Non-Renewable Resources, Extraction Technology and Endogenous Growth*

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Abstract

How can total output and living standards continue to grow over time in the presence of essential non-renewable resources? We develop a theory of innovation in non-renewable resource extraction and economic growth. Firms increase their economically extractable reserves of non-renewable resources through investment in new extraction technology and reduce their reserves through extraction. Our model allows us to study the interaction between geology and technological change, and its effects on prices, total output growth, and the resource intensity of the economy. The model accommodates long-term trends in non-renewable resource markets – namely stable prices and exponentially increasing extraction – for which we present data on 65 non-renewable resources extending back to 1700. The paper suggests that over the long term, increasing consumption of non-renewable resources fosters the development of new extraction technologies and hence offsets the exhaustion of higher quality resource deposits. (JEL codes: O30, O41, Q30)

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1 Introduction

Over the past three hundred years, increasing living standards have gone hand in hand with rising inputs of non-renewable resources, such as metals and fossil fuels, in the world economy. Average per capita real GDP is more than 17 times larger today compared to 1700. Over the same period, the per capita extraction and consumption of non-renewable resources increased from about 5 kg in 1700 to roughly 3,000 kg today. What's more, inflation adjusted prices for major non-renewable resources have strongly fluctuated, but around flat or declining long-run trends (see Nordhaus et al., 1992; Krautkraemer, 1998; Jones and Vollrath, 2002; Livernois, 2009).

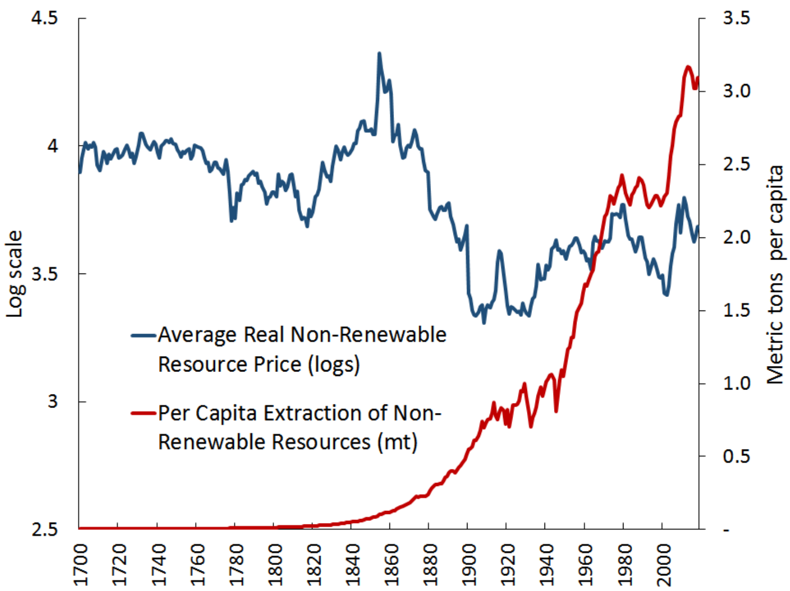


Figure 1: Per Capita Primary Production and Average Real Prices (log) of Non-Renewable Resources, 1700-2018. The figure is based on a new data-set that covers 65 non-renewable resources. See the appendix for sources.

However, we obviously live on a finite planet. The amount of non-renewable resources in the earth's crust can not be increased. Based on this presumption, standard economic theory tells us that the production and consumption of non-renewable resources should decline at a constant rate and prices go up at the rate of interest (Hotelling, 1931). Total output growth is possible, if resource saving technology offsets the drag from depletion (see Groth, 2007; Aghion and Howitt, 1998; Jones and Vollrath, 2002; Nordhaus et al., 1992).

Do the contradictory directions of historical trends and theoretical predictions imply that we live on borrowed time, essentially consuming future generations' resources? Are price signals distorted such that they do not reflect the increasing scarcity of non-renewable resources? Or can we explain these long-run trends by the coincidental exploration of new resources? As billions of people are expected to catch-up with living standards in industrialized countries over the next decades (Organization for Economic Cooperation and Development, 2019), these questions are important with regard to conflicts over resources (see Acemoglu et al., 2012b) and the environment. Long-run trends in non-renewable resource markets are key inputs in modeling the economic consequences of climate change and the transition towards clean energy (see Acemoglu et al., 2012a; Golosov et al., 2014; Hassler and Sinn, 2012; van der Ploeg and Withagen, 2012).

The main contribution of our paper is to develop a model that combines technological change in resource extraction, geology and endogenous growth to explain these long-run trends. We integrate a more realistic extraction sector – which is based on geological evidence and includes firms that can invest into technological change in resource extraction – into a two sector endogenous growth model (Romer (1987, 1990) and Acemoglu (2002)).

Specifically, the model's extraction sector has three key features: First, the non-renewable resource is located in heterogeneous deposits of different ore grades. Ore grade is the concentration of the resource in the rock and is the most important feature of mineral deposits. As a local approximation to Ahrens (1953, 1954) fundamental law of geochemistry, we assume that the resource quantity is distributed such that it exhibits increasing returns as the ore grade of deposits decreases.

Second, extraction firms can invest in technological change to expand their reserves. Reserves are deposits of the resource in the Earth's crust that are economically recoverable with current technologies. We hereby formalize Nordhaus (1974) suggestion that innovation in extraction technology helps overcome scarcity by turning mineral deposits into reserves. We make a sharp distinction between these reserves, which need to be replenished by the extractive firms, and other occurrences in the Earth's crust. We argue based on stylized facts that it is reasonable to assume that the amount of resources in these other occurrences is infinitely large for all practical purposes.

Third, technological change in resource extraction is the result of investment by extraction technology firms. Each extraction technology is specific to deposits of particular grades. This picks up the idea that firms need to adjust technology to extract the resource from deposits of lower grades. Extraction technology is hence rivalrous. Most similar to this understanding of innovation is Desmet and Rossi-Hansberg (2014), where non-replicable factors of production, like land, provide the incentive for innovation despite perfect competition.

The extractive sector is added to an endogenous lab-equipment model of growth with input varieties and directed technological change. The model also features a standard intermediate goods sector with goods producing firms and sector-specific technology firms. The final good is produced from the intermediate good and the non-renewable resource.

In our model, total output growth generates increased demand for resources. This incentivizes extraction firms to invest into new extraction technologies to convert lower grade deposits of non-renewable resources into reserves, leading to continuous growth in resource extraction.

Resource prices are the result of competitive markets for resources and extraction technologies. They are equal to the sum of the marginal costs of extraction and of inventing new extraction machines. Diminishing returns from the technology in making deposits of lower grades extractable are offset by the distribution of the resource in the Earth's crust, which exhibits exponentially increasing resource quantities as grades of the deposits decline. This leads to a constant price over the long run, increasing per capita resource extraction, and a constant growth rate of the economy.

Total output growth depends, among others, on the distribution of the non-renewable resource in the earth's crust and the specification of the technology function. For example, a higher average concentration of the resource in the earth's crust leads to a higher growth rate of total output. This geological feature is in contrast to conventional models, where there is a drag to growth driven by the rate of depletion (see Nordhaus et al., 1992; Weitzman, 1999). In the case where the elasticity of substitution is below one, the economy can stop growing under certain conditions. This happens, for example, when the average concentration of the non-renewable resource in the Earth's crust goes close to zero.

The extractive sector is not the engine of economic growth in our model, because it features only constant returns to scale at the aggregate. In contrast to the intermediate good sector, where firms can make use of the entire *stock* of technology for production, firms in the extractive sector can only use the *flow* of new technology to convert deposits of lower grades into new reserves. This is driven by the rivalrous nature of technology, where technology is grade specific and the related deposits are exhausted.

Our model suggests that it constant prices and exponentially increasing resource extraction are reasonable assumptions when modeling long-run trends. If increases in resource demand continue to incentivize technological change in extraction, the world economy will be able to continue to grow in the presence of essential non-renewable resources. This would allow billions of people to increase their living standards, while the risk for resource conflicts would not increase.

At the same time, fossil fuel consumption could continue to grow, as innovation in extraction technology leads to steady prices in the long-run. Low fossil fuel prices tend to increase climate change and make the transition more costly in the models of transition towards clean energy (Acemoglu et al., 2012a; Golosov et al., 2014; Hassler and Sinn, 2012; van der Ploeg and Withagen, 2012). Our model does not include negative externalities and cannot model these dynamics. However, an increasing carbon tax could reduce the demand for fossil fuels and decrease the incentives to develop new extraction technologies for fossil fuels. This would help the transition towards clean sources of energy.[1] The availability of critical metals that are needed for the energy transition would not face constraints based on our model.

[1] This idea was first formulated by Romer (2016) in his blog on our paper

Our approach builds on a small literature that introduces extraction technology and heterogenous deposits to models with non-renewable resources. Rausser (1974) establishes that improvements in the extraction technology due to learning-by-doing can endogenously increase the extraction of non-renewable resources in a partial equilibrium model. Heal (1976) models the implications of a non-renewable resource, which is inexhaustible, but extractable at different grades and costs in partial equilibrium. Extraction costs increase with cumulative extraction, but then remain constant when a "backstop technology" is reached. Slade (1982) adds exogenous technological change in extraction technology to the Hotelling (1931) model and predicts a U-shaped relative price curve. Cynthia-Lin and Wagner (2007) use a similar model with an inexhaustible non-renewable resource and exogenous technological change. They obtain a constant relative price with increasing extraction. Hart (2016) models resource extraction with heterogeneous deposits in a growth model with exogenous labor productivity increases. After a temporary "frontier phase" with a constant resource price and extraction rising at the rate of aggregate output, firms needs to extract resources from greater depths. Subsequently, a long-run balanced growth path is reached with constant resource extraction and prices that rise in line with wages. Fourgeaud et al. (1982) focuses on explaining sudden fluctuations in the development of non-renewable resource prices by allowing the resource stock to grow in a stepwise manner through technological investment.

There is only one paper, to our knowledge, that like ours includes technological change in the extraction of a non-renewable resource in an endogenous growth model. Tahvonen and Salo (2001) model the transition from a non-renewable energy resource to a renewable energy resource. Their model follows a learning-by-doing approach as technological change is linearly related to the level of extraction and the level of productive capital. It explains decreasing prices and the increasing use of a non-renewable energy resource over a particular time period before non-renewable energy resource prices increase in the long term.

In section 2, we document stylized facts on the long-term development of non-renewable resource prices, production, and world real GDP, on technological change in resource extraction and on the abundance and distribution of non-renewable resources in the Earth's crust. Section 3 describes key assumptions about geology and technology and how they interact in our model. Section 4 outlines the microeconomic foundations of the extractive sector and its innovation process. Section 5 presents the growth model, and section 6 derives theoretical results, which are discussed in section 7. In section 8 we draw conclusions.

2 Stylized Facts

The interaction between technology and geology makes the extractive sector distinct from other sectors in the economy and shapes its long-run development. In the following, we document that non-renewable resource extraction has increased over the long term, while non-renewable resource prices have non-increasing or even declining trends. We also present data that non-renewable resources are abundant in the Earth's crust, but that continued extraction is only possible due to technological change.

2.1 Resource Extraction, Real Prices, and Aggregate Output over the Long Term

The extraction and hence consumption[2] of non-renewable resources increased strongly with economic development over the past three hundred years. Figure 2 shows that global extraction rose from about 3.3 million metric tons in 1700 to 21 billion metric tons in 2015. This is an increase by a factor of more than 6000. Global real GDP increased at a factor of about 190 over the same period, while real GDP on a per capita basis multiplied by 15. A closer statistical examination confirms that the mine production of non-renewable resources exhibits significantly positive growth rates in the long term (see table 2 in the appendix). These results also hold by-and-large for per capita production of the respective commodities over the long run (see table 3 in the appendix).

[2]Over the long term, extraction and consumption of resources are about equal, as stockholdings vary over the business-cycle and are generally relatively small compared to consumption. In some cases, where recycling is important, consumption could be higher. Our data is therefore a lower bound estimate for metals and non-metals.

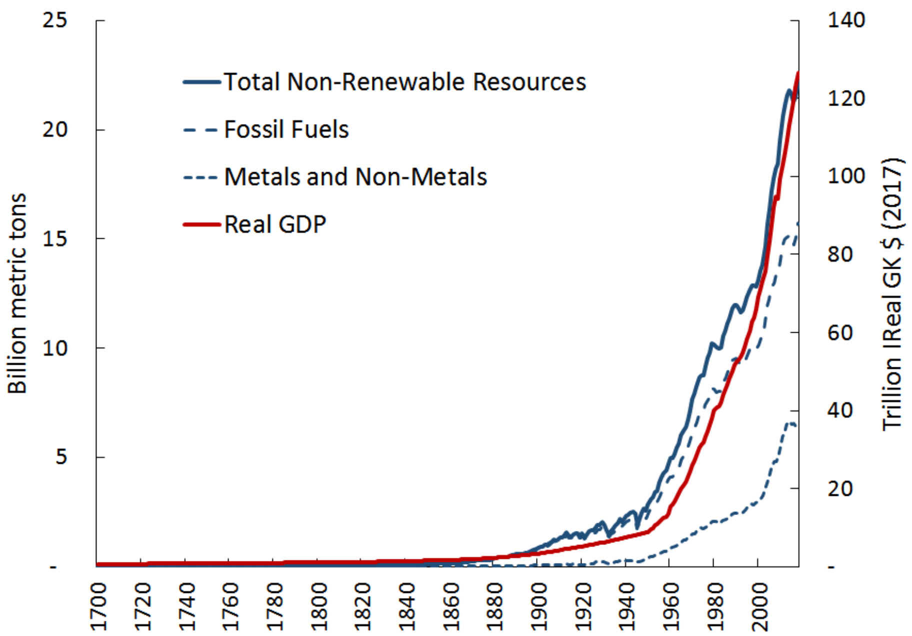


Figure 2: World Primary Production of Non-Renewable Resources and World Real GDP, 1700-2018. The figure shows that the total quantity of extracted non-renewable resources increased roughly in line with world real GDP over the last 300 years. About two thirds of the non-renewable resource production is driven by fossil fuels, including crude oil, coal and natural gas, and the other third by metals and non-metals.

