Liquidity Fluctuations in Over the Counter Markets*

Vincent Maurin†

Job Market Paper

This Draft: November 2015

[Most recent version]

Abstract

This paper shows that lemon markets exhibit liquidity fluctuations whereby the ease to sell an asset varies endogenously over time. In the model, agents meet in a decentralized market and bargain under asymmetry of information about the quality of the asset. Liquidity increases with the average quality of the pool of sellers but the composition of the pool responds negatively to past liquidity. When this effect is strong, cyclical equilibria arise where prices and volume of trade oscillate without aggregate shocks. These fluctuations are generally inefficient and call for policy interventions. When the economy is in a cycle, a revertible asset purchase program can jump-start the market and smooth out fluctuations. Finally, I show that increasing transparency in the Over The Counter market harms liquidity provision and may be undesirable.

Keywords : Adverse Selection, Over-The Counter, Cycles, Market Structure.

JEL Codes : D47, D82, G01.

*I am indebted to my supervisor Piero Gottardi for his invaluable help and support. I would also like to thank Andrea Attar, Alp Atakan, Francis Bloch, Rahul Deb, Vitor Farinha Luz, Manolis Galenianos, Andrea Galeotti, Sergei Kovbasyuk, Pablo Kurlat, Ben Lester, Konrad Mierendorff, Humberto Moreira, Artem Neklyudov, Dan Quint, Jean Tirole, Venky Venkateswaran, Pierre-Olivier Weill, Ariel Zetlin-Jones for useful discussions as well as the audience of the EUI Microeconomics Working Group for their comments.

†European University Institute. Email: vincent.maurin@eui.eu

1 Introduction

The recent 2007 crisis started with a widespread shortage of liquidity in the financial system after a prolonged boom. Difficulties to sell or finance securities on secondary markets triggered a collapse in the issuance of many assets, impacting credit and ultimately the real economy. The severity of the bust prompted a two stage response from policy-makers acting both as market participants and market designers. During the crisis, the US Fed provided credit lines and purchased assets such as Mortgage Backed Securities to prop up trading and liquidity. In Europe, fears of government losses or uncertainty regarding their optimal design sometimes delayed the implementation of these programs[1]. In a second ongoing phase, regulators have started to overhaul various segments of financial markets. For instance, the Market in Financial Instruments Directive[2] requires *"all standardised derivatives to be traded on organized and transparent venues"*. For many assets, transactions indeed take place Over The Counter (OTC) where trading frictions and opacity may cause illiquidity and unstability[3]. Increased transparency and competition are generally seen as desirable features of centralized platforms.

In this paper, I propose a theory based on asymmetry of information to explain why liquid OTC markets can become illiquid. Endogenous variations in the supply of high quality assets generate price and volume swings. I use the model to study an asset purchase program designed to revive market liquidity in bad times. I show that large interventions might be too costly and that tight government budget constraints dampen the effect of the policy. Finally, in the presence of asymmetry of information, limited competition can actually foster liquidity. The introduction of a more transparent market structure, in line with the target of current reforms, might thus decrease welfare.

In the model, agents have different valuations for a long-term asset and thus gain from trade. Preferences switch over time so that buyers may ultimately need to resell an asset previously acquired. In the OTC market, agents match bilaterally and the buyer makes a Take It Or Leave It offer. There are two qualities of the asset which either pays a high or a low dividend. The key friction is that only the seller knows the quality of the asset he holds as well as his own valuation. Hence, trade may fail to occur because of adverse selection as in Akerlof (1970). A pooling price offer to attract high quality sellers entails

[1] "*Too little, too late*" was the financial press widespread reception to the ECB 60 billion-a-month bond buying plan announced in January 2015.

[2] MIFID II, Regulation 600/2014 of the European Parliament. In the US, Title VII of the Dodd Frank in the US contains similar provisions.

[3] On its website, the IMF referring to OTC markets explains that *"some types of market arrangements can very quickly become disorderly, dysfunctional, or otherwise unstable"*

losses on low quality assets. When the share of high quality sellers is too low, a buyer thus reduces his bid to trade only with low quality sellers. We then call market liquidity the ease to sell a good quality asset. Liquidity can fluctuate because both the value of a lemon to a buyer and the composition of the pool of sellers are endogenous. First, high future prices raise the resale value of a lemon. This increases buyers' willingness to pay today for an asset of unknown quality. Hence high future liquidity begets present liquidity since then, buyers are more likely to offer a price at which high quality assets trade. Dynamics can be more intricate, however, since the composition of the pool of sellers responds negatively to market liquidity. If the market was liquid in the past, high quality assets are in the hands of high valuation agents who do not wish to sell. Buyers thus mostly meet low quality sellers who want to flip their lemon independently of their valuation. A low price offer becomes more profitable, so that liquidity can generate illiquidity. This *composition effect* also leaves room for recovery. If the market is illiquid, selling pressure from high quality asset owners accumulates over time as high valuation agents switch to low valuation. Buyers meet increasingly more good quality sellers and, after some time, may offer a pooling price.

When the discount factor and the probability of switching type are low, the *composition effect* is strong and equilibrium cycles exist in the absence of any aggregate shock. For $T \geq 2$, a T period cycle consists of $T - 1$ trough periods where only lemons trade and 1 peak period where both qualities trade. The market price for the low quality asset increases during the trough to reflect high offers at the next peak. Indeed, traders know that the accumulation of selling pressure for high quality assets will lead future buyers to offer a pooling price. To the best of my knowledge, this paper is the first to characterize cycles in this environment. Foucault et al. (2013) describes "'make-take" liquidity cycles in electronic markets which share many features with the dynamics in my model[4]. The model also sheds light on boom and bust episodes commonly associated to financial crises. During the trough of the cycle, investors exhibit *speculative behavior* in the words of Harrison and Kreps (1978) as they know they buy a lemon but pay an increasing premium that captures future resale gains. The low quality asset appears like a hot potato that agents pass to the next investor in line. Since the pool quality may take time to reach the peak whereas a pooling offer immediately clears the market, the model can rationalize slow build-ups followed by fast crashes.

[4]The sharp fall and progressive build-up of liquidity after a period with high volume and high price also evokes Duffie (2010)'s account of price movements after good news. In my model, at the peak of a cycle, there is "good news" about the quality of assets for sale as the pool contains many high quality assets. Liquidity then falls because of the composition effect.

Cycles emerge for an intermediate share of high quality assets in the economy. In that region, there also exists a steady state equilibrium in mixed strategy where buyers randomize between the pooling and the separating price. Intuitively, fundamentals are neither favorable enough for the market to be fully liquid nor so bad that good quality assets never trade. Partial illiquidity materializes either through buyers' randomization in a steady state or through cyclical dynamics. I show that fluctuations in a cycle entail a surplus loss with respect to the steady state. There is too much trade at the peak of the cycle and too little at the bottom. Surplus would improve by propping up (resp. taming) liquidity in the trough (resp. at the peak).

Illiquid and unstable markets call for policy interventions. I study an asset purchase program by a benevolent government who is bound to resell the assets purchased. The combination of an asset purchase program together with a resale constraint fits the description of many policies implemented during the financial crisis[5]. In my model, the resale constraint proceeds from a natural assumption that the government values assets less than private agents[6]. In addition, a revertible intervention does not affect the fundamentals of the economy in the long run. I show that this asset purchase program can still increase surplus when the economy is in a cycle. Buying lemons jump-starts the OTC market as it improves the average asset quality private buyers face in the trough. Reselling lemons at the peak may then reduce prices and trading volume in line with steady state levels. When the objective is to maximize aggregate surplus, the government weighs the benefits from jump-starting and then stabilizing the market with the asset holdings costs. My numerical analysis shows that achieving the first objective sometimes requires buying too many assets and the intervention becomes undesirable.

I show that the program is not self-financing. Although he eventually resells assets, the government runs a loss. Indeed, he enjoys a lower utility for the dividends of the asset but also pays a premium to induce participation in the program. As we observed, buying lemons increases liquidity. However, better conditions in the OTC market raise the outside (market) option of lemon holders and in turn the price the government must pay. To finance the shortfall, I allow the government to tax transactions in the resale period. Then, I show that Pareto improvements are possible but budget-neutral inter-

[5]In 2014, the Federal Reserve trimmed down his Mortgage Backed Security Purchase Program. In recent years, The British government has sold back shares of private banks acquired in 2009.

[6]Intuitively, a public entity does not value the potential services (borrowing, hedging..) attached to holding assets. Asset purchase programs would then generate some misallocation. To quote Singh (2013), *"some central banks purchases of good collateral have contributed to shrinkage in the pledged collateral market"*

ventions need be smaller. Indeed, while flattening fluctuations raises surplus, riding the liquidity cycle relaxes the government budget constraint in two ways. Maintaining high liquidity in the resale period reduces the capital loss of the government who can quote a high resale price. It also increases the tax base to make up for this loss.

Finally, Section 5 introduces a more transparent trading infrastructure called Exchange. Buyers now post and commit to terms of trade prior to meeting a counterparty. Sellers thus observe prices posted by all buyers. With bilateral matching, they may face a different level of rationing for each price. I show that the resulting increase in competition in the Exchange lowers liquidity and may decrease welfare. On the upside, price posting economizes on search costs. Indeed buyers and sellers can coordinate on a sub-market and the former compete directly in price. However, the availability of multiple offers induces sellers to try and signal their asset quality. High quality sellers should thus choose higher prices where they face a lower probability of trading. I show that in equilibrium, high quality assets do not trade at all. Both dimensions of private information - valuation and asset quality - matter for this extreme result. The key ingredient is the presence of high valuation lemon owners who have no gains from trade with buyers - a "no gap at the middle condition". These traders block transactions of high quality assets as they would otherwise mimic high quality sellers. Hence, competition generated by a transparent centralized market may harm liquidity provision. In the opaque OTC market, local monopsony power protects buyers from competition, allowing for a pooling outcome.

Aggregate surplus is higher in the OTC market than in the Exchange when the share of high quality assets is sufficiently large. In that region, the realized gains from trade on high quality assets overcome the inefficiencies attached to random search and bargaining. This comparison suggests that, to some extent, opaqueness of the asset traded and the trading structure are complement. In the presence of asymmetry of information about the asset value, a transparent and competitive exchange is not necessarily desirable.

Relation to the literature

A strand of literature has identified self-fulfilling expectations as a mechanism for cycles and chaotic dynamics in the absence of aggregate shocks. Boldrin and Woodford (1990) provides a survey of early endogenous business cycle models. More recently, a series of works including Gu et al. (2013) or Rocheteau and Wright (2013) highlighted the contribution of credit constraints in generating such dynamics. In my model, cycles rather hinge on variations in the composition of the pool of sellers - a backward-looking variable -, and equilibrium multiplicity is not as severe.

This paper relates more closely to the growing literature on dynamic markets with asymmetric information. My contribution is to show that this environment is prone to liquidity fluctuations. Recent works (e.g. Deneckere and Liang, 2006, Camargo and Lester, 2014, Moreno and Wooders, 2013) have emphasized that, as a screening device, trading delay is tantamount to rationing in a static environment. Thus, lemon markets eventually thaw endogenously or thanks to the arrival of news as in Daley and Green (2012). These dynamics leading to separation of sellers are no longer present with retrade. My paper is thus closer to Chiu and Koeppl (2014) since there also, the lemon problem does not vanish over time[7]. Monopsonist buyers also offer terms of trade after matching and may pool sellers if chances to obtain a good quality asset are high. However, their model does not disentangle the preference switching process from the trading process. This natural feature is an important element to identify cycles. In addition, as I discuss later, my revertible policy differs from their permanent asset purchase program. Finally, their paper does not discuss the role of the market structure or transparency.

I compare indeed the OTC equilibrium to that of an Exchange where agents post and commit to terms of trade before meeting in a directed search environment. There, as in Guerrieri and Shimer (2014) and Chang (2014), building on Guerrieri et al. (2010) and the pioneering work of Gale (1996), separation obtains through rationing at different prices[8]. As a difference with these works, I show that the possibility to resell assets with two dimensions of private information exacerbates market illiquidity. High valuation owners who try to flip their lemon form a middle type with whom buyers do not gain from trade. Adverse selection is thus more severe and high quality assets do not trade. As a consequence, liquidity is lower than in the OTC market when the equilibrium is pooling in that market. In a common value environment, Hörner and Vieille (2009) and Fuchs and Skrzypacz (2015) showed that pre-trade information may come at the expense of liquidity. If buyers can observe the offers a seller rejected, the latter can use this information as a signal. In my model of the Exchange, it is rather the possibility to observe current terms of trade and not inconclusive past offers that leads to separation.

Although the Exchange improves trading efficiency for low quality assets, market centralization and transparency can thus reduce aggregate surplus. In the seminal search model of Duffie et al. (2005), trading takes place under symmetric information and only

[7]Hellwig and Zhang (2013) add endogenous information acquisition to this framework and show that equilibria with different degree of adverse selection and liquidity can coexist. In my model, liquidity varies over time in a given equilibrium

[8]Exclusivity for sellers is crucial to generate separation in this environment. Kurlat (2015) relaxes this assumption and obtains a pooling outcome. See Wilson (1980) for a useful discussion on this issue.

the first effect is present. There, the centralized benchmark unambiguously dominates the market with frictions. My result complements the findings of Malamud and Rostek (2014) who highlight the surprising role of market power as a potential force against centralization. In their model, the market structure is also exogenous. Michelacci and Suarez (2006) or Bolton et al. (2014) endogenize traders' choice with symmetric information in the OTC market, while I maintain asymmetric information in both platforms.

Finally, the government purchase program for lemon markets I study shares many features with the policy experiment of Philippon and Skreta (2012) Tirole (2012) or Chiu and Koeppl (2014). Unlike these papers and some mentioned above, I impose a realistic constraint on the government to revert the policy, that is to resell the assets purchased. Although the intervention may not change the fundamentals of the economy in the long run, it can prop up and then stabilize liquidity in a cycle. If implemented in steady state, this program would have at best no effect on aggregate surplus. As in Fuchs and Skrzypacz (2015) or Faria-e-Castro et al. (2015) with a different focus, I discuss the interaction between designing and funding the policy. To generate a high taxing profit in the resale period, the government somewhat rides the liquidity cycle and the intervention does not flatten fluctuations as much. The government then leans against the wind by buying low and selling high, like the market-maker of Weill (2007).

The rest of the paper is organized as follows. Section 2 introduces the model. In Section 3, I describe the main dynamic effects and solve for stationary equilibria including cycles. Section 4 discusses the welfare implications of liquidity fluctuations and studies an asset purchase program aimed at restoring liquidity. Section 5 analyzes a market structure change by allowing agents to post prices before meetings. Finally, Section 6 concludes. Proofs are in Appendix B.

2 The Model

2.1 Environment

Time is discrete and runs forever $t = 0, 1, .., \infty$. The economy is populated by a large continuum of infinitely-lived agents with discount factor $\delta < 1$. They consume a non-storable numeraire good c and dividends d from assets. Agents can have either low ($i = 1$) or high valuation ($i = 2$) for the dividends with the following instantaneous preferences:

$$u^i(c, d) = c + \tau^i d$$

where $1 = \tau^1 < \tau^2 = \tau$. Agents with a higher private valuation like the (dividends of the) asset more. Valuation is persistent but may switch from one period to the next with probability $\gamma \in (0, 1/2)$. This Markov Process is identical and independently distributed across agents. The valuation of an agent is private information[9]. Agents are endowed with e units of the numeraire good every period. There is an infinitely lived asset in fixed supply S with two varieties denoted H (High) and L (Low) with share q and $1-q$ respectively. Variety L pays dividend $d_L > 0$ in every period while variety H pays dividend $d_H > d_L$. The variety or quality is private information to the current holder of the asset. The asset is indivisible and agents may hold either zero or one unit[10].

Asset owners enter date $t \geq 1$ carrying their holdings from date $t-1$. At the beginning of the period, valuations can switch and agents may wish to trade. Non asset owners must pay a cost $\kappa > 0$ to enter a decentralized market where they offer terms of trade to asset owners. Section 2.2 describes the market structure in detail. The key friction is the asymmetry of information between buyers and sellers. At the end of the period, dividends pay off and buyers discover the quality of the asset purchased if any. The economy then moves on to period $t+1$.

With two valuations and three possible asset holding status, there are effectively six types of agents in the economy. Let thus (τ^i, a) denote an agent with private valuation $\tau^i \in \{\tau^1, \tau^2\}$ and asset holding $a \in \{0, L, H\}$ where by convention $a = 0$ means no asset. Importantly, there are two dimensions of private information for an asset owner: valuation and asset quality. Let now $\mu_a^i(t)$ be the mass of of (τ^i, a) agents in period t after the type switch but before the market opens. With this notation, $\{\mu_a^i(0)\}_{a=0,L,H}^{i=1,2}$ is the initial distribution of asset holdings. These quantities verify the following balance equations:

$$\begin{cases} \mu_H^1(t) + \mu_H^2(t) = Sq \\ \mu_L^1(t) + \mu_L^2(t) = S(1-q) \end{cases} \tag{1}$$

The total supply of each variety of the asset must match the total holdings of this variety across the population in any period t. I now impose a series of assumptions on the

[9] Private valuation may capture different services attached to holding an asset (hedging, collateral). Since their type switch agents with high valuations may need to resell an asset purchased as it is usual in secondary markets.

