\right).
\end{aligned}$$

Note first that

$$\frac{\partial U^S(a_e^*(\widetilde{t}_i(b,N),\widetilde{t}_{i+1}(b,N),\widetilde{t}_{i+1}(b,N)),\omega,b)}{\partial b} = -\frac{\partial U^S(a_e^*(\widetilde{t}_i(b,N),\widetilde{t}_{i+1}(b,N),\widetilde{t}_{i+1}(b,N)),\omega,b)}{\partial a}.$$

The above is true because we have assumed that $U^S(a,\omega,b(\omega)) = -(a-\omega+b)^2$.for some concave and single peaked function G. Note now that

$$\int_{\widetilde{t}_i(b,N)}^{\widetilde{t}_{i+1}(b,N)} \frac{\partial U^S(a_e^*(\widetilde{t}_i(b,N),\widetilde{t}_{i+1}(b,N),\widetilde{t}_{i+1}(b,N)),\omega,b)}{\partial a} f(\omega)\,d\omega > 0. \tag{18}$$

Call this Fact A. This is true for reasons shown in the next step. Finally, note that

$$\begin{aligned} \frac{\partial \widetilde{t}_i(b,N)}{\partial b} &< 0,\ \frac{\partial \widetilde{t}_{i+1}(b,N)}{\partial b} < 0, \\ \frac{\partial a_e^*(\widetilde{t}_i(b,N),\widetilde{t}_{i+1}(b,N),\widetilde{t}_{i+1}(b,N))}{\partial \widetilde{t}_i} &> 0,\ \frac{\partial a_e^*(\widetilde{t}_i(b,N),\widetilde{t}_{i+1}(b,N),\widetilde{t}_{i+1}(b,N))}{\partial \widetilde{t}_{i+1}} > 0. \end{aligned}$$

Step 5 This proves Fact A. Note that given $[x,y] \subseteq [0,1]$ and some action a,

$$\frac{\partial\left(-\int_x^y (a-(\omega+b))^2\right)}{\partial a} = -2\,(x-y)\left(\left(\frac{x+y}{2}+b\right)-a\right).$$

The sign of the above expression thus simply depends on the sign of $\left(\frac{x+y}{2}+b\right)-a$. Recall furthermore that

$$a_e^*(\widetilde{t}_i(b,N),\widetilde{t}_{i+1}(b,N),\widetilde{t}_{i+1}(b,N)) = \frac{\widetilde{t}_i(b,N)+2\widetilde{t}_{i+1}(b,N)}{3}.$$

Now, note that for $r = 0,.,N-1$,

$$\left(\frac{\widetilde{t}_r(b,N)+\widetilde{t}_{r+1}(b,N)}{2}+b\right) - \left(\frac{\widetilde{t}_r(b,N)+2\widetilde{t}_{r+1}(b,N)}{3}\right) = \frac{1}{12}2^{N-r}\frac{6Nb-1}{2^N-1},$$

which is equal to 0 if $b = b_s(N)$ and positive if $b > b_s(N)$.

Step 6 We now prove Point c) for R. Recall that $N \geq 2$, $\pi^R(D(b,N)) = -\frac{1}{12N^2} - b^2$ and is thus decreasing in b. Note now the following facts:

Fact I: $\forall N \geq 2$, $\pi^R(M(b_s(N),N)) = \pi^R(D(b_s(N),N))$.

Fact II: $\forall N \geq 2$, $\pi^R(M(b_s(N),N+1)) > \pi^R(D(b_s(N),N))$.

Fact III: $\forall N \geq 2$ and $b \in (b_s(N+1), b_s(N)]$, $\pi^R(M(b,N+1))$ is strictly decreasing in b.

Fact IV: $\forall N \geq 2$, $\pi^R(M(b_s(N), N+1)) > \pi^R(D(b_s(N+1), N))$.

Consider thus any $b \leq b_s(2)$. We know that there is a N s.t. $b \in (b_s(N+1), b_s(N)]$ and $b \leq b_m(N+1)$. Furthermore, it follows from facts I, II and III that $\pi^R(M(b, N+1)) > \pi^R(D(b, N))$. We simply prove Facts I, II in what follows. The (algebraically tedious but conceptually simple) proof of Fact III is omitted.

Step 7 The proof of Fact I is identical to the proof of Fact 1. This step proves Fact II. Simply note that

$$\pi^R(M(b_s(N), N+1)) - \pi^R(D(b_s(N), N)) = \frac{2(11 - 2^N 27 + 4^{2+N})}{63(2^{N+1} - 1)N^3} > 0.$$

Step 8 This step proves Fact IV. Note that

$$\pi^R(M(b_s(N), N+1)) - \pi^R(D(b_s(N+1), N)) = \frac{\frac{12}{(2^{1+N}-1)^2} - \frac{44}{(2^{1+N}-1)} + \frac{32+3N(19+6N)}{(1+N)^2}}{252N^3} > 0.$$

■

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