Non-renewable resource prices exhibit strong fluctuations but are mostly non-increasing or even declining over the long term. Figure 3 presents the simple average of log prices for the 63 non-renewable resources, which is downward trending over the long term. The upper line shows the simple average log price for metals and non-metals, which exhibits a steeper decline, especially around the beginning of the 20th century. The lower line is the simple average of real prices for the three fossil minerals. It seems to follow no trend over the long-term, but there is a significant uptick since the 1970s. This is driven by oil and natural gas prices and oligopolistic behaviour in these markets. Dvir and Rogoff (2010) shows that the real price for crude oil exhibits structural breaks over the long term, related to the Texas Railroad Commission and OPEC. Coal prices have not persistently gone up over the same time period. Our evidence is in line with the literature, see e.g. Krautkraemer (1998), Von Hagen (1989), Cynthia-Lin and Wagner (2007), Stuermer (2016). The literature is certainly not conclusive on price trends (see Pindyck, 1999; Lee et al., 2006; Slade, 1982; Jacks, 2013; Harvey et al., 2010), but we conclude that prices do generally not show increasing trends over the long term.

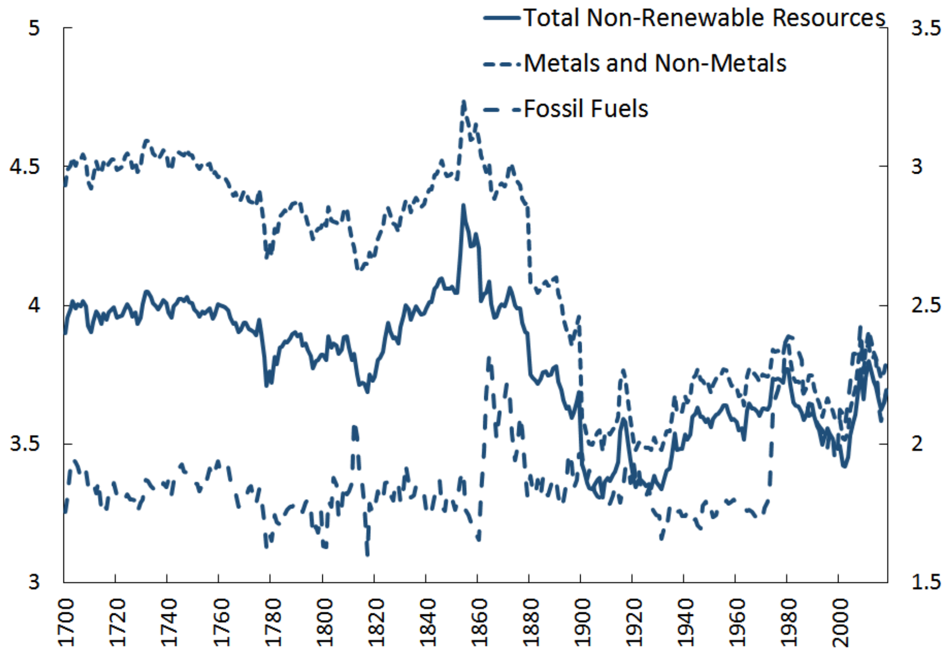


Figure 3: Average Real Commodity Prices (logs), 1700-2018.

Figure 4 plots the factor share of non-renewable resources. The factor share stayed below 0.5 percent until the beginning of the 19th century. It then went up to roughly 3 percent at the beginning of the 20th century, strongly driven by the increase in the intensity of use of both metals and non-metals but also fossil minerals. Since then it has spiked up sharply twice, in the 1970s and 1980s during the two oil crisis and then again around 2005 to 2010, when oil and other commodity prices were high due to unexpectedly strong demand from China (Kilian, 2009; Stuermer, 2018). Note that the factor share of metals and non-metals certainly fluctuates but around no trend over the past 120 years.

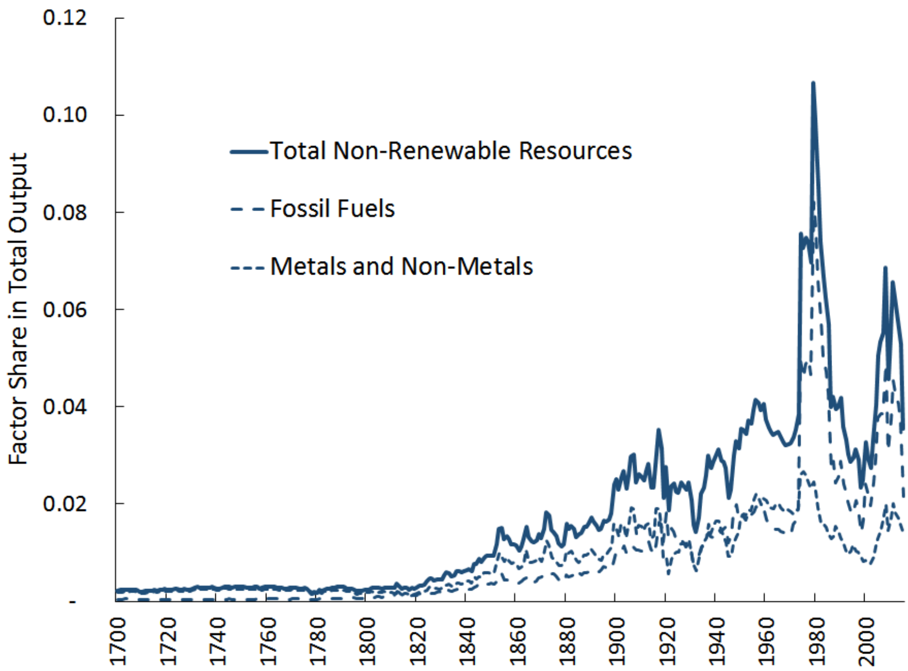


Figure 4: Factor Share of Non-Renewable Resources in World GDP, 1700-2018.

Overall, we take these stylized facts as motivation to build a model that exhibits constant growth in the worldwide production of non-renewable resources, trend-less resource prices and constant growth in world aggregate output.

2.2 Non-Renewable Resources are Abundant in the Earth's Crust

We update and extent a data-set by Nordhaus (1974) on the abundance (or total quantity) of mineral non-renewable resources in the Earth's crust. Table 1, second column, shows that the crustal abundance of major non-renewable resources, which is the estimated total quantities in the Earth's crust, are substantial.[3] The forth column shows the annual mine production, which is several orders of magnitude smaller than the quantities in the Earth's crust. The fifth column illustrates that current annual extraction could be continued for millions or billions of years depending on the resource.[4]

Hydrocarbons are also quite abundant in the Earth's crust. Even though reserves of conventional oil resources – the highest grade fossil fuel – may be exhausted someday, deposits of unconventional oil, natural gas, and coal, which could substitute for conventional oil in the long run, are plentiful in the Earth's crust. These results are in line with numerous studies that conclude that fossil fuels will last far longer than many expect (see Aguilera et al. (2012), Rogner (1997) and Covert et al. (2016)).

Of course, extraction of most of these resource quantities in the Earth's crust is impossible or extremely costly with current technology. Only a small fraction is proven to be economically extractable with current technology. This fraction is defined as reserves (see U.S. Geological Survey (2018)). The term "economic" implies that profitable

[3]Note also that the Earth's crust makes up less than one percent of the Earth's mass. There are hence more non-renewable resources in other layers of the Earth.

[4]Table 5 in the appendix illustrates that the assumption of exponentially increasing extraction of non-renewable resources does not alter the overall conclusion of table 1.

extraction under defined investment assumptions (typically including some assumption about current and expected prices) has been established with reasonable certainty. Table 1, column three, shows that reserves are relatively small compared to the crustal abundance. They amount to only a couple of decades of current extraction (see column six).

	Crustal Abundance (Bil. mt)	Reserves (Bil. mt)	Annual Output (Bil. mt)	Crustal Abundance/ Annual Output (Years)	Reserves/ Annual Output (Years)
Aluminum	1,990,000,000[e]	30[b1]	0.06[a]	33,786,078,000	100[1]
Copper	1,510,000[e]	0.8[b]	0.02[b]	76,650,000	40
Iron	1,392,000,000[e]	83[b2]	0.06[a]	1,200,000,000	55[2]
Lead	290,000[e]	0.1[b]	0.005[b]	61,702,000	18
Tin	40,000[e]	0.005[b]	0.0003[b]	137,931,000	16
Zinc	2,250,000[e]	0.23[b]	0.013[b]	170,445,000	17
Gold	70[e]	0.0001[b]	0.000003[b]	22,076,000	17
Coal[3]		510[d]	3.9[d]		131
Crude Oil[4]	15,000,000[6h]	241[d]	4.4[d]	1,297,529	55
Nat. Gas[5]		179[d]	3.3[d]		54

Notes: [1]Data for bauxite, [2]data for iron ore, [3]includes lignite and hard coal, [4]includes conventional and unconventional oil, [5]includes conventional and unconventional gas, [6]all organic carbon in the earth's crust. Sources: [a]U.S. Geological Survey (2016), [b]U.S. Geological Survey (2018), [c]British Petroleum (2017), [d]Federal Institute for Geosciences and Natural Resources (2017), [e]Perman et al. (2003), [h]Littke and Welte (1992).

Table 1: Quantities of selected non-renewable resources in the crustal mass and in reserves, measured in metric tons and in years of production based on current annual mine production.

2.3 Technological Change Increases Reserves

The boundary between reserves and other occurrences in the earth's crust is dynamic due to technological change and exploration.

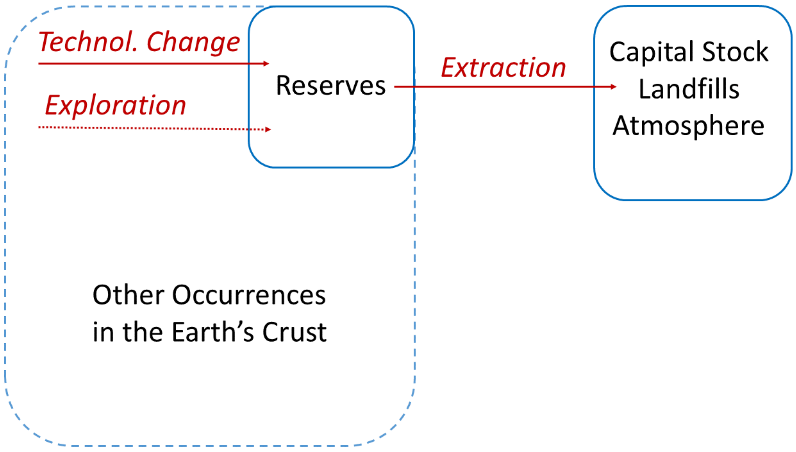


Figure 5: Non-Renewable Resource Flows.

This is a stylized version of the official Resource/Reserve Classification System for Minerals as used by the U.S. Geological Survey (see U.S. Geological Survey (2018)).

Figure 5 shows how resources are classified as either reserves or other occurrences in the Earth's crust.[5] As reserves get depleted though extraction, firms explore new deposits and develop new technology to convert other occurrences into reserves. This allows firms to continue extraction. The extracted resource becomes either part of the capital stock or is discharged after utilization into land-fills or the atmosphere.

Technological change in the extractive sector is different from other sectors due to its interaction with geology. It allows for the extraction of more resources that are a factor of production and is hence factor extracting technological change. In other sectors, technological change is typically factor augmenting, meaning that a fixed factor is used in a more efficient way. This is also how non-renewable resources and technological change have traditionally been modeled, as non-renewable resources are typically not regarded as produced factor.

[5]Please note that we have left out a major category, the reserve base, to ease understanding. The reserve base encompasses those parts of the resource in the earth's crust that have a reasonable potential for becoming economically available within planning horizons beyond those that assume proven technology and current economics (see U.S. Geological Survey (2018))

2.4 Non-Renewable Resources are Log-Normally Distributed in the Earth's Crust

To better understand the interaction between geology and technological change, we first take a closer look at the distribution of non-renewable resources in the Earth's crust.

Non-renewable resources are not uniformly concentrated in the earth's crust. Variations in the geochemical processes have shaped the characteristics of non-renewable resource occurrences in the Earth's crust over billions of years. Deposits differ in their geological characteristics along many dimensions, for example, ore grades, thickness and depths. We focus on ore grade, as this is the most important characteristic. Some deposits are highly concentrated with a specific resource (high grade, close to 100 percent ore grade), and other deposits are less so (low grade, close to 0 percent ore grade). The grade distinguishes the difficulty of extraction, where a low grade is very difficult.

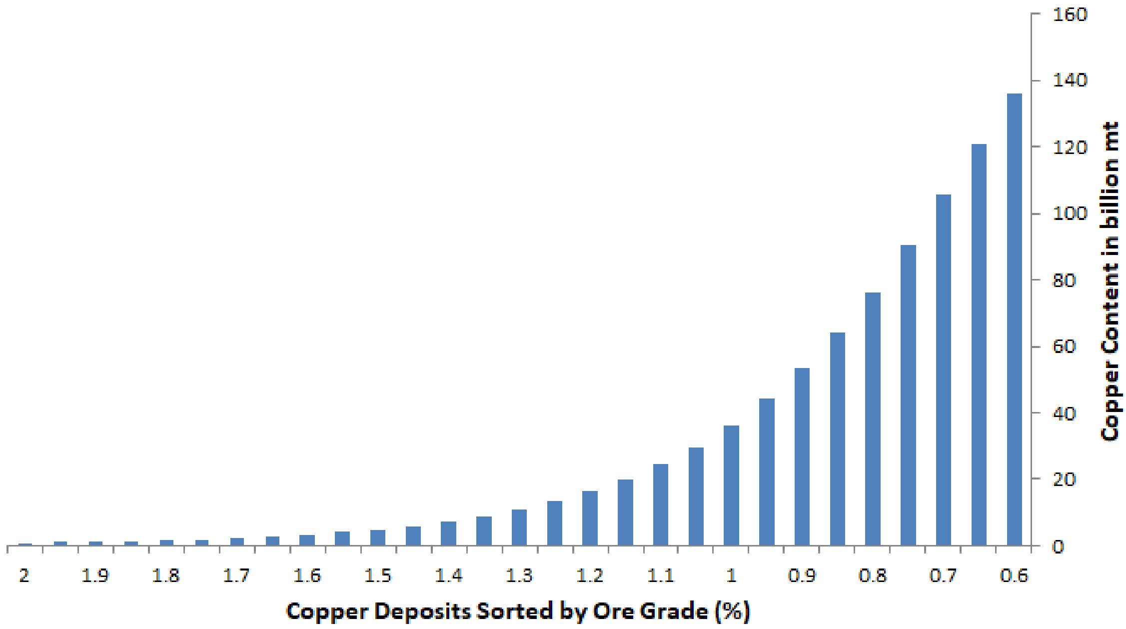


Figure 6: Grade-quantity distribution of copper in the Earth's crust. The total copper content increases, as the ore grades of copper deposits decline. Note that the x-axis has been reversed for illustrative purposes. Source: Gerst (2008).