[10] I make this assumption for tractability. The indivisibility comes without loss of generality as buyers may offer contracts with probabilities of trade.

parameters of the model. The main restriction disciplines the degree of adverse selection:

$$\tau d_L < d_H \tag{LC}$$

I will refer to (LC) as the lemon condition. In a static environment, the value of a L asset to a type τ^2 agent lies below the value of a H asset to a type τ^1 agent. As I will show, this assumption translates into a monotonicity condition on asset owner types in the dynamic model. The following assumptions are technical:

$$\frac{\tau d_H}{1-\delta} \leq e \tag{A1}$$

$$\kappa \leq \bar{\kappa}(\gamma, \delta, \tau, d_L, d_H), \tag{A2}$$

Condition (A1) ensures that trade does not fail for lack of funds. Observe indeed that the left hand side is the present discounted value of asset H when held in every period by a high valuation agent. Condition (A2) guarantees that search costs are small enough to preserve gains from trade and non-owners find it optimal to search. The expression of $\bar{\kappa}(\gamma, \delta, \tau, d_L, d_H)$ is in Appendix A. The important observation is that $\bar{\kappa}$ does not depend on the share of H assets q.

2.2 The OTC market

Until Section 5 with price posting, I consider a market structure with random search and ex-post offers, called Over The Counter or OTC. Trading is decentralized. A non-owner must pay the search cost $\kappa > 0$ to match with at most one asset owner. In a match, the non-owner makes a Take It Or Leave It (TIOLI) offer[11] to the asset owner. I first describe the matching and bargaining stage in a given period $t \geq 0$ and then turn to the model dynamics. Definition 2 then introduces the concept of the OTC equilibrium.

Matching

Asset owners with total mass S simply wait for a match. The mass of active non-owners or buyers in period t, $\mu^B(t)$ results from an entry decision detailed later. The matching function is of the Leontieff type. Precisely, the probability $\lambda^S(t)$ (resp. $\lambda^B(t)$) for a seller

[11] We follow most of the literature on bargaining with common value in giving the bargaining power to the uninformed party. The search costs matters primarily to compare the OTC and the Exchange structure in a meaningful way. In equilibrium, buyers will indeed always make zero profit. Since they do not compete simultaneously in price in the OTC market, this would not be possible without the cost.

to meet a buyer (resp. for a buyer to meet a seller) in period t is:

$$\lambda^S(t) = \min\left\{\frac{\mu^B(t)}{S}, 1\right\}, \quad \lambda^B(t) = \min\left\{\frac{S}{\mu^B(t)}, 1\right\}$$

Search frictions are minimal because the short side of the market finds a counterparty for sure[12]. Search is random so that a matched buyer meets an owner with type (τ^i, a) with probability $\mu_a^i(t)/S$ where $i \in \{1,2\}$ and $a \in \{L, H\}$. This is the fraction of that type in the population of asset owners. Private information and randomness generate inefficiencies because agents with no gains from trade may meet.

Stage Bargaining Game

In a match, the buyer does not know the quality of the asset held by the seller. He offers a price[13] to trade that the seller may accept or refuse. Formally, a strategy for buyer $(\tau^k, 0)$ in period t is a distribution $\Pi^k(t, .)$ over the real line for $k \in \{1,2\}$. We let $Supp(\Pi^k(t, .))$ denote the support of this distribution. A strategy for owner (τ^i, a) in period t is a probability $\alpha_a^i(t,p) \in [0,1]$ to accept offer p. To introduce the primitives of the bargaining game, let $\{v_a^i(t)\}_{a=0,L,H}^{i=1,2}$ be the value functions in period t before the market opens, net of the value of the endowment stream. Then:

$$\bar{v}_a^i(t+1) := (1-\gamma)v_a^i(t+1) + \gamma v_a^j(t+1), \quad a \in \{0, L, H\}, i \in \{1,2\}, j \neq i$$

is agent (τ^i, a) expected future utility from holding asset a, given that he may switch valuation. We can now write down payoffs in the bargaining game. When offered p, owner with type (τ^i, a) solves:

$$\max_{\alpha_a^i(t,p)} \alpha_a^i(t,p)\left[p + \delta\bar{v}_0^i(t+1)\right] + (1-\alpha_a^i(t,p))\left[\tau^i d_a + \delta\bar{v}_a^i(t+1)\right] \tag{2}$$

If he accepts, a seller obtains the price p and the future utility from being a non owner $\delta\bar{v}_0^i(t+1)$. If he refuses the offer, he enjoys the dividend $\tau^i d_a$ and obtains next period

[12] I make this assumption for tractability. Most of the search models set in continuous time a la Duffie et al. (2005) use a purely random search technology but specific functional forms as well. In discrete time however, Guerrieri and Shimer (2014) use a similar matching function.

[13] We can extend the set of possible offers to contracts formed by a price and a probability to trade. An offer would thus be a menu of such contracts. It can be shown that buyers do not use this extra dimension to screen sellers, that is all proposed contract have probability of trade equal to 1. The result is from Samuelson (1984).

expected utility from the asset $\delta\bar{v}_a^i(t+1)$. For buyer $(\tau^k, 0)$, price p^* is optimal if:

$$p^* \in \arg\max_p \left\{ \sum_{\substack{i=1,2 \\ a=L,H}} \frac{\mu_a^i(t)}{S} \Big(\alpha_a^i(t,p) \left[\tau^k d_a + \delta\bar{v}_a^k(t+1) - p \right] + (1-\alpha_a^i(t,p))\delta\bar{v}_0^k(t+1) \Big) \right\} \tag{3}$$

Under asymmetry of information, the buyer forms expectations over the asset owner type he matched with. When offering p, he obtains the asset with probability $\alpha_a^i(t,p)$ if he meets seller (τ^i, a). He then enjoys the current dividend from the asset $\tau^k d_a$ and its future value $\delta\bar{v}_a^k(t+1)$ minus the price he pays p. Otherwise, the buyer goes on to the next period where his utility is $\bar{v}_0^k(t+1)$. We can now introduce the solution concept for the stage bargaining game.

Definition 1 (Bargaining Equilibrium)**.** *For value functions $\{v_a^i(t+1)\}_{a=0,L,H}^{i=1,2}$, the bargaining equilibrium of period t is given by probabilities $\{\alpha_a^i(t,p)\}_{a=L,H}^{i=1,2}$ and distribution $\{\Pi^k(t,.)\}^{k=1,2}$ such that*

1. *Probability $\alpha_a^i(t,p)$ solves seller's problem (2) for any $p \in \mathbb{R}$.*

2. *A buyer offers p, that is $p \in Supp(\Pi^k(t,.))$ if p solves (3).*

Subgame perfection follows from the requirement that sellers reply optimally to any price, including out of equilibrium offers. Although the primitives of the game are ultimately endogenous, we may partially characterize the bargaining equilibrium of period t, using the reservation values for each type of asset owner. Define:

$$r_a^i(t) := \tau^i d_a + \delta(\bar{v}_a^i(t+1) - \bar{v}_0^i(t+1)), \quad i = 1,2, \quad a = L, H \tag{4}$$

In words, $r_a^i(t)$ is the net value attached to holding asset a for agent i in period t over not owning an asset. The label reservation value comes from the seller problem (2). Indeed, an asset owner (τ^i, a) would never accept an offer below $r_a^i(t)$. Reservation values are thus inversely related to the eagerness to sell the asset which is the relevant statistic for each type of asset owner. The following Lemma simplifies the description of the bargaining equilibrium, anticipating on Definition 2:

Lemma 1. *In any OTC equilibrium (Definition 2), the following statements hold:*

1. *Type ranking:*
$$r_L^1(t) < r_L^2(t) < r_H^1(t) < r_H^2(t) \tag{5}$$

2. *Only type* $(\tau^2, 0)$ *search and* $Supp(\Pi^2(t,.)) \in \{r_L^1(t), r_H^1(t)\}$.
Let $\pi(t) := \Pi^2(t, r_H^1(t)) - \Pi^2(t, r_L^1(t))$ *be the probability of a pooling offer* $r_H^1(t)$.

To prove this Lemma, we anticipate on equilibrium definition 2 and use the free entry condition. Buyers make zero profit which simplifies the expression of (4) for the reservation values[14]. Agents with low valuation or low asset quality accept a lower price to sell. In particular, Assumption (LC) implies that $r_L^2(t) < r_H^1(t)$. Even high valuation owners of lemons are more eager to sell than low valuation owners of H assets. From (5), it is clear that $(\tau^1, 0)$ non-owners do not gain from trade with any asset owner type. Hence they do not pay the cost κ to search. A buyer is thus an agent of type τ^2. Buyers only target low valuation asset owners with whom they have gains from trade. From (5) however, an offer addressed to (τ^1, H) owners attracts all L asset owners. In the following, we refer to the probability of a pooling offer $\pi(t)$ as market liquidity. Indeed it measures the probability that a (τ^1, H) asset owner sells his asset. Building on Lemma 1, the buyer's problem boils down to a binary choice between a separating price offer $r_L^1(t)$ and a pooling offer $r_H^1(t) > r_L^1(t)$. Let us write his profit $v^B(t, \pi)$ from randomization π in period t:

$$\begin{aligned} v^B(t,\pi) := \frac{1}{S}\Bigg\{ & \pi\Big[\mu_H^1(t)(r_H^2(t) - r_H^1(t)) + S(1-q)(r_L^2(t) - r_H^1(t))\Big] \\ & + (1-\pi)\mu_L^1(t)(r_L^2(t) - r_L^1(t))\Bigg\} + \delta\bar{v}_0^2(t+1) \end{aligned} \tag{6}$$

Equation (6) captures the standard rent-efficiency trade-off. The pooling offer $r_H^1(t)$ (weight π) attracts high quality assets but generates losses $r_H^1 - r_L^2(t)$ on low quality assets. With a separating offer $r_L^1(t)$ (weight $1-\pi$), buyers forgo gains from trade on the H asset to extract rents $r_L^2(t) - r_L^1(t)$ from (τ^1, L) sellers. Illiquidity materializes in this case. For simplicity, we let $v^B(t)$ denote the value of $v^B(t, \pi)$ at the optimum, which is the utility of a matched buyer.

[14] In Chiu and Koeppl (2014), this is not an issue because the only way a type τ^1 asset owner switches back to type τ^2 is by selling his asset. Observe that the difficulty also vanishes in the case where types are iid across time, that is $\gamma = 1/2$ since then $\bar{v}_a^1(t) = \bar{v}_a^2(t)$ for $a \in \{0, L, H\}$

Buyers entry

Non-owners decide whether to search for an asset, given the matching probability $\lambda^B(t)$ and the outcome of the bargaining game $v^B(t)$. Non-owner $(\tau^2, 0)$ obtains a net payoff equal to $-\kappa + \lambda^B(t)v^B(t) + (1-\lambda^B(t))\delta\bar{v}_0^2(t+1)$ from searching. Otherwise, he goes on to the next period with utility $\delta\bar{v}_0^2(t+1)$. The equilibrium mass of buyers derives from the optimal search choice of non-owners:

$$\mu^B(t) = \begin{cases} 0 & \text{if } -\kappa + \left[v^B(t) - \delta\bar{v}_0^2(t+1)\right] < 0 \\ \frac{S(v^B(t)-\delta\bar{v}_0^2(t+1))}{\kappa} & \text{otherwise} \end{cases} \tag{7}$$

With assumption (A2) on the search cost κ, the last case will prevail in equilibrium, that is non-owners do enter as buyers.

Dynamics

An agent valuation for an asset depends both on its reservation value and the price he may obtain for the asset in the OTC market. Precisely, for $i \in \{1, 2\}$,

$$v_H^i(t) = \tau^i d_H + \delta\bar{v}_H^i(t+1) \tag{8}$$

$$v_L^i(t) = \tau^i d_L + \delta\bar{v}_L^i(t+1) + \lambda^S(t)\pi(t)[r_H^1(t) - r_L^i(t)] \tag{9}$$

Equation (8) shows that H asset owners are at most indifferent between trading today and waiting next period. Indeed, they never receive an offer above their reservation value. Low quality asset owners earn information rents when matched - the second term in (9). These are proportional to the probability $\pi(t)$ of a pooling offer and the difference between the pooling price $r_H^1(t)$ and the reservation value $r_L^i(t)$. For non asset owners, we obtain:

$$v_0^1(t) = \delta\bar{v}_0^1(t+1) \tag{10}$$

$$v_0^2(t) = \max\left\{0, -\kappa + \lambda^B(t)\left[v^B(t) - \delta\bar{v}_0^2(t+1)\right]\right\} + \delta\bar{v}_0^2(t+1) \tag{11}$$

Non-owners $(\tau^1, 0)$ might only become active buyers if they switch type. Non-owners $(\tau^2, 0)$ decide whether to search and become buyers.

Finally, we need to characterize the evolution of asset ownership across time. For a given asset, the owner type might change because the original owner sold the asset or switched valuation. Figure 1 describes these dynamics for the H asset. Consider the $\mu_H^1(t)$ assets initially held by (τ^1, H) agents at the beginning of period t. Some agents fail

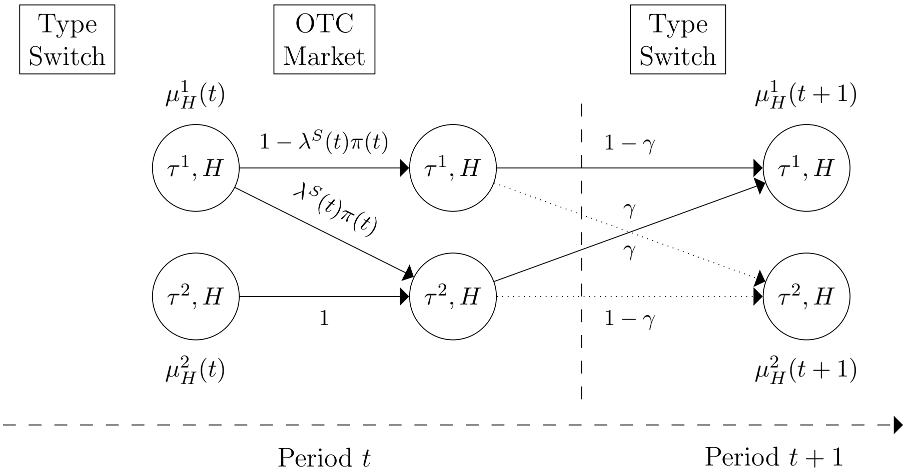


Figure 1: Law of motion for the H asset

to find a match with probability $1 - \lambda^S(t)$ or do not trade in a match with probability $\lambda^S(t)(1 - \pi(t))$. Overall, a fraction $1 - \lambda^S(t)\pi(t)$ of these assets is not traded. Summing over the solid lines in Figure 1, we thus obtain:

$$\begin{aligned} \mu_H^1(t+1) &= \left[(1-\gamma)(1-\lambda^S(t)\pi(t)) + \gamma\lambda^S(t)\pi(t)\right]\mu_H^1(t) + \gamma\mu_H^2(t) \\ &= \gamma S q + (1-2\gamma)(1-\lambda^S(t)\pi(t))\mu_H^1(t) \end{aligned} \tag{12}$$

where I used the balance condition $\mu_H^2(t) = \gamma S q - \mu_H^1(t)$ in the last line. We may similarly derive the law of motion for L assets with the difference that (τ^1, L) agents fail to trade only when they do not find a match. We obtain:

$$\mu_L^1(t+1) = \gamma S(1-q) + (1-2\gamma)(1-\lambda^S(t))\mu_L^1(t) \tag{13}$$

Equation (12) highlights the effect of past buyers' offer on the composition of the pool of sellers. The mass $\mu_H^1(t+1)$ of H asset owners looking to sell decreases with the probability $\pi(t)$ of a pooling price in period t. For (τ^1, L) owners, $\pi(t)$ affects the price received but not the trading probability. Overall, market liquidity thus affects negatively the pool of sellers, and hence future market liquidity. We can now introduce the definition of a stationary OTC equilibrium.

Definition 2 (OTC Equilibrium)**.** *An OTC equilibrium is a collection of value functions* $\{v_a^i(t)\}_{a=0,L,H}^{i=1,2}$ *and reservation functions* $\{r_a^i(t)\}_{a=L,H}^{i=1,2}$*, a distribution of asset owners* $\{\mu_a^i(t)\}_{a=L,H}^{i=1,2}$*, a mass of buyers* $\mu^B(t)$ *and a probability* $\pi(t)$ *of a high price offer* $r_H^1(t)$ *for any* t *such that:*

1. *Buyers' offers verify* $\pi(t) \in \arg\max_\pi v^B(t,\pi)$ *and* $\mu^B(t)$ *verifies condition* (7)*.*

2. *Functions* $\{v_a^i(t)\}_{a=0,L,H}^{i=1,2}$ *and* $\{r_a^i(t)\}_{a=L,H}^{i=1,2}$ *verify equations* (9)*-*(11) *and* (4)*.*

3. *Distribution* $\{\mu_a^i(t)\}_{a=L,H}^{i=1,2}$ *verifies equations* (1) *and law of motion* (12)*-*(13)

4. ***Stationary property****:* $\exists\, T \in \mathbb{N}_+$ *such that for all endogenous variables* z

$$z(t+T) = z(T)$$

The lowest T *for which this property holds is the period of the equilibrium.*

By imposing the stationary property, I focus on the long-run dynamics of the model, hence the absence of reference to the initial distribution $\{\mu_a^i(0)\}_{a=L,H}^{i=1,2}$. The permanent component of liquidity fluctuations captures most of the model dynamics. Observe that an equilibrium with period $T = 1$ is a steady sate. The main result of the paper is to show that there can be other (cyclical) stationary equilibria with period $T \geq 2$.

3 Equilibrium Liquidity Dynamics

We first present a series of preliminary results. We then characterize equilibrium cycles in Section 3.1 and steady states in Section 3.2.

Lemma 2. *In any OTC equilibrium, the following statements hold*

i) Agents (τ^i, H) *valuation is:*

$$\forall\, t, i \in \{1,2\}, \quad v_H^i(t) = r_H^i := \frac{(1-\delta)\tau^i + \delta\gamma(\tau+1)}{(1-\delta)\big[1-\delta(1-2\gamma)\big]} d_H$$

ii) Buyers make zero profit: $\forall\, t$, $v_0^i(t) = 0$*, for* $i = 1,2$ *and equilibrium entry is*

$$\mu^B(t) = \frac{S v^B(t)}{\kappa}$$

iii) Sellers find a match with probability 1*:* $\forall\, t,\ \lambda^S(t) = 1$.

As he receives offers through sequential matching, an H asset owners enjoys the same utility than in autarky[15]. Part *ii*) follows from free-entry since buyers enter as long as they earn a positive profit. For part *iii*), assumption (A2) further ensures that buyers' entry is enough to match all asset owners. This implies that trading does not fail for lack of buyers and that market illiquidity is fully driven by asymmetry of information and thus adverse selection[16].

We now identify the two main dynamic forces at play in the model. Using the expression for buyers' profit $v^B(t,\pi)$ from (6), let us derive the net profit from a pooling offer over a separating offer:

$$v^B(t,1) - v^B(t,0) = \frac{\mu_H^1(t)}{S}(r_H^2 - r_H^1) - (1-q)\big[r_H^1 - r_L^2(t) + \gamma(r_L^2(t) - r_L^1(t))\big] \qquad (14)$$

where we replaced $\mu_L^1(t)$ and $r_H^i(t)$ by $\gamma S(1-q)$ and r_H^i respectively, using Lemma 2. Pooling becomes more advantageous as the share of high quality assets for sale $\mu_H^1(t)/S$ or the reservation values $\{r_L^i(t)\}^{i=1,2}$ for the L asset increase. The following Lemma shows how these quantities react to past and future prices to shape today buyers' trade-off (14).

Lemma 3. *In any OTC equilibrium, the following statements hold:*

i) Competition effect: $\pi(t)$ *increases with* $\{\pi(t+l)\}_{l=1,..,\infty}$

ii) Composition effect: $\pi(t)$ *decreases with* $\pi(t-1)$*.*

Part *i*) states that the probability of a pooling offer today increases with the probability of pooling offers in the future. The crucial insight is that higher future prices raise the value of a lemon both to the prospective buyer and the current owner. First, the ex-post loss on lemons $r_H^1 - r_L^2(t)$ from a pooling offer decreases as the buyer's valuation $r_L^2(t)$ goes up. When he knows he can resell the lemon at a good price tomorrow, the buyer is inclined to offer a (high) pooling price although the quality is uncertain. Second, intertemporal competition also squeezes the margin $r_L^2(t) - r_L^1(t)$ on the alternative separating offer.

[15] The market would then shut down if asset owners were to pay the search cost κ, a result known as the Diamond paradox. Interestingly, this result also arises as an equilibrium outcome in Section 5 where buyers compete simultaneously for sellers.

[16] The fact that sellers find a buyer with probability 1 is a byproduct of the efficient rationing property attached to the matching function. The analysis can accommodate a matching function of the form $\alpha \min\{\mu^B(t), \mu^S(t)\}$ with $\alpha < 1$. Under appropriate modifications to A2, we would obtain similar results.

To see this, suppose that a lemon owner receives a pooling offer the next period, that is $\pi(t+1) = 1$. Then this resale price determines the future value of the lemon both for the current and the new potential owner. Hence, the gains from a separating offer boil down to the difference in the current dividend valuation $(\tau - 1)d_L$. Gains from trade would be larger if agents were to stay in autarky forever after (or $\pi(t+l) = 0$ for $l \geq 1$). Both components of the *competition effect* work in the same direction so that $\pi(t)$ increases with $\{\pi(t+l)\}_{l=1,..,\infty}$, that is future liquidity begets present liquidity. The *competition effect* captures complementarity across time in decisions to pool sellers, which is a source of equilibrium multiplicity.

Part $ii)$ establishes that the probability of a pooling offer today $\pi(t)$ depends negatively on the probability $\pi(t-1)$ in the last period. The *composition effect* hinges on the endogenous asset holdings dynamics captured by law of motion (12). Suppose indeed that liquidity was high in the last period, or $\pi(t-1) = 1$. This pooling offer clears the market for all assets. In particular, the implicit supply of H assets, $\mu_H^1(t) = \gamma Sq$ reaches its lowest point in period t. As the pool of sellers now contains mostly L assets, buyers should find it more profitable to make a low separating offer. But if liquidity is indeed low today, that is $\pi(t) = 0$, we obtain:

$$\mu_H^1(t+1) - \mu_H^1(t) = 2\gamma(Sq/2 - \mu_H^1(t)) > 0$$

The distribution tomorrow becomes more favorable to a pooling offer because the implicit supply of H assets for sale increases. Delaying trade thus improves the pool of sellers through the accumulation of selling pressure of H assets. The composition effect creates a negative relationship between present and future liquidity which is key to equilibrium fluctuations. The relative strength of the composition and the competition effect then determines the nature of equilibrium.

3.1 Liquidity Cycles

For $T \geq 2$, I solve for OTC equilibria involving pure strategies[17] for buyers, that is where $\pi(t) \in \{0, 1\}$ for any t. Lemma 4 first shows that liquidity cannot be high in two consecutive dates.

[17] If anything, cycles with pure strategies generates starker price fluctuations and are thus harder to sustain. In the next section, I also characterize mixed-strategy equilibria steady state ($T = 1$).

Lemma 4. *In an OTC equilibrium with period $T \geq 2$, if $\pi(t) = 1$, then $\pi(t+1) = 0$, that is a period with low liquidity always follows a period with high liquidity.*

The intuition for Lemma 4 follows from our previous discussion. After a pooling offer in period t, $\mu_H^1(t+1) = \gamma Sq$ using law of motion (12). The distribution of assets becomes least favorable to pooling[18] and buyers prefer to offer the low separating price. When they do so, the quality in the pool of sellers improves but it might take time before buyers find it optimal to pool sellers again depending on the speed of the type switching process. Hence, there can be several consecutive periods with low liquidity $\pi = 0$.

For a given period T, we label without ambiguity by 0 the peak dates $t, t+T, t+2T$ where $\pi(t) = 1$. In these periods, buyers offer a pooling price r_H^1. Labels $1, 2, ..., T-1$ are for the intermediate "trough" dates when buyers offer the separating price $r_{L,T}^1(t)$. The additional subscript T captures the dependence of endogenous variables on the cycle length when relevant. We may now state the main Proposition of the paper.

Proposition 1. *Let $T \geq 2$. There exists thresholds $(\underline{q}_T, \bar{q}_T)$ such that an OTC equilibrium of period T exists if and only if $q \in [\underline{q}_T, \bar{q}_T]$ and*

$$\frac{1-(1-2\gamma)^T}{1-(1-2\gamma)^{T-1}} \geq \frac{r_H^1 - (1-\gamma) r_{L,T}^2(0) - \gamma r_{L,T}^1(0)}{r_H^1 - (1-\gamma)\tau d_L - \gamma d_L - \delta r_H^1} \tag{E_T}$$

where for $i \in \{1, 2\}$ and $t = 0, .., T$

$$r_{L,T}^i(t) = \left[\frac{1-\delta^{T-t}}{1-\delta}(\tau+1) + (-1)^i \frac{1-(\delta(1-2\gamma))^{T-t}}{1-\delta(1-2\gamma)}(\tau-1)\right]\frac{d_L}{2} + \delta^{T-t} r_H^1 \tag{15}$$

For $T \geq 2$, $(E_{T+1}) \Rightarrow (E_T)$ and $\bar{q}_{T+1} < \underline{q}_T$.
In a cycle of period T, the mass of (τ^1, H) agents is

$$\mu_{H,T}^1(t) = \frac{1-(1-2\gamma)^t}{2} Sq, \quad t = 1, .., T \tag{16}$$

Observe first that, in a cycle, agents' valuation for a lemon $r_{L,T}^i(t)$ weighs the holding value over the remaining trough periods and the pooling price they will receive at the peak, in $T-t$ periods. During the trough, equation (16) captures the accumulation of selling pressure of H assets that leads to the peak. Let us now interpret the existence condition

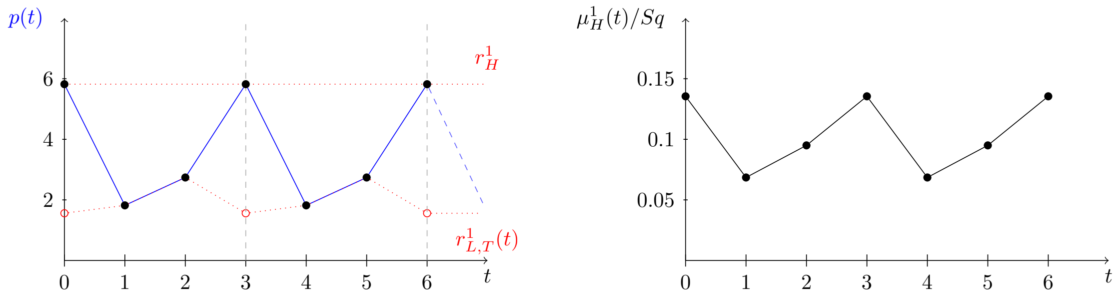
[18] The argument thus relies partially on the fact that search frictions are mild, or $\lambda^S(t) = 1$. Intuitively though, entry by buyers should precisely be larger and frictions less severe for sellers at the peak of the cycle.

(E_T) for a cycle of period T. The left hand side measures the relative improvement in the pool quality between the last date of the trough and the peak as can be seen from (16). It thus captures the benefits in date $T-1$ from waiting one more period to make a pooling offer. The right hand side represents the costs from waiting. Indeed, in period $T-1$, losses on lemons from a pooling offer would be low since a buyer can resell at the pooling price next period. In period 0 however, acquiring lemons is costly because a buyer can flip them at a high price only T periods after the purchase. A cycle thus exists if the improvement in the pool quality from the composition effect overcomes the loss increase from the competition effect. Adding one period to the cycle leaves more time for the pool to improve. Hence longer period cycles exist for lower values of the share of H assets q. Observe that as T increases though, the last period marginal improvement in the pool quality $\mu^1_{H,T}(T) - \mu^1_{H,T}(T-1)$ goes down. On the other hand, buyers' incentives to anticipate the gains from trade with H assets go up. Low periods equilibrium cycles are thus easier to sustain, that is $(E_{T+1}) \Rightarrow (E_T)$ for $T \geq 2$.

Figure 2 illustrates price and composition dynamics for a 3 period cycle. In Figure 2a, the solid blue line represents the transaction price over the cycle. It coincides with the reservation value $r^1_{L,T}(t)$ (lower dashed red line) of (τ^1, L) owners during the trough and equals the pooling offer r^1_H (upper dashed red line) at the peak. After the peak, Figure 2b illustrates the drop in the supply of H assets $\mu^1_{H,T}(t)$ measured as a fraction of the total quantity of H assets. During the trough, only L assets are traded but the price increases. Indeed, it must reflect (τ^1, L) sellers' outside option of waiting for a better offer in $t = 3$. In particular, the asset trades above the full information price although buyers know for sure they are buying a L asset. We can interpret this premium as a bubble component reflecting the future high value of the lemon at the peak of the cycle. These dynamics then evoke the "hot potato" story for a financial crisis. Agents know they purchase bad assets at inflated prices but ride the bubble to resell them at an even higher price in the future. When they do (at the peak of the cycle), the price drops significantly and the bubble bursts. More generally, our analysis shows that liquidity fluctuations arise naturally in markets with asymmetry of information. Prices reflect the average quality of assets offered for sale but the supply responds endogenously to past prices.

3.2 Steady State Equilibria

This section describes steady state equilibria, that are OTC equilibria of period $T = 1$, this time both in mixed and in pure buyers' strategy. I shorten the presentation since the



(a) Prices and Reservation Values

(b) Asset Supply of quality H

Figure 2: A 3 Period Cycle ($\delta = 0.3$, $d_L = 1$, $d_H = 4$, $\tau = 2$, $\gamma = 0.05$)

characterization mostly serves as a basis for welfare comparison with liquidity cycles. I drop the time arguments for endogenous variables.

Proposition 2. *There exists two thresholds in the share of H assets in the economy $(\underline{q}, \bar{q}) \in [0,1]^2$ such that the only steady state equilibria are the following:*

i) *When the share q is low, $q \leq \underline{q}$, there is a separating equilibrium $\pi = 0$.*

ii) *When the share q is high, $q \geq \bar{q}$, there is a pooling equilibrium $\pi = 1$.*

iii) *For $q \in (\min\{\underline{q}, \bar{q}\}, \max\{\underline{q}, \bar{q}\})$, there exists an equilibrium in mixed strategy $\pi(q) \in (0,1)$.*

When $\underline{q} \leq \bar{q}$, there is a unique equilibrium for any value of q. When $\underline{q} \geq \bar{q}$, equilibria i), ii) and iii) coexist on $[\bar{q}, \underline{q}]$.

Intuitively, a pooling equilibrium with high liquidity $\pi = 1$ may only exist if the share of H assets q is high enough. When q is too low, buyers thus cater only to (τ^1, L) owners and the equilibrium is separating with $\pi = 0$. For intermediate values of q, a partial equilibrium exists with $\pi(q) \in (0,1)$. The mixed strategy equilibrium highlights the tension between the composition and the competition effect. As π goes up, lemons become more valuable and pooling offers more profitable. The competition effect thus favors pooling. However, the steady state share of H quality assets decreases with π. Indeed, from law of motion (12), we obtain:

$$\mu_H^1(\pi) = \frac{\gamma}{2\gamma + \pi(1-2\gamma)} Sq \tag{17}$$

Hence, through the composition effect, higher market liquidity makes it less profitable for buyers to offer a pooling price. These forces work against one another so that mixed strategy equilibria exist for an open interval of values of q.

When the upper bound of the pooling region $\underline{q}$ exceeds the lower bound $\bar{q}$ of the pooling region, a separating, a pooling and a mixed equilibrium coexist. The proof to Proposition 2 shows that the multiplicity condition $\underline{q} \geq \bar{q}$ writes:

$$\frac{\delta\gamma(1-\delta)(1-2\gamma)(\tau-1)d_L}{(1-\delta)(d_H-\tau d_L)+\gamma\delta(\tau+1)(d_H-d_L)+\gamma(1-\delta)(\tau-1)d_L} \geq 1-\delta-2\gamma \tag{18}$$

In the limit case where $d_L \to 0$, the condition writes[19] $\delta > 1-2\gamma$. From equation (17), $1-2\gamma$ measures the sensitivity of the H asset supply μ_H^1 to liquidity π and thus captures the strength of the *composition effect*. The discount factor δ determines the weight assigned to future payoff and thus the strength of the competition effect. Hence, in the parameter region with steady state multiplicity, the *competition effect* dominates the *composition effect*. We now gather the existence results from Propositions 1 and 2 to provide a complete picture of OTC equilibria. In particular, we are interested in the nature of the steady state equilibrium in the region where cycles exist.

Corollary 1. *For $T \geq 2$, an OTC equilibrium of period T exists when there is a unique steady state equilibrium, that is (E_T) $\Rightarrow$ $\underline{q} \leq \bar{q}$. A cyclical equilibrium exists in the region where the steady state is in mixed strategy, that is $[\underline{q}_T, \bar{q}_T] \subset [\underline{q}, \bar{q}]$ with $\bar{q}_2 = \bar{q}$.*

The proof of Proposition 1 establishes these results. First, cycles exist when the composition effect is strong whereas steady state multiplicity relies on the competition effect. Second, there is a natural relationship between mixed strategy steady state and cycles. When the share of H quality assets is intermediate, neither pooling nor separating can be sustained in every period. In a steady state, liquidity spreads out evenly across periods as sellers face a constant probability to receive a high price. In a cycle, liquidity fluctuates with the average quality in the pool of sellers and terms of trade change over time. Figure 3 describes the equilibrium regions in the (γ, δ) parameter space. The uppermost downward sloping line represents equation (18) and breaks the parameter space into two regions. In the bottom left part, there is a unique steady state for each value of q. Within this sub-region, equilibrium cycles may exist. Higher value of the period T corresponds to darker shades. The figure shows that cycles with period $T \geq 3$ require low discount factors[20]. Hence, in the rest of the analysis we focus on period 2

[19] This is the case analyzed by Chiu and Koeppl (2014) in a continuous time environment. Although, strictly speaking, this case is not well-defined in my model because of Assumption (A2), it is useful to from intuition.

[20] This is in part driven by our focus on equilibrium cycles in pure buyers' strategies.