The fundamental law of geochemistry (Ahrens (1953, 1954)) states that each chemical element exhibits a log-normal grade-quantity distribution in the earth's crust, postulating a decided positive skewness. Hence, the total resource quantity in low grade deposits is large, while the total resource quantity in high grade deposits is relatively small. The reason for this is that low grade deposits have a far larger volume of rock than high grade deposits. For example, figure 6 shows that the total copper content increases, as the ore grades of copper deposits in the Earth's crust decline.

While a log-normal distribution for the distribution of certain resources is the textbook standard assumption in geochemistry, this is still a developing literature, especially regarding very low concentrations of metals, which might be mined in the distant future. For example, Skinner (1979) and Gordon et al. (2007) propose a discontinuity in the distribution due to the so-called "mineralogical barrier," the approximate point below which metal atoms are trapped by atomic substitution.

Gerst (2008) concludes in his geological study of copper deposits that he can neither confirm nor refute this hypotheses. However, based on worldwide data on copper deposits over the past 200 years, he finds evidence for a log-normal relationship between copper production and ore grades. Mudd (2007) analyzes the historical evolution of extraction and grades of deposits for different base metals in Australia. He also finds that production has increased at a constant rate, while grades have consistently declined.

We conclude that there remains uncertainty about the geological distribution, specially regarding hydrocarbons with their distinct formation processes. However, it is reasonable to assume that non-renewable resources are distributed according to a log-normal relationship between the grade of its deposits and its quantity based on geochemical theory and evidence.

2.5 Extraction Technology

Empirical evidence suggests that technological change affects the extractable ore grade with diminishing returns (see Lasserre and Ouellette, 1991; Mudd, 2007; Simpson, 1999; Wellmer, 2008). For example, Radetzki (2009) and Bartos (2002) describe how technological changes in mining equipment, prospecting, and metallurgy have gradually made possible the extraction of copper from lower grade deposits. The average ore grades of copper mines have decreased from about twenty percent 5,000 years ago to currently below one percent (Radetzki, 2009). Figure 7 illustrates this development using the example of global copper mines from 1800 to 2000. Mudd (2007) and Scholz and Wellmer (2012) come to similar results for different base metal mines in Australia and for copper mines in the U.S, respectively.

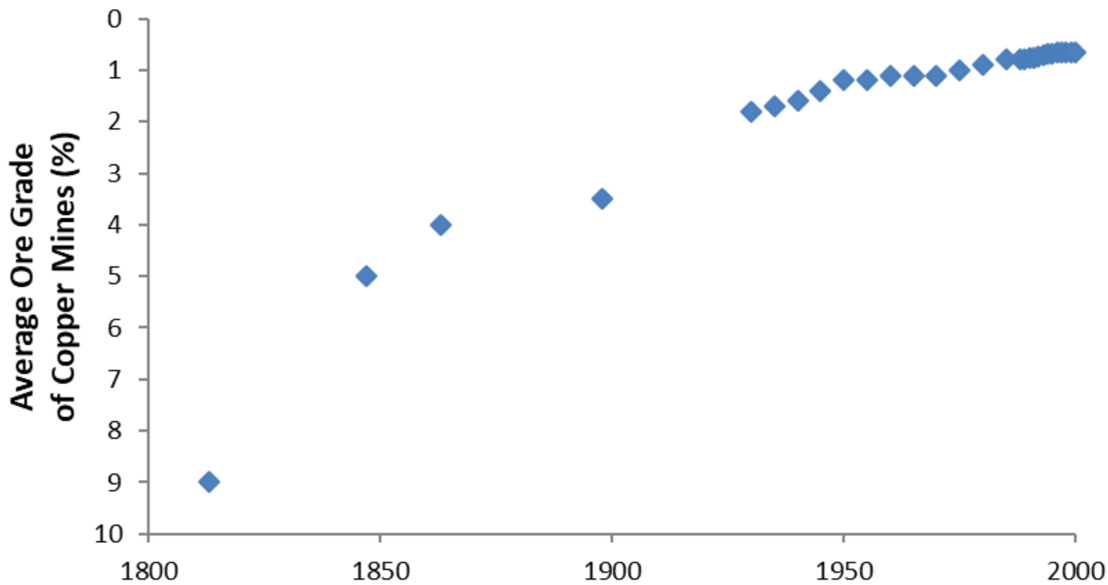


Figure 7: The historical development of average ore grades of copper mines in the world suggests diminishing returns of technological change on extractable ore grades. Note that the y-axis has been reversed for illustrative purposes. Source: Gerst (2008)

However, Figure 7 also shows that decreases in grades have slowed as technological development progressed. Under the reasonable assumption that global real R&D spending in extraction technology and its impact on technological change has stayed constant or increased over the long term, there are decreasing returns to R&D in terms of making mining from deposits of lower grades economically feasible.

We observe similar developments for hydrocarbons. Using the example of the offshore oil industry, Managi et al. (2004) finds that technological change has offset the cost-increasing degradation of resources. Crude oil has been extracted from ever deeper sources in the Gulf of Mexico, as Figure 12 in the appendix shows. Furthermore, technological change and high prices have made it profitable to extract hydrocarbons from unconventional sources, such as tight oil or oil sands (International Energy Agency, 2012).

Overall, we conclude that the long-run data suggests that there are no constant returns from technological change in resource extraction in terms of ore grades. Historical evidence rather suggests diminishing returns to technological development.

3 The Interaction Between Geology and Technology

The stylized facts highlighted the importance for understanding the interaction between geology and technology. In the following, we describe key assumptions based on these stylized facts. We point out that there are offsetting effects between geology and technology, which lead to constant returns from technological development in terms of new reserves.

3.1 Geological Function

The stylized facts showed that non-renewable resources are likely log-normally distributed in the Earth's crust, which can be approximated by an increasing relationship between ore grades and resource quantities for the foreseeable future. In our model, we define the grade O of a deposit as the average concentration of the resource; the grade ranges from 0 to 1. We approximate the distribution of the non-renewable resource in the Earth's crust as a function of ore grades by the following (see also Figure 8):

$$R(O) = \frac{\delta}{O}, \quad \delta \in \mathbb{R}_+ \ , \ O^\star \in (0,1) \ . \tag{1}$$

Parameter δ controls the curvature of the function. If δ is high, the total quantity of the non-renewable resources is large, whereas a low δ indicates relatively small quantity of the non-renewable resource in the crustal mass.

The functional form implies that the resource quantity goes to infinity as grades approach zero. Although we recognize that non-renewable resources are ultimately finite in supply, we follow Nordhaus (1974) in his assessment that non-renewable resources are so abundant in the earth's crust that given technological change, "the future will not be limited by sheer availability of important materials." The assumption that the underlying resource quantity goes to infinity for any time frame that is relevant for human economic activity is analogue to households maximizing over an infinite horizon.

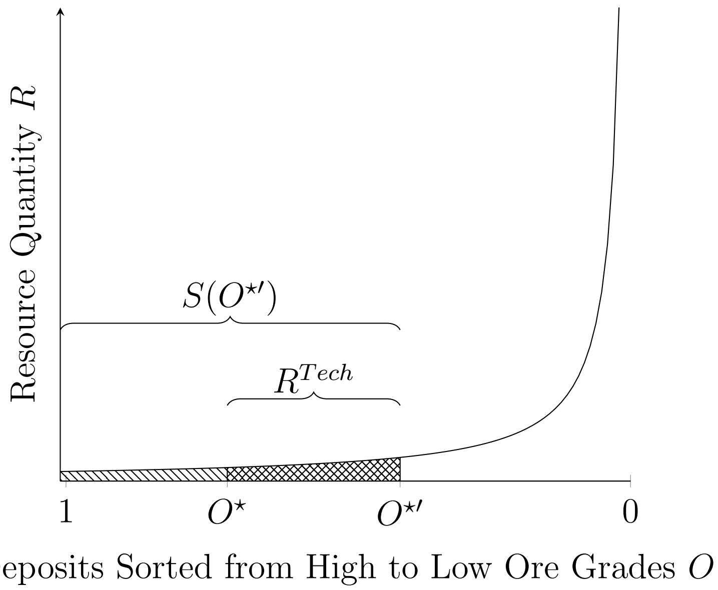


Figure 8: Geological Function: Distribution of the Non-Renewable Resource in the Earth's Crust. Note that the x-axis has been reserved for illustrative purposes.

Technological development makes resource extraction from lower ore grades possible and converts deposits into reserves. For example, a new technology shifts the extractable deposits from grade $O^\star$ down to grade $O^{\star\prime}$. Note that $O^\star$ indicates the lowest grade that firms can extract with the new technology level. This technological change adds resources to the reserves that are equal to: $R^{Tech} = \int_{O^{\star\prime}}^{O^\star} R(O^\star) dO^\star, \;\; \delta \in \mathbb{R}_+ \;, \; O^\star \in (0,1) \;.$

The total amount of resources that has been converted to reserves due to technological change in the interval $[O^{\star\prime}, 1)$ is defined by:

$$S(O^{\star\prime}) = \int_{O^{\star\prime}}^{1} \frac{\delta}{O^\star} dO = -\delta ln(O^{\star\prime}), \;\; \delta \in \mathbb{R}_+ \;, \; O^\star \in (0,1) \tag{2}$$

Note that this is under the assumption that there is no resource extraction and hence no flows out of the reserves.

3.2 Technological Function

The stylized facts suggest that there are diminishing returns of technological change in terms of making deposits of lower ore grades extractable. We accommodate this by an extraction technology function, which maps the state of the extraction technology N onto the extractable grade $O^\star$ of the deposits (see figure 9), takes the form:

$$O^\star(N_R) = e^{-\mu N_R}, \ \mu \in \mathbb{R}_+ \ \ N_R \in (0, \infty) \ . \tag{3}$$

The grade $O^\star$ is the lowest grade that firms can extract with technology level N_R. Technological change, N_R, expands the range of grades that can be extracted. The extractable grade is a decreasing convex function of technology. Technological development makes deposits economically extractable, but there are decreasing marginal returns in terms of grades. The curve in Figure 9 starts with deposits of close to a 100 percent ore grade, which represents the state of the world several thousand years ago. We assume that extractable ore grades only get closer to zero in the long term.

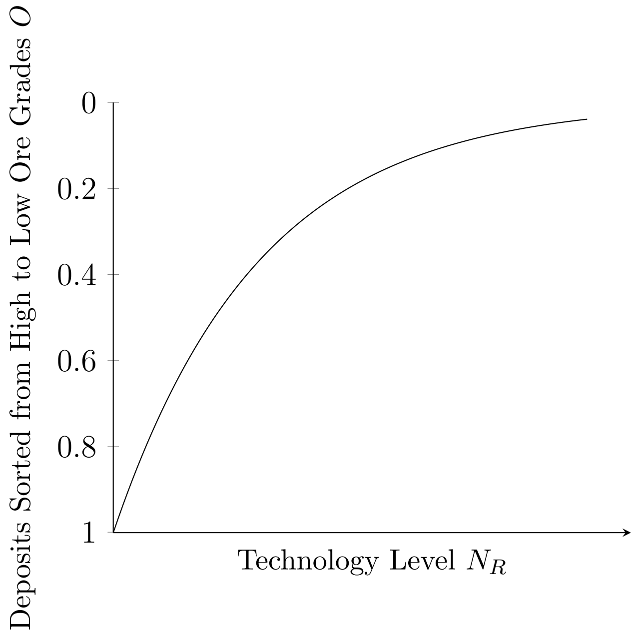


Figure 9: The Extraction Technology Function Assumes Diminishing Returns to Technological Development in Terms of Ore Grades. Note that the y-axis has been reserved for illustrative purposes.

The curvature parameter of the extraction technology function is μ. If, for example, μ is high, the average effect of new technology on converting deposits to reserves in terms of grades is relatively high.

3.3 Marginal Effect of Extraction Technology on Reserves

The interaction of the two functions leads to a linear relationship between technological development and reserves. Figure 10, Panel A, shows the technology function. Two equal steps in advancing technology from 0 to N and from N to N', lead to diminishing returns in terms of extractable ore grades $O^{\star}$ and $O^{\star\prime}$, where $O^{\star\prime} - O^{\star} \prec O^{\star}$.

Panel B depicts equation 2, which is the integral of the geological function. The figure shows how the different advances in extractable ore grades $O^{\star}$ and $O^{\star\prime}$ map into equally increases in the reserve levels S and S', where $S' - S = S$. Finally, Panel C summarizes the linear relationship between the technology level and the reserve level as a result of the two functions.

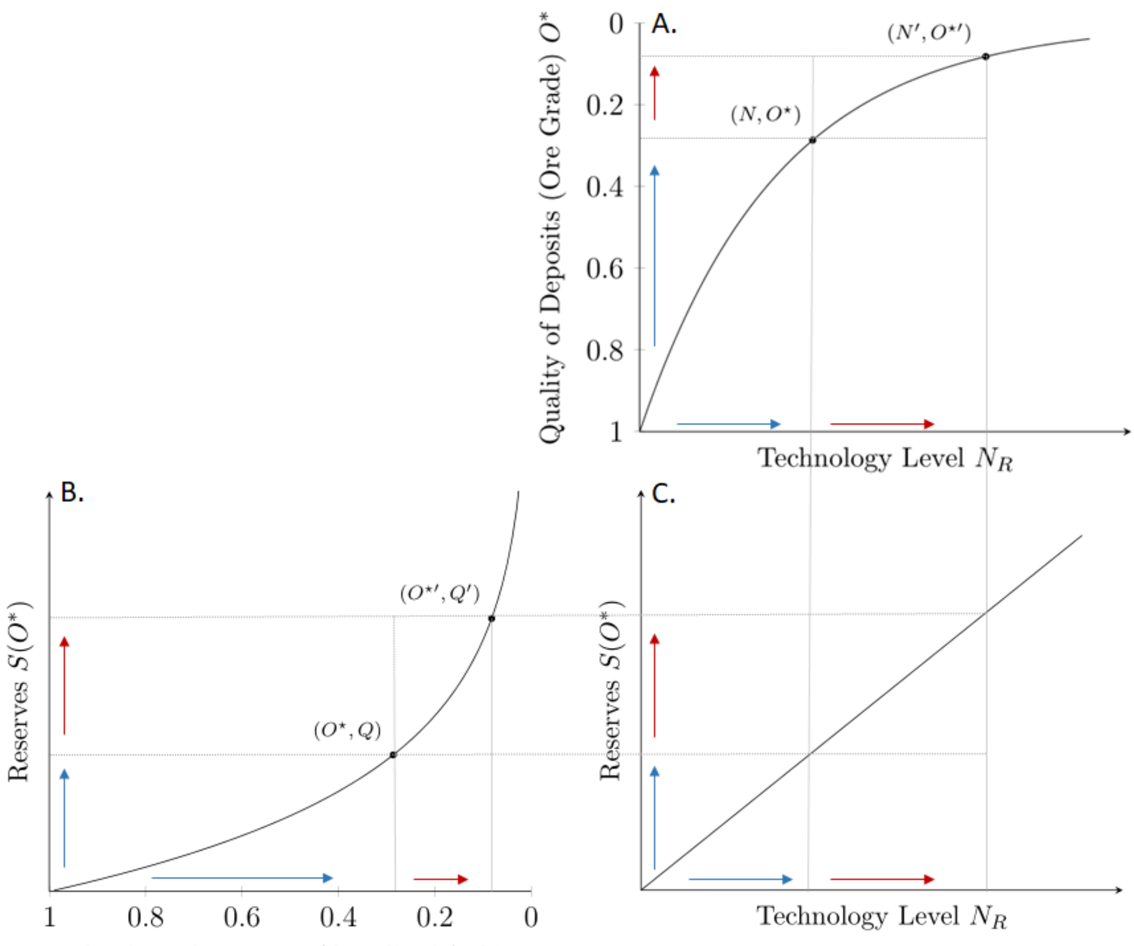


Figure 10: The interaction between the extraction technology function (Panel A) and the integrated geological function (Panel B) leads to a linear relationship between technology level N_R and reserves S (Panel C). Note that the y-axis in panel A and the x-axis in panel B have been reversed for illustrative purposes.

The integrated geology function, equation (2), and the technology function, equation (3), have offsetting effects. This leads to a constant marginal effect of new technology on new reserves.

Proposition 1 *Reserves S increase proportionally to the level of extraction technology N_R:*

$$S(O^\star(N_{Rt})) = \delta\mu N_{Rt} \ .$$

The marginal effect of new extraction technology on reserves equals:

$$\frac{dS(O^\star(N_{Rt}))}{dN_R} = \delta\mu \ .$$

As the natural exponential in equation (3) and the natural log in equation (2) cancel out, there is a linear relationship between the state of technology N_R and the total quantity of the resource converted into reserves S.

Proof of Proposition 1

$$\begin{aligned} S(O^\star(N_{Rt})) &= -\delta \ln(O^\star(N_{Rt})) \\ &= -\delta \ln(e^{-\mu N_{Rt}}) \\ &= \mu\delta N_{Rt} \end{aligned}$$

□

The intuition is that two offsetting effects cause this result: (i) the resource is geologically distributed such that it implies increasing returns in terms of new reserves as the grade of deposits decline; (ii) new extraction technology exhibits decreasing returns in terms of making lower grade deposits extractable.

The equations in Proposition 1 depend on the shapes of the geological function and the technology function. If the respective parameters δ and μ are high, the marginal return on new extraction technology will also be high.

The constant effect of technology on new reserves is a sensible assumption, as we base the functional form of the two functions on stylized facts. It implies that the social value of an innovation is equal to the private value. R&D development does not cause an exhaustion of the resource. Future innovations are not reduced in profitability. No positive or negative spill-overs occur in our model.

However, the constant effect of technology on new reserves is only a first approximation. Of course, there could be other functional forms for the two functions, especially other forms for the technology function. Note that we take a very conservative stand with the functional form of the technological function, as we assume that it technology shows decreasing returns to technological change in terms of lower ore grades. In addition, we allow for wide parameter spaces for the functional forms of the two functions. We discuss other function forms in section 7.

4 The Extractive Sector

Firms determine the rate of technological change in general equilibrium in an endogenous growth model. In this chapter, we outline a simple extractive sector with two different types of firms, extraction firms and technology firms. The former buy technology from the technology firms and extract the resource from deposits of declining grades, while the latter innovate and produce extraction technology.[6] Both types of

[6]To ease comparison, the extractive sector is constructed in analogy to the intermediate goods sector in Acemoglu (2002), which we briefly describe in the next chapter.

firms know fully about the distribution of the resource in the earth's crust, as described in equation 1, and about the technology function, equation 3.

4.1 Extraction Firms

We consider a large number of infinitely small extractive firms.[7] As we model long-run trends in the extractive sector, we assume that the sector is fully competitive and firms take the demand for the non-renewable resource as given.[8] The non-renewable resource is assumed to be a homogeneous good.

Firms can hold reserves S. Reserves are defined as non-renewable resources in underground deposits that can be extracted with *grades-specific* technology (or machine varieties) at a constant extraction cost $\phi > 0$. We assume that the marginal extraction cost for deposits not classified as reserves are infinitely high, $\phi = \infty$. Firms' reserves $\dot{S}$ evolve according to:

$$\dot{S}_t = -R_t^{Extr} + R_t^{Tech}, \quad S_t \geq 0, R_t^{Tech} \geq 0, R_t^{Extr} \geq 0. \tag{4}$$

On the one hand, Firms can extract the resource from its reserves using grade-specific technology, a flow that we denote as R_t^{Extr}. Note that machines fully depreciates after use. On the other hand, firms can also expand the quantity of its reserves by

[7] We assume that the firm level production functions exhibit constant returns to scale, so there is no loss of generality in focusing on aggregate production functions.

[8] Historically, producer efforts to raise prices were successful in some non-oil commodity markets, though short-lived as longer-run price elasticities proved to be high (see Radetzki, 2008; Herfindahl, 1959; Rausser and Stuermer, 2016). Similarly, a number of academic studies discard OPEC's ability to raise prices over the long term (see Aguilera and Radetzki, 2016, for an overview). This is in line with historical evidence that OPEC has never constrained members' capacity expansions, which would be a precondition for long-lasting price interventions (Aguilera and Radetzki, 2016)

investing in new *grades-specific* technology, a flow denoted as R_t^{Tech}.

Extraction firms can purchase the new technologies from sector-specific technology firms at price χ_R. A new grades-specific technology allows firms to claim ownership of all the non-renewable resource in the respective additional deposits. Firms declare these deposits their new reserves.

In our setup, reserves are a function of geology and extraction technology. They are more akin to working capital in the spirit of Nordhaus (1974), as they are inventories of resources in the ground that can be used as input to production. The non-renewable resource is not defined as a fixed, primary factor, but it is a production factor that is "produced" by technological change, making it resembling to working capital.

Combining equations (4), Proposition 1, and the assumptions that each variety of technology is only used for specific deposits of particular grades, firms' flow of new reserves due to technological change is:[9]

$$R_t^{Tech} = \delta \mu \dot{N}_R. \tag{5}$$

Extractive firms' profits are:

$$\pi_R^E = p_R R^{Extr} - \phi R^{Extr} - \chi_R \delta \mu \dot{N}_R, \tag{6}$$

where the cost of extraction and the investment into new technology are subtracted from the income derived from selling the resource.

[9]Please see Appendix 1.2 for the derivation of this equation.

4.2 Technology Firms in the Extractive Sector

Sector-specific technology firms j invent *grades-specific* extraction technology j and sell it to extraction firms.[10]

Each new machine variety makes particular deposits of lower grades O extractable and can only be used for this specific geological formation. This picks up the idea that technology needs to be at least slightly adjusted in order to extract resources from deposits of different grades. The use of a machine by one extraction firm prevents another extraction firm from using it because of this feature. Once these deposits are extracted, new machine types have to be invented. Technology is hence rivalrous in the context of extracting non-renewable resources.[11]

Throughout we assume that there is free entry of technology firms into research. Technology firms observe the demand for grades-specific machine varieties by the extraction firms. The innovation possibilities frontier, which determines how new technologies are created, is assumed to take the form:[12]

$$\dot{N}_R = \eta_R M_R \; . \tag{7}$$

Each technology firm can spend one unit of the final good for R&D investment M at time t to generate a flow rate $\eta_R > 0$ of new patents, respectively. The cost of inventing

[10] This notation is chosen for consistency with the general equilibrium model, where the cost of innovation is $\frac{1}{\eta}$ in terms of final output. We use j to denominate both, new technologies and firms, because each firm can only invent one new technology in line with Acemoglu (2002).

[11] This is in contrast to the intermediate goods sector, where technology is non-rivalrous.

[12] We assume in line with Acemoglu (2002) that there is no aggregate uncertainty in the innovation process. There is idiosyncratic uncertainty, but with many different technology firms undertaking research, equation 7 holds deterministically at the aggregate level.

a new machine variety is $\frac{1}{\eta}$. Each firm can invent only one new machine variety at a time.

A firm that invents a new extraction machine receives a perpetual patent. The patent grants the firm the right to build the respective machine at a fixed marginal cost $\psi_R > 0$. The know-how about building the machine diffuses to all technology firms and can be used to invent new machine varieties for lower ore grades. The economy is assumed to start at the initial technology level $N_R(0) > 0$.

As each machine variety is specific to deposits of certain grades, only one machine is build and sold per variety. As a consequence, each technology firm stays in the market for only one time period. The value of a technology firm that discovers a new machine depends hence on instantaneous profits:

$$V_R(j) = \pi_R(j) = \chi_R(j)x_R(j) - \psi_R x_R(j) \ , \tag{8}$$

which is the difference between the machine price $\chi_R(j)$ and the cost to produce a machine ψ_R times the number of produced machines $x_R(j) = 1$.

This formulation allows us to boil down a dynamic optimization problem to a static one. It makes the model solvable and computable. At the same time, the model stays rich enough to derive meaningful theoretical predictions about the relationship between technological change, geology and economic growth.

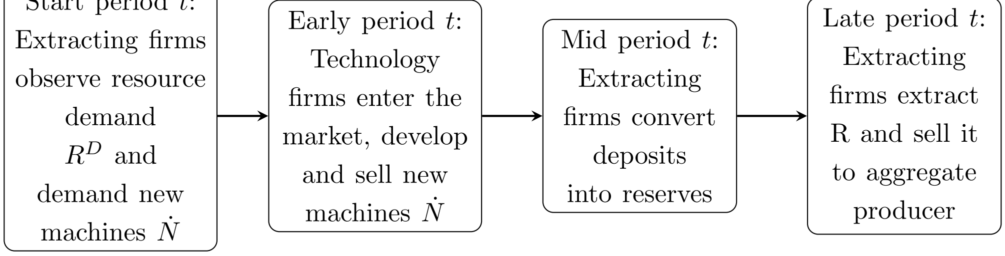
4.3 Timing

Figure 11 illustrates the timing in our model. At the start of period t, extraction firms observe the resource demand from the aggregate production sector and they demand new machine varieties from the technology firms to convert a equivalent number of deposits of lower ore grades into reserve.

In the early period of t, technology firms observe this demand. They invest into new machines that are specific to the grades of the respective deposits. Firms enter the market until the value of entering, namely profits, equals market entry cost, which is the cost to invent a new technology. Each technology firm obtains a patent for their newly developed machine variety, produces one machine based on the patent and sells it to the extraction firms. The knowledge about the machine directly diffuses to the other firms.

In the mid-period of t, extraction firms convert deposits to reserves based on the new machines. In the later period of t, extraction firms extract the resource and sell it to the final good producers.

Figure 11: Timing and Firms' Problem



5 Extraction Technology in an Endogenous Growth Model

We embed the extractive sector in an endogenous growth model with two sectors, and take the framework byAcemoglu (2002) as a starting point. The intermediate goods sector is modeled with expanding input varieties. The greater variety of inputs increases the division of labor and raises the productivity of final good firms (see Romer, 1987, 1990).

5.1 Setup

We consider a standard setup of an economy with a representative consumer that has constant relative risk aversion preferences:

$$\int_0^\infty \frac{C_t^{1-\theta} - 1}{1-\theta} e^{-\rho t} dt \ .$$

The variable C_t denotes consumption of aggregate output at time t, ρ is the discount rate, and θ is the coefficient of relative risk aversion.

The aggregate production function combines two inputs, namely an intermediate good Z and a non-renewable resource R, with a constant elasticity of substitution:

$$Y = \left[\gamma Z^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma) R_{Extr}^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} \ . \tag{9}$$

The distribution parameter $\gamma \in (0,1)$ indicates their respective importance in producing aggregate output Y. The elasticity of substitution is $\varepsilon > 0$, when the resource is not essential for aggregate production (see Dasgupta and Heal, 1980).

The budget constraint of the representative consumer is: $C+I+M \leq Y$. Aggregate spending on machines is denoted by I and aggregate R&D investment by M, where $M = M_Z + M_R$. The usual no-Ponzi game conditions apply.

Setting the price of the final good as the numeraire gives:

$$\left[\gamma^{\varepsilon} p_Z^{1-\varepsilon} + (1-\gamma)^{\varepsilon} p_R^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} = 1 \ , \tag{10}$$

where p_Z is the price index of the intermediate good and p_R is the price index of the non-renewable resource. Intertemporal prices of the intermediate good are given by the interest rate $[r_t]_{T=0}^{\infty}$.