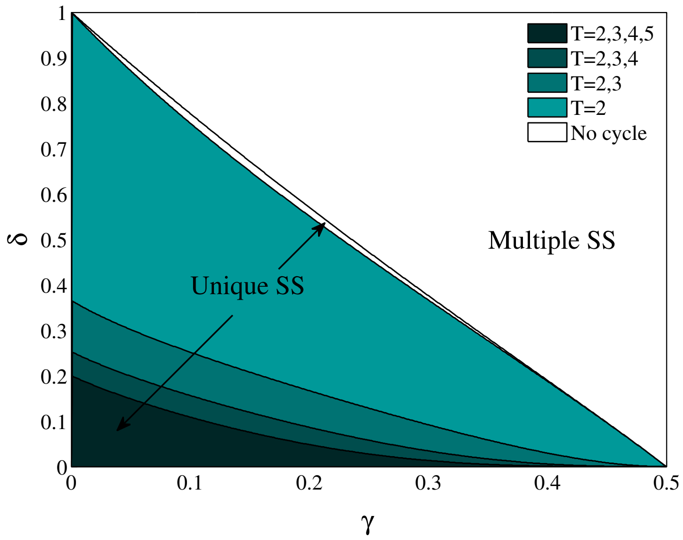


Figure 3: Equilibrium Existence ($d_L = 1$, $d_H = 4$, $\tau = 3$)

cycles (lighter shade on Figure 3), that is (E_T) holds for $T = 2$ only and $q \in [\underline{q}_2, \bar{q}]$

3.3 Welfare Comparison

Aggregate surplus $W(t)$ is defined recursively as follows:

$$W(t) = S(1-q)\tau d_L + \mu_H^1(t)(1-\pi(t))d_H + \left[\mu_H^1(t)\pi(t) + \mu_H^2(t)\right]\tau d_H - \mu^B(t)\kappa + \delta W(t+1) \quad (19)$$

The first three terms correspond to allocative efficiency. With symmetric information, type τ^2 agents would hold all the assets after trading. Market illiquidity, generates misallocation of a fraction $\mu_H^1(t)(1-\pi(t))$ of the H assets. The fourth term measures trade costs which are proportional to equilibrium entry. Since buyers get to make TIOLI offers, entry is typically inefficient[21]. The last term is self-explanatory.

In this section, the subscript ss (resp. cy) refers to endogenous variables in the steady state (resp. 2 period cycle). We thus denote W_{ss} welfare in steady state and $(W_{cy}(0), W_{cy}(1))$ welfare in the high and low date of cycle. Since liquidity fluctuates between $\pi_{cy}(0) = 1 > \pi_{ss}$ and $\pi_{cy}(1) = 0 < \pi_{ss}$ in a cycle, one could guess that welfare in a cycle also fluctuates around the steady state level W_{ss}. We show however that fluctuations entail a dynamic welfare loss with respect to the steady state.

[21]In general search frictions could also affect allocative efficiency if sellers can fail to meet buyers, that is $\lambda^S(t) < 1$, which is not the case in equilibrium here.

Proposition 3. *There exists $\hat{q} \in [\underline{q}_2, \bar{q}]$ such that for all $q \in (\hat{q}, \bar{q}]$, $W_{ss} > W_{cy}(0) > W_{cy}(1)$, that is the surplus in a steady state is greater than in a cycle in every date.*

Proposition 3 shows that when $q \in [\hat{q}, \bar{q}]$ the steady state equilibrium dominates the cyclical equilibrium, irrespectively of the "starting date" for the cycle. The striking part of the result is that surplus may be lower at the peak of the cycle than in steady state, that is $W_{ss} \geq W_{cy}(0)$, although trading volume and liquidity are maximal at a peak date. However, high liquidity at the peak lowers future market liquidity through the composition effect and generates dynamic misallocation of the H asset - the fourth term in (19). When q is close to $\bar{q}$, $\pi_{ss}(q) \to 1$, that is the steady state equilibrium features full liquidity in the limit. In a cycle however, every two periods some H assets are not traded. With the composition effect, there is too much trade at the peak and too little in the trough. When $q \in [\underline{q}, \hat{q}]$, the first inequality is reversed and surplus evaluated at the peak is higher than in steady sate. The comparison then depends on the reference date for the cycle although the average value of W_{cy} lies below W_{ss}. The unambiguous welfare ranking when $q \in [\hat{q}, \bar{q}]$ suggests that policy interventions can be desirable when the economy is in a cycle. Intuitively, a benevolent policy maker would seek to prop up (resp. tame) trading in the trough (resp. peak) of the cycle. In the following Section, we show how an asset purchase program can achieve this objective.

4 Liquidity and Policy Intervention

In this section, I study an asset purchase program by a benevolent government who seeks to jump-start the market but must resell the assets he purchased. Unlike previous works including Tirole (2012) and Chiu and Koeppl (2014), I can thus capture a realistic policy constraint because the government must eventually close his position. With respect to a permanent policy, a revertible program does not change the fundamentals of the economy in the long-run. In the following, I characterize feasible policies analytically and provide numerical results for the surplus maximizing policy. Although its effects are smaller, I also show that a budget-neutral policy is feasible and can increase welfare in the Pareto sense. Formally, the benevolent government is a large agent with the same discount factor δ as private agents and preferences:

$$u^G(c, d) = c + \tau^G d$$

over the consumption good and the dividends where $\tau^G \in [0, \tau]$. The government derives a lower utility from the dividends than type τ^2 agents, as for instance, he would not value potential services (borrowing, hedging) from holding assets. The government has deep pockets and can hold many assets.

4.1 Timing and Policy Design

Prior to the intervention, the economy is in an equilibrium cycle of period $T = 2$. Let t_{int} be the intervention date which corresponds to a low date of the cycle. At the beginning of period t_{int}, the distribution of agents across assets is thus $\{\mu^i_{a,cy}(1)\}^{i=1,2}_{a=L,H}$. We divide periods t_{int} and $t_{int}+1$ into phases $i)$ and $ii)$ as follows:

1. Date t_{int}: Purchase.

    i) The government announces that he will buy up to S^G assets at unit price P^G. Asset owners may apply and sell their asset to the government.

    ii) OTC market with asset owners who have not participated in the program.

2. Period $t_{int}+1$: Resale.

    i) The government quotes a resale price R^G at which he resells all assets purchased in t_{int}. Non-asset owners may apply to purchase these assets.

    ii) OTC market with asset owners including buyers of step $i)$.

The division of period t_{int} into steps $i)$ and $ii)$ is important. By removing assets before the OTC market opens, the government can affect the distribution of sellers faced by buyers[22] and hence the probability of a pooling offer. For simplicity, we abstract from issues related to the timing of the exit strategy since the government must resell assets one period after. A policy is a triplet $(S^G, P^G, R^G) \in \mathbb{R}^3_+$ where S^G is the program size, P^G the purchase price and R^G the resale price. Besides the obligation to resell assets, I impose a medium-run stabilization constraint on the intervention.

In order to explicit these constraints, I take as given the sequence $(\pi(t_{int}), \pi(t_{int}+1), ...)$ of buyers' offers. In equilibrium, this sequence will be consistent with the policy. The

[22] Buying assets in the private OTC market would not have an impact if sellers are captive. If they are not, they could use the government program as a credible threat to induce more competitive offers from buyers which generates higher liquidity. We are interested in a policy that is actually implemented in equilibrium rather than an equilibrium selection device.

stabilization constraint sets the mass of (τ^1, H) agents in period $t_{int}+2$ to:

$$\mu^1_H(t_{int}+2) = \mu^1_{H,ss} \tag{SC}$$

When (SC) holds, we know from Proposition 2 that the steady state is a continuation equilibrium from period $t_{int}+2$ onward. We can thus avoid dealing with equilibrium transitions and focus on the short-term policy trade-off. The resale constraint puts an upper bound on the price R^G the government can ask to sell his assets. This bound depends on the selection of assets acquired in period t_{int} and thus on agents' participation decisions. Agent (τ^i, a) for $i \in \{1, 2\}$ and $a \in \{L, H\}$ opts in the program if and only if $P^G \geq v^i_a(t_{int})$ that is when[23] the price offered exceeds the utility he expects from trading in the OTC market at t_{int}. Observe that the outside option $v^i_a(t_{int})$ is endogenous since it depends on the equilibrium induced by the intervention. The crucial insight from Lemma 1 is that L asset owners have a lower market utility and would thus be the first to opt in. This leaves room for the policy to improve the distribution of assets in the OTC market. Since the government has no more information than private buyers, he obtains a random selection (S^G_L, S^G_H) of assets held by those agents who opt in where for $a \in \{L, H\}$:

$$S^G_a = \min\left\{1, \frac{S^G}{\sum_{j,a'} \mu^j_{a',cy}(1)\mathbb{1}_{\{P^G \geq v^j_{a'}(t_{int})\}}}\right\} \sum_{i=1,2} \mathbb{1}_{\{P^G \geq v^i_a(t_{int})\}}\mu^i_{a,cy}(1) \tag{20}$$

where the term between curly brackets captures rationing of sellers when the program is over-subscribed. We assume that S^G can adjust so that the program is never under-subscribed. The government cannot quote a resale price R^G higher than the buyer's valuation for the average asset from the government pool:

$$R^G \leq v^2_L(t_{int}+1)S^G_L/S + v^2_H(t_{int}+1)S^G_H/S \tag{RC}$$

Although conditions (SC) and (RC) bear on endogenous objects, they ultimately constrain the policy that induces this equilibrium. Let us now define the government payoff v^G as:

$$v^G = S^G(\delta R^G - P^G) + \left[S^G_H d_H + S^G_L d_L\right]\tau^G \tag{GP}$$

The first term is the capital gain. The second term measures the government valuation

[23] If $P^G = v^i_a(t_{int})$, the agent is indifferent between opting in or out of the program and could randomize. We can dismiss this concern because the government could induce participation strictly by raising the price by an infinitesimal amount

for the dividends and depends on the selection of assets acquired by the government. In the numerical analysis of Section 4.2, we first allow the government to run a loss $v^G < 0$ and then impose budget-neutrality. We may now define formally a feasible policy

Definition 3. *A policy (S^G, P^G, R^G) is feasible if there exists $(\pi(t_{int}), \pi(t_{int}+1)) \in [0,1]^2$ such that:*

1. *Given the sequence of buyers' strategies $(\pi(t_{int}), \pi(t_{int}+1), \pi_{ss}, \pi_{ss}, ...)$, the resale constraint (RC) and the stabilization constraint (SC) hold.*
2. *Strategies $(\pi(t_{int}), \pi(t_{int}+1))$ are optimal for buyers, that is for $l \in \{0, 1\}$*

$$\pi(t_{int}+l) \in \arg\max_{\pi} v^B(t_{int}+l, \pi)$$

given that $\pi(t+l) = \pi_{ss}$ for $l \geq 2$.

We now take as granted that the equilibrium with a feasible intervention is the steady state π_{ss} from period $t_{int}+2$ onward. In words, a feasible policy satisfies the resale (RC) and the stabilization constraints (SC) in an equilibrium induced by this policy[24]. The first objective of the policy is to maximize the net surplus gain ΔW_{int} from the intervention:

$$\begin{aligned}\Delta W(t_{int}) = & \underbrace{\left[\gamma Sq - (1-\pi(t_{int}))\mu_H^1(t_{int}) - \delta(1-\pi(t_{int}+1))\mu_H^1(t_{int}+1)\right](\tau-1)d_H}_{\text{Short-Term Trading Gains}} \\ & - \underbrace{(S_H^G d_H + S_L^G d_L)(\tau - \tau^G)}_{\text{Holding Costs}} + \underbrace{\left[\mu_C^B(1) - \mu^B(t_{int}) + \delta(\mu_C^B(0) - \mu^B(t_{int}+1))\right]\kappa}_{\text{Trading Costs Difference}} \\ & + \underbrace{\delta^2(W_{ss} - W_{cy}(1))}_{\text{Long-Run Gains}} \end{aligned} \tag{21}$$

The short term trading gains account for the increase in liquidity from 0 to $\pi(t_{int})$ in period t_{int}, but also its potential decrease from 1 to $\pi(t_{int}+1)$ in period $t_{int}+1$ when the government resells assets. The holding costs are negative because the government must hold asset he values less than private agents. The long-run gains are positive because surplus is higher in steady state (reached after 2 periods) than in the low date of the cycle. The purchase and resale prices P^G and R^G do not enter expression (21) since transfers are neutral with linear utility. Still, the level of the intervention price P^G matters as it induces a particular selection of applicants through the participation constraint.

[24] A feasible policy and an induced equilibrium thus solve a fixed-point problem. Although this is a relevant concern, we cannot claim the policy uniquely implements this equilibrium.

4.2 Welfare Improving Policy: Numerical Analysis

In this section, I use the numerical values reported in Table 1. Deep parameters are in bold characters. Although the 2 period cycle exists for less than 2.5% of the possible range of values of q, the features of the intervention may change significantly over this range as we show below. Appendix C describes the step to construct and rank feasible

$\boldsymbol{\delta}$	$\boldsymbol{\gamma}$	$\boldsymbol{\tau}$	$\boldsymbol{d_L}$	$\boldsymbol{d_H}$	$\boldsymbol{\tau^G}$	$\underline{q}_2$	$\bar{q}_2$
0.7	0.05	3	1	4	0	0.610	0.632

Table 1: Benchmark parameter values

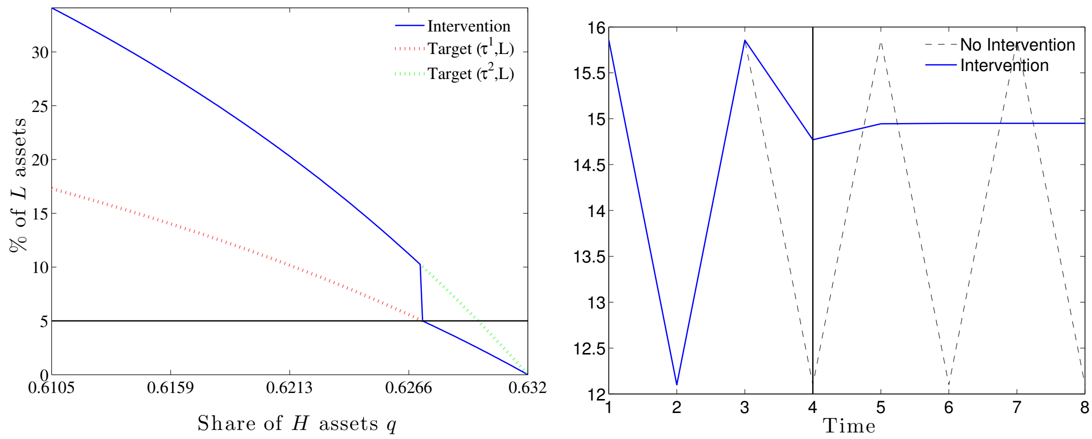
policies. Essentially, with discrete types of asset owner, the policy selection boils down to a discrete choice over these four types. The selection then reduces to a binary choice as a welfare maximizing policy should not target H asset owners. Intuitively, removing H from the market does not improve liquidity in the intervention period t_{int}.

Figure 4 illustrates the results. On the left panel, the red dotted line shows the intervention size targeting (τ^1, L) agents whereas the green dotted line is for (τ^2, L) agents. The blue line is the selection of the surplus maximizing intervention. The most efficient intervention targets (τ^1, L) agents. However, these agents only hold $\gamma = 5\%$ of the L assets - the horizontal line on the graph. For a larger intervention, the government needs to target (τ^2, L) owners. The pecking order from low to high valuation owners of lemons arises because buying L assets from (τ^1, L) is more efficient to jump-start the market. Indeed, it decreases both the cost from a pooling offer and the benefits from a separating offer for buyers. The latter effect is not present for (τ^2, L) agents. We see that the size of the intervention decreases with the share of H quality assets q. To understand this result, observe that the gap between the target supply of H quality assets $\mu^1_{H,ss}(q)$ imposed by the stabilization constraint (SC) and that in the trough of a cycle $\mu^1_{cy,H}(q)$ is

$$\mu^1_{H,ss}(q) - \mu^1_{cy,H}(q) = (1 - 2\gamma)\frac{\gamma S q(1 - \pi_{ss}(q))}{2\gamma + \pi_{ss}(q)(1 - 2\gamma)} \xrightarrow[q \to \bar{q}]{} 0$$

An intervention targeting exclusively (τ^1, L) agents suffices to bridge this gap if q is high.

Figure 4b represents the average price path in the OTC market with the intervention (solid line) for the median value $q = 0.6213$. Before $t_{int} = 4$, it coincides with the no-



(a) Size of the Intervention (% L assets) **(b)** OTC Market Price path ($q = 0.6213$)

Figure 4: Revertible Asset Purchase Program

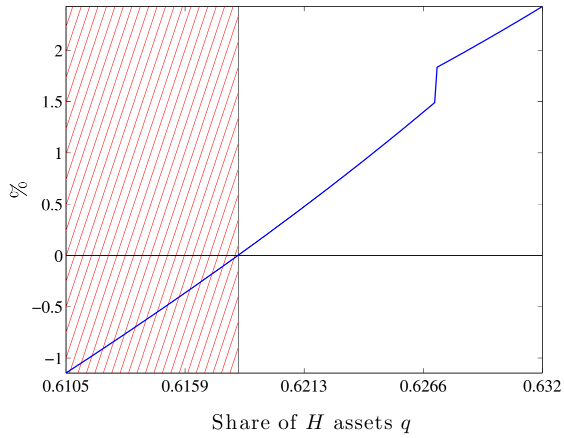
intervention price[25] (dashed line). The asset purchase program smoothes out fluctuations very quickly. Indeed, the price in the intervention period nearly reaches the equilibrium steady state price of period $t_{int} + 2$. The picture looks similar for other values of q. The next Section shows how these results change with a budget-balanced intervention.

On figure 5, the left panel plots the net surplus gain ΔW_{int} whereas the right panel highlights the contribution of each component according to the decomposition of equation (21). Surplus gains increase in q together with the inverse of the program size. A large intervention might even be undesirable (the hatched area in figure 5a). Figure 5b shows indeed that for low values of q, government holding costs proportional to $(\tau - \tau^G)S^G$ dominate. As the intervention size decreases for higher values of q, the positive components take over. In particular, welfare improves with the long-term stabilization gains and the short-term trading gains.