5.2 Intermediate Good Sector

The intermediate good sector follows the basic setup of Acemoglu (2002). It consists of a large number of infinitely small firms that produce the intermediate good, and technology firms that produce sector-specific technologies.[13]

Firms produce an intermediate good Z according to the production function:

$$Z = \frac{1}{1-\beta_Z} \left(\int_0^{N_z} x_z(j)^{1-\beta_Z} dj \right) L_Z^{\beta} \ , \tag{11}$$

[13]Like in the extractive sector, we assume that the firm level production functions exhibit constant returns to scale, so there is no loss of generality in focusing on aggregate production functions. Firms in the extractive and in the intermediate sectors use different types of machines to produce the non-renewable resource and the intermediate good, respectively. Firms are owned by the representative household.

where $x_Z(j)$ refers to the number of machines used for each machine variety j in the production of the intermediate good, L is labor, which is in fixed supply, and β_Z is $\in (0,1)$. This implies that machines in the intermediate good sector are partial complements.[14]

All intermediate good machines are supplied by sector-specific technology firms that each have one fully enforced perpetual patent on the respective machine variety. As machines are partial complements, technology firms have some degree of market power and can set the price for machines. The price charged by these firms at time t is denoted $\chi_Z(j)$ for $j \in [0, N_Z(t)]$. Once invented, machines can be produced at a fixed marginal cost $\psi_Z > 0$.

The innovation possibilities frontier is assumed to take a similar form like in the extractive sector: $\dot{N}_Z = \eta_R M_Z$. Technology firms can spend one unit of the final good for R&D investment M_Z at time t to generate flow rate $\eta_Z > 0$ of new patents. Each firm hence needs $\frac{1}{\eta_Z}$ units of final output to develop a new machine variety. Technology firms can freely enter the market if they develop a patent for a new machine variety. They can only invent one new variety.

6 Characterization of Equilibrium

We now solve the model in general equilibrium such that extractive firms determine the rate of change in the extraction technology. We define the allocation in this econ-

[14]While machines of type j in the intermediate sector can be used infinitely often, a machine of variety j in the resource sector is grade-specific and essential to extracting the resource from deposits of certain grades O. A machine of variety j in the extractive sector is therefore only used once, and the range of machines employed to produce resources at time t is $\dot{N}_R$. In contrast, the intermediate good sector can use the full range of machines $[0, N_Z(t)]$ complementing labor.

omy by the following objects: time paths of consumption levels, aggregate spending on machines, and aggregate R&D expenditure, $[C_t, I_t, M_t]_{t=0}^{\infty}$; time paths of available machine varieties, $[N_{Rt}, N_{Zt},]_{t=0}^{\infty}$; time paths of prices and quantities of each machine, $[\chi_{Rt}(j), x_{Rt}(j)]_{j\in[0,N_{Rt}]t}^{\infty}$ and $[\chi_{Zt}(j), x_{Zt}(j)]_{j\in[0,N_{Zt}],t}^{\infty}$; the present discounted value of profits V_R and V_Z, and time paths of interest rates and wages, $[r_t, w_t]_{t=0}^{\infty}$.

An equilibrium is an allocation in which all technology firms in the intermediate good sector choose $[\chi_{Zt}(j), x_Z t(j)]_{j\in[0,N_Z(t)],t}^{\infty}$ to maximize profits. Machine prices in the extractive sector $\chi_{Rt}(j)$ result from the market equilibrium, because extraction technology firms are in full competition and only produce one machine per patent.

The evolution of $[N_{Rt}, N_{Zt}]_{t=0}^{\infty}$ is determined by free entry; the time paths of factor prices, $[r, w]_{t=0}^{\infty}$, are consistent with market clearing; and the time paths of $[C_t, I_t, M_t]_{t=0}^{\infty}$ are consistent with household maximization.

6.1 The Final Good Producer

The final good producer demands the intermediate good and the resource for aggregate production. Prices and quantities for both are determined in a fully competitive equilibrium. Taking the first order condition with respect to the intermediate good and the non-renewable resource in equation (9), we obtain the demand for the intermediate good

$$Z = \frac{Y(1-\gamma)^{\varepsilon}}{p_Z^{\varepsilon}} \ ,$$

and the demand for the resource

$$R^D = \frac{Y(1-\gamma)^\varepsilon}{p_R^\varepsilon} \,. \tag{12}$$

6.2 Extraction Firms

To characterize the (unique) equilibrium, we first determine the demand for machine varieties in the extractive sector.[15] Machine prices and the number of machine varieties are determined in a market equilibrium between extractive firms and technology firms. Firms optimization problem is static since machines depreciate fully after use.

In equilibrium, it is profit maximizing for firms to not keep reserves, $S(j) = 0$.[16] It follows that the production function of extractive firms is

$$R_t^{Extr} = R_t^{Tech} = \delta\mu\dot{N}_{Rt}. \tag{13}$$

Extractive firms face a cost for producing R_t^{Extr} units of resource given by $\Omega(R_t^{Extr}) = R_t^{Extr}\chi_R\frac{1}{\delta\mu}$, where χ_R is the machine price charged by the extraction technology firms. The marginal cost is $\Omega'(R_t^{Extr}) = \chi_R\frac{1}{\delta\mu}$. The inverse supply function of the resource is hence constant and we obtain a market equilibrium at

$$p_R = \chi_R\frac{1}{\delta\mu}$$

[15] Please see Appendix 1.4 for the respective derivations regarding intermediate good firms.

[16] See appendix Appendix 1.3 for the derivation of this result. If we assumed stochastic technological change, extractive firms would keep a positive stock of reserves S_t to insure against a series of bad draws in R&D. Reserves would grow over time in line with aggregate growth. The result would, however, remain the same: in the long term, resource extraction equals new reserves.

and

$$R_t^D = \frac{Y(1-\gamma)^{\varepsilon}}{(\chi_R \frac{1}{\delta\mu})^{\varepsilon}} \ . \tag{14}$$

Using (13) and (14), we obtain the demand for machines:

$$\dot{N}_R = \frac{1}{\delta\mu} \frac{Y(1-\gamma)^{\varepsilon}}{(\chi_R \frac{1}{\delta\mu})^{\varepsilon}} \ . \tag{15}$$

6.3 Technology Firms in the Extractive Sector

In the extractive sector, the demand function for extraction technologies (15) is isoelastic, but there is perfect competition between the different suppliers of extraction technologies, as machine varieties are perfect substitutes in terms of producing the homogenous resource.[17]

Because only one machine is produced for each machine variety j, the constant rental rate χ_R that the monopolists charge includes the cost of machine production ψ_R and a mark-up that refinances R&D costs. The rental rate is the result of a competitive market and derived from (14). It equals:

$$\chi_R(j) = \left(Y/R^{Extr}\right)^{\frac{1}{\varepsilon}} (1-\gamma)\delta\mu. \tag{16}$$

To complete the description of equilibrium on the technology side, we impose the free-entry condition:

$$\pi_{Rt} = \frac{1}{\eta_R} \text{if} M_R \succ 0 \ . \tag{17}$$

[17]Please see Appendix 1.5 for the respective derivations for technology firms in the intermediate good sector.

Firms enter the market until the value of entering, namely profits, equals market entry cost, which is the cost to develop a new technology.

Like in the intermediate sector, markups are used to cover technology expenditure in the extractive sector. Combining equations (8) and (16), we obtain that the net present discounted value of profits of technology firms from developing one new machine variety is:

$$V_R(j) = \pi_R(j) = \chi_R(j) - \psi_R = \left(Y/R^{Extr}\right)^{\frac{1}{\varepsilon}} (1-\gamma)\delta\mu - \psi_R \,. \tag{18}$$

To compute the equilibrium quantity of machines and machine prices in the extractive sector, we first rearrange (18) with respect to R and consider the free entry condition. We obtain

$$R_t^{Extr} = \frac{Y(1-\gamma)^{\varepsilon}}{\left(\left(\frac{1}{\eta_R} + \psi_R\right)\frac{1}{\delta\mu}\right)^{\varepsilon}} \,. \tag{19}$$

We insert (19) into (16) and obtain the equilibrium machine price.

$$\chi_R(j) = \frac{1}{\eta_R} + \psi_R \,. \tag{20}$$

6.4 Equilibrium Resource Price

The resource price equals marginal production costs due to perfect competition in the resource market. Equation (20) implies the following proposition:[18]

Proposition 2 *The resource price depends negatively on the average crustal concentration of the non-renewable resource and the average effect of extraction technology on*

[18]Please see Appendix 1.6 for the equilibrium price of the intermediate good.

ore grades:

$$p_R = \left(\frac{1}{\eta_R} + \psi_R\right)\frac{1}{\delta\mu}\ , \tag{21}$$

where ψ_R reflects the marginal cost of producing the machine and η_R is a markup that serves to compensate technology firms for R&D cost.

The intuition is as follows: If, for example, δ is high, the average crustal concentration of the resource is high (see equation (1)) and the price is low. If μ is high, the average effect of new extraction technology on converting deposits of lower grades to reserves is high (see equation (3)). This implies a lower resource price. The resource price level also depends negatively on the cost parameter of R&D development η_R.

6.5 Resource Intensity of the Economy

Substituting equation (21) into the resource demand equation (12), we obtain the ratio of resource consumption to aggregate output.

Proposition 3 *The resource intensity of the economy is positively affected by the average crustal concentration of the resource and the average effect of extraction technology:*

$$\frac{R^{Extr}}{Y} = (1-\gamma)^{\varepsilon}\left[\left(\frac{1}{\eta_R} + \psi_R\right)\frac{1}{\delta\mu}\right]^{-\varepsilon}\ .$$

The resource intensity of the economy is negatively affected by the elasticity of substitution if $(1-\gamma)^{\varepsilon}\left[(\frac{1}{\eta_R} + \psi_R)\frac{1}{\delta\mu}\right]^{-\varepsilon} < 1$ *and positively otherwise.*

6.6 The Growth Rate on the Balanced Growth Path

We define the BGP equilibrium as an equilibrium path where consumption grows at the constant rate g^* and the relative price p is constant. From (10) this definition implies that p_{Zt} and p_{Rt} are also constant.

Proposition 4 *There exists a unique BGP equilibrium in which the relative technologies are given by equation* (39) *in the appendix, and consumption and output grow at the rate*[19]

$$g = \theta^{-1}\left(\beta\eta_Z L\left[\gamma^{-\varepsilon} - \left(\frac{1-\gamma}{\gamma}\right)^{\varepsilon}\left(\frac{1}{\eta_R\delta\mu} + \frac{\psi_R}{\delta\mu}\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}\frac{1}{\beta}} - \rho\right). \tag{22}$$

The growth rate of the economy is positively influenced by (i) the crustal concentration of the non-renewable resource δ and (ii) the effect of R&D investment in terms of lower ore grades μ.

Adding the extractive sector to the standard model by Acemoglu (2002), changes the interest part of the Euler equation, $g = \theta^{-1}(r - \rho)$.[20] Instead of two exogenous production factors, the interest rate r in our model only includes labor, but adds the resource price, as p_Z depends on p_R according to equation (37).

If $(1-\gamma)^{\varepsilon}(\eta_R\delta\mu)^{1-\varepsilon} < 1$ holds, then the substitution between the intermediate good and the resource is low and R&D investment in extraction technology have a small yield

[19]Starting with any $N_R(0) > 0$ and $N_Z(0) > 0$, there exists a unique equilibrium path. If $N_R(0)/N_Z(0) < (N_R/N_Z)^*$ as given by (39), then $M_{Rt} > 0$ and $M_{Zt} = 0$ until $N_{Rt}/N_{Zt} = (N_R/N_Z)^*$. If $N_R(0)/N_Z(0) > (N_R/N_Z)^*$, then $M_{Rt} = 0$ and $M_{Zt} > 0$ until $N_{Rt}/N_{Zt} = (N_R/N_Z)^*$. It can also be verified that there are simple transitional dynamics in this economy whereby starting with technology levels $N_R(0)$ and $N_Z(0)$, there always exists a unique equilibrium path, and it involves the economy monotonically converging to the BGP equilibrium of (22) like in Acemoglu (2002).

[20]There is no capital in this model, but agents delay consumption by investing in R&D as a function of the interest rate.

in terms of additional reserves. The effect that economic growth is impossible if the resource cannot be substituted by other production factors is known as the "limits to growth" effect in the literature (see Dasgupta and Heal, 1979, p. 196 for example). When the effect occurs, growth is *limited* in models with a positive initial stock of resources, because the initial resource stock can only be consumed in this case. In our model, growth is *impossible*, because there is no initial stock and the economy is not productive enough to generate the necessary technology. When the inequality does not hold, the economy is on a balanced growth path.

6.7 Technology Growth

We derive the growth rates of technology in the two sectors from equations (13), (12), and (21). The stock of technology in the intermediate good sector grows at the same rate as the economy.

Proposition 5 *The stock of extraction technology grows proportionally to output according to:*

$$\dot{N}_R = (1-\gamma)^{\varepsilon} Y \left(1/\eta_R + \psi_R\right)^{-\varepsilon} (\delta\mu)^{\varepsilon-1} .$$

In contrast to the intermediate good sector, where firms can make use of the *stock* of technology, firms in the extractive sector can only use the *flow* of new technology to convert deposits of lower grades into new reserves. Previously developed technology cannot be employed because it is grade specific, and deposits of that particular grade have already been depleted. Note also that firms in the extractive sector need to invest a larger share of total output to attain the same rate of growth in technology in comparison to firms in the intermediate good sector.

The effects of the two parameters δ from the geological function and μ from the extraction technology function on $\dot{N}_R$ depend on the elasticity of substitution ε. Like in Acemoglu (2002), there are two opposing effects at play: the first is a price effect. Technology investments are directed towards the sector of the scarce good. The second is a market size effect, meaning that technology investments are directed to the larger sector.

If the goods of the two sectors are complements ($\varepsilon < 1$), the price effect dominates. An increase in δ or μ lowers the cost of resource production and the resource price, but the technology growth rate in the resource sector decelerates, because R&D investment is directed towards the complementary intermediate good sector. If the resource and the intermediate good are substitutes ($\varepsilon > 1$), the market size effect dominates. An increase in δ or μ makes resources cheaper and causes an acceleration in the technology growth rate in the resource sector, because more of the lower cost resource is demanded.

7 Discussion

We discuss the assumptions made in section 5, the comparison to other models with non-renewable resources, and the ultimate finiteness of the resource.

We chose the functional forms of the geological function and the extraction technology based on empirical evidence. Our model provides theoretical results that are consistent with the historical evolution resource prices and production. However, for making long-term predictions based on our model, a natural question is how other functional forms of the two functions would affect the predictions of the model.

Our model can also be generalized to different functional forms of the geological function and the extraction technology function. If one or both of the functions have different forms, the effects on resource price, resource intensity of the economy, and growth rate will depend on the resulting changes in proposition 1. In the first case, where increasing returns in the geology function more than offset the decreasing returns in the technology function, the unit extraction cost declines and the resource becomes more abundant. As a result, the resource price is lower, the resource intensity higher, and the growth rate of the economy also higher. The condition that resource prices equal marginal resource extraction cost would still extend to this case. Prices cannot be below marginal extraction cost, since firms would make negative profits.

In the second case where the increasing returns in the geology function do not offset the decreasing returns in the technology function, the resource price increases over time as the unit extraction technology cost goes up, the resource intensity decline and the growth rate of the economy declines as well. There would still be no scarcity rent like in Hotelling (1931)[21], but an additional social cost if extraction firms hold infinite property rights (Heal, 1976). This social cost reflects that present extraction pushes up future unit extraction technology cost. This would drive a wedge between the resource price and the unit extraction cost.

However, extraction firms typically do not hold property rights but instead lease extraction rights for a definite period of time from private owners in the U.S. or governments elsewhere. These leases typically require the firm to start production at

[21]Note that a scarcity rent has not yet been found empirically (see e.g. Hart and Spiro, 2011)

some time or the lease is terminated early. In addition, there is a substantial risk of ex-appropriation for extractive firms in many countries (see e.g. Stroebel and Van Benthem, 2013). If there is no exclusive property right of extraction firms in the resource, and there is free entry and exit like in our model, firms will increase their production until the resource price equals the unit extraction cost (Heal, 1976).[22]

Finally, if any of the two functions is discontinuous with an unanticipated break, at which the respective parameters change to either $\delta' \in \mathbb{R}_+$ or $\mu' \in \mathbb{R}_+$, there will be two balanced growth paths: one for the period before, and one for the period after the break. Both paths would behave according to the model's predictions.

How does our model compare to other models with non-renewable resources? We make the convenient assumption that the quantity of non-renewable resources is for all practical economic purposes approaches infinite. As a consequence, resource availability does not limit growth if there is investment in technological change. Substitution of capital for non-renewable resources, technological change in the use of the resource, and increasing returns to scale are therefore not necessary for sustained growth as in Groth (2007) or Aghion and Howitt (1998). If the resource was finite in our model, the extractive sector would behave in the same way as in standard models with a sector based on Hotelling (1931). As Dasgupta and Heal (1980) point out, in this case the growth rate of the economy depends strongly on the degree of substitution between the resource and other economic inputs. For $\varepsilon > 1$, the resource is non-essential; for $\varepsilon < 1$, the total output that the economy is capable of producing is finite. The production

[22] Note that there are also reserves in our setup, which could lead to hording of reserves in anticipation of higher prices. However, this again is only realistic if mining and extraction firms hold infinite ownership rights of the reserves.

function is, therefore, only interesting for the Cobb-Douglas case.

Our model suggests that the non-renewable resource can be thought of as a form of capital: if the extractive firms invest in R&D in extraction technology, the resource is extractable without limits as an input to aggregate production. This feature marks a distinctive difference from models such as the one of Bretschger and Smulders (2012). They investigate the effect of various assumptions about substitutability and a decentralized market on long-run growth, but keep the assumption of a finite non-renewable resource. Without this assumption, the elasticity of substitution between the non-renewable resource and other input factors is no longer central to the analysis of limits to growth.

Some might argue that the relationship described in proposition 1 cannot continue to hold in the future as the amount of non-renewable resources in the earth's crust is ultimately finite. Scarcity will become increasingly important, and the scarcity rent will be positive even in the present. However, for understanding current prices and consumption patterns, current expectations about future developments are important. Given that the quantities of available resources indicated in table 1 are very large, their ultimate end far in the future should approximately not affect economic behavior today and in the near future. The relationship described in proposition 1 seems to have held in the past and looks likely to hold for the foreseeable future. Since in the long term, extracted resources equal the resources added to reserves due to R&D in extraction technology, the price for a unit of the resource will equal the extraction cost plus the per-unit cost of R&D and hence, stay constant in the long term. This may explain why scarcity rents cannot be found empirically.

8 Conclusion

This paper examines the interaction between geology and technology and its impact on the resource price, total output growth, and the resource intensity of the economy. We argue that economic growth causes the production and use of a non-renewable resource to increase at a constant rate. The marginal production cost of non-renewable resources stay constant in the long term. Economic growth enables firms to invest in extraction technology R&D, which makes resources from deposits of lower grades economically extractable. We help explain the long-term evolution of non-renewable resource prices and world production for more than 200 years.

Our model makes simplifying but reasonable assumptions, which render our model analytically solvable. However, we believe that a less simple model would essentially provide the same results. There are four major simplifications in our model, which should be examined in more detail in future extensions. First, there is no uncertainty in R&D development, and therefore no incentive for firms to keep a positive amount of the non-renewable resource in their reserves. If R&D development is stochastic as in Dasgupta and Stiglitz (1981), there would be a need for firms to keep reserves. We leave it to future work to lift the assumption of no aggregate uncertainty and to model positive reserve holdings, as we observe than empirically.

Second, our model features perfect competition in the extractive sector. We could obtain a model with monopolistic competition in the extractive sector by introducing explicitly privately-owned deposits. A firm would need to pay a certain upfront cost or exploration cost in order to acquire a mineral deposit. This upfront cost would give technology firms a certain monopoly power as they develop machines that are specific to a single deposit.

Third, extractive firms could face a trade-off between accepting high extraction costs due to a lower technology level and investing in R&D to reduce extraction costs. A more general extraction technology function would provide the basis to generalize this assumption.

Fourth, our model does not include recycling. Recycling has become more important for metal production over time due to the increasing abundance of recyclable materials and the comparatively low energy requirements (see Wellmer and Dalheimer, 2012). Introducing recycling into our model would further strengthen the argument of this paper, as it increases the economically extractable stock of the non-renewable resource.

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Appendix 1

Appendix 1.1 Data Description

We include the following 63 non-renewable resources in the data-set: fossil minerals: coal, natural gas, petroleum; metals: aluminum, antimony, arsenic, beryllium, bismuth, boron, cadmium, cesium, chromium, cobalt, copper, gallium, germanium, gold, indium, lead, lithium, magnesium (compounds and metal), manganese, mercury, molybdenum, nickel, niobium, platinum-group metals, pig iron, rare earths, rhenium, selenium, silver, strontium, tantalum, thorium, tin, tungsten, vanadium, zinc, zirconium; non-metals: asbestos, barite, bromine, cement, diatomite, feldspar, fluorspar, garnet, graphite, gypsum, iodine, kyanite, nitrogen, phosphate rock, potash, pumice, silicon, sulfur, talc& pyrophyllite, tellurium, thallium, uranium, vermiculite, wolalstonite.

We currently do not include the following metals: bauxite, iron ore, hafnium, cesium; non-metals: natural abrasives, clays, coal combustion products (ashes), diamond (industrial), gemstones, iron oxide pigments, lime, peat, perlite, quartz, salt, sand, soda ash, sodium sulfate, stone, titanium (pigments, metal, mineral concentrates), and helium. These non-renewable resources are excluded for a variety of different reasons, including lack of global historical data, e.g. for stones, no clear separation in the data between natural and synthetic materials like in the case of industrial diamonds, and prevention of double-counting due to different products in the value chain, e.g. iron ore is not but pig iron is included. Most of the excluded commodities would not change the results of our analysis, because the extracted quantities and market value are negligible.

Two exceptions are stones and salt, which exhibit relatively large extracted quantities.

The number of resources increases over time, as previously more and more mineral commodities are explored and employed in the manufacturing of goods. In 1700, our data-set includes copper, gold, mercury, pig iron, silver, tin, and coal. These are all non-renewable resources that were in broad use in the global economy at the time with the exceptions of stones and salt. The number non-renewable resources increases to 33 in 1900 in our data-set, including petroleum, natural gas and a broad variety of metals and non-metals, and 63 in 2000.

Appendix 1.2 Derivation of Extraction Firms' New Reserves

Equation (5) is derived in the following way: Firms can buy machine varieties j to increase their reserves by:

$$R_t^{Tech} = \delta\mu \lim_{h\to 0} \frac{1}{h} \int_{N_R(t-h)}^{N_R(t)} x_R(j)^{(1-\beta)} dj \ , \tag{23}$$

where $x_R(j)$ refers to the number of machines used for each machine variety j.

We assume that $\beta = 0$ in the extractive sector, because firms invest into technology to continue resource production. If firms do not invest, extraction cost becomes infinitely high. Firms invest into technology for the next lowest grade deposits. However, firms are ultimately indifferent about the specific deposit from which they extract, because conditioned on new technology the same homogeneous resource can be produced from all deposits. That's why machine varieties are full complements in our setup. This is in contrast to the intermediate goods sector, where machine varieties are partial complements and firms invest into machine varieties to increase the division of labor.

As a machine variety j in the resource sector is grade-specific and essential to extracting the resource from deposits of certain grades, each variety j in the extractive sector is only used once, and the range of machines employed to produce resources at time t is $\dot{N}_R$. In contrast, the intermediate good sector can use machine types infinitely often and hence the full range of machines $[0, N_Z(t)]$ complementing labor. Under the assumption that $x_R(j) = 1$, equation (23) turns into:

$$\begin{aligned} R_t^{Tech} &= \delta\mu \lim_{h\to 0} \frac{1}{h} \int_{N_R(t-h)}^{N_R(t)} 1dj \\ &= \delta\mu \dot{N}_R \; . \end{aligned}$$

Appendix 1.3 Solving for the Equilibrium: Extraction Firms

To show that it is profit maximizing for extraction firms to not keep any reserves if there is no uncertainty, we first assume that firms have already invested in technology and accessed new reserves R^{Tech}. Firms can either extract the resource for immediate sale R^{Extr} or build reserves S. We obtain the following optimization problem of a firm:

$$\max_{R^{Extr}} (p_R - \phi) R^{Extr} \text{ such that } R^{Tech} \geq R^{Extr}. \tag{24}$$

The maximization problem can be expressed with the following Lagrangian:

$$L = (p_R - \phi) R^{Extr} + \lambda [R^{Tech} - R^{Extr}]. \tag{25}$$

This leads to the following first order conditions:

$$(p_R - \phi)R^{Extr} - \lambda = 0 \quad (26)$$

$$\lambda[R^{Tech} - R^{Extr}] = 0 \quad (27)$$

Consider the case that the constraint is not binding. Given (27), we obtain $\lambda = 0$, and from (26) follows $p_R - \phi = 0$. This is a contradiction, since the market entry condition ensures $\pi_R > 0$, which is not in line with $p_R - \phi = 0$. Therefore, the constraint must be binding and $R^{Tech} = R^{Extr}$. In equilibrium, it is thus profit maximizing for firm j to not keep reserves, $S(j) = 0$.

It follows that the production function of the extractive firms is

$$R_t^{Extr} = \delta\mu\dot{N}_{Rt}. \quad (28)$$

Appendix 1.4 Solving for the Equilibrium: Intermediate Good Firms

For the intermediate good firms, the maximization problem can be written as

$$\max_{L,\{x_Z(j)\}j\in[0,N_{Zt}]} p_Z Z - wL - \int_0^{N_Z} \chi_Z(j)x_Z(j)dj \ .$$

The problem is static, as machines depreciate fully.

The FOC with respect to $x_Z(j)$ immediately implies the following isoelastic demand function for machines:

$$x_{Zt}(j) = \left(\frac{p_{Zt}}{\chi_{Zt}(j)}\right)^{1/\beta} L \; , \tag{29}$$

for all $j \in [0, N_Z(t)]$ and all t,

Appendix 1.5 Solving for the Equilibrium: Technology Firms in the Intermediate Good Sector

Substituting (29) into (30), we calculate the FOC with respect to machine prices in the intermediate good sector: $\chi_Z(j)$: $\left(\frac{p_Z}{\chi_Z(j)}\right)^{\frac{1}{\beta}} L - (\chi_Z(j) - \psi_R) p_Z^{\frac{1}{\beta}} \frac{1}{\beta} \chi_Z(j)^{\frac{1}{\beta}-1} L = 0$. Hence, the solution of the maximization problem of any monopolist $j \in [0, N_Z]$ involves setting the same price in every period according to

$$\chi_{Zt}(j) = \frac{\psi_R}{1-\beta} \text{ for all } j \text{ and } t \; .$$

The value of a technology firm in the intermediate good sector that discovers one of the machines is given by the standard formula for the present discounted value of profits:

$$V_Z(j) = \int_t^\infty exp\left(-\int_t^s r(s')ds'\right) \pi_Z(j) ds \; .$$

Instantaneous profits are denoted

$$\pi_Z(j) = (\chi_Z(j) - \psi_Z)x_Z(j) \ , \tag{30}$$

where r is the market interest rate, and $x_Z(j)$ and $\chi_Z(j)$ are the profit-maximizing choices for the technology monopolist in the intermediate good sector.

All monopolists in the intermediate good sector charge a constant rental rate equal to a markup over their marginal cost of machine production, ψ_R. We normalize the marginal cost of machine production to $\psi_R \equiv (1-\beta)$ (remember that the elasticity of substitution between machines is $\epsilon \equiv \frac{1}{\beta}$), so that

$$\chi_{Zt}(j) = \chi_Z = 1 \text{ for all } j \text{ and } t \ . \tag{31}$$

In the intermediate good sector, substituting the machine prices (31) into the demand function (29) yields: $x_{Zt}(j) = p_{Zt}^{1/\beta} L$ for all j and all t.

Since the machine quantities do not depend on the identity of the machine, only on the sector that is being served, profits are also independent of machine variety in both sectors. Firms are symmetric.

In particular profits of technology firms in the intermediate good sector are $\pi_{Zt} = \beta p_{Zt}^{1/\beta} L$. This implies that the net present discounted value of monopolists only depends on the sector and can be denoted by V_{Zt}.