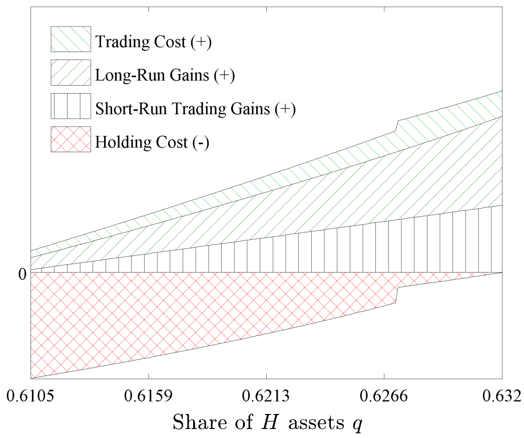
4.3 Budget-Neutral and Pareto Improving Intervention

We find that the government earns a negative net return of -30% across values of q. Holding costs that are proportional to the difference in valuation $\tau - \tau^G = 3$ contribute significantly to this loss. However, even when $\tau^G = \tau = 3$, the average net return is still around -7%. Indeed, this number also reflects a premium the government pays to induce participation from asset owners. Suppose indeed that q is low (below the kink on Figure

[25] Announcing the policy in $t_{int} - 1 = 3$ would have no effect here. If anything, a pooling offer in that period then becomes even more profitable but the equilibrium would not change.



(a) Welfare Gains (%)



(b) Contributions of Welfare Components

Figure 5: Efficiency properties

4a). The government must attract (τ^2, L) owners and offer at least their market value of $P^G = v_L^2(t_{int})$. This price also compensates asset owners for the information rents earned in the OTC market while the government obtains L assets for sure. In comparison, the maximum price a buyer would pay in the OTC market for a L asset is $r_L^2(t_{int})$. Hence, the government pays a premium:

$$v_L^2(t_{int}) - r_L^2(t_{int}) = \pi(t_{int})(r_H^1 - r_L^2(t_{int})) > 0$$

Interestingly, the higher liquidity $\pi(t_{int})$ in the market following the purchase, the bigger this premium. The objective to increase liquidity thus raises the government losses. It is then natural to ask whether a budget-neutral policy is feasible. Indeed, a loss-making intervention does not constitute a Pareto improvement as private agents who would contribute those funds are worse-off despite surplus gains.

To finance the intervention, the goverment may now tax transactions in the OTC market of period $t_{int} + 1$. Every buyer who purchases an asset must pay $\sigma^G > 0$ units of the reference good c. The main point of this part of the analysis is to stress the interaction between designing and financing the policy. Indeed, a transaction tax distorts buyers' trade-off between a pooling and a separating offer. To see this, let us write the expression for the net gains from a pooling offer with tax σ^G adapting equation (14):

$$v^B(t,1) - v^B(t,0) = \frac{\mu_H^1(t)}{S}(r_H^2 - r_H^1) - (1-q)(r_H^1 - r_L^2(t)) + \frac{\mu_L^1(t)}{S}(r_L^2(t) - r_L^1(t)) \\ - \sigma^G\left[\frac{\mu_H^1(t) + \mu_L^2(t)}{S}\right] \tag{22}$$

The first line is similar to equation (14) while the second line shows the effect of the tax. When offering a pooling price r_H^1, a buyer increases his trading probability and thus its tax payment proportionally to the mass of traders who only accept that offer, that is agents (τ^2, L) and (τ^1, H). Ceteris paribus, the tax thus lowers the benefits from a pooling offer. Hence, if the government were to set the tax naively so as to offset the loss, he would actually destroy liquidity in period $t_{int}+1$. Hence, all the parameters $(S^G, P^G, R^G, \sigma^G)$ of the policy must now be determined jointly. The government payoff with tax verifies:

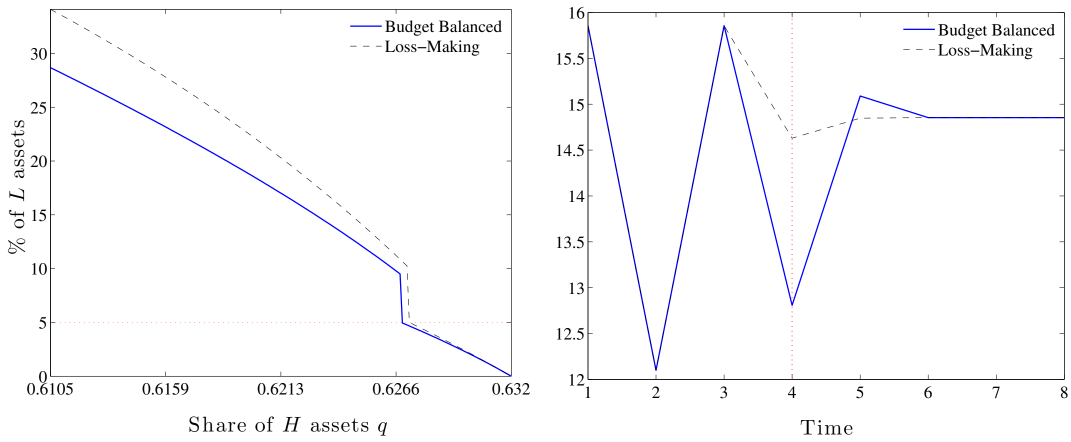
$$v^G := S^G(\delta R^G - P^G) + \sigma^G\left[\mu_L^1(t_{in}+1) + \pi(t_{int}+1)(\mu_H^1(t_{int}+1) + \mu_L^2(t_{int}+1))\right]$$

where in expression (GP), we used $\tau^G = 0$. The term between brackets is the volume of trade in period $t_{int}+1$. A feasible budget-neutral policy is a policy feasible according to Definition 3 with the additional budget-neutrality constraint $v^G = 0$.

Numerical Results

Across values of $q \in [0.6195, 0.6302]$ where the intervention increases aggregate welfare, an average 20% of surplus is lost because of the budget-balance constraint[26]. Figure 6 provides some intuition by comparing intervention size and price paths with the constraint (solid line) and without (dashed line). The intervention becomes smaller in order to reduce the holding costs. Interestingly, the asset purchase does not flatten fluctuations as much. Indeed, maintaining fluctuations between t_{int} and $t_{int}+1$ relax the budget-balance constraint in two ways. First, it allows for a high resale price R^G for the government assets. Second, increasing liquidity in period t_{int} with a large intervention would decrease the mass of (τ^1, H) asset owners in period $t_{int}+1$ through the composition effect. These asset owners precisely belong to the implicit tax base in period $t_{int}+1$ as show by equation (22). The budget balance condition thus creates a trade-off between jump-starting the market to increase surplus and riding the cycle to finance the intervention.

[26]The range of values of q where the policy increases surplus is $q \geq 0.6195$ instead of $q \geq 0.6183$ previously,which is 6% smaller. Hence the effect appears larger on the intensive margin.



(a) Size of the Intervention **(b)** OTC Market Price path ($q = 0.62$)

Figure 6: Budget Balanced Program

Asset Purchase vs. Subsidy

We finally compare[27] the asset purchase program to another feasible budget-neutral intervention: a subsidy χ^G in period t_{int}, also financed by a tax σ^G in period $t_{int}+1$. As we discussed, purchasing lemons modifies the composition of the pool of sellers since the relative probability to find a H asset in the OTC market is:

$$\frac{\mu_H^1(t_{int})}{\mu_H^1(t_{int}) + \mu_L^2(t_{int}) + \mu_L^1(t_{int})} = \frac{\gamma S q}{\gamma S q + S(1-q) - S^G}$$

The effect of the purchase size S^G increases in q. Instead, the subsidy increases the net gain from a pooling offer by:

$$\frac{\Delta v^B(t_{int})}{\partial \chi^G} = \frac{\mu_H^1(t_{int}) + \mu_L^2(t_{int})}{S} = \gamma q + (1-\gamma)(1-q)$$

where we used equation (22) because a subsidy is merely a negative tax. This time, the effect is larger when q is small. In addition, the government needs not hold asset in this case. Figure 7 plots the surplus gains for the subsidy and for the asset purchase program under various values of τ^G. Besides our benchmark $\tau^G = 0$, we also consider the case where the government has the same valuation for the asset as low valuation ($\tau^G = 1$)

[27]In the US, during the recent financial crisis, the Public Private Investment Program for legacy Mortgage Backed Securities was a form of subsidy while the SBA 7(a) Securities Purchase Program for small business loans was a direct asset purchase.

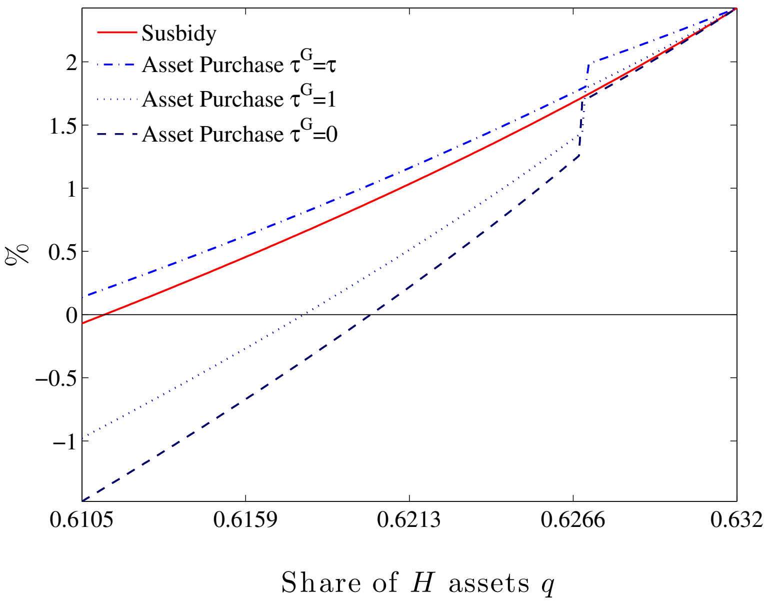


Figure 7: Welfare gains (%): Subsidy vs. Asset Purchase

and high valuations ($\tau^G = \tau$) agents. The results show that for a given value of τ^G, the asset purchase becomes relatively more attractive as q increases, in line with our informal analysis. Second, the asset purchase program performs better for high values of τ^G since the costs from holding assets go down.

We have shown that a revertible asset purchase program can jump-start the market in the short-run and stabilize it in the long-run. However, large interventions entail important misallocation costs for the government and budget neutral policies have more limited effects. In the next section, we study the impact of a structural change to the OTC market on equilibrium liquidity.

5 Liquidity and Market Structure

In the wake of the financial crisis, regulators pointed at the very structure of OTC markets as a source of illiquidity and instability[28]. In my model, random search with ex-post bargaining generates several inefficiencies. First, buyers enter as long as they can make profit while fewer buyers could match all sellers. Resources are thus wasted on search

[28]On its website, the IMF referring to OTC markets explains that *"some types of market arrangements can very quickly become disorderly, dysfunctional, or otherwise unstable"*. Ongoing Dodd-Frank and EMIR reforms notably mandate central clearing of OTC instruments such as derivatives and swaps with a stated objective to increase transparency and competition.

costs. Second, with captive sellers, buyers reduce demand to extract rent, lowering trading volume and liquidity. In a centralized market place, a trader implicitly observes demand and supply from the rest of the market simultaneously and may signal his interest in trading. To capture these differences, I modify the model to allow buyers to post and commit to prices before meeting a counterparty while keeping the matching technology unchanged. Pre-trade transparency improves as sellers may pick their trading price from those posted by buyers. I show however that while the Exchange brings traders together more efficiently, liquidity shuts down for high quality assets.

5.1 Exchange

Formally, an exchange in period t is a continuum of markets $p \in \mathbb{R}^+$ where p is a price for the asset[29]. Each market p in period t is characterized by the ratio of buyers to sellers $\theta(t,p)$ and a belief vector $\{\gamma_a^i(t,p)\}_{a=L,H}^{i=1,2}$ about the share of each type of asset owner in market p. Each agent takes these quantities as given.

Matching

The bilateral matching technology is identical. Buyers and sellers choose the market they want to trade in, taking $\theta(t,p)$ as given. The probability for a seller (resp. a buyer) to meet a buyer (resp. a seller) in market p, in period t is:

$$\lambda^S(t,p) = \min\{\theta(t,p), 1\}, \quad \lambda^B(t,p) = \min\left\{\theta(t,p)^{-1}, 1\right\}$$

Hence, for a seller, $\theta(t,p)$ measures the extent of rationing in market p. Importantly, owners can not sell their asset in two different markets p and $\hat{p} \neq p$ in the same period. Hence, an attempt to sell at a price p is a commitment not to try and sell at a different price $p' < p$. This can act as a signal of quality if sellers expect rationing at high prices[30]. As in the OTC market though, exclusivity only restricts intra-period trades.

[29]There is no loss in generality in assuming that buyers post prices and not contracts since rationing plays the same role as probabilities of trade. Implicitly, the OTC market has only one such sub-market where all asset owners and buyers must go.

[30]Models of competitive adverse selection such as Gale (1996), (Guerrieri and Shimer, 2014), Chang (2014) also impose this exclusivity assumption. Kurlat (2015) allows for non-exclusivity in a static model and obtains pooling. For an analysis of non-exclusivity with a strategic equilibrium concept, see Biais et al. (2000) and Attar et al. (2011).

Sellers Problem

Asset owner (τ^i, a) chooses the market which maximizes his utility:

$$v_a^i(t) = \max_{p \in \mathbb{R}^+} v_a^i(t,p)$$
$$\text{where} \quad v_a^i(t,p) = \lambda^S(t,p)(p - r_a^i(t)) + \tau^i d_a + \delta \bar{v}_a^i(t+1) \tag{23}$$

For an asset owner, $v_a^i(t,p)$ is the utility from trading in market p. Asset owners may always choose a very high price where $\theta(t,p) = 0$ if they do not want to trade. One can interpret a decision to sell at a high price with rationing as a limit order while a decision to sell at a low price without rationing would be a market order.

Buyers Problem

Let $\gamma_a^i(t,p) \in [0,1]$ denote buyers' belief about the share of type (τ^i, a) in market p in period t. The buyer's payoff from market p for non asset owner τ^2 writes:

$$v^B(t,p) = \lambda^B(t,p)\Big[\big(\gamma_H^2(t,p) + \gamma_H^1(t,p)\big) r_H^2(t) + \big(\gamma_L^2(t,p) + \gamma_L^1(t,p)\big) r_L^2(t) - p\Big] + \delta \bar{v}_0^2(t+1,p) \tag{24}$$

A buyer cares about the quality of the asset a not the type τ^i of the seller. We let $\mu^B(t,.)$ be the measure of buyers over markets $p \in \mathbb{R}_+$ with support $\mathcal{P}(t)$ and define:

$$\mu^B(t) = \int_{\mathcal{P}(t)} \mu^B(t,p)\mathrm{d}p$$

as the total mass of buyers.

Law of Motion

The law of motion for types (τ^1, a) writes:

$$\mu_a^1(t+1) = \Big[1 - \gamma - (1-2\gamma)\int \gamma_a^1(t,p)\lambda^B(t,p)\mu^B(t,p)\mathrm{d}p\Big] + \gamma \mu_a^2(t) \tag{25}$$

The expression is similar to the one derived for OTC markets except that agents might visit different markets p with different trading probabilities[31].

[31] The formula above seems convoluted but economizes on notation as we do not need to introduce measure of sellers $\mu_a^i(t,p)$ in the market. We would have

$$\lambda^S(t,p)\mu_a^i(t,p) = \lambda^B(t,p)\mu^B(t,p)\gamma_a^i(t,p)$$

Beliefs

On markets where trade takes place, beliefs $\{\gamma_a^i(t,p)\}_{a=L,H}^{i=1,2}$ shall reflect the distribution of sellers choosing this market. A complete description of the exchange requires buyers to form expectations about inactive markets $p \notin \mathcal{P}(t)$ as well. Many pessimistic equilibria can be sustained if buyers believe sellers would supply the L asset in inactive markets. We thus impose a refinement similar to Gale (1996) and Guerrieri et al. (2010). On inactive markets, buyers expect to see asset owners who find it most profitable to deviate to that market. This belief refinement formalized in Point 2 of Definition 4 essentially adapts Cho and Kreps (1987) to a competitive environment. We refer to the papers mentioned above for a more extensive discussion.

Definition 4 (Exchange Equilibrium)**.** *An Equilibrium of the Exchange is given by value functions $\{v_a^i(t)\}_{a=L,H}^{i=1,2}$, distributions $\{\mu_a^i(t)\}_{a=L,H}^{i=1,2}$ a measure $\mu^B(p,t)$ with support $\mathcal{P}(t)$ and total mass $\mu^B(t)$, a rationing function $\theta(t,p) : \mathbb{R}^+ \mapsto \mathbb{R}^+ \cup \{\infty\}$ and belief function $\gamma(t,p) : \mathbb{R}_+ \mapsto \Delta^4$ for any t such that*

1. *Buyers optimality and free entry. For all $p \in \mathbb{R}^+$, $\mathcal{P}(t) = \arg\max_p v^B(t,p) - \kappa$ and $\mu^B(t)$ is determined by (7).*

2. *Sellers optimality. For all $p \in \mathbb{R}^+$, $i = 1,2$ and $a \in \{L,H\}$, $v_a^i(t) \geq v_a^i(t,p)$ with equality if $\theta(t,p) < \infty$ and $\gamma_a^i(t,p) > 0$.*

3. *Market Clearing. For $i = 1,2$ and $a = L,H$*

$$\int_{\mathcal{P}(t)} \frac{\gamma_a^i(t,p)}{\theta(t,p)} \mu^B(t,p)\mathrm{d}p \leq \mu_a^i(t)$$

   with equality if $v_a^i(t) > \tau^i d_a + \delta \bar{v}_a^i(t+1)$

4. *Law of motion : $\{\mu_a^i(t)\}_{a=L,H}^{i=1,2}$ verify (25) and balance conditions (1).*

5. ***Stationary Property** : $\exists T \in \mathbb{N}_+$ such that for all endogenous variables z*

$$z(t+T) = z(t)$$

The definition follows closely Guerrieri et al. (2010) and Guerrieri and Shimer (2014). Active markets $\mathcal{P}(t)$ are those buyers choose to visit. Point 2 formalizes the requirement that sellers choose the market(s) which maximizes their utility. In addition, on markets where $\theta(t,p) < \infty$, buyers should expect to see sellers who are indifferent between that market and their optimal choice. This is formally the refinement we discussed above. Point 3 ensures that supply on active markets is consistent with buyers beliefs. When ome asset owners might find it optimal not to trade, that is $v_a^i(t) = \tau^i d_a + \delta \bar{v}_a^i(t+1)$, they can supply their asset on inactive markets where $\theta(t,p) = 0$. Finally, point 4 says that the mass of owners depends on past trading decisions since asset owners face different levels of rationing on each market. Point 5 is the stationary property already present in Definition 2 for an OTC equilibrium.