Combining the demand for machines (29) with the production function of the intermediate good sector (11) yields the *derived* production function:

$$Z(t) = \frac{1}{1-\beta} p_{Zt}^{\frac{1-\beta}{\beta}} N_{Zt} L, \tag{32}$$

The equivalent equation in the extractive sector is (13), because there is no optimization over the number of machines by the extraction technology firms, as the demand for machines per machine variety is one.

Appendix 1.6 Equilibrium Prices

Prices of the intermediate good and the non-renewable resource are derived from the marginal product conditions of the final good technology, (9), which imply

$$\begin{aligned} p &\equiv \frac{p_R}{p_Z} = \frac{1-\gamma}{\gamma} \left(\frac{R^{Extr}}{Z} \right)^{-\frac{1}{\varepsilon}} \\ &= \frac{1-\gamma}{\gamma} \left(\frac{\delta \mu \dot{N}_R}{\frac{1}{1-\beta} p_L^{\frac{1-\beta}{\beta}} N_Z L} \right)^{-\frac{1}{\varepsilon}} \end{aligned}$$

There is no derived elasticity of substitution in analogy to Acemoglu (2002), because there is only one fixed factor, namely L in the intermediate good sector. In the extractive sector, resources are produced by machines from deposits. The first line of this expression simply defines p as the relative price between the intermediate good and the non-renewable resource, and uses the fact that the ratio of the marginal productivities of the two goods must be equal to this relative price. The second line substitutes from (32) and (13). There are no relative factor prices in this economy like in Acemoglu (2002), because there is only one fixed factor in the economy, namely L in the intermediate good sector.

Appendix 1.7 Proof for the Balanced Growth Path

We define the BGP equilibrium as an equilibrium path where consumption grows at the constant rate g^* and the relative price p is constant. From (10) this definition implies that p_{Zt} and p_{Rt} are also constant.

Household optimization implies

$$\frac{\dot{C}_t}{C_t} = \frac{1}{\theta}(r_t - \rho),$$

and

$$\lim_{t \to \infty} \left[\exp\left(- \int_0^t r(s)ds \right) (N_{Zt}V_{Zt} + \dot{N}_{Rt}V_{Rt}) \right] = 0,$$

which uses the fact that $N_{Zt}V_{Zt} + \dot{N}_{Rt}V_{Rt}$ is the total value of corporate assets in the economy. In the resource sector, only *new* machine varieties produce profit.

The consumer earns wages from working in the intermediate good sector and earns interest on investing in technology M_Z. The budget constraint thus is $C = wL + rM_Z$. Maximizing utility in equation (5.1) with respect to consumption and investments yields the first order conditions $C^{-\theta}e^{-\rho t} = \lambda$ and $\dot{\lambda} = -r\lambda$ so that the growth rate of consumption is

$$g_c = \theta^{-1}(r - \rho) \ . \tag{33}$$

This is equal to output growth on the balanced growth path. We can thus solve for the interest rate and obtain $r = \theta g + \rho$. The free entry condition for the technology firms imposes that profits from investing in patents must be zero. Revenue per unit of

R&D investment is given by V_Z, cost is equal to $\frac{1}{\eta_Z}$. Consequently, we obtain $\eta_Z V_Z = 1$. Making use of equation (34), we obtain $\frac{\eta_Z \beta p_Z^{\frac{1}{\beta}} L}{r} = 1$. Solving this for r and substituting it into equation (33) we obtain the following proposition:

$$g = \theta^{-1}(\beta \eta_Z L p_Z^{\frac{1}{\beta}} - \rho) \ .$$

Adding the extractive sector to the standard model by Acemoglu (2002), changes the interest part of the Euler equation, $g = \theta^{-1}(r - \rho)$.[23] Instead of two exogenous production factors, the interest rate r in our model only includes labor, but adds the resource price, as p_Z depends on p_R according to equation (37). Together with (21), this yields the growth rate on the balanced growth path.

Proposition 6 *Suppose that*

$$\beta \left[(1-\gamma)_R^{\varepsilon}(\eta_R R^{Extr})^{\sigma-1} + \gamma_Z^{\varepsilon}(\eta_Z L)^{\sigma-1}\right]^{\frac{1}{\sigma-1}} > \rho, \textit{ and}$$
$$(1-\theta)\beta \left[\gamma_R^{\varepsilon}(\eta_R R^{Extr})^{\sigma-1} + \gamma_Z^{\varepsilon}(\eta_Z L)^{\sigma-1}\right]^{\frac{1}{\sigma-1}} < \rho.$$

If $(1-\gamma)^{\varepsilon}(\eta_R \delta \mu)^{1-\varepsilon} < 1$ *the economy cannot produce. Otherwise, there exists a unique BGP equilibrium in which the relative technologies are given by equation* (39), *and consumption and output grow at the rate in equation* (22).[24]

[23] There is no capital in this model, but agents delay consumption by investing in R&D as a function of the interest rate.

[24] Starting with any $N_R(0) > 0$ and $N_Z(0) > 0$, there exists a unique equilibrium path. If $N_R(0)/N_Z(0) < (N_R/N_Z)^*$ as given by (39), then $M_{Rt} > 0$ and $M_{Zt} = 0$ until $N_{Rt}/N_{Zt} = (N_R/N_Z)^*$. If $N_R(0)/N_Z(0) > (N_R/N_Z)^*$, then $M_{Rt} = 0$ and $M_{Zt} > 0$ until $N_{Rt}/N_{Zt} = (N_R/N_Z)^*$. It can also be verified that there are simple transitional dynamics in this economy whereby starting with technology levels $N_R(0)$ and $N_Z(0)$, there always exists a unique equilibrium path, and it involves the economy monotonically converging to the BGP equilibrium of (22) like in Acemoglu (2002).

Appendix 2 Directed Technological Change

Let V_Z and V_R be the BGP net present discounted values of new innovations in the two sectors. Then the Hamilton-Jacobi-Bellman Equation version of the value function for the intermediate good sector $r_t V_Z(j) - \dot{V}_Z(j) = \pi_Z(j)$ and the free entry condition of extraction technology firms imply that

$$V_Z = \frac{\beta p_Z^{1/\beta} L}{r^*}, \text{ and } V_R = \chi_R(j) - \psi_R \ , \tag{34}$$

where r^* is the BGP interest rate, while p_Z is the BGP price of the intermediate good and $\chi_R(j)$ is the BGP machine price in the extractive sector.

The greater is V_R relative to relative to V_Z, the greater are the incentives to develop machines in the extractive sector rather than developing machines in the intermediate good sector. Taking the ratio of the two equations in (34) and including the equilibrium machine price (20) yields

$$\frac{V_R}{V_Z} = \frac{\chi_R(j) - \psi_R}{\frac{1}{r}\beta p_Z^{\frac{1}{\beta}} L} = \frac{\frac{1}{\eta_R}}{\frac{1}{r}\beta p_Z^{\frac{1}{\beta}} L} \ . \tag{35}$$

This expression highlights the effects on the direction of technological change

1. The price effect manifests itself because V_R/V_Z is decreasing in p_Z. The greater is the intermediate good price, the smaller is V_R/V_Z and thus the greater are the incentives to invent technology complementing labor. Since goods produced by

the relatively scarce factor are relatively more expensive, the price effect favors technologies complementing the scarce factor. The resource price p_R does not affect V_R/V_Z due to perfect competition among extraction technology firms and a flat supply curve.

2. The market size effect is a consequence of the fact that V_R/V_Z is decreasing in L. Consequently an increase in the supply of labor translates into a greater market for the technology complementing labor. The market size effect in the intermediate good sector is defined by the exogenous factor labor. There is no equivalent in the extractive sector.

3. Finally, the cost of developing one new machine variety in terms of final output also influences the direction of technological change. If the parameter η increases, the cost goes down, the relative profitability V_R/V_Z decreases, and therefore the incentive to invent extraction technology declines.

Since the intermediate good price is endogenous, combining (33) with (35) the relative profitability of the technologies becomes

$$\frac{V_R}{V_Z} = \frac{\frac{1}{\eta_R}}{\frac{1}{r}\beta \left(p_R \frac{\gamma}{1-\gamma} \left(\frac{\delta \mu \dot{N}_R}{\frac{1}{1-\beta} p_Z^{\frac{1-\beta}{\beta}} N_Z L} \right)^{\frac{1}{\varepsilon}} \right)^{\frac{1}{\beta}} L} \tag{36}$$

Rearranging equation (10) we obtain

$$p_Z = \left(\gamma^{-\varepsilon} - \left(\frac{1-\gamma}{\gamma} \right)^{\varepsilon} p_R^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}} . \tag{37}$$

Combining (37) and (21), we can eliminate relative prices, and the relative profitability of technologies becomes:

$$\frac{V_R}{V_Z} = \frac{\frac{1}{\eta_R}}{\frac{1}{r}\beta\left(\left(\gamma^{-\varepsilon} - \left(\frac{1-\gamma}{\gamma}\right)^{\varepsilon}\left(\left(\frac{1}{\eta_R} + \psi_R\right)\frac{1}{\mu\delta}\right)^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}}\right)^{\frac{1}{\beta}} L}.$$

Using the free-entry conditions and assuming that both of them hold as equalities, we obtain the following BGP *technology market clearing* condition:

$$\eta_Z V_Z = \eta_R V_R. \tag{38}$$

Combining 38 with 36, we obtain the following BGP ratio of relative technologies and solving for $\frac{\dot{N}_R}{N_Z}$ yields:

$$\left(\frac{\dot{N}_R}{N_Z}\right)^* = \left(\left(\frac{r}{\eta_Z \beta L}\right)^{\beta} \frac{1-\gamma}{\gamma p_R}\right)^{\varepsilon} \frac{L p_Z^{\frac{1-\beta}{\beta}}}{(1-\beta)\delta\mu} \tag{39}$$

where the asterisk ($*$) denotes that this expression refers to the BGP value. The relative productivities are determined by both prices and the supply of labor.

Appendix 3 The Case of Multiple Resources

We now extend the model and replace the generic resource with a set of distinct resources. We do so in analogy to a generic capital stock as in many growth models. We define resource extraction R^{Extr}, resource prices p_R and resource investments M_R as aggregates of the respective variables of different resources $i \in [0, G]$,

$$
\begin{aligned}
R^{Extr} &= \left(\sum_i R_i^{Extr\,\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \\
p_R &= \left(\sum_i \frac{R_i^{Extr}}{R^{Extr}} p_{R_i}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{1}{1-\sigma}}, \\
M_R &= \sum_i M_{R_i}, \\
\frac{R^{Extr}}{Y} &= (1-\gamma)^{\epsilon} {p_R}^{-\epsilon}, \\
g &= \theta^{-1} \left(\beta \eta_Z L \left[\gamma^{-\varepsilon} - \left(\frac{1-\gamma}{\gamma} \right)^{\varepsilon} {p_R}^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}\frac{1}{\beta}} - \rho \right),
\end{aligned}
$$

where σ is the elasticity of substitution between the different resources. Note that the aggregate resource price consists of the average of the individual resources weighted by their share in physical production.

This extension can be used to make theoretical predictions. As an example, we focus here on the relative price of two resources, aluminum a and copper c. Using equation (21) and assuming that the cost of producing machines ψ_R and the flow rate of innovations η_R are uniform across resources, we obtain that prices depend solely on geological and technological parameters:

$p_R^c = (\delta^c \mu^c)^{-1}$ and $p_R^a = (\delta^a \mu^a)^{-1}$.

Total resource production equals

$$R^{Extr} = \left(R^{Extr\,c\,\frac{\sigma-1}{\sigma}} + R^{Extr\,a\,\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \quad ,$$

From this, we derive the following theoretical predictions:

$\frac{p_R^c}{p_R^a} \;\; = \frac{(\delta^a \mu^a)}{(\delta^c \mu^c)}$

and $\frac{R^{Extr\,c}}{R^{Extr\,a}} \;\; = (\frac{(\delta^c \mu^c)}{(\delta^a \mu^a)})^{\sigma}$,

$\frac{p_R^c R^{Extr\,c}}{p_R^a R^{Extr\,a}} \;\; = (\frac{\delta^a \mu^a}{\delta^c \mu^c})^{\sigma-1}$ and $\frac{\dot{N}_R^c}{\dot{N}_R^a} \;\; = (\frac{\delta^c \mu^c}{(\delta^a \mu^a)})^{\sigma-1} (\frac{\eta_R^c}{\eta_R^a})^{\sigma}$

We can investigate what happens when a new resource gets used (e.g. aluminum was not used until the end of the XIX^{th}). If we assume that $\sigma > 1$ and that the resource is immediately at its steady-state price, the price of the resource aggregate will immediately decline and the growth rate of the economy will increase: $p_R = ((\delta_1^c \delta_2^c)^{\sigma-1} + (\delta_1^a \delta_2^a)^{\sigma-1})^{\frac{1}{1-\sigma}}$.

Alternatively, a progressive increase in aluminum technology, $\dot{N}_R^a = \eta_R^a \min\,(N_R^a/\overline{N}, 1)\, M_R^a$, would generate an initial decline in the real price (as $\eta_R^a \min\,(N_R^a/\overline{N}, 1)$ increases) and faster growth in the use of aluminum initially. This is in line with historical evidence from the copper and aluminum markets.

Appendix 4 Tables and Figures

		Aluminum	Copper	Lead	Tin	Zinc	Crude Oil	World GDP
Range		1855-2009	1821-2009	1802-2009	1792-2009	1821-2009	1861-2009	1792-2009
Constant	Coeff.	48.464	4.86	16.045	4.552	30.801	35.734	0.128
	t-stat.	*** 3.810	*** 2.694	*** 3.275	* 2.231	** 2.58	*** 4.365	0.959
Lin.Trend	Coeff.	-0.221	-0.006	-0.087	-0.016	-0.174	-0.182	0.018
	t-stat.	** -2.568	-0.439	** -2.294	-0.999	* -1.975	*** -3.334	*** 16.583
Range		1855-2009	1850-2009	1850-2009	1850-2009	1850-2009	1861-2009	1850-2009
Constant	Coeff.	48.464	5.801	6.032	3.569	5.579	25.198	0.995
	t-stat.	*** 3.810	*** 3.461	***3.371	* 2.185	*** 3.774	*** 4.81	*** 5.49
Lin.Trend	Coeff.	-0.221	-0.018	-0.038	-0.015	-0.021	-0.182	0.019
	t-stat.	** -2.568	-1.007	-1.938	-0.833	-1.308	*** -3.334	*** 9.797
Range		1900-2009	1900-2009	1900-2009	1900-2009	1900-2009	1900-2009	1900-2009
Constant	Coeff.	19.703	5.965	2.980	2.844	4.44	9.883	2.004
	t-stat.	*** 5.498	*** 2.651	* 2.043	1.361	* 2.225	*** 6.912	*** 7.8
Trend	Coeff.	-0.178	0.035	-0.019	-0.015	-0.018	-0.083	0.018
	t-stat.	*** 3.174	-0.995	-0.853	-0.464	-0.592	***-3.711	***4.549
Range		1950-2009	1950-2009	1950-2009	1950-2009	1950-2009	1950-2009	1950-2009
Constant	Coeff.	10.781	5.043	13.205	0.051	5.675	9.897	4.729
	t-stat.	*** 7.169	*** 4.979	*** 2.936	0.028	*** 4.619	*** 9.574	*** 12.89
Lin.Trend	Coeff.	-0.171	-0.057	-0.48	0.04	-0.078	-0.196	-0.028
	t-stat.	*** -3.999	-1.978	-1.553	0.768	* -2.255	*** -6.64	*** -2.724
Range		1875-1975	1875-1975	1875-1975	1875-1975	1875-1975	1875-1975	1875-1975
Constant	Coeff.	50.75	6.307	3.851	3.762	4.384	12.272	1.244
	t-stat.	*** 4.846	** 2.543	1.938	1.664	* 2.032	*** 4.060	*** 5.509
Lin.Trend	Coeff.	-0.53	-0.024	-0.018	-0.026	-0.005	-0.072	0.027
	t-stat.	*** -2.974	-0.566	-0.536	-0.66	-1.26	-1.403	***7.045

Notes: The table presents coefficients and t-statistics for regressions of the growth rates on a constant and a linear trend. ***, **, and * indicate significance at the 1%, 2.5% and 5% level, respectively.

Table 2: Tests for the stylized facts that growth rates of world primary production and world real GDP are equal to zero and trendless. As our model does not include population growth, we run the same tests for the per capita data as a robustness check. The results are roughly in line with the results described above. See table 3 on the next page.

		Aluminum	Copper	Lead	Tin	Zinc	Crude Oil	World GDP
Range		1855-2009	1821-2009	1802-2009	1792-2009	1821-2009	1861-2009	1792-2009
Constant	Coeff.	48.301	5.474	20.57	4.427	30.7	35.689	0.032
	t-stat.	*** 3.824	*** 3.06	*** 3.845	* 2.181	** 2.584	*** 4.379	0.276
Lin.Trend	Coeff.	-0.229	-0.018	-0.125	-0.023	-0.182	-0.19	0.01
	t-stat.	*** -2.677	-1.367	*** -3.025	-1.457	* -2.071	*** -3.499	*** 11.066
Range		1855-2009	1850-2009	1850-2009	1850-2009	1850-2009	1861-2009	1850-2009
Constant	Coeff.	48.301	5.399	5.629	3.179	5.18	24.681	0.628
	t-stat.	*** 3.824	*** 3.254	***3.169	1.961	*** 3.541	*** 4.733	*** 4.052
Lin.Trend	Coeff.	-0.229	-0.027	-0.047	-0.024	-0.03	-0.19	0.01
	t-stat.	*** -2.677	-1.523	** -2.442	-1.348	-1.895	*** -3.499	*** 5.876
Range		1900-2009	1900-2009	1900-2009	1900-2009	1900-2009	1900-2009	1900-2009
Constant	Coeff.	18.595	4.985	2.028	1.903	3.473	8.869	1.071
	t-stat.	*** 5.242	* 2.241	1.41	0.918	1.763	*** 6.306	*** 4.862
Trend	Coeff.	-0.184	-0.042	-0.027	-0.023	-0.026	-0.09	0.01
	t-stat.	*** -3.315	-1.214	-1.186	-0.694	-0.404	*** -4.084	*** 3.01
Range		1950-2009	1950-2009	1950-2009	1950-2009	1950-2009	1950-2009	1950-2009
Constant	Coeff.	8.583	2.952	1.141	-1.954	3.578	7.716	2.632
	t-stat.	*** 5.742	*** 2.892	1.04	1.086	*** 2.87	*** 7.493	*** 7.444
Lin.Trend	Coeff.	-0.156	-0.044	-0.35	0.051	-0.065	-0.18	-0.016
	t-stat.	*** -3.667	-1.515	-1.129	0.997	-1.819	*** -6.14	-1.551
Range		1875-1975	1875-1975	1875-1975	1875-1975	1875-1975	1875-1975	1875-1975
Constant	Coeff.	50.004	5.854	3.413	3.317	3.942	11.789	0.834
	t-stat.	*** 4.81	** 2.386	1.738	1.480	1.851	*** 3.933	*** 4.509
Lin.Trend	Coeff.	-0.542	-0.038	-0.032	-0.039	-0.019	-0.086	0.013
	t-stat.	*** -3.06	-0.908	-0.959	-1.028	-0.517	-1.691	***4.004

Notes: The table presents coefficients and t-statistics for regressions of the growth rates on a constant and a linear trend. ***, **, and * indicate significance at the 1%, 2.5% and 5% level, respectively.

Table 3: Tests for the stylized fact that growth rates of world per capita mine production and world per capita real GDP are equal to zero and trendless.

		Aluminum	Copper	Lead	Tin	Zinc	Crude Oil
Range		1905-2009	1792-2009	1792-2009	1792-2009	1824-2009	1862-2009
Constant	Coeff.	-1.774	0.572	0.150	1.800	1.072	8.242
	t-stat.	(-0.180)	(0.203)	(0.052)	(0.660)	(0.205)	(0.828)
Lin.Trend	Coeff.	0.008	0.009	0.016	0.001	0.014	-0.021
	t-stat.	(0.137)	(0.428)	(0.714)	(0.069)	(0.357)	(-0.317)
Range		1905-2009	1850-2009	1850-2009	1862-2009	1850-2009	1850-2009
Constant	Coeff.	-1.299	0.109	-0.268	2.439	1.894	7.002
	t-stat.	(-0.200)	(0.030)	(-0.073)	(0.711)	(0.407)	(1.112)
Lin.Trend	Coeff.	0.008	0.020	0.030	-0.004	0.013	-0.021
	t-stat.	(0.137)	(0.518)	(0.755)	(-0.109)	(0.267)	(-0.317)
Range		1900-2009	1900-2009	1900-2009	1900-2009	1900-2009	1900-2009
Constant	Coeff.	-0.903	-1.428	-0.490	1.068	2.764	-1.974
	t-stat.	(-0.239)	(-0.332)	(-0.102)	(0.269)	(0.443)	(-0.338)
Lin.Trend	Coeff.	0.008	0.055	0.054	0.010	0.010	0.100
	t-stat.	(0.137)	(0.820)	(0.713)	(0.168)	(0.099)	(1.106)
Range		1950-2009	1950-2009	1950-2009	1950-2009	1950-2009	1950-2009
Constant	Coeff.	2.269	1.556	-3.688	-0.061	-0.515	3.445
	t-stat.	(0.479)	(0.240)	(-0.505)	(-0.011)	(-0.062)	(0.354)
Lin.Trend	Coeff.	-0.055	0.041	0.198	0.049	0.103	0.090
	t-stat.	(-0.411)	(0.225)	(0.958)	(0.307)	(0.441)	(0.326)
Range		1875-1975	1875-1975	1875-1975	1875-1975	1875-1975	1875-1975
Constant	Coeff.	-0.549	1.323	0.370	3.719	1.136	-1.111
	t-stat.	(-0.088)	(0.266)	(0.081)	(0.812)	(0.176)	(-0.176)
Lin.Trend	Coeff.	-0.003	0.011	0.030	-0.012	0.051	0.094
	t-stat.	(-0.033)	(0.135)	(0.383)	(-0.152)	(0.468)	(0.875)

Notes: The table presents coefficients and *t*-statistics for regressions of the growth rates on a constant and a linear trend.***, **, and * indicate significance at the 1%, 2.5% and 5% level, respectively.

Table 4: Tests of the stylized fact that the growth rates of real prices of mineral commodities equal zero and do hence not follow a statistically significant trend.

	Crustal Abundance (Bil. mt)	Reserves (Bil. mt)	Annual Output (Bil. mt)	Crustal Abundance/ Annual Output (Years)	Reserves/ Annual Output (Years)
Aluminum	1,990,000,000[e]	30[b1]	0.06[a]	491[g]	42[f1]
Copper	1,510,000[e]	0.8[b]	0.02[b]	483	26[f]
Iron	1,392,000,000[e]	83[b2]	1.2[a]	580[g]	39[f2]
Lead	290,000[e]	0.1[b]	0.005[b]	1,099	16[f]
Tin	40,000[e]	0.005[b]	0.0003[b]	1,405	14[f]
Zinc	2,250,000[e]	0.23[b]	0.013[b]	668	14[f]
Gold	70[e]	0.00005[b]	0.000003[b]	925	15[f]
Coal[3]		511[d]	3.9[d]		63[c]
Crude Oil[4]	15,000,000[h]	241[d]	4.4[d]	558	41[c]
Nat. Gas[5]		179[d]	3.3[d]		34[c]

Notes: We have used the following average annual growth rates of production from 1990 to 2010: Aluminum: 2.5%, Iron: 2.3%, Copper: 2%, Lead: 0.7%, Tin: 0.4%, Zinc: 1.6%, Gold: 0.6%, Crude oil: 0.7%, Natural gas: 1.7%, Coal: 1.9%, Hydrocarbons: 1.4%.
Aluminum's 'crustal abundance/annual output' statistic uses aluminum while its 'reserves/annual output' uses bauxite. [1]Data for bauxite, [2]data for iron ore, [3]includes lignite and hard coal, [4]includes conventional and unconventional oil, [5]includes conventional and unconventional gas, [6]all organic carbon in the earth's crust. Sources: [a]U.S. Geological Survey (2016), [b]U.S. Geological Survey (2018), [c]British Petroleum (2017), [d]Federal Institute for Geosciences and Natural Resources (2017), [e]Perman et al. (2003), [f]U.S. Geological Survey (1992), [g]U.S. Geological Survey (1992), [h]Littke and Welte (1992).

Table 5: Availability of selected non-renewable resources in years of production left in the reserve and crustal mass based on an exponentially increasing annual mine production (based on the average growth rate over the last 20 years).

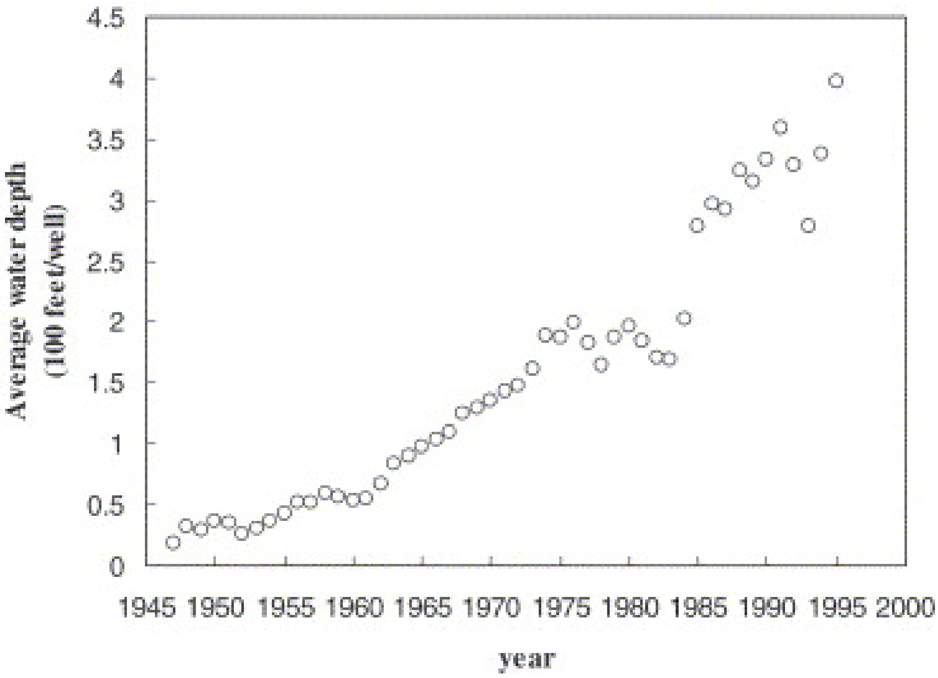

Figure 12: Average water depth of wells drilled in the Gulf of Mexico. Source: Managi et al. (2004).

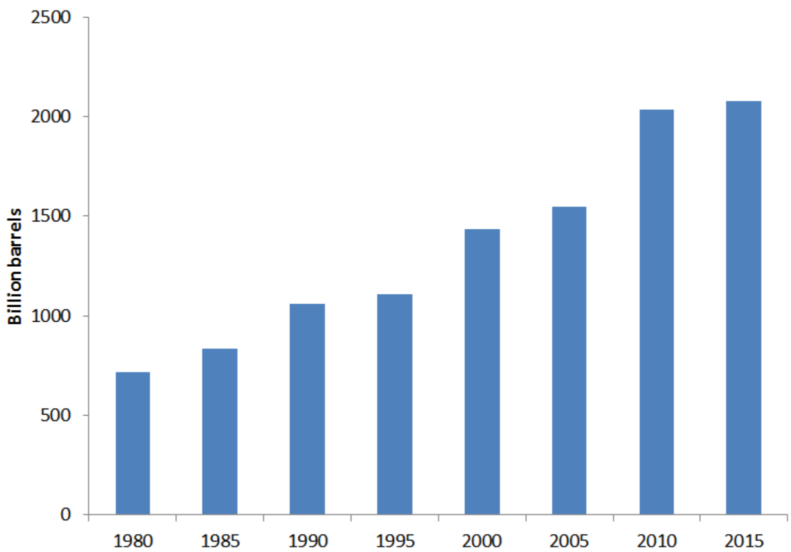

Figure 13: Historical evolution of oil reserves, including Canadian oil sands from 1980 to 2015. Source: BP, 2017.