5.2 Equilibrium

As in the construction of the OTC equilibrium, an important statistic is the ordering of types of asset owners.

Lemma 5. *In any exchange equilibrium, for all t,*

$$r_L^1(t) \leq r_L^2(t) < r_H^1(t) \leq r_H^2(t) \tag{26}$$

I omit the proof which can be readily adapted from that of Lemma 1 and does not rely on the price formation process. The important information for our analysis is the monotone relationship between types: agents with a lower quality assets are more eager to sell, independently of their private valuation for the asset[32]. In an Exchange, it means that sellers with a higher type in the sense of Lemma 5 would accept (more) rationing to trade at higher prices and signal their quality. When type τ^2 agents hold L assets, which they do to realize gains from trade, Proposition 4 shows that this logic leading to separation has an extreme consequence as no market opens for H assets.

Proposition 4. *The unique exchange equilibrium is a steady state where*

i) Buyers make zero profit $\forall t$, $v_0^2 = 0$.

ii) Owners (τ^1, L) trade in the only open market $\mathcal{P} = \{p_L\}$ where

$$p_L := \frac{\tau d_L - \delta\gamma\kappa}{1-\delta} - \kappa$$

[32] Guerrieri and Shimer (2014) only consider private information about the asset dividend. In Guerrieri and Shimer (2015), the two dimensions actually collapse to one with a monotonicity assumption similar to my lemon condition (LC). Chang (2014) relaxes this assumption to obtain bunching in equilibrium. In my model, while the monotonicity condition follows closely from (LC), it is endogenous.

Other asset owners do not trade. Reservation values for L asset owners are

$$r_L^1 = d_L + \delta p_L + \delta\gamma\kappa$$
$$r_L^2 = p_L + \kappa$$

iii) The rationing function θ satisfies $\theta(p) = \frac{p_L - r_L^1}{p - r_L^1}$ if $p \in [p_L, p_L + \kappa]$ and $\theta(p) = 0$ otherwise. The belief function is

$$(\gamma_L^1, \gamma_H^1 \gamma_L^2, \gamma_H^2)(p) = \begin{cases} (1,0,0,0) & \text{if} \quad p \in [p_L, p_L + \kappa] \\ (0,0,1,0) & \text{if} \quad p > r_L^2 \end{cases}$$

iv) The masses of traders are

$$\begin{cases} \mu_L^1 = \gamma S(1-q) \\ \mu_L^2 = (1-\gamma)S(1-q) \end{cases} \qquad \begin{cases} \mu_H^1 = Sq/2 \\ \mu_H^2 = Sq/2 \end{cases}$$

while equilibrium entry $\mu^B(.,.)$ is an atom of mass μ_L^1 at p_L.

The last part of the result shows that the mass of buyers is equal to the mass of asset owners who sell in equilibrium, that is (τ^1, L) agents. Since asset owners can signal their willingness to trade, search from buyers is not random as in the OTC market. Competition between buyers drives equilibrium entry and search costs to the minimal level to support trade[33]. However, the existence of different prices with different level of rationing allows agents to signal the quality of their asset. The first consequence is that a pooling price p cannot be sustained as otherwise, H asset owners would want to deviate to a higher price $p' > p$. Higher rationing at that price, $\theta(., p') > \theta(., p)$ makes this signal credible for buyers who would then propose price p', a logic similar to the cream-skimming deviation in strategic models. The equilibrium is thus separating. In this environment, equilibrium rationing of (τ^1, H) owners is extreme since they do not trade: liquidity π is 0. Indeed, the monotonicity result $r_L^2 < r_H^1$ in Lemma 5 shows that (τ^2, L) agents would then like to trade in any market chosen by type (τ^1, H) where $p > r_H^1$. These L asset owners with no gains from trade thus block trading for (τ^1, H) agents. As a result, misallocation is severe as half of the H assets are held by low valuation τ^1 agents. Selling pressure is large but

[33]The result that $\mu^B = \mu_L^1$ exactly comes from the matching function and the positive search costs $\kappa > 0$. If $\kappa = 0$, there could additional entry at no aggregate cost.

does not lead to trade.

Both components of private information are important for the result. When buyers only ignore the quality of the asset proposed, a similar logic leads to separation between (τ^1, L) and (τ^1, H) agents but the latter trade with positive probability as long as $d_L > 0$. Otherwise, the so-called no gap at the bottom condition of the Akerlof (1970) model shut downs all trade. Asymmetry of information about valuation creates a middle type (τ^2, L) which prevents trade of the H asset. The lemon condition $\tau d_L < d_H$ is much weaker than $d_L = 0$ but also generates trading freeze of the H asset while the low quality market functions smoothly. In the OTC market as well, bi-dimensional private information reduces liquidity since buyers face disproportionately more L assets. However, ex-post offers protect buyers from cream-skimming deviations who may thus propose a pooling price if the share of H assets is high enough. Interestingly, the inter-temporal *competition effect* reinforces buyers' incentives to pool sellers in the OTC market and increases liquidity but intra-temporal competition reduces liquidity.

5.3 Welfare

We formalize the discussion above by comparing aggregate surplus across market structures. Let $W_E(q)$ be our measure of welfare in the unique stationary equilibrium of the exchange. In the OTC market, we focus on the steady state equilibrium and denote welfare by $W_{OTC}(q)$. This is the unique stationary equilibrium for $q \in [0, \underline{q}_2) \cup (\bar{q}, 1]$. In $q \in [\underline{q}_2, \bar{q}]$, our results would change quantitatively but not qualitatively when considering the cycle. The aggregate surplus gains from trading in an exchange rather than in the OTC market as a function of q are:

$$W_E(q) - W_{OTC}(q) = \frac{\mu^B(\pi(q)) - \gamma S(1-q)}{1-\delta}\kappa - \frac{Sq - 2\mu_H^1(\pi(q))(1-\pi(q))}{2(1-\delta)}(\tau - 1)d_H \quad (27)$$

The first term is positive and captures the gains from improving the meeting process with price posting. These gains have two sources. First, with random search, it must be that at least S buyers enter to match all sellers. Second, OTC equilibrium entry $\mu^B(\pi(q))$ might even be higher because buyers do not compete in price. In the Exchange, the price adjusts so that entry matches exactly the mass of asset owners that are selling. The second term is negative and measures the difference in misallocation of the H asset. While half of the H assets are not properly allocated in the Exchange, this fraction falls to $\mu_H^1(\pi(q))(1-\pi(q))/S$ in the OTC market since buyers may offer a pooling price

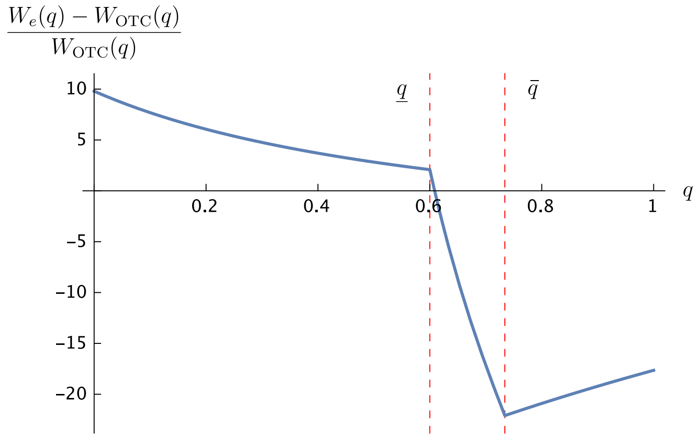


Figure 8: Welfare: Exchange vs. OTC (%)
Parameter Values: $\delta = 0.8$, $\gamma = 0.05$, $d_L = 2$, $d_H = 6$, $\tau = 2$

$(\pi(q) > 0)$. The following proposition derives the sign of expression (27) as a function of the share of H assets q.

Proposition 5. *There exists $q_W \in [\underline{q}, \bar{q}]$ such that $W_E(q) - W_{OTC}(q) \geq 0$ for $q \leq q_W$ and $W_E(q) - W_{OTC}(q) \leq 0$ otherwise.*

Proposition 5 states that when the share of H assets is above the threshold q_W aggregate surplus is higher in the OTC market. When $q \leq \underline{q}$, the OTC equilibrium is also separating $(\pi(q) = 0)$ and only lemons are traded. Hence only the component related to trading costs plays a role and favors the Exchange. Pooling arises in the equilibrium of the OTC market when $q \geq \underline{q}$ and reduces the misallocation of H assets - the second term in (27). The threshold q_W is such that the realized gains from trade on the H assets overcome the difference in trading costs. Figure 8 plots the welfare difference of equation (27) in percentage as a function of q for specific numeric values of the other parameters. The dashed red lines delimit the region where the OTC steady state equilibrium is in mixed strategy, that is $q \in [\underline{q}, \bar{q}]$. In that region, the welfare difference becomes negative and decreases steeply because H assets start trading and the difference in trading costs per unit traded decrease with intertemporal competition in the OTC market. Observe that in the region where the OTC equilibrium is pooling in pure strategy, that is $[\bar{q}, 1]$, the welfare difference might increase. Although trade in the OTC market improves the allocation of the increasing mass of H quality assets, the lack of intertemporal competition for that asset also raises trading costs significantly. Still, the first effect dominates and in the limit $W_E(1) - W_{OTC}(1) < 0$.

Our analysis thus emphasizes the ambiguous role of pre-trade information for market efficiency in the presence of asymmetry of information. The centralization of the trading platform saves on trading costs as buyers do not have to search for potential sellers. Sellers in turn may observe all buyers' offer simultaneously rather than sequentially through search which reduces the inefficiencies due to monopoly pricing in the OTC market. However, competition lowers liquidity as sellers are now able to signal their type which hinders the correct allocation of high quality assets[34].

6 Conclusion

This paper presents a theory of endogenous liquidity fluctuations based on asymmetry of information and re-trade in secondary markets. I show that Over the Counter Market are prone to fluctuations where prices and trading volume vary in the absence of aggregate shocks. Equilibrium cycles are inefficient because of the dynamic externality attached to the composition effect. Hence although market conditions will eventually improve, it is desirable to bring liquidity forward in the short-run and stabilize the market in the long-run. I show that a revertible asset purchase program can achieve these objectives. However, our analysis highlights several limitations. First, the government asset purchase program interferes with the efficient allocation of assets in the economy. Indeed, private agents also value assets for their convenience to realize transactions while the government does not. In addition, absent taxation, the government runs a loss because he must pay a premium to convince asset owners to part with their assets. The need to finance the intervention interacts with the design of the policy and mitigates its effect. The last part of the paper draws mixed conclusions about the current set of structural reforms of the OTC market. When buyers may post prices before meetings, matching is more efficient to bring buyers and sellers together but high quality assets become illiquid. Hence, the lack of competition and information in OTC markets can be desirable to foster liquidity.

This analysis still leaves many interesting questions open for policy design in asset markets with adverse selection. I imposed an immediate resale constraint to capture realistic constraint for policy makers in a tractable way. In practice, the timing and the pace of these exit strategies seems important. In addition, the supply of asset is fixed and constant in my model while it could react endogenously to liquidity in secondary

[34]This comparison leaves open the question of the endogenous choice of platforms by traders. Observe however that in our environment, (τ^1, H) sellers do not gain from trade. In the OTC market, the trading price is at most equal to their reservation value while they fail to trade in the Exchange. Hence pure OTC and pure Exchange market structures can be interpreted as a particular selection of equilibrium in the larger game where traders would choose their platform.

markets. On the issue of competition and information, the comparison between OTC and Exchange highlights a negative role of transparency on liquidity. More work is needed to understand the coexistence of platforms with different degree of competition and opacity and its implication for efficiency. ■

Appendices

A Assumptions

A.1 Upper bound on search costs

I give the expression for the upper bound $\bar{\kappa}(\gamma, \delta, \tau, d_L, d_H)$ of the search cost κ in Assumption (A2). The condition ensure that buyers find it profitable to enter if they match for sure $\lambda^B = 1$ and face the least favorable prospects. Building on expression (14), we can provide a lower bound for the net gains from entering the market. Precisely

$$v^B \geq M(q) := \max \Bigg\{ \gamma(1-q)(\tau-1)d_L, \gamma q \frac{\tau - 1}{1-\delta(1-2\gamma)} d_H \\ -(1-q)\frac{(1-\delta)(d_H - \tau d_L) + \delta\gamma(\tau+1)(d_H - d_L}{(1-\delta)\big[1-\delta(1-2\gamma)\big]} \Bigg\}$$

Intuitively, the first (resp. second) argument is the minimum possible payoff from a separating (resp. pooling) offer. Hence we define $\bar{\kappa}(\gamma, \delta, \tau, d_L, d_H) := \min_q M(q)$. It is easy to check that this expression is strictly positive if $d_L > 0$ and $\gamma > 0$.

B Proofs

B.1 Proof of Lemma 1

Part 1

We want to show that for any (τ^i, τ^j, a, a') such that $\tau^i d_a \geq \tau^j d_{a'}$, we have $r_a^i(t) \geq r_a^j(t)$. Anticipating equilibrium free entry condition, we know that non-owners make zero profit so that $r_a^i(t) = \tau^i d_a + \delta \bar{v}_a^i(t)$. Hence, we establish the following sufficient condition for the result: $\bar{v}_a^i(t) \geq \bar{v}_a^j(t)$. For this proof, let $\pi(t,p)$ be the probability that an owner receive offer p in period t. Since the matching technology is symmetric and buyers ignore seller's type, $\{\pi(t,p)\}_p$ is the same across asset owners. By optimality, agent (i, a) obtains a higher utility than if he behaves from t on-wards like agent (j, a') for $j \in \{i, -i\}$. Hence,

$$v_a^i(t) \geq r_a^i(t) + \lambda^S(t) \sum_{p \in \Gamma(t)} \pi(t,p) \alpha_{a'}^j(t,p)[p - r_a^i(t)]$$

where $\alpha_{a'}^j(t,p)$ is the acceptance probability of type (j,a'). Hence

$$v_a^i(t) - v_{a'}^j(t) \geq r_a^i(t) - r_{a'}^j(t) - \lambda^S(t) \sum_{p \in \Gamma(t)} \pi(t,p)\alpha_{a'}^j(t,p)[r_a^i(t) - r_{a'}^j(t)]$$

Using the expression for $r_a^i(t)$ and denoting $f_{a'}^j(t) = \lambda^S(t) \sum_{p\in\Gamma(t)} \pi(t,p)\alpha_{a'}^j(t,p) \leq 1$ we obtain

$$\begin{aligned} v_a^i(t) - v_{a'}^j(t) \geq & (1 - f_{a'}^j(t))(\tau^i d_a - \tau^j d_{a'}) + \delta(1 - f_{a'}^j(t))(\bar{v}_a^i(t) - \bar{v}_{a'}^j(t)) \\ = & (1 - f_{a'}^j(t))(\tau^i d_a - \tau^j d_{a'}) + \delta(1-\gamma)(1 - f_{a'}^j(t))(v_a^i(t+1) - v_{a'}^j(t+1)) \\ & + \delta\gamma(1 - f_{a'}^j(t))(v_a^{-i}(t+1) - v_{a'}^{-j}(t+1)) \end{aligned}$$

Hence we can rewrite the following expression as

$$\begin{bmatrix} v_L^2(t) - v_L^1(t) \\ v_H^1(t) - v_L^2(t) \\ v_H^2(t) - v_H^1(t) \end{bmatrix} \geq \begin{bmatrix} (\tau-1)d_L \\ d_H - \tau d_L \\ (\tau-1)d_H \end{bmatrix} + \delta M(t) \begin{bmatrix} v_L^2(t+1) - v_L^1(t+1) \\ v_H^1(t+1) - v_L^2(t+1) \\ v_H^2(t+1) - v_H^1(t+1) \end{bmatrix} \tag{28}$$

where

$$M(t) = \begin{bmatrix} (1 - f_L^1(t))(1-2\gamma) & 0 & 0 \\ \gamma(1 - f_L^2(t)) & 1 - f_L^2(t) & \gamma(1 - f_L^2(t)) \\ 0 & 0 & (1 - f_H^1(t))(1-2\gamma) \end{bmatrix}$$

Iterating on the inequality above and using the transversality condition $\lim_{t\to\infty} \delta^t v_a^i(t) = 0$, it follows that the left hand side of (28) is positive. It is then straightforward to show that the result extends to reservation values.

Part 2

From subgame perfection of the bargaining game, seller (τ^i, a) strategy is simply $\alpha_a^i(t,p) = \mathbf{1}_{p \geq r_a^i(t)}$ for $i = 1,2$ and $a = L, H$. A seller accepts any offer weakly above his reservation value. It follows immediately that a buyer may only offer one of these reservation values. To characterize the buyers' offer, let us rewrite the buyer's problem (3) as

$$Supp\left(\Pi^k(t,.)\right) = \arg\max_p \left\{ \sum_{\substack{i=1,2 \\ a=L,H}} \frac{\mu_a^i(t)}{S} \alpha_a^i(t,p)\big(r_a^k(t,p) - p\big) \right\} + \delta \bar{v}_0^k(t+1)$$

Consider first an agent $(\tau^1, 0)$. Since $r_a^2(t) - r_a^1(t) \geq 0$ for $a = L, H$, this agent weakly prefers not to make an offer. This means that not participating in the market is a strictly dominant strategy since searching costs $\kappa > 0$. Let us turn now to type $(\tau^2, 0)$ agents. Using law of motion (13)-(12), observe first that $\mu_a^1(t) > 0$ for $a = L, H$. In any period, there is a strictly positive mass of each type of asset owners due to the type switching process. We argue now that offer

$r_a^2(t)$ is strictly dominated by offer $r_a^1(t)$. Indeed, by lowering his offer, the buyer makes a strictly greater profit on all types (τ^i, a') for which $r_{a'}^i(t) \leq r_a^1(t)$. In addition, while the lower offer fails to attract (τ^2, a) agents anymore, the buyer was breaking even on this group with the higher offer. Hence the only possible offers are $r_L^1(t)$ and $r_H^1(t)$.

B.2 Proof of Lemma 2

Point i). Since agent (τ^2, H) never trades and agent (τ^1, H) only receives offer (weakly) below his reservation value. By stationarity, we can drop the time argument in dynamic equation (8) to obtain the following system

$$r_H^1 = d_H + \delta\big[(1-\gamma)r_H^1 + \gamma r_H^2(t+1)\big]$$
$$r_H^2 = d_H + \delta\big[(1-\gamma)r_H^2 + \gamma r_H^1(t+1)\big]$$

Straightforward manipulations give:

$$r_H^i = \frac{(1-\delta)\tau^i + \delta\gamma(\tau+1)}{(1-\delta)\big[1-\delta(1-2\gamma)\big]} d_H, \quad i = 1, 2$$

Point ii). From the free entry condition (7) and dynamic equations (10)-(11) we obtain for $i = 1, 2$

$$v_0^i(t) = \delta \bar{v}_0^i(t+1)$$

By stationary property 5 of Definition 2 , it is immediate that for all t, and $i \in \{1, 2\}$, $v_0^i(t) = 0$.

Point iii). We want to show that in equilibrium $\mu^B(t) \geq S$. Suppose that the opposite inequality holds. Then the net profit from searching for a type τ^2 agent is $-\kappa + v^B(t)$ since the matching probability is 1 for a buyer. If this expression is strictly positive, buyers would enter and since there is no rivalry in the matching technology as long as $\mu^B(t) \leq S$, entry would be $\mu^B(t) \geq S$, proving the conjecture wrong. Using expression (6), a lower bound on $v^B(t)$ is

$$v^B(t) \geq \max\Bigg\{\gamma(1-q)(\tau-1)d_L, \gamma q \frac{\tau-1}{1-\delta(1-2\gamma)} d_H$$
$$-(1-q)\frac{(1-\delta)(d_H - \tau d_L) + \delta\gamma(\tau+1)(d_H - d_L}{(1-\delta)\big[1-\delta(1-2\gamma)\big]}\Bigg\} := M(q)$$

Our assumption that $\kappa \geq \min_q M(q) = \bar{\kappa}(\gamma, \delta, \tau, d_L, d_H)$ ensures that $-\kappa + v^B(t) > 0$ so that in equilibrium $\mu^B(t) \geq S$, that is $\lambda^S(t) = 1$.

B.3 Proof of Lemma 3

Observe first that $\pi(t)$ weakly increases when $v^B(t,1) - v^B(t,0)$ increases.

Competition effect

From equation (14), $v^B(t,1) - v^B(t,0)$ increases with $\{r_L^i(t)\}^{i=1,2}$. For $i = 1,2$, $r_L^i(t) = \tau^i d_L + \delta \bar{v}_L^i(t+1)$ where $\bar{v}_L^i(t+1) = (1-\gamma)v_L^i(t+1) + \gamma v_L^j(t+1)$ for $j \neq i$. From dynamic equations (9), we have

$$v_L^i(t+1) = \pi(t+1)r_H^1 + (1-\pi(t+1))\big[\tau^i d_L + \bar{v}^i(t+2)\big]$$

Since $r_H^1 \geq r_L^i(t+1) := \tau^i d_L + \bar{v}^i(t+2)$ for $i = 1,2$ by Lemma 1, the result follows.

Composition effect

From equation (14), $v^B(t,1) - v^B(t,0)$ increases with $\mu_H^1(t)$. From law of motion (12), we have

$$\mu_H^1(t) = \gamma Sq + (1-2\gamma)(1-\pi(t-1))\mu_H^1(t-1)$$

Since $\gamma < 1/2$, $1 - 2\gamma > 0$ and thus $\mu_H^1(t)$ decreases with $\pi(t-1)$.

B.4 Proof of Lemma 4

$\triangleright$ The proof is by contradiction. Let t_0 be such that $\pi(t_0) = 1$. Suppose then that $\pi(t_0+1) = 1$. We want to establish that $\pi(t_0+2) = 1$. Then by induction, it means that for all $t \geq t_0$, $\pi(t) = 1$, a contradiction with having a cycle of period $T \geq 2$.

Using law of motion (12), we have that $\mu_H^1(t_0+1) = \gamma Sq$, that is the distribution of assets is least favorable to a pooling offer in period t_0+1. Since $\pi(t_0+1) = 1$, we also have $\mu_H^1(t_0+2) = \gamma Sq$. Suppose now $\pi(t_0+2) = 0$. Then , the sequence of offers $\{\pi(t)\}_{t>t_0+1}$ is weakly less favorable to a pooling offer than the sequence $\{\pi(t)\}_{t>t_0+2}$ since it starts with 0. Hence, using Lemma 3, since $\pi(t)$ is weakly increasing in $\{\pi(t+l\}_{l=1,..,\infty}$, if $\pi(t_0+1) = 1$, it must be that $\pi(t_0+2) = 1$, a contradiction. $\triangleleft$

B.5 Proof of Proposition 1

In the following, we characterize a T period cycle and then derive conditions for it to be an equilibrium. Let us first write down the endogenous variables in the conjectured equilibrium. Using the results in the main text, we have

$$\mu_{H,T}^1(t) = \frac{1-(1-2\gamma)^t}{2}Sq, \qquad t = 1,..,T$$

Reservation values $\{r^i_{L,T}(.)\}^{i=1,2}$ verify the following equations:

$$r^i_{L,T}(t) = \begin{cases} \tau^i d_L + \delta\big[(1-\gamma)r^i_{L,T}(t+1) + \gamma r^j_{L,T}(t+1)\big] & \text{if} \quad t = 0, ..., T-2 \\ \tau^i d_L + \delta r^1_H & \text{if} \quad t = T-1 \end{cases}$$

where $j \neq i$. We obtain for $t = 0, .., T-1$

$$\begin{aligned} r^1_{L,T}(t) + r^2_{L,T}(t) &= \frac{1-\delta^{T-t}}{1-\delta}(\tau+1)d_L + 2\delta^{T-t} r^1_H \\ r^2_{L,T}(t) - r^1_{L,T}(t) &= \frac{1-\big[\delta(1-2\gamma)\big]^{T-t}}{1-\delta(1-2\gamma)}(\tau-1)d_L \end{aligned}$$

from which we get for $t = 0, .., T-1$

$$\begin{aligned} r^1_{L,T}(t) &= \left[\frac{1-\delta^{T-t}}{1-\delta}(\tau+1) - \frac{1-(\delta(1-2\gamma))^{T-t}}{1-\delta(1-2\gamma)}(\tau-1)\right]\frac{d_L}{2} + \delta^{T-t} r^1_H \\ r^2_{L,T}(t) &= \left[\frac{1-\delta^{T-t}}{1-\delta}(\tau+1) + \frac{1-(\delta(1-2\gamma))^{T-t}}{1-(\delta(1-2\gamma))}(\tau-1)\right]\frac{d_L}{2} + \delta^{T-t} r^1_H \end{aligned}$$

Since for all t and $i = 1, 2$, $r^i_{L,T}(t) \leq r^1_H$, we obtain by backward induction that

$$r^i_{L,T}(t) \leq r^i_{L,T}(t+1), \qquad t = 0, .., T-2$$

The net gain from a pooling offer writes

$$v^B_T(t,1) - v^B_T(t,0) = \mu^1_H(t)(r^2_H - r^1_H) - S(1-q)\big[r^1_H - (1-\gamma)r^2_{L,T}(t) - \gamma r^1_{L,T}(t)\big]$$

The conjecture is an equilibrium if this expression is strictly negative in periods $t = 1, .., T-1$ and strictly positive in period 0. From the analysis above, this expression is increasing over $[|1, T-1|]$. Hence, we need only to verify that $v^B_T(0,1) - v^B_T(0,0) > 0$ and $v^B_T(T-1,1) - v^B_T(T-1,0) < 0$. These conditions are respectively equivalent to

$$\begin{aligned} q \geq \underline{q}_T &:= \frac{2(r^1_H - r^2_{L,T}(0)) + 2\gamma\big(r^2_{L,T}(0) - r^1_{L,T}(0)\big)}{\big[1-(1-2\gamma)^T\big](r^2_H - r^1_H) + 2(r^1_H - r^2_{L,T}(0)) + 2\gamma\big(r^2_{L,T}(0) - r^1_{L,T}(0)\big)} \\ q \leq \bar{q}_T &:= \frac{2(r^1_H - r^2_{L,T}(T-1)) + 2\gamma\big(r^2_{L,T}(T-1) - r^1_{L,T}(T-1)\big)}{\big[1-(1-2\gamma)^{T-1}\big](r^2_H - r^1_H) + 2(r^1_H - r^2_L(T-1)) + 2\gamma\big(r^2_{L,T}(T-1) - r^1_{L,T}(T-1)\big)} \end{aligned}$$

Hence the T periods cycle exists if and only if $\underline{q}_T \leq \bar{q}_T$ that is

$$\frac{1-(1-2\gamma)^T}{1-(1-2\gamma)^{T-1}} \geq \frac{r^1_H - r^2_{L,T}(0) + \gamma\big(r^2_{L,T}(0) - r^1_{L,T}(0)\big)}{(1-\delta)r^1_H - \tau d_L + \gamma(\tau-1)d_L} \tag{E_T}$$

The LHS decreases with T. On the RHS, the denominator does not depend on T while the numerator is equal to $r_H^1 - (1-\gamma)r_{L,T}^2(0) - r_{L,T}^1(0)$ and increases in T. This proves that for $T' \geq T$ $(E_{T'}) \Rightarrow (E_T)$

It is clear that $\bar{q}_T$ only depends on T through the first term of the denominator $1-(1-2\gamma)^{T-1}$. It is then immediate that the sequence $\{\bar{q}_T\}$ is decreasing in T. Cumbersome but straightforward computations show that $\bar{q}_2 = \bar{q}$ where $\bar{q}$ is defined in Proposition 2. Finally, we are left to show that $\bar{q}_{T+1} \leq \underline{q}_T$ for $T \geq 2$. From, the expression above, this is true if $r_{L,T+1}^i(T) \geq r_{L,T}^i(0)$ for $i = 1, 2$. By definition, we have $r_{L,T+1}^i(T) = r_{L,T}^i(T-1)$ and the result follows from the monotonicity of $r_{L,T}^i$.

B.6 Proof of Proposition 2

To prove Proposition 1, we proceed as follows. First, we write all endogenous variables as a function of π. Then we solve for a fixed point equation in π.

Step 1

Using law of motion (12), we obtain

$$\begin{aligned}\mu_H^1(\pi) &= \gamma Sq + (1-2\gamma)(1-\pi)\mu_H^1(\pi)\\ &= \frac{(1-\gamma)\pi + (1-\pi)\gamma}{\pi + 2\gamma(1-\pi)} Sq\end{aligned}$$

We determine the reservation values for L asset owners $(r_L^1(\pi), r_L^2(\pi))$ which solve

$$\begin{aligned}r_L^1(\pi) &= d_L + \delta\Big(\pi r_H^1 + (1-\pi)\big[(1-\gamma)r_L^1(\pi) + \gamma r_L^2(\pi)\big]\Big)\\ r_L^2(\pi) &= \tau d_L + \delta\Big(\pi r_H^1 + (1-\pi)\big[(1-\gamma)r_L^2(\pi) + \gamma r_L^1(\pi)\big]\Big)\end{aligned}$$

Hence

$$\begin{aligned}r_L^2(\pi) + r_L^1(\pi) &= (\tau+1)d_L + 2\delta\pi r_H^1 + \delta(1-\pi)\big[r_L^2(\pi) + r_L^1(\pi)\big]\\ r_L^2(\pi) - r_L^1(\pi) &= (\tau-1)d_L + \delta(1-\pi)(1-2\gamma)(r_L^2(\pi) - r_L^1(\pi))\end{aligned}$$

From which we obtain for $i = 1, 2$

$$r_L^i(\pi) = \tau^i d_L + \frac{1}{1-(1-\pi)\delta}\left[\pi\delta r_H^1 + (1-\pi)\delta\left(\tau^i d_L + \gamma\frac{\tau^j - \tau^i}{1-\delta(1-\pi)(1-2\gamma)} d_L\right)\right]$$

Step 2

Using the buyer's problem (14), with a slight abuse of notation, let us write $v^B(\pi, \hat{\pi})$ where the first argument is the strategy played by other buyers and the second argument is the strategy of an individual buyer. An equilibrium π must verify

$$\pi = \begin{cases} 0 & \text{if } v^B(0,0) \geq v^B(0,1) \\ \in (0,1) & \text{if } v^B(\pi,0) = v^B(\pi,1) \\ 1 & \text{if } v^B(1,1) \geq v^B(1,0) \end{cases} \tag{29}$$

We can first characterize too cutoffs $(\underline{q}, \bar{q})$ for the existence of the pure strategy equilibria $\pi^* = 0$ and $\pi^* = 1$. Plugging the expressions obtained above, we have

$$\begin{aligned} v^B(0,1) - v^B(0,0) = & \frac{q}{2}\frac{\tau - 1}{1 - \delta(1-2\gamma)} d_H - (1-q)\frac{(1-\delta)(d_H - v d_L) + \delta\gamma(v+1)(d_H - d_L)}{(1-\delta)(1-\delta(1-2\gamma))} \\ & - \gamma(1-q)\frac{\tau - 1}{1-\delta(1-2\gamma)} d_L \end{aligned}$$

and thus $v^B(0,1) - v^B(0,0) \leq 0$ if and only if

$$\begin{aligned} & q(1-\delta)(\tau-1)d_H - 2(1-q)(1-\delta)(d_H - \tau d_L) - 2(1-q)\delta\gamma(\tau+1)(d_H - d_L) \\ & - 2\gamma(1-q)(1-\delta)(\tau-1)d_L \leq 0 \end{aligned}$$

We thus obtain the threshold $\underline{q}$ introduced in the main text:

$$\begin{aligned} & \underline{q} = \frac{2a}{(1-\delta)(\tau-1)d_H + 2a} \\ & a = (1-\delta)(d_H - \tau d_L) + \gamma\delta(\tau+1)(d_H - d_L) + \gamma(1-\delta)(\tau-1)d_L \end{aligned}$$

Similarly, we have

$$v^B(1,1) - v^B(1,0) = \gamma q \frac{\tau-1}{1-\delta(1-2\gamma)} d_H - (1-q)\left[\frac{1-\delta+\delta\gamma(\tau+1)}{1-\delta(1-2\gamma)} d_H - \tau d_L\right] - \gamma(1-q)(\tau-1)d_L$$

and thus $v^B(1,1) \geq v^B(1,0)$ if and only if

$$\begin{aligned} & \gamma q(\tau-1)d_H - (1-q)(1-\delta)d_H - (1-q)\delta\gamma(\tau+1)d_H + (1-q)(1-\delta(1-2\gamma))\tau d_L \\ & - \gamma(1-q)(1-\delta(1-2\gamma))(\tau-1)d_L \geq 0 \end{aligned}$$

We obtain

$$\bar{q} = \frac{a-b}{\gamma(\tau-1)d_H + a - b}, \quad \text{where} \quad b = \delta\gamma(1-2\gamma)(\tau-1)d_L$$

To derive mixed strategy equilibria, let us focus on the case $\underline{q} < \bar{q}$ (a similar argument applies when the multiplicity condition holds). Then for $q \in (\underline{q}, \bar{q})$, we have $v^B(0,1) - v^B(0,0) > 0$ and $v^B(1,1) - v^B(1,0) < 0$. Hence by continuity of $v^B(.,1) - v^B(.,0)$ in its first argument, there exists $\pi(q) \in (0,1)$ such that $v^B(\pi(q),0) = v^B(\pi(q),1)$. We are left to prove uniqueness of this mixed strategy equilibrium. We have

$$v^B(\pi,1) - v^B(\pi,0) = \frac{\gamma Sq}{2\gamma + \pi(1-2\gamma)}(r_H^2 - r_H^1) - S(1-q)\frac{(1-\delta)r_H^1 - (\gamma + (1-\gamma)\tau)d_L}{1-(1-\pi)\delta} - S(1-q)\frac{(1-\pi)(1-2\gamma)(\tau-1)\delta\gamma d_L}{\left[1-(1-\pi)\delta\right]\left[1-\delta(1-\pi)(1-2\gamma)\right]}$$

The expression above shows that the zeros of $v^B(.,1) - v^B(.,0)$ are solutions to a second order equation in π which may have at most 2 real roots. Hence, the second root cannot belong to $(0,1)$ as otherwise the expression would need to change sign twice on $(0,1)$ and have a third root. This concludes the proof.

B.7 Proof of Proposition 3

Using equation (19), we obtain the following expression for steady state welfare:

$$W_{ss}(q) = \frac{S(1-q)\tau d_L + Sq\tau d_H}{1-\delta} - \frac{(1-\pi(q))\mu_H^1(\pi(q))(\tau-1)d_H + \mu_{ss}^B\kappa}{1-\delta}$$

For the high date of the cycle we obtain:

$$W_{cy}(0,q) = \frac{S(1-q)\tau d_L + Sq\tau d_H}{1-\delta} - \frac{\delta\gamma Sq(\tau-1)d_H + \mu_{cy}^B(0)\kappa + \delta\mu_{cy}^B(1)\kappa}{1-\delta^2}$$

Since buyers make zero profit in equilibrium, trade costs cover the gains from trade that is $\mu^B(t)\kappa = Sv^B(t)$ using equation (7). in equilibrium. In steady state, the trade costs are thus equal to $\mu_{ss}^B\kappa = \gamma S(1-q)(r_L^2(\pi(q)) - r_L^1(\pi(q)))$. In a cycle, trade costs are respectively $\mu_{cy}^B(1)\kappa = \gamma S(1-q)(\tau-1)d_L$ in the trough and $\mu_{cy}^B(0)\kappa = \mu_{H,C}^1(0)(r_H^2 - r_H^1) + S(1-q)(r_{L,cy}^2(0) - r_H^1) > \mu_{cy}^B(1)\kappa$ at the peak. Hence we obtain

$$W_{ss,q} - W_{cy}(0,q) > \frac{1}{1-\delta}\left[\left(\frac{\delta}{1+\delta} - \frac{1-\pi(q)}{2\gamma + \pi(q)(1-2\gamma)}\right)\gamma Sq(\tau-1)d_H + (\mu_{cy}^B(1) - \mu_{ss}^B)\kappa\right]$$

As $q \to \bar{q}$, $\pi(q) \to 1$. Hence $\mu_{ss}^B \to \mu_{cy}^B(1)$. The second term in the brackets thus converges to 0. The first term however is bounded away from 0 as q converges towards $\bar{q}$. This proves the result.

B.8 Proof of Proposition 4

The proof is in two steps. First, I show that there cannot be a market where $\theta(t,p) > 0$ and $\gamma_H^i(t,p) > 0$. Finally, I show that the allocation in Proposition 4 is the only equilibrium possible.

Step 1

Observe first that in a stationary equilibrium, it must be that $\mu_L^2(t) > 0$ since agents have a positive probability to switch type The argument is by contradiction. Observe first that $\max \mathcal{P}(t) < r_H^2(t)$. Indeed, the maximum price buyers will pay for an asset is $r_H^2(t) - \kappa$, that is the value of a H asset minus the search cost. Hence, (τ^2, H) will never sell their asset. Practically, they choose a market $p > r_H^2(t)$ where $\theta(t,p) = 0$. Define now

$$\mathcal{P}_H(t) = \{p \in \mathcal{P}(t) \mid \gamma_H^1(t,p) > 0\}$$

The set $\mathcal{P}_H(t)$ is the set of active markets where H assets are for sale. We want to show that $\mathcal{P}_H(t) = \emptyset$. By (τ^1, H) sellers optimality condition, we have $\min \mathcal{P}_H(t) \geq r_H^1(t)$. Let $\bar{p}_H(t) = \max \mathcal{P}_H(t) = \max \mathcal{P}(t)$.

Suppose first that $\gamma_H^1(t,p) = 1$. It must be that (τ^2, L) sellers are at most indifferent about trading at that price. Let thus be

$$\bar{p}_L^2(t) = \max\{p \in \mathcal{P}(t) \mid \gamma^2(t,p) > 0\} < \bar{p}_H(t)$$

Market $\bar{p}_L^2(t)$ is the maximum price at which agents (τ^2, L) trade. For $\bar{p}_L^2(t)$ to be optimal, it must be that:

$$\lambda^S(t, \bar{p}_L^2(t))(\bar{p}_L^2(t) - r_L^2(t)) \geq \lambda^S(t, \bar{p}_H(t))(\bar{p}_H(t) - r_L^2(t))$$

In particular, we must have $\bar{p}_L^2(t) > r_L^2(t)$. Since $r_L^2(t) - \kappa$ is the maximum price a L asset can command, it must be that $\gamma_H^1(t,p') > 0$. By seller's optimality, agents (τ^1, H) must weakly prefer market $p_L^2(t)$ to any market $p' > \bar{p}_L^2(t)$, that is

$$\lambda^S(t, \bar{p}_L^2(t))(\bar{p}_L^2(t) - r_H^1(t)) \geq \lambda^S(t,p')(p' - r_H^1(t))$$

This is only possible if $\lambda^S(t,p') < \lambda^S(t, \bar{p}_L^2(t)) \leq 1$. Then since $r_L^2(t) < r_H^1(t)$, agents (τ^2, L) strictly prefer market $\bar{p}_L^2(t)$ over p'. Hence, using Part 2 of the Equilibrium definition, $\gamma_H^1(t,p') = 1$. Let us now write buyers profit in market $\bar{p}_L^2(t)$ and $p' > \bar{p}_L^2(t)$

$$\begin{aligned} v_0^2(t, \bar{p}_L^2(t)) &= -\kappa + \lambda^B(t, \bar{p}_L^2(t))\big[(\gamma_L^1(t) + \gamma_L^2(t,p))r_L^2(t) + \gamma_H^1(t,p)r_H^2(t) - \bar{p}_L^2(t)\big] \\ v_0^2(t,p') &= -\kappa + r_H^2(t) - p' \end{aligned}$$

Hence,

$$\lim_{p' \to \bar{p}_L^2(t)} v_0^2(t,p') > v_0^2(t, \bar{p}_L^2(t))$$

which is incompatible with buyers' optimality condition.

But if we suppose now that $\gamma_H^1(t, \bar{p}_H) < 1$, the same argument applies. Hence, we have shown that $\mathcal{P}_H(t) = \emptyset$.

Step 2

We have shown that only (τ^1, L) asset owners might trade. To conclude the proof we must derive the equilibrium price $p_L(t)$ for trade as well as equilibrium entry from non-owners. With free entry, buyers make zero profit so that $v_0^2(t, p_L) = 0$. This implies that

$$p_L(t) = \tau d_L + \delta \bar{v}_L^2(t+1) - \kappa$$

Asset owners (τ^1, L) find a match for sure while (τ^2, L) asset owners do not trade, so that

$$\begin{aligned} v_L^1(t) &= p_L(t) \\ v_L^2(t) &= \tau d_L + \delta v_L^2(t+1) \end{aligned}$$

Using these equations together with the stationary condition of Definition 4, we obtain that (p_L, v_L^2) are constant over time and equal to

$$\begin{aligned} v_L^2 &= \frac{\tau - \delta\gamma\kappa}{1-\delta} d_L \\ p_L &= v_L^2 - \kappa \end{aligned}$$

Finally, non-owners make zero profit upon entering if and only if $\lambda^B(t) = 1$. Hence $\mu^B(t) = \mu_L^1(t) = \gamma S(1-q)$.

B.9 Proof of Proposition 5

Consider first the case where $q \in [0, \underline{q}]$. In this region, expression (27) become:

$$W_E(q) - W_{OTC}(q) = \frac{\gamma S(1-q)}{1-\delta} \left[\frac{\tau - 1}{1 - \delta(1-2\gamma)} - \kappa \right]$$

which is positive and decreasing in q. When $q \in [\underline{q}, \bar{q}]$, we obtain

$$\begin{aligned} W_E(q) - W_{OTC}(q) = & \frac{\gamma S(1-q)}{1-\delta} \left[\frac{\tau - 1}{1 - \delta(1-\pi(q))(1-2\gamma)} - \kappa \right] \\ & - \frac{Sq\pi(q)}{(1-\delta)\left[2\gamma + \pi(q)(1-2\gamma)\right]} (\tau - 1) d_H \end{aligned}$$

We have that $\pi(.)$ is strictly increasing in q over $q \in [\underline{q}, \bar{q}]$. Hence both terms of the expression above are increasing in q. We establish now that that this expression is negative when evaluated in $\bar{q}$

$$W_E(\bar{q}) - W_{OTC}(\bar{q}) = \frac{\gamma S(1-\bar{q})}{1-\delta}\big[(\tau-1)d_L - \kappa\big] - \frac{S\bar{q}}{2(1-\delta)}(\tau-1)d_H$$

It is sufficient to establish that $2\gamma(1-\bar{q})d_L - \bar{q}d_H \leq 0$. Using the expression derived in the proof of Proposition 2, straightforward computations show that this is the case. Finally, on the interval $[\bar{q}, 1]$, we have

$$\begin{aligned} W_E(q) - W_{OTC}(q) = &- \frac{Sq}{2}(\tau-1)\frac{(1-\delta)(1-2\gamma)}{(1-\delta)\big[1-\delta(1-2\gamma)\big]}d_H \\ &- S(1-q)\left[\frac{\kappa - \tau d_L}{1-\delta} + \frac{1-\delta+\delta\gamma(\tau+1)}{(1-\delta)\big[1-\delta(1-2\gamma)\big]}d_H\right] \end{aligned}$$

It is immediate to see that this expression is negative in $q = 1$. Since it is also linear in q, it is negative over $[\bar{q}, 1]$.

C Numerical Exercise

We describe the method used to solve for equilibrium induced by feasible policies in Section 4.2. We show that each policy ultimately depends the identity of a targeted type and on variables $(S^G, \pi(t_{int}), \pi(t_{int}+1))$ which solve three equations imposed by Definition 3. Let us first express the reservation values for $i \in \{1, 2\}$. We have

$$\begin{aligned} r_L^i(t_{int}+1) &= r_{L,ss}^i \\ r_L^i(t_{int}) &= \tau^i d_L + \delta\Big(\pi(t_{int}+1)r_H^1 + (1-\pi(t_{int}+1))\big[(1-\gamma)r_L^i(t_{int}+1) + \gamma r_L^j(t_{int}+1)\big]\Big) \end{aligned}$$

The first equality follows from the fact that the economy is in a steady state equilibrium from period $t_{int}+2$ onward. We now turn to agents participation constraint. Reservation values in period t_{int} are determine exactly as before. From Lemma 1, we have that $v_a^i(t_{int}) \geq v_a^j(t_{int})$ if $\tau^i d_a \geq \tau^j d_{a'}$ for $(i, j) \in \{1, 2\}^2$ and $(a, a') \in \{L, H\}^2$. We thus call targeted type the highest type willing to participate in the program. Given a targeted type, (τ^i, a), the purchase price P^G must verify

$$v_a^i(t_{int}) \leq P^G < v_{a'}^j(t_{int}), \quad \forall j \in \{1, 2\},\ a' \in \{L, H\}, \text{ such that } \tau^j d_{a'} > \tau^i d_a$$

For a given size S^G, the value within the range does not change the effect of the policy and we thus set $P^G = v_a^i(t_{int})$. Similarly, we set R^G to the upper bound defined by the resale constraint (RC). For a given targeted type, the size of the intervention also pins down the selection of assets (S_H^G, S_L^G) through (20). Let us then express the masses of each type of trader in period

$t_{int} + 1$:

$$\mu_L^1(t_{int}+1) = \gamma(S(1-q) - S_L^G),$$
$$\mu_H^1(t_{int}+1) = \gamma(Sq - S_H^G) + (1-2\gamma)(1-\pi(t_{int}))\mu_H^1(t_{int})$$

In phase i) of period $t_{int}+1$, the government sold his assets to τ^2 buyers. The masses of type τ^1 traders in phase ii) of period $t_{int}+1$ thus obtain from law of motions (13)-(12) substituting total supply by non-government supply. It is now clear that for a given targeted type, an equilibrium with intervention is pinned down by a triplet $(S^G, \pi(t_{int}), \pi(t_{int}+1))$ verifying the three conditions in Definition 3 the stabilization constraint SC and the optimality of offer $(\pi(t_{int}), \pi(t_{int}+1))$ for buyers. We thus adopt the following numerical procedure:

1. Select the highest type (τ^i, a) to attract where $i \in \{1, 2\}$, $a \in \{L, H\}$.

2. Derive the lower bound on the purchase price P^G and the upper bound on the resale price R^G as a function of $(\pi(t_{int}), \pi(t_{int}+1))$ thanks to (RC).

3. Solve for a fixed point in $(S^G, \pi(t_{int}), \pi(t_{int}+1))$ following Definition 3.

4. Rank the policies derived in Point 3. across types using surplus criterion (21).

References

Akerlof, G. A. (1970). The market for "lemons": Quality uncertainty and the market mechanism. *The Quarterly Journal of Economics 84*(3), pp. 488–500.

Attar, A., T. Mariotti, and F. Salanié (2011). Nonexclusive Competition in the Market for Lemons. *Econometrica 79*(6), 1869–1918.

Biais, B., D. Martimort, and J.-C. Rochet (2000). Competing mechanisms in a common value environment. *Econometrica 68*(4), pp. 799–837.

Boldrin, M. and M. Woodford (1990). Equilibrium models displaying endogenous fluctuations and chaos : A survey. *Journal of Monetary Economics 25*(2), 189–222.

Bolton, P., T. Santos, and J. A. Scheinkman (2014). Cream Skimming in Financial Markets. NBER Working Papers 16804, National Bureau of Economic Research, Inc.

Camargo, B. and B. Lester (2014). Trading dynamics in decentralized markets with adverse selection. *Journal of Economic Theory 153*(C), 534–568.

Chang, B. (2014). Adverse Selection and Liquidity Distortion. Discussion Papers 1513, University of Wisconsin-Madison.

Chiu, J. and T. Koeppl (2014). Trading Dynamics with Adverse Selection and Search: Market Freeze, Intervention and Recovery. Working Papers 1267, Queen's University, Department of Economics.

Cho, I.-K. and D. M. Kreps (1987). Signaling Games and Stable Equilibria. *The Quarterly Journal of Economics 102*(2), 179–221.

Daley, B. and B. Green (2012). Waiting for news in the market for lemons. *Econometrica 80*(4), 1433–1504.

Deneckere, R. and M.-Y. Liang (2006). Bargaining with interdependent values. *Econometrica 74*(5), 1309–1364.

Duffie, D. (2010). Presidential address: Asset price dynamics with slow-moving capital. *The Journal of Finance 65*(4), 1237–1267.

Duffie, D., N. Garleanu, and L. H. Pedersen (2005). Over-the-counter markets. *Econometrica 73*(6), 1815–1847.

Faria-e-Castro, M., J. Martinez, and T. Philippon (2015). Runs versus Lemons: Information Disclosure and Fiscal Capacity. NBER Working Papers 21201, National Bureau of Economic Research, Inc.

Foucault, T., O. Kadan, and E. Kandel (2013). Liquidity cycles and make/take fees in electronic markets. *The Journal of Finance 68*(1), 299–341.

Fuchs, W. and A. Skrzypacz (2015). Government interventions in a dynamic market with adverse selection. *JET*.

Gale, D. (1996). Equilibria and pareto optima of markets with adverse selection. *Economic Theory 7*(2), 207–235.

Gu, C., F. Mattesini, C. Monnet, and R. Wright (2013). Endogenous Credit Cycles. *Journal of Political Economy 121*(5), 940 – 965.

Guerrieri, V. and R. Shimer (2014). Dynamic Adverse Selection: A Theory of Illiquidity, Fire Sales, and Flight to Quality. *American Economic Review 104*(7), 1875–1908.

Guerrieri, V. and R. Shimer (2015). Markets with Multidimensional Private Information. Working papers, University of Chicago.

Guerrieri, V., R. Shimer, and R. Wright (2010). Adverse Selection in Competitive Search Equilibrium. *Econometrica 78*(6), 1823–1862.

Harrison, J. M. and D. M. Kreps (1978). Speculative investor behavior in a stock market with heterogeneous expectations. *The Quarterly Journal of Economics 92*(2), 323–336.

Hellwig, K.-P. and S. Zhang (2013). Market Runs: Liquidity and the Value of Information. Technical report, New York University.

Hörner, J. and N. Vieille (2009). Public vs. private offers in the market for lemons. *Econometrica 77*(1), 29–69.

Kurlat, P. (2015). Asset markets with heterogeneous information. *Econometrica (forthcoming)*.

Malamud, S. and M. Rostek (2014). Decentralized Exchange. Working Papers 12-18, Swiss Finance Institute.

Michelacci, C. and J. Suarez (2006). Incomplete Wage Posting. *Journal of Political Economy 114*(6), 1098–1123.

Moreno, D. and J. Wooders (2013). Dynamic markets for lemons: Performance, liquidity, and policy intervention. Working Paper Series 5, Economics Discipline Group, UTS Business School, University of Technology, Sydney.

Philippon, T. and V. Skreta (2012). Optimal interventions in markets with adverse selection. *American Economic Review 102*(1), 1–28.

Rocheteau, G. and R. Wright (2013). Liquidity and asset-market dynamics. *Journal of Monetary Economics 60*(2), 275 – 294.

Samuelson, W. F. (1984). Bargaining under asymmetric information. *Econometrica 52*(4), 995–1005.

Singh, M. (2013). The changing collateral space. IMF Working Papers 13/25, International Monetary Fund.

Tirole, J. (2012). Overcoming adverse selection: How public intervention can restore market functioning. *American Economic Review 102*(1), 29–59.

Weill, P.-O. (2007). Leaning against the wind. *The Review of Economic Studies 74*(4), 1329–1354.

Wilson, C. (1980). The nature of equilibrium in markets with adverse selection. *The Bell Journal of Economics 11*(1), pp. 108–